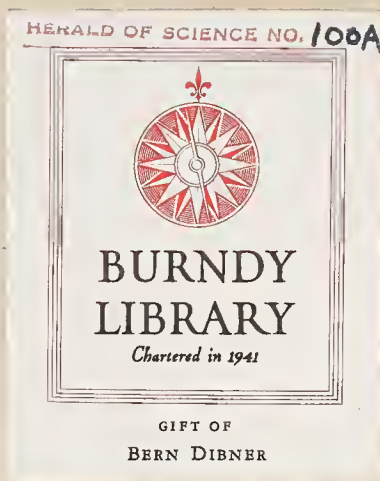
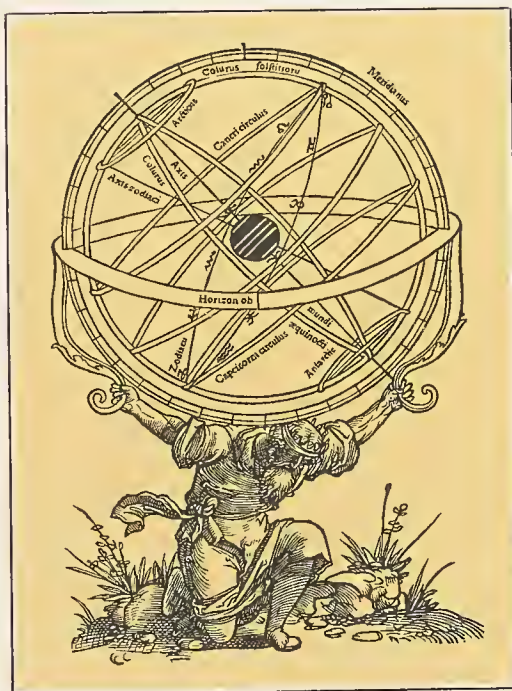




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inner margin of lillipage and one or two other leaves
restored and a leaf in the Grounplot skillfully
renewed



Imprinted at London by Iohn Daye.



Here is (gentle Reader) nothing
(the word of God onely set apart)
which so much beautifieth and a-
dorneth the soule and minde of
mā, as doth the knowledge of good
artes and sciences: as the know-
ledge of naturall and morall Phi-
losophie. The one setteth before
our eyes, the creatures of God,

both in the heauens aboue, and in the earth beneath: in which as
in a glasse, we beholde the exceding maiestie and wisedome of
God, in adorning and beautifying them as we see: in geuing vn-
to them such wonderfull and manifolde proprieties, and naturall
workinges, and that so diuersly and in such varietie: farther in
maintaining and conseruing them continually, whereby to praise
and adore him, as by S. Paule we are taught. The other tea-
cheth vs rules and preceptes of vertue, how, in common life a-
mongest men, we ought to walke vprightly: what dueties per-
taine to our selues, what pertaine to the gouernment or good or-
der both of an housholde, and also of a citie or common wealth.
The reading likewise of histories, conduceth not a litle, to the ad-
orning of the soule & minde of man, a studie of all men cōmen-
ded: by it are seene and known the artes and doinges of infinite
wise men gone before vs. In histories are contained infinite ex-
amples of heroicall vertues to be of vs followed, and horrible ex-
amples of vices to be of vs eschewed. Many other artes also
there are which beautifie the minde of man: but of all other none
do more garnishe & beautifie it, then those artes which are cal-
led Mathematicall. Vnto the knowledge of which no man can
attaine, without the perfect knowledge and instruction of the
principles, groundes, and Elementes of Geometrie. But per-

The Translator to the Reader.

fectly to be instructed in them, requireth diligent studie and reading of olde auncient authors. Amongest which, none for a beginner is to be preferred before the most auncient Philosopher Euclide of Megara. For of all others he hath in a true methode and iuste order, gathered together whatsoeuer any before him had of these Elementes written: inuenting also and adding many thinges of his owne: wherby he hath in due forme accomplished the arte: first geuing definitions, principles, & groundes, wherof he deduceth his Propositions or conclusions, in such wonderfull wise, that that which goeth before, is of necessitie required to the proufe, of that which followeth. So that without the diligent studie of Euclides Elementes, it is impossible to attaine vnto the perfecte knowledge of Geometrie, and consequently of any of the other Mathematicall sciences. Wherefore considering the want & lacke of such good authors hitherto in our English tounge, lamenting also the negligence, and lacke of zeale to their countrey in those of our nation, to whom God hath geuen both knowledge, & also abilitie to translate into our tounge, and to publishe abroad such good authors, and bookes (the chiefe instrumentes of all learninges): seing moreouer that many good wittes both of gentlemen and of others of all degrees, much desirous and studious of these artes, and seeking for them as much as they can, sparing no paines, and yet frustrate of their intent, by no meanes attaining to that which they seeke: I haue for their sakes, with some charge & great trauaile, faithfully translated into our vulgare touge, & set abroad in Print, this booke of Euclide. Whereunto I haue added easie and plaine declarations and examples by figures, of the definitions. In which booke also ye shall in due place finde manifolde additions, Scholies, Annotations, and Inuentions: which I haue gathered out of many of the most famous & chiefe Mathematiciēs, both of old time, and in our age: as by diligent reading it in course, ye shall well

 The Tranſlater to the Reader.

well perceauē. The fruite and gaine which I require for theſe my paines and trauaile, ſhall be nothing els, but onely that thou gentle reader, will gratefully accept the ſame: and that thou mayeſt thereby receauē ſome profite: and moreouer to excite and ſtirre up others learned, to do the like, & to take paines in that behalfe. By meanes wherof, our Engliſhe tounge ſhall no leſſe be enriched with good Authors, then are other ſtraunge toungeſ: as the Dutch, French, Italian, and Spaniſhe: in which are red all good authors in a maner, found amongeſt the Grekes or Latines. Which is the chiefeſt cauſe, that amongeſt the do flouriſhe ſo many cunning and ſkilfull men, in the inuentions of ſtraunge and wonderfull thinges, as in theſe our daies we ſee there do. Which fruite and gaine if I attaine vnto, it ſhall encourage me hereafter, in ſuch like ſort to tranſlate, and ſet abroad ſome other good authors, both pertaining to religion (as partly I haue already done) and alſo pertaining to the Mathematicall Artes. Thus gentle reader farewell.

(?)

 .ij.



TO THE VNFAINED LOVERS
 of truthe , and constant Studentes of Noble
Sciences, **IOHN DEE** of *London*, hartily
 wifheth grace from heauen, and most prospere-
 rous successe in all their honest attemptes and
 exercifes.



Iuine *Plato*, the great Master
 of many worthy Philosophers,
 and the constant auoucher, and
 pithy perswader of *Vnum*, *Bo-
 num*, and *Ens*: in his Schole and
 Academie, sundry times (besides
 his ordinary Scholers) was visited
 of a certaine kinde of men, allured
 by the noble fame of *Plato*, and
 the great commendation of hys
 profound and profitable doctrine.
 But when such Hearers, after long
 harkening to him, perceaued, that
 the drift of his discourses issued
 out, to conclude, this *Vnum*, *Bo-
 num*, and *Ens*, to be Spirituall, Infi-
 nite, Æternall, Omnipotent, &c.

Nothyng beyng alledged or expressed, How, worldly goods: how, worldly digni-
 tie: how, health, Stréngth or lustines of body: nor yet the meanes, how a merueilous
 sensible and bodyly blysse and felicitie hereafter, might be atteyned: Straightway,
 the fantasies of those hearers, were dampnt: their opinion of *Plato*, was clene chaun-
 ged: yea his doctrine was by them despised: and his schole, no more of them visi-
 ted. Which thing, his Scholer, *Aristotle*, narrowly cōsidering, founde the cause ther-
 of, to be, For that they had no forwarnyng and information, in generall, whereto
 his doctrine tended. For, so, might they haue had occasion, either to haue forborne
 his schole hauntyng: (if they, then, had misliked his Scope and purpose) or con-
 stantly to haue continued therein: to their full satisfaction: if such his finall scope &
 intent, had ben to their desire. Wherefore, *Aristotle*, euer, after that, vsed in brief, to
 forewarne his owne Scholers and hearers, both of what matter, and also to what
 ende, he tooke in hand to speake, or teach. While I consider the diuerse trades of
 these two excellent Philosophers (and am most sure, both, that *Plato* right well, o-
 therwise could teach: and that *Aristotle* mought boldely, with his hearers, haue
 dealt in like sorte as *Plato* did) I am in no little pang of perplexitie: Bycause, that,
 which I mislike, is most easy for me to performe (and to haue *Plato* for my exāple.)
 And that, which I know to be most commendable: and (in this first bringyng, into
 common handling, the *Artes Mathematicall*) to be most necessary: is full of great
 difficultie and sundry daungers. Yet, neither do I think it mete, for so straunge mat-
 ter (as now is ment to be published) and to so straunge an audience, to be bluntly,
 at first, put forth, without a peculiar Preface: Nor (Imitatyng *Aristotle*) well can I
 hope, that accordyng to the amplexes and dignitie of the *State Mathematicall*, I
 am able, either playnly to prescribe the materiall boundes: or precisely to expresse
 the chief purposes, and most wonderfull applications therof. And though I am
 sure, that such as did shrink from *Plato* his schole, after they had perceiued his fi-
 nall

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nall conclusion, would in these things haue ben his most diligent hearers) so infinitely mought their desires, in fine and at length, by our *Artes Mathematicall* be satisfied) yet, by this my Præface & forewarnyng, Aswell all such, may (to their great behofe) the soner, hither be allured: as also the *Pythagoricall*, and *Platonicall* perfect scholer, and the constant profound Philosopher, with more ease and spede, may (like the Bee,) gather, hereby, both wax and hony.

The intent of
this Preface.

Wherefore, seying I finde great occasion (for the causes alleged, and farder, in respect of my *Art Mathematicke generall*) to vse a certaine forewarnyng and Præface, whose content shalbe, that mighty, most plesant, and frutefull *Mathematicall Tree*, with his chief armes and second (gifted) braunches: Both, what euery one is, and also, what commodity, in generall, is to be looked for, aswell of griff as stocke. And forasmuch as this enterprise is so great, that, to this our tyme, it neuer was (to my knowledge) by any achieved: And also it is most hard, in these our drery dayes, to such rare and straunge Artes, to wyn due and common credit: Neuertheles, if, for my sincere endeiuour to satisfie your honest expectation, you will but lend me your thâkefull mynde a while: and, to such matter as, for this time, my penne (with spede) is hable to deliuer, apply your eye or eare attentifely: perchance, at once, and for the first salutynge, this Preface you will finde a lesson long enough. And either you will, for a second (by this) be made much the apter: or shortly become, well hable your selues, of the Lyons claw, to coniecture his royall symmetrie, and farder propertie. Now then, gentle, my frendes, and countrey men, Turne your eyes, and bend your myndes to that doctrine, which for our present purpose, my simple talent is hable to yeld you.

All things which are, & haue beyng, are found vnder a triple diuersitie generall. For, either, they are demed Supernaturall, Naturall, or, of a third being. Things Supernaturall, are immateriall, simple, indiuisible, incorruptible, & vnchangeable. Things Naturall, are materiall, compounded, diuisible, corruptible, and chaungeable. Things Supernaturall, are, of the minde onely, comprehended: Things Naturall, of the sense exterior, are hable to be perceiued. In things Naturall, probabilitie and coniecture hath place: But in things Supernaturall, chief demonstration, & most sure Science is to be had. By which properties & comparafons of these two, more easily may be described, the state, condition, nature and property of those things, which, we before termed of a third being: which, by a peculier name also, are called *Thynges Mathematicall*. For, these, beyng (in a maner) middle, betwene things supernaturall and naturall: are not so absolute and excellent, as things supernatural: Nor yet so base and grosse, as things naturall: But are things immateriall: and neuerthelesse, by materiall things hable somewhat to be signified. And though their particular Images, by Art, are aggregable and diuisible: yet the generall *Formes*, notwithstanding, are constant, vnchaungeable, vntrâfformable, and incorruptible. Neither of the sense, can they, at any tyme, be perceiued or iudged. Nor yet, for all that, in the royall mynde of man, first conceiued. But, surmountyng the imperfectiõ of coniecture, weenyng and opinion: and commyng short of high intellectuall conceitiõ, are the Mercuriall fruite of *Dianeticall* discourse, in perfect imagination subsistynge. A meruaylous newtralitye haue these things *Mathematicall*: and also a straunge participatiõ betwene things supernaturall, immortall, intellectuall, simple and indiuisible: and thynges naturall, mortall, sensible, compounded and diuisible. Probabilitie and sensible profe, may well serue in things naturall: and is commendable: In Mathematicall reasoninges, a probable Argument, is nothyng regarded: nor yet the testimony of sense, any whit credited: But onely a perfect demonstration, of truthes certaine, necessary, and inuincible: vniuersally and necessarily concluded:

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cluded: is allowed as sufficient for an Argument exactly and purely Mathematical. "

Of *Mathematicall* thinges, are two principall kinde: namely, *Number*, and *Magnitude*. *Number*, we define, to be, a certayne Mathematicall Sūme, of *Units*. And, an *Unit*, is that thing Mathematicall, Indiuisible, by participation of some likenes of whose property, any thing, which is in deede, or is counted One, may resonably be called One. We account an *Unit*, a thing *Mathematicall*, though it be no *Number*, and also indiuisible: because, of it, materially, *Number* doth consist: which, principally, is a thing *Mathematicall*. *Magnitude* is a thing *Mathematicall*, by participation of some likenes of whose nature, any thing is iudged long, broad, or thicke. A thicke *Magnitude* we call a *Solide*, or a *Body*. What *Magnitude* so euer, is *Solide* or *Thicke*, is also broad, & long. A broad magnitude, we call a *Superficies* or a *Plaine*. Euery playne magnitude, hath also length. A long magnitude, we terme a *Line*. A *Line* is neither thicke nor broad, but onely long: Euery certayne *Line*, hath two endes: The endes of a line, are *Pointes* called. A *Point*, is a thing *Mathematicall*, indiuisible, which may haue a certayne determined situation. If a Poynt moue from a determined situation, the way wherein it moued, is also a *Line*: mathematically produced. whereupon, of the auncient Mathematiciens, a *Line* is called the race, or course of a *Point*. A Poynt we define, by the name of a thing *Mathematicall*: though it be no *Magnitude*, and indiuisible: because it is the propre ende, and bound of a *Line*: which is a true *Magnitude*. And *Magnitude* we may define to be that thing *Mathematicall*, which is diuisible for euer, in partes diuisible, long, broad, or thicke. Therefore though a Poynt be no *Magnitude*, yet *Terminatiuely* we reckon it a thing *Mathematicall* (as I sayd) by reason it is properly the end, and bound of a line.

Number.

Note the worde, *Unit*, to expresse the Greke *Monas*, & not *Vnitie*: as we haue all, commonly, till now, used.

Magnitude.

"

A point.

"

A Line.

Magnitude.

Neither *Number*, nor *Magnitude*, haue any Materialitie. First, we will consider of *Number*, and of the Science *Mathematicall*, to it appropriate, called *Arithmetike*: and afterward of *Magnitude*, and his Science, called *Geometrie*. But that name contenteth me not: whereof a word or two hereafter shall be sayd. How Immateriall and free from all matter, *Number* is, who doth not perceauē: yea, who doth not wonderfully wōder at it. For, neither pure *Element*, nor *Aristoteles*, *Quinta Essentia*, is hable to serue for *Number*, as his propre matter. Nor yet the puritie and simplenes of Substance *Spirituall* or *Angelicall*, will be found propre enough thereto. And therefore the great & godly Philosopher *Annius Boetius*, sayd: *Omnia quacūq; a prima rerū natura constructa sunt, Numerorum videntur ratione formata. Hoc enim fuit principale in animo Conditoris Exemplar*. That is:

All thinges (which from the very first originall being of thinges, haue bene framed and made) do appeare to be Formed by the reason of Numbers. For this was the principall example or patterne in the minde of the Creator.

O comfortable allurements, O rauishing perswasion, to deale with a Science, whose Subiect, is so Auncient, so pure, so excellent, so surmounting all creatures, so vsed of the Almighty and incomprehensible wisdom of the Creator, in the distinct creation of all creatures: in all their distinct partes, properties, natures, and vertues, by order, and most absolute number, brought from *Nothing*, to the *Formalitie* of their being and state. By *Numbers* propertie therefore, of vs, by all possible meanes, (to the perfection of the Science) learned, we may both winde and draw our selues into the inward and deepe search and vew, of all creatures distinct vertues, natures, properties, and *Formes*: And also, farder, arise, clime, ascend, and mount vp (with *Speculative winges*) in spirit, to behold in the Glas of Creation, the *Forme* of *Formes*, the *Exemplar Number* of all thinges *Numerable*: both visible and inuifible: mortall and

immortall

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immortall, Corporall and Spirituall. Part of this profound and diuine Science, had *Ioachim* the Prophetier atteyned vnto: by *Numbers Formall, Naturall, and Rationall*, forseyng, concludyng, and forshewyng great particular euent, long before their comming. His booke yet remainyng, hereof, are good profe: And the noble Earle of *Mirandula*, (besides that,) a sufficient witnesse: that *Ioachim*, in his propheties, proceeded by no other way, then by *Numbers Formall*. And this Earle hym selfe, in Rome, *set vp 900. Conclusions, in all kinde of Sciences, openly to be disputed of: and among the rest, in his Conclusions *Mathematicall*, (in the eleuenth Conclusion) hath in Latin, this English sentence. *By Numbers, a way is had, to the searchyng out, and understanding of euery thyng, hable to be knowne. For the verifysing of which Conclusion, I promise to aunswere to the 74. Questions, vnder written, by the way of Numbers.* Which Conclusions, I omit here to rehearse: as well auoidyng superfluous prolixitie: as, by cause *Ioannes Picus*, workes, are commonly had. But, in any case, I would wish that those Conclusions were red diligently, and perceiued of such, as are earnest Obseruers and Considerers of the constant law of nūbers: which is planted in thyngs Naturall and Supernaturall: and is prescribed to all Creatures, inuiolably to be kept. For, so, besides many other thynges, in those Conclusions to be marked, it would appeare; how sincerely, & within my boundes, I disclose the wonderfull mysteries, by numbers, to be atteyned vnto.

Of my former wordes, easy it is to be gathered, that *Number* hath a treble state: One, in the Creator: an other in euery Creature (in respect of his complete constitution:) and the third, in Spirituall and Angelicall Myndes, and in the Soule of mā. In the first and third state, *Number*, is termed *Number Numbring*. But in all Creatures, otherwise, *Number*, is termed *Nūber Numbred*. And in our Soule, *Nūber* beareth such a swaye, and hath such an affinitie therewith, that some of the old *Philosophers* taught, *Māns Soule*, to be a *Number mouyng it selfe*. And in dede, in vs, though it be a very Accident: yet such an Accident it is, that before all Creatures it had perfect beyng, in the Creator, Sempiternally. *Number Numbring* therfore, is the discretion discerning, and distincting of thynges. But in God the Creator, This discretion, in the beginnyng, produced orderly and distinctly all thynges. For his *Numbring*, then, was his Creatyng of all thynges. And his Continuall *Numbring*, of all thynges, is the Conseruation of them in being. And, where and when he will lacke an *Vnit*: there and then, that particular thyng shalbe *Discreated*. Here I stay. But our Seuerallyng, distinctyng, and *Numbring*, createth nothyng: but of Multitude considered, maketh certaine and distinct determination. And albeit these thynges be waighty and truthe of great importance, yet (by the infinite goodnes of the Almighty *Ternarie*,) Artificiall Methods and easy wayes are made, by which the zealous Philosopher, may wyn here this *Riuierish Ida*, this Mountayne of Contemplation: and more then Contemplation. And also, though *Number*, be a thyng so Immateriall, so diuine, and eternall: yet by degrees, by litle and litle, stretchyng forth, and applyng some likenes of it, as first, to thynges Spirituall: and then, bryngyng it lower, to thynges sensibly perceiued: as of a momentanye founde iterated: then to the least thynges that may be seen, numerable: And at length, (most grossely,) to a multitude of any corporall thynges seen, or felt: and so, of these grosse and sensible thynges, we are trayned to learne a certaine Image or likenes of numbers: and to vse Arte in them to our pleasure and proffit. So grosse is our conuersation, and dull is our apprehension: while mortall Sense, in vs, ruleth the common wealth of our litle world. Hereby we say, Three Lyons, are three: or a *Ternarie*. Three Egles, are three, or a *Ternarie*. Which **Ternaries*, are eche, the *Vnion*, knot, and *Vniformitie*, of three discrete and distinct *Vnits*. That is, we may in eche *Ternarie*, thrise, seuerally pointe, and shew a part, *One, One, and One*. Where, in *Numbring*, we say *One, two, Three,*

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Three. But how farre, these visible Ones, do differre from our Indiuifible Vnits (in pure *Arithmetike*, principally considered) no man is ignorant. Yet from these grosse and materiall thynges, may we be led vpwrd, by degrees, so, informing our rude Imagination, toward the cōceiuyng of *Numbers*, absolutely (: Not supposing, nor admixtyng any thyng created, Corporall or Spirituall, to support, conteyne, or represent those *Numbers* imagined:) that at length, we may be hable, to finde the number of our owne name, gloriously exemplified and registred in the booke of the *Trinitie* most blessed and aternall.

But farder vnderstand, that vulgar Practisers, haue *Numbers*, otherwise, in sundry Considerations: and extend their name farder, then to *Numbers*, whose least part is an *Vnit*. For the common Logist, Reckenmaster, or Arithmeticien; in hys vsing of *Numbers*: of an *Vnit*, imagineth lesse partes: and calleth them *Fractions*. As of an *Vnit*, he maketh an halfe, and thus noteth it, $\frac{1}{2}$. and so of other, (infinitely diuerse) partes of an *Vnit*. Yea and farder, hath, *Fractions of Fractions*. &c. And, forasmuch, as, *Addition*, *Substraction*, *Multiplication*, *Diuision* and *Extraction of Rotes*, are the chief, and sufficient partes of *Arithmetike*: which is, the Science that demonstrateth the properties, of *Numbers*, and all operatiōs, in numbers to be performed: How often, therefore, these fīue sundry sortes of Operations, do, for the most part, of their execution, differre from the fīue operations of like generall property and name, in our Whole numbers practisable, So often, (for a more distinct doctrine) we, vulgarly account and name it, an other kynde of *Arithmetike*. And by this reason: the Consideration, doctrine, and working, in whole numbers onely: where, of an *Vnit*, is no lesse part to be allowed: is named (as it were) an *Arithmetike* by it selfe. And so of the *Arithmetike of Fractions*. In lyke sorte, the necessary, wonderfull and Secret doctrine of Proportion, and proportionalitye hath purchased vnto it selfe a peculier maner of handlyng and working: and so may seme an other forme of *Arithmetike*. Moreouer, the *Astronomers*, for spede and more commodious calculation, haue deuised a peculier maner of orderyng nūbers, about theyr circular motions, by Sexagenes, and Sexagesimes. By Signes, Degrees and Minutes &c. which commonly is called the *Arithmetike of Astronomical* or *Phisicall Fractions*. That, haue I briefly noted, by the name of *Arithmetike Circular*. Bycause it is also vsed in circles, not *Astronomical*. &c. Practise hath led *Numbers* farder, and hath framed them, to take vpon them, the shew of *Magnitudes* propertie: Which is *Incommensurabilitie* and *Irrationalitie*. (For in pure *Arithmetike*, an *Vnit*, is the common Measure of all *Numbers*.) And here, Nūbers are become, as Lynes, Playnes and Solides: some tymes *Rationall*, some tymes *Irrationall*. And haue propre and peculier characters, (as $\sqrt{8}$. $\sqrt{11}$. and so of other. Which is to signifie *Rote Square*, *Rote Cubik*: and so forth.) & propre and peculier fashions in the fīue principall partes: Wherefore the practiser, este meth this, a diuerse *Arithmetike* from the other. Practise bryngeth in, here, diuerse compoundyng of *Numbers*: as some tyme, two, three, foure (or more) *Radicall* nūbers, diuently knit, by signes, of More & Lesse: as thus $\sqrt{8} \cdot 12 + \sqrt{11} \cdot 15$. Or thus $\sqrt{8} \cdot 8 \cdot 19 + \sqrt{11} \cdot 12 - \sqrt{8} \cdot 2$. &c. And some tyme with whole numbers, or fractions of whole Number, amōg them: as $20 + \sqrt{8} \cdot 24 \cdot \sqrt{11} \cdot 16 + 33 - \sqrt{8} \cdot 10 \cdot \sqrt{8} \cdot 44 + 12 - + \sqrt{11} \cdot 9$. And so, infinitely, may hap the varietie. After this: Both the one and the other hath fractions incident: and so is this *Arithmetike* greatly enlarged, by diuerse exhibiting and vse of Compositions and mixtynges. Consider how, I (beyng desirous to deliuer the student from error and Cauillation) do giue to this Practise, the name of the *Arithmetike of Radicall numbers*: Not, of *Irrationall* or *Surd Numbers*: which other while, are *Rationall*: though they haue the Signe of a Rote before them,

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them, which, *Arithmetike* of whole Numbers most vsuall, would say they had no such Roote: and so account them *Surd Numbers*: which, generally spokē, is vntrue: as *Euclides* tenth booke may teach you. Therefore to call them, generally, *Radical Numbers*, (by reason of the signe $\sqrt{}$. prefixed,) is a sure way: and a sufficient generall distinction from all other ordryng and vsing of Numbers: And yet (beside all this) Consider: the infinite desire of knowledge, and incredible power of mans Search and Capacitye: how, they, ioyntly haue waded farder (by mixtyng of speculation and practise) and haue found out, and atteyned to the very chief perfection (almost) of *Numbers* Practicall vse. Which thing, is well to be perceiued in that great Arithmeticall Arte of *Æquation*: commonly called the *Rule of Coss.* or *Algebra*. The Latines termed it, *Regulam Rei & Censuræ*, that is, the *Rule of the thyng and his value*. With an apt name: comprehending the first and last pointes of the worke. And the vulgar names, both in Italian, Frenche and Spanish, depend (in namyng it,) vpon the signification of the Latin word, *Res*: *A thing*: vnleast they vse the name of *Algebra*. And therin (commonly) is a dubble error. The one, of them, which thinke it to be of *Geber* his inuentyng: the other of such as call it *Algebra*. For, first, though *Geber* for his great skill in Numbers, Geometry, Astronomy, and other maruailous Artes, mought haue seemed hable to haue first deuised the sayd Rule: and also the name carryeth with it a very nere likenes of *Geber* his name: yet true it is, that a *Greke* Philosopher and Mathematicien, named *Diophantus*, before *Geber* his tyme, wrote 13. bookes therof (of which, six are yet extant: and I had them to * vse, of the famous Mathematicien, and my great frende, *Petrus Montau-*

* Anno. 1550. *reus*;) And secondly, the very name, is *Algiebar*, and not *Algebra*: as by the Arabien *Auicen*, may be proued: who hath these precise wordes in Latine, by *Andreas Alpagus* (most perfect in the Arabik tung) so translated. *Scientia faciendi Algiebar & Almachabel. i. Scientia inueniendi numerum ignotum, per additionem Numeri, & diuisionem & æquationem*. Which is to say: *The Science of workyng Algiebar and Almachabel*, that is, the *Science of findyng an vknownen number, by Addyng of a Number, & Diuision & æquation*. Here haue you the name: and also the principall partes of the Rule, touched. To name it, *The rule, or Art of Æquation*, doth signifie the middle part and the State of the Rule. This Rule, hath his peculier Characters: and the principal partes of *Arithmetike*, to it appertayning, do differre from the other *Arithmeticall operations*. This *Arithmetike*, hath *Nubers* Simple, Cōpound, Mixt: and Fractions, accordingly. This Rule, and *Arithmetike* of *Algiebar*, is so profound, so generall and so (in maner) conteyneth the whole power of Numbers Application practicall: that mans witt, can deale with nothyng, more proffitable about numbers: nor match, with a thyng, more mete for the diuine force of the Soule, (in humane Studies, affaires, or exercises) to be tryed in. Perchaunce you looked for, (long ere now,) to haue had some particular profe, or euident testimony of the vse, proffit and Commodity of *Arithmetike* vulgar, in the Common lyfe and trade of men. Therto, then, I will now frame my selfe: But herein great care I haue, least length of sundry profes, might make you deme, that either I did misdoute your zelous mynde to vertues schole: or els mistrust your hable witts, by some, to gesse much more. A profe then, foure, fiue, or six, such, will I bryng, as any reasonable man, therewith may be perswaded, to loue & honor, yea learne and exercise the excellent Science of *Arithmetike*.

And first: who, nerer at hand, can be a better witnessse of the frute receiued by *Arithmetike*, then all kynde of Marchants? Though not all, alike, either nede it, or vse it. How could they forbear the vse and helpe of the Rule, called the Golden Rule:

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Rule: Simple and Compound: both forward and backward? How might they misse *Arithmeticall* helpe in the Rules of Felowshyp: either without tyme, or with tyme: and betwene the Marchant & his Factor? The Rules of Bartering in wares onely: or part in wares, and part in money, would they gladly want? Our Marchant venturers, and Trauaylers ouer Sea, how could they order their doynge iustly and without losse, vnleaste certaine and generall Rules for Exchaunge of money, and Rechaunge, were, for their vse, deuised? The Rule of Alligation, in how sundry cases, doth it conclude for them, such precise verities, as neither by naturall witt, nor other experience, they, were hable, els, to know? And (with the Marchant then to make an end) how ample & wonderfull is the Rule of False positions? especially as it is now, by two excellent Mathematiciens (of my familier acquaintance in their life time) enlarged? I meane *Gemma Frisius*, and *Simon Iacob*. Who can either in brief conclude, the generall and Capitall Rules? or who can I imagine the Myriades of sundry Cases, and particular examples, in Act and earnest, continually wrought, tried and concluded by the forenamed Rules, onely? How sundry other *Arithmeticall practises*, are commonly in Marchantes handes, and knowledge: They them selues, can, at large, testifie.

The Mintmaster, and Goldsmith, in their Mixture of Metals, either of diuerse kindes, or diuerse values: how are they, or may they, exactly be directed, and meruailously pleased, if *Arithmetike* be their guide? And the honorable Phisiciās, will gladly confesse them selues, much beholding to the Science of *Arithmetike*, and that sundry wayes: But chiefly in their Art of Graduation, and compounde Medicines. And though *Galenus*, *Auerrois*, *Arnoldus*, *Lullus*, and other haue published their positions, aswell in the quantities of the Degrees about Temperament, as in the Rules, concluding the new *Forme* resulting: yet a more precise, commodious, and easy *Method*, is extant: by a Countreyman of ours (about 200. *R. B.* yeares ago) inuented. And forasmuch as I am vncertaine, who hath the fame: or when that litle Latin treatise, (as the Author writ it,) shall come to be Printed: (Both to declare the desire I haue to pleasure my Countrey, wherein I may: and also, for very good profe of Numbers vse, in this most subtile and frutefull, Philosophicall Conclusion,) I intend in the meane while, most briefly, and with my farther helpe, to communicate the pith therof vnto you.

First describe a circle: whose diameter let be an inch. Diuide the Circumference into foure equall partes. Frō the Center, by those 4. sections, extend 4. right lines: eche of 4. inches and a halfe long: or of as many as you liste, about 4. without the circumference of the circle: So that they shall be of 4. inches long (at the least) without the Circle. Make good euident markes, at euery inches end. If you list, you may subdiuide the inches againe into 10. or 12. smaller partes, equall. At the endes of the lines, write the names of the 4. principall elementall Qualities. *Hotte* and *Colde*, one against the other. And likewise *Moyst* and *Dry*, one against the other. And in the Circle write *Temperate*. Which *Temperature* hath a good Latitude: as appeareth by the Complexion of man. And therefore we haue allowed vnto it, the foresayd Circle: and not a point Mathematicall or Physicall.

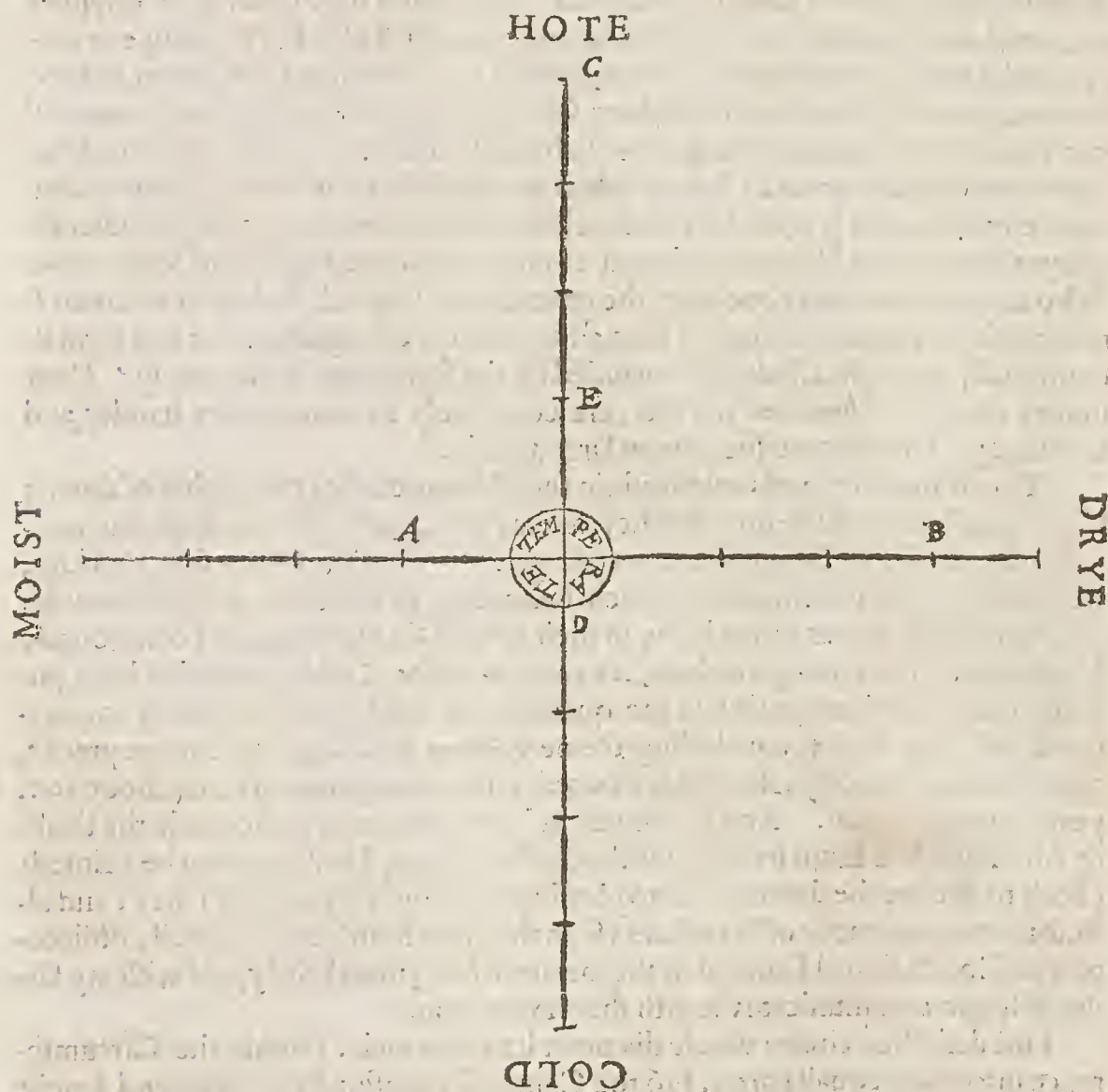
Now, when you haue two thinges Miscible, whose degrees are * truly known: Of necessitie, either they are of one Quantitie and waight, or of diuerse. If they be of one Quantitie and waight: whether their formes, be Contrary Qualities, or of one kinde (but of diuerse intentions and degrees) or a *Temperate*, and a Contrary, *The forme resulting of their Mixture, is in the Middle betwene the degrees of*

*Take some part of Lullus counsaile in his booke de Q. Essentia.

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the formes mixt. As for example, let *A*, be *Moist* in the first degree: and *B*, *Dry* in the third degree. Adde 1. and 3. that maketh 4: the halfe or middle of 4. is 2.

**Note.* This 2. is the middle, equally distant from *A* and *B* (for the **Temperament* is counted none. And for it, you must put a Ciphre, if at any time, it be in mixture).



Counting then from *B*, 2. degrees, toward *A*: you finde it to be *Dry* in the first degree: So is the *Forme resulting* of the Mixture of *A*, and *B*, in our example. I will geue you an other example. Suppose, you haue two thinges, as *C*, and *D*: and of *C*, the Heate to be in the 4. degree: and of *D*, the Colde, to be remisse, euen vnto the *Temperament*. Now, for *C*, you take 4: and for *D*, you take a Ciphre: which, added vnto 4, yeldeth onely 4. The middle, or halfe, whereof, is 2. Wherefore the *Forme resulting* of *C*, and *D*, is Hote in the second degree: for, 2. degrees, accounted from *C*, toward *D*, ende iuste in the 2. degree of heate. Of the third manner, I will geue also an example: which let be this: I haue a liquid Medicine whose

Note. Qualitie of heate is in the 4. degree exalted: as was *C*, in the example foregoing: and an other liquid Medicine I haue: whose Qualitie, is heate, in the first degree. Of eche of these, I mixt a like quantitie: Subtract here, the lesse frõ the more: and the residue diuide into two equall partes: whereof, the one part, either added to the lesse, or subtracted from the higher degree, doth produce the degree of the *Forme*

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Forme resulting, by this mixture of C, and E. As, if from 4. ye abate 1. there resteth 3. the halfe of 3. is $1\frac{1}{2}$. Adde to 1. this $1\frac{1}{2}$. you haue $2\frac{1}{2}$. Or subtract from 4. this $1\frac{1}{2}$. you haue likewise $2\frac{1}{2}$ remayning. Which declareth, the *Forme resulting*, to be *Heate*, in the middle of the third degree.

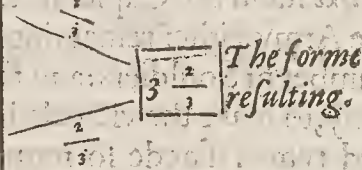
But if the Quantities of two things Commixt, be diuerse, and the Intensions (of their Formes Miscible) be in diuerse degrees, and heighthes. (Whether those Formes be of one kinde, or of Contrary kindes, or of a Temperate and a Contrary, *What proportion is of the lesse quantitie to the greater, the same shall be of the difference, which is betwene the degree of the Forme resulting, and the degree of the greater quantitie of the thing miscible, to the difference, which is betwene the same degree of the Forme resulting, and the degree of the lesse quantitie.* As for example. Let two pound of Liquor be geuen, hote in the 4. degree: & one pound of Liquor be geuen, hote in the third degree. I would gladly know the Forme resulting, in the Mixture of these two Liquors. Set downe your nũbers in order, thus.

P. 2.	Hote. 4.
P. 1.	Hote. 3.

Now by the rule of Algiebar, haue I deuised a very easie, briefe, and generall maner of working in this case. Let vs first, suppose that *Middle Forme resulting*, to be $1ze$: as that Rule teacheth. And because (by our Rule, here geuen) as the waight of 1. is to 2: So is the difference betwene 4. (the degree of the greater quantitie) and $1ze$: to the difference betwene $1ze$ and 3. (the degree of the thing, in lesse quãtitie. And with all, $1ze$, being alwayes in a certaine middell, betwene the two heighthes or degrees). For the first difference, I set $4-1ze$: and for the second, I set $1ze-3$. And, now againe, I say, as 1. is to 2. so is $4-1ze$ to $1ze-3$. Wherefore, of these foure proportionall numbers, the first and the fourth Multiplied, one by the other, do make as much, as the second and the third Multiplied the one by the other. Let these Multiplications be made accordingly. And of the first and the fourth, we haue $1ze-3$. and of the second & the third, $8-2ze$. Wherefore, our Equation is betwene $1ze-3$: and $8-2ze$. Which may be reduced, according to the Arte of Algiebar: as, here, adding 3. to eche part, geueth the Equation, thus, $1ze=11-2ze$. And yet againe, contracting, or Reducing it: Adde to eche part, $2ze$: Then haue you $3ze$ æquall to 11: thus represented $3ze=11$. Wherefore, diuiding 11. by 3: the Quotient is $3\frac{2}{3}$: the *Valew* of our $1ze$, *Coß*, or *Thing*, first supposed. And that is the heighth, or Intension of the *Forme resulting*: which is, *Heate*, in two thirdes of the fourth degree: And here I set the shew of the worke in conclusion, thus. The proufe hereof is easie. by subtracting 3. from $3\frac{2}{3}$, resteth

$\frac{2}{3}$. Subtracte the same heighth of the *Forme resulting*, (which is $3\frac{2}{3}$) frõ 4: then resteth $\frac{10}{3}$.

P. 2.	Hote. 4.
P. 1.	Hote. 3.



You see, that $\frac{10}{3}$ is double to $\frac{2}{3}$.

as 2. P. is double to 1. P. So should it be: by the rule here geuen. Note. As you added to eche part of the Equation, 3: so if ye first added to eche part $2ze$, it would stand, $3ze-3=8$. And now adding to eche part 3: you haue (as afore) $3ze=11$.

And though I, here, speake onely of two things Miscible: and most commonly, mo then three, foure, fīue or six, (&c.) are to be Mixed: (and in one Compound

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Note. to be reduced: & the Forme resultyng of the same, to serue the turne) yet these Rules are sufficient: duely repeated and iterated. In procedyng first, with any two: and then, with the Forme Resultyng, and an other: & so forth. For, the last worke, concludeth the Forme resultyng of them all. I nede nothing to speake, of the Mixture (here supposed) what it is. Common Philosophie hath defined it, saying, *Mixtio est miscibilium, alteratorum, per minima coniunctorum, Vnio.* Euery word in the definition, is of great importance. I nede not also spend any time, to shew, how, the other manner of distributing of degrees, doth agree to these Rules. Neither nede I of the farder vse belonging to the Crosse of Graduation (before described) in this place declare, vnto such as are capable of that, which I haue all ready sayd. Neither yet with examples specifie the Manifold varieties, by the foresayd two generall Rules, to be ordered. The witty and Studious, here, haue sufficient: And they which are not hable to attein to this, without liuely teaching, and more in particular: would haue larger discoursing, then is mete in this place to be dealt withall: And other (perchaunce) with a proude snuffe will disdain this litle: and would be vnthankfull for much more. I, therefore conclude: and wish such as haue modest and earnest Philosophicall mindes, to laude God highly for this: and to Meruayle, that the profoundest and subtilest point, concerning *Mixture of Formes and Qualities Naturall*, is so Matcht and maryed with the most simple, easie, and short way of the noble Rule of *Algiebar*. Who can remaine, therefore vnpersuaded, to loue, allow, and honor the excellent Science of *Arithmetike*? For, here, you may perceiue that the litle finger of *Arithmetike*, is of more might and contriuing, then a hundred thousand mens wittes, of the middle sorte, are hable to perfourme, or truely to conclude, with out helpe thereof.

Taxilix. Now will we farder, by the wise and valiant Capitaine, be certified, what helpe he hath, by the Rules of *Arithmetike*: in one of the Artes to him appertaining: And of the Grekes named *Taxilix*. That is, the Skill of Ordryng Souldiers in Battell ray after the best maner to all purposes. This Art so much dependeth vppon Numbers vse, and the Mathematicals, that *Ælianus* (the best writer therof,) in his worke, to the *Emperour Hadrianus*, by his perfection, in the Mathematicals, (beyng greater, then other before him had,) thinketh his booke to passe all other the excellent workes, written of that Art, vnto his dayes. For, of it, had written *Æneas: Cyneas of Thessaly: Pyrrhus Epirota: and Alexander his sonne: Clearchus: Pausanias: Euangelus: Polybius*, familer frende to *Scipio: Eupolemus: Iphicrates, Possidonius*: and very many other worthy Capitaines, Philosophers and Princes of Immortall fame and memory: Whose fayrest floure of their garland (in this feat) was *Arithmetike*: and a litle perceiuerance, in *Geometricall Figures*. But in many other cases doth *Arithmetike* stand the Capitaine in great stede. As in proportionyng of vittayles, for the Army, either remaining at a stay: or suddenly to be encreased with a certaine number of Souldiers: and for a certain tyme. Or by good Art to diminish his company, to make the victuals, longer to serue the remanent, & for a certaine determined tyme: if nede so require. And so in sundry his other accountes, Reckenynges, Measurynges, and proportionynges, the wise, expert, and Circumspect Capitaine will affirme the Science of *Arithmetike*, to be one of his chief Counsaylors, directers and aiders. Which thing (by good meanes) was euident to the Noble, the Couragious, the loyall, and Curteous *Iohn*, late Earle of Warwicke. Who was a yong Gentleman, throughly knowne to very few. Albeit his lusty valiantnes, force, and Skill in Chiualrous feates and exercises: his humblenes, and frendelynes to all men, were thinges, openly, of the world perceiued. But what rotes (otherwise,) vertue had fastened in his brest, what Rules of godly and honorable life

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life he had framed to him selfe: what vices, (in some then liuing) notable, he tooke great care to eschew: what manly vertues, in other noble men, (flourishing before his eyes,) he Sythingly aspired after: what prowesses he purposed and ment to achieve: with what feats and Artes, he began to furnish and fraught him selfe, for the better seruice of his Kyng and Countrey, both in peace & warre. These (I say) his Heroicall Meditations, forecastinges and determinations, no twayne, (I thinke) beside my selfe, can so perfectly, and truly report. And therefore, in Conscience, I count it my part, for the honor, preferment, & procuring of vertue (thus, briefly) to haue put his Name, in the Register of *Fame Immortall*.

To our purpose. This *John*, by one of his actes (besides many other: both in England and Fraunce, by me, in him noted.) did disclose his hartie loue to vertuous Sciences: and his noble intent, to excell in Martiall prowesse: When he, with humble request, and instant Solliciting: got the best Rules (either in time past by Greke or Romaine, or in our time vsed: and new Stratagemes therein deuised) for ordning of all Companies, summes and Numbers of mē, (Many, or few) with one kinde of weapon, or mo, appointed: with Artillery, or without: on horsebacke, or on fote: to giue, or take onset: to seem many, being few: to seem few, being many. To marche in battaile or Iornay: with many such feates, to Foughten field, Skarmoush, or Ambushe appartaining: And of all these, liuely designementes (most curiously) to be in velame parchment described: with Notes & peculier marks, as the Arte requireth: and all these Rules, and descriptions Arithmeticall, inclosed in a riche Case of Gold, he vsed to weare about his necke: as his Iuell most precious, and Counsaylour most trusty. Thus, *Arithmetike*, of him, was shryned in gold: Of *Numbers* frute, he had good hope. Now, Numbers therefore innumerable, in *Numbers* prayse, his shryne shall finde.

This noble Earle, dyed Anno. 1554. skarse of 24. yeares of age: hauing no issue by his wife: Daughter to the Duke of Somerset.

What nede I, (for farder profe to you) of the Scholemasters of Iustice, to require testimony: how nedefull, how frutefull, how skillfull a thing *Arithmetike* is? I meane, the Lawyers of all sortes. Vndoubtedly, the Ciuilians, can meruaylously declare: how, neither the Auncient Romaine lawes, without good knowledge of *Numbers art*, can be perceiued: Nor (Iustice in infinite Cases) without due proportion, (narrowly considered,) is hable to be executed. How Iustly, & with great knowledge of Arte, did *Papinianus* institute a law of partition, and allowance, betwene man and wife after a diuorce? But how *Accursius*, *Baldus*, *Bartolus*, *Iason*, *Alexander*, and finally *Alciatus*, (being otherwise, notably well learned) do iumble, gesse, and erre, from the æquity, art and Intent of the lawmaker: *Arithmetike* can detect, and conuince: and clerely, make the truth to shine. Good *Bartolus*, tyred in the examining & proportioning of the matter: and with *Accursius* Glosses, much cumbered: burst out, and sayd: *Nulla est in toto libro; hac glossa difficilior: Cuius computationem nec Scholastici nec Doctores intelligunt. &c.* That is: In the whole booke, there is no Glosses harder then this: Whose accoumpt or rekenyng, neither the Scholars, nor the Doctours vnderstand. &c. What can they say of *Iulianus* law, *Sita Scriptum. &c.* Of the Testators will iustly performing, betwene the wife, Sonne and daughter? How can they perceiue the æquitie of *Aphricanus*, *Arithmeticall* Reckening, where he treateth of *Lex Falcidia*? How can they deliuer him, from his Reprouers: and their maintainers: as *Ioannes*, *Accursius*, *Hypolitus* and *Alciatus*? How Iustly and artificially, was *Africanus* reckening made: Proportionating to the Sommes bequeathed, the Contributions of eche part? Namely, for the hundred presently receiued, $17\frac{1}{7}$. And for the hundred, receiued after ten monethes, $12\frac{6}{7}$: which make the 30: which were to be cōtributed by the legataries to the heire.

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For, what proportion, 100 hath to 75: the same hath 17 $\frac{1}{2}$ to 12 $\frac{1}{2}$: Which is Sequitertia: that is, as 4, to 3. which make 7. Wonderfull many places, in the Ciuile law, require an expert *Arithmeticien*, to vnderstand the deepe Iudgemēt, & Iust determination of the Auncient Romaine Lawmakers. But much more expert ought he to be, who should be hable, to decide with æquitie, the infinite varietie of Cases, which do, or may happen, vnder euery one of those lawes and ordinances Ciuile. Hereby, easely, ye may now coniecture: that in the Canon law: and in the lawes of the Realme (which with vs, beare the chief Authoritie), Iustice and equity might be greatly preferred, and skilfully executed, through due skill of *Arithmetike*, and proportions appertainyng. The worthy Philosophers, and prudent lawmakers (who haue written many bookes *De Republica*: How the best state of Common wealthes might be procured and mainteined,) haue very well determined of Iustice: (which, not onely, is the Base and foundation of Common weales: but also the totall perfection of all our workes, words, and thoughtes:) defining it, to be that vertue, by which, to euery one, is rendred, that to him appertaineth. God challengeth this at our handes, to be honored as God: to be loued, as a father: to be feared as a Lord & master. Our neighbours proportiō, is also prescribed of the Almighty lawmaker: which is, to do to other, euen as we would be done vnto. These proportions, are in Iustice necessary: in duety, commendable: and of Common wealthes, the life, strength, stay and flourishing. *Aristotle* in his *Ethikes* (to fatch the sede of Iustice, and light of direction, to vse and execute the same) was fayne to fly to the perfection, and power of Numbers: for proportions *Arithmeticall* and *Geometricall*. *Plato* in his booke called *Epinomis* (which booke, is the Threasury of all his doctrine) where, his purpose is, to seke a Science, which, when a man had it, perfectly: he might seme, and so be, in dede, *Wise*. He, briefly, of other Sciences discoursing, findeth them, not hable to bring it to passe: But of the Science of Numbers, he sayth. *Illā, quæ numerum mortalium generi dedit, id profecto efficiet. Deum autem aliquem, magis quam fortunam, ad salutem nostram, hoc munus nobis arbitror contulisse. &c. Nam ipsum bonorum omnium Authorem, cur non maximi boni, Prudentie dico, causam arbitramur?* That Science, verely, which hath taught mankynde number, shall be able to bryng it to passe. And, I thinke, a certaine God, rather then fortune, to haue giuen vs this gift, for our blisse. For, why should we not Iudge him, who is the Author of all good things, to be also the cause of the greatest good thyng, namely, *Wisedome*? There, at length, he proueth *Wisedome* to be atteyned, by good Skill of *Numbers*. With which great Testimony, and the manifold profes, and reasons, before exprest, you may be sufficiently and fully perswaded: of the perfect Science of *Arithmetike*, to make this account: That of all Sciences, next to *Theologie*, it is most diuine, most pure, most ample and generall, most profounde, most subtile, most commodious and most necessary. Whose next Sister, is the Absolute Science of *Magnitudes*: of which (by the Direction and aide of him, whose *Magnitude* is Infinite, and of vs Incomprehensible) I now entend, so to write, that both with the *Multitude*, and also with the *Magnitude* of Meruaylous and frutefull verities, you (my frendes and Countreymen) may be stirrd vp, and awaked, to behold what certaine Artes and Sciences, (to our vn-speakable behofe) our heaucnly father, hath for vs prepared, and reuealed, by sundry *Philosophers* and *Mathematiciens*.

Both, *Number* and *Magnitude*, haue a certaine Originall sede, (as it were) of an incredible property: and of man, neuer hable, Fully, to be declared. Of *Number*, an Vnit, and of *Magnitude*, a Poynte, doo seeme to be much like Originall

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nall causes: But the diuerfitie neuerthelesse, is great. We defined an *Vnit*, to be a thing Mathematicall Indiuifible: A Point, likewise, we sayd to be a Mathematicall thing Indiuifible. And farder, that a Point may haue a certaine determined Situation: that is, that we may assigne, and prescribe a Point, to be here, there, yonder. &c. Herein, (behold) our *Vnit* is free, and can abyde no bondage, or to be tyed to any place, or seat: diuifible or indiuifible. Agayne, by reason, a Point may haue a Situation limited to him: a certaine motion, therfore (to a place, and from a place) is to a Point: incident and appertainyng. But an *Vnit*, can not be imagined to haue any motion. A Point, by his motion, produceth; Mathematically, a line: (as we sayd before) which is the first kinde of Magnitudes, and most simple: An *Vnit*, can not produce any number. A Line, though it be produced of a Point moued, yet, it doth not consist of pointes: Number, though it be not produced of an *Vnit*, yet doth it Consist of vnits, as a materiall cause. But formally, Number, is the Vnion, and Vnitie of Vnits. Which vnytyng and knitting, is the workmanship of our minde: which, of distinct and discrete Vnits, maketh a Number: by vniformitie, resulting of a certaine multitude of Vnits. And so, euery number, may haue his least part, giuen: namely, an *Vnit*: But not of a Magnitude, (no, not of a Lyne,) the least part can be giue: bycause, infinitely, diuision thereof, may be conceiued. All Magnitude, is either a Line, a Plaine, or a Solid. Which Line, Plaine, or Solid, of no Sense, can be perceiued, nor exactly by hād (any way) represented: nor of Nature produced: But, as (by degrees) Number did come to our perceiuerance: So, by visible formes, we are holpen to imagine, what our Line Mathematicall, is. What our Point, is. So precise, are our Magnitudes, that one Line is no broader then an other: for they haue no bredth: Nor our Plaines haue any thicknes. Nor yet our Bodies, any weight: be they neuer so large of dimensiō. Our Bodyes, we can haue Smaller, then either Arte or Nature can produce any: and Greater also, then all the world can comprehend. Our least Magnitudes, can be diuided into so many partes, as the greatest. As, a Line of an inch long, (with vs) may be diuided into as many partes, as may the diameter of the whole world, from East to West: or any way extended: What priuiledges, aboue all manual Arte, and Natures might, haue our two Sciences Mathematicall, to exhibite, and to deale with thinges of such power, liberty, simplicity, puritie, and perfection: And in them, so certainly, so orderly, so precisely to procede: as, excellent is that workemā Mechanicall Iudged, who nereft can approche to the representing of workes, Mathematically demonstrated? And our two Sciences, remaining pure, and absolute, in their proper termes, and in their owne Matter: to haue, and allowe, onely such Demonstrations, as are plaine, certaine, vniuerfall, and of an æternall veritye: This Science of *Magnitude*, his properties, conditions, and appertanances: commonly, now is, and from the beginnyng, hath of all Philosophers, ben called *Geometrie*. But, veryly, with a name to base and scant, for a Science of such dignitie and amplexes. And, perchaunce, that name, by cōmon and secret consent, of all wisemen, hitherto hath ben suffred to remayne: that it might carry with it a perpetuall memorye, of the first and notablest benefite, by that Science, to common people shewed: Which was, when Boundes and meres of land and ground were lost, and confounded (as in *Egypt*, yearly, with the ouerflowyng of *Nilus*, the greatest and longest riuer in the world) or, that ground bequeathed, were to be assigned: or, ground sold, were to be layd out: or (when disorder preuailed) that Commōs were distributed into feueralties. For, where, vpon these & such like occasiōs, Some by ignorāce, some by negligēce, Some by fraude, and some by violence, did wrongfully limite, measure, encroach, or challenge (by

Numbers

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Geometrie

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pretence of iust content, and measure) those landes and groundes : great losse, disquietnes, murder, and warre did (full oft) ensue: Till, by Gods mercy, and mans Industrie, The perfect Science of Lines, Plaines, and Solides (like a diuine Iusticier,) gaue vnto euery man, his owne. The people then, by this art pleased, and greatly relieued, in their landes iust measuring: & other Philosophers, writing Rules for land measuring: betwene them both, thus, confirmed the name of *Geometria*, that is, (according to the very etimologie of the word) Land measuring. Wherin, the people knew no farder, of Magnitudes vse, but in Plaines: and the Philosophers, of the, had no feet hearers, or Scholers: farder to disclose vnto, then of flat, plaine *Geometrie*. And though, these Philosophers, knew of farder vse, and best vnderstode the etymologie of the worde, yet this name *Geometria*, was of them applyed generally to all sortes of Magnitudes : vnleast, otherwhile, of *Plato*, and *Pythagoras* : When

*Plato. 7. de
Rep.

0. 7. de they would precisely declare their owne doctrine. Then, was * *Geometria*, with them, *Studium quod circa planum versatur*. But, well you may perceiue by *Euclides Elementes*, that more ample is our Science, then to measure Plaines: and nothyng lesse therin is taught (of purpose) then how to measure Land. An other name, therefore, must nedes be had, for our Mathematicall Science of Magnitudes: which regardeth neither clod, nor turff: neither hill, nor dale: neither earth nor heauen: but is absolute *Megethologia*: not creping on ground, and daffeling the eye, with pole
 33 perche, rod or lyne: but lifyng the hart aboute the heauens, by inuisible lines, and
 33 immortall beames: meteth with the reflexions, of the light incomprehensible: and
 33 so procureth Ioye, and perfection vnspeakable. Of which true vse of our *Megethica*, or *Megethologia*, *Diuine Plato* seemed to haue good taste, and iudgement: and (by the name of *Geometrie*) so noted it: and warned his Scholers therof: as, in hys seuenth *Dialog*, of the Common wealth, may euidently be sene. Where (in Latin) thus it is: right well translated: *Profecto, nobis hoc non negabunt, Quicumq; vel paululum quid Geometrie gustarunt, quin hac Scientia, contra, omnino se habeat, quam de ea loquuntur, qui in ipsa versantur*. In English, thus. *Verely* (sayth *Plato*) *whosoever haue, (but euen very litle) tasted of Geometrie, will not denye vnto vs, this: but that this Science, is of an other condicion, quite contrary to that, which they that are exercised in it, do speake of it.* And there it followeth, of our *Geometrie*, *Quod quaeritur cognoscendi illius gratia, quod semper est, non & eius quod oritur quandoq; & interit. Geometria, eius quod est semper, Cognitio est. Attollet igitur (o Generose vir) ad Veritatem, animum: atq; ita, ad Philosophandum preparabit cogitationem, ut ad supera conuertamus: quae, nunc, contra quam decet, ad inferiora deijcimus. &c. Quam maxime igitur praecipendum est, ut qui praeclarissimam hanc habitant Civitatem, nullo modo, Geometriam spernant. Nam & quae praeter ipsius propositum, quodam modo esse videntur, haud exigua sunt. &c.* It must nedes be confessed (saith *Plato*) *That [Geometrie] is learned, for the knowyng of that, which is euer: and not of that, which, in tyme, both is bred and is brought to an ende. &c. Geometrie is the knowledge of that which is euerlastyng. It will lift vp therefore (O Gentle Syr) our mynde to the Veritie: and by that meanes, it will prepare the Thought, to the Philosophicall loue of wisdome: that we may turne or conuert, toward heauenly thinges [both mynde and thought] which now, otherwise then becommeth vs, we cast down on base or inferior things. &c. Chiefly, therefore, Commaundement must be giuen, that such as do inhabit this most honorable Citie, by no meanes, despise Geometrie. For euen those thinges [done by it] which, in manner, seame to be, beside the purpose of Geometrie: are of*

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no small importance. &c. And besides the manifold vses of *Geometrie*, in matters appertaining to warre, he addeth more, of second vnpurposed frute, and commoditye, arising by *Geometrie*: saying: *Scimus quin etiam, ad Disciplinas omnes facilius perdiscendas, interesse omnino, attigerit ne Geometriam aliquis, an non.* &c. *Hanc ergo Doctrinam, secundo loco discendam Iuuenibus statuamus.* That is. But, also, we know, that for the more easy learning of all *Artes*, it importeth much, whether one haue any knowledge in *Geometrie*, or no. &c. Let vs therefore make an ordinance or decree, that this Science, of young men shall be learned in the second place. This was *Diuine Plato* his Iudgement, both of the purposed, chief, and perfect vse of *Geometrie*: and of his second, dependyng, deriuatiue commodities. And for vs, Christen men, a thousand thousand mo occasions are, to haue nede of the helpe of * *Megethologicall* Contemplations: wherby, to trayne our Imaginations and Myndes, by litle and litle, to forsake and abandon, the grosse and corruptible Obiectes, of our vtward senses: and to apprehend, by sure doctrine demonstratiue, Things Mathematicall. And by them, readily to be holpen and conducted to conceiue, discourse, and conclude of things Intellectual, Spirituall, æternall, and such as concerne our Blisse euerlasting: which, otherwise (without Speciall priuiledge of Illumination, or Reuelation frō heauen) No mortall mans wyt (naturally) is hable to reach vnto, or to Compasse. And, veryly, by my small Talent (from aboue) I am hable to proue and testifie, that the litterall Text, and order of our diuine Law, Oracles, and Mysteries, require more skill in Numbers, and Magnitudes: then (commonly) the expositors haue vttered: but rather onely (at the most) so warned: & shewed their own want therin. (To name any, is nedeles: and to note the places, is, here, no place: But if I be duely asked, my answer is ready.) And without the litterall, Grammaticall, Mathematicall or Naturall verities of such places, by good and certaine Arte, perceiued, no Spirituall sense (propre to those places, by Absolute *Theologie*) will thereon depend. No man, therefore, can doute, but toward the atteyning of knowledge incomparable, and Heauenly Wisedome: Mathematicall Speculations, both of Numbers and Magnitudes: are meanes, aydes, and guides: ready, certaine, and necessary. From henceforth, in this my Preface, will I frame my talke, to *Plato* his fugitiue Scholers: or, rather, to such, who well can, (and also wil,) vse their vtward senses, to the glory of God, the benefite of their Countrey, and their owne secret contentation, or honest preferment, on this earthly Scaffold. To them, I will orderly recite, describe & declare a great Number of *Artes*, from our two Mathematicall fountaines, deriued into the fieldes of *Nature*. Wherby, such Sedes, and Rotes, as lye depe hyd in the ground of *Nature*, are refreshed, quickened, and prouoked to grow, shote vp, floure, and giue frute, infinite, and incredible. And these *Artes*, shalbe such, as vpon Magnitudes properties do depende, more, then vpon Number. And by good reason we may call them *Artes*, and *Artes Mathematicall Deriuatiue*: for (at this tyme) I Define An Arte, to be a Methodicall cōplete Doctrīne, hauing abundance of sufficient, and peculier matter to deale with, by the allowance of the Metaphisicall Philosopher: the knowledge whereof, to humane state is necessarye. And that I account, An Art Mathematicall deriuatiue, which by Mathematicall demonstratiue Method, in Nūbers, or Magnitudes, ordieth and confirmeth his doctrine, as much & as perfectly, as the matter subiect will admit. And for that,

I. D.
* Herein, I
would gladly
shake of, the
earthly name,
of *Geometrie*.

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An Arte.

Art Mathematicall Deriuatiue.

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A Mechanicien.

I intend to vse the name and propertie of a *Mechanicien*, otherwise, then (hitherto) it hath ben vsed, I thinke it good, (for distinction sake) to giue you also a brief description, what I meane therby. A *Mechanicien*, or a *Mechanicall* workman is he, whose skill is, without knowledge of Mathematicall demonstration, perfectly to worke and finishe any sensible worke, by the *Mathematicien* principall or deriuatiue, demonstrated or demonstrable. Full well I know, that he which inuenteth, or maketh these demonstrations, is generally called *A Speculatiue Mechanicien*: which differreth nothing from a *Mechanicall Mathematicien*. So, in respect of diuerse actions, one man may haue the name of sundry artes: as, some tyme, of a *Logicien*, some tymes (in the same matter otherwise handled) of a *Rethoricien*. Of these trifles, I make, (as now, in respect of my Preface,) small account: to fyle the for the fine handlyng of subtile curious disputers. In other places, they may commaunde me, to giue good reason: and yet, here, I will not be vnreasonable.

Geometrie vulgar.

- First, then, from the puritie, absolutenes, and Immaterialitie of Principall *Geometrie*, is that kinde of *Geometrie* deriued, which vulgarly is counted *Geometrie*: and is the Arte of Measuring sensible magnitudes, their iust quãtities and contentes. This, teacheth to measure, either at hand: and the practiser, to be by the thing Measured: and so, by due applying of Cumpase, Rule, Squire, Yarde, Ell, Perch, Pole, Line, Gaging rod, (or such like instrument) to the Length,
1. Plaine, or Solide measured, *to be certified; either of the length, perimetry, or distance lineall: and this is called, *Mecometrie*. Or *to be certified of the content of any plaine Superficies: whether it be in ground Surueyed, Borde, or Glasse measured, or such like thing: which measuring, is named *Embadometrie*. *Or els to vnderstand the Soliditie, and content of any bodily thing: as of Tymber and Stone, or the content of Pits, Pondes, Wells, Vessels, small & great, of all fashions. Where, of Wine, Oyle, Beere, or Ale vessells, &c, the Measuring, commonly, hath a peculiar name: and is called *Gaging*. And the generall name of these Solide measures,
 2. is *Stereometrie*. Or els, this *vulgar Geometrie*, hath consideration to teach the practiser, how to measure things, with good distance betwene him and the thing measured: and to vnderstand thereby, either *how Farre, a thing seene (on land or water) is from the measurer: and this may be called *Apomecometrie*: Or, how High or depe, aboue or vnder the leuel of the measurers standing, any thing is, which is sene
 3. on land or water, called *Hypsometrie*. *Or, it informeth the measurer, how Broad any thing is, which is in the measurers vew: so it be on Land or Water, situated: and may be called *Platometrie*. Though I vse here to condition, the thing measured, to be on Land, or Water Situated: yet, know for certaine, that the sundry heigthe of Cloudes, blasing Starres, and of the Mone, may (by these meanes) haue their distances from the earth: and, of the blasing Starres and Mone, the Soliditie (aswell as distances) to be measured: But because, neither these things are vulgarly taught: nor of a common practiser so ready to be executed: I, rather, let such measures be reckened incident to some of our other Artes, dealing with thinges on high, more purposely, then this vulgar Land measuring *Geometrie* doth: as in *Perspectiue* and *Astronomie*, &c.

Note.

OF these feates (Farther applied) is Sprong the feate of *Geodesie*, or Land Measuring: more cunningly to measure & Suruey Land, Woods, and Waters, a farre of. More cunningly, I say: But God knoweth (hitherto) in these Realmes of England and Ireland (whether through ignorance or fraude, I can not tell; in euery particular) how great wrong and iniurie hath (in my time) bene committed

Note.

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by vntrue measuring and surueying of Land or Woods, any way . And, this I am sure: that the Value of the difference, betwene the truth and such Surueyes, would haue bene hable to haue found (for euer) in eche of our two Vniuersities, an excellent Mathematicall Reader: to eche, allowing (yearly) a hundred Markes of lawfull money of this realme: which, in dede, would seme requisit, here, to be had (though by other wayes prouided for) as well, as, the famous Vniuersitie of Paris, hath two Mathematicall Readers : and eche, two hundreth French Crownes yearly, of the French Kinges magnificent liberalitie onely . Now, againe, to our purpose returning : Moreouer, of the former knowledge Geometricall, are growen the Skills of *Geographie* , *Chorographie* , *Hydrographie* , and *Stratarithmetrie* .

Geographie teacheth wayes, by which, in sundry formes, (as *Spharique*, *Plaine* , or other), the Situation of Cities, Townes, Villages, Fortes, Castells, Mountaines, Woods, Hauens, Riuers, Crekes, & such other things, vpon the outface of the earthly Globe (either in the whole, or in some principall member and portion thereof contained) may be described and designed, in commensurations Analogicall to Nature and veritie: and most aptly to our view, may be represented. Of this Arte how great pleasure, and how manifold commodities do come vnto vs, daily and houely: of most men, is perceaued . While, some, to beautifie their Halls, Parlers, Chambers, Galleries, Studies, or Libraries with: other some, for thinges past, as battels fought, earthquakes, heauenly fyringes, & such occurrentes, in histories mentioned: therby liuely, as it were, to vewe the place, the region adioyning, the distance from vs : and such other circumstances . Some other, presently to vewe the large dominion of the Turke : the wide Empire of the Moschouite: and the litle morsell of ground, where Christendome (by profession) is certainly knowen. Litle, I say, in respecte of the rest, &c . Some, either for their owne iorneyes directing into farre landes: or to vnderstand of other mens trauailes . To conclude, some, for one purpose : and some, for an other, liketh, loueth, getteth, and vseth, Mappes, Chartes, & Geographical Globes . Of whose vse, to speake sufficiently, would require a booke peculiar.

Chorographie seemeth to be an vnderling, and a twig, of *Geographie*: and yet neuerthelesse, is in practise manifolde, and in vse very ample . This teacheth Analogically to describe a small portion or circuite of ground, with the contents: not regarding what commensuration it hath to the whole, or any parcell, without it, contained . But in the territory or parcell of ground which it taketh in hand to make description of, it leaueth out (or vndescribed) no notable, or odde thing, aboue the ground visible . Yea and sometimes, of thinges vnder ground, geueth some peculiar marke : or warning : as of Mettall mines, Cole pittes, Stone quarries, &c . Thus, a Dukedome, a Shiere, a Lordship, or lesse, may be described distinctly . But marueilous pleasant, and profitable it is , in the exhibiting to our eye, and commensuration, the plat of a Citie, Towne, Forte, or Pallace , in true Symmetry : not approaching to any of them : and out of Gunne shot, &c . Hereby, the *Architect* may furnishe him selfe, with store of what patterns he liketh : to his great instruction: euen in those thinges which outwardly are proportioned: either simply in them selues : or respectiue, to Hilles, Riuers, Hauens, and Woods adioyning . Some also, terme this particular description of places , *Topographie*.

Hydrographie, deliuereth to our knowledge, on Globe or in Plaine, the perfect Analogicall description of the Ocean Sea coastes, through the whole world : or in the chiefe and principall partes thereof : with the Iles and chiefe

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particular places of daungers, conteyned within the boundes, and Sea coastes described: as, of Quicksand, Bankes, Pittes, Rockes, Races, Countertides, Whorle-pooles, &c. This, dealeth with the Element of the water chiefly: as *Geographie* did principally take the Element of the Earthes description (with his appertanances) to taske. And besides thys, *Hydrographie*, requireth a particular Register of certaine Landmarkes (where markes may be had) from the sea, well hable to be skried, in what point of the Seacumpase they appeare, and what apparent forme, Situation, and bignes they haue, in respecte of any daungorous place in the sea, or nere vnto it, assigned: And in all Coastes, what Mone, maketh full Sea: and what way, the Tides and Ebbes, come and go, the *Hydrographer* ought to recorde. The Soundinges likewise: and the Channells wayes: their number, and depthes ordinarily, at ebbe and flud, ought the *Hydrographer*, by obseruation and diligence of *Measuring*, to haue certainly knowen. And many other pointes, are belonging to perfecte *Hydrographie*, and for to make a *Rutter*, by: of which, I nede not here speake: as of the describing, in any place, vpon Globe or Plaine, the 32. pointes of the Compase, truely: (wherof, scarcely foure, in England, haue right knowledge: bycause, the lines therof, are no straight lines, nor Circles.) Of making due projection of a Sphere in plaine. Of the Variacion of the Compas, from true Northe: And such like matters (of great importance, all) I leaue to speake of, in this place: bycause, I may seame (al ready) to haue enlarged the boundes, and duety of an *Hydrographer*, much more, then any man (to this day) hath noted, or prescribed. Yet am I well hable to proue, all these thinges, to appertaine, and also to be proper to the *Hydrographer*. The chief vse and ende of this Art, is the Art of Navigation: but it hath other diuerse vses: euen by them to be enioyed, that neuer lacke sight of land.

Stratarithmetrie, is the Skill, (appertainyng to the warre,) by which a man can set in figure, analogicall to any *Geometricall* figure appointed; any certaine number or summe of men: of such a figure capable: (by reason of the vsuall spaces betwene Souldiers allowed: and for that, of men, can be made no Fractions. Yet, neuertheles, he can order the giuen summe of men, for the greatest such figure, that of them, can be ordred) and certifie, of the ouerplus: (if any be) and of the next certaine summe, which, with the ouerplus, will admit a figure exactly proportionall to the figure assigned. By which Skill, also, of any army or company of men: (the figure & sides of whose orderly standing, or array, is knowen) he is able to expresse the iust number of men, within that figure contained: or (orderly) able to be contained. * And this figure, and sides therof, he is hable to know: either beyng by,

*Note: and at hand: or a farre of. Thus farre, stretcheth the description and property of

The difference betwene Stratarithmetrie and Tacticie. „ *Stratarithmetrie*: sufficient for this tyme and place. It differreth from the Feate „ *Tactickall*, *De aciebus instruendis*. bycause, there, is necessary the wisedome and fore- „ fight, to what purpose he so ordreth the men: and Skillfull hability, also, for any „ occasion, or purpose, to deuise and vse the aptest and most necessary order, array „ and figure of his Company and Summe of men. By figure, I meane: as, either of a „ *Perfect Square*, *Triangle*, *Circle*, *Ouale*, *long square*, (of the Grekes it is called *Eteromekes*) *Rhombe*, *Rhomboïd*, *Lunular*, *Ryng*, *Serpentine*, and such other *Geometricall* „ figures: Which, in warres, haue ben, and are to be vsed: for commodiousnes, ne- „ cessity, and auantage &c. And no small skill ought he to haue, that should make „ true report, or nere the truth, of the numbers and Summes, of footemen or horse- „ men, in the Enemyes ording. A farre of, to make an estimate, betwene nere „ termes of More and Lesse, is not a thyng very rife, among those that gladly would

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do it. Great pollicy may be vsed of the Capitaines, (at tymes fete, and in places conuenient) as to vse Figures, which make greatest shew, of so many as he hath: and vsing the aduantage of the three kindes of vsuall spaces: (betwene footemen or horsemen) to take the largest: or when he would seme to haue few, (beyng many:) contrarywise, in Figure, and space. The Herald, Purseuant, Sergeant Royall, Capitaine, or who soeuer is carefull to come nere the truth herein, besides the Iudgement of his expert eye, his skill of Ordering *Tacticall*, the helpe of his Geometricall instrument: Ring, or Staffe Astronomicall: (commodiously framed for cariage and vse) He may wonderfully helpe him selfe, by perspective Glasses. In which, (I trust) our posterity will proue more skillfull and expert, and to greater purposes, then in these dayes, can (almost) be credited to be possible.

Thus haue I lightly passed ouer the Artificiall Feates, chiefly dependyng vpon vulgar *Geometrie*: & commonly and generally reckened vnder the name of *Geometrie*. But there are other (very many) *Methodicall Artes*, which, declynyng from the purity, simplicitie, and Immateriality, of our Principall Science of *Magnitudes*: do yet neuertheles vse the great ayde, direction, and Method of the sayd principall Science, and haue propre names, and distinct: both from the Science of *Geometrie*, (from which they are deriued) and one from the other. As *Perspective*, *Astronomie*, *Musike*, *Cosmographie*, *Astrologie*, *Statike*, *Anthropographie*, *Trochilike*, *Helicosophie*, *Pneumatithmie*, *Menadrie*, *Hypogeiodie*, *Hydragogie*, *Horometrie*, *Zographie*, *Architecture*, *Nauigation*, *Thaumaturgike* and *Archemastrie*. I thinke it necessary, orderly, of these to giue some peculier descriptions: and withall, to touch some of their commodious vses, and so to make this Preface, to be a little swete, pleasant Nofegaye for you: to comfort your Spirites, beyng almost out of courage, and in despayre, (through brutish brute) Weenyng that *Geometrie*, had but serued for buildyng of an house, or a curious bridge, or the rouse of Westminster hall, or some witty pretty deuise, or engyn, appropriate to a Carpenter, or a Ioyner &c: That the thing is farre otherwise, then the world, (commonly) to this day, hath demed, by worde and worke, good profe wilbe made.

Among these Artes, by good reason, *Perspective* ought to be had, ere of *Astronomicall Apparences*, perfect knowledge can be atteyned. And bycause of the prerogatiue of *Light*, beyng the first of *Gods Creatures*: and the eye, the light of our body, and his Sense most mighty, and his organ most Artificiall and *Geometricall*: At *Perspective*, we will begyn therefore. *Perspective*, is an Art Mathematicall, which demonstrateth the maner, and properties, of all Radiations Direct, Broken, and Reflected. This Description, or Notation, is brief: but it reacheth so farre, as the world is wyde. It concerneth all Creatures, all Actions, and passions, by Emanation of beames perfourmed. Beames, or naturall lines, (here) I meane, not of light onely, or of colour (though they, to eye, giue shew, witnes, and profe, wherby to ground the Arte vpon) but also of other *Formes*, both *Substantiall*, and *Accidental*, the certaine and determined actiue Radiall emanations. By this Art (omitting to speake of the highest pointes) we may vse our eyes, and the light, with greater pleasure: and perfecter Iudgement: both of things, in light seene, & of other: which by like order of Lightes Radiations, worke and produce their effectes. We may be ashamed to be ignorant of the cause, why so sundry wayes our eye is deceiued, and abused: as, while the eye weeneth a round Globe or Sphere (beyng farre of) to be a flat and plaine Circle, and so likewise iud-

1.5.
Frende,
you will finde
it hard, to performe
my description of
this Feate. But by
Chorographie, you
may helpe your selfe
some what: where the
Figures knowne (in
Sides and Angles)
are not Regular:
And where, Resolu-
tion into Triangles
can serue. &c. And
yet you will finde it
strange to deale thus
generally with A-
rithmeticall figures:
and, that for Bat-
taye ray. Their con-
tentes, differ so much
from like Geometri-
call Figures.

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geth a plaine Square, to be roūd: supposeth walles parallels, to approche, a farre of: rose and floure parallels, the one to bend downward, the other to rise vpward, at a little distance from you. Againe, of thinges being in like swiftnes of mouing, to thinke the nerer, to moue faster: and the farder, much slower. Nay, of two thinges, wherof the one (incomparably) doth moue swifter then the other, to deme the slower to moue very swift, & the other to stand: what an error is this, of our eye? Of the Raynbow, both of his Colours, of the order of the colours, of the bignes of it, the place and heith of it, (&c) to know the causes demonstratiue; is it not pleasant, is it not necessary: of two or three Sonnes appearing: of Blasing Sterres: and such like thinges: by naturall causes, brought to passe, (and yet neuertheles, of farder matter, Significatiue) is it not commodious for man to know the very true cause, & occasion Naturall: Yea, rather, is it not, greatly, against the Souerainty of Mans nature, to be so ouerhot and abused, with thinges (at hand) before his eyes: as with a Pecockes tayle, and a Doues necke: or a whole ore, in water, holden, to seme broken. Thynges, farre of, to seme nere: and nere, to seme farre of. Small thinges, to seme great: and great, to seme small. One man, to seme an Army: Or a man to be curstly affrayed of his owne shadow. Yea, so much, to feare, that, if you, being (alone) nere a certaine glasse, and proffer, with dagger or sword, to foyne at the glasse, you shall suddenly be moued to giue backe (in maner) by reason of an Image, appearing in the ayre, betwene you & the glasse, with like hand, sword or dagger, & with like quicknes, foyning at your very eye, likewise as you do at the Glasse. Straunge, this is, to heare of: but more meruailous to behold, then these my wordes can signifie. And neuerthelesse by demonstration Opticall, the order and cause therof, is certified: euen so, as the effect is consequent. Yea, thus much more, dare I take vpon me, toward the satisfiying of the noble courage, that longeth ardently for the wisdom of Causes Naturall: as to let him vnderstand, that, in London, he may with his owne eyes, haue profe of that, which I haue sayd herein. A Gentleman, (which, for his good seruice, done to his Countrey, is famous and honorable: and for skill in the Mathematicall Sciences, and Languages, is the Od man of this land. &c.) euen he, is hable: and (I am sure) will, very willingly, let the Glasse, and profe be sene: and so I (here) request him: for the encrease of wisdom, in the honorable: and for the stopping of the mouthes malicious: and repressing the arrogancy of the ignorant. Ye may easily gesse, what I meane. This Art of *Perspectiue*, is of that excellency, and may be led, to the certifiying, and executing of such thinges, as no man would easily beleue: without Actuall profe perceiued. I speake nothing of *Naturall Philosophie*, which, without *Perspectiue*, can not be fully vnderstanded, nor perfectly attained vnto. Nor, of *Astronomie*: which, without *Perspectiue*, can not well be grounded: Nor *Astrologie*, naturally Verified, and auouched. That part hereof, which dealeth with Glasses (which name, Glasse, is a generall name, in this Arte, for any thing, from which, a Beame reboundeth) is called *Catoptrike*: and hath so many vses, both merueilous, and profitabile: that, both, it would hold me to long, to note therein the principall conclusions, all ready knowne: And also (perchaunce) some thinges, might lacke due credite with you: And I, therby, to leese my labor: and you, to slip into light Iudgement*, Before you haue learned sufficiently the powre of Nature and Arte.

Now, to procede: *Astronomie*, is an Arte Mathematicall, which demonstrateth the distance, magnitudes, and all naturall motions, apparences, and passions propre to the Planets and fixed Sterres: for

any

A merueilous
Glasse. E

S.W.P.

E

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any time past, present and to come: in respect of a certaine Horizon, or without respect of any Horizon. By this Arte we are certified of the distance of the Starry Skye, and of eche *Planete* from the Centre of the Earth: and of the greatnes of any Fixed starre sene, or *Planete*, in respect of the Earthes greatnes. As, we are sure (by this Arte) that the Solidity, Massines and Body of the *Sonne*, conteineth the quantitie of the whole Earth and Sea, a hundred thre score and two times, lesse by $\frac{1}{8}$ one eight parte of the earth. But the Body of the whole earthly globe and Sea, is bigger then the body of the *Mone*, three and forty times lesse by $\frac{1}{8}$ of the *Mone*. Wherefore the *Sonne* is bigger then the *Mone*, 7000 times, lesse, by $59\frac{32}{64}$ that is, precisely $6940\frac{25}{64}$ bigger then the *Mone*. And yet the vnskillfull man, would iudge them a like bigge. Wherefore, of Necessity, the one is much farder from vs, then the other. The *Sonne*, when he is fardest from the earth (which, now, in our age, is, when he is in the 8. degree, of Cancer) is, 1179 Semidiameters of the Earth, distante. And the *Mone* when she is fardest from the earth, is 68 Semidiameters of the earth and $\frac{1}{4}$. The nerest, that the *Mone* cometh to the earth, is Semidiameters $52\frac{1}{4}$. The distance of the Starry Skye is, frō vs, in Semidiameters of the earth $20081\frac{1}{2}$. Twenty thousand fourescore, one, and almost a halfe. Subtract from this, the *Mones* nerest distance, from the Earth: and therof remaineth Semidiameters of the earth $20029\frac{1}{4}$. Twenty thousand nine and twenty and a quarter. So thicke is the heauenly Palace, that the *Planetes* haue all their exercise in, and most meruailously perfourme the Commaūdement and Charge to them giuen by the omnipotent Maiestie of the king of kings. This is that, which in *Genesis* is called *Ha Rakia*. Consider it well. The Semidiameter of the earth, cōteineth of our common miles $3436\frac{1}{4}$ three thousand, foure hundred thirty six and foure eleuenth partes of one inyle: Such as the whole earth and Sea, round about, is 21600. One and twenty thousand six hundred of our myles. Allowyng for euery degree of the greatest circle, thre score myles. Now if you way well with your selfe but this litle parcell of frute *Astronomicall*, as concerning the bignesse, Distances of *Sonne*, *Mone*, *Sterry Sky*, and the huge massines of *Ha Rakia*, will you not finde your Consciences moued, with the kingly Prophet, to sing the confession of Gods Glory, and say, *The Heauens declare the glory of God, and the Firmament [Ha Rakia] sheweth forth the workes of his bandes*. And so forth, for those fiue first staues, of that kingly Psalme. Well, well, It is time for some to lay hold on wisdome, and to Iudge truly of thinges: and not so to expound the Holy word, all by Allegories: as to Neglect the wisdome, powre and Goodnes of God, in, and by his Creatures, and Creation to be seen and learned. By parables and Analogies of whose natures and properties, the course of the Holy Scripture, also, declareth to vs very many Mysteries. The whole Frame of Gods Creatures, (which is the whole world,) is to vs, a bright glasse: from which, by reflexion, reboundeth to our knowledge and perceiuerance, Beames, and Radiations: representing the Image of his Infinite goodnes, Omnipotēcy, and wisdome. And we therby, are taught and perswaded to Glorifie our Creator, as God: and be thankfull therfore. Could the Heathenistes finde these vses, of these most pure, beawtifull, and Mighty Corporall Creatures: and shall we, after that the true *Sonne* of rightwisenesse is risen aboue the *Horizon*, of our temporall *Hemissphærie*, and hath so abundantly streamed into our hartes, the direct beames of his goodnes, mercy, and grace: Whose heat All Creatures feele: Spirituall and Corporall: Visible and

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Inuisible: Shall we (I say) looke vpon the *Heauen, Sterres, and Planets*, as an Oxe and an Asse doth: no furder carefull or inquisitiue, what they are: why were they Created, How do they execute that they were Created for: Seing, All Creatures, were for our sake created: and both we, and they, Created, chiefly to glorifie the Almighty Creator: and that, by all meanes, to vs possible. *Nolite ignorare* (saith *Plato in Epinomis*) *Astronomiam, Sapientissimū quiddam esse.* Be ye not ignorant, *Astronomie* to be a thyng of excellent wisdom. *Astronomie*, was to vs, from the beginning commended, and in maner commaunded by God him selfe. In asmuch as he made the *Sonne, Mone, and Sterres*, to be to vs, for *Signes*, and knowledge of Seasons, and for Distinctions of Dayes, and yeares. Many wordes nede not. But I wish, euery man should way this word, *Signes*. And besides that, conferre it also with the tenth Chapter of *Hieremie*. And though Some thinke, that there, they haue found a rod: Yet Modest Reason, will be indifferent Iudge, who ought to be beaten therewith, in respect of our purpose. Leauing that: I pray you vnderstand this: that without great diligence of Obseruation, examination and Calculation, their periods and courses (wherby *Distinction* of Seasons, yeares, and New Mones might precisely be knowne) could not exactly be certified. Which thing to performe, is that *Art*, which we here haue Defined to be *Astronomie*. Wherby, we may haue the distinct Course of Times, dayes, yeares, and Ages: aswell for Consideratiō of Sacred Prophecies, accomplished in due time, foretold: as for high Myfticall Solemnities holding: And for all other humaine affaires, Conditions, and couenantes, vpon certaine time, betwene man and man: with many other great vses: Wherin, (verely), would be great incertainty, Confusion, vntruth, and brutish Barbarousnes: without the wonderfull diligence and skill of this Arte: continually learning, and determining Times, and periodes of Time, by the Record of the heavenly booke, wherein all times are written: and to be read with an *Astronomicall staffe*, in stede of a festue.

Musike, of Motion, hath his Originall cause: Therefore, after the motions most swift, and most Slow, which are in the Firmament, of Nature performed: and vnder the *Astronomers Consideration*: now I will Speake of an other kinde of Motion, producing sound, audible, and of Man numerable. *Musike* I call here that *Science*, which of the Grekes is called *Harmonice*. Not meddling with the Controuersie betwene the auncient *Harmonistes*, and *Canonistes*. *Musike* is a Mathematicall Science, which teacheth, by sense and reason, perfectly to iudge, and order the diuersities of soundes, hye and low. *Astronomie* and *Musike* are Sisters, saith *Plato*. As, for *Astronomie*, the eyes: So, for *Harmonious Motion*, the eares were made. But as *Astronomie* hath a more diuine Contemplation, and commodity, then mortall eye can perceiue: So, is *Musike* to be considered; that the

1. * Minde may be preferred, before the eare. And from audible sound, we ought to ascende, to the examination: which numbers are *Harmonious*, and which not.
2. And why, either, the one are: or the other are not. I could at large, in the heavenly
3. * motions and distances, describe a meruailous Harmonie, of *Pythagoras Harpe*
4. with eight stringes. Also, somewhat might be sayd of *Mercurius* * two Harpes,
5. eche of foure Stringes Elementall. And very straunge matter, might be alledged
6. of the *Harmonie*, to our * Spirituall part appropriate. As in *Ptolomæus* third boke, in the fourth and sixth Chapters may appeare. * And what is the cause of the apt bonde, and frendly felowship, of the Intellectuall and Mentall part of vs, with our grosse & corruptible body: but a certaine Meane, and *Harmonious Spiritualitie*, with

both

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both participatyng, & of both (in a maner) resulyng: In the * Tune of Mans voyce, and also * the sound of Instrument, what might be sayd, of Harmonie. No common Musicien would lightly beleue. But of the sundry Mixture (as I may terme it) and concurse, diuerse collation, and Application of these Harmonies: as of thre, foure, fve, or more: Maruailous haue the effectes ben: and yet may be founde, and produced the like: with some proportionall consideration for our time, and being : in respect of the State, of the thinges then : in which, and by which, the wondrous effectes were wrought. Democritus and Theophrastus affirmed, that, by Musike, griefes and diseases of the Minde, and body might be cured, or inferred. And we finde in Records, that Terpander, Arion, Ismenias, Orpheus, Amphion, David, Pythagoras, Empedocles, Asclepiades and Timotheus, by Harmonicall Consonācy, haue done, and brought to pas, thinges, more then meruailous, to here of. Of them then, making no farther discourse, in this place : Sure I am, that Common Musike, commonly vsed, is found to the Musiciens and Hearers, to be so Commodious and pleasant, That if I would say and dispute, but thus much: That it were to be otherwise vsed, then it is, I should finde more repreeuers, then I could finde priuy, or skilfull of my meaning. In thinges therfore euident, and better knowen, then I can expresse: and so allowed and liked of, (as I would wish, some other thinges, had the like hap) I will spare to enlarge my lines any farder, but consequently follow my purpose.

Of Cosmographie, I appointed briefly in this place, to geue you some intelligence. Cosmographie, is the vhole and perfect description of the heauenly, and also elementall parte of the world, and their homologall application, and mutuall collation necessarie. This Art, requireth Astronomie, Geographie, Hydrographie and Musike. Therefore, it is no small Arte, nor so simple, as in common practise, it is (slightly) considered. This matcheth Heauen, and the Earth, in one frame, and aptly applieth parts Correspondent: So, as, the Heauenly Globe, may (in practise) be duely described vpon the Geographicall, and Hydrographicall Globe. And there, for vs to consider an Equinoctiall Circle, an Ecliptike line, Colures, Poles, Sterres in their true Longitudes, Latitudes, Declinations, and Verticalitie: also Climes, and Parallels: and by an Horizon annexed, and reuolution of the earthly Globe (as the Heauen, is, by the Primouant, caried about in 24. æquall Houres) to learne the Risings and Settings of Sterres (of Virgill in his Georgikes: of Hesiod: of Hippocrates in his Medicinall Sphere, to Perdicca King of the Macedonians: of Diocles, to King Antigonus, and of other famous Philosophers prescribed) a thing necessary, for due manuring of the earth, for Navigation, for the Alteration of mans body: being, whole, Sicke, wounded, or bruised. By the Reuolution, also, or mouing of the Globe Cosmographically, the Rising and Setting of the Sonne: the Lengthes, of dayes and nightes : the Houres and times (both night and day) are knowne : with very many other pleasant and necessary vses : Wherof, some are knowne: but better remaine, for such to know and vse: who of a sparke of true fire, can make a wonderfull bonfire, by applying of due matter, duely.

Of Astrologie, here I make an Arte, seuerall from Astronomie: not by new deuise, but by good reason and authoritie: for, Astrologie, is an Arte Mathematicall, which reasonably demonstrateth the operations and effectes, of the naturall beames, of light, and secrete influence: of the Sterres and Planets: in euery element and elementall body:

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Read in Aristotle his 8. booke of Politikes: the 5, 6, and 7. chapters. Where you shall haue some occasion farder to thinke of Musike, than commonly is thought.

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at all times, in any Horizon assigned. This Arte is furnished with many other great Artes and experiences: As with perfecte *Perspectiue*, *Astronomic*, *Cosmographie*, *Naturall Philosophie* of the 4. Elementes, the Arte of Graduation, and some good vnderstanding in *Musike*: and yet moreouer, with an other great Arte, hereafter following, though I, here, set this before, for some considerations me mouing. Sufficient (you see) is the stuffe, to make this rare and secrete Arte, of: and hard enough to frame to the Conclusion Syllogisticall. Yet both the manifolde and continuall trauailes of the most auncient and wise Philosophers, for the atteyning of this Arte: and by examples of effectes, to confirme the same: hath left vnto vs sufficient proufe and witnesse: and we, also, daily may perceauē, That mans body, and all other Elementall bodies, are altered, disposed, ordred, pleased, and displeased, by the Influentiall working of the *Sunne*, *Mone*, and the other Starres and Planets. And therefore, sayth *Aristotle*, in the first of his *Meteorologicall* bookes, in the second Chapter: *Est autem necessario Mundus iste, supernis lationibus ferè continuus: Vt, inde, vis eius vniuersa regatur. Ea siquidem Causa prima putanda omnibus est, vnde motus principium existit.* That is: This [Elementall] World is of necessitie, almost, next adioyning, to the heauenly motions: That, from thence, all his vertue or force may be gouerned. For, that is to be thought the first Cause vnto all: from which, the beginning of motion, is. And againe, in the tenth Chapter. *Oportet igitur & horum principia sumamus, & causas omnium similiter. Principium igitur ut mouens, præcipuumq; & omnium primum, Circulus ille est, in quo manifeste Solis latio, &c.* And so forth. His *Meteorologicall* bookes, are full of argumentes, and effectuell demonstrations, of the vertue, operation, and power of the heauenly bodies, in and vpon the fower Elementes, and other bodies, of them (either perfectly, or vnperfectly) composēd. And in his second booke, *De Generatione & Corruptione*, in the tenth Chapter. *Quocirca & prima latio, Ortus & Intertus causa non est: Sed obliqui Circuli latio: ea namq; & continua est, & duobus motibus fit:* In English, thus. *Wherefore the vppermost motion, is not the cause of Generation and Corruption, but the motion of the Zodiacke: for, that, both, is continuall, and is caused of two mouinges.* And in his second booke, and second Chapter of hys *Physikes*. *Homo namq; generat hominem, atq; Sol.* For Man (sayth he) and the Sonne, are cause of mans generation. Authorities may be brought, very many: both of 1000. 2000. yea and 3000. yeares Antiquitie: of great Philosophers, Expert, Wise, and godly men, for that Conclusion: which, daily and hourly, we men, may discerne and perceauē by sense and reason: All beastes do feelee, and simply shew, by their actions and passions, outward and inward: All Plants, Herbes, Trees, Flowers, and Fruites. And finally, the Elementes, and all thinges of the Elementes composēd, do geue Testimonie (as *Aristotle* sayd) that theyr Whole Dispositions, vertues, and naturall motions, depend of the Actiuitie of the heauenly motions and Influences. Whereby, beside the specificall order and forme, due to euery seede: and beside the Nature, propre to the Indiuiduall Matrix, of the thing produced: What shall be the heauenly Impression, the perfect and circumspecte Astrologien hath to Conclude. Not onely (by *Apotelesmes*) τὸ ὄν, but by Naturall and Mathematicall demonstration τὸ δῖόν. Whereunto, what Sciences are requisite (without exception) I partly haue here warned: And in my *Proædæmes* (besides other matter there disclosed) I haue Mathematically furnished vp the whole Method: To this our age, not so carefully handled by any, that

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cuer I saw, or heard of. I was, (for * 21. yeares ago) by certaine earnest disputations, of the Learned *Gerardus Mercator*, and *Antonius Gogaua*, (and other,) therto so prouoked: and (by my constant and inuincible zeale to the veritie) in obseruations of Heauenly Influencies (to the Minute of time,) than, so diligent: And chiefly by the Supernaturall influence, from the Starre of Iacob, so directed: That any Modest and Sober Student, carefully and diligently seeking for the Truth, will both finde & cōfesse, therein, to be the Veritie, of these my wordes: And also become a Reasonable Reformer, of three Sortes of people: about these Influentiall Operations, greatly erring from the truth. Wherof, the one, is Light Beleuers, the other, Light Despisers, and the third Light Practisers. The first, & most cōmon Sort, thinke the Heauen and Sterres, to be answerable to any their doubts or desires: which is not so: and, in dede, they, to much, ouer reache. The Second sorte thinke no Influentiall vertue (frō the heauenly bodies,) to beare any Sway in Generation and Corruption, in this Elementall world. And to the *Sunne*, *Mone* and *Sterres* (being so many, so pure, so bright, so wonderfull bigge, so farre in distance, so manifold in their motions, so constant in their periodes. &c.) they assigne a sleight, simple office or two, and so allow vnto the (according to their capacities) as much vertue, and power Influentiall, as to the Signe of the *Sunne*, *Mone*, and seuen *Sterres*, hanged vp (for Signes) in London, for distinction of houses, & such grosse helpes, in our wordly affaires: And they vnderstand not (or will not vnderstand) of the other workinges, and vertues of the Heauenly *Sunne*, *Mone*, and *Sterres*: not so much, as the Mariner, or Husband man: no, not so much, as the *Elephant* doth, as the *Cynocephalus*, as the *Porpentine* doth: nor will allow these perfect, and incorruptible mighty bodies, so much vertuall Radiation, & Force, as they see in a litle peece of a *Magnes Stone*: which, at great distance, sheweth his operation. And perchance they thinke, the Sea & Riuers (as the *Thames*) to be some quicke thing, and so to ebbe, and flow, run in and out, of them selues, at their owne fantasies. God helpe, God helpe. Surely, these men, come to short: and either are to dull: or willfully blind: or, perhaps, to malicious. The third man, is the common and vulgare *Astrologien*, or Practiser: who, being not duely, artificially, and perfectly furnished: yet, either for vaine glory, or gayne: or like a simple dolt, & blinde Bayard, both in matter and maner, erreth: to the discredit of the *Wary*, and modest *Astrologien*: and to the robbing of those most noble corporall Creatures, of their Naturall Vertue: being most mighty: most beneficiall to all elementall Generation, Corruption and the appartenances: and most Harmonious in their Monarchie: For which thinges, being knowen, and modestly vsed: we might highly, and continually glorifie God, with the princely Prophet, saying. *The Heauens declare the Glorie of God: who made the Heauens in his wisdom: who made the Sonne, for to haue dominion of the day: the Mone and Sterres to haue dominion of the nyght: whereby, Day to day vttereth talke: and night, to night declareth knowledge: Prayse him, all ye Sterres, and Light. Amen.*

IN order, now foloweth, of Statike, somewhat to say, what we meane by that name: and what commodity, doth, on such Art, depend. Statike, is an Arte Mathematicall, which demonstrateth the causes of heauynes, and lightnes of all thynges: and of motions and properties, to heauynes and lightnes, belonging. And for asmuch as, by the Bilanx, or Balance (as the chief sensible Instrument,) Experience of these demonstrations may

* Anno. 1548
and 1549.^{ms}
Louayn.

Note.

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3.

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be had: we call this Art, *Statike*: that is, *the Experimentes of the Balance*. Oh, that men wist, what profit, (all maner of wayes) by this Arte might grow, to the hable examiner, and diligent practiser. Thou onely knowest all thinges precisely (O God) who hast made weight and Balance, thy Iudgement: who hast created all thinges in *Number, Waight, and Measure*: and hast wayed the mountaines and hills in a Balance: who hast peysed in thy hand, both Heauen and earth. We therefore warned by the Sacred word, to Consider thy Creatures: and by that consideration, to wyne a glyms (as it were,) or shaddow of perceiuerance, that thy wisdome, might, and goodnes is infinite, and ynspeakable, in thy Creatures declared: And being farther aduertised, by thy mercifull goodnes, that, three principall wayes, were, of the, vsed in Creation of all thy Creatures, namely, *Number, Waight and Measure*, And for as much as, of *Number and Measure*, the two Artes (auncient, famous, and to humaine vses most necessary,) are, all ready, sufficiently knowen and extant: This third key, we beseeche thee (through thy accustomed goodnes,) that it may come to the nedefull and sufficient knowledge, of such thy Seruauntes, as in thy workemanship, would gladly finde, thy true occasions (purposely of the vsed) whereby we should glorifie thy name, and shew forth (to the weaklinges in faith) thy wondrous wisdome and Goodnes. Amen.

Meruaile nothing at this pang (godly frend, you Gentle and zelous Student.) An other day, perchaunce, you will perceiue, what occasion moued me. Here, as now, I will giue you some ground, and withall some shew, of certaine commodities, by this Arte arising. And bycause this Arte is rare, my wordes and practises might be to darke: vnleast you had some light, holden before the matter: and that, best will be, in giuing you, out of *Archimedes* demonstrations, a few principal Conclusions, as foloweth.

1.
The Superficies of euery Liquor, by it selfe consistyng, and in quyet, is Sphæricall: the centre whereof, is the same, which is the centre of the Earth.

2.
If Solide Magnitudes, being of the same bignes, or quãtitie, that any Liquor is, and hauyng also the same Waight: be let downe into the same Liquor, they will settle downeward, so, that no parte of them, shall be aboue the Superficies of the Liquor: and yet neuertheles, they will not sinke vtterly downe, or drowne.

3.
If any Solide Magnitude beyng Lighter then a Liquor, be let downe into the same Liquor, it will settle downe, so farre into the same Liquor, that so great a quantitie of that Liquor, as is the parte of the Solid Magnitude, settled downe into the same Liquor: is in Waight, æquall, to the waight of the whole Solid Magnitude.

4.
Any Solide Magnitude, Lighter then a Liquor, forced downe into

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into the same Liquor, will moue vpward, with so great a power, by how much, the Liquor hauyng æquall quantitie to the whole Magnitude, is heauyer then the same Magnitude.

5.

Any Solid Magnitude, heauyer then a Liquor, beyng let downe into the same Liquor, will sinke downe vtterly: And wilbe in that Liquor, Lighter by so much, as is the waight or heauynes of the Liquor, hauing bygnes or quantitie, æquall to the Solid Magnitude.

6.

If any Solide Magnitude, Lighter then a Liquor, be let downe into the same Liquor, the waight of the same Magnitude, will be, to the Waight of the Liquor. (Which is æquall in quantitie to the whole Magnitude,) in that proportion, that the parte, of the Magnitude settled downe, is to the whole Magnitude.

I.D.

The Cutting of a Sphere according to any proportion assigned may by this proposition be done Mechanically by tempering Liquor to a certayne waight in respect of the waight of the Sphere therein Swymming.

BY these verities, great Errors may be reformed, in Opinion of the Naturall Motion of thinges, Light and Heauy. Which errors, are in Naturall Philosophie (almost) of all men allowed: to much trusting to Authority: and false Suppositions. As, Of any two bodyes, the heauyer, to moue downward faster then the lighter. This error, is not first by me, Noted: but by one *John Baptist de Benedictis*. The chief of his propositions, is this: which seemeth a Paradox.

A common error, noted.

If there be two bodyes of one forme, and of one kynde, æquall in quantitie or vnæquall, they will moue by æquall space, in æquall tyme: So that both theyr mouynges be in ayre, or both in water: or in any one Middle.

A paradox.

Hereupon, in the feate of Gunnyng, certaine good discourses (otherwise) may receiue great amendement, and furdurance. In the entended purpose, also, allowing somewhat to the imperfection of Nature: not aunswerable to the precisenes of demonstration. Moreouer, by the foresaid propositions (wisely vsed.) The Ayre, the water, the Earth, the Fire, may be nerely, knowen, how light or heauy they are (Naturally) in their assigned partes: or in the whole. And then, to thinges Elementall, turning your practise: you may deale for the proportion of the Elementes, in the thinges Compounded. Then, to the proportions of the Humours in Man: their waightes: and the waight of his bones, and flesh. &c. Than, by waight, to haue consideration of the Force of man, any maner of way: in whole or in part. Then, may you, of Ships water drawing, diuersly, in the Sea and in fresh water, haue pleasant consideration: and of waying vp of any thing, sonken in Sea or in fresh water &c. And (to lift vp your head a loft:) by waight, you may, as precisely, as by any instrument els, measure the Diameters of *Sonne* and *Mone*. &c. Frende, I pray you, way these thinges, with the iust Balance of Reason. And you will finde Meruailes vpon Meruailes: And esteeme one Drop of Truth (yea in Naturall Philosophie) more worth, then whole Libraries of Opinions, vndemonstrated: or not aunswering to Natures Law, and your experience. Leauing these

N. T.

The wonderfull use of these Propositions.

c.j.

thinges,

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*The practise
Staticall, to
know the pro-
portion, be-
twene the
Cube, and the
Sphere*

*I. D.
*For, so, haue
you. 256.
partes of a
Graine.*

things, thus: I will giue you two or three, light practises, to great purpose: and so finish my Annotation *Staticall*. In Mathematicall matters, by the Mechaniciens ayde, we will behold, here, the Cominodity of waight. Make a Cube, of any one Vniforme: and through like heauy stuffe: of the same Stuffe, make a Sphære or Globe, precisely, of a Diameter æquall to the Radicall side of the Cube. Your stuffe, may be wood, Copper, Tinne, Lead, Siluer. &c. (being, as I sayd, of like nature, condition, and like waight throughout.) And you may, by Say Balance, haue prepared a great number of the smallest waightes: which, by those Balance can be discerned or tryed: and so, haue proceeded to make you a perfect Pyle, company & Number of waightes: to the waight of six, eight, or twelue pound waight: most diligently tryed, all. And of euery one, the Content knowen, in your least waight, that is wayable. [They that can not haue these waightes of precisenes: may, by Sand, Vniforme, and well dusted, make them a number of waightes, somewhat nere precisenes: by halving euer the Sand: they shall, at length, come to a least common waight. Therein, I leaue the farder matter, to their discretion, whom nede shall pinche.] The *Venetians* consideration of waight, may seme precise enough: by eight descentes progressionall, * halving, from a grayne. Your Cube, Sphære, apt Balance, and conuenient waightes, being ready: fall to worke. First, way your Cube. Note the Number of the waight. Way, after that, your Sphære. Note likewise, the Nüber of the waight. If you now find the waight of your Cube, to be to the waight of the Sphære, as 21. is to 11. Then you see, how the Mechanicien and *Experimenter*, without Geometrie and Demonstration, are (as nerely in effect) taught the proportion of the Cube to the Sphere: as I haue demonstrated it, in the end of the twelfth boke of *Euclide*. Often, try with the same Cube and Sphære. Then, chaunge, your Sphære and Cube, to an other matter: or to an other bignes: till you haue made a perfect vniuersall Experience of it. Possible it is, that you shall wyne to nerer termes, in the proportion.

When you haue found this one certaine Drop of Naturall veritie, procede on, to Inferre, and duely to make assay, of matter depending. As, bycause it is well demonstrated, that a Cylinder, whose heith, and Diameter of his base, is æquall to the Diameter of the Sphære, is Sesquialter to the same Sphære (that is, as 3. to 2.) To the number of the waight of the Sphære, adde halfe so much, as it is: and so haue you the number of the waight of that Cylinder. Which is also Comprehended of our former Cube: So, that the base of that Cylinder, is a Circle described in the Square, which is the base of our Cube. But the Cube and the Cylinder, being both of one heith, haue their Bases in the same proportion, in the which, they are, one to an other, in their Massines or Soliditie. But, before, we haue two numbers, expressing their Massines, Solidities, and Quantities, by waight: wherfore, we haue * the proportion of the Square, to the Circle, inscribed in the same Square. And so are we fallen into the knowledge sensible, and Experimentall of *Archimedes* great Secret: of him, by great trauaile of minde, sought and found. Wherfore, to any Circle giuen, you can giue a Square æquall: * as I haue taught, in my Annotation, vpon the first proposition of the twelfth boke, And likewise, to any Square giuen, you may giue a Circle æquall: * If you describe a Circle, which shall be in that proportion, to your Circle inscribed, as the Square is to the same Circle. This, you may do, by my Annotations, vpon the second proposition of the twelfth boke of *Euclide*, in my third Probleme there. Your diligence may come to a proportion, of the Square to the Circle inscribed, nerer the truth, then is the proportion of 14. to 11. And consider, that you may begyn at the Circle and Square, and so come to conclude of the Sphære, & the Cube, what their

**The proportion of
the Square to the
Circle inscribed.*

**The Squaring of
the Circle, Mecha-
nically:*

**To any Square
giuen, to geue a
Circle, æquall.*

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their proportion is: as now, you came from the Sphere to the Circle. For, of Silver, or Gold, or Latton Lamyns or plates (thorough one hole drawe, as the manner is) if you make a Square figure: & way it: and then, describing thereon, the Circle inscribed: & cut of, & file away, precisely (to the Circle) the ouerplus of the Square: you shall then, waying your Circle, see, whether the waight of the Square, be to your Circle, as 14. to 11. As I haue Noted, in the beginning of *Euclides* twelfth boke. &c. after this resort to my last proposition, vpon the last of the twelfth. And there, helpe your selfe, to the end. And, here, Note this, by the way. That we may Square the Circle, without hauing knowledge of the proportion, of the Circumference to the Diameter: as you haue here perceined. And otherwayes also, I can demonstrate it. So that, many haue cumberd them selues superfluously, by trauailing in that point first, which was not of necessitie, first: and also very intricate. And easily, you may, (and that diuersly) come to the knowledge of the Circumference: the Circles Quantitie, being first knowen. Which thing, I leaue to your consideration: making hast to despatch an other Magistrall Probleme: and to bring it, nerer to your knowledge, and readier dealing with, then the world (before this day,) had it for you, that I can tell of. And that is, *A Mechanicall Dubbling of the Cube: &c.* Which may, thus, be done: Make of Copper plates, or Tyn plates, a foursquare vpright Pyramis, or a Cone: perfectly fashioned in the holow, within. Wherin, let great diligence be vsed, to approche (as nere as may be) to the Mathematicall perfection of those figures. At their bases, let them be all open: euery where, els, most close, and iust to. From the vertex, to the Circumference of the base of the Cone: & to the sides of the base of the Pyramis: Let 4. straight lines be drawen, in the inside of the Cone and Pyramis: making at their fall, on the perimeters of the bases, equall angles on both sides them selues, with the sayd perimeters. These 4. lines (in the Pyramis: and as many, in the Cone) diuide: one, in 12. æquall partes: and an other, in 24. an other, in 60, and an other, in 100. (reckenyng vp from the vertex.) Or vse other numbers of diuision, as experience shall teach you. Then, * set your Cone or Pyramis, with the vertex downward, perpendicularly, in respect of the Base. (Though it be otherwayes, it hindreth nothyng.) So let the most stedily be stayed. Now, if there be a Cube, which you wold haue Dubbled. Make you a prety Cube of Copper, Silver, Lead, Tynne, Wood, Stone, or Bone. Or els make a hollow Cube, or Cubik coffen, of Copper, Silver, Tynne, or Wood &c. These, you may so proportio in respect of your Pyramis or Cone, that the Pyramis or Cone, will be hable, to containe the waight of them, in water, 3. or 4. times: at the least: what stuff so euer they be made of. Let not your Solid angle, at the vertex, be to sharpe: but that the water may come with ease, to the very vertex, of your hollow Cone or Pyramis. Put one of your Solid Cubes in a Balance apt: take the waight therof exactly in water. Powre that water, (without losse) into the hollow Pyramis or Cone, quietly. Marke in your lines, what numbers the water Cutterth: Take the waight of the same Cube againe: in the same kinde of water, which you had before: put that * also, into the Pyramis or Cone, where you did put the first. Marke now againe, in what number or place of the lines, the water Cutterth them. Two

Note
Squaring of
the Circle
without know-
ledge of the
proportion be-
twene Cir-
cumference
and Diame-
ter.

*To Dubble
the Cube re-
dily: by Art
Mechanicall:
depending vpon
Demon-
stration Ma-
thematicall.*

I. D.
The 4. sides of this
Pyramis must be 4.
Isosceles Triangles &
like and equall.

I. D.
* In all workinges
with this Pyramis
or Cone, Let their
Situations be in all
Pointes and Condi-
tions, a like, or all
one: while you are
about one worke. Els
you will erre.

I. D.
* Consider well when
you must put your
waters together: and
when, you must emp-
ty your first water,
out of your Pyramis
or Cone. Els you
will erre.

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wayes you may conclude your purpose : it is to wete, either by numbers or lines. By numbers : as, if you diuide the side of your Fundamentall Cube into so many æquall partes, as it is capable of, conueniently, with your ease, and precisenes of the diuision. For, as the number of your first and lesse line (in your hollow Pyramis or Cone,) is to the second or greater (both being counted from the vertex) so shall the number of the side of your Fundamentall Cube, be to the nūber belonging to the Radical side, of the Cube, dubble to your Fundamentall Cube: Which being multiplied Cubik wise, will sone shew it selfe, whether it be dubble or no, to the Cubik number of your Fundamentall Cube. By lines, thus: As your lesse and first line, (in your hollow Pyramis or Cone,) is to the second or greater, so let the Radical side of your Fundamētall Cube, be to a fourth proportionall line, by the 12. proposition, of the sixth boke of *Euclide*. Which fourth line, shall be the Rote Cubik, or Radical side of the Cube, dubble to your Fundamentall Cube: which is the thing we desired. For this, may I (with ioy) say, ΕΥΦΗΚΑ, ΕΥΦΗΚΑ, ΕΥΦΗΚΑ: thanking the holy and glorious Trinity: hauing greater cause therto, then * *Archimedes* had (for finding the fraude vsed in the Kinges Crowne, of Gold): as all men may easily Iudge: by the diuersitie of the frute following of the one, and the other. Where I spake before, of a hollow Cubik Coffen: the like vse, is of it: and without waight. Thus. Fill it with water, precisely full, and poure that water into your Pyramis or Cone. And here note the lines cutting in your Pyramis or Cone. Againe, fill your coffen, like as you did before. Put that Water, also, to the first. Marke the second cutting of your lines. Now, as you proceded before, so must you here procéde. * And if the Cube, which you should Double, be neuer so great: you haue, thus, the proportion (in small) betwene your two litle Cubes: And then, the side, of that great Cube (to be doubled) being the third, will haue the fourth, found, to it proportionall: by the 12. of the sixth of *Euclide*.

* Vitruvius.
Lib. 9. Cap. 3.

God be than-
ked for this
Inuention, &
the fruite en-
suing.

*Note.

Note, as concerning the Spharicall Superficies of the Water.



*Note.

Note this A-
bridgement of
Dubbling the
Cube. &c.

Note, that all this while, I forget not my first Proposition Staticall, here rehearsed: that, the Superficies of the water, is Sphaericall. Wherein, vse your discretion: to the first line, adding a small heere breadth, more: and to the second, halfe a heere breadth more, to his length. For, you will easily perceave, that the difference can be no greater, in any Pyramis or Cone, of you to be handled. Which you shall thus trye. *For finding the swelling of the water aboue leuell.* Square the Semidiameter, from the Centre of the earth, to your first Waters Superficies. Square then, halfe the Subtendent of that watry Superficies (which Subtendent must haue the equall partes of his measure, all one, with those of the Semidiameter of the earth to your watry Superficies) : Subtracte this square, from the first : Of the residue, take the Rote Square. That Rote, Subtracte from your first Semidiameter of the earth to your watry Superficies : that, which remaineth, is the heith of the water, in the middle, aboue the leuell. Which, you will finde, to be a thing insensible. And though it were greatly sensible,* yet, by helpe of my sixt Theoreme vpon the last Proposition of Euclides twelfth booke, noted: you may reduce all, to a true Leuell. But, farther diligence, of you is to be vsed, against accidentall causes of the waters swelling: as by hauing (somewhat) with a moyst Sponge, before, made moyst your hollow Pyramis or Cone, will preuent an accidentall cause of Swelling, &c. Experience will teach you abundantly: with great ease, pleasure, and comoditie.

Thus, may you Double the Cube Mechanically, Treble it, and so forth, in any proportion. Now will I Abridge your paine, cost, and Care herein. Without all preparing of your Fundamentall Cubes: you may (alike) worke this Conclusion. For, that, was rather a kinde of Experimentall demonstration, then the shortest way:

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and all, vpon one Mathematicall Demonstration, depending . Take water (as much as conueniently will serue your turne : as I warned before of your Fundamentall Cubes bignes) Way it precisely . Put that water, into your Pyramis or Cone . Of the same kinde of water, then take againe, the same waight you had before : put that likewise into the Pyramis or Cone . For, in eche time, your marking of the lines, how the Water doth cut them, shall geue you the proportion between the Radicall sides, of any two Cubes, wherof the one is Double to the other: working as before I haue taught you: *sauiug that for you Fundamentall Cube his Radicall side: here, you may take a right line, at pleasure.

Note. * 

Yet farther proceeding with our droppe of Naturall truth : you may (now) geue Cubes, one to the other, in any proportiō geue: Rationall or Irrationall : on this maner. Make a hollow Parallelipipedon of Copper or Tinne: with one Base wating, or open: as in our Cubike Coffen. Frō the bottome of that Parallelipipedon, raise vp, many perpendiculars, in euery of his fower sides. Now if any proportion be assigned you, in right lines: Cut one of your perpendiculars (or a line equall to it, or lesse then it) likewise : by the 10. of the sixth of Euclide. And those two partes, set in two sundry lines of those perpendiculars (or you may set them both, in one line) making their beginniges, to be, at the base : and so their lengthes to extend vpward . Now, set your hollow Parallelipipedon, vp right, perpendicularly, steadie . Poure in water, handsomly, to the heith of your shorter line . Poure that water, into the hollow Pyramis or Cone . Marke the place of the rising . Settle your hollow Parallelipipedon againe . Poure water into it: vnto the heith of the second line, exactly . Poure that water *duely into the hollow Pyramis or Cone : Marke now againe, where the water cutteth the same line which you marked before . For, there, as the first marked line, is to the second : So shall the two Radicall sides be, one to the other, of any two Cubes: which, in their Soliditie, shall haue the same proportion, which was at the first assigned : were it Rationall or Irrationall.

To giue Cubes one to the other in any proportion, Rationall or Irrationall.

* Emptying the first.

Thus, in sundry waies you may furnishe your selfe with such straunge and profitable matter: which, long hath bene wished for. And though it be Naturally done and Mechanically : yet hath it a good Demonstration Mathematicall . Which is this : Alwaies, you haue two Like Pyramids : or two Like Cones, in the proportions assigned : and like Pyramids or Cones, are in proportion, one to the other, in the proportion of their Homologall sides (or lines) tripled. Wherefore, if to the first, and second lines, found in your hollow Pyramis or Cone, you ioyne a third and a fourth, in continuall proportion : that fourth line, shall be to the first, as the greater Pyramis or Cone, is to the lesse : by the 33. of the eleuenth of Euclide . If Pyramis to Pyramis, or Cone to Cone, be double, then shall * Line to Line, be also double, &c. But, as our first line, is to the second, so is the Radicall side of our Fundamentall Cube, to the Radicall side of the Cube to be made, or to be doubled : and therefore, to those twaine also, a third and a fourth line, in continuall proportion, ioyned : will geue the fourth line in that proportion to the first, as our fourth Pyramidall, or Conike line, was to his first : but that was double, or treble, &c. as the Pyramids or Cones were, one to an other (as we haue proued) therefore, this fourth, shalbe also double or treble to the first, as the Pyramids or Cones were one to an other : But our made Cube, is described of the second in proportion, of the fower proportionall lines : therefore * as the fourth line, is to the first, so is that Cube, to the first Cube : and we haue proued the fourth line, to be to the first, as the Pyramis or Cone, is to the Pyramis or Cone : Wherefore the Cube is

The demonstrations of this Dubbling of the Cube, and of the rest.

I.D.
* Hereby, helpe your selfe to become a precise practiser. And so consider, how, nothing at all, you are hindered (sensibly) by the Conueniencie of the water.

* By the 33. of the eleuenth booke of Euclide.

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I.D.
* And your diligence
in practise, can so
(in waight of wa-
ter) performe it:
Therefore, now, you
are able to geue good
reason of your whole
doing.

* Note this
Corollary.

* The great
Commodities
following of
these new In-
ventions.

☞*

Such is the
Fruite of the
Mathemati-
call Sciences
and Artes.

to the Cube, as Pyramis is to Pyramis, or Cone is to Cone. But we* Suppose Pyramis to Pyramis, or Cone to Cone, to be double or treble. &c. Therefore Cube, is to Cube, double, or treble, &c. Which was to be demonstred. And of the Parallelipipedō, it is euident, that the water Solide Parallelipipedons, are one to the other, as their heithes are, seing they haue one base. Wherefore the Pyramids or Cones, made of those water Parallelipipedons, are one to the other, as the lines are (one to the other) betwene which, our proportion was assigned. But the Cubes made of lines, after the proportiō of the Pyramidal or Conik *homologall* lines, are one to the other, as the Pyramides or Cones are, one to the other (as we before did proue) therefore, the Cubes made, shalbe one to the other, as the lines assigned, are one to the other: Which was to be demonstred. Note. * This, my Demonstratiō is more generall, then onely in Square Pyramis or Cone: Consider well. Thus, haue I, both Mathematically and Mechanically, ben very long in wordes: yet (I trust) nothing tedious to them, who, to these thinges, are well affected. And verily I am forced (auoiding prolixitie) to omit sundry such things, easie to be practised: which to the Mathematicien, would be a great Threasure: and to the Mechanicien, no finall gaine. * Now may you, Betwene two lines giuen, finde two middle proportionals, in Continuall proportion: by the hollow Parallelipipedon, and the hollow Pyramis, or Cone. Now, any Parallelipipedon rectangle being giuen: thre right lines may be found, proportionall in any proportion assigned, of which, shal be produced a Parallelipipedon, aequal to the Parallelipipedon giuen. Hereof, I noted somwhat, vpon the 36. proposition, of the 11. booke of *Euclide*. Now, all those thinges, which *Vitruuius* in his Architecture, specified hable to be done, by dubbling of the Cube. Or, by finding of two middle proportionall lines, betwene two lines giuen, may easely be performed. Now, that Probleme, which I noted vnto you, in the end of my Addition, vpon the 34. of the 11. booke of *Euclide*, is proued possible. Now, may any regular body, be Transformed into an other, &c. Now, any regular body: any Sphere, yea any Mixt Solid: and (that more is) Irregular Solides, may be made (in any proportiō assigned) like vnto the body, first giuen. Thus, of a *Manneken*, (as the *Dutch* Painters terme it) in the same *Symmetrie*, may a Giant be made: and that, with any gesture, by the *Manneken* vsed: and contrarywise. Now, may you, of any Mould, or Modell of a Ship, make one, of the same Mould (in any assigned proportion) bigger or lesser. Now, may you, of any* Gunne, or little peece of ordinaunce, make an other, with the same *Symmetrie* (in all pointes) as great, and as little, as you will. Marke that: and thinke on it. Infinitely, may you apply this, so long sought for, and now so easily concluded: and withall, so willingly and frankly communicated to such, as faithfully deale with vertuous studies. Thus, can the Mathematicall minde, deale Speculatiuely in his own Arte: and by good meanes, Mount aboue the cloudes and sterres: And thirdly, he can, by order, Descend, to frame Naturall thinges, to wonderfull vses: and when he list, retire home into his owne Centre: and there, prepare more Meanes, to Ascend or Descend by; and, all, to the glory of God, and our honest delectation in earth.

Although, the Printer, hath looked for this Præface, a day or two, yet could I not bring my pen from the paper, before I had giuen you comfortable warning, and brief instructions, of some of the Commodities, by *Statike*, hable to be reaped: In the rest, I will therefore, be as brief, as it is possible: and with all, describing them, somwhat accordingly. And that, you shall perceiue, by this, which in order com-
meth

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meth next. For, wheras, it is so ample and wonderfull, that, an whole yeare long, one might finde fruitfull matter therin, to speake of: and also in practise, is a Threasure endeles: yet will I glanse ouer it, with wordes very few.

THIS do I call **Anthropographie**. Which is an Art restored, and of my preferment to your Seruice. I pray you, thinke of it, as of one of the chief pointes, of Humane knowledge. Although it be, but now, first Cōfirmed, with this new name: yet the matter, hath from the beginning, ben in consideration of all perfect Philosophers. Anthropographie, is the description of the Number, Measure, Waight, figure, Situation, and colour of euery diuerse thing, conteyned in the perfect body of MAN: with certain knowledge of the Symmetrie, figure, waight, Characterization, and due locall motion, of any parcell of the sayd body, asigned: and of Nūbers, to the sayd parcell appertainyng. This, is the one part of the Definition, mete for this place: Sufficient to notifie, the particularitie, and excellency of the Arte: and why it is, here, ascribed to the Mathematics. Yf the description of the heauenly part of the world, had a peculier Art, called *Astronomie*: If the description of the earthly Globe, hath his peculier arte, called *Geographie*. If the Matching of both, hath his peculier Arte, called *Cosmographie*: Which is the Descriptiō of the whole, and vniuersall frame of the world: Why should not the description of him, who is the Lesse world: and, frō the beginning, called *Microcosmus* (that is. *MAN* is the Lesse World.) And for whose sake, and seruice, all bodily creatures els, were created: Who, also, participateth with Spirites, and Angels: and is made to the Image and similitude of *God*: haue his peculier Art: and be called the *Arte of Artes*: rather, then, either to want a name, or to haue to base and improprie a name? You must of sundry professions, borow or challenge home, peculier partes hereof: and farder procede: as, *God*, *Nature*, *Reason* and *Experience* shall informe you. The Anatomistes will restore to you, some part: The Physiognomistes, some: The Chyromantistes some. The Metaposcopistes, some: The excellent, *Albert Durer*, a good part: the Arte of Perspective, will somwhat, for the Eye, helpe forward: *Pythagoras*, *Hipocrates*, *Plato*, *Galenus*, *Meletius*, & many other (in certaine thinges) will be Contributaries. And farder, the Heauen, the Earth, and all other Creatures, will eche shew, and offer their Harmonious seruice, to fill vp, that, which wanteth hereof: and with your own Experience, concluding: you may Methodically register the whole, for the posteritie: Whereby, good profe will be had, of our Harmonious, and Microcosmicall constitution. The outward Image, and vew hereof: to the Art of *Zographie* and Painting, to Sculpture, and Architecture: (for Church, House, Fort, or Ship) is most necessary and profitable: for that, it is the chiefe base and foundation of them. Looke in * *Vitruuius*, whether I deale sincerely for your behoufe, or no. Looke in *Albertus Durerus*, *De Symmetria humani Corporis*. Looke in the 27. and 28. Chapters, of the second booke, *De occulta Philosophia*. Consider the Arke of *Noe*. And by that, wade farther. Remember the *Delphicall Oracle* *NO SCETE IPSVM* (*Knowe thy selfe*) so long agoe pronounced: of so many a Philosopher repeated: and of the *Wiseſt* attempted: And then, you will perceauē, how long agoe, you haue bene called to the Schole, where this Arte might be learned. Well. I am nothing affrayde, of the disdayne of some such, as thinke Sciences and Artes, to be but Seuen. Perhaps, those Such, may, with ignorance, and shame enough, come short of them Seuen also: and yet neuerthelesse

MAN is
the Lesse
World.

* *ED*
Microcos-
mus.
* *Lib. 3.*
Cap. 1.

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they can not prescribe a certaine number of Artes: and in eche, certaine vnpassable boundes, to God, Nature, and mans Industrie. New Artes, dayly rise vp: and there was no such order taken, that, All Artes, should in one age, or in one land, or of one man, be made knowne to the world. Let vs embrace the giftes of God; and wayes to wisdome, in this time of grace, from aboue, continually bestowed on them, who thankfully will receiue them: *Et bonis Omnia Cooperabuntur in bonum.*

Trochilike, is that Art Mathematicall, which demonstrateth the properties of all Circular motions, Simple and Compounde. And bycause the frute hercof, vulgarly receiued, is in Wheles, it hath the name of *Trochilike*: as a man would say, *Whele Art*. By this art, a Whele may be geuen which shall moue ones about, in any tyme assigned. Two Wheles may be giuen, whose turnynges about in one and the same tyme, (or equall tymes), shall haue, one to the other, any proportion appointed. By Wheles, may a straight line be described: Likewise, a Spirall line in plaine, Conicall Section lines, and other Irregular lines, at pleasure, may be drawn. These, and such like, are principall Conclusions of this Arte: and helpe forward many pleasant and profitable Mechanicall workes: As Milles, to Saw great and very long Deale bordes, no man being by. Such haue I seene in Germany: and in the Citie of Prage: in the kingdome of Bohemia: Coyning Milles, Hand Milles for Corne grinding: And all maner of Milles, and Whele worke: By Winde, Smoke, Water, Waight, Spring, Man or Beast, moued. Take in your hand, *Agricola De re Metallica*: and then shall you (in all Mines) perceaue, how great nede is, of Whele worke. By Wheles, straunge workes and incredible, are done: as will, in other Artes hereafter, appeare. A wonderfull example of farther possibilitie, and present commoditie, was sene in my time, in a certaine Instrument: which by the Inuenter and Artificer (before) was solde for xx. Talentes of Golde: and then had (by misfortune) receaued some iniurie and hurt: And one *Ianellus* of *Cremona* did mend the same, and presented it vnto the Emperour *Charles* the fifth. *Hieronymus Cardanus*, can be my witnesse, that therein, was one Whele, which moued, and that, in such rate, that, in 7000. yeares onely, his owne periode should be finished. A thing almost incredible: But how farre, I keepe me within my boundes: very many men (yet aliue) can tell.

Helicosophie, is nere Sister to *Trochilike*: and is, An Arte Mathematicall, which demonstrateth the designing of all Spirall lines in Plaine, on Cylinder, Cone, Sphære, Conoid, and Sphæroid, and their properties appertayning. The vse hereof, in *Architecture*, and diuerse Instrumentes and Engines, is most necessary. For, in many thinges, the Skruie worketh the feate, which, els, could not be performed. By helpe hereof, it is recorded, that, where all the power of the Citie of *Syracusa*, was not hable to moue a certaine Ship (being on ground) mightie *Archimedes*, setting to, his Skruish Engine, caused *Hiero* the king, by him self, at ease, to remoue her, as he would. Wherat, the King wondring: *Από τούτης τῆς ἡμέρας, περί παντός, Αρχιμήδης λέγοντι πεισευτέρω.* From this day, forward (said the King) Credit ought to be giuen to *Archimedes*, what soeuer he sayth.

Pneumatithmie demonstrateth by close hollow Geometricall Figures, (regular and irregular) the straunge properties (in motion or stay) of the Water, Ayre, Smoke, and Fire, in theyr cōtinuitie, and

Saw Milles.

* *Athenens*
Lib. 5. cap. 8.

Proclus.
Pag. 18.

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and as they are ioyned to the Elementes next them. This Arte, to the Naturall Philosopher, is very profitable: to proue, that *Vacuum*, or *Emptines* is not in the world. And that, all Nature, abhorreth it so much: that, contrary to ordinary law, the Elementes will moue or stand. As, Water to ascend: rather then betwene him and Ayre, Space or place should be left, more then (naturally) that quantitie of Ayre requireth, or can fill. Againe, Water to hang, and not descend: rather then by descending, to leaue Emptines at his backe. The like, is of Fire and Ayre: they will descend: when, either, their Cōtinuitie should be dissolued: or their next Element forced from them. And as they will not be extended, to discontinuitie: So, will they not, nor yet of mans force, can be prest or pent, in space, not sufficient and aunswerable to their bodily substance. Great force and violence will they vse, to enioy their naturall right and libertie. Hereupon, two or three men together, by keping Ayre vnder a great Cauldron, and forcyng the same downe, orderly, may without harme descend to the Sea bottome: and continue there a tyme &c. Where, Note, how the thicker Element (as the Water) giueth place to the thynner (as, is the ayre:) and receiue violence of the thinner, in maner. &c. Pumps and all maner of Bellowes, haue their ground of this Art: and many other straunge deuises. As, *Hydraulica*, Organes goyng by water. &c. Of this Feat, (called commonly *Pneumatica*,) goodly workes are extant, both in Greke, and Latin. With old and learned Schole men, it is called *Scientia de pleno & vacuo*.

To go to the
bottom of the
Sea without
daunger.

Menadrie, is an Arte Mathematicall, which demonstrateth, how, aboue Natures vertue and power simple: Vertue and force may be multiplied: and so, to direct, to lift, to pull to, and to put or cast fro, any multiplied or simple, determined Vertue, Waight or Force: naturally, not, so, directible or moueable. Very much is this Art furdred by other Artes: as, in some pointes, by *Perspectiue*: in some, by *Statike*: in some, by *Trochilike*: and in other, by *Helicosophie*: and *Pneumatikie*. By this Art, all Cranes, Gybbettes, & Ingines to lift vp, or to force any thing, any maner way, are ordred: and the certaine cause of their force, is knowne. As, the force which one man hath with the Duche wagen Racke: therwith, to set vp agayne, a mighty wagen laden, being ouerthrowne. The force of the Crossebow Racke, is certainly, here, demonstrated. The reason, why one mā, doth with a leauer, lift that, which Sixe men, with their handes onely, could not, so easily do. By this Arte, in our common Cranes in London, where powre is to Crane vp, the waight of 2000. pound: by two Wheles more (by good order added) Arte concludeth, that there may be Craned vp 200000. pound waight &c. So well knew *Archimedes* this Arte: that he alone, with his deuises and engynes, (twise or thrise) spoyled and discomfited the whole Army and Hoste of the Romaines, besieging *Syracusa*, *Marcus Marcellus* the Consul, being their Generall Capitaine. Such huge Stones, so many, with such force, and so farre, did he with his engynes hayle among them, out of the Citie. And by Sea likewise: though their Ships might come to the walls of *Syracusa*, yet hee vtterly confounded the Romaine Nauye. What with his mighty Stones hurlyng: what with Pikes of* 18 fote long, made like shaftes: which he forced almost a quarter of a myle. What, with his catchyng hold of their Shyps, and hoysing them vp aboue the water, and suddenly letting them fall into the Sea againe: what with his* Burning Glasses: by which he fired their other Shippes a far-off: what, with his other pollicies, deuises, and engines, he so manfully acquit himselfe: that all the Force, courage, and pollicie of the Romaines (for a great season)

*Plutarchus in Mar-
co Marcello.
Synesius in Episto-
lis.
Polybius.
Plinius.
Quintilianus.
T. Liuius.
* Athenæus,*

* *Galenus.
Anthemius.*

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Burning
Glasses.

Gunnes.

could nothing preuaile, for the winning of Syracusa. Wherupon, the Romanes named *Archimedes*, *Briareus*, and *Centimanus*. *Zonaras* maketh mention of one *Proclus*, who so well had perceiued *Archimedes* Arte of *Menadrie*, and had so well inuented of his owne, that with his Burning Glasses, being placed vpon the walles of Byfance, he multiplied so the heate of the Sunne, and directed the beames of the same against his enemies Nauie with such force, and so sodeinly (like lightening) that he burned and destroyed both man and ship. And *Dion* specifieth of *Priscus*, a *Geometricien* in Byfance, who inuented and vsed sondry Engines, of Force multiplied: Which was cause, that the *Emperour Seuerus* pardoned him, his life, after he had wonne Byfance: Bycause he honored the Arte, wytt, and rare industrie of *Priscus*. But nothing inferior to the inuention of these engines of Force, was the inuention of Gunnes. Which, from an English man, had the occasion and order of first inuenting: though in an other land, and by other men, it was first executed. And they that should see the record, where the occasion and order generall, of Gunning, is first discoursed of, would thinke: that, small thinges, slight, and comon: comming to wise mens consideration, and industrious mens handling, may grow to be of force incredible.

Hypogeiodie, is an Arte Mathematicall, demonstratyng, how, vnder the Sphæricall Superficies of the earth, at any depth, to any perpendicular line assigned (whose distance from the perpendicular of the entrance: and the Azimuth, likewise, in respect of the said entrance, is knowen) certaine way may be præscribed and gone: And how, any way aboue the Superficies of the earth designed, may vnder earth, at any depth limited, be kept: goyng alwayes, perpendicularly, vnder the way, on earth designed: And, contrarywise, Any way, (straight or croked,) vnder the earth, beyng giuen: vppon the vtface, or Superficies of the earth, to Lyne out the same: So, as, from the Centre of the earth, perpendiculars drawen to the Sphæricall Superficies of the earth, shall precisely fall in the Correspondent pointes of those two wayes. This, with all other Cases and circumstances herein, and appertenances, this Arte demonstrateth. This Arte, is very ample in varietie of Conclusions: and very profitable sundry wayes to the Common Wealth. The occasion of my Inuenting this Arte, was at the request of two Gentlemen, who had a certaine worke (of gaine) vnder ground: and their groundes did ioine ouer the worke: and by reason of the crokednes, diuers depthes, and heithes of the way vnder ground, they were in doubt, and at controuersie, vnder whose ground, as then, the worke was. The name onely (before this) was of me published, *De Itinere Subterraneo*: The rest, be at Gods will. For Pioners, Miners, Diggers for Mettalls, Stone, Cole, and for secrete passages vnder ground, betwene place and place (as this land hath diuerse) and for other purposes, any man may easily perceauē, both the great fruite of this Arte, and also in this Arte, the great aide of Geometrie.

Hydragogie, demonstrateth the possible leading of Water, by Natures lawe, and by artificiall helpe, from any head (being a Spring, standing, or running Water) to any other place assigned.

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Long, hath this Arte bene in vse : and much thereof written : and very marueilous workes therein, performed : as may yet appeare, in Italy: by the Ruynes remaining of the Aqueductes . In other places, of Riuers leading through the Maine land, Nauigable many a Mile. And in other places, of the marueilous forcings of Water to Ascend . which all, declare the great skill, to be required of him, who should in this Arte be perfecte, for all occasions of waters possible leading . To speake of the allowance of the Fall, for euery hundred foote: or of the Ventills (if the waters labour be farre, and great) I neede not : Seing, at hand (about vs) many expert men can sufficiently testific, in effecte, the order : though the Demonstration of the Necessitie thereof, they know not : Nor yet, if they should be led, vp and downe, and about Mountaines, from the head of the Spring: and then, a place being assigned : and of them, to be demaunded, how low or high, that last place is, in respecte of the head, from which (so crokedly, and vp and downe) they be come: Perhaps, they would not, or could not, very redily, or nerely assoyle that question. Geometrie therefore, is necessary to *Hydragogie* . Of the sundry wayes to force water to ascend, cyther by *Tympane*, *Kettell mills*, *Skrue*, *Ctesibike*, or such like : in *Vitruuius*, *Agricola*, (and other,) fully, the maner may appeare . And so, thereby, also be most euident, how the Artes, of *Pneumatithmie*, *Helicosophie*, *Statike*, *Trochilike*, and *Menadrie*, come to the furniture of this, in Speculation, and to the Commoditie of the Common Wealth, in practise.

Horometrie, is an Arte Mathematicall, which demonstrateth, how, at all times appointed, the precise vsuall denominatiõ of time, may be knowen, for any place assigned . These wordes, are sinit and plaine easie Englishe, but the reach of their meaning, is farther, then you woulde lightly imagine . Some part of this Arte, was called in olde time, *Gnomonice*: and of late, *Horologiographia* : and in Englishe, may be termed, *Dialling* . Auncient is the vse, and more auncient, is the Inuention . The vse, doth well appeare to haue bene (at the least) aboue two thousand and three hundred yeare agoe : in * King *Achaz* Diall, then, by the Sunne, shewing the distinction of time . By Sunne, Mone, and Sterres, this Dialling may be performed, and the precise Time of day or night knowen . But the demonstratiue delineation of these Dials, of all sortes, requireth good skill, both of *Astronomie*, and *Geometrie* Elementall, Sphæricall, Phænomennall, and Conikall . Then, to vse the groundes of the Arte, for any regular Superficies, in any place offred : and (in any possible apt position therof) thereon, to describe (all maner of wayes) how, vsuall howers, may be (by the *Sunnes* shadow) truely determined : will be found no sleight Painters worke . So to Paint, and prescribe the *Sunnes* Motion, to the breadth of a heare . In this Feate (in my youth) I Inuented a way, How in any Horizontall, Murall, or *Æquinoctiall* Diall, &c. At all howers (the *Sunne* shining) the Signe and Degree ascendent, may be knowen . Which is a thing very necessary for the Rising of those fixed Sterres : whose Operation in the Ayre, is of great might, euidently . I speake no further, of the vse hereof . But forasmuch as, Mans affaires require knowledge of Times & Momentes, when, neither *Sunne*, *Mone*, or *Sterre*, can be sene: Therefore, by Industrie Mechanicall, was inuented, first, how, by Water, running orderly, the Time and howers might be knowen: whereof, the famous *Ctesibius*, was Inuentor : a man, of *Vitruuius*, to the Skie (iustly) extolled . Then, after that, by Sand running, were howers measured : Then, by *Trochilike* with waight : And of late time, by *Trochilike* with Spring : without waight . All these,

4. Reg. 20.

d.ij.

by

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*A perpetual
Motion.*

by Sunne or Sterres direction (in certaine time) require ouersight and reformation, according to the heauenly Æquinoctiall Motion : besides the inæqualitie of their owne Operation . There remayneth (without parabolicall meaning herein) among the Philosophers, a more excellent, more commodious, and more marueilous way, then all these : of hauing the motion of the Primouant (or first æquinoctiall motion,) by Nature and Arte, Imitated: which you shall (by furder search in waightier studyes) hereafter, vnderstand more of. And so, it is tyme to finish this Annotation, of Tymes distinction, vsed in our common, and priuate affaires: The commoditie wherof, no man would want, that can tell, how to bestow his tyme.

Zographie, is an Arte Mathematicall, which teacheth and demonstrateth, how, the Interfection of all visuall Pyramides, made by any playne assigned, (the Centre, distance, and lightes, beyng determined) may be, by lynes, and due propre colours, represented. A notable Arte, is this: and would require a whole Volume, to declare the property thereof: and the Commodities ensuyng. Great skill of *Geometrie*, *Arithmetike*, *Perspectiue*, and *Anthropographie*, with many other particular Artes, hath the *Zographer*, nede of, for his perfection. For, the most excellent Painter, (who is but the propre Mechanicien, & Imitator sensible, of the *Zographer*) hath attained to such perfection, that Sense of Man and beast, haue iudged thinges painted, to be things naturall, and not artificiall: aliue, and not dead. This Mechanicall *Zographer* (commonly called the Painter) is meruailous in his skill: and seemeth to haue a certaine diuine power: As, of frendes absent, to make a frendly, present comfort: yea, and of frendes dead, to giue a continuall, silent presence: not onely with vs, but with our posteritie, for many Ages. And so procedyng, Consider, How, in Winter, he can shew you, the liuely vew of Sommers Ioy, and riches: and in Sommer, exhibite the countenance of Winters dolefull State, and nakednes. Cities, Townes, Fortes, Woodes, Armyes, yea whole Kingdomes (be they neuer so farre, or greate) can he, with ease, bring with him, home (to any mans Iudgement) as Paternes liuely, of the thinges rehearsed. In one little house, can he, enclose (with great pleasure of the beholders,) the portrayture liuely, of all visible Creatures, either on earth, or in the earth, liuing: or in the waters lying, Creeping, flyding, or swimming: or of any foule, or fly, in the ayre flying. Nay, in respect of the Starres, the Skie, the Cloudes: yea, in the shew of the very light it selfe (that Diuine Creature) can he match our eyes Iudgement, most nerely. What a thing is this: thinges not yet being, he can represent so, as, at their being, the Picture shall seame (in maner) to haue Created them. To what Artificer, is not Picture, a great pleasure and Commoditie: Which of them all, will refuse the Direction and ayde of Picture: The Architect, the Goldsmith, and the Arras Weauer: of Picture, make great account. Our liuely Herbals, our portraitures of birdes, beastes, and fishes: and our curious Anatomies, which way, are they most perfectly made, or with most pleasure, of vs beholden? Is it not, by Picture onely? And if Picture, by the Industry of the Painter, be thus commodious and meruailous: what shall be thought of *Zographie*, the Scholemaster of Picture, and chief gouernor? Though I mencion not *Sculpture*, in my Table of Artes Mathematicall: yet may all men perceiue, How, that *Picture* and *Sculpture*, are Sisters germaine: and both, right profitable, in a Commō wealth, and of *Sculpture*, as well as of Picture, excellent Artificers haue written great bokes in commendation. Witnesse I take, of *Georgio Vasari*, *Pittore Aretino*: of *Pomponius Gauricus*: and other. To these two Artes, (with other,) is a certaine od Arte, called *Althamafat*, much beholdyng: more, then the common *Sculptor*, *Entayler*, *Keruer*, *Cutter*, *Grauer*, *Founder*,

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der, or Paynter (&c) know their Arte, to be commodious.

Architecture, to many may seeme not worthy, or not mete, to be reckned among the *Artes Mathematicall*. To whom, I thinke good, to giue some account of my so doying. Not worthy, (will they say,) bycause it is but for building, of a house, Pallace, Church, Forte, or such like, grosse workes. And you, also, defined the *Artes Mathematicall*, to be such, as dealt with no Materiall or corruptible thing: and also did demonstratiuely procede in their faculty, by Number or Magnitude. First, you see, that I count, here, *Architecture*, among those *Artes Mathematicall*, which are Deriued from the Principals: and you know, that such, may deale with Naturall thinges, and sensible matter. Of which, some draw nerer, to the Simple and absolute Mathematicall Speculation, then other do. And though, the *Architect* procureth, enformeth, & directeth, the *Mechanicien*, to handworke, & the building actuall, of house, Castell, or Pallace, and is chief Iudge of the same: yet, with him selfe (as chief *Master* and *Architect*,) remaineth the Demonstratiue reason and cause, of the *Mechanicien*s worke: in Lyne, plaine, and Solid: by *Geometricall*, *Arithmetickall*, *Opticall*, *Musickall*, *Astronomicall*, *Cosmographicall* (& to be brief) by all the former Deriued *Artes Mathematicall*, and other Naturall Artes, hable to be confirmed and stablished. If this be so: then, may you thinke, that *Architecture*, hath good and due allowance, in this honest Company of *Artes Mathematicall* Deriuatiue. I will, herein, craue Iudgement of two most perfect *Architectes*: the one, being *Vitruuius*, the Romaine: who did write ten bookes thereof, to the Emperour *Augustus* (in whose daies our Heauenly Archemaster, was borne): and the other, *Leo Baptista Albertus*, a Florentine: who also published ten bookes therof. *Architectura* (sayth *Vitruuius*) est *Scientia pluribus disciplinis & varijs eruditionibus ornata: cuius Iudicio probantur omnia, quæ ab ceteris Artificibus perficiuntur opera*. That is. Architecture, is a Science garnished with many doctrines & diuerse instructions: by whose Iudgement, all workes, by other workmen finished, are Iudged. It followeth. *Ea nascitur ex Fabrica, & Ratiocinatione. &c. Ratiocinatio autem est, quæ res fabricatas, Solertia ac ratione proportionis, demonstrare atq; explicare potest*. Architecture groweth of Framing, and Reasoning. &c. Reasoning, is that, which of thinges framed, with forecast, and proportion: can make demonstration, and manifest declaration. Againe. *Cum, in omnibus enim rebus, tum maxime etiam in Architectura, hæc duo insunt: quod significatur, & quod significat. Significatur proposita res, de qua dicitur: hanc autem Significat Demonstratio, rationibus doctrinarum explicata*. Forasmuch as, in all thinges: therefore chiefly in Architecture, these two thinges are: the thing signified: and that which signifieth. The thing propounded, whereof we speake, is the thing Signified. But Demonstration, expressed with the reasons of diuerse doctrines, doth signifie the same thing. After that. *Vt literatus sit, peritus Graphidos, eruditus Geometrie, & Optices non ignarus: instructus Arithmetica: historias complures nouerit, Philosophos diligenter audiuerit: Musicam sciuerit: Medicinæ non sit ignarus, responsa Iuris peritorum nouerit: Astrologiam, Calig. rationes cognitæ habeat*. An Architect (sayth he) ought to vnderstand Languages, to be skilfull of Painting, well instructed in Geometrie, not ignorant of Perspective, furnished with Arithmetike, haue knowledge of many histories, and diligently haue heard Philosophers, haue skill of Musike, not ignorant of Physike, know the aunsweres of Lawyers, and haue Astro-

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nomie, and the courses Cæleſtiall, in good knowledge. He geueth reason, orderly, wherefore all theſe Artes, Doctrines, and Inſtructions, are requiſite in an excellent *Architeēt*. And (for breuitie) omitting the Latin text, thus he hath. Secondly, it is behofefull for an *Architeēt* to haue the knowledge of Painting: that he may the more eaſilie faſhion out, in patternes painted, the forme of what worke he liketh. And Geometrie, geueth to *Architeēture* many helpes: and firſt teacheth the *Uſe* of the Rule, and the Cumpasse: wherby (chiefly and eaſilie) the deſcriptions of Buildinges, are deſpatched in Groundplats: and the direCTIONS of Squires, Leuells, and Lines. Likewise, by *Perspective*, the Lightes of the heauen, are well led, in the buildinges: from certaine quarters of the world. By *Arithmetike*, the charges of Buildinges are ſummed together: the meaſures are expreſſed, and the hard queſtions of Symmetries, are by Geometricall Meanes and Methods diſcourſed on. &c. Beſides this, of the Nature of thinges (which in Greke is called *φυσικολογία*) *Philophie* doth make declaration. Which, it is neceſſary, for an *Architeēt*, with diligence to haue learned: becauſe it hath many and diuers naturall queſtions: as ſpecially, in *Aqueduētes*. For in their courses, leadinges about, in the leuell ground, and in the mountinges, the naturall Spirites or breathes are ingendred diuers wayes: The hindrances, which they cauſe, no man can helpe, but he, which out of *Philophie*, hath learned the originall cauſes of thinges. Likewise, who ſoener ſhall read *Cteſibius*, or *Archimedes* bookes, (and of others, who haue written ſuch Rules) can not thinke, as they do: vnleſſe he ſhall haue receaued of *Philophers*, inſtructions in theſe thinges. And Muſike he muſt nedes know: that he may haue vnderſtanding, both of Regular and Mathematicall Muſike: that he may temper well his Baſiſtes, Catapultes, and Scorpions. &c. Moreouer, the Braſen Veſſels, which in Theatres, are placed by Mathematicall order, in ambries, vnder the ſteppes: and the diuerſities of the ſoundes (which y Greicians call *ἡρμία*) are ordred according to Muſicall Symphonies & Harmonies: being diſtributed in y Circuites, by *Diateſſaron*, *Diapente*, and *Diapaſon*. That the conuenient voyce, of the players ſound, whē it came to theſe preparations, made in order, there being increaſed: with y increaſing, might come more cleare & pleaſant, to y eares of the lokers on. &c. And of *Aſtronomie*, is knowē y Eaſt, Weſt, South, and North. The faſhion of the heauen, the *Æquinox*, the *Solſticie*, and the courſe of the ſterres. Which thinges, vnleaſt one know: he can not perceiue, any thyng at all, the reaſon of *Horologies*. Seyng therfore this ample Science, is garniſhed, beautified and ſtored, with ſo many and ſundry ſkils and knowledges: I thinke, that none can iuſtly account them ſelues *Architeētes*, of the ſuddeyne. But they onely, who from their childeſ yeares, aſcendyng by theſe degrees of knowledges, beyng foſtered vp with the atteynnyng of many Languages and Artes, haue wonne to the high Tabernacle of *Architeēture*. &c. And to whom Nature hath giuen ſuch quicke Circumſpection, ſharpnes of witt, and Memorie, that they may be very abſolutely ſkillfull in *Geometrie*, *Aſtronomie*, *Muſike*, and the reſt of the Artes Mathematicall.

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call: Such surmount and passe the calling, and state, of Architectes: and are become Mathematiciens: &c. And they are found, seldome. As, in tymes past, was Aristarchus Samius: Philolaus, and Archytas, Tarentynes: Apollonius Pergens: Eratosthenes Cyreneus: Archimedes, and Scopas, Syracusians. Who also, left to theyr posteritie, many Engines and Gnomonicall workes: by numbers and naturall meanes, inuented and declared.

Thus much, and the same wordes (in sense) in one onely Chapter of this Incōparable Architect Vitruuius, shall you finde. And if you should, but take his boke in your hand, and slightly loke thorough it, you would say straight way: This is Geometrie, Arithmetike, Astronomie, Musike, Anthropographie, Hydragogie, Horometrie. &c. and (to cōclude) the Storehouse of all workmāship. Now, let vs listen to our other Iudge, our Florentine, Leo Baptista: and narrowly consider, how he doth determine of Architecture. *Sed anteq̃ ultra progrediar. &c.* But before I procede any further (sayth he) I thinke, that I ought to expresse, what man I would haue to bee allowed an Architect. For, I will not bryng in place a Carpenter: as though you might Compare him to the Chief Masters of other Artes. For the hand of the Carpenter, is the Architectes Instrument. But I will appoint the Architect to be that man, who hath the skill, (by a certaine and meruailous meanes and way,) both in minde and Imagination to determine: and also in worke to finish: what workes so euer, by motion of waight, and cuppling and framynge together of bodies, may most aptly be Commodious for the worthiest Vses of Man. And that he may be able to performe these thinges, he hath nede of atteynyng and knowledge of the best, and most worthy thynges. &c. The whole Feate of Architecture in buildyng, consisteth in Lineamentes, and in Framynge. And the whole power and skill of Lineamentes, tendeth to this: that the right and absolute way may be had, of Coptyng and ioyning Lines and angles: by which, the face of the buildyng or frame, may be comprehended and concluded. And it is the property of Lineamentes, to prescribe vnto buildynges, and euery part of them, an apt place, & certaine nūber: a worthy maner, and a femely order: that, so, y whole forme and figure of the buildyng, may rest in the very Lineamentes. &c. And we may prescribe in mynde and imagination the whole formes, all materiall stuffe being secluded. Which point we shall atteyne, by Notyng and forepointyng the angles, and lines, by a sure and certaine direction and connexion. Seyng then, these thinges, are thus: Lineamente, shalbe the certaine and constant prescribing, conceiued in mynde: made in lines and angles: and finished with a learned minde and wyt. We thanke you Master Baptist, that you haue so aptly brought your Arte, and phraze therof, to haue some Mathematicall perfection: by certaine order, nūber, forme, figure, and Symmetrie mentall: all naturall & sensible stuffe set a part. Now, then, it is euident, (Gentle reader) how aptely and worthely, I haue preferred Architecture, to be bred and fostered vp in the Dominion of the percles Princeſſe; Mathematica: and to be a naturall Subiect of hers. And the name of Architecture, is of the principalitie, which this Science hath, aboue all other Artes. And Plato affirmeth, the Architect to be Master ouer all, that make any worke. Whereupon, he is neither Smith, nor Builder: nor, separately, any Artificer: but the

A Mathematicien.

Vitruuius.

Who is an Architect.

* The Immaterialitie of perfect Architecture.

What, Lineament is.

Note.

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Hed, the Prouest, the Director, and Iudge of all Artificiall workes, and all Artificers. For, the true *Architect*, is hable to teach, Demonstrate, distribute, describe, and Iudge all workes wrought. And he, onely, searcheth out the causes and reasons of all Artificiall thynges. Thus excellent, is *Architecture*: though few (in our dayes) attayne thereto: yet may not the Arte, be otherwise thought on, then in very dede it is worthy. Nor we may not, of auncient Artes, make new and imperfect Definitions in our dayes: for scarcitie of Artificers: No more, than we may pynche in, the Definitions of *Wisedome*, or *Honestie*, or of *Friendeshyp* or of *Iustice*: No more will I consent, to Diminish any whit, of the perfection and dignitie; (by iust cause) allowed to absolute *Architecture*. Vnder the Direction of this Arte, are thre principall, necessary *Mechanicall Artes*. Namely, *Howsing*, *Fortification*, and *Naupegie*. *Howsing*, I vnderstand, both for Diuine Seruice, and Mans common vsage: publike, and priuate. Of *Fortification* and *Naupegie*, straunge matter might be told you: But perchaunce, some will be tyred, with this Bederoll, all ready rehearsed: and other some, will nycely nip my grosse and homely discoursing with you: made in post hast: for feare you should wante this true and frendly warnyng, and tast giuyng, of the *Power Mathematicall*. Lyfe is short, and vncertaine: Tymes are perilouse: &c. And still the Printer awayting, for my pen staying: All these thinges, with farder matter of Ingratefulnes, giue me occasion to passe away, to the other Artes remainyng, with all speede possible.

THE Arte of *Nauigation*, demonstrateth how, by the shortest good way, by the aptest Directiō, & in the shortest time, a sufficient Ship, betwene any two places (in passage Nauigable,) assigned: may be cōducted: and in all stormes, & naturall disturbances chauncyng, how, to vse the best possible meanes, whereby to recouer the place first assigned. What nede, the *Master Pilote*, hath of other Artes, here before recited, it is easie to know: as, of *Hydrographie*, *Astronomie*, *Astrologie*, and *Horometrie*. Presupposing continually, the common Base, and foundation of all: namely, *Arithmetike* and *Geometrie*. So that, he be hable to vnderstand, and Iudge his own necessary Instrumentes, and furniture Necessary: Whether they be perfectly made or no: and also can, (if nede be) make them, hym selfe. As Quadrantes, The Astronomers Ryng, The Astronomers staffe, The Astrolabe vniuersall. An Hydrographicall Globe. Charts Hydrographicall, true, (not with parallel Meridians). The Common Sea Compas: The Compas of variacion: The Proportionall, and Paradoxall Compasses (of me Inuented, for our two Moscouy Master Pilotes, at the request of the Company) Clockes with spryng: houre, halfe houre, and three houre Sandglassses: & sundry other Instrumētes: And also, be hable, on Globe, or Playne to describe the Paradoxall Compasse: and duely to vse the same, to all maner of purposes, whereto it was inuented. And also, be hable to Calculate the Planetes places for all tymes.

Moreouer, with Sonne Mone or Sterre (or without) be hable to define the Longitude & Latitude of the place, which he is in: So that, the Longitude & Latitude of the place, from which he sayled, be giuen: or by him, be knowne. whereto, appertayneth expert meanes, to be certified euer, of the Ships way. &c. And by foreseeing the Rising, Settyng, Nonestedyng, or Midnightyng of certaine tempestuous fixed Sterres: or their Coniunctions, and Anglynges with the Planetes, &c. he ought to haue expert coniecture of Stormes, Tempestes, and Spoutes: and such lyke Meteorologicall effectes, daungerous on Sea. For (as *Plato* sayth,) *Mutationes,*

oppor-

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opportunitatesq; temporum presentire, non minus rei militari, quàm Agricultura, Navigationiq; conuenit. To foresee the alterations and opportunities of tymes, is conuenient, no lesse to the Art of Warre, then to Husbandry and Nauigation. And besides such cunningg meanes, more euident tokens in Sonne and Mone, ought of hym to be knowen: such as (the Philosophicall Poëte) Virgilius teacheth, in hys Georgikes. Where he sayth,

*Sol quoq; & exoriens & quum se condet in undas,
Signa dabit, Solem certissima signa sequuntur. &c.*

*Nam sæpe videmus,
Ipsius in vultu varios errare colores.*

Ceruleus, pluuiam denunciat, igneus Euros.

Sin macula incipient rutilo immiscerier igni,

Omnia tum pariter vento, nimbisq; videbis

Feruere: non illa quisquam me nocte per altum

Ire, neq; a terra moueat conuellere funem. &c.

Sol tibi signa dabit. Solem quis dicere falsum

Audeat: ————— &c.

Georgic. I.

And so of Mone, Sterres, Water, Ayre, Fire, Wood, Stones, Birdes, and Beastes, and of many thynges els, a certaine Sympathicall forewarnyng may be had: sometymes to great pleasure and proffit, both on Sea and Land. Sufficiently, for my present purpose, it doth appeare, by the premisses, how *Mathematicall*, the *Arte of Nauigation*, is: and how it nedeth and also vseth other *Mathematicall Artes*: And now, if I would go about to speake of the manifold Commodities, commyng to this Land, and others, by Shypps and *Nauigation*, you might thinke, that I catch at occasions, to vse many wordes, where no nede is.

Yet, this one thyng may I, (iustly) say. In *Nauigation*, none ought to haue greater care, to be skillfull, then our English Pylotes. And perchaunce, Some, would more attempt: And other Some, more willingly would be aydyng, if they wist certainly, What Priuiledge, God had endued this Iland with, by reason of Situation, most commodious for *Nauigation*, to Places most Famous & Riche. And though, (of* Late) a young Gentleman, a Courragious Capitaine, was in a great readiness, with good hope, and great causes of persuation, to haue ventured, for a Discouerye, (either *Westerly*, by *Cape de Paramantia*: or *Esterly*, aboue *Noua Zemla*, and the *Cyremisses*) and was, at the very nere tyme of Attemptyng, called and employed otherwise (both then, and since,) in great good seruice to his Countrey, as the Irish Rebels haue* tasted: Yet, I say, (though the same Gentleman, doo not hereafter, deale therewith) Some one, or other, should listen to the Matter: and by good aduise, and discrete Circumspection, by little, and little, wyne to the sufficient knowledge of that Trade and Voyage: Which, now, I would be sory, (through Carelesnesse, want of Skill, and Courrage,) should remayne Vnknowne and vnheard of. Seyng, also, we are herein, halfe Challenged, by the learned, by halfe request, published. Therof, verely, might grow Commoditye, to this Land chiefly, and to the rest of the Christen Common wealth, farre passing all riches and worldly Threasure.

** Anno. 1567
S. H. G.*

** Anno. 1569*

Thaumaturgike, is that Art Mathematicall, which giueth certaine order to make straunge workes, of the sense to be perceiued, and of men greatly to be wondred at. By sundry meanes, this *Wonderworke* is wrought. Some, by *Pneumatithmie*. As the workes of *Ctesibius* and *Hero*,

A.j.

Some

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Some by waight: wherof *Timaeus* speaketh. Some, by Stringes strayned, or Springs, therewith Imitating liuely Motions. Some, by other meanes, as the Images of Mercurie: and the brasen hed, made by *Albertus Magnus*, which dyd seme to speake. *Boethius* was excellent in these feates. To whom, *Cassiodorus* writyng, sayth. *Your purpose is to know profound thynges: and to shew meruayles.* By the disposition of your *Arte*, Metals do low: *Diomedes* of brasse, doth blow a Trumpet loude: a brasen Serpent hisseth: byrdes made, sing sweetely. Small thynges we rehearse of you, who can Imitate the heauen. &c.

* Anno. 1551

Of the straunge Selfmouyng, which, at Saint Denys, by Paris, * I saw, ones or twise (*Orontius* beyng then with me, in Company) it were to straunge to tell. But some haue written it. And yet, (I hope) it is there, of other to be sene. And by *Perspectiue* also straunge thinges, are done. As partly (before) I gaue you to vnderstand in *Perspectiue*. As, to see in the Ayre, a loft, the lyuely Image of an other man, either walkyng to and fro: or standyng still. Likewise, to come into an house, and there to see the liuely shew of Gold, Siluer or precious stones: and commyng to take them in your hand, to finde nought but Ayre. Hereby, haue some men (in all other matters counted wise) fouly ouershot the selues: misdeaming of the meanes. Therefore sayd *Claudius Celestinus*. *Hodie magna literaturae uis & magnae reputationis uidemus, opera quaedam quasi miranda, supra Naturā putare: de quibus in Perspectiua doctus causam facilliter reddidisset.* That is. Now a dayes, we see some men, yea of great learnyng and reputation, to Iudge certain workes as meruaylous, aboue the power of Nature: Of which workes, one that were skillfull in *Perspectiue* might easely haue giuen the Cause. Of *Archimedes*

*De his qua
Mundo mi-
rabiliter eue-
niunt. cap. 8.*

Tusc. 1.

Sphere, *Cicero* witnesseth. Which is very straunge to thinke on. For when *Archimedes* (sayth he) did fasten in a Sphere, the mouynges of the Sonne, Mone, and of the fine other Planets, he did, as the God, which (in *Timaeus* of Plato) did make the world. That, one turnyng, should rule motions most vnlike in slownes, and swiftnes. But a greater cause of meruayling we haue by *Claudianus* report hereof. Who affirmeth this *Archimedes* worke, to haue ben of Glasse. And discourseth of it more at large: which I omit. The Doue of wood, which the Mathematicien *Archytas* did make to flye, is by *Agellius* spoken of. Of *Daedalus* straunge Images, Plato reporteth. *Homere* of *Vulcans* Selfmouers, (by secrete wheles) leaueth in writyng. *Aristotle*, in hys *Politikes*, of both, maketh mention. Meruaylous was the workmanship, of late dayes, performed by good skill of *Trochilike*. &c. For in Noremberge, A flye of Iern, beyng let out of the Artificers hand, did (as it were) fly about by the gestes, at the table; and at length, as though it were weary, retourne to his masters hand agayne. Moreouer, an Artificiall Egle, was ordred, to fly out of the same Towne, a mighty way, and that a loft in the Ayre, toward the Emperour comming thether: and followed hym, beyng come to the gate of the towne. * Thus, you see, what, *Arte Mathematicall* can performe, when Skill, will, Industry, and Hability, are duely applyed to profe.

✎ *

*A Digression.
Apologeticall.*

AND for these, and such like marueilous Actes and Feates, Naturally, Mathematically, and Mechanically, wrought and contriued: ought any honest Student, and Modest Christian Philosopher, be counted, & called a Coniurer? Shall the folly of Idiotes, and the Mallice of the Scornfull, so much preuaile, that He, who seeketh no worldly gaine or glory at their handes: But onely, of God, the threasor of heauenly wisedome, & knowledge of pure veritie: Shall he (I say) in the meane

space,

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space, be robbed and spoiled of his honest name and fame : He that seeketh (by S. Pauls aduertisement) in the Creatures Properties, and wonderfull vertues, to finde iuste cause, to glorifie the Æternall, and Almighty Creator by : Shall that man, be (in hugger mugger) condemned, as a Companion of the Helhoundes, and a Caller, and Coniurer of wicked and damned Spirites : He that bewaileth his great want of time, sufficient (to his contentation) for learning of Godly wisdom, and Godly Verities in : and onely therein setteth all his delight : Will that mā leese and abuse his time, in dealing with the Chiefe enemy of Christ our Redemer: the deadly foe of all mankind : the subtile and impudent peruerter of Godly Veritie: the Hypocriticall Crocodile : the Enuious Basiliske, continually desirous, in the twinke of an eye, to destroy all Mankind, both in Body and Soule, æternally : Surely (for my part, somewhat to say herein) I haue not learned to make so brutish, and so wicked a Bargaine. Should I, for my xx. or xxv. yeares Studie : for two or three thousand Markes spending : seuen or eight thousand Miles going and trauailing, onely for good learninges sake : And that, in all maner of wethers : in all maner of waies and passages : both early and late : in daunger of violence by man : in daunger of destruction by wilde beastes : in hunger : in thirst : in perillous heates by day, with toyle on foote : in dangerous dampes of colde, by night, almost bereuing life : (as God knoweth) : with lodgings, oft times, to small ease : and sometime to lesse securitie. And for much more (then all this) done & suffred, for Learning and attaining of Wisedome : Should I (I pray you) for all this, no otherwise, nor more warily : or (by Gods mercifulnes) no more luckily, haue fished, with so large, and costly, a Nette, so long time in drawing (and that with the helpe and aduise of Lady Philosophie, & Queene Theologie) : but at length, to haue caught, and drawen vp, * a Frog : Nay, a Deuill : For, so, doth the Common peuishe Pratler. Imagine and Iangle: And, so, doth the Malicious skorner, secretly wishe, & brauely and boldly face down, behinde my backe. Ah, what a miserable thing, is this kinde of Men : How great is the blindnes & boldnes, of the Multitude, in thinges aboue their Capacitie : What a Land : what a People : what Maners : what Times are these : Are they become Deuils, them selues: and, by false witnesse bearing against their Neighbour, would they also, become Murderers : Doth God, so long geue them respite, to reclaime them selues in, from this horrible slaundering of the guiltlesse : contrary to their owne Consciences : and yet will they not cease : Doth the Innocent, forbear the calling of them, Iuridically to aunswere him, according to the rigour of the Lawes : and will they despise his Charitable pacience : As they, against him, by name, do forge, fable, rage, and raise slaunder, by Worde & Print: Will they prouoke him, by worde and Print, likewise, to Note their Names to the World : with their particular deuises, fables, beastly Imaginations, and vnchristenlike slaunders : Well : Well. O (you such) my vnkinde Countrey men. O vn-naturall Countrey men. O vnthankfull Countrey men. O Brainicke, Rashe, Spitefull, and Disdainfull Countrey men. Why oppresse you me, thus violently, with your slaundering of me : Contrary to Veritie : and contrary to your owne Consciences : And I, to this hower, neither by worde, deede, or thought, haue bene, any way, hurtfull, damageable, or iniurious to you, or yours : Haue I, so long, so dearly, so farre, so carefully, so painfully, so daungerously sought & trauailed for the learning of Wisedome, & atteyning of Vertue : And in the end (in your iudgement) am I become, worse, then when I begā : Worse, thē a Mad man? A dangerous Member in the Common Wealth: and no Member of the Church of Christ? Call you this, to be Learned? Call you this, to be a Philosopher? and a louer of Wisedome? To forsake the straight heavenly way : and to wallow in the broad way of

* A prouerb.
Fayre fisht,
and caught a
Frog.

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damnation? To forsake the light of heauenly Wisedome: and to lurke in the dungeon of the Prince of darkenesse? To forsake the Veritie of God, & his Creatures: and to fawne vpon the Impudent, Craftie, Obstinate Lier, and continuall disgracer of Gods Veritie, to the vttermoſt of his power? To forsake the Life & Blisse Æternall: and to cleaue vnto the Author of Death euerlasting? that Murderous Tyrant, most greedily awaiting the Pray of Mans Soule? Well: I thanke God and our Lorde Iesus Christ, for the Comfort which I haue by the Examples of other men, before my time: To whom, neither in godlines of life, nor in perfection of learning, I am worthy to be compared: and yet, they sustained the very like Iniuries, that I do: or rather, greater. Patient *Socrates*, his *Apologie* will testifie: *Apuleius* his *Apologies*, will declare the Brutishnesse of the Multitude. *Ioannes Picus*, Earle of Mirandula, his *Apologie* will teach you, of the Raging slaunder of the Malicious Ignorant against him. *Ioannes Trithemius*, his *Apologie* will specifie, how he had occasion to make publike Protestation: as well by reason of the Rude Simple: as also, in respect of such, as were counted to be of the wisest sort of men. Ma-

- ny could I recite: But I deferre the precise and determined handling of this matter: being loth to detect the Folly & Mallice of my Natiue Countrey men.* Who, so hardly, can digest or like any extraordinary course of Philosophicall Studies: not falling within the Cumpasse of their Capacitie: or where they are not made priuie of the true and secrete cause, of such wonderfull Philosophicall Feates. These men, are of fower sortes, chiefly. The first, I may name, *Vaine pratling busie bodies*: The second, *Fond Friends*: The third, *Imperfectly zelous*: and the fourth, *Malicious Ignorant*. To eche of these (briefly, and in charitie) I will say a word
1. or two, and so returne to my Præface. *Vaine pratling busie bodies*, vse your idle assemblies, and conferences, otherwise, then in talke of matter, either aboue your Capacities, for hardnesse: or contrary to your Consciences, in Veritie. *Fond Friends*, leaue of, so to commend your vnacquainted friend, vpon blinde affection: As, because he knoweth more, then the common Student: that, therefore, he must needes be skilfull, and a doer, in such matter and maner, as you terme *Coniuring*. Weening, thereby, you aduaunce his fame: and that you make other men, great marueilers of your hap, to haue such a learned friend. Cease to ascribe Impietie, where you preterid Amitie. For, if your tounge were true, then were that your friend, *Vntrue*, both to God, and his Soueraigne. Such *Friends* and *Fondlinges*, I shake of, and renounce you: Shake you of, your Folly. *Imperfectly zelous*, to you,
 3. do I say: that (perhaps) well, do you Meane: But farre you misse the Marke: If a Lambe you will kill, to feede the flocke with his bloud. Sheepe, with Lambes bloud, haue no naturall sustenance: No more, is Christes flocke, with horrible slaunders, duely ædified. Nor your faire pretense, by such rashe ragged Rhetorike, any whit, well graced. But such, as so vse me, will finde a fowle Cracke in their Credite. Speake that you know: And know, as you ought: Know not, by Heare say, when life lieth in daunger. Search to the quicke, & let Charitie be your guide.
 4. *Malicious Ignorant*, what shall I say to thee? *Prohibe linguam tuam a malo. A detractione parcite lingua. Cause thy tounge to refraine fro euill. Refraine your tounge from slaunder*. Though your tounge be sharpned, Serpent like, & Adders poison lye in your lippes: yet take heede, and thinke, betimes, with your selfe, *Vir linguosus non stabilietur in terra. Virum violentum venabitur malum, donec precipitetur*. For, sure I am, *Quia faciet Dominus Iudicium afflicti: & vindictam pauperum*.

Psal. 140.

Thus, I require you, my assured friendes, and Countrey men (you Mathematiciens, Mechaniciens, and Philosophers, Charitable and discrete) to deale in my behalfe,

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behalf, with the light & vnttrue tounge, my enuious Aduersaries, or Fond friends. And farther, I would wishe, that at leysor, you would consider, how *Basilius Magnus*, layeth *Moses* and *Daniel*; before the eyes of those, which count all such Studies Philosophicall (as mine hath bene) to be vngodly, or vnprofitable. Waye well *S. Stephen* his witnesse of *Moses*. *Eruditus est Moses omni Sapientia Egyptiorū: & erat potens in verbis & operibus suis.* *Moses* was instructed in all maner of wisedome of the *Ægyptians*: and he was of power both in his wordes, and workes. You see this Philosophicall Power & Wisedome, which *Moses* had, to be nothing misliked of the Holy Ghost. Yet *Plinius* hath recorded, *Moses* to be a wicked *Magicien*. And that (of force) must be, either for this Philosophicall wisedome, learned, before his calling to the leading of the Children of *Israel*: or for those his wonders, wrought before King *Pharao*, after he had the conducting of the *Israelites*. As concerning the first, you perceave, how *S. Stephen*, at his Martyrdome (being full of the Holy Ghost) in his Recapitulation of the olde Testament, hath made mention of *Moses* Philosophie: with good liking of it: And *Basilius Magnus* also, auoucheth it, to haue bene to *Moses* profitable (and therefore, I say, to the Church of God, necessary). But as cōcerning *Moses* wonders, done before King *Pharao*: God, him selfe, sayd: *Vide ut omnia ostenta, quæ posui in manu tua, facias coram Pharaone.* See that thou do all those wonders before *Pharao*, which I haue put in thy hand. Thus, you evidently perceave, how rashly, *Plinius* hath slandered *Moses*, of vayne fraudulent *Magike*, saying: *Est & alia Magices Factio, a Mose, Iamne, & Iotape, Iudeis pendens: sed multis millibus annorum post Zoroastrem.* &c. Let all such, therefore, who, in Iudgement and Skill of Philosophie, are farre Inferior to *Plinie*, take good heede, least they ouershoot them selues rashly, in Iudging of *Philosophers* *straunge Actes*: and the Meanes, how they are done. But, much more, ought they to beware of forging, deuising, and imagining monstrous feates, and wonderfull workes, when and where, no such were done: no, not any sparke or likelihode, of such, as they, without all shame, do report. And (to conclude) most of all, let them be ashamed of Man; and afraide of the dreadfull and Iuste Iudge: both Follishly or Maliciously to deuise: and then, deuilishly to father their new fond Monsters on me: Innocent, in hand and hart: for trespassing either against the lawe of God, or Man, in any my Studies or Exercises, Philosophicall, or Mathematicall: As in due time, I hope, will be more manifest.

Now end I, with *Archemastrie*. Which name, is not so new, as this Arte is rare. For an other Arte, vnder this, a degree (for skill and power) hath bene indued with this English name before. And yet, this, may serue for our purpose, sufficiently, at this present. This Arte, teacheth to bryng to actuall experience sensible, all worthy conclusions by all the Artes Mathematicall purposed; & by true Naturall Philosophie concluded: & both addeth to them a farder scope, in the termes of the same Artes, & also by hys propre Method, and in peculier termes, procedeth, with helpe of the foresayd Artes, to the performance of complet Experiēces, which of no particular Art, are hable (Formally) to be challenged. If you remember, how we considered *Architecture*, in respect of all common handworkes: some light may you haue, therby, to vnderstand the Souerainty and propertie of this Science. Science I may call it, rather, then an Arte: for the excellency and Mastershyp it hath, ouer so many, and so mighty Artes and

Art. 7. C.

Lib. 30.

Cap. 1.

1.

2.

3.

2.

3.

A. iij.

Sciences.

John Dee his Mathematicall Præface.

Sciences. And bycause it procedeth by *Experiences*, and searcheth forth the causes of Conclusions, by *Experiences*: and also putteth the Conclusions them selues, in *Experience*, it is named of some, *Scientia Experimentalis*. The *Experimentall Science*. *Nicolaus Cusanus* termeth it so, in hys *Experimentes Statikall*. And an other *Philosopher*, of this land Natiue (the floure of whose worthy fame, can neuer dye nor wither) did write therof largely, at the request of *Clement the sixth*. The Arte carrieth with it, a wonderfull Credit: By reason, it certifieth, sensibly, fully, and completely to the vtmost power of Nature, and Arte. This Arte, certifieth by *Experience* complete and absolute: and other Artes, with their Argumentes, and Demonstrations, persuaide: and in wordes, proue very well their Conclusions. * But wordes, and Argumentes, are no sensible certifying: nor the full and finall frute of Sciences practisable. And though some Artes, haue in them, *Experiences*, yet they are not complete, and brought to the vttermost, they may be stretched vnto, and applyed sensibly. As for example: the Naturall *Philosopher* disputeth and maketh goodly shew of reason: And the *Astronomer*, and the *Opticall Mechanicien*, put some thynges in *Experience*: but neither, all, that they may: nor yet sufficiently, and to the vtmost, those, which they do. There, then, the *Archemaster* steppeth in, and leadeth forth on, the *Experiences*, by order of his doctrine *Experimentall*, to the chief and finall power of Naturall and Mathematicall Artes. Of two or three men, in whom, this Description of *Archemastrie* was *Experimentally*, verified, I haue read and hard: and good record, is of their such perfection. So that, this Art, is no fantastickall Imagination: as some Sophister, might, *Cum suis Insolubilibus*, make a flourish: and daseill your Imagination: and dash your honest desire and Courage, from beleuing these thinges, so vnheard of, so meruaylous, & of such Importance. Well: as you will. I haue forewarned you. I haue done the part of a frende: I haue discharged my Duety toward God: for my small Talent, at hys most mercyfull handes receiued. To this Science, doth the *Science Alnirangiat*, great Seruice. Muse nothyng of this name. I chaunge not the name, so vsed, and in Print published by other: beyng a name, propre to the Science. Vnder this, commeth *Ars Sintrillia*, by *Artephius*, briefly written. But the chief Science, of the *Archemaster*, (in this world) as yet knowen, is an other (as it were) OPTICAL Science: wherof, the name shall be told (God willyng) when I shall haue some, (more iust) occasion, therof, to Discourse.

Here, I must end, thus abruptly (Gentle frende, and vnfayned louer of honest and necessary verities.) For, they, who haue (for your sake, and vertues cause) requested me, (an old forworne Mathematicien) to take pen in hand: (through the confidence they reposed in my long experience: and tryed sincerity) for the declaring and reportyng somewhat, of the frute and commodity, by the Artes Mathematicall, to be atteyned vnto: euen they, Sore agaynst their willes, are forced, for sundry causes, to satisfie the workemans request, in endyng forthwith: He, so feareth this, so new an attempt, & so costly: And in matter so slenderly (hitherto) among the common Sorte of Studentes, considered or esteemed.

And where I was willed, somewhat to alledge, why, in our vulgare Speche, this part of the Principall Science of *Geometrie*, called *Euclides Geometricall Elementes*, is published, to your handlyng: being vnlatined people, and not Vniuersitie Scholers: Verily, I thinke it nedelesse.

I. For, the Honour, and Estimation of the Vniuersities, and Graduates, is, hereby, nothing diminished. Seing, from, and by their Nurse Children, you receaue all this Benefite: how great soeuer it be.

Neither

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Neither are their Studies, hereby, any whit hindred. No more, then the Italian *Vniuersities*, as *Academia Bononiensis, Ferrariensis, Florentina, Mediolanensis, Patauina, Papiensis, Perusina, Pisana, Romana, Senensis*, or any one of them, finde them selues, any deale, disgraced, or their Studies any thing hindred, by *Frater Lucas de Burgo*, or by *Nicolaus Tartalea*, who in vulgar Italian language, haue published, not onely *Euclides Geometrie*, but of *Archimedes* somewhat: and in Arithmetike and Practicall Geometrie, very large volumes, all in their vulgar speche. Nor in Germany haue the famous *Vniuersities*, any thing bene discontent with *Albertus Durerus*, his Geometricall Institutions in Dutch: or with *Gulielmus Xylander*, his learned translation of the first sixe bookes of *Euclide*, out of the Greke into the high Dutch. Nor with *Gualterus H. Riffius*, his Geometricall Volume: very diligently translated into the high Dutch tounge, and published. Nor yet the *Vniuersities* of Spaine, or Portugall, thinke their reputation to be decayed: or suppose any their Studies to be hindred by the Excellent *P. Nonnius*, his Mathematicall workes, in vulgare speche by him put forth. Haue you not, likewise, in the French tounge, the whole Mathematicall Quadriuie? and yet neither Paris, Orleance, or any of the other *Vniuersities* of Fraunce, at any time, with the Translators, or Publishers offended: or any mans Studie thereby hindred?

And surely, the Common and Vulgar Scholer (much more, the Gramarian) before his comming to the *Vniuersitie*, shall (or may) be, now (according to *Plato* his Counsell) sufficiently instructed in *Arithmetike* and *Geometrie*, for the better and easier learning of all maner of *Philosophie, Academicall, or Peripateticall*. And by that meanes, goe more cherefully, more skilfully, and spedily forward, in his Studies, there to be learned. And, so, in lesse time, profite more, then (otherwise) he should, or could do.

Also many good and pregnant Englishe wittes, of young Gentlemen, and of other, who neuer intend to meddle with the profound search and Studie of *Philosophie* (in the *Vniuersities* to be learned) may neuerthelesse, now, with more ease and libertie, haue good occasion, vertuously to occupie the sharpnesse of their wittes: where, els (perchance) otherwise, they would in fond exercises, spend (or rather leese) their time: neither seruing God: nor furduring the Weale, common or priuate.

And great Comfort, with good hope, may the *Vniuersities* haue, by reason of this *Englishe Geometrie, and Mathematicall Præface*, that they (hereafter) shall be the more regarded, esteemed, and resorted vnto. For, when it shall be knownen and reported, that of the *Mathematicall Sciences* onely, such great Commodities are ensuing (as I haue specified): and that in dede, some of you vnlained Studentes, can be good witnesse, of such rare fruite by you enioyed (thereby): as either, before this, was not heard of: or els, not so fully credited: Well, may all men coniecture, that farre greater ayde, and better furniture, to winne to the Perfection of all *Philosophie*, may in the *Vniuersities* be had: being the Storehouses & Threasury of all Sciences, and all Artes, necessary for the best, and most noble State of Common Wealthes.

Besides this, how many a Common Artificer, is there, in these Realmes of England and Ireland, that dealeth with Numbers, Rule, & Cumpasse: Who, with their owne Skill and experience, already had, will be hable (by these good helpes and informations) to finde out, and deuise, new workes, straunge Engines, and Instrumentes: for sundry purposes in the Common Wealth? or for priuate pleasure? and for the better maintayning of their owne estate? I will not (therefore)

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fight against myne owne shadowe. For, no man (I am sure) will open his mouth against this Enterprise. No mā (I say) who either hath Charitie toward his brother (and would be glad of his furtherance in vertuous knowledge) : or that hath any care & zeale for the bettering of the Cōmon state of this Realme. Neither any, that make accompt, what the wiser sort of men (Sage and Stayed) do thinke of them. To none (therefore) will I make any *Apologie*, for a vertuous acte doing : and for cōmending, or setting forth, Profitable Artes to English men, in the English tounge.

„ But, vnto God our Creator, let vs all be thankefull : for that, *As he, of his Good-*
„ *nes, by his Powre, and in his wisdom, hath Created all thynges, in Number,*
„ *Waight, and Measure*: So, to vs, of hys great Mercy, he hath reuealed Meanes,
„ whereby, to attayne the sufficient and necessary knowledge of the foresayd hys
„ three principall Instrumentes : Which Meanes, I haue abundantly proued vnto
„ you, to be the *Sciences and Artes Mathematicall*.

And though I haue ben pinched with straightsnes of tyme: that, no way, I could so pen downe the matter (in my Mynde) as I determined: hopng of conuenient laysure. Yet, if vertuous zeale, and honest Intent prouoke and bryng you to the readyng and examinyng of this Compendious treatise, I do not doute, but, as the veritie therof (accordng to our purpose) will be euident vnto you : So the pith and force therof, will persuaide you : and the wonderfull frute therof, highly pleasure you. And that you may the easier perceiue, and better remember, the principall pointes, whereof my Preface treateth, I will giue you the Groundplatt of my whole discourse, in a Table annexed: from the first to the last, somewhat Methodically contriued.

*The Ground
platt of this
Preface in a
Table.*

If Haft, hath caused my poore pen, any where, to stumble : You will, (I am sure) in part of recompence, (for my earnest and sincere good will to pleasure you), Consider the rockish huge mountaines, and the perilous vnbeaten wayes, which (both night and day, for the while) it hath toyled and labored through, to bryng you this good Newes, and Comfortable profe, of Vertues frute.

So, I Commit you vnto Gods Mercyfull direction, for the rest : hartely besechyng hym, to prosper your Studyes, and honest Intentes: to his Glory, & the Commodity of our Countrey. *Amen.*

*Written at my poore House
At Mortlake.*

Anno. 1579. Februarij. 9.

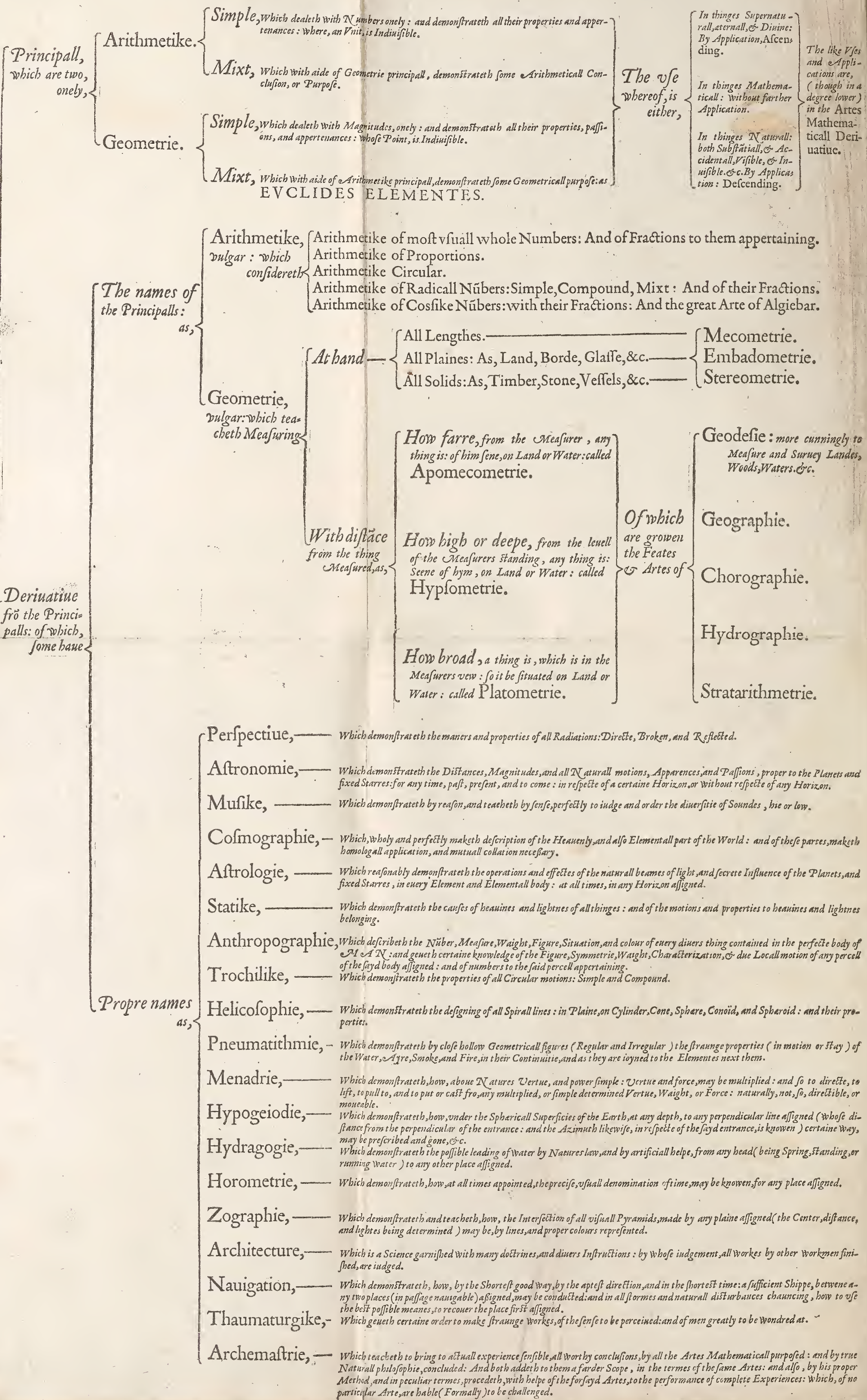


The first booke of Eu-

clid's principles of geometry, in which are contained the demonstrations of all the propositions in the first six books of Euclid's Elements.

Here haue you (according to my promise) the Groundplat of my MATHEMATICALL Praeface: annexed to *Euclide* (now first) published in our Englishetounge. An. 1570. Febr. 3.

Sciences,
and Artes
Mathe-
maticall,
are, either



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fight against myne owne shadowe. For, no man (I am sure) will open his mouth
against this Enterprise. No mā (I say) who either hath Charitie toward his brother

The first booke of Eu- clides Elementes.



IN THIS FIRST BOOKE is intreated of the most simple, easie, and first matters and groundes of Geometry, as, namely, of Lynes, Angles, Triangles, Parallels, Squares, and Parallelogrammes. First of their definitions, shewyng what they are. After that it teacheth how to draw Parallel lynes, and how to forme diuersly figures of three sides, & foure sides, according to the varietie of their sides, and Angles : & cōpareth them all with Triangles, & also together the one with the other. In it also is taught how a figure of any forme may be chaunged into a Figure of an other forme. And for that it entreateth of these most com-

*The argument
of the first
Booke.*

mon and generall thynges, thys booke is more vniuersall then is the seconde, third, or any other, and therefore iustly occupieth the first place in order : as that without which, the other bookes of *Euclide* which follow, and also the workes of others which haue written in Geometry, cannot be perceaued nor vnderstanded. And forasmuch as all the demonstrations and proofes of all the propositions in this whole booke, depende of these groundes and principles following, which by reason of their playnnes neede no greate declaration, yet to remoue all (be it neuer so litle) obscuritie, there are here set certayne shorte and manifest expositions of them.

Definitions.

1. *A signe or point is that, which hath no part.*

*Definition of
a poynt.*

The better to vnderstand what maner of thing a signe or point is, ye must note that the nature and propertie of quantitie (wherof Geometry entreateth) is to be deuided, so that whatsoeuer may be deuided into sundry partes, is called quantitie. But a point, although it pertayne to quantitie, and hath his beyng in quantitie, yet is it no quantitie, for that it cannot be deuided. Because (as the definition saith) it hath no partes into which it should be deuided. So that a pointe is the least thing that by minde and vnderstanding can be imagined and conceyued : then which, there can be nothing lesse, as the point *A* in the margent.

A.

A signe or point is of *Pithagoras* Scholers after this manner defined: *A poynt is an vnitie which hath position.* Nūbers are conceaued in mynde without any forme & figure, and therefore without matter wheron to receaue figure, & consequently without place and position. Wherfore vnitie beyng a part of number, hath no position, or determinate place. Wherby it is manifest, that number is more simple and pure then is magnitude, and also immateriall: and so vnity which is the beginning of number, is lesse materiall then a signe or poynt, which is the beginnyng of magnitude. For a poynt is materiall, and requireth position and place, and therby differeth from vnitie.

*Definition of
a poynt after
Pithagoras.*

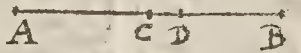
2. *A line is length without breadth.*

*Definition of
a line.*

There pertaine to quantitie three dimensions, length, bredth, & thicknes, or depth: and by these thre are all quātities measured & made known. There are also, according

The first Booke

to these three dimensions, three kyndes of continuall quantities : a lyne, a superficies, or plaine, and a body. The first kynde, namely, a line is here defined in these wordes, *A lyne is length without breadth.* A point, for that it is no quantitie nor hath any partes into which it may be deuided, but remaineth indiuisible, hath not, nor can haue any of these three dimensions. It neither hath length, breadth, nor thickenes. But to a line, which is the first kynde of quantitie, is attributed the first dimension, namely, length, and onely that, for it hath neither breadth nor thicknes, but is conceaued to be drawne in length onely, and by it, it may be deuided into partes as many as ye list, equall, or vnequall. But as touching breadth it remaineth indiuisible. As the lyne *AB*, which is onely drawen in length, may be deuided in the pointe *C* equally, or in the point *D* vnequally, and so into as many partes as ye list. There are also of diuers other geuen other definitions of a lyne : as



An other definition of a line.

A lyne is the mouyng of a poynte, as the motion or draught of a pinne or a penne to your fence maketh a lyne.

An other.

Agayne, *A lyne is a magnitude hauing one onely space or dimension, namely, length wantyng breadth and thickenes.*

The endes of a line.

3 The endes or limites of a lyne, are pointes.

For a line hath his beginning from a point, and likewise endeth in a point: so that by this also it is manifest, that pointes, for their simplicitie and lacke of composition, are neither quantitie, nor partes of quantitie, but only the termes and endes of quantitie. As the pointes *A*, *B*, are onely the endes of the line *AB*, and no partes thereof. And herein differeth a poynte in quantitie, from vnitie in number: for that although vnitie be the beginning of nombers, and no number (as a point is the beginning of quantitie, and no quantitie) yet is vnitie a part of number. For number is nothyng els but a collection of vnities, and therefore may be deuided into them, as into his partes. But a point is no part of quantitie, or of a lyne: neither is a lyne composed of pointes, as number is of vnities. For things indiuisible being neuer so many added together, can neuer make a thing diuisible, as an instant in time, is neither tyme, nor part of tyme, but only the beginning and end of time, and coupleth & ioyneth partes of tyme together.



Difference of a point frō vnitie. Vnitie is a part of number.

A poynt is no part of quantitie.

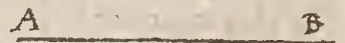
Definition of a right line.

4 A right lyne is that which lieth equally betwene his pointes.

As the whole line *AB* lyeth straight and equally betwene the poyntes *AB* without any going vp or comming downe on eyther side.

Definition of a right line after Campanus.

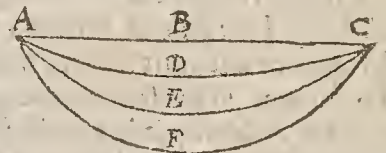
Campanus and certain others, define a right line thus:



A right line is the shortest extension or draught, that is or may be from one poynt to an other. Archimedes defineth it thus.

Definitio thereof after Archimedes.

*A right line is the shortest of all lines, which haue one and the self same limites or endes: which is in maner al one with the definitio of Campanus. As of all these lines *ABC*, *ADC*, *AEC*, *AFC*, which are all drawn from the point *A*, to the poynte *C* as Campanus speaketh, or which haue the self same limites or endes, as Archimedes speaketh, the lyne *ABC*, beyng a right line, is the shortest.*



Definitio thereof after Plato.

Plato defineth a right line after this maner: A right line is that whose middle part shadoweth the extremes. As if

you put any thyng in the middle of a right lyne, you shall not see from the one ende to the other, which thyng happeneth not in a crooked lyne. The Ecclipse of the Sunne (say Astronomers) then happeneth, when the Sunne, the Moone, & our eye are in one right line. For the Moone then being in the midst betwene vs and the Sunne, causeth it to be darkened. Diuers other define a right line diuersly, as followeth.

An other definition.

A right lyne is that which standeth firme betwene his extremes.

An other.

Agayne, *A right line is that which with an other line of lyke forme cannot make a figure.*

Agayne,

Agayne, *A right lyne is that which hath not one part in a plaine superficies, and an other erected on high.* *Another.*

Agayne, *A right lyne is that, all whose partes agree together with all his other partes.* *Another.*

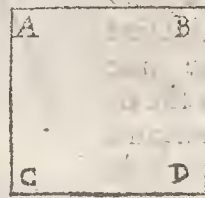
Agayne, *A right lyne is that, whose extremes abiding, cannot be altered.* *Another.*

Euclide doth not here define a crooked lyne, for it neded not. It may easely be vnderstand by the definition of a right lyne, for euery contrary is well manifested & set forth by hys contrary. One crooked lyne may be more crooked then an other, and from one poynt to an other may be drawn infinite crooked lynes: but one right lyne cannot be righter then an other, and therfore from one point to an other, there may be drawn but one right lyne. As by the figure aboue set, you may see. *Why Euclida here defineth not a crooked lyne.*

5 *A superficies is that, which hath onely length and breadth.*

Definition of a superficies.

A superficies is the second kinde of quantitie, and to it are attributed two dimensions, namely length, and breadth. As in the superficies *ABCD*, whose length is taken by the lyne *AB*, or *CD*, and breadth by the lyne *AC*, or *BD*: and by reason of those two dimensions a superficies may be deuided two wayes, namely by his length, and by hys breadth, but not by thickeffe, for it hath none. For, that is attributed onely to a body, which is the third kynde of quantitie, and hath all three dimensions, length, breadth, and thicknes, and may be deuided according to any of them.



A superficies may be deuided two wayes.

Others define a superficies thus: *A superficies is the terme or ende of a body.* As a line is the ende and terme of a superficies.

Another definition of a superficies.

6 *Extremes of a superficies, are lynes.*

The extremes of a superficies.

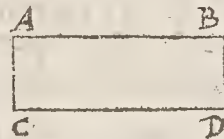
As the endes, limites, or borders of a lyne, are pointes, inclosing the line: so are lines the limites, borders, and endes inclosing a superficies. As in the figure aforesayde you maye see the superficies inclosed with foure lynes. The extremes or limites of a bodye, are superficieses. And therfore a superficies is of some thus defined: *A superficies is that, which endeth or incloseth a body:* as is to be sene in the sides of a die, or of any other body.

Another definition of a superficies.

7 *A plaine superficies is that, which lieth equally betwene his lines.*

Definition of a plaine superficies.

As the superficies *ABCD* lyeth equally and smoothe betwene the two lines *AB*, and *CD*: or betwene the two lines *AC*, and *BD*: so that no part thereof eyther swelleth vpward, or is depressed downward. And this definitiō much agreeth with the definition of a right line. A right line lieth equally betwene his points, and a plaine superficies lyeth equally betwene his lynes. Others define a plaine superficies after this maner:



A plaine superficies, is the shortest extension or draught from one lyne to an other: like as a right lyne is the shortest extension or draught from one point to an other.

Another definition of a plaine superficies.

Euclide also leaueth out here to speake of a crooked and hollow superficies, because it may easely be vnderstand by the diffinition of a plaine superficies, being hys contrary. And euen as from one point to an other may be drawn infinite crooked lines, & but one right line, which is the shortest: so from one lyne to an other may be drawn infinite crooked superficieses, & but one plain superficies, which is the shortest. Here must you consider when there is in Geometry mention made of pointes, lines, circles, triangles, or of any other figures, ye may not conceyue of them as they be in matter, as in woode, in mettall, in paper, or in any such lyke, for so is there no lyne, but hath some breadth, and may be deuided: nor points, but that shal haue some partes, and may also be deuided, and so of others. But you must conceiue them in mynde, plucking them by imagination from all matter, so shall ye vnderstande them truely and perfectly, in their owne nature as they are defined. As a lyne to be long, and not broad: and a poynte to

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Another definition of a playne superficies.

An other definition.

An other definition.

An other definition.

Definition of a playne angle.

be so little, that it shall haue no part at all.

Others otherwyse define a playne superficies: *A playne superficies is that, which is firmly set betwene his extremes, as before was sayd of a right lyne.*

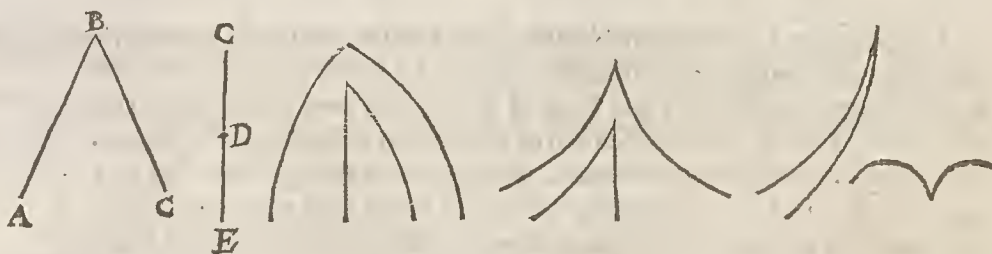
Agayne, *A playne superficies is that, vnto all whose partes a right line may well be applied.*

Again, *A playne superficies is that, which is the shortest of al superficies, which haue one & the self extremes: As a right line was the shortest line that can be drawen betwene two pointes.*

Agayne, *A playne superficies is that, whose middle darkeneth the extremes, as was also sayd of a right lyne.*

8 *A playne angle is an inclination or bowing of two lines the one to the other and the one touching the other, and not beyng directly ioyned together.*

As the two lines AB , & BC , incline the one to the other, and touch the



one the other in the point B , in which point by reason of the inclination of the sayd lines, is made the angle ABC . But if the two lines which touch the one the other, be without all inclination of the one to the other, and be drawne directly the one to the other, then make they not any angle at all, as the lines CD , and DE , touch the one the other in the point D , and yet as ye see they make no angle.

Definition of a rectilined angle.

9 *And if the lines which containe the angle be right lynes, then is it called a rightlyned angle.*

As the angle ABC , in the former figures, is a rightlined angle, because it is contained of right lines: where note, that an angle is for the most part described by three letters, of which the second or middle letter representeth the very angle, and therefore is set at the angle.

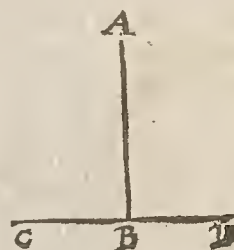
By the contrary, a crooked lyned angle, is that which is contained of crooked lines, which may be diuersly figured. Also a mixt angle is that which is caused of them both, namely, of a right line and a crooked, which may also be diuersly figured, as in the figures before set ye may see. There are of angles three kindes, a right angle, an acute angle, and an obtuse angle, the definitions of which now follow.

Three kindes of angles.

What a right angle, & what also a perpendicular lyne is.

10 *When a right line standing vpon a right line maketh the angles on either side equall: then either of those angles is a right angle. And the right lyne which standeth erected, is called a perpendicular line to that vpon which it standeth.*

As vpon the right line CD , suppose there do stand an other line AB , in such sort, that it maketh the angles on either side therof equall: namely, the angle ABC on the one side equall to the angle ABD on the other side: then is eche of the two angles ABC , and ABD a right angle, and the line AB , which standeth erected vpon the line CD , without inclination to either part is a perpendicular line, commonly called among artificers a plumbelyne.



What an obtuse angle is.

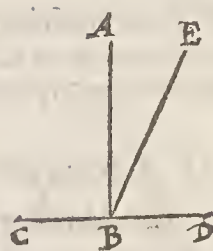
11 *An obtuse angle is that which is greater then a right angle.*

As

As the angle CBE in the example is an obtuse angle, for it is greater then the angle ABC , which is a right angle, because it containeth it, and containeth moreouer the angle ABE .

12 An acute angle is that, which is lesse then a right angle.

As the angle EBD in the figure before put is an acute angle, for that it is lesse then the angle ABD , which is a right angle, for the right angle containeth it, and moreouer the angle ABE .



What an acute angle is.

13 A limite or terme, is the ende of euery thing.

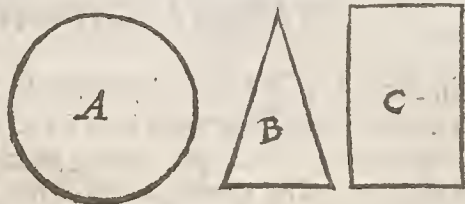
For as much as of things infinite (as Plato saith) there is no science, therefore must magnitude or quantitie (wherof Geometry entreateth) be finite, and haue borders and limites to inclose it, which are here defined to be the endes therof. As a point is the limite or terme of a line, because it is thend therof: A line likewise is the limite & terme of a superficies: and likewise a superficies is the limite and terme of a body, as is before declared.

The limite of any thing.

No science of things infinite

14 A figure is that, which is containd vnder one limite or terme, or many.

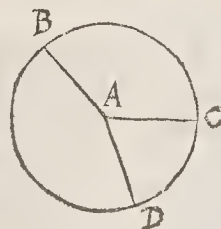
As the figure A is contained vnder one limit, which is the round line. Also the figure B is containd vnder three right lines. And the figure C vnder foure, and so of others, which are their limites or termes.



Definition of a figure.

15 A circle is a plaine figure, conteyned vnder one line, which is called a circumference, vnto which all ^{right lines} lines drawn from one poynt within the figure and falling vpon the circumference therof are equall the one to the other.

As the figure here set is a plaine figure, that is a figure without groffenes or thicknes, and is also containd vnder one line, namely, the crooked lyne BCD , which is the circumference therof, it hath moreouer in the middle therof a point, namely, the point A , from which, all the lines drawn to the superficies, are equal: as the lines AB , AC , AD , and other how many foeuer.



Definition of a circle.

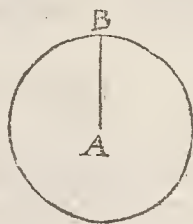
Of all figures a circle is the most perfect, and therefore is it here first defined.

A circle the most perfect of all figures.

16 And that point is called the centre of the circle, as is the point A , which is set in the middes of the former circle.

The centre of a circle.

For the more easy declaration, that all the lines drawn from the centre of the circle to the circumference, are equall, ye must note, that although a line be not made of pointes: yet a point, by his motion or draught, describeth a line. Likewise a line drawn, or moued, describeth a superficies: also a superficies being moued maketh a folide or bodie. Now the imagine the line AB , (the point A being fixed) to be moued about in a plaine superficies, drawing the point B continually about the point A , till it returne to the place where it began first to moue: so shall the point B , by this motion, describe the circumference of the circle, and the point A being fixed, is the centre of the circle. Which in



B.iii.

all

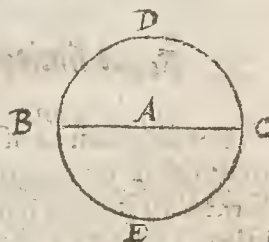
The first Booke

all the time of the motion of the line, had like distance: from the circumference, namely, the length of the line AB . And for that all the lines drawn from the centre to the circumference are described of that line, they are also equal vnto it, & betwene theselues.

Definition of a
diameter.

- 17 *A diameter of a circle, is a right line, which drawn by the centre thereof, and ending at the circumference on either side, deuideth the circle into two equall partes.*

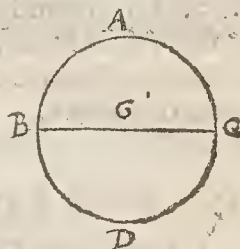
As the line BAC in this circle present is the diameter, because it passeth from the point B , of the one side of the circumference, to the point C , on the other side of the circumference, & passeth also by the point A being the centre of the circle. And moreover it deuideth the circle into two equall partes: the one, namely BDC , being on the one side of the line, & the other namely, BEC , on the other side, which thing did *Thales Miletius* (which brought Geometry out of Egypt into Grece) first obserue and proue. For if a line drawn by the centre, do not deuide a circle into two equal partes: all the lines drawn from the centre to the circumference should not be equall.



Definition of a
semicircle.

- 18 *A semicircle, is a figure which is containd vnder the diameter, and vnder that part of the circumference which is cut of by the diametre.*

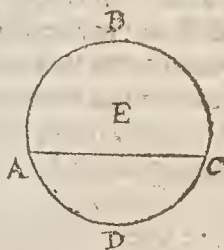
As in the circle $ABCD$ the figure BAC is a semicircle, because it is contained of the right line BGC , which is the diameter, and of the crooked line BAC , being that part of the circumference, which is cut of by the diameter BGC . So likewise the other part of the circle, namely BDC , is a semicircle as the other was.



Definition of a
section of a circle.

- 19 *A section or portion of a circle, is a figure whiche is containd vnder a right lyne, and a parte of the circumference, greater or lesse then the semicircle.*

As the figure ABC , in the example, is a section of a circle, & is greater then halfe a circle, and the figure ADC , is also a section of a circle, and is lesse then a semicircle. A section, portion, or part of a circle is all one, and signifieth such a part which is either more or lesse, then a semicircle: so that a semicircle is not here called a section or portion of a circle. A right lyne drawn from one side of the circumference of a circle to the other, not passyng by the centre, deuideth the circle into two vnequall partes, which are two sections, of which that which containeth the centre is called the greater section, and the other is called the lesse section. As in the example, the part of the circle ABC , which containeth in it the centre E , is the greater section, being greater then the halfe circle: the other part, namely ADC , which hath not the centre in it, is the lesse section of the circle, being lesse then a semicircle.



Definition of
rectilined figures.

- 20 *Rightlined figures are such which are containd vnder right lynes.*

As are such as followeth, of which some are containd vnder three right lines, some vnder foure, some vnder fise, and some vnder mo.

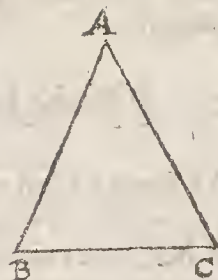
Definition of
three sided figures.

- 21 *Thre sided figures, or figures of thre sydes, are such which are containd vnder three right lines.*

As

As the figure in the example ABC , is a figure of three sides, because it is contained vnder thre right lines, namely, vnder the lines AB, BC, CA .

A figure of three sides, or a triangle, is the first figure in order of all right lined figures, and therefore of all others it is first defined. For vnder lesse then three lines, can no figure be comprehended.

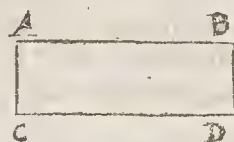


22. *Four sided figures or figures of four sides are such, which are contained vnder four right lines.*

Definition of four sided figures.

As the figure here set, is a figure of four sides, for that it is comprehended vnder four right lines, namely, AB, BD, DC, CA .

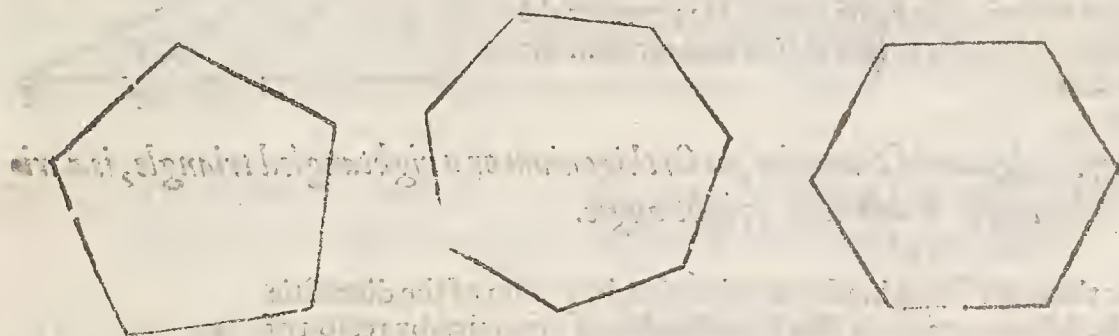
Triangles, and four sided figures serue commonly to many vses in demonstrations of Geometry. Wherefore the nature and properties of them, are much to be obserued, the vse of other figures is more obscure.



23. *Many sided figures are such which haue mo sides then foure.*

Definition of many sided figures.

Right lined figures hauing mo sides then foure, by continual adding of sides may be

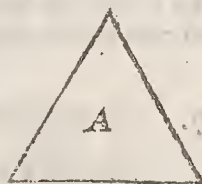


infinite. Wherefore to define them all seuerally, according to the number of their sides, should be very tedious, or rather impossible. Therefore hath *Euclide* comprehended the vnder one name, and vnder one diffinition: calling them many sided figures, as many as haue mo sides then foure: as if they haue five sides, sixe, seuen, or mo. Here note ye, that euery rightlined figure hath as many angles, as it hath sides, & taketh his denomination aswell of the number of his angles, as of the number of his sides. As a figure contained vnder three right lines, of the number of his three sides, is called a thre sided figure: euen so of the number of his three angles, it is called a triangle. Likewise a figure contained vnder foure right lines, by reason of the number of his sides, is called a foure sided figure: and by reason of the number of his angles, it is called a quadrangled figure, and so of others.

24. *Of three sided figures or triangles, an equilatre triangle is that, which hath three equall sides.*

Definition of an equilateral triangle.

Triangles haue their differences partly of their sides, and partly of their angles. As touching the differences of their sides, there are three kindes. For either all three sides of the triangle are equall, or two onely are equall, & the third vnequal: or els all three are vnequall the one to the other. The first kind of triangles, namely, that which hath three equall sides, is most simple, and easiest to be known: and is here first defined, and



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is called an equilater triangle, as the triangle *A* in the example, all whose sides are of one length.

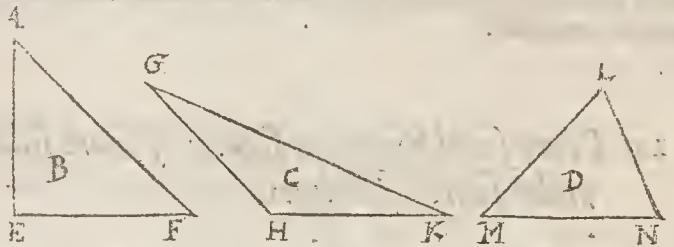
Definition of
an Ifofceles.

25. Ifofceles, is a triangle, which hath onely two sides equall.

The second kinde of triangles hath two sides of one length, but the third side is either longer or shorter thē the other two, as are the triangles here figured, *B, C, D*

In the triangle *B*, the two sides *AE* and *EF* are equal the one to the other, and the side *AF*, is longer

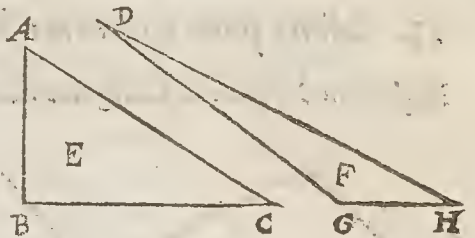
then any of them both: Likewise in the triangle *C* the two sides *GH* and *HK*, are equall, and the side *GK* is greater. Also in the triangle *D*, the sides *LM* and *MN*, are equall, and the side *LN* is shorter.



Definition of
a Scalenum.

26. Scalenum is a triangle, whose three sides are all vnequall.

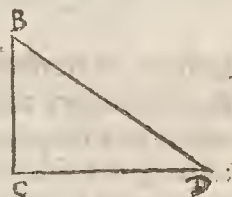
As are the triangles *E, F*, in which there is no one side equal to any of the other. For in the triangle *E*, the side *AC* is greater then the side *BC*, and the side *BC* is greater thē the side *AB*: Likewise in the triangle *F*, the side *DH*, is greater thē the side *DG*, and the side *DG*, is greater then the side *GH*.



An Orthigonium
triangle.

27. Again of triangles, an Orthigonium or a rightangled triangle, is a triangle which hath a right angle.

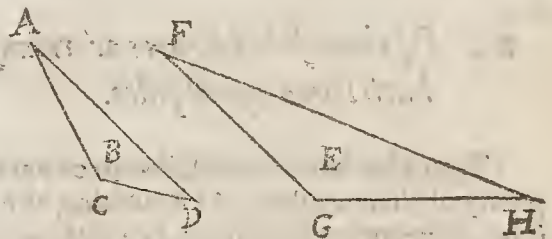
As there are three kinds of triangles, by reason of the diuersitie of the sides, so are there also three kinds of triangles, by reason of the varietie of the angles. For euery triangle either containeth one right angle, & two acute angles: or one obtuse angle, & two acute: or three acute angles: for it is impossible that one triangle should containe two obtuse angles, or two right angles, or one obtuse angle, and the other a right angle. All which kinds are here defined. First a rightangled triangle which hath in it a right angle. As the triangle *BCD*, of which the angle *BCD*, is a right angle.



An Ambligonium
triangle.

28. An ambliogonium or an obtuse angled triangle, is a triangle which hath an obtuse angle.

As is the triangle *B*, whose angle *ACD*, is an obtuse angle, and is also a Scalenum, hauing his three sides vnequall: the triangle *E*, is likewise an Ambliogonium, whose angle *FGH*, is an obtuse angle, & is an Ifosceles, hauing two of his sides equall, namely *FG* and *GH*.

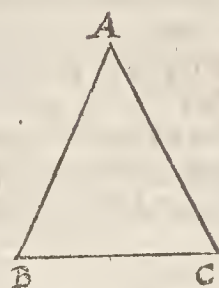
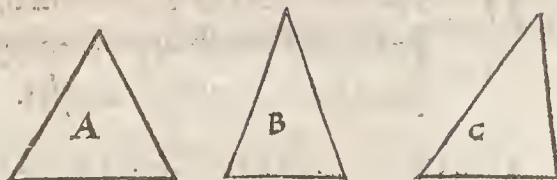


An Oxigonium
triangle.

29. An oxigonium or an acuteangled triangle, is a triangle which hath all his three angles acute.

As

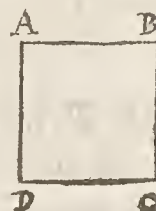
As the triangles A, B, C , in the example, all whose angles are acute: of which A is an equilateral triangle, B , an Ilosceles, and C a Scalenon. An equilateral triangle is most simple, and hath one vniforme construction, and therefore all the angles of it are equall, and neuer hath in it either a right angle, or an obtuse: but the angles of an Ilosceles or a Scalenon, may diuersly vary. It is also to be noted that in comparison of any two sides of a triangle, the third is called a base. As of the triangle ABC in respect of the two lines AB and AC , the line BC , is the base: and in respect of the two sides AC and CB , the line AB , is the base, and likewyse in respect of the two sides CB & BA , the line AC , is the base.



- 30 Of foure syded figures, a quadrate or square is that, whose sydes are equall, and his angles right.

Definition of a square.

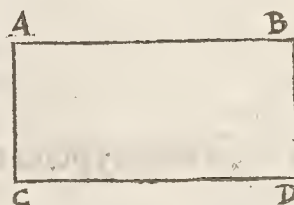
As triangles haue their difference and varietie by reason of their sides and angles: so likewise do figures of foure sides, take their varietie and difference partly by reason of their sides, & partly by reason of their angles, as appeareth by their definitions. The four sided figure $ABCD$ is a square or a quadrate, because it is a right angled figure, all his angles are right angles, and also all his four sides are equall.



- 31 A figure on the one syde longer, or squarelike, or as some call it, a long square, is that which hath right angles, but hath not equall sydes.

Definition of a long square.

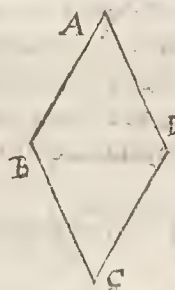
This figure agreeth with a square touching his angles, in that either of them hath right angles, and differeth from it onely by reason of his sides, in that the sides thereof be not equall, as are the sides of a square. As in the example, the angles of the figure $ABCD$, are right angles, but the two sides thereof AB , and CD , are longer then the other two sides AC , & BD .



- 32 Rhombus (or a diamonde) is a figure hauing foure equall sydes, but it is not right angled.

Definition of a Diamond figure

This figure agreeth with a square, as touching the equallitie of lines, but differeth from it in that it hath not right angles, as hath the square. As of this figure, the foure lines AB, BC, CD, DA , be equall, but the angles thereof are not right angles. For the two angles ABC and ADC , are obtuse angles, greater then right angles, & the other two angles BAD and BCD , are two acute angles lesse then two right angles. And these foure angles are yet equall to foure right angles: for, as much as the acute angle wanteth of a right angle, so much the obtuse angle exceedeth a right angle.

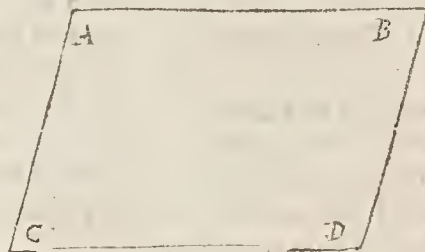


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Definition of a
diamondlike fi-
gure.

- 33 Rhomboides (or a diamond like) is a figure, whose opposite sides are equall, and whose opposite angles are also equall, but it hath neither equall sides, nor right angles.

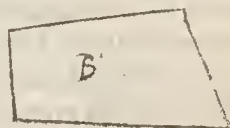
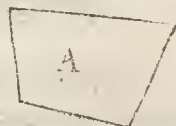
As in the figure $ABCD$, all the four sides are not equall, but the two sides AB and CD , being opposite the one to the other, also the other two sides AC and BD , being also opposite, are equall the one to the other. Likewise the angles are not right angles, but the angles CAB , and CDB , are obtuse angles, and opposite and equall the one to the other. Likewise the angles ABD , and ACD , are acute angles, and opposite, and also equall the one to the other.



Trapezia or
tables.

- 34 All other figures of four sides besides these, are called trapezia, or tables.

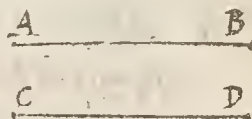
Such are all figures, in which is obserued no equallitie of sides nor angles: as the figures A and B , in the margēt, which haue neither equall sides, nor equal angles, but are described at all aduenture without obseruation of order, and therefore they are called irregular figures.



Definition of
Parallel lines.

- 35 Parallel or equidistant right lines are such, which being in one and the selfe same superficies, and produced infinitely on both sydes, do neuer in any part concur.

As are the lines AB , and CD , in the example.

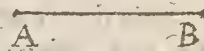


Petitions or requestes.

- 1 From any point to any point, to draw a right line.

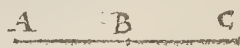
What Peti-
tions are.

After the definitions, which are the first kind of principles, now follow petitions, which are the second kynd of principles: which are certain general sentences, so plain, & so perspicuous, that they are perceiued to be true as soone as they are vttered, & no man that hath but common sence, can, nor will deny them. Of which, the first is that, which is here set. As from the point A , to the point B , who wil deny, but easily graunt that a right line may be drawn? For two points howsoeuer they be set, are imagined to be in one and the selfe same plaine superficies, wherfore from the one to the other there is some shortest draught, which is a right line. Likewise any two right lines howsoeuer they be set, are imagined to be in one superficies, and therefore from any one line to any one line, may be drawn a superficies.



- 2 To produce a right line finite, straight forth continually.

As to draw in length continually the right line AB , who will not graunt? For there is no magnitude so great, but that there may be a greter, nor any so litle, but that there may be a lesse. And

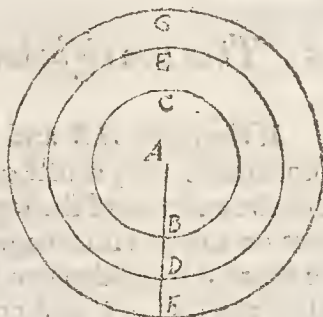


a line

a line is a draught from one point to an other, therefore from the point B , which is the ende of the line AB , may be drawn a line to some other point, as to the point C , and from that to an other, and so infinitely.

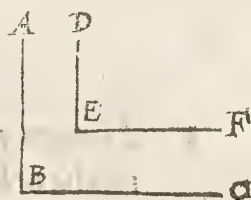
3 Vpon any centre and at any distance, to describe a circle.

A playne superficies may in compasse be extended infinitely: as from any pointe to any pointe may be drawn a right line, by reason wherof it commeth to passe that a circle may be described vpon any centre and at any space or distance. As vpon the centre A , and vpon the space AB , ye may describe the circle BC , & vpon the same centre, vpon the distance AD , ye may describe the circle DE , or vpon the same centre A , according to the distaunce AF , ye may describe the circle FG , and so infinitely extendyng your space.

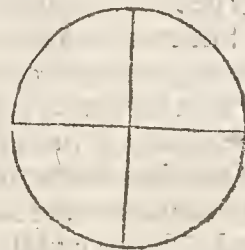


4 All right angles are equall the one to the other.

This petition is most plaine, and offreth it selfe euen to the fence. For as much as a right angle is caused of one right lyne falling perpendicularly vpon an other, and no one line can fall more perpendicularly vpon a line then an other: therefore no one right angle can be greater the an other: neither do the length or shortenes of the lines alter the greatnes of the angle. For in the example, the right angle ABC , though it be made of much longer lines then the right angle DEF , whose lines are much shorter, yet is that angle no greater then the other. For if ye set the point E iust vpon the point B , then shal the line ED , euenly and iustly fall vpon the line AB , and the line EF , shall also fall equally vpon the line BC , and so shal the angle DEF , be equall to the angle ABC , for that the lines which cause them, are of like inclination.

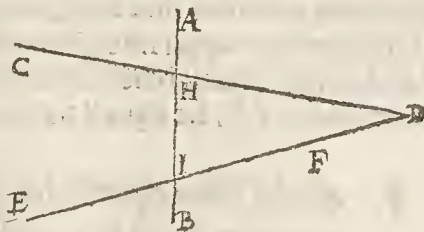


It may euidently also be sene at the centre of a circle. For if ye draw in a circle two diameters, the one cutting the other in the centre by right angles, ye shall deuide the circle into fowre equall partes, of which eche contayneth one right angle, so are all the foure right angles about the centre of the circle equall.



5 When a right line falling vpon two right lines, doth make on one & the selfe same syde, the two inwarde angles lesse then two right angles, then shal these two right lines beyng produced at length concurre on that part, in which are the two angles lesse then two right angles.

As if the right line AB , fall vpon two right lines, namely, CD and EF , so that it make the two inward angles on the one side, as the angles DHI & FIH , lesse then two right angles (as in the example they do) the said two lines CD , and EF , being drawn forth in lēgth on that part, wheron the two angles being lesse the two right angles consist, shal at lēgth concurre and meete together: as in the point D , as it is easie to see. For the partes of the lines towardes D , are more enclined the one to



C.ij.

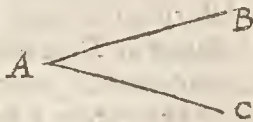
the

The first Booke

the other, then the partes of the lines towards CE are. Wherefore the more these parts are produced, the more they shall approach neare and neare, till at length they shal mete in one point. Contrariwise the same lines drawn in légth on the other side, for that the angles on that side, namely, the angle CHB , and the angle EIA , are greater then two right angles, so much as the other two angles are lesse then two right angles, shall neuer mete, but the further they are drawn, the further they shalbe distant the one from the other.

6 That two right lines include not a superficies.

If the lines AB and AC , being right lines, should inclose a superficies, they muste of necessitie bee ioyned together at both the endes, and the superficies must be betwene thé. Ioyne them on the one side together in the pointe A , and imagine the point B to be drawn to the point C ; so shall the line AB , fall on the line AC , and couer it, and so be all one with it, and neuer inclose a space or superficies.



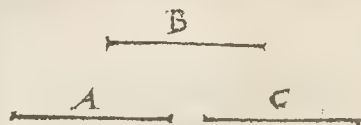
Common sentences.

1 Things equall to one and the selfe same thyng: are equall also the one to the other.

What common sentences are.

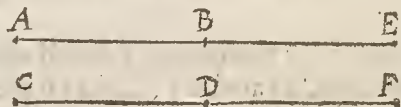
Difference betwene petitions & common sentences.

After definitions and petitions, now are set common sentences, which are the third and last kynd of principles. Which are certaine general propositions, commonly known of all men, of themselves most manifest & cleare, & therefore are called also dignities not able to be denied of any. Petitions also are very manifest, but not so fully as are the common sentences, and therefore are required or desired to be graunted. Petitions also are more peculiar to the arte whereof they are: as those before put are proper to Geometry: but common sentences are generall to all things wherunto they can be applied. Agayne, petitions consist in actions or doing of somewhat most easy to be done: but common sentences consist in consideration of mynde, but yet of such things which are most easy to be vnderstanded, as is that before set. As if the line A be equall to the line B , And if the line C be also equall to the line B , then of necessitie the lines A and C , shalbe equal the one to the other. So is it in all superficieses, angles, & numbers, & in all other things (of one kynde) that may be compared together.



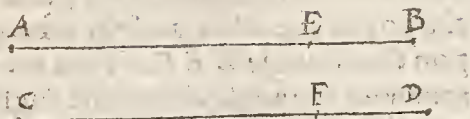
2 And if ye adde equall thinges to equall thinges: the whole shalbe equall.

As if the line AB be equal to the line CD , & to the line AB , be added the line BE , & to the line CD , be added also an other line DF , being equal to the line BE , so that two equal lines, namely, BE , and DF , be added to two equall lynes AB , & CD : then shal the whole lyne AE , be equall to the whole lyne CF , and so of all quantities generally.



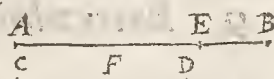
3 And if from equall thinges, ye take away equall thinges: the thinges remaining shall be equall.

As if from the two lines AB and CD , being equal, ye take away two equall lines, namely, EB and FD , then maye you conclude by this common sentence, that the partes remayning, namely, AE , and CF are equall the one to the other: and so of all other quantities.



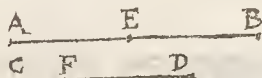
- 4 And if from vnequall thinges ye take away equall thinges: the thynges which remayne shall be vnequall.

As if the lines AB , and CD , be vnequall, the line AB , beyng longer then the line CD , & if ye take frō them two equall lines, as EB , and FD : the partes remayning, which are the lines AE and CF , shall be vnequall the one to the other, namely, the lyne AE , shall be greater then the line CF , which is euer true in all quantities whatfoeuer.



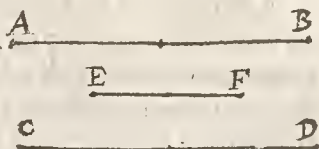
- 5 And if to vnequall thinges ye adde equall thinges: the whole shall be vnequall.

As if ye haue two vnequal lines, namely, AE the greater, and CF the lesse, & if ye adde vnto thē two equall lines, EB & FD , then maye ye conclude that the whole lines composd are vnequall: namely, that the whole lyne AB , is greater then the whole line CD , and so of all other quantities.



- 6 Thinges which are double to one and the selfe same thing: are equall the one to the other.

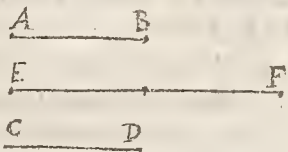
As if the line AB be double to the line EF , and if also the line CD , be double to the same line EF : thē may you by this common sentence conclude, that the two lines AB , & CD , are equall the one to the other. And this is true in all quantities, and that not only, when they are double, but also if they be triple or quadruple, or in what proportion foeuer it be of the greater inequalitie. Which is when the greater quantitie is compared to the lesse.



What proportion of the greater inequality is

- 7 Thinges which are the halfe of one and the selfe same thing: are equal the one to the other.

As if the line AB , be the halfe of the line EF , and if the lyne CD , be the halfe also of the same line EF : then may ye conclude by this common sentence, that the two lines AB and CD , are equall the one to the other. This is also true in all kyndes of quantitie, and that not onely when it is a halfe, but also if it be a third, a quarter, or in what proportion foeuer it be of the lesse inequalitie. Which is when the lesse quantitie is cōpared to the greater.



What proportion of the lesse inequality is

- 8 Thinges which agree together: are equall the one to the other.

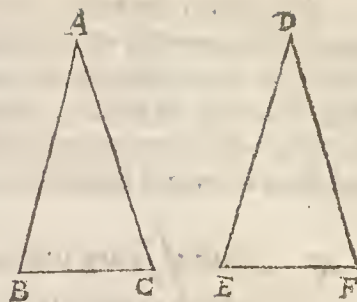
Such thinges are sayd to agree together, whiche when they are applied the one to the other: or set the one vpon the other, the one excedeth not the other in any thyng.

C.iiij.

As

The first Booke

As if the two triangles ABC , and DEF , were applied the one to the other, and the triangle ABC , were set vpon the triangle DEF , if then the angle A , do iustly agree with the angle D , and the angle B , with the angle E , and also the angle C , with the angle F : and moreouer if the line AB , do iustly fall vpon the line DE , and the line AC , vpon the line DF , and also the line BC , vpon the line EF , so that on euery part of these two triangles, there is iust agreement, then may ye conclude that the two triangles are equall.



9 Every whole is greater then his part.

As the whole is equal to all his partes taken together, so is it greater then any one part therof. As if the line CB be a part of the line AB , then by this common sentence ye may conclude that the whole line AB , is greater then the part, namely, the line CB . And this is generall in all thinges.



What a Proposition is.



He principles thus placed & ended, now follow the propositions, which are sentences set forth to be proued by reasoning and demonstrations, and thetfore they are agayne repeated in the end of the demonstration. For the proposition is euer the conclusion, and that which ought to be proued.

Propositions of two sortes.

What a Probleme is.

Propositions are of two sortes, the one is called a Probleme, the other a Theoreme.

A Probleme, is a proposition which requireth some action, or doing: as the making of some figure, or to deuide a figure or line, to apply figure to figure, to adde figures together, or to subtrah one from an other, to describe, to inscribe, to circumscribe one figure within or without another, and suche like. As of the first proposition of the first booke is a probleme, which is thus: *Upon a right line geuen not being infinite, to describe an equilater triangle, or a triangle of three equall sides.* For in it, besides the demonstration and contemplation of the mynde, is required somewhat to be done: namely, to make an equilater triangle vpon a line geuen. And in the ende of euery probleme, after the demonstration, is concluded after this manner, *Which is the thing, Which was required to be done.*

What a Theoreme is.

A Theoreme, is a proposition, which requireth the searching out and demonstration of some propertie or passion of some figure: Wherin is onely speculation and contemplation of minde, without doing or working of any thing. As the fifth proposition of the first booke, which is thus, *An Isosceles or triangle of two equall sides, hath his angles at the base, equall the one to the other, &c.* is a Theoreme. For in it is required only to be proued and made plaine by reason and demonstratiō, that these two angles be equall, without further working or doing. And in the ende of euery Theoreme, after the demonstration is concluded after this manner, *Which thyng was required to be demonstrated or proued.*



☞ The first Probleme. The first Proposition.

Upon a right line geuen not beyng infinite, to describe an equilater triangle, or a triangle of three equall sides.

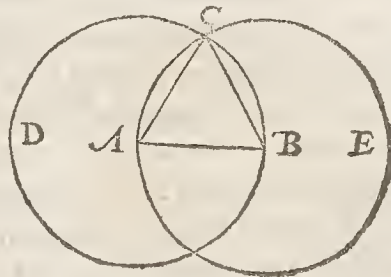


Suppose that the right line geuen be AB .

It is required vpon the line AB , to describe an equilater triangle, namely, a triangle of three equall sides. Now therefore making the centre the point A and the space AB , describe (by the third petition) a circle BCD : and agayne (by the same) making the centre the point B , and the space BA , describe an other circle ACE . And (by the first petition) from the point C , wherein the circles cut the one the

Construction.

other, draw one right line to the point A , and an other right line to the point B . And forasmuche as the point A is the centre of the circle BCD , therefore (by the 15. definition) the line AC is equall to the line AB : Agayne forasmuch as the point B is the centre of the circle CAE , therefore (by the same definition) the line BC is equall to the line BA . And it is proued, that the line AC is equall to the line AB : wherefore either of these lines CA and CB , is equall to the line AB : but thinges which are equall to one and the same thing, are also equall the one to the other (by the first common sentence) wherefore the line CA , also is equall to the line CB . Wherefore these three right lines CA , AB , and BC are equal the one to the other. Wherefore the triagle ABC is equilater. Wherefore vpon the line AB , is described an equilater triangle ABC . Wherefore vpon a line geuen not being infinite, there is described an equilater triangle, Which is the thing, which was required to be done.



Demonstration

A triangle or any other rectilined figure is then said to be set or described vpon a line, when the line is one of the sides of the figure.

This first proposition is a *Probleme*, because it requireth acte or doying, namely, to describe a triangle. And this is to be noted, that euery *Proposition*, whether it be a *Probleme*, or a *Theoreme*, commonly containeth in it a thing geuen, and a thing required to be searched out: although it be not alwayes so. And the thing geuen, is euer set before the thing required. In some propositions there are more things geuen then one, and mo thinges required then one. In some there is nothing geuen at all.

Thing geuen.

Thing required

Moreover euery *Probleme* & *Theoreme*, beyng perfect and absolute, ought to haue all these partes, namely. First the *Proposition*, to be proued. Then the *exposition*

Proposition.

Exposition.

The first Booke

Determination. which is the explication of the thing geuen. After that followeth the *determination*, which is the declaration of the thing required. Then is set the *construction* of suche things which are necessary ether for the doing of the proposition, or for the demonstration. Afterward followeth the *demonstration*, which is the reason and prooue of the proposition. And last of all is put the *conclusion*, which is inferred & proued by the demonstration, and is euer the proposition. But all those partes are not of necessity required in euery Probleme and Theoreme. But the *Proposition*, *demonstration*, and *conclusion*, are necessary partes, & can neuer be absent: the other partes may sometymes be away.

Case. Further in diuers propositions, there happen diuers cases: which are nothing els, but varietie of delineation and construction, or chaunge of position, as when pointes, lines, superficiesses, or bodies are chaunged. VVhich thinges happen in diuers propositions.

The thing geuen in this Probleme.

The thing required.

The proposition.

The exposition.

The determination.

The construction.

NOW then in this *Probleme*, the *thing geuen*, is the line geue: the *thing required*, to be ferched out is, how vpon that line to describe an equilater triangle. The *Proposition* of this *Probleme* is, *Vpon a right line geuen not beyng infinite, to describe an equilater triangle.*

The *exposition* is, *Suppose that the right line geuen be AB*, and this declareth onely the *thing geuen*. The *determination* is, *It is required vpon the line AB, to describe an equilater triangle: for therby as you see, is declared onely the thing required.* The *construction* beginneth at these wordes, *Now therefore making the centre the point A, & the space AB, describe (by the third petition) a circle & c, and continueth vntil you come to these wordes, And forasmuch as the point A & c. For the herto are described circles and lines, necessarye both for the doying of the proposition, and also for the demonstration therof.*

The demonstration.

VVhich *demonstration* beginneth at these wordes: *And forasmuche as the point A is the centre of the circle C B D & c: And so procedeth till you come to these wordes, VVherfore vpon the line AB is described an equilater triangle ABC.* For vntill you come thether,

The perticular conclusion.

is, by groundes before set and constructions had, proued, and made euident, that the triangle made, is equilater. And then in these wordes, *Wherfore vpon the line AB, is described an equilater triangle ABC*, is put the first *conclusion*. For there are commonly in euery proposition two conclusions: the one perticular, the other vniuersal: and from the first you go to the last. And this is the first and perticular conclusion, for that it concludeth, that vpon the lyne AB is described an equilater triangle, which is according to the exposition. After it, followeth the last and vniuersal conclusion, *Wherfore vpon a right line geuen not being infinite is described an equilater triangle.* For whether the line geuen be greater or lesse then thys lyne, the same constructions and demonstrations proue the same conclusion. Last of all is added this clause,

The vniuersall conclusion.

Which is the thing which was required to be done: wherby as we haue before noted, is declared, that this proposition is a Probleme and not a Theoreme. As for varietie of cases in this proposition there is none, for that the line geuen, can haue no diuersitie of position.

The note where by it is knowne so be a Probleme.

No cases in thys proposition.

As you haue in this *Probleme* sene plainelye set foorth the *thing geuen*, and the *thing required*, moreouer the *proposition*, *exposition*, *determination*, *construction*, *demonstration*, and *conclusion* (which are generall also to many other both *Problemes* and *Theoremes*) so may you by the example therof distinct them, and searche them out in other *Problemes*, and also *Theoremes*.

Three kindes of demonstration.

This also is to be noted, that there are three kyndes of demonstration, The one is called *Demonstratio a priori*, or composition. The other is called *Demonstratio a posteriori*.

a posteriori, or resolution. And the third is a demonstration leading to an impossibility.

A demonstration *a priori*, or composition is, when in reasoning, from the principles and first groundes, we passe descending continually, till after many reasons made, we come at the length to conclude that, which we first chiefly entend. And this kinde of demonstration yfeth Euclide in his bookes for the most part.

Demonstration a priori, or composition.

A demonstration *a posteriori*, or resolution is, when contrariwise in reasoning, we passe from the last conclusion made by the premisses, and by the premisses of the premisses, continually ascending, til we come to the first principles and grounds, which are indemonstrable, and for theyr simplicity can suffer no farther resolution.

Demonstration a posteriori, or resolution.

A demonstration leading to an impossibility is that argument, whose conclusion is impossible: that is, when it concludeth directly against any principle, or against any proposition before proved by principles, or propositions before proved.

Demonstration leading to an impossibility.

Premisses in an argument, are propositions goyng before the conclusion by which the conclusion is proved.

Premisses what they are.

Composition passeth from the cause to the effect, or from things simple to things more compounded. Resolution contrariwise passeth from things compounded to things more simple, or from the effect to the cause.

Composition or the first kynde of demonstration, which passeth from the principles, may easely be sene in this first proposition of Euclide. The demonstration wherof beginneth thus. And forasmuch as the point A is the centre of the circle C B D, therefore the line A C, is equal to the line A B. This reason (you see) taketh his beginnyng of a principle, namely, of the definition of a circle. And this is the first reason. A gayne forasmuch as B is the centre of the circle C A E, therefore the line B C is equall to the lyne B A: which is the second reason. And it was before proved that the lyne A C is equal to the line A B, wherfore either of these lines C A & C B is equal to the lyne A B. And this is the third reason. But things which are equal to one & the selfe same thying, are also equal the one to the other. VVherfore the line C A is equal to the line C B. And this is the fourth argument. VVherfore these three lines C A, A B, and B C are equall the one to the other which is the conclusion, and the thing to be proved.

An example of composition in the first proposition.

First reason.

Second reason.

Third reason.

Fourth reason.

Conclusion.

You may also in the same first Proposition, easely take an exaple of Resolution: vsing a contrary order passyng backward fro the last conclusio of the former demonstration, til you come to the first principle or ground wheron it began. For the last argument or reason in composition, is the first in Resolution: & the first in composition, is the last in resolution. Thus therfore must ye procede. The triangle A B C is contained of three equall right lines, namely; A B, A C, and B C, and therefore it is an equilater triangle by the definition of an equilater triangle: and this is the first reason. That the three lines be equall, is thus proved. The lines A C and C B are equall to the line A B, wherfore they are equall the one to the other: and this is the second reason. That the lines A B and B C, are equal is thus proved: The lines A B and A C, are drawn from the centre of the circle A C E, to the circumference of the same: wherfore they are equall by the definition of a circle: and this is the third reason. Likewise that the lines A C and A B, are

Example of resolution in the first proposition.

First reason.

Second reason.

Third reason.

Fourth reason which is the end of the whole resolution.

D. i.

equall

The first Booke

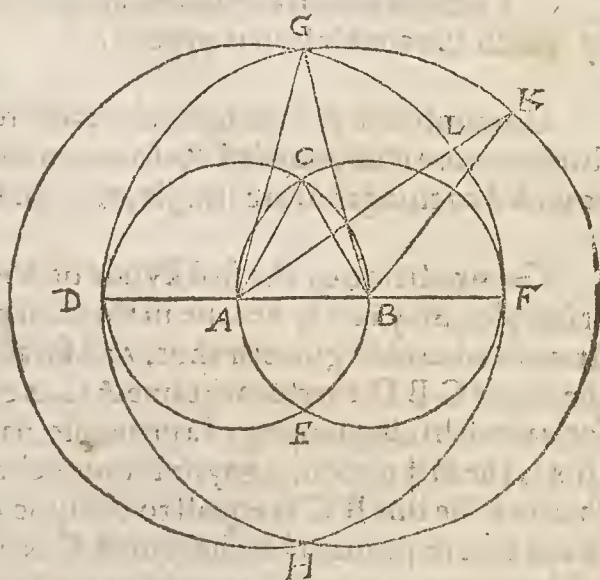
equall, is proued by the same reason. For the lines AC and AB are drawn from the centre of the circle BCD : wherefore they are equall by the same definition of a circle: this is the fourth reason or sillogisme. And thus is ended the whole resolution: for that you are come to a principle, which is indemonstrable, & can not be resolved.

Of a demonstration leading to an impossibilitie, or to an absurditie, you may haue an example in the fourth proposition of this booke.

An addition of Campanus.

How to describe
an Isosceles
triangle.

BUt nowe if vpon the same line geuen, namely, AB , ye wil describe the other two kinds of triangles, namely, an *Isosceles* or a triagle of two equal sides, & a *Scalenon*, or a triangle of three vnequall sides. First for the describing of an *Isosceles* triangle produce the line AB on ether side, vntill it concur with the circumferences of both the circles in the pointes D and F , and making the centre the point A , describe a circle HFG according to the quantity of the line AF . Likewise making the centre the poynte B , describe a circle HDG , according to the quantitie of the line BD . Now the these circles shall cut the one the other in two poyntes, which let be H , and G : And let the endes of the line geuen be ioyned with one of the sayd sections by two right lines, which let be AG and BG . And forasmuche as these two lines AB and AD are drawn frō the centre of the circle CDE vnto the circumference therof, therefore ar they equal. Likewise, the lines BA and BF , for that they are drawn from the centre of the circle $EACF$ to the circumference therof, are equal. And forasmuch as ether of the lines AD and BF is equall to the line AB , therefore they are equal the one to the other. Wherefore putting the line AB comō to the both, the whole line BD shall be equall to the whole line AF . But BD is equal to BG , for they are both drawn frō the centre of the circle HDG to the circumference therof. And likewise by the same reason the line AF is equal to the line AG . Wherefore by the comō sentēce the lines AG and BG are equal the one to the other, and either of them is greater then the line AB , for that either of the two lines BD and AF is greater then the line AB . Wherefore vpon the line geuen is described an *Isosceles* or triangle of two equall sides.



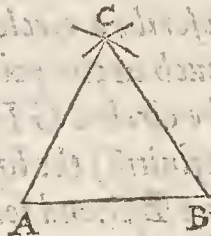
How to describe
a Scalenum.

Ye may also describe vpon the selfe same line a *Scalenon*, or triangle of three vnequall sides, if by two right lines, ye ioynē both the endes of the line geuen to some one point that is in the circumference of one of the two greater circles, so that that poynt be not in one of the two sections, and that the line DF do not concur with it, when it is on either side produced continually and directly. For let the poynte K be taken in the circumference of the circle HDG , and let it not be in any of the sections, neyther let the line DF concur with it, when it is produced continually and directly vnto the circumference therof. And draw these lines AK and BK , and the line AK shall cut the circumference of the circle HFG . Let it cut it in the poynte L : now then by the common sentence the line BK shall be equal to the line AL , for (by the definition of a circle) the line BK is equall to the line BG , and the line AL is equall to the line AG which is equal to the line BG . Wherefore the line AK is greater then the line BK and by the same reason maye it be proued that the line BK is greater then the line AB .

Wherefore

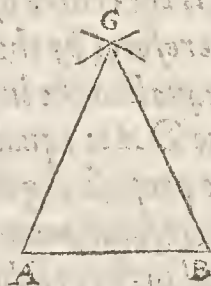
Wherefore the triangle ABK consisteth of three vnequal sides. And so haue ye vpon the line geuen, described all the kindes of triangles.

This is to be noted, that if a man will mechanically and redely, not regarding demonstration vpon a line geuen describe a triangle of three equall sides, he needeth not to describe the whole foresayd circle, but onely a little part of eche: namely, where they cut the one the other, and so from the point of the section to draw the lines to the endes of the line geuen. As in this figure here put.



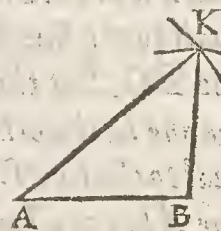
How to describe an equilateral triangle redily & mechanically.

And likewise, if vpon the said line he will describe a triangle of two equall sydes, let him extend the compasse according to the quantitie that he will haue the syde to be, whether longer then the line geuen or shorter: and so draw onely a little part of eche circle, where they cut the one the other, & fro the point of the section draw the lines to the endes of the line geuen. As in the figures here put. Note that in this the two sydes must be such, that beyng ioyned together, they be longer then the line geuen.



How to describe an Isosceles triangle redily.

And so also if vpon the sayd right line he will describe a triangle of three vnequal sydes, let him extend the compasse. First, according to the quantitie that he will haue one of the vnequall sydes to be, and so draw a little part of the circle, & then extend it according to the quantitie that he wil haue the other vnequal syde to be, and draw likewyse a little part of the circle, and that done, from the point of the section draw the lines to the endes of the line geuen, as in the figure here put. Note that in this the two sides must be such, that the circles described according to their quatitie may cut the one the other.



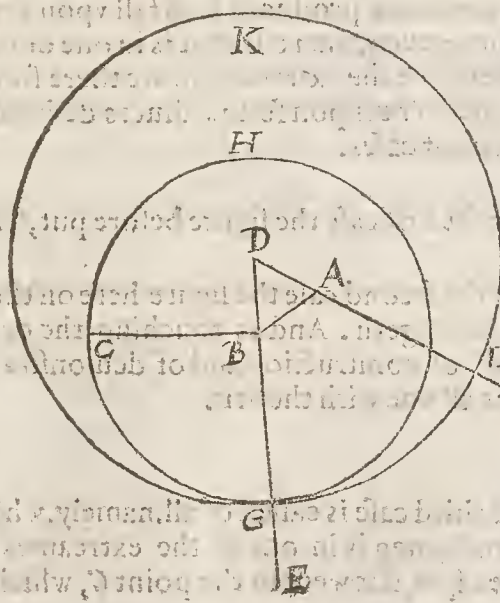
How to describe a Scalenum triangle redily.

The second Probleme. The second Proposition.

From a point geuen, to draw a right line equal to a right line geuen.



Suppose that the point geue be A , & let the right line geuen be BC . It is required from the point A , to draw a right lyne equall to the line BC .



Draw (by the first peticio) from the point A to the poynte B a right line AB : and vpon the line AB describe (by the first propositio) an equilateral triangle, and let the same be $DA B$, and exted, by the second peticio, the right lines DA & DB , to the poyntes E

Di. and

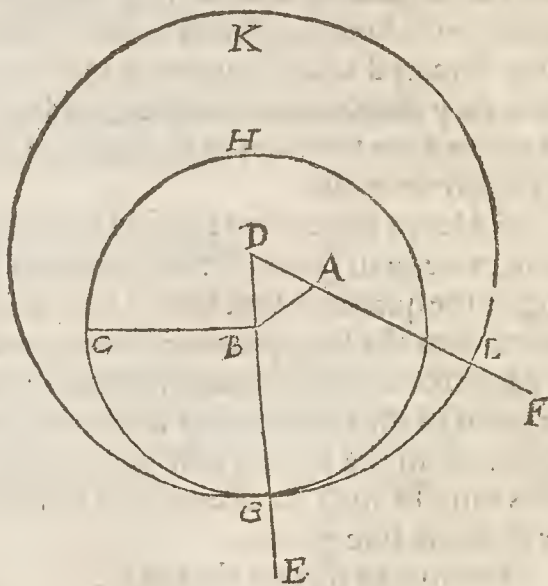
Construction.

The first Booke

and F, G (by the third petitiō) making the centre B and the space BC describe a circle CGH : & againe (by the same) making the centre D and the space DG describe a circle GKL . And foras-

Demonstration:

much as the pointe B is the centre of the circle CGH , therefore (by the 15. definitiō) the line BC is equal to the line BG : and forasmuch as the poynt D is the centre of the circle GKL : therefore (by the same) the line DL is equall to the line DG : of which the line DA is equall to a line DB (by the propositiō going before) wherfore the residue, namely, the line AL is equal to the residue, namely, to the line BG (by the third common sentence) And it is proued that the line BC is e-



quall to the line BG . VVherfore eyther of these lines AL & BC is equal to the line BG . But things which are equall to one and the same thing are also equall the one to the other (by the first commō sentence.) VVherfore the line AL is equal to the line BC . VVherfore from the poynt geuen, namely, A , is drawn a right line AL equall to the right line geuen BC : which was required to be done.

Of Problemes and Theoremes, as we haue before noted, some haue no cases at all, which are those which haue onely one position and construction: and other some haue many and diuers cases: which are such propositions which haue diuers descriptions & constructions, and change their positions. Of which sorte is this second proposition, which is also a Probleme. This proposition hath two things geuen: Namely, a pointe, and a line: the thing required is, that from the pointe geuen wherefoeuer it be put, be drawn a line equall to the line geuen. Now this poynt geuen may haue diuers positiōs. For it may be placed eyther without the right line geuen, or in some point in it. If it be without it, either it is on the side of it, so that the right line drawn from it to the ende of the right line geuen maketh an angle: or els it is put directly vnto it, so that the right line geuen being produced shall fall vpon the point geuen which is without. But if it be in the line geuen, then either it is in one of the endes or extreames thereof: or in some place betwene the extremes. So are there foure diuers positions of the poynt in respect of the line. Wherupon follow diuers delineations and constructions, and consequent-ly varietie of cases.

*Two things ge-
uen in this pro-
position.
Fower cases in
this proposition.*

The first case.

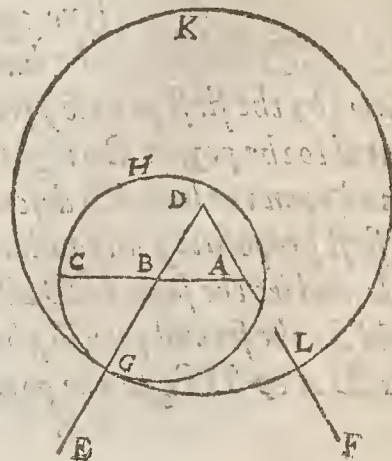
For the first case the figure before put, serueth.

The second case.

To the second case the figure here on the side set belongeth. And as touching the order both of construction and of demonstra- tion it is all one with the first.

The third case.

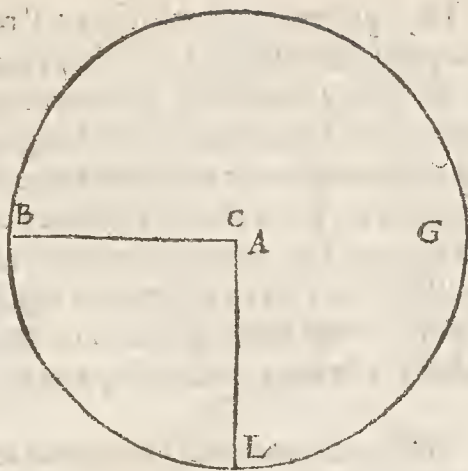
The third case is easiest of all, namely, whē the poynt geuen is in one of the extreames. As for exāple, if it were in the point C , which



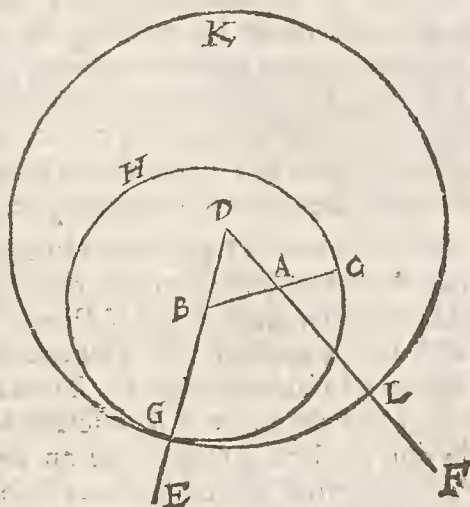
is one of the extreames of the line BC . Then making the centre the poynt C , and the space CB describe a circle BLG : and from the centre C draw a line vnto the circumference, which let the CL , which by the definition of a circle, shalbe equall to the line geuen BC .

The fourth case as touching construction herein differeth from the two firste, for that whereas in the you are willed to draw a right line from the poynt geuen, namely, A , to the poynt B which is one of the endes of the line geuen, here you shal not nede to draw that line, for that it is already drawn. As touching the rest, both in construction and demonstration you may proceede as in the two firste. As it is manifeste to see in thys figure here on the side put.

This proposition for the playnes & easines thereof, seemeth to be as it were a principle, and may easily mechanically be done. For opening the compasse to the quantitie of the line geuen, and setting on foote of it fixed in the poynt geuen and marking with the other another poynt whersoever it fall, & so by the first petition drawing a right line from the one of those poyntes to the other, the sayd right line shall beequall to the right line geuen: yet in deede is it no principle, for that it may by demonstration be proued: but principles can not be proued, as we haue before declared.



The fourth case.



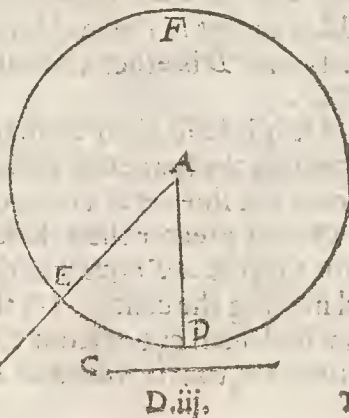
This proposition though it be very easie to be done mechanically, yet is no principle.

The 3. Probleme. The 3. Proposition.

Two vnequal right lines being geuen, to cut of from the greater, a right lyne equall to the lesse.



Suppose that the two vnequal right lines geuen be AB & C , of which let the lyne AB be the greater. It is required from the line AB being the greater, to cut of a right line equal to the right line C , which is the lesse line. drawt en (by the second proposition) from the point A a right line equall to the line C , and let the same be AD : and making the centre A and the space AD describe (by the third petition) a circle DEF . And forasmuche as the point A is the centre of y circle DEF , therefore AE is equal to AD , but the line C is equal to the line AD . Wherefore either of these lines AE and C is equall to AD , wherefore the line AE is equall to the line C , wherefore two vnequall right lines being geuen, namely, AB and C , there is cut of from AB being y greater, a right line AE equall to the lesse lyne, namely, to C : which was required to be done.



This

The first Booke

Two things ge-
uen in this pro-
position.
Diuers cases in
it.

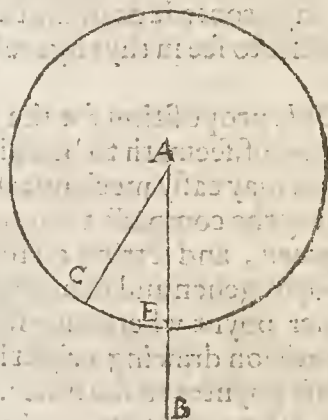
This proposition, which is a Probleme, hath two things geuen, namely, two vnequall right lines: the thing required is, from the greater to cut of a line equal to the lesse. It hath also diuers cases. For the lines geuen either are distinct th'one from the other: or are ioyned together at one of their endes: or they cut the one the other, or the one cutteth the other in one of the extreames. VVhich may be two wayes. For ether the greater cutteth the lesse, or the lesse the greater. If they cut the one the other, either ech cutteth th'other into equall partes: or into vnequall partes: or the one into equall partes, and the other into vnequall partes. VVhich may happen in two sorts, for the greater may be cut into equall partes, and the lesse into vnequall partes: or contrariwise.

The first case.

When the vnequall lines geuen are distinct the one from the other, the figure before put serueth.

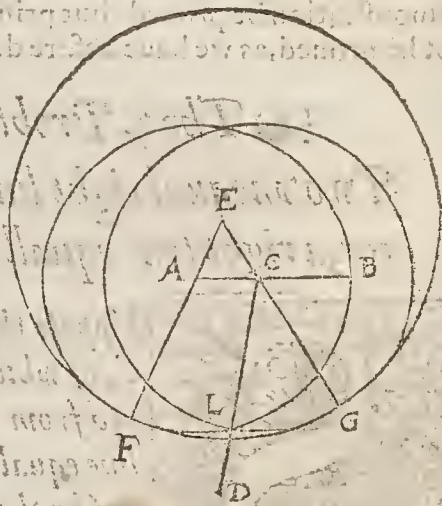
The second case.

If they be ioyned together at one of their endes, it is easie to do. For making the centre that end where they are ioyned together, & the space the lesse line, describe a circle: whiche shall of necessitie (by the definition of a circle) cut of from the greater line a line equal to the lesse line, as it is playne to see in the figure here put.



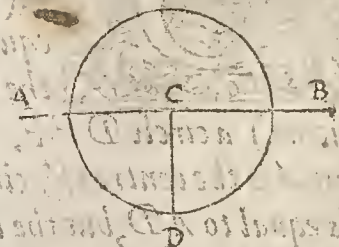
The third case.

But if the one cut the other in one of the extreames. As for exāple: Suppose that the vnequall right lines geuen be AB and CD , of which let the line CD be the greater: And let the line CD cut the line AB in his extreame C . Then making the centre A and the space AB , describe a circle BF . And ypon the line AC describe an equilater triangle (by the first) which let be AEC : & produce the lines EA and EC . And againe making the centre E and the space EA describe a circle GF . Likewise making the centre C and the space CE , describe a circle GL . Now foras much as the line EA is equal to the line EC (For E is the centre) of which the line EA is equal to the line EC : therefore the residue AF is equal to the residue CG . But the line AF is equal to the line AB , for A is the centre, wherefore also the line CG is equal to the line AB . But the line CG is also equal to the line CL , for the point C is the centre. Wherefore the line AB is equal to the line CL . Wherefore from the line CD is cut of the line CL which is equal to the line AB .



The fourth case.

But now let CD be lesse then AB , and let it cut AB by his extreame C . Now then eyther it cutteth it in the middest or not in the middest. First let it cut it in the middest: then CD is ether the halfe of AB , & so is AC equal to CD .

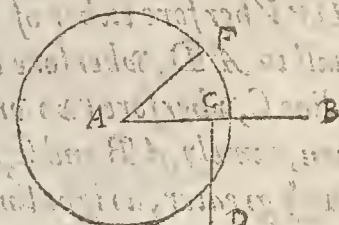


The fifth case.

Or it is lesse then the halfe: and then making the centre C & the space CD describe a circle, which shall cut of from the line AB a line equal to the line CD .

The sixth case.

Or it is greater then the half. And then vnto the point A put the line AF equal to the line CD , by the second. And making the centre A & the space AF describe a circle, which shall cut of from the line AB a line equal to the line AF , that is, vnto the line CD .



But

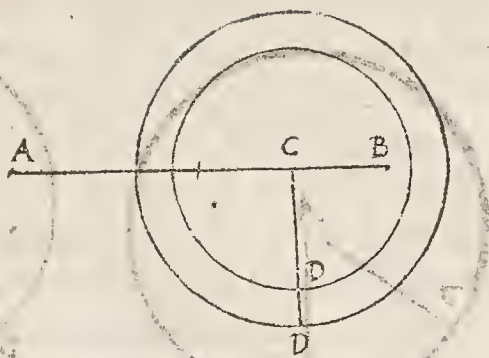
But if the line CD do not cut the line AB in the midft: CD fhall either be the halfe of the line AB ; or greater then the halfe, or leffe. If CD be the halfe of AB , or leffe then the half of AB , the making the centre C , and the fpace CD describe a circle whiche fhall cut of from the line AB a line equal to the line CD .

But if it be greater then the halfe, then againe vnto the point A put the line AF equal to the line CD (by the fecond propofitiō:) & making the centre A , and the fpace AF describe a circle which fhall cut of from the line AB a line equall to the line AF , that is, to the line CD .

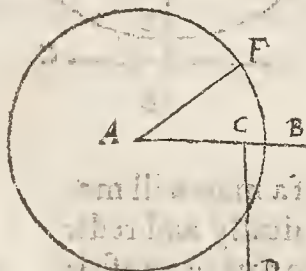
But if they cut the one the other as the lines CD & AB do. The making the centre B & the fpace BA describe a circle AF , & draw a line from the point B to the point C , & produce it to the point F . And forasmuch as the two right lines BF and CD are vnequall, and the line CD cutteth the line BF by one of his extreames, therefore it is poffible to cut of from CD a line equal to the line BF . For how to do it we haue before declared, wherefore it is poffible from the line CD to cut of a line equal to the line AB : or AB and BF are equal the one to the other.

This is to be noted, that in all thefe cafes, a man may both as touching construction and demonstration, procede as in the firft cafe. For it is poffible in any pofition to put to the ende of the greater lyne a line equal to the leffe lyne, and fo making the centre the fayd ende, and the fpace the leffe line, to describe a circle, which fhall cut of frō the greater lyne a lyne equal to the line put, namely, to the leffe line geuen, as it is manifeft to feem the figures partly here vnder fet, and partly in the beginning of the other fide put.

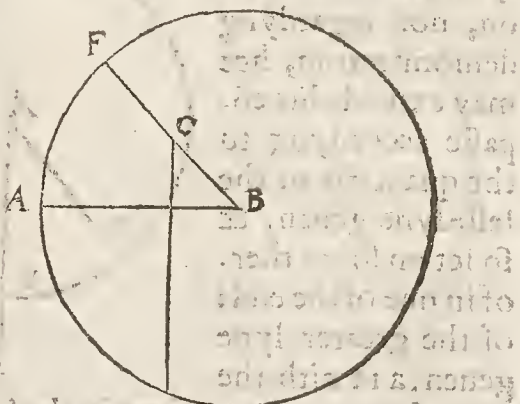
The feuenth & eight cafes.



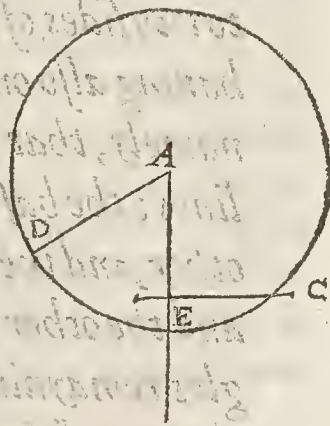
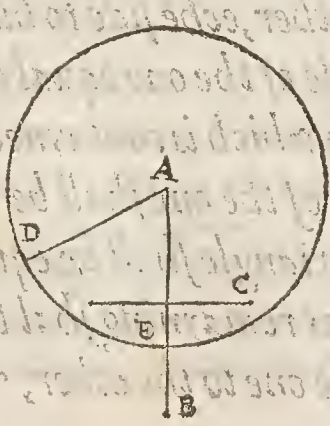
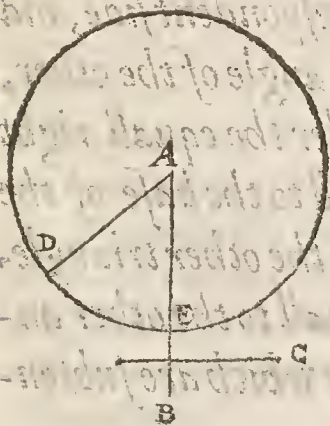
The ninth cafe.

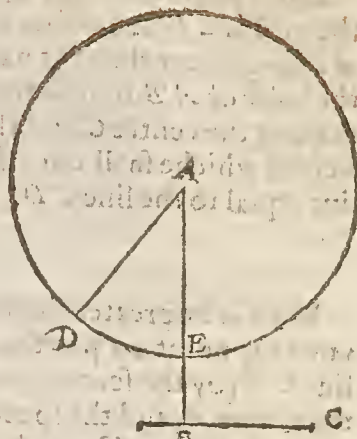
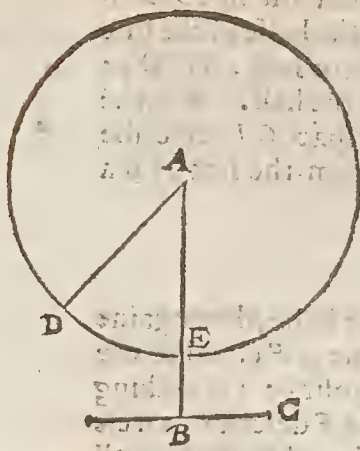
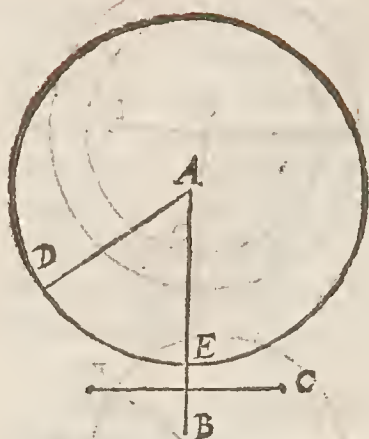


The tenth cafe.



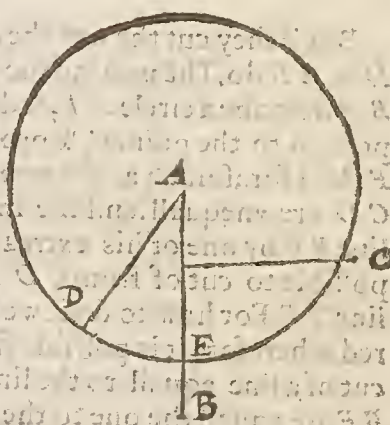
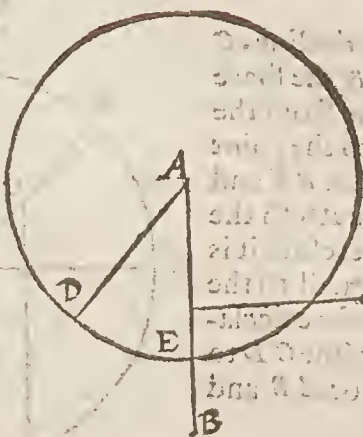
In all thefe cafes the construction and demonstration of the firft cafe will ferue.





This Proposition, though it be most easie to be done mechanically, yet is no principle.

If a man will mechanically and redily do this proposition, not regarding demonstration, hee may extend his compasse accordyng to the quantitie of the lesse lyne geuen, & so set on foote thereof in one of the ends of the greater lyne geuen, and with the



other foote marke a pointe in the said greater line, which shall cutte of from the greater line a line equall to the lesse. The easines of doing wherof may cause this proposition also to seeme vnto some to be rather a principle, then a proposition. But to that we haue before in the former proposition answered.

The first Theoreme.

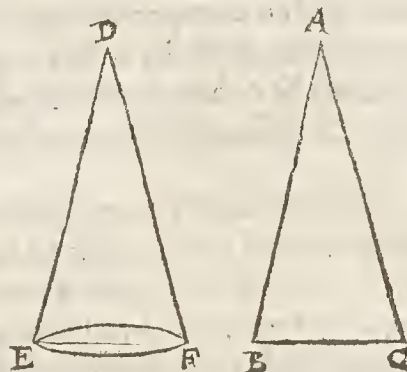
The 4. Proposition.

If there be two triangles, of which two sides of th^e one be equal to two sides of the other, eche side to his correspondent side, and hauing also on angle of the one equal to one angle of the other, namely, that angle which is containd vnder the equall right lines: the base also of the one shall be equal to the base of the other, and the one triangle shall be equal to the other triangle, and the other angles remayning shal be equal to the other angles remayning, the one to the other, vnder which are subtended equall sides.

Suppose



Suppose that there be two triangles ABC , & DEF , ha-
 uing two sides of the one, namely AB , and AC , equall to
 two sides of the other, namely, to DE and DF , the one to
 the other, that is, AB to DE , and AC to DF : hauning al-
 so the angle BAC , equall to the angle EDF . Then I say
 that the base also BC is equall to EF : & $\angle$ the tri-
 angle ABC , is equall to the triangle DEF : and $\angle$ the o-
 ther angles remainyng are equall to the other angles re-
 mayning, the one to the other, vnder which are subtended equall sydes: that is, $\angle$
 the angle ABC is equall to the angle DEF , and $\angle$ the angle ACB is equall to
 to the angle DFE . For the triangle ABC ex-
 actly agreyng with the triangle DEF , and the
 point A being put vpon the point D , & the right
 line AB vpon the right line DE , the point B
 also shall exactly agree with the point E : for
 that (by supposition) the line AB is equal to the
 line DE . And the line AB exactly agreyng
 with the line DE , the right line also AC exact-
 ly agreeth with the right line DF , for that (by
 supposition) the angle BAC is equall to the an-
 gle EDF . And forasmuch as the right line AC is also (by supposition) equall to
 the right line DF , therefore the point C exactly agreeth with the point F . A-
 gaine forasmuch as the point C exactly agreeth with the point F , and the
 point B exactly agreeth with the point E : therefore the base BC shall exactly
 agree with the base EF . For if the point B do exactly agree with the point E ,
 and the point C with the point F , and the base BC do not exactly agre wyth the
 base EF , then two right lines do include a superficies: which (by the 10. comon
 sentence) is impossible. VVherfore the base BC exactly agreeth w the base EF ,
 and therefore is equall vnto it. VVherfore the whole triangle ABC exactly a-
 greeth with the whole triangle DEF , & therefore (by the 8. common sentence)
 is equall vnto it. And (by the same) the other angles remainyng exactly agree
 with the other angles remainyng, and are equall the one to the other: that is, the
 angle ABC to the angle DEF , and the angle ACB to the angle DFE . If
 therefore there be two triangles, of which two sides of the one, be equall to two
 sydes of the other, eche to his correspondent side, and hauning also one angle of the
 one equall to one angle of the other, namely, that angle which is contayned vnder
 the equall right lines: the base also of the one shall be equall to the base of the
 other, and the one triangle shall be equal to the other triangle, and the other an-
 gles remainyng shall be equall to the other angles remainyng, the one to the o-
 ther, vnder which are subtended equall sydes: whiche thing was required to be
 demonstrated.



Demonstration
 leading to an
 absurditie.

This Proposition which is a Theoreme, hath two things geuen: namely, the
 E.1. equality

Two things ge-
 uen in this proa-
 position.

The first Booke

Three things
required in it.

equality of two sides of the one triangle, to two sides of the other triangle, and the equalitie of two angles containd vnder the equall sydes. In it also are three things required. The equality of base to base: the equality of field to field: and the equality of the other angles of the one triangle to the other angles of the other triangle, vnder which are subtended equall sides.

How one side is
equall to an o-
ther, & so gene-
rally how one
right line is e-
quall to an o-
ther.

One side of a playne figure is equall to an other, and so generally one right lyne is equall to an other, when the one being applied to the other, theyr extreames agree together. For otherwise euey righte line applied to any right lyne, agreeth therwith: but equall right lines only, agree in the extremes.

How one rectilin-
ed angle is e-
quall to an other

One rectilined angle is equall to an other rectilined angle, when one of the sides which comprehendeth the one angle, being set vpon one of the sides which comprehendeth the other angle, the other side of the one agreeth with the other syde of the other. And that angle is the greater, whose syde falleth without: and that the lesse, whose syde falleth within.

Why this parti-
cle, each to his
correspondent
side, is put.

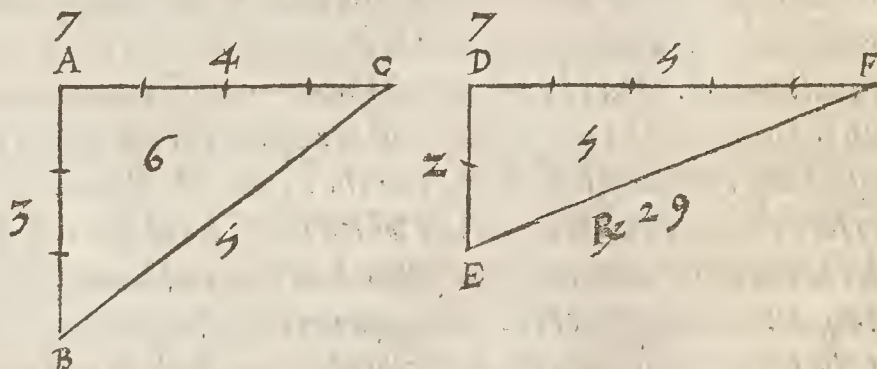
VVhere as in this proposition is put this particule *each to his correspondent side*, (in stede wherof often times after ward is vsed this phrase *the one to the other*) it is of necessity so put. For otherwise two sydes of one triangle added together, may be equall to two sydes of an other triangle added together, and the angles also containd vnder the equall sydes may be equall: and yet the two triangles may notwithstanding be vnequall. VVhere note that a triangle is sayd to be equall to an other triangle, when the field or area of the one is equall to the area of the other. And the area of a triangle, is that space, which is containd within the sydes of a triangle. And the circuite or compasse of a triangle is a line composed of all the sides of a triangle. And so may you think of all other rectilined figures. And now to proue that there may be two triangles, two sydes of one of which being added together, may be equall to two sydes of the other added together, and the angles containd vnder the equall sydes may be equall, and yet notwithstanding the two triangles vnequall. Suppose that there be two rectangle triangles: namely, A B C, and D E F, and let their right angles be B A C and E D F. And in the triangle A B C let the syde A B be 3. and the syde A C 4. which both added together make 7.

How one trian-
gle is equall to
an other.

What the field
or area of a tri-
angle is, and so
of any rectilined
figure.

What the cir-
cuite or compasse
of a triangle is,
and so also of a-
ny rectilined fi-
gure.

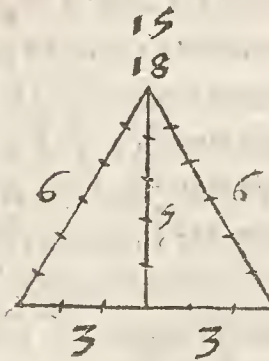
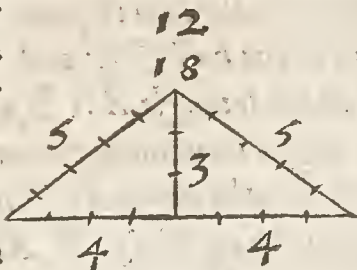
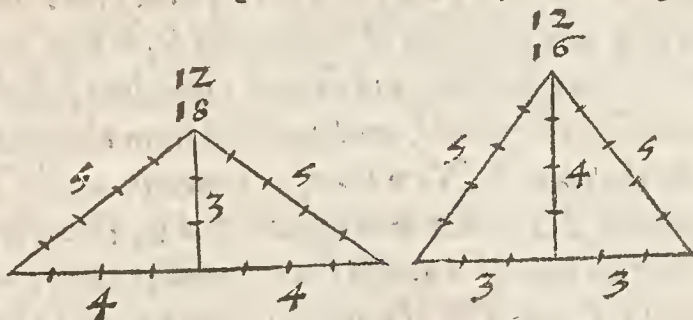
And in the triagle D E F, let the side D E be 2, and the side D F be 5, whiche added together make also 7. & so the



sydes of the one triangle added together, are equall to the sides of the other triagle added together. Yet are both the triangles vnequall, and also their bases. For the area of the triangle A B C is 6. and his base is 5. And the area of the triangle D E F is 5: and his base is 2.9. So that to haue the areas of two triangles to be equall, it is requisite that all the sydes of the two triangles be equall, each to his correspondent syde. It happeneth also sometymes in triangles, that the areas of them beyng equall, their sydes added together shall be vnequall. And contrari-

wise

wise, their sides beyng equall, their areas be vnequall. As in these figures here put it is plaine to see. In the first example the areas of the two triangles are equal, for they are eche 12, and the sides in ech added together are vnequall, for in the one triangle the sides added together make 18. and in the other they make 16. But in the second exāple the areas of the two triāgles are vnequal, for the one is 12, and th' other is 15. But the sides added together in eche are equall, for in eche they make 18.



How an angle is sayd to subtend a side: and a side an angle.

That angle is said to subtend a side of a triāgle, which is placed directly opposite, & against that side. That side also is sayd to subtend an angle, which is opposite to the angle. For euery angle of a triangle is contayned of two sydes of the triangle, and is subtended to the third side.

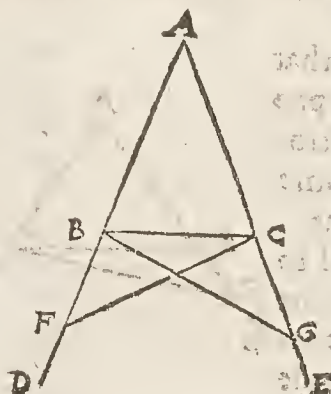
This is the first Proposition in which is vsed a demonstration leading to an absurditie, or an impossibilitie. VVhich is a demonstration that proueth not directly the thing entended, by principles, or by things before proued by these principles: but proueth the contrary therof to be impossible, & so doth indirectly proue the thing entended.

This proposition proued by a demonstration leading to an absurdity.

The 2. Theoreme. The 5. Proposition.

An Isofceles, or triangle of two equal sides, hath his angles at the base equall the one to the other. And those equal sides being produced, the angles which are vnder the base are also equall the one to the other.

Suppose that ABC be a triangle of two equall sydes called Isofceles, hauing the syde AB equall to the side AC . And (by the second peticio) produce the lines AB & AC directly to y points D & E . The I say, that y angle ABC is equal to the angle ACB : and y angle $CB D$ is equal to th' angle BCE . Take in the line BD a point at all aduentures, and let the

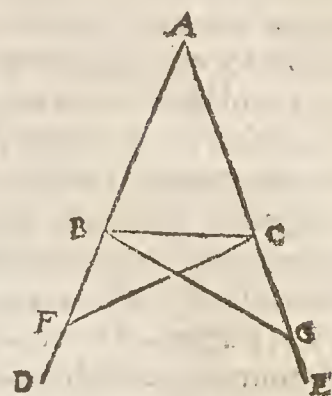


Constructions.

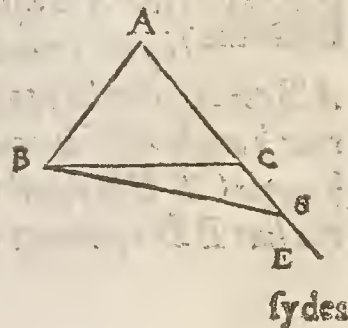
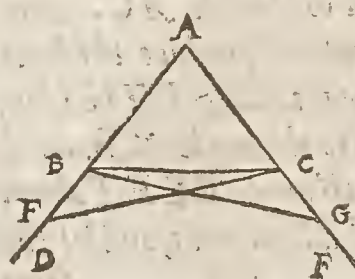
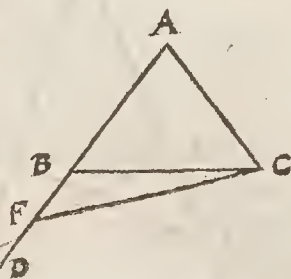
The first Booke

Demonstration.

same be F , and (by the third proposition) from the greater line, namely, AE , cut off a line equall to AF being the lesse line, and let the same be AG : and draw a right line frō the point F to the point C , and an other from the point G to the point B . Now then for as much as AF is equall to AG , and AB is equall to AC , therefore these two lines FA and AC are equall to these two lines GA and AB , the one to the other, and they containe a common angle, namely, that which is contained vnder FAG : wherfore (by the fourth proposition) the base FC is equall to the base GB : and the triangle AFC is equall to the triangle AGB , and the other angles remaining, are equall to the other angles remaining the one to the other, vnder which are subtended equall sides: that is, the angle ACF is equall to the angle ABG , and the angle AFC is equall to the angle AGB . And forasmuch as the whole line AF is equall to the whole line AG , of which the line AB is equal to AC , therefore the residue of the line AF , namely, the line BF , is equal to the residue of the line AG , namely, to the line CG (by the third common sentence) And it is proued that CF is equal to GB . Now therefore these two BF & FC are equall to these two CG and GB the one to the other, and the angle BFC is equall to the angle CGB , and they haue one base, namely, BC , common to them both: wherfore (by the 4. proposition) the triangle BFC is equall to the triangle CGB , and the other angles remainyng are equall to the other angles remaining eche to other, vnder which are subtended equall sides. VVherfore the angle FBC is equall to the angle GCB , and the angle BCF is equall to the angle CBG . Now forasmuch as the whole angle ABG is equall to the whole angle ACF (as it hath bene proued) of which the angle CBG is equal to the angle BCF : therefore the angle remainyng: namely, ABC is equall to the angle remaining, namely, to ACB (by the third common sentence) And they are the angles at the base of the triangle ABC . And it is proued that the angle FBC is equall to the angle GCB , and they are angles vnder the base. VVherfore a triangle of two equall sides hath his angles at the base equall the one to the other. And those equall sides being produced, the angles which are vnder the base are also equall the one to the other: which was required to be proued.



For that
this pro
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what
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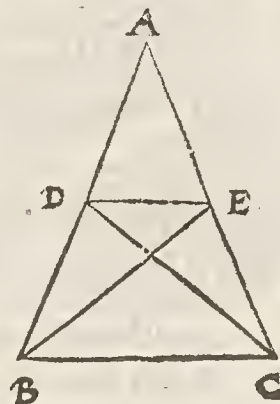


sydes

sides of the triangles AFC and AGB & also the sydes of the triāgles BFC & CGB run so one within an other, therefore I haue here put the distinctly, names ly, the triangles FAC and BFC on one syde of the figure of the propositiō & the triangles AGB and CGB on the other syde: so that you may with lesse labor see the demonstration playnely.

That in an Isoceles triangle, the two angles aboue the base are equall, may otherwise be demonstrated without drawing lines beneath the base somewhat altering the construction. Namely, drawing the lines within the triangle, whiche before were without it after this manner.

Suppose that ABC be an Isoceles triangle: and in the line AB take a point at all aduentures, and let the same be D , And from the line AC cut of a line equall to the line AD . Which let be AE . And draw these right lines BE , DC , and DE . Now forasmuch as in the triangles, ABE , and ACD , the side AB is equall to the side AC , by supposition, and the sides AD and AE are also equall by construction, and the angle at the poynt A is common to them both: therefore, by the fourth proposition, the base BE is equall to the base DC . And, by the same, the angles remayning of the one triangle are equall to the angles remayning of the other triangle. Wherefore the angle ABE is equal to the angle ACD . Againe forasmuch as in the triangles BDE , and CED the side DB is equall to the side EC , and the side BE to the side DC , and the angle DBE is equal to the angle ECD , and the base DE being common to both triangles is equall to it selfe: therefore the angles remayning of the one triangle, are equall to the angles remayning of the other triangle. Wherefore the angle EDB is equall to the angle DEC : & the angle DEB is equal to the angle EDC . And forasmuch as the angle EDB is equal to the angle DEC , frō which are taken away equall angles DEB , & EDC , therefore by the common sentence the angles remayning, namely, BDC and CEB are equall: And as it was before manifest the sides BD and DC are equall to the sides CE and EB the one to the other, that is, ech to his correspondent side: and the base BC is common to both the triangles: wherefore the angles remayning are equall to the angles remayning the one to the other, vnder which are subtēded equall sides. Wherefore the angle DBC is equall to the angle ECB . For the line DC subtēdeth the angle DBC , and the line EB subtēdeth the angle ECB : which two lines are as we haue before proued equall. Wherefore in an Isoceles triangle, the angles at the base are equall, though the right lines be not produced.



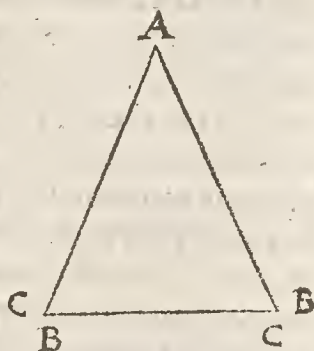
An other demonstration inuened by Proclus.

To proue this also, there is an other demonstration of Pappus much shorter which needeth no kind of addition of any thing at all: as followeth.

Suppose that ABC be an Isoceles triangle, & let the side AB be equall to the side AC . Now then vnderstand this one triangle to be as it were two triangles. And thus reason. Forasmuch as in the two triangles ABC and ACB , AB is equal to AC & AC to AB , therefore two sides of the one are equall to two sides of the other, ech to his correspondent side, & the angle BAC is equall to the angle CAB , for it is one and the selfe same angle. Wherefore by the 4. proposition the base CB is equall to the base BC , and the triangle ABC is equall to the triangle ACB : and the angle ABC is equall to the angle ACB , and the angle ACB to the angle ABC : for vnder them are subtēded equall sides, naniely, the lines AB & AC . Wherefore in an Isoceles triangle, the angels at the base are equall.

E.iii.

The



An other demonstration inuened by Pappus.

The first Booke

Thales Milesius
the inuenter of
this proposition.

The old Philosopher Thales Milesius was the first inuenter of this fifth proposition, as also of many other.

The third Theoreme. The sixth Proposition.

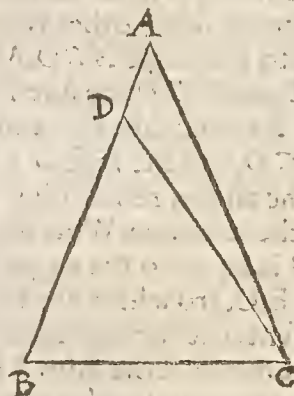
If a triangle haue two angles equall the one to the other: the sides also of the same, which subtend the equall angles, shall be equall the one to the other.

Construction.



Demonstration
leading to an
impossibility.

Suppose that ABC be a triangle, hauing the angle ABC equall to the angle ACB . Then I say that the side AB is equall to the side AC . For if the side AB be not equal to the side AC , then one of them is greater. Let AB be the greater. And by the third proposition, from AB being the greater cut of a line equal to the lesse line, which is AC . And let the same be DB . And drawe a line from the poynt D to the poynt C . Now forasmuch as the side DB is equall to the syde AC , and the line BC is common to the both: therefore these two sydes DB and BC are equall to these two sydes AC & CB the one to the other. And the angle DBC is by supposytion equall to the angle ACB . Wherefore (by the 4. proposytion) the base DC is equall to the base AB : & (by the same) the triangle DBC is equall to the triangle ACB : namely, the lesse triangle vnto the greater triagle, which is impossible. Wherefore the syde AB is not vnequal to the side AC . Wherefore it is equal. If therefore a triangle haue two angles equall the one to the other: the sydes also of the same, which subtende the equall angles, shall be equall the one to the other: which was required to be demonstrated.



The chiefeft and
most proper kind
of conuerſion.

In Geometrie is oftentimes vsed conuerſion of propositions. As this proposition is the conuerſe of the proposition next before. The chiefeft and moſt proper kind of conuerſion is, when that which was the thing ſuppoſed in the former proposition, is the concluſion of the conuerſe and ſecond proposition: and contrarywiſe that which was concluded in the firſt, is the thing ſuppoſed in the ſecond. As in the fifth proposition it was ſuppoſed the two ſides of a triangle to be equal, the thing concluded is, that the two angles at the baſe are equal & in this proposition, which is the conuerſe therof is ſuppoſed that the angles at the baſe be equal. Which in the former proposition was the concluſion. And the concluſion is, that the two ſydes ſubtending the two angles are equal, which in the former proposition was the ſuppoſition. This is the chiefeft kind of conuerſion vniſorme and certayne.

There

There is an other kind of conuerſion, but not ſo full a conuerſion nor ſo perfect as the firſt is, VVhich happeneth in compoſed propoſitions, that is, in ſuch, which haue mo ſuppoſitions then one, and paſſe from theſe ſuppoſitions to one concluſion. In the conuerſes of ſuch propoſitiōs, you paſſe from the concluſion of the firſt propoſition, with one or mo of the ſuppoſitions of the ſame: & conclude ſome other ſuppoſition of the ſelfe firſt propoſition: of this kinde there are many in Euclide. Therof you may haue an example in the 8. propoſition being the conuerſe of the fourth. This conuerſion is not ſo vniforme as the other, but more diuers and vncertaine according to the multitude of the things geuen, or ſuppoſitions in the propoſition.

An other kind of conuerſion not ſo perfect as the firſt.

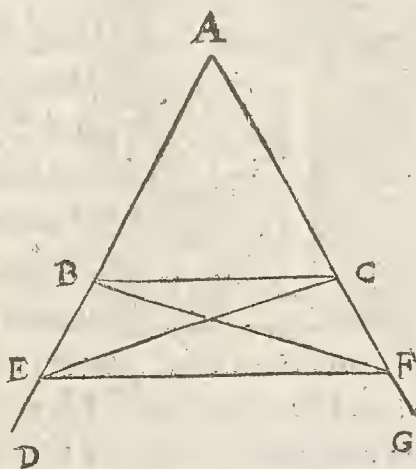
But becauſe in the fifth propoſition there are two concluſions, the firſt, that the two angles at the baſe be equall: the ſecond, that the angles vnder the baſe are equall: this is to be noted, that this fixt propoſition is the conuerſe of the ſame fifth as touching the firſt concluſion onely. You may in like maner make a conuerſe of the ſame propoſition touching the ſecond concluſion therof. And that after this maner.

Two concluſions in the fifth propoſition.

The fixt propoſition is the conuerſe as touching the firſt concluſion onely. The conuerſe as touching the ſecond concluſion.

THe two ſides of a triangle beyng produced, if the angles vnder the baſe be equall, the ſaid triangle ſhall be an Iſoſceles triangle. In which propoſitiō the ſuppoſition is, that the angles vnder the baſe are equall: which in the fifth propoſition was the concluſion: & the concluſion in this propoſition is, that the two ſides of the triangle are equal, which in the fiſt propoſition was the ſuppoſition. But now for prooſe of the ſaid propoſition:

Suppoſe that there be a triangle ABC , & let the ſides AB , and AC be produced to the poyntes D and G , and let the angles vnder the baſe be equall, namely, the angles DBC , and GCB . Then I ſay that the triangle ABC is an Iſoſceles triangle. For take in the line AD a point which let be E , And vnto the line BE put the line CF equall (by the 3. propoſitiō). And draw theſe lines EC , BF , and EF . Now for aſmuch as BE is equall to CF , and BC is common to the both, therefore theſe two lines BE & BC , are equall to theſe two lines CF and CB the one to the other, & the angle $EB C$ is equall to the angle $FC B$ by ſuppoſition. Wherefore (by the 4. propoſition) the baſe of the one is equall to the baſe of the other, and the one triangle is equal to the other triangle, & the other angles remayning are equal vnto the other angles remayning, the one to the other, vnder which are ſubtended equall ſides. Wherefore the baſe EC is equall to the baſe FB , and the angle BEC to the angle CFB , and the angle CBF to the angle BCE . But the whole angle $EB C$ is equall to the whole angle $FC B$, of which the angle $FC B$ is equall to the angle ECB : wherefore the angle remayning EBF is equall to the angle remayning FCE . But the line BE is equall to the line CF , & the line BF to the line CE , and they contayne equall angles: wherefore by the ſame fourth propoſition the angle BEF is equall to the angle CFE . Wherefore by this fixt propoſition the ſide AE is equall to the ſide AF : of which BE is equall to CF , by conſtruction: wherefore (by the third common ſentence) the reſidue AB is equall to the reſidue AC . Wherefore the triangle ABC is an Iſoſceles triangle. If therefore the two ſides of a triangle being produced, the angles vnder the baſe be equall, the ſayd triangle ſhall be an Iſoſceles triangle: which was required to be proued.



Conſtruction.

Demonſtration.

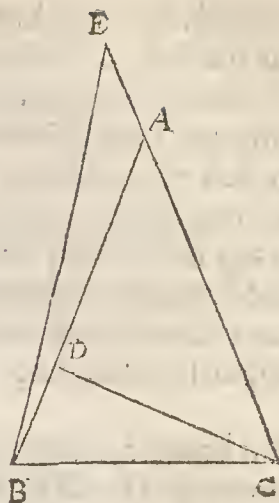
This moreouer is to be noted, that in this propoſition there may be an other caſe: for in taking an equall line to AC from AB , you may take it from the poynte A and

An other caſe in this fixt propoſition.

The first Booke

not from the poynt *B*. And yet though this supposition also be put the selfe same absurdity will follow.

For suppose that *AC* be equall to *AD*: and produce the line *CA* to the poynt *E*: and put the line *AE* equall to the line *DB* (by the third proposition) wherefore the whole line *CE* is equall to the whole line *AB* (by the second common sentēce) Draw a line from the poynt *E* to the point *B*. And forasmuch as the line *AB* is equall to the line *EC*, and the line *BC* is common to them both, and the angle *ACB* is supposed to be equall to the angle *ABC*: Wherefore (by the fourth proposition) the triangle *ECB* is equall to the triangle *ABC*, namelye, the whole to the part: which is impossible.



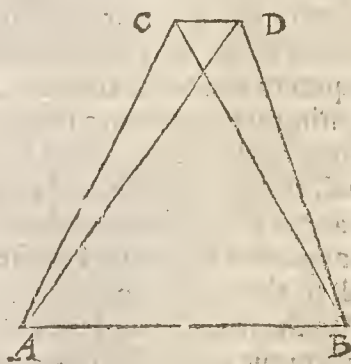
The 4. Theoreme. The 7. Proposition.

ffrom the endes of one line, be drawn two right lynes to any point: there can not frō the self same endes on the same side, be drawn two other lines equal to the two first lines, the one to the other, vnto any other point.



Or if it be possible: then from the endes of one & the self same right line, namely, *AB*, from the pointes (I say) *A* and *B*, let there be drawn two right lines *AC* and *CB* to the point *C*: and from the same endes of the line *AB*, let there be drawn two other right right lines *AD* and *DB* equall to the lines *AC* and *CB* the one to the other, is, eche to his correspondent line, and on one and the same

side, and to an other point, namely, to *D*: so that let *CA* be equall to *DA* beyng both drawn from one end, that is, *A*: & let *CB* be equall to *DB*, beyng both also drawn from one ende, that is, *B*. And (by the first petition) draw a right line from the point *C* to the point *D*. Now forasmuch as *AC* is equal to *AD*, the angle *ACD* also is (by the 5. proposition) equall to the angle *ADC*: wherefore the angle *ACD* is lesse thē the angle *BCD*. VVherefore the angle *BCD* is much lesse then the angle *BCD*. Againe forasmuch as *BC* is equall to *BD*, and therefore also the angle *BCD* is equall to the angle *BDC*. And it is proued that it is much lesse then it: which is impossible. If therefore from the endes of one line, be drawn two right lines to any point: there can not from the selfe same endes on the same side, be drawn two other lines equall to the two first lines, the one to the other, vnto any other point: VVhich was required to be demonstrated.



*Demonstration
leading to an
absurditie.*

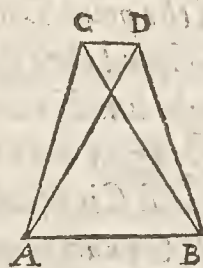
In

In this proposition the conclusion is a negation, which very rarely happeneth in the mathematicall artes. For they euer for the most part vse to conclude affirmatiuely, & not negatiuely. For a proposition vniuersall affirmative is most agreeable to sciences, as saith *Aristotle*, and is of it selfe strong, and needeth no negatiue to his prooffe. But an vniuersall proposition negatiue must of necessitie haue to his prooffe an affirmative. For of onely negatiue propositions there can be no demonstrations. And therefore sciences vsing demonstration, conclude affirmatiuely, and very seldome vse negatiue conclusions.

Negatiue conclusions rarely vsed in the mathematicall artes.

An other demonstration after Campanus.

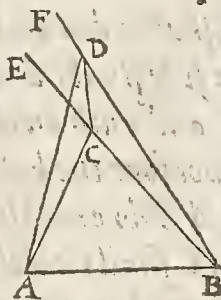
Suppose that there be a line AB , from whose ends A and B , let there be drawn two lines AC and BC on one side, which let concur in the poynt C . Then I say that on the same side there cannot be drawn two other lines, from the endes of the line AB , which shall concur at any other poynt, so that that which is drawe from the point A shall be equall to the line AC , and that which is drawn from the point B shall be equall to the line BC . For if it be possible, let there be drawn two other lines on the selfe same side, which let concur in the point D , and let the line AD be equall to the line AC , & the line BD equall to the line BC . Wherefore the poynt D shall fall either within the triangle ABC , or without. For it cannot fall in one of the sides, for then a parte should be equall to his whole. If therefore it fall without; then either one of the lines AD and DB shall cut one of the lines AC and CB , or els neither shall cut neyther. Firste let one cut the other and draw a right line from C to D . Now forasmuch as in the triangle ACD , the two sides AC and AD are equall, therefore the angle ACD is equall to the angle ADC , by the fifth proposition: likewise forasmuch as in the triangle BCD , the two sides BC and BD are equall, therefore by the same, the angles BCD & BDC are also equall. And forasmuch as the angle BDC is greter then the angle ADC , it followeth that the angle BCD is greater then the angle ACD , namely, the part greater then the whole: which is impossible.



Diuers cases in this demonstration.

First case.

But if the point D fall without the triangle ABC , so that the lines cut not the one the other, draw a line from D to C . And produce the lines BD & BC beyond the base CD , vnto the points E & F . And forasmuch as the lines AC and AD are equall, the angles ACD and ADC shall also be equall, by the fifth proposition: likewise forasmuch as the lines BC and BD are equal, the angles vnder the base, namely, the angles FDC and EDC are equall, by the seconde part of the same proposition. And for as much as the angle ECD is lesse then the angle ACD : It followeth that the angle FDC is lesse then the angle ADC : which is impossible: for that the angle ADC is a part of the angle FDC . And the same inconuenience will follow if the poynt D fall within the triangle ABC ,



Second case.

The fift Theoreme. The 8. Proposition.

If two triangles haue two sides of th' one equall to two sides of the other, eche to his correspondent side, & haue also the base of the one equall to the base of the other: they shall haue also the angle contained vnder the equall right lines of the one, equall to the angle containd vnder the equall right lynes of the other.

Fi.

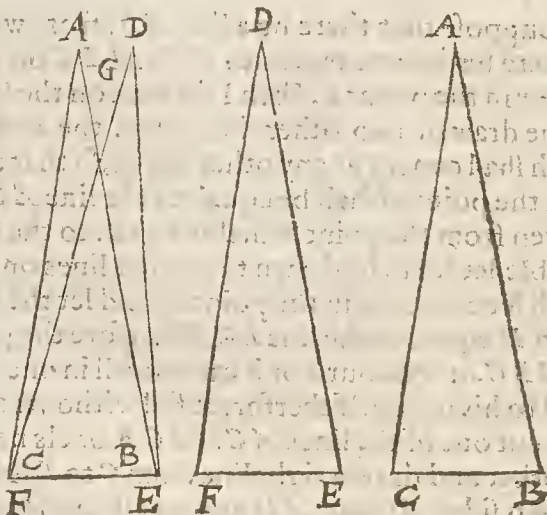
Suppose

The first Booke



*Demonstration
leading to an
impossibility.*

Suppose that there be two triangles ABC and DEF : & let these two sides of the one AB and AC , be equall to these two sides of the other DE , and DF , ech to his correspondent side, that is, AB to DE , and AC to DF , & let the base of the one, namely, BC be equal to the base of the other, namely, to EF . Then I say, that the angle BAC is equall to the angle EDF . For the triangle ABC exactly agreeing with the triangle DEF , and the point B being put vpon the point E , and the right line BC vpon the right line EF : the point C shall exactly agree with the point F (for the line BC is equall to the line EF). And BC exactly agreeing with EF the lines also BA and AC shall exactly agree with the lines ED & DF . For if the base BC do exactly agree with the base FE , but the sides BA & AC doo not exactly agree wth the sides ED & DF , but differ as FG & GF do: the from y^e endes of one



lyne shalbe drawn two right lines to a poynt, & from the self same endes on the same side shalbe drawn two other lines, equal to the two first lines, y^e one to the other, and vnto an other poynt: but that is impossible, (by the seventh proposition) VVherfore the base BC exactly agreeing with the base EF , the sides also BA and AC do exactly agree with the sides ED and DF . VVherfore also the angle BAC shall exactly agree wth the angle EDF , and therefore shall also be equal to it. If therefore two triangles haue two sides of the one equall to two sides of the other, ech to his correspondent side, and haue also the base of y^e one equall to the base of the other: they shall haue also the angle containd vnder the equall right lines of the one, equall to the angle containd vnder the equall right lines of the other: which was required to be proued.

*This proposition
is the conuerse
of the fourth,
but not the chiefest
kind of con-
uerſion.*

This Theoreme is the conuerſe of the fourth, but it is not the chiefest and principall kind of conuerſion. For it turneth not the whole ſuppoſition into the concluſion, and the whole concluſion into the ſuppoſition. For the fourth propoſitiō whoſe conuerſe this is, is a cōpound theoreme, hauing two things geue^d or ſuppoſed, which are theſe: the one, that two ſides of the one triāgle be equal to two ſides of the other triāgle: th^e other, that the angle cōtaind of the two ſides of th^e one is equal to the angle contained of the two ſides of th^e other: but hath amongſt other one thing required, whiche is, that the baſe of the one, is equal to the baſe of the other. Now in this 8. propoſitiō, being the conuerſe thereof: that the baſe of the one is equal to the baſe of th^e other, is the ſuppoſition, or the thing geue^d; which in the former propoſitiō was the concluſiō. And this, that two ſides of
the

the one are equall to two sides of the other, is in this proposition also a supposition, like as it was in the former proposition; so that it is a thing geuen in either proposition. The conclusion of this proposition is that the angle enclosed of the two equall sides of the one triangle is equall to the angle enclosed of the two equall sides of the other triangle: which in the former proposition was one of the things geuen.

Philo and his scholas demonstrate this proposition without the helpe of the former proposition, in this maner.

Suppose that there be two triangles ABC and DEF , hauing two sydes of the one equall to two sydes of the other, namely, AB and AC equall to DE and DF , the one to the other, & the base BC equall to the base EF . And for that the base BC is equall to the base EF , therefore the one being applied to the other they agree. Place the two triangles ABC & DEF in one & the self same plaine superficies, & apply the base of the one to the base of the other: But yet so that the triangle ABC be set one the other side of the right line EF , that the top of the one may be opposite to the top of the other. And in stead of the triangle ABC put the triangle EFG as in the figure. And let DE be equall to EG , and DF to FG . Nowe then by this meanes shall happen diuers cases. For the line FG may fall directly vpo the line DF , or it may so fall that it may make with the line DF an angle within the figure, or with out.

An other demonstration inuented by Philo.

After this demonstration three cases in this proposition.

First let it fall directly. And forasmuche as the line DE is equall to the line EG , and DF to FG , is one right line: therefore DEG is an Isoceles triangle: and so, by the fifth proposition, the angle at the point D is equal to the angle at the poynt G : which was required to be proued.

The first case.

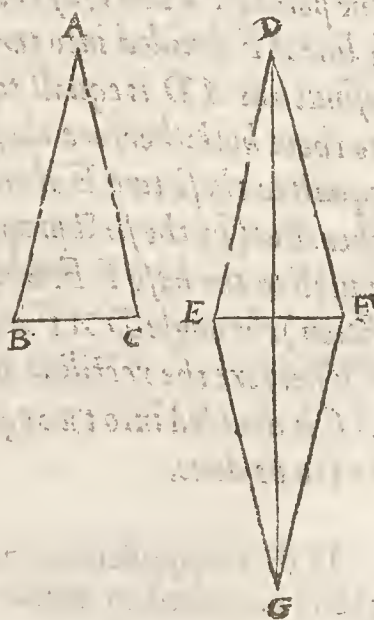
But if it fall not directly, but make with the line DF an angle within the figure, draw a line from D to G . Now forasmuch as ED and EG are equall, and the line DG is the base: therefore by the fifth proposition, the angle EDG is equall to the angle EGD . Agayne forasmuch as DF is equall to FG , and DG is the base: therefore by the same, the angle FDG is equal to the FGD : and it was proued that the angle EDG is equall to the angle EGD : wherfore the whole angle EDF is equal to the whole angle FGE : which was required to be proued.

The second case.

But if the line FG make with the line DF an angle without the figure draw a right line without the figure

F.ij.

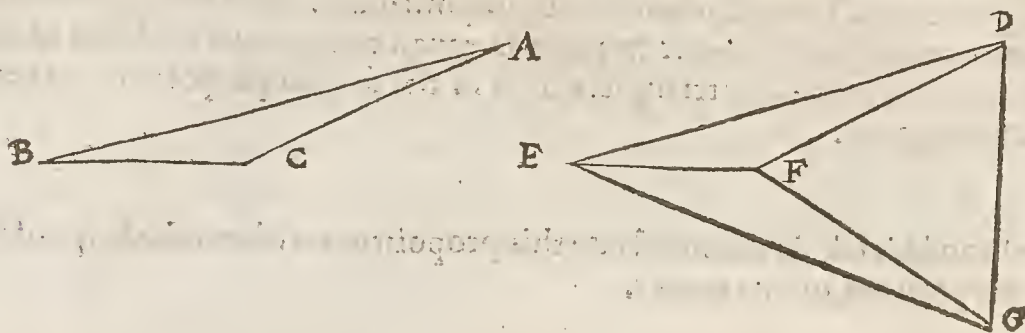
from



The third case.

The first Booke

from the poynt D to the poynt G. And forasmuch as D E and E G are equall, and D G is the base, therefore by the fifth proposition, the angles E D G and D G E are equall. A-



gaine forasmuch as D F is equall to F G, and D G is the base, therefore, by the same, the angle F D G is equall to the angle F G D. And it was proued that the whole angles E D G & D G E are equall the one to the other: wherfore the angles remayning E D F & E G F are equall the one to the other, which was required to be proued.

The 4. Probleme. The 9. Proposition.

To deuide a rectiline angle geuen, into two equall partes.

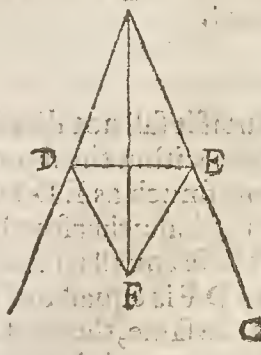
Construction.



*S*uppose that the rectiline angle geuen be B A C. It is required to deuide the angle B A C into two equal partes. In the line A B take a point at all aduentures, & let the same be D. And (by the third proposition) from the lyne A C cutte of the line A E equall to A D. And (by the first petition) draw a right line from the point D to the point E. And (by the first proposition) vpon the line D E

Demonstration.

describe an equilater triangle and let the same be D F E, and (by the first petition) drawe a right line from the poynte A to the point F. Then I say that the angle B A C is by lyne A F deuided into two equal partes. For, forasmuch as A D is equall to A E, and A F is comon to them both: therefore these two D A and A F, are equall to these two E A and A F, the one to the other. But (by the first proposition) the base D F is equall to the base E F: wherfore (by the 8. proposition) the angle D A F is equal to the angle F A E. Wherfore the rectiline angle geuen, namely, B A C is deuided into two equal partes by the right line A F. Which was required to be done.



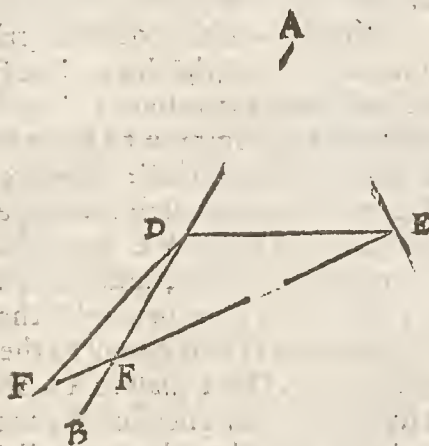
In this proposition is not taught to deuide a right lined angle into mo partes then two: albeit to deuide an angle, so it be a right angle, into three partes, it is not

not hard. And it is taught of *Vitellio* in his first booke of Perspective, the 28. Proposition. For to deuide an acute angle into three equal partes, is (as saith *Proclus*) impossible: vnles it be by the helpe, of other lines which are of a mixt nature. Which thing *Nicomedes* did by such lines which are called *Concoides linea*, who first serched out the inuention, nature, & properties of such lines. And others did it by other meanes, as by the helpe of quadrant lines inuented by *Hippias* & *Nicomedes*. Others by *Helices* or *Spiral* lines inuented of *Archimedes*. But these are things of much difficulty and hardnes, and not here to be intreated of.

It is impossible to deuide an acute rectiline angle into three equall partes without the helpe of lines which are of a mixt nature.

Here against this proposition may of the aduersary be brought an instance. For he may cauill that the hed of the equilater triangle shall not fall betwene the two right lines, but in one of them, or without them both. As for example.

Suppose that the angle to be deuided into two equall partes be BAC , and in the line BA take the poynt D , and vnto the line DA put the line AE equal (by the third proposition.) And draw a line frō D to E . And vpon the line DE describe (by the first) an equilater triangle, which let be DFE . Now then if it be possible that the point F do not fall betwene the lines AB & AC , then it shal fall either in the line AB or AC , or without them both. Suppose that the point F be fall vpon line AB , so that let DFE be an equilater triangle. Wherefore the line DF is equal to the line FE : & the angles at the base are equall, namely, the angles EDF and DEF . Wherefore the whole angle DEC is greater then the angle EDF . Againe forasmuch as AD is equall to AE , therefore ADE is an Isoceles triangle. Wherefore (by the fifth proposition) the angles vnder the base are equall. Wherefore the angle DEC is equall to the angle EDB . But it was also greater; which is impossible. Wherefore the top of the equilater triangle cānot be in the right line AB . And in like sort also may we proue that it cānot be in the right line AC . Wherefore suppose that it be without them both, if it be possible. And forasmuch as DF is equal to FE , the angles at the base are equal, namely, the angles DEF & EDF . Wherefore the angle DEF is greater then the angle EDF . Wherefore the angle DEC is much greater then the angle EDF . But it is also equal vnto it. For they are angles vnder the base DE of the Isoceles triangle ADE . Which is impossible. Wherefore the poynt F shall not fall without the two right lines on that side. And in like sorte may we proue that it shall not fall without them on the other side. Wherefore it shall of necessity fall betwene them: which was required to be proued.



*An instance is an obiection or a doubt, whereby is lett ed or troubled the construction, or demonstration, & containeth an vntruth, and an impossibility: and therefore it must of necessity be answered vnto, and the falsehood thereof made manifest.

There may also in this proposition be diuers cases, if it so happen that there be no space vnder the base DE to describe an equilater triangle, but that of necessity, you must describe it on the same side that the lines AB and AC are. For then the sides of the equilater triangle either exactly agree with the lines AD and AE , if the said lines AD and AE be equall with the base DE . Or they fall without them, if the lines AD and AE be lesse then the base DE . Or they fall within them, if the said lines be greater then the base DE .

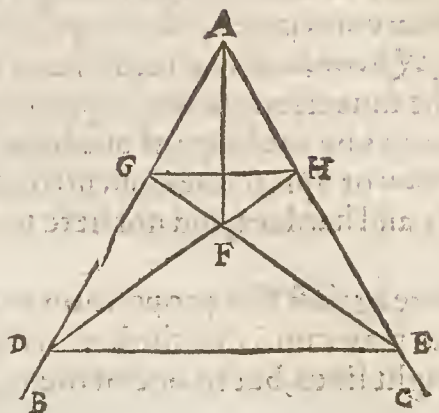
Diuers cases in this proposition.

First let them exactly agree, And let DAE be an equilater triangle. And in the side AD take the poynt G . And from the side AE cut of a line equal to the line AG (by the third proposition) which let be AH . And draw these right lines GE , HD and GH , and AF . Now forasmuch as AD is equal to AE , and AG vnto AH , therefore these two lines

The first case.

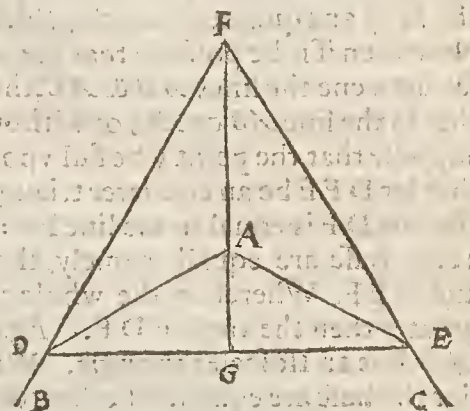
The first Booke

DA and AH are equall to these two lines EA and AG : and they contayne one and the selfe same angle. Wherefore by the fourth proposition, the angle GDH is equal to the angle HEG . And the base DH is equall to the base EG . But the line DG is equal to the line EH : wherefore againe by the fourth proposition, the angle EGH is equal to the angle DHG . Wherefore by the sixt proposition, the base GF is equal to the base HF . And forasmuch as AH is equall to AG , and AF is common to the both, and the base GF is equal to the base HF , therefore the angle GAF is equal to the angle HAF . Wherefore the angle GAH is devided into two equall partes: which was required to be done.



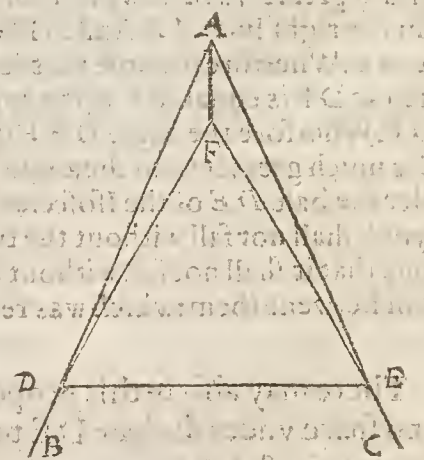
The second case.

But if the sides of the equilateral triangle fall without the right lines BA & AC , as do the lines DF & EF , the draw a line from F to A & produce the line FA to the point G . Now forasmuch as the lines DF and FE are equal, & the line FA is common to them both, & the bases DA and AE are equall: therefore (by the eight) the angle DFA is equal to the angle EFA . Againe forasmuch as DF and FE are equall, and FG is common to them both, and they containe equall angles (as it hath bene proued) therefore (by the fourth) the base DG is equall to the base GE . And forasmuch as AD is equall to AE , and AG is common to them both. Therefore (by the eight) the angle DAG is equal to the angle EAG . wherefore the angle DAE is devided into two equall partes: Which was required to be done.



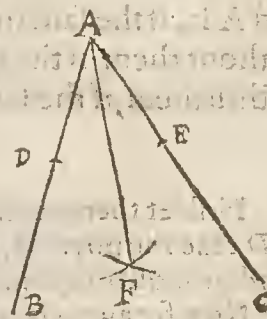
The third case.

But if the sides of the equilateral triangle fall within the right lines BA and AC , as do the lines DF and FE , then againe draw a line from A to F . And forasmuch as DA is equall to AE , and AF is common to them both, and the base DF is equal to the base FE : therefore the angle DAF is (by the eight) equall to the angle EAF . Wherefore the angle at the point A is devided into two equall partes, howsoeuer the equilateral triangle be placed: which was required to be done.



To deuide a rectiline angle into two equall partes Mechanically.

This is to be noted, that if a man will mechanically or readily, not regarding demonstration, deuide the foresaid rectiline angle BAC , and so any other rectiline angle geuen whatsoeuer, into two equall partes, he shall neede onely with one opening of the compasse taken at all aduentures to marke the two pointes D and E , which cut of equal partes of the lines AB and AC , howsoeuer they happen, and so making the centres the two pointes D and E , to describe two circles according to the openyng of the compasse: and from the point A to their intersection, which let be the point F to draw a rightline: which shall deuide the angle BAC into two equall partes. And here note, that you shall not neede to draw the circles all whole, but one:



ly a portion where they cut the one the other: As in the figure here in the end of the other side put.

The 5. Probleme. The 10. Proposition.

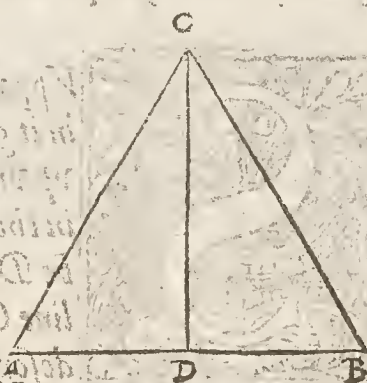
To deuide a right line geuen being finite, into two equall partes.



Suppose that the right line geuen be AB . It is required to deuide the line AB into two equal partes. Describe (by the first proposition) vpon the line AB an equilateral triangle and let the same be ABC . And (by the former proposition) deuide the angle ACB into two equall partes by the right line CD . Then I say that the

Construction.

right line AB is deuided into two equall partes in the poynt D . For forasmuch as (by the first proposition) AC is equall to CB , and CD is common to the both: therefore these two lines AC & CD are equal to these two lines BC & CD , y one to y other, and the angle ACD is equall to the angle BCD . Wherefore (by the 4. proposition) the base AD is equall to the base BD . Wherefore the right line geuen AB , is deuided into two equall partes in the poynt D : which was required to be done.



Demonstration.

Apollonius teacheth to deuide a right line being finite into two equall partes after this manner.

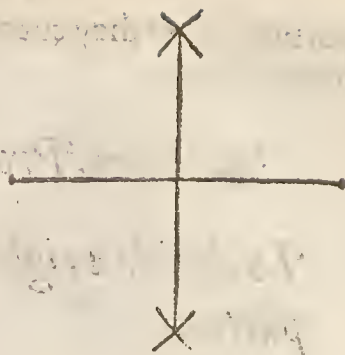
An other way to deuide a right line being finite, inuented by Apollonius.

Suppose that the right line being finite be AB : which it is required to deuide in to two equal parts. Now the making the centre the point A & the space AB describe a circle. Again making the centre the poynt B & the space BA describe another circle: and from the common sections draw the right line CD , which let cut the line AB in the poynt E . Then I say, that the right line CD deuideth the line AB into two equall partes in the point E . For draw these right lines DA , DB , CA and CB . Which shal be equal the one to the other, for that they are drawne from the centres to the circumferences of equall circles. And forasmuch as the lines CA & AD are equal to the lines CB and BD , and the line CD is common to either of them: therefore, by the eighth, the angle ACD is equall to the angle BCD . Again forasmuch as the line CE is common to the lines CA and CB , which are equal the one to the other: therefore these two lines CA and CE are equal to these two lines CB and CE , the one to the other, and they contayne equall angles, as it hath bene proued. Wherefore by the 4. proposition the base AE is equal to the base BE . Wherefore the line AB is deuided into two equall partes in the poynt E : which was required to be done.

F. liii.

By

By this way of deuinding a right line, into two equall parts inuented by Apollonius, it is manifest, that if a man wil mechanically, or redely, not considering the demonstratiō, deuide the said rightline, and so any right line geuen whatsoeuer, into two equall: partes he nede onely to marke the poynts of the interfections of the circles, & to draw a line frō the sayd interfections, which shall deuide the right line geuen into two equall partes: as in the figure here put.



The 6. Probleme. The 11. Proposition.

Vpon a right line geuen, to rayse vp from a poynt geuen in the same line a perpendicular line.



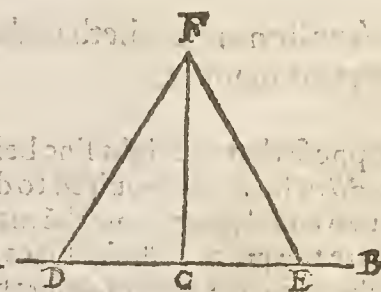
Construction.

Suppose that the right line geuen be AB , & let the point in it geuen be C . It is required from the poynte C to rayse vp vnto the right line AB a perpendicular line. Take in the line AC a poynt at all aduentures, & let the same be D , and (by the 3. proposition) put vnto DC an equall line CE . And by the first proposition) vpon the line DE describe an equilateral triangle FDE , & draw a line frō

F to C . Then I say that vnto the right line geuen AB , and from the poynte in it geuen, namely, C is raysed vp a perpendicular line FC . For forasmuch as DC is equal to CE , & the line CF is comon to them both: therefore these two DC and CE , are equal to these two EC & CF , the one to the other: and (by the first proposition) the base DF is equal to the base EF : wherefore (by the 8. proposition) the angle DCF is equal to the angle ECF : and they be side angles. But when a right line standing vpon a right line doth make the two side angles equall the one to the other, either of those equall angles is (by the 10. definition) a right angle: & the line standing vpon the right line is called a perpendicular line. Wherefore the angle DCF , & the angle ECF are right angles. Wherefore vnto the right line geuen AB , & frō the poynt in it C , is raysed vp a perpendicular line CF : which was required to be done.

Demonstration.

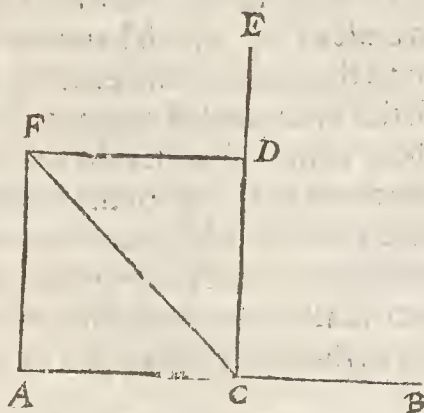
et poynt doli. 11.
adign n. d. 11.
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adign n. d. 11.
adign n. d. 11.



Although the poynte geuen should be set in one of the endes of the right line geuen, it is easy so to do it as it was before. For producing the line in length from the poynt by the second petition, you may worke as you did before. But if one require to erect a right line perpendicularly from the poynt at the end of the line

lyne, without producing the rightlyne, that also may well bee done after this maner.

Suppose that the right line geuen be AB , & let the point in it geuen be in one of the endes therof, namely, in A . And take in the line AB a point at all aduentures, and let the same be C . And from the said point raise vp (by the foresaid proposition) vnto AB a perpendicular line, which let be CE . And (by the 3. proposition) from the line CE cut of the line CD equall to the line CA . And (by the 9. Proposition) denide the angle ACD into two equall partes by the line CF . And from the point D raise vp vnto the line CE a perpendicular line, DF , which let concur with the line CF in the point F . And draw a right line from F to A . Then I say that the angle at the point A is a right angle. For, forasmuch as DC is equall to CA , and CF is common to them both, and they containe equall angles (for the angle at the point C is deuided into two equall partes) therefore (by the 4. Proposition) the line DF is equall to the line FA , and so the angle at the point A is equal to the angle at the point D . But the angle at the point D is a right angle. Wherefore also the angle at the point A is a right angle. Wherefore from the point A vnto the line AB , is raised vp a perpendicular line AF , without producing the line AB . Which was required to be done.



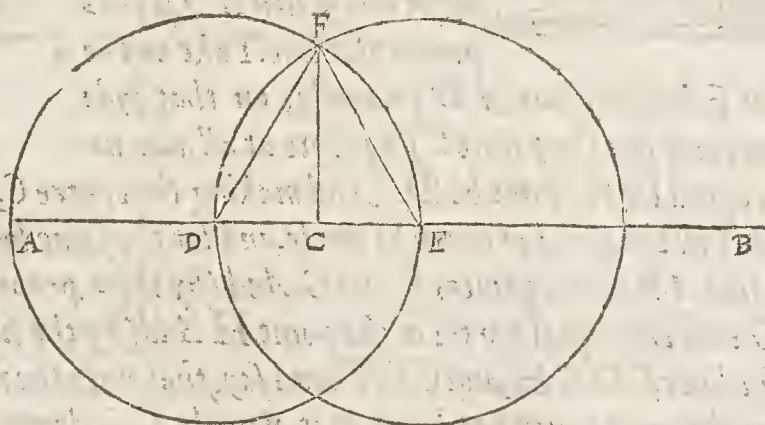
An other case in this proposition.

Construction.

Demonstration.

Appollonius teacheth to raise vp vnto a line geuen, from a point in it geuen, a perpendicular line, after this maner.

Suppose that the right line geuen be AB . And let the point in it geuen be C . And in the line AB take a point at all aduentures, & let the same be D . And from the lyne CB , cut of a line equall to the line CD , which let be CE : and making the centre D , and the space DE , describe a circle. And againe making the centre C , & the space ED , describe an



An other way to erect a perpendicular line inuented by Appollonius.

Construction.

other circle, and let the point of their interfection be F . And draw a right line from F to C . Then I say that the line FC is erected perpendicularly vnto the line AB . For draw these lines FD and FE : which shal by the definition of a circle be either of them equal to the line DE : and therefore (by the first common sentence) are equall the one to the other. But the lines DC and CE are by construction equall, and the line FC is common to them both. Wherefore the angles also at the point C are equal (by the 8. proposition:) wherefore they are right angles. Wherefore the line FC is erected perpendicularly vnto the line AB from the point C : which was required to be done.

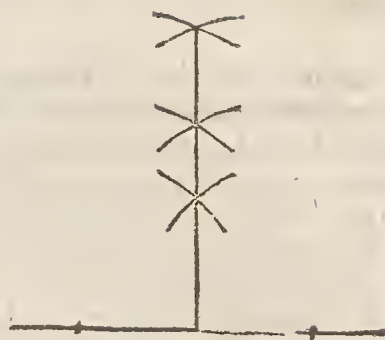
Demonstration.

By this way of erecting a perpendicular line inuented by Appollonius, it is also manifest, that if a man will mechanically, without demonstration, erect vnto a line

How to erect a perpendicular line mechanically.

The first Booke

a line geuen from a point geuē in it a perpendicular line: he neede onely on either side of the pointe geuen, to cut of equall lines: and so making either of the endes of the said lines (either of th' endes I say, which haue not one point cōmon to them both) the centres, and the space both the lynes added together, or wider then both, or at the lest wider thē one of them, to describe those portions of the circles where they cut the one the other, and from the point of the intersection to the point geuen, to draw a lync, which shall be perpendicular vnto the lync geuē: as in the figure here put it is manifest to see.



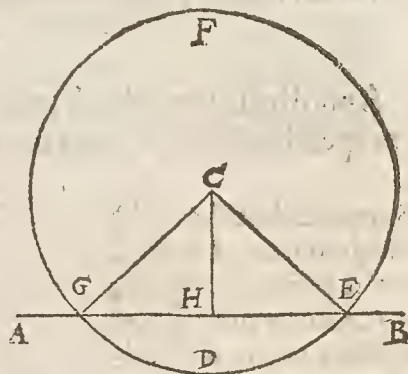
The 7. Probleme. The 12. Proposition.

Vnto a right line geuen being infinite, and from a point geuen not being in the same line, to draw a perpendicular line.



Construction.

Et the right line geuen being infinite be AB , & let y point geuen not being in the said line AB , be C . It is required from the point geuē, namely, C , to draw vnto the right line geuen AB , a perpendicular line. Take on the



other syde of the line AB (namely, on that syde wherein is not the pointe C) a pointe at all aduentures, and let the same be D . And making the centre C , and the space CD , describe (by the third petition) a circle, and let the same be EFG , which let cutte the line AB in the pointes E and G . And (by the x. proposition) deuide the lync EG into two equal partes in the point H . And (by the first petition) draw these right lines, CG , CH , and CE . Then I say, that vnto the right line geuen AB , & from the point geuen not being in it, namely, C , is drawn a perpendicular lync CH . For forasmuch as GH is equall to HE , and HC is common to them both: therefore these two sydes GH and HC , are equal to these two sydes EH & HC , the one to the other: and (by the 15. definitiō) the base CG is equal to the base CE : wherfore (by the 8. proposition) the angle CHG is equall to the angle CHE : and they are syde angles: but when a right line standing vpon a right line maketh the two syde angles equall the one to the other, either of those equall angles is (by the 10. definition) a right angle, and the line standing vpon the sayde right line is called a perpendicular line. VVherfore vnto the right line geuē AB , and from the point geuen C , which is not in the line AB , is drawn a perpendicular line CH : which was required to be done. This

Demonstration.

This Probleme did *Oenopides* first finde out, considering the necessary vse thereof to the study of Astronomy.

Oenopides the first inuenter of this probleme.

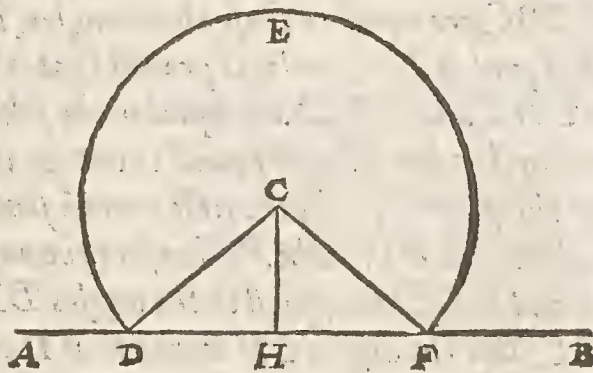
There are two kindes of perpendicular lines: wherof one is a plaine perpendicular line, the other is a solide. A plaine perpendicular line is, when the point from whence the perpendicular line is drawn, is in the same plaine superficies with the line wherunto it is a perpendicular. A solide perpendicular line is, when the point, from whence the perpendicular is drawne, is on high, and without the plaine superficies. So that a plaine perpendicular line is drawn to a right line: & a solide perpendicular line is drawn to a superficies. A plaine perpendicular line causeth right angles with one onely line, namely, with that vpon whome it falleth. But a solide perpendicular line causeth right angles, not only with one line, but with as many lynes as may be drawn in that superficies, by the touch thereof. This proposition teacheth to draw a plaine perpendicular line, for it is drawn to one line, and supposed to be in the selfe same plaine superficies.

Two kindes of perpendicular lines, namely, a plaine perpendicular line and a solide.

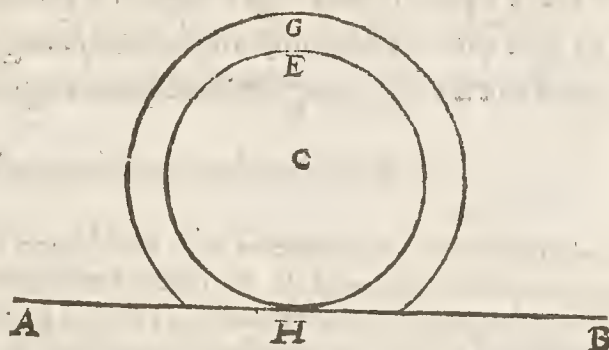
This proposition teacheth to draw a plaine perpendicular line.

An other case in this proposition.

There may be in this proposition another case. For if it be so, that on the other side of the line *AB*, there be no space to take a point in but onely on that side wherein is the point *C*. Then take some certaine point in the line *AB*, which let be *D*. And making the centre the point *C*, and the space *CD*, describe a part of the circumference of a circle, which let be *DEF*: which let cut the line *AB* in the two pointes *D* and *F*. And deuide the line *DF* into two equall partes in the point *H*. And draw these lines *CD*, *CH* and *CF*. And forasmuch as *DH* is equal to *HF*, and *CH* is common to them both, and *CD* is equal to *CF* (by the 15. definition:) therefore the angles at the point *H* are equal the one to the other (by the 8. proposition:) & they are side angles, wherefore they are right angles. Wherefore the line *CH* is a perpendicular to the line *DF*. But if it happen so that the circle which is described do not cutte the lyne, but touche it, then takyng a point without the point *E*, namely, the point *G*, and making the centre the point *C*, and the space *CG*, describe a part of the circumference of a circle: which shall of necessitie cut the line *AB*: and so may you proceede as you did before. As you see in the second figure.



Construction.



Demonstration.

The 6. Theoreme. The 13. Proposition.

When a right line standing vpon a right line maketh any angles: those angles shall be either two right angles, or equall to two right angles.

G.ij.

Suppose

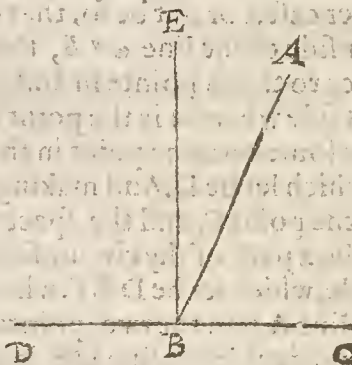
The first Booke



Construction

Demonstration

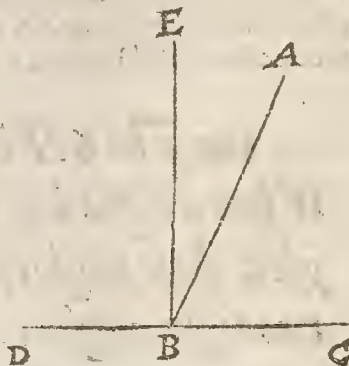
Suppose that the right line AB standing vppon the right line CD do make these angles CBA and ABD . Then I say; that the angles CBA and ABD are eyther two right angles, or els equall to two right angles. If the angle CBA be equall to the angle ABD : then are they two right angles (by the tenth definition.) But if not, raise vp (by the 11. proposition) vnto the right line CD , and from the pointe geuen in it, namely, B , a perpendicular line BE . VVherfore (by the x. definition) the angle CBE and EBD are right angles. Now forasmuch as the angle CBE is equall to these two angles CBA and ABE , put the angle EBD common to them both: wherfore the angles CBE and EBD are equal to these three angles CBA , ABE , and EBD . Agayne forasmuch as the angle DBA is equall vnto these two angles DBE and EBA , put the angle ABC common to them both: wherfore the angles DBA and ABC are equal to these three angles, DBE , EBA , and ABC . And it is proued that the angles CBE and EBD are equal to the selfesame three angles: but thinges equall to one & the selfsame thing, are also (by the first commō sentence) equall the one to the othe. VVherfore the angles CBE and EBD are equall to the angles DBA & ABC . But the angles CBE and EBD are two right angles: wherfore also the angles DBA and ABC are equall to two right angles. VVherfore when a right line standing vpon a right line maketh any angles: those angles shalbe either two right angles, or equall to two right angles: which was required to be demonstrated.



An othet demonstration after Pelitarinus.

An other demonstration after Pelitarinus.

Suppose that the right line AB do stand vpon the right line CD . Then I say, that the two angles ABC and ABD , are either two right angles, or equal to two right angles. For if AB be perpēdiculer vnto CD : thē is it manifest, that they are right angles (by the conuerſion of the definition) But if it incline towardes the end C , then (by the 11. proposition) from the point B , erect vnto the line CD a perpendicular line BE . By whiche construction the propositiō is very manifest. For forasmuch as the angle ABD is greater then the right angle DBE by the angle ABE , and the other angle ABC is lesse then the right angle CBE by the selfesame angle ABE : if from the greater bee taken away the excesse, and the same bee added to the lesse, they shall be made two right angles. That is, if from the obtuse angle ABD be taken away the angle ABE , there shal remayne the right angle DBE . And then if the same angle ABE be added to the acute angle CBA , there shall bee made the right angle CBE . Wherefore it is manifest, that the two angles, namely, the obtuse angle ABD , & the acute angle ABC , are equall to the two right angles CBE and DBE : which was required to be proued. The



The 7. Theoreme. The 14. Proposition.

If unto a right line, and to a point in the same line, be drawn two right lines, not both on one and the same side, making the side angles equall to two right angles: those two right lynes shall make directly one right line.



Unto the right line AB , & to y point in it B , let there be drawn two right lines BC , and BD , vnto contrary sides, making the syde angles, namely, ABC & ABD , equall to two right angles. Then I say, that y right

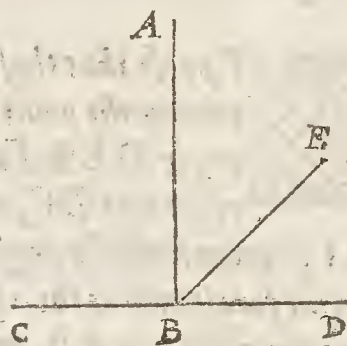
lines BD and BC make both one right line. For if CB and BD do not make both one right line, let the right line BE be so drawn to BC , that they both make one right line. Now forasmuch as the right line AB standeth vpon the right line CBE , therefore the angles ABC and ABE are equall to two right angles (by the 13 proposition) But (by supposition) the angles ABC and ABD are equall to two right angles: wherfore the angles CBA , and ABE , are equall to the angles CBA , and ABD : take away the angle ABC , which is common to them both. Wherfore the angle remayning ABE , is equall to the angle remaining ABD , namely, the lesse to the greater, which is impossible. Wherfore the line BE is not so directly drawn to BC , that they both make one right line. In like sorte may we proue, that no other line, besides BD , can so be drawne. Wherfore the lines CB and BD , make both one right line. If therefore vnto a right line, & to a point in the same line, be drawn two right lines, not both on one and the same side, making the side angles equall to two right angles: those two lines shal make directly one right line: which was required to be proued.

Another demonstration after Pelitarius.

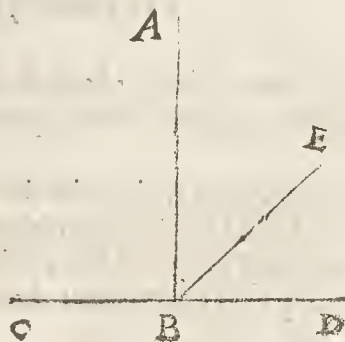
Suppose that there be a right line AB , vnto whose pointe B , let there be drawn two right lines CB and BD , vnto contrary sides: and let the two angles CBA , and DBA , be either two right angles, or equall to two right angles. Then I say, that the two lines CB and BD , do make directly one right line, namely, CD . For if they do not, the let BE be so drawn vnto CB , that they both make directly one right line CBE : which shall passe either aboue the line BD , or vnder it. First let it passe aboue it. And for as much as the two angles CBA and ABE , are (by the former proposition) equall to two right angles, and are a part of the two angles, CBA and ABD : but the angles CBA and ABD are by (supposition) equall also to two right angles: therefore the parte is equall to the whole which is impossible. And the like absurditie will follow if CB

G.iii.

E



Demonstration leading to an absurditie.



Another demonstration after Pelitarius.

The first Booke

E passe vnder the line BD : namely, that the whole shalbe equall to the part: which is also impossible. Wherefore CD is one right line: which was required to be proued.

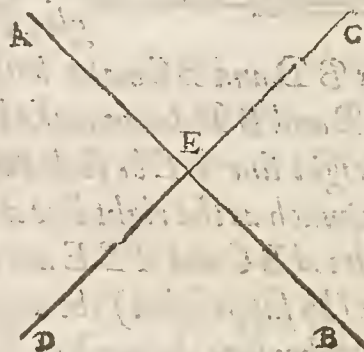
The 8. Theoreme. The 15. Proposition.

If two right lines cut the one the other: the hed angles shal be equal the one to the other.

Demonstration.



Suppose that these two right lines AB and CD , do cut the one the other in the point E . Then I say, that the angle AEC , is equall to the angle DEB . For forasmuch as the right line AE , standeth vpon the right line DC , making these angles CEA , and AED : therefore (by the 13. proposition) the angles CEA , and AED , are equall to two right angles. Agayne forasmuch as the right line DE , standeth vpon the right line AB , making these angles AED , and DEB : therefore (by the same proposition) the angles AED , and DEB , are equall to two right angles. and it is proued, that the angles CEA , and AED , are also equall to two right angles. Wherefore the angles CEA , and AED , are equall to the angles AED , and DEB . Take away the angle AED , which is common to them both. Wherefore the angle remayning CEA , is equall to the angle remayning DEB . And in like sort may it be proued, that the angles CEB , and DEA , are equall the one to the other. If therefore two right lines cut the one the other, the hed angles shalbe equall the one to the other: which was required to be demonstrated.



Thales Milesius the first inuenter of this proposition. No construction in this proposition. What hed angles are.

Thales Milesius the Philosopher was the first inuenter of this Proposition, as witnesseth Eudemius, but yet it was first demonstrated by Euclide. And in it there is no construction at all. For the exposition of the thing geue, is sufficient inough for the demonstration.

Hed Angles, are apposite angles, caused of the intersection of two right lines: and are so called, because the hedges of the two angles are ioyned together in one pointe.

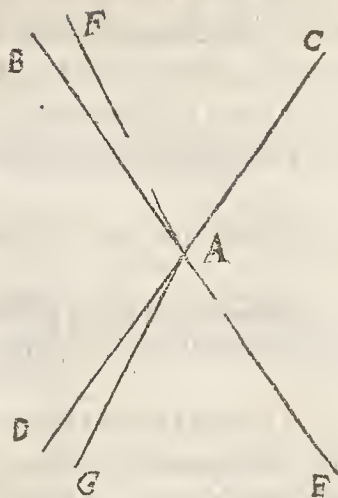
The conuerse of this proposition after Pelitarius.

The conuerse of this proposition after Pelitarius.

If fower right lines being drawn from one point, do make fower angles, of which the two opposite angles are equall: the two opposite lines shalbe drawn directly, and make one right line.

Suppose that there be fower right lines AB , AC , AD , and AE , drawn from the poynt A , making fower angles at the point A : of which let the angle BAC be equall to the angle DAE , and the angle BAD to the angle CAE . Then I say, that BE and CD are onely two right lines: that is, the two right lines BA and AE are drawn directly, and

and doo make one right line, and likewise the two right lines CA and AD are drawen directly, and do make one right line. For otherwise if it be possible, let EF be one right line, and likewise let CG be one right line. And forasmuch as the right line EA standeth vpon the right line CG , therefore the two angles EAC and EAG , are (by the 13 proposition) equall to two right angles. And forasmuch as the right line GA standeth vpon the right line EF : therefore (by the selfe same) the two angles EAG and FAG , are also equall to two right angles. Wherefore taking away the angle EAG , which is common to them both, the angle EAC , shall (by the thirde common sentence) be equall to the angle FAG : but the angle EAC is supposed to be equall to the angle BAD . Wherefore the angle BAD is equall to the angle FAG , namely a part to the whole: which is impossible. And the selfe same absurditie will follow, on what side soeuer the lines be drawen. Wherefore BE is one line, and CD also is one line: which was required to be proued.



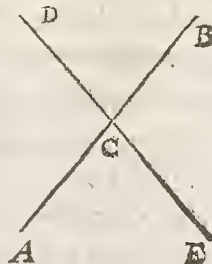
Demonstration leading to an absurditie.

The same conuerse after Proclus.

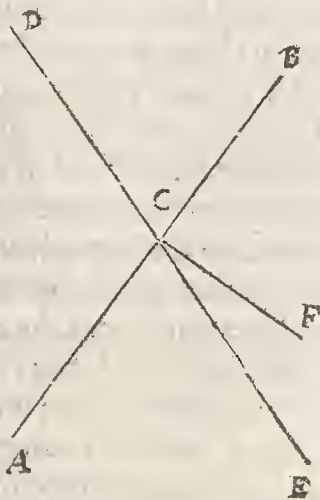
If vnto a right line, and to a point thereof be drawen two right lines, not on one and the same side, in such sort that they make the angles at the toppe equall: those right lines shalbe drawen directly one to the other, and shal make one right line.

The same conuerse after Pelmarius, which is demonstrated directly.

Suppose that there be a right line AB , and take a point in in C . And vnto the point in it C , draw these two right lines CD and CE vnto contrary sides, making the angles at the hed equal; namely, the angles ACD and BCE . Then I say, that the lines CD and CE are drawen directly, and do make one right line. For forasmuch as the right line CD standing vpon the right line AB , doth make the angles DCA and DCB equall to two right angles (by the 13 proposition:) and the angle DCA is equall to the angle BCE : therefore the angles DCB and BCE are equal to two right angles. And forasmuch as vnto a certayne right line BC , and to a point thereof C , are drawen two right lines not both on one and the same side, making the side angles equall to two right angles, therefore (by the 14 proposition) the lines CD and CE are drawen directly, & do make one right line, which was required to be proued.



The same may also be demonstrated by an argument leading to an absurditie. For if CE be not drawn directly to CD , so that they both make one right line, then (if it be possible) let CF be drawne directly vnto it. So that let DCF be one right line. And forasmuch as the two right lines AB and DF do cutte the one the other, they make the hed angles equall (by the 15. proposition) Wherefore the angles ACD and BCF are equall: but (by supposition) the angles ACD and BCE are also equall. Wherefore (by the first common sentence) the angle BCE is equall to the angle BCF : namely, the greater to the lesse: which is impossible. Wherefore no other right line besides CE is drawen directly to CD . Wherefore the lines CD and CE are drawen directly, and make one right line: which was required to be proued.



The same conuerse after Proclus demonstrated indirectly.

The first Booke

What a Corollary is.

Of this fuetenth Proposition followeth a *Corollary*. VVhere note that a *Corollary* is a Proposition, whose demonstration dependeth of the demonstration of an other Proposition, and it appeareth sodenly, as it were by chance offering it selfe vnto vs: and therefore is reckoned as lucre or gayne. The *Corollary* which followeth of this proposition, is thus.

A Corollary following of this proposition.

If fower right lines cut the one the other: they make fower angles equall to fower right angles.

This *Corollary* gaue great occasion to finde out that wonderful proposition inuented of Pithagoras, which is thus.

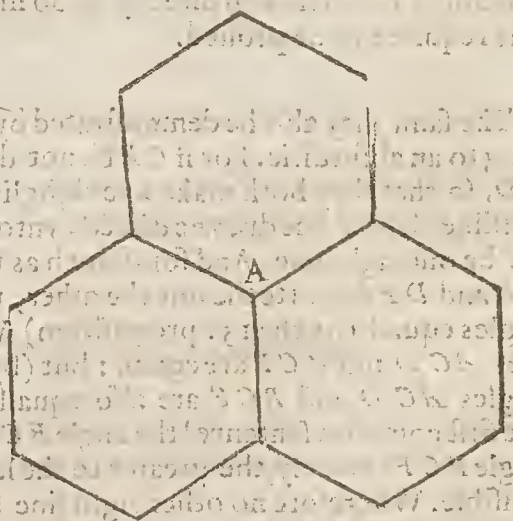
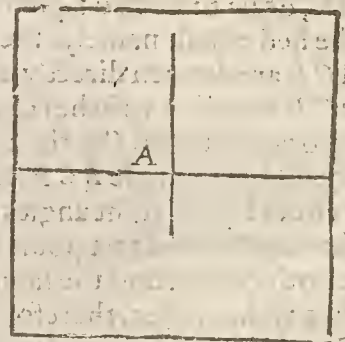
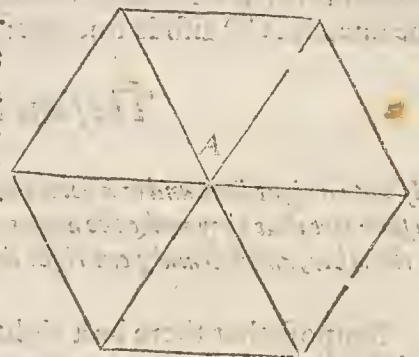
A wonderfull proposition inuented by Pithagoras.

Only three kindes of figures of many angles, namely, an equilater triangle, a right angled figure of fower sides, and a figure of sixe sides, hauing equall sides and equal angles, can fill the whole space about a point, their angles touching the same point.

Euery angle of an equilater tri- angle is equal to two third partes of a right angle

Euery angle of a sixe angled figure is equall to a right angle, and to a third part of a right angle.

Euery angle of an equilater triangle containeth two third partes of a right angle: sixe tymes two thirdes of a right angle make fower right angles. VVherefore sixe equilater triangles fill the whole space about a point which is equal to fower right angles, as in the 1. figure. Also euery angle of a right angle quadrilater figure is a right angle: wherefore fower of them fill the whole space as in the 2. figure. Euery angle of a sixe angled figure, is equal to a right angle, and moreouer to a third part of a right angle. But a right angle, and a third part of a right angle, take three tymes, make 4. right angles: wherefore three equilater sixe angled figures fill the whole space about a point: which space (by this *Corollary*) is equall to fower right angles: as in the third figure. Any other figure of many sides, howsoeuer you ioyneth together at the angles, shal either want of fower angles, or excede them. By this *Corollary* also it is manifest that if more then two lines, that is, three, or fower, or how many soeuer do cut the one the other in one point, all the angles by them made at the point shalbe equall to fower right angles. For they fill the place of fower right angles. And it is also many fest, that the angles by those right lines made are double in number to the right lines which cutte the one the other. So that if there be two lines which cut the one the other, there are there made fower angles equall to fower right angles: but if three, then are there made sixe angles: if fower, eight angles, and so infinitely. For euen the multitude, or number of of the angles is dumbled to the multitude of the right lines which cut the one the other. And as the angles increase in multitude, so



diminish

diminish they in magnitude. For that that which is deuided is alwayes one and the selfe same thing, namely, two right angles.

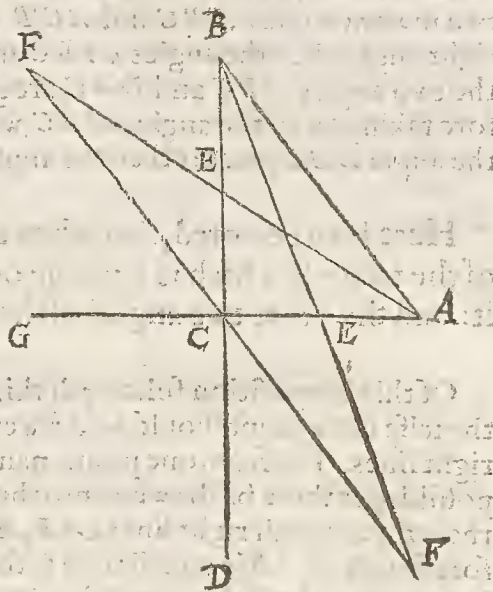
The 9. Theoreme. The 16. Proposition.

Whensoever in any triangle, the lyne of one syde is drawen forth in length: the outwarde angle shall be greater then any one of the two inwarde and opposite angles.



Suppose that ABC be a triangle:
let one of y sides therof, namely
 BC be produced vnto the point
 D . Then I say, that the outwarde

angle ACD , is greater then any one of y two
inward and opposite angles, that is, then the
angle CBA , or then the angle BAC . Deuide
the line AC (by the 10. proposition) into two
equall partes, in the point E . And draw a line
from the point B to the point E . And (by the
2. petition) extend BE to the point F . And
(by the 2. proposition) vnto the line BE put
an equall line EF . And (by the first petiti-
on) draw a line from F to C : and (by the 2. petiti-



Constructiō.

on) extend the line AC to the point G . Now forasmuch as the line AE , is equall
to the line EC , and BE is equall to EF : therefore these two sides AE and EB ,
are equall to these two sides CE and EF , the one to the other: and the angle AEB ,
is (by the 15. proposition) equall to the angle FEC , for they are hed an-
gles: wherefore (by the 4. proposition) the base AB is equall to the base FC :
And the triangle ABE is equall to the triangle FEC : and the other angles re-
mayning are equall to the other angles remayning, the one to the other, vnder
which are subtended equall sides. Wherefore the angle BAE is equall to the
angle ECF . But the angle ECD , is greater then the angle ECF . Wherefore
the angle ACD , is greater then the angle BAC . In like sort also if the line BC
be deuided into two equall partes, may it be proued, that the angle BCG , that
is, the angle ACD , is greater then the angle ABC . Whensoever therefore in
any triangle, the line of one side is drawen forth in length: the outward angle
shalbe greater then any one of the two inward and opposite angles: which was
required to be demonstrated.

Demonstratiō.

An other demonstration after Pelitarius.

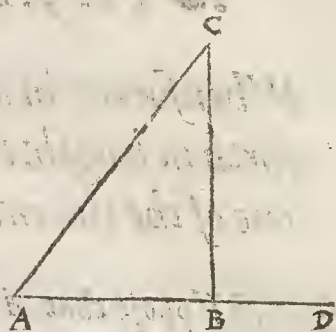
Suppose that the triangle geuen be ABC . Whose side AB let be produced vnto
the

H.j.

*An other De-
monstration af-
ter Pelitarius.*

The first Booke

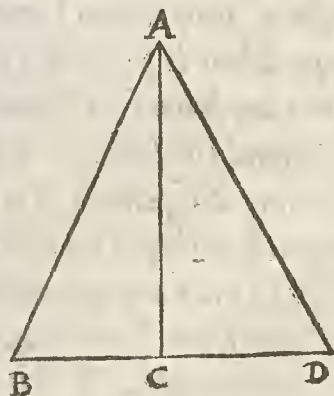
the point D . Then I say, that the angle DBC is greater then either of the angles BAC and ACB . For forasmuch as the two lines AC and BC do concur in the point C , and vpon them falleth the line AB : therefore (by the conuerse of the first petition) the two inward angles on one and the selfe same side, are lesse then two right angles. Wherefore the angles ABC and CAB are lesse then two right angles: but the angles ABC and DBC are (by the 13 proposition) equal to two right angles. Wherefore the two angles ABC and DBC are greater then the two angles ABC and BAC . Wherefore taking away the angle ABC , which is common to them both, there shall be left the angle DBC greater then the angle BAC . And by the same reason, forasmuch as the two lines BA and CA concur in the point A , and vpon them falleth the right line CB , the two inward angles ABC and ACB are lesse then two right angles. But the angles ABC and DBC are equal to two right angles. Wherefore the two angles ABC and DBC , are greater then the two angles ABC & ACB . Wherefore taking away the angle ABC , which is common to them both, there shall remaine the angle DBC greater then the angle ACB : which was required to be proued.



Here is to be noted, that when the side of a triangle is drawn forth, the angle of the triangle which is next the outward angle, is called an angle in order vnto it: and the other two angles of the triangle are called opposite angles vnto it.

A Corollary following of this proposition.

Of this Proposition followeth this *Corollary*, that it is not possible that from one & the selfe same point should be drawn to one and the selfe same right line, three equall right lines. For from one point, namely, A , if it be possible, let there be drawn vnto the right line BD , these three equall right lines AB , AC , & AD . And forasmuch as AB is equall to AC , the angles at the base are (by the fifth proposition) equall. Wherefore the angle ABC is equal to the angle ACB . Agayne forasmuch as AB is equall to AD , the angle ABD is (by the same) equall to the angle ADB : but the angle ABC was equall to the angle ACB . Wherefore the angle ACB is equall to the angle ADB : namely, the outward angle to the inwarde & opposite angle: which is impossible. Wherefore from one and the selfe same point, can not be drawn to one & the selfe same right line three equall right lines: which was required to be proued.

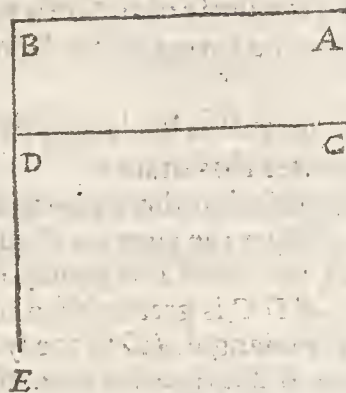


Another Corollary following also of the same.

By this Proposition also may this be demonstrated, that if a right line falling vpon two right lines, do make the outward angle equall to the inward and opposite angle, those right lines shall not make a triangle, neither shall they concur. For otherwise one & the selfe same angle should be both greater, and also equal: which is impossible. As for example.

Suppose that there be two right lines AB and CD , and vpon them let the right line BE fall, making the angles ABD and CDE equall. Then I say, that the right lines AB and CD shall not concur. For if they concur, the foresaide angles abiding equall, namely, the angles CDE and ABD : Then forasmuch as the angle CDE is the outward angle it is of necessitie greater then the inward and opposite angle, & it is also equal vnto it: which is impossible. Wherefore if the said lines cōcurre, they shall not the angles remayne equall, but the angle at the point D shall be encreased. For whether AB abiding

abiding fixed you suppose the line CD to be moued vnto it, so that they concurre, the space and distance in the angle will be greater: for how much more C D approacheth to AB , so much farther of goeth it from DE . Or whether CD abiding fixed, you imagine the line AB to be moued vnto it, so that they concurre, the angle ABD will be lesse, for therewith all it commeth nere vnto the lines CD & BD . Or whether you imagine either of them to be moued the one to the other, you shall finde that the line AB comming nere to CD , maketh the angle ABE lesse, and CD going farther from DE by reason of his motion to the line BD , maketh the angle CDE to increase. Wherefore it followeth of necessitie, that if it be a triangle, and that the right lines AB and CD do concurre, the outward angle also shall be greater then the inward and opposite angle. For either the inward and opposite angle abiding fixed, the outward is increased: or the outwarde abiding fixed, the inward and opposite is diminished: or els both of them being moued till they concurre, the inwarde is diminished, and the outwarde is more increased. And the cause hereof is the motion of the right lines the one tending to that parte where it diminisheth the inwarde angle, the other tending to that part where it increaseth the outward angle.

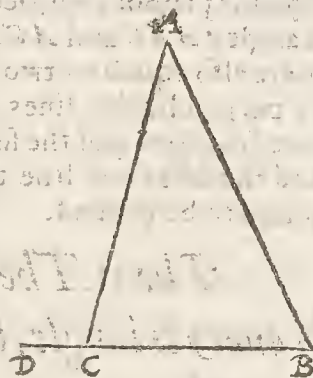


The 10. Theoreme. The 17. Proposition.

In euery triangle, two angles, which two soeuer be taken, are lesse then two right angles.



Suppose that ABC be a triangle. Then I saye that two angles of the sayd triangle ABC , which two angles soeuer be taken, are lesse the two right angles. Extend (by the 2. peticio) the line BC , to the point



Construction.

Demonstration.

D . And forasmuch as (by the proposition going before) the outward angle of the triangle ABC , namely, the angle ACD is greater then the inward and opposite angle ABC : put the angle ACB commō to them both: wherefore the angles ACD and ACB are greater then the angles ABC and BCA . But (by the 13. proposition) the angles ACD and ACB are equall to two right angles. Wherefore the angles ABC and BCA are lesse then two right angles. In like sort also may we proue, that the angles BAC and ACB are lesse then two right angles: and also, that the angles CAB & ABC are lesse then two right angles. Wherefore in euery triangle, two angles, which two soeuer be taken, are lesse then two right angles: which was required to be proued.

H.ii.

This

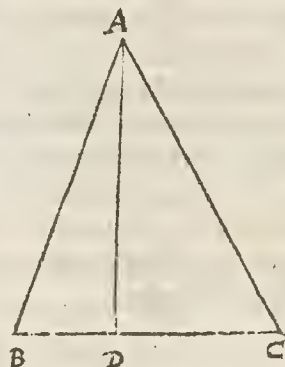
The first Booke

This may also be demonstrated without the helpe of the former proposition, by the conuerse of the fifth petition, and by the 13. proposition as you saw was done in the former after *Pelitarinus*.

It may also be demonstrated without producing any of the sides of the triangle, after this maner.

An other demonstration inuented by Proclus.

Suppose that there be triangle ABC . And in the side BC take a point at all aduentures, and let the same be D , and draw the line AD . And forasmuch as in the triangle ABD , the side BD is produced, therefore (by the former proposition) the outward angle ADC , is greater then the inward and opposite angle ABD . Agayne forasmuch as in the triangle ADC , the syde CD is produced, therefore (by the same) the outward angle ADB , is greater then the inward and opposite angle ACD : but the angles at the point D are equall to two right angles (by the 13. proposition:) wherfore the angles ABC and ACB are lesse then two right angles. And by the same reason may we proue that the angles BAC and BCA are lesse then two right angles, if we take a point in the line AC , and draw a right line from it to the point B : and so also may it be proued that the angles CAB and ABC are lesse the two ryght angles, if there be taken in the lyne AB a point, and from it be a line drawn to the point C .



A Corollary following this Proposition.

By this proposition also may be proued this Corollary, that from one and the selte same point to one and the selte same right line, can not be drawn two perpendicular lines.

For if it be possible, from the point A , let there be drawn vnto the right line BC , two perpendicular lines AB , and AC : wherfore the angles ABC and ACB are right angles. But forasmuch as ABC is a triangle: therefore any two angles thereof are (by this proposition) lesse then two right angles. Wherefore the angles ABC and ACB are lesse then two right angles: but they are also equall to two right angles, by reason AB and AC are perpendicular lines vpon BC : which is impossible. Wherefore from one and the selte same point cannot be drawe to one and the selte same line two perpendicular lines: which was required to be proued.



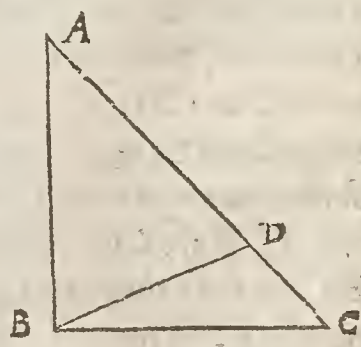
The 11. Theoreme. The 18. Proposition.

In euery triangle, to the greater side is subtended the greater angle.

Construction.



Suppose that ABC be a triangle, hauing the side AC greater then the side AB . Then I say that the angle ABC is greater then the angle BAC . For forasmuch as AC is greater then AB , put (by the 3. proposition) vnto AB an equall line AD . And (by



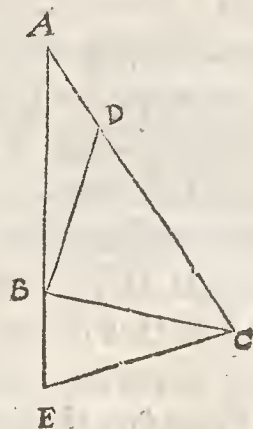
the

the first petition) draw a line from the point B to the point D . And forasmuch as the outward angle of the triangle DBC , namely, the angle ADB is greater then the inward and opposite angle DCB (by the 16. proposition,) but (by the 5. proposition) the angle ADB is equall to the angle ABD , for the syde AB is equall to the syde AD : therefore the angle ABD is greater then the angle ACB . Wherefore the angle ABC is much greater then the angle ACB . Wherefore in every triangle, to the greater syde is subtended the greater angle: which was required to be proued.

Demonstration.

You may also proue the angle at the point B greater then the angle at the point C (the syde AC being greater then the syde AB) if from the line AC you cut of a line equall to the line AB , beginning at the point C , as before you beganne at the point A : and that after this manner. Let the line DC be equall to the line AB and draw the line BD : and produce AB to the point E : and put the line BE equal to the line AD . Wherefore the whole line AE is equall to the whole line AC : draw a line from E to C . And forasmuch as AE is equal to AC , therefore the angle AEC is also equall to the angle ACE (by the 5. proposition:) but the angle ABC is greater then the angle AEC . For one of the sides of the triangle CBE , namely, the syde BE is produced; and so the outward angle ABC is greater then the inward and opposite BEC (by the 16 proposition:) wherefore the angle ABC is much greater then the angle ACB : which was required to be proued.

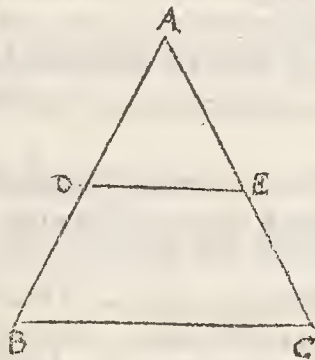
An other demonstration after Prophyrius.



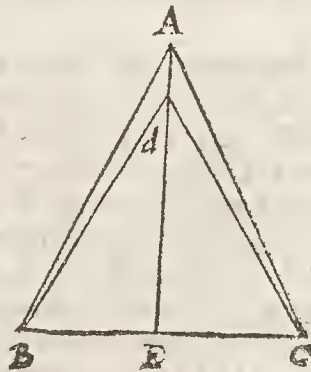
Note that that which is here spoken in this proposition, is to be vnderstanded in one and the self same triangle. For it is possible that one and the selfe same angle may be subtended of a greater line, and of a lesse line: and one and the selfe same right line may subtend a greater angle, and a lesse angle. As for example.

That which is spoken in this Proposition is to be vnderstanded in one and the selfe same triangle.

Suppose that there be an Isoceles triangle ABC , & in the syde AB take the point D at all aduentures: & fro the line AC cut of (by the 3. proposition) the lyne AE equall to the line AD . And draw a right line from D to E . Wherefore the right lines DE and BC do subtend the angle at the point A , & of them the one is greater, and the other lesse. And after the selfe same manner a man may put infinite right lines greater & lesse, subtending the angle at the point A .



Agayne suppose that ABC be an Isoceles triangle. And let BC be lesse then either of the lines BA and AC . And vpon BC describe (by the first) an equilateral triangle BCD . And draw a line from A to D , and produce it to the point E . And forasmuch as in the triangle ABD , the outward angle BDE , is greater then the inward & opposite angle BAD (by the 16. proposition) And by the same in the triagle ACD , the outward angle CDE , is greater then the inward & opposite angle CAD : therefore the whole angle BDC is greater the the whole angle BAC . And one and the selfe same right line subtendeth both these angles, namely, the greater angle & the lesse. And it is also proued, that greater right lines & lesse subtende one and the selfe same angle. But in



The first Booke

one and the selfe same triangle one right line subtendeth one angle, and the great right line euer subtendeth the great angle, and the lesse the lesse, as it was proued in the proposition.

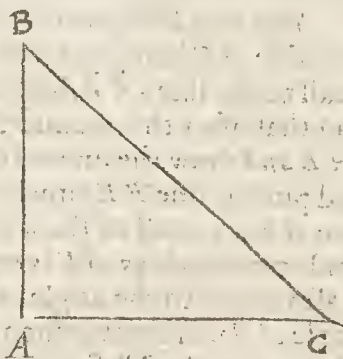
The 12. Theoreme. The 19. Proposition.

In euery triangle, vnder the greater angle is subtended the greater side.

Suppose that ABC be a triangle, hauing the angle ABC greater then the angle BCA . Then I say that the side AC is greater

*Demonstration
leading to an
impossibilitie.*

then $\dot{y}$ side AB . For if not, the side AC is either equal to $\dot{y}$ side AB , or els it is lesse the it. The side AC is not equal to $\dot{y}$ side AB , for then (by the 5. proposition) $\dot{y}$ angle ABC should be equall to the angle ACB : but (by suppositiō) it is not. *Wherefore* the side AC is not equal to the side AB . And the side AC can not be lesse then the side AB , for then the angle ABC should be lesse then the angle ACB (by the proposition next goyng before). But (by supposition it is not) *Wherefore* the side AC is not lesse then the side AB . *Wherefore* the side AC is greater then the side AB . *Wherefore* in euery triangle, vnder the greater angle is subtended the greater side: which was required to be demonstrated.



*This proposition
is the conuerse
of the former.*

This proposition is the conuerse of the proposition next going before. *Wherefore* as you see, that which was the conclusion in the former, is in this the supposition, or thing geuen: and that which was there the thing geuen, is here the thing required or conclusion. And it is proued by an argument leading to an impossibilitie, as commonly all conuerfes are.

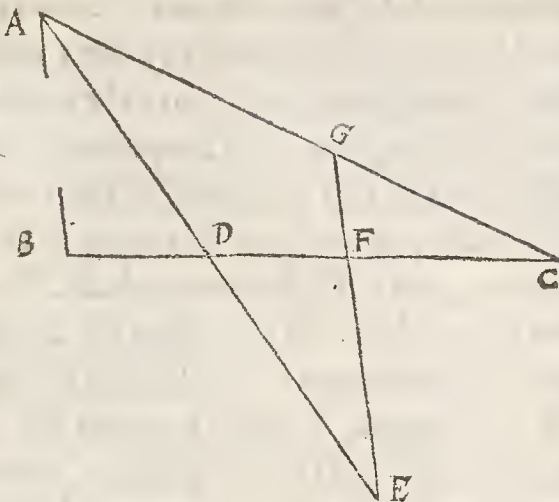
Proclus demonstrateth this proposition after an other way: but first he putteth this **Assumpt* following.

**An Assumpt is
a Proposition taken
of necessitie
to the helpe of a
demonstration,
the certainty
whereof is not
so plaine, and
therefore needeth
it selfe first to be
demonstrated.
An assumpt put
by Proclus for
the demonstration
of this Proposition.*

If an angle of a triangle be deuided into two equall partes, and if the line which deuideth it being drawen to the base, do deuide the same into two vnequall partes: the sides which contayne that angle shalbe vnequall, and that shalbe the greater side, which falleth on the grater side of the base, and that the lesse which falleth on the lesse side of the base.

Suppose ABC to be a triangle, and (by the 9. proposition) deuide the angle at the point A , into two equall partes, by the right line AD . And let the line AD deuide the base BC , into two vnequall partes, and let the part CD be greater then the parte BD . Then I say, that the side AC is greater then the side AB . Produce the line AD to the point E , and (by the third) put the line DE equall to the line DA . And forasmuch as DC is by supposition greater then DB , put (by the 3. proposition) DF equal to BD , and draw a line frō E to F , and produce it to the point G : Now forasmuch as AD is equall to ED and DB is equall to DF , therefore in the two triangles ABD , and EDF , two sides of the one are equall to two sides of the other, eche to his correspondent side: and (by the 15. proposition) they contayne equall angles, namely, the hed angles

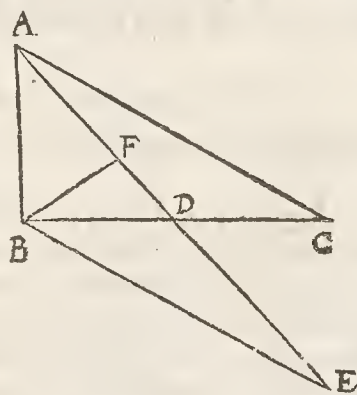
gles : wherefore (by the fourth proposition) the base BA is equall to the base EF : and the angle DEF is equall to the angle DAB . But the angle DAG is by construction equall to the same angle DAB : wherefore (by the first common sentence) the angles EAG and AEG are equall. Wherefore (by the 6. proposition) the side AG is equall to the side EG . Wherefore the side AC is greater then the side EG . Wherefore it is much greater then the side EF . But the side EF is equall to the side AB , as it hath bene proued. Wherefore the side AC is greater then the side AB : which was required to be proued.



This assumpt being put, this Proposition is of Proclus thus demonstrated.

Suppose ABC to be a triangle, hauing his angle at the point B greater then the angle at the point C . Then I say that the side AC is greater then the side AB . Deuide the line BC into two equall partes in the point D , and draw a line from A to D . And produce the line AD to the point E : and put the line DE equall to the line AD , and draw a line from B to E . Now forasmuch as BD is equall to DC , and AD is equall to DE therefore in the two triangles ADC and BDE , two sides of the one are equall to two sides of the other, ech to his correspondent side, and they containe equall angles (by the 15. proposition): wherefore (by the fourth proposition) the base BE is equall to the base AC , and the angle DBE is equal to the angle at the point C . Deuide also th'angle ABE into two equal partes by the line BF : wherefore the line EF is greater then the line FA . And forasmuch as in the triangle ABE , the angle at the point B is deuided into two equall partes by the right line BF , and the line EF is greater then the line AF : therefore by the former Assumpt the side BE is greater then the side BA : but the line BE is equall to the line AC . Wherefore the syde AC is greater then the side AB : which was required to be proued.

An other demonstration of the same Proposition as by Proclus.

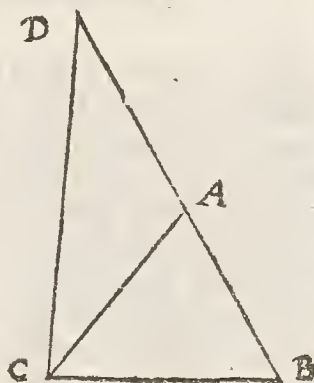


The 13. Theoreme. The 20. Proposition.

In euery triangle two sides, which two sides soeuer be taken, are greater then the side remayning.



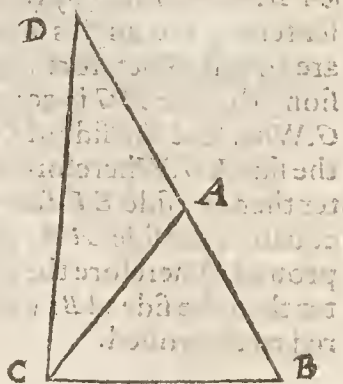
Suppose that ABC be a triangle. Then I say that two sides of the triangle ABC , which two sides soeuer be taken, are greater: then the side remayning that is, the sides BA and AC are greater then the side BC .



Construction.

Demonstration.

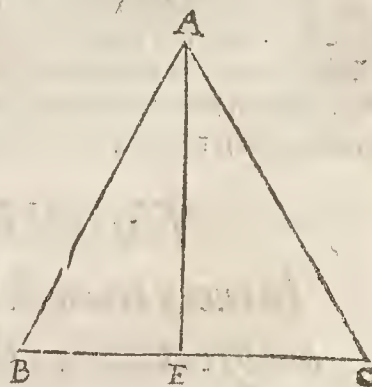
side BC : and the sides AB and BC then the side AC : and the sides AC and BC then the side BA . Produce (by the 2. petition) the line BA to the point D . And (by the third proposition) vnto the line AC put an equall line AD : and drawe a line from the point D to the pointe C . And forasmuch as the line DA is equall to the line AC , therefore (by the 5. proposition) the angle ADC is equall to the angle ACD . But the angle BCD is greater then the angle ACD , therefore the angle BCD is greater then the angle ADC . And forasmuch as DCB is a triangle, hauing the angle BCD greater then the angle ADC , but (by the 18. proposition) vnder the greater angle is subtended the greater side: wherfore DB is greater then BC . But the line DB is equall to the lines AB and AC (for the line AD is equall to the line AC) wherfore the sides BA and AC , are greater then the side BC . And in like sorte may we proue, that the sides AB and BC are greater then the side AC : & that the sides BC and CA are greater then the side AB . Wherfore in euery triangle two sides, which two sides soeuer be taken, are greater then the side remayning: which was required to be demonstrated.



This Proposition may also be demonstrated without producing any of the sides, after this maner.

Another demonstration without producing one of the sides.

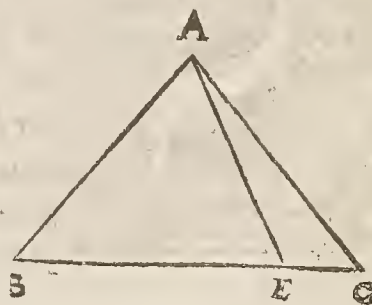
Suppose ABC to be triangle. Then I say, that the two sides AB and AC are greater then the side BC : deuide the angle at the point A (by the 9. proposition) into two equall partes by the right line AE . And forasmuch as in the triangle ABE , the outward angle AEC is greater then the angle BAE (by the 16 proposition), and the angle BAE is put to be equall to the angle EAC , therefore the side AC is greater then the side CE . And by the same reason the side AB is greater then the side BE . For in the triangle AEC the outward angle AEB , is greater then the angle CAE , that is then the angle EAB . Wherfore also the side AB is greater then the side BE . Wherfore the sides AB and AC are greater then the whole side BC . And after the same maner may you proue touching the other sides also.



The same may yet also be demonstrated another way.

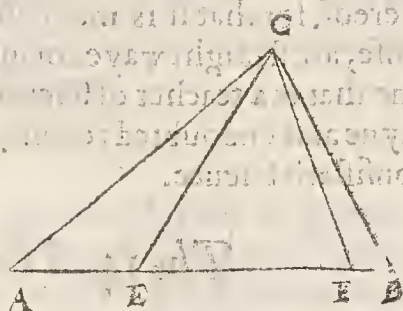
Another Demonstration.

Suppose ABC to be a triangle. Now if ABC be an equilateral triangle, then without doubt any two sides thereof are greater then the third. For the three sides being equall any two sides of them are double to the third. But if it be an Ifoseles triangle, either the base is lesse then either of the equall sides or it is greater. If the base be lesse, then againe two of them are greater then the thirde, but if the base be greater: let BC being the base of the Ifoseles triangle ABC be greater then either of the sides AB & AC and from it cut of (by the 3. proposition



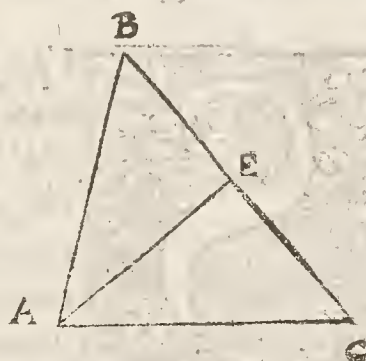
position) a line equall to any one of the other sides, whiche let bee BE , and drawe a line from A to E . And forasmuch as in the triangle AEB , the angle AEC is an outward angle: therefore it is greater then the angle BAE (by the 16. proposition). And by the same reason, the angle AEB is greater then the angle CAE . Wherefore the angles at the point E are greater then the whole angle at the pointe A . But the angle BEA is equal to the angle BAE (by the 5. proposition) for AB is put to be equall to BE . Wherefore the angle remayning AEC is greater then the angle CAE . Wherefore also the side AC is greater then the side EC . But the side AB is equall to the side BE . Wherefore the sides AB and AC are greater then the side BC .

But if the triangle ABC be a Scalenum, let the side AB be the greatest, and let AC be the meane, and BC the least. Wherefore the greatest side being added to any one of the two sides must nedes be greater then the third. For of it selfe it is greater then any of them. But if AB being the greatest, you would proue the sides AC and CB to be greater then it. Then as you did in the I-focles triangle, cut of from the greatest a line equall to one of them, and from the point C to the point of the interfection draw a right line, and reason as you did before by the outward angles of the triangle, and you shal haue your purpose.



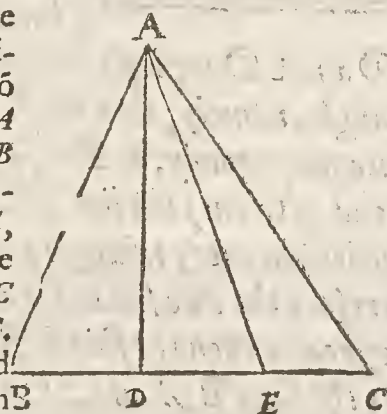
This proposition may yet moreouer be demonstrated by an argument leading to an absurditie, and that after this manner.

Suppose ABC to be a triangle. Then I say that the sides AB and AC , are greater then the side BC . For if they be not greater, they are either equall or lesse. First let them be equall, and from the line BC cut of the line BE equall to the line AB (by the 3. proposition) wherefore the residue EC is equall to AC . Now forasmuch as AB is equall to BE they subtend equall angles. Likewise forasmuch as AC is equall to CE they subtend equal angles. Wherefore the angles which are at the point E are equall to the angles whiche are at the pointe A , which is impossible (by the 16. proposition).



Another demonstration leading to an absurditie

But now let the sides AB and AC be lesse then the side BC , and from the line BC cut of (by the 3. proposition) the line BD equall to the line AB , and likewise from the same line BC cut of the line CE equall to the line AC . And forasmuch as AB is equall to BD , the angle BDA also is equall to the angle BAD (by the fifth proposition). Again forasmuch as AC is equall to CE , therefore (by the same) the angle CEA is equall to the angle EAC . Wherefore these two angles BDA and CEA are equall to these two angles BAD and EAC . Agayne forasmuch as the angle BDA is the outward angle of the triangle ADC , therefore it is greater then the angle EAC . For it is greater then the angle DAC (by the 16. proposition). And by the same reason, forasmuch as CEA is the outward angle of the triangle ABE , therefore it is greater then the angle BAD (for it is greater then the angle BAE). Wherefore the angles BDA and CEA are greater then the two angles BAD and EAC . But they were also proued equall vnto them: which is impossible. Wherefore the sides AB and AC are neither equall to the side BC , nor lesse then it, but greater. And so also may it be proued of the rest.



The first Booke

A briefe demonstration by the definition of a right line.

Not all things manifest to the sense, are straight way manifest to reason and understanding.

A man may also more briefly demonstrate this proposition by Campanus definition of a right line, which as we haue before declared is thus: *A right line is the shortest extension or draught that is or may be from one point to another.* Wherefore any one side of a triangle, for that it is a right line drawn from some one point to some other one point, is of necessitie shorter then the other two sides drawn from and to the same pointes.

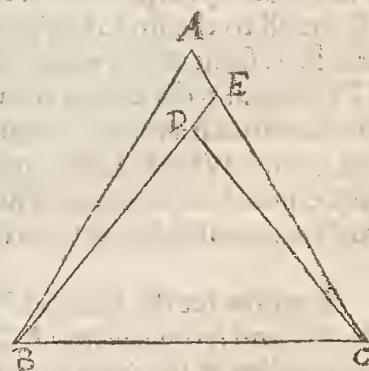
Epicurus and such as followed him derided this proposition, not counting it worthy to be added in the number of propositions of Geometry for the easines thereof, for that it is manifest euen to the sense. But not all things manifest to sense, are straight wayes manifest to reason and vnderstanding. It pertayneth to one that is a teacher of sciences, by prose and demonstration to render a certayne and vndoubted reason, why it so appeareth to the sense: and in that onely consisteth science.

The 14. Theoreme. The 21. Proposition.

If from the endes of one of the sides of a triangle, be drawen to any point within the sayde triangle two right lines: those right lines so drawen, shalbe lesse then the two other sides of the triangle, but shall containe the greater angle.



Suppose that ABC be a triangle: and from the endes of the side BC , namely, from the pointes B and C , let there be drawen within γ triangle two right lines BD and CD to γ point D . Then I say, that the lines



Demonstration.

BD and CD are lesse then the other sides of the triangle, namely, then the sides BA and AC : and that the angle which they containe, namely, $BD C$, is greater then the angle BAC . Extend (by the second petition) the line BD to the point E . And forasmuch as (by the 20. proposition) in euery triangle the two sides are greater then the side remayning, therefore the two sides of the triangle ABE , namely, the sides AB and AE , are greater then the side EB . Put the line EC common to them both. Wherefore the lines BA and AC , are greater then the lines BE and EC . Againe forasmuch as (by the same) in the triangle CED , the two sides CE and ED , are greater then γ side DC , put γ line DB common to them both: wherefore γ lines CE and EB , are greater then the lines CD and DB . But it is proued that the lines BA and AC , are greater then the lines BE and EC . Wherefore the lines BA and AC , are much greater then the lines BD and DC . Agayne forasmuch

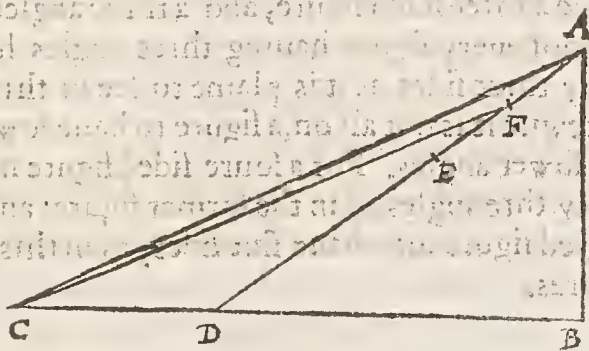
as

as (by the 16. proposition) in euery triangle, the outward angle is greater then the inward and opposite angle, therefore the outward angle of the triangle CDE , namely, BDC , is greater then the angle CED . Wherefore also (by the same) the outward angle of the triangle ABE , namely, the angle CEB is greater then the angle BAC . But it is proued, that the angle BDC is greater then the angle CEB . Wherefore the angle BDC is much greater then the angle BAC . Wherefore if from the endes of one of the sides of a triangle, be drawn to any point within the sayde triangle two right lines: those right lines so drawn shalbe lesse then the two other sides of the triangle, but shall contayne the greater angle: which was required to be demonstrated.

In this proposition is expressed, that the two right lines drawn within the triangle, haue their beginning at the extremes of the side of the triangle. For fro the one extreme of the side of the triangle, and from some one point of the same side, may be drawn two right lines within the triangle, which shall be longer then the two outward lines: which is wonderfull and seemeth straunge, that two right lines drawn vpon a parte of a line, should be greater then two right lines drawn vpon the whole line. And agayne it is possible from the one extreme of the side of a triangle, and from some one point of the same side to drawe two right lynes within the triangle which shall containe an angle lesse then the angle containd vnder the two outward lines.

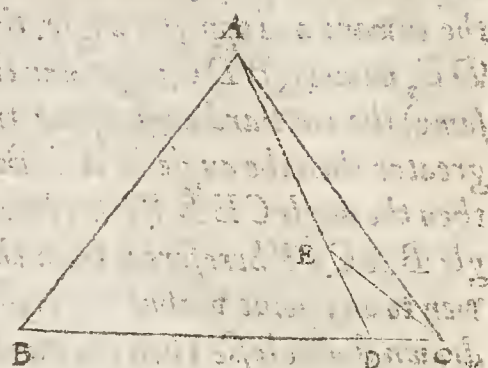
As touching the first part.

Suppose ABC to be a right angle triangle, whose right angle let be at the point B . And in the side BC take a point at al aduentures, which let be D : and draw a right line fro A to D . Wherefore the line AD is greater then the line AB (by the 19. proposition) From the line AD cut of (by the thirde) a line equall to the line AB , which let be DE . And deuide the line E into two equall partes in the point F (by the 10. proposition) And draw a line fro F to C . Now forasmuch as AFC is a triangle, therefore the lines AF and FC are greater then the line AC (by the former proposition): but AF is equal to FE : wherefore the right lines FE and FC are greater then the line AC . And the line DE is equall to the line AB . Wherefore the right lines FC and FD are greater then the right lines AB and AC : and they are drawn within the triangle ABC , the one from one extreme of the side BC , and the other from a point in the same side BC : which was required to be proued.

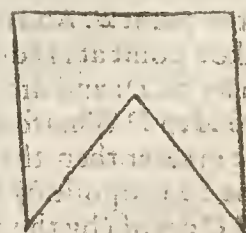
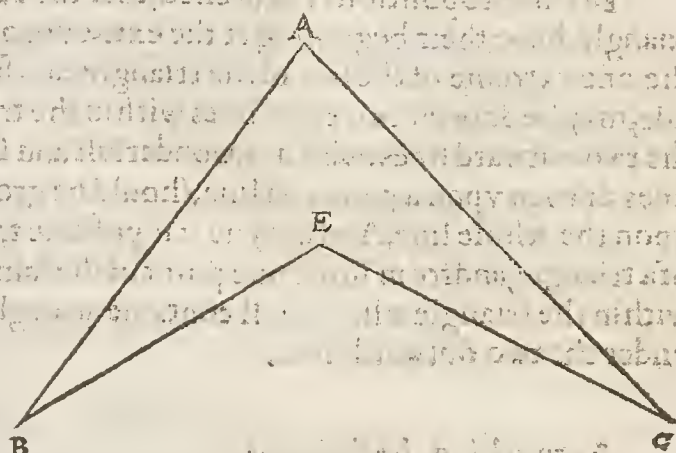


As touchyng the seconde parte. Suppose ABC to be an Ilosceles triangle, and let the base thereof namelye, BC be greater then either of the equall sydes, and from the lyne BC cutte of a line equall to the lyne AB (by the thirde proposition) whiche let be BD : and drawe a line from A to D : and in the

line AD take a point E at adventure, which let be E , & draw a line from C to E . Now forasmuch as the line AB , is equal to the line BD , therefore (by the fifth proposition) the angle BAD is equal to the angle BDA . And forasmuch as in the triangle EDC the angle EDB is an outward angle, therefore (by the 16. proposition) it is greater then the inward and opposite angle DEC . Wherefore the angle BAD is greater then the angle DEC . Wherefore the angle BAC is much greater then the angle DEC : and the angle BAC is contained of the outward right lines BA and AC , and the angle DEC is contained of the inward right lines DE and EC : which was required to be proved.



By meanes of this proposition also is described that kynd of triangles, which containeth foure sides. As for example this figure $ABCE$. For it is contained of foure sides BA , AC , CE , & EB . But it hath onely three angles, one at the point B , another at the point A , and the third at the point C . Wherefore this present figure $ABCE$ is a quadrilateral triangle: which of old philosophers hath euer bene counted wonderfull. And here is to be noted, that there is difference betweene a three sided figure, and a three angled figure. For not euery figure hauing three angles hath also onely three sides, as it is plaine to see in this figure. Likewise it is not all on, a figure to haue foure sides, and foure angles. For a foure sided figure may haue onely three angles, as in the former figure: and a foure angled figure may haue five sides, as in this figure folowing. And so of all other figures.

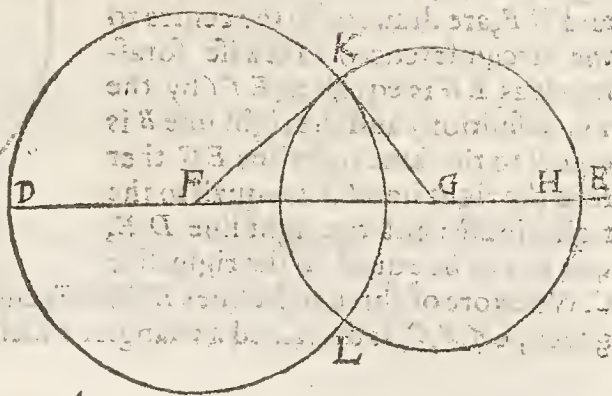


The 8. Probleme. The 22. Proposition.

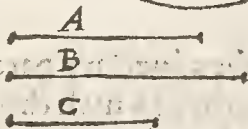
*Of thre right lines, which are equall to thre right lines geue;
to make a triangle. But it behoueth two of those lines, which
two soeuer be taken, to be greater then the third. For that in
euery triangle two sides, which two sides soeuer be taken, are
grea-*

greater then the side remayning,

Suppose that the three right lines geue be A, B, C : of which let two of them, which two soeuer be taken, be greater then the third, that is, let the lines A, B , be greater then the line C , and the lines A, C , then the line B , and the lines B, C , then the line A . It is required of three right lines equall to the right lines A, B, C , to make a triangle. Take a right line hauing an appointed ende on the side D , and being infinite on the side E . And (by the 3. proposition) put vnto the line A , an equall line



Construction.



DF , and put vnto the line B , an equall line FG , and vnto γ line C , an equall line GH . And making the centre F , and the space DF , describe (by the 3. peticiō) a circle DKL . Agayne making the centre G , and the space GH , describe (by the same) a circle HKL : and let the point of the intersection of the sayd circles be K , and (by the first peticion) draw a right line from the point K to γ point F , & an other from the point K to the point G . Then I say, that of thre right lines equall to the lines A, B, C , is made a triangle KFG . For forasmuch as the point F is the centre of the circle DKL , therefore (by the 15. definition) the line FD is equall to the line FK . But the line A is equall to the line FD wherefore (by the first common sentence) the line FK is equall to the line A . Agayne forasmuch as the point G , is the centre of the circle LKH , therefore (by the same definition) the line GK is equall to the line GH . But the line C is equall to the line GH : wherefore (by the first common sentence) the line KG is equall to the line C . But the line FG is by supposition equal to the line B : wherefore these three right lines GF, FK , and KG , are equall to these three right lines A, B, C . Wherefore of three right lines, that is, KF, FG , and GK , which are equall to the thre right lines geuen, that is to A, B, C , is made a triangle KFG : which was required to be done.

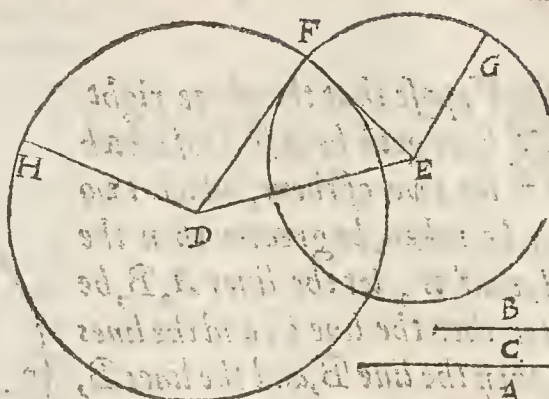
Demonstration.

An other construction, and demonstration after Fluffates.

Suppose that the three right lines be A, B, C . And vnto some one of them, namely, to C , put an equall line DE , and (by the second proposition) from the point E , draw the line EG , equall to the line B : and (by the same) vnto the point D put the line DH equal to the line A . And making the centre the point E , & the space EG , describe a circle FG : like wise making the centre the point D , and the space DH , describe an other circle HF : which circles let cutte the one the other in the point F . And draw

An other construction and demonstration after Fluffates.

these lines DF and EF . Then I saye that DFE is a triangle described of 3. right lines equal to the right lines A, B, C . For forasmuch as the line DH is equal to the right line A , the line DF , shall also be equal to the same right line A , (For that the lines DH and DF , are drawn frō the centre to the circumference). Likewise forasmuch as EG is equal to EF (by the 15. definition) and the right line B is equal to the same right line EG : therfore the right line EF is equal to the right line B : but the right line DE , was put to be equal to the right line C . Wherefore of three right lines ED, DF , and FE , which are equal to three right lines geuen, A, B, C , is described a triangle: which was required to be done.

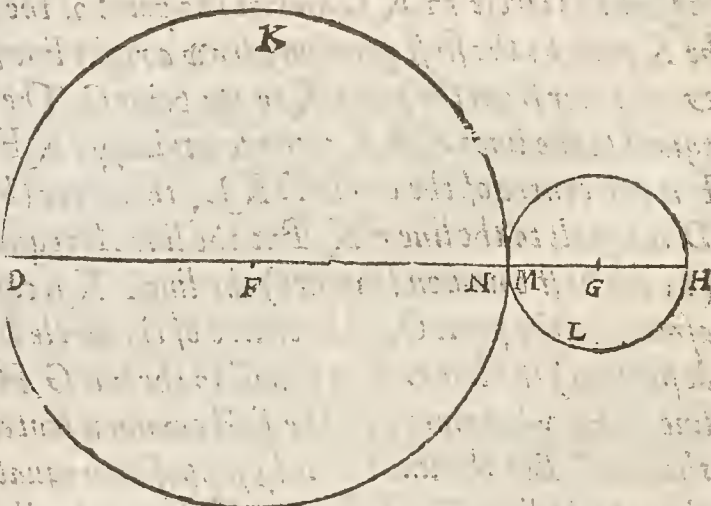


Instances in this Probleme.

In this proposition the aduersary paradventure will caull, that the circles shall not cut the one the other (which thing *Euclide* putteth them to do) But now if they cutte not the one the other, either they touch the one the other, or they are distant the one from the other. First if it be possible let them tooche the one the other, as in the figure here put (the construction whereof answereth to the construction of *Euclide*).

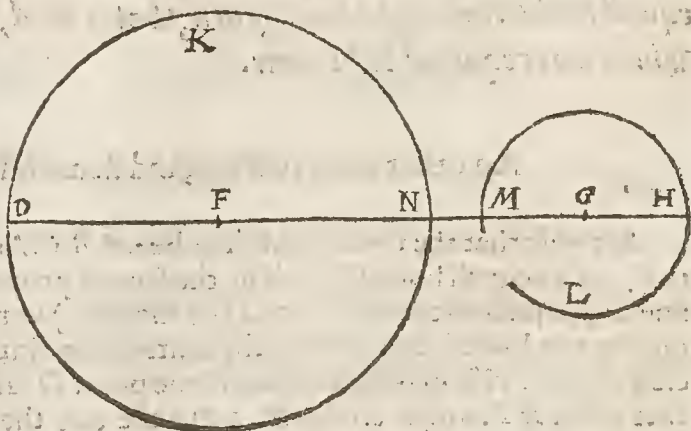
First instance.

And forasmuch as F is the centre of the circle DK , therefore the line DF is equal to the line FN . And forasmuch as the point G , is the centre of the circle HL , therefore the line HG , is equal to the line GM . Wherefore these two lines DF , and HG , are equal to one line, namely, to FG . But they were put to be greater then it: for the lines DF, FG , and HG , were put to be equal to the lines A, B, C , every two of which are supposed to be greater then the thirde: wherefore they are both greater, and also equal, which is impossible. Agayne if it be possible, let the circles be distant the one from the other, as



Second instance

as are the circles DK and HL . And forasmuch as F is the centre of the circle DK , therefore the line DF is equal to the line FN . And forasmuch as G is the centre of the circle HL , therefore the line HG is equal to the line GM : wherefore the whole line FG is greater then the two lines DF , and HG , (for the line FG , exceedeth the lines DF , and HG , by the line NM) but it was suppo-

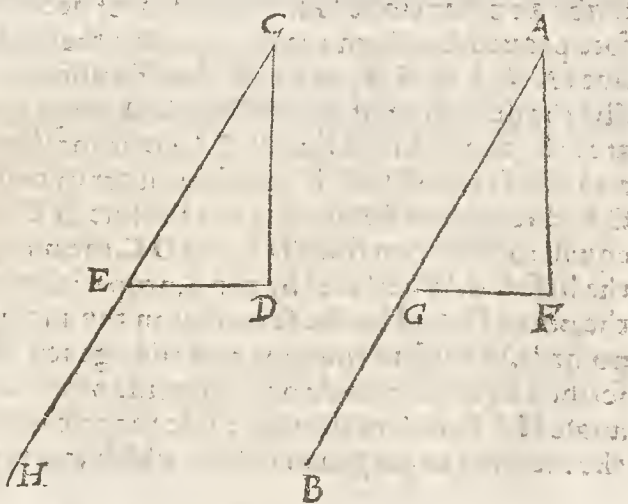


fed that the lines DE and HG are greater then the line FG : as also it was supposed that the lines A and C , were greater then the line B (for the line DE is put to be equall to the line A , and the line FG to the line B , and the line HG to the line C .) Wherefore they are both greater and also equall: which is impossible. Wherefore the circles neyther tooch the one the other, nor are distant the one from the other. Wherefore of necessity they cut the one the other: which was required to be proued.

The 9. Probleme. The 23. Proposition.

Upon a right line geuen, and to a point in it geuen: to make a rectiline angle equall to a rectiline angle geuen.

Suppose $\angle$ the right line geuen be AB , & let $\angle$ point in it geuen be A . And let also the rectiline angle geuen be DCH . It is required vpon the right line geuen AB , and to the point in it geuen A , to make a rectiline angle equall to the rectiline angle geuen DCH . Take in either of the lines CD and CH a point at all aduentures, & let the same be D and E . And (by the first peticion) draw a right line fro D to E . And of thre right lines, AF , FG and GA , which let be equall to the three right lines geuen, that is, to CD , DE , and EC , make (by $\angle$ proposition goyng before) a triangle, and let the same be AFG . so that let the line CD be equall to the line AF , and the line CE to the line AG , and moreover the lyne DE to the line FG . And forasmuch as these two lines DC and CE are equall to these two lines FA and AG , the one to the other, and the base DE is equall to the base FG : therefore (by the 8. proposition) the angle DCE is equall to the angle FAG . Wherefore vpon the right line geuen AB , and to the point in it geuen namely A , is made a rectiline angle FAG , equal to the rectiline angle geuen DCH : which was required to be done.



Construction.

Demonstration.

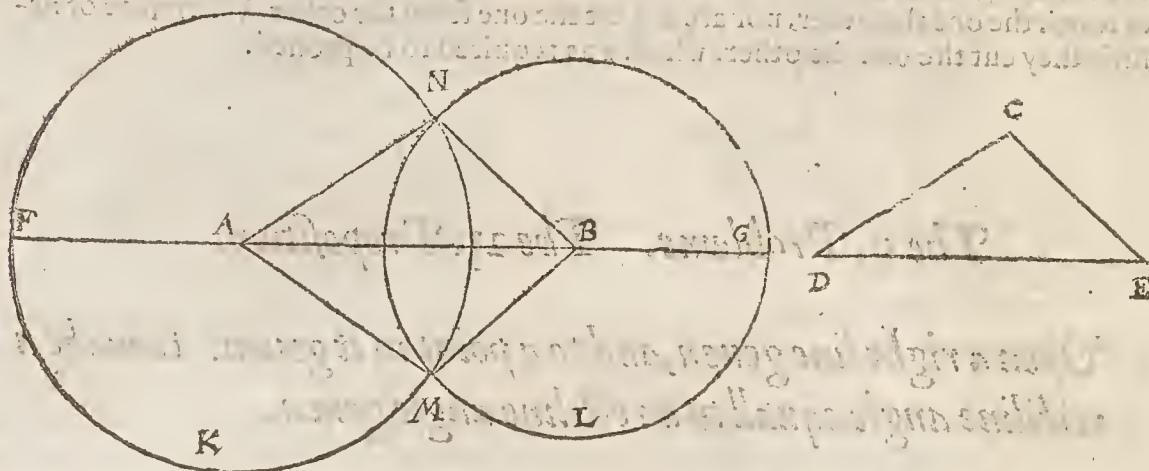
An other construction and demonstration after Proclus.

Suppose that the right line geuen be AB : & let the point in it geuen be A , & let the rectiline angle geuen be CDE . It is required vpon the right line geuen AB , & to the point in it geuen A , to make a rectiline angle equal to the rectiline angle geuen CDE . Draw a line fro C to E . And produce the line AB on either side to the points F and G . And vn-

An other construction and demonstration after Proclus.

The first Booke

to the line CD , put the line FA equal, & vnto the line DE let the line AB be equal, & vnto the line EC put the line BG equal. And making the centre the point A , & the space AF , describe a circle $K F$. And agayne making the centre the point B and the space $B G$ de-

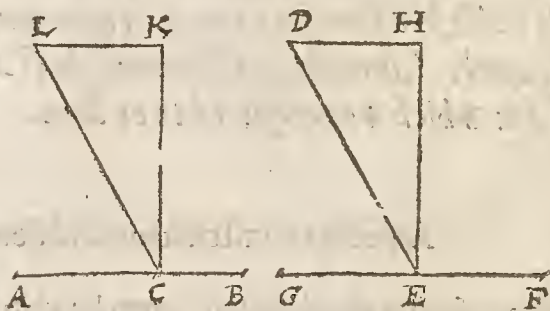


scribe an other circle GL : which shal of necessitie cut the one the other, as we haue before proued. Let them cut the one the other in the pointes M & N . And draw these right lines AN , AM , BN , and BM . And forasmuch as FA is equall to AM : and also to AN (by the definition of a circle) but CD is equall to FA , wherfore the lines AM and AN are eche equall to the line DC . Agayne forasmuch as BG , is equall to BM ; and to BN , and BG is equall to CE : therfore either of these lines BM and BN is equall to the line CE . But the line BA is equall to the line DE . Wherfore these two lines BA & AM , are equall to these two lines DE and DC , the one to the other, and the base BM is equal to the base CE . Wherfore (by the 8. proposition) the angle MAB , is equall to the angle at the point D . And by the same reason the angle NAB , is equall to the same angle at the point D . Wherfore vpon the right line geuen AB , and to the point in it geuen A , is described a rectiline angle on either side of the line AB : namely, on one side the rectiline angle NAB , and on the other side the rectiline angle MAB , either of which is equall to the rectiline angle geuen CDE : which was required to be done.

An other construction also, and demonstration after Pelitarius.

Another Demonstration after Pelitarius.

Suppose that the right line geuen be AB : and let the point in it geuen be C , and let the rectiline angle geuen be DEF . It is required vpon the line geuen AB , and to the point in it geuen C , to describe a rectiline angle equall to the rectiline angle geuen DEF . Produce the line FE to the point G : and from the point E erect (by the 11. proposition) vnto the line GF a perpendicular line EH , which if it exactly agree with the line ED , then was the angle geue a right angle. Wherfore if from the point C you erecte a perpendicular line vnto the line AB , that shall be done which was required to be done. But if it do not, then from the point H , erect vnto the line HE , a perpendicular line HD , whiche being produced shall (by the fifth petition) concurre with the line ED being also produced: for the angle DEH is lesse then a right angle (when as GEH is a right angle). Wherfore let them concurre in the point D , and so is made the triangle DEH . After the same maner frō the point geuen C , erect vnto the line AB a perpendicular line CK : which let be equall to the



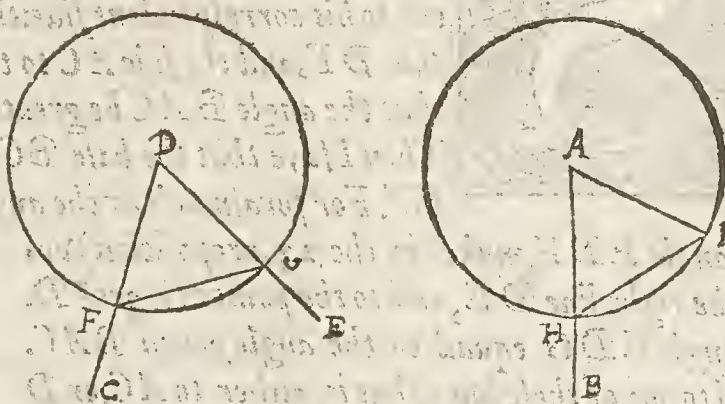
the perpendicular line EH (by the 3. proposition): and from the point K erect vnto the line KC , a perpendicular line KL , whiche let be equall to the perpendicular lyne HD . And draw a line from C to L . Then I say that the angle LCB , is equall to the angle geuen DEF . For the two triangles HED and KGL , are (by the fourth proposition) equal, and equilater the one to the other: and the two angles LCK and DEH are equal. And the two angles BCK and FEH are equal, for either of them is a right angle. Wherefore (by the 2. common sentence) the whole angle LCB , is equall to the whole angle DEF . Which was required to be done.

And if the perpendicular line chaunce to fall without the angle geuen, namely, if the angle geuen be an acute angle, the selfe same manner of demonstration will serue: but onely that in stede of the second common sentence, must be vsed the 3. common sentence.

Appollonius putteth another construction & demonstration of this proposition: which (though the demonstration thereof depende of propositions put in the third booke, yet for that the construction is very good for him that wil redely, and mechanically, without demonstration, describe vpon a line geuen, and to a point in it geuen, a rectiline angle equall to a rectiline angle geuen) I thought not amisse here to place it. And it is thus.

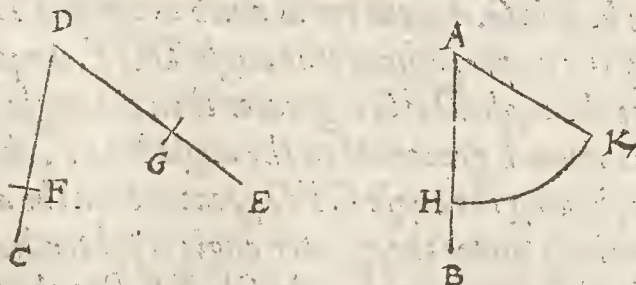
Suppose that the rectiline angle geuen be CDE , and let the right line geuen be AB , and let the point in it geuen be A . Take in the line CD , a point at all adventures, which let be F . And making the centre the point D , and the space DF , describe a circle FG , cutting the line DE , in the point G , and draw a right line from F to G . Likewise from the line AB , cut of a line equall to the line DF , which let be AH . And making the centre the point A , and the space AH describe a circle HK , and from the point H , subtend vnto the circumference of the circle a right lyne equall to the right line FG , which let be HK : and draw a right lyne from A to K . Then I say that the angle HAK , is equall to the angle CDE . The prooffe whereof I now omitte for that it dependeth of the 28 and 27 propositions of the third booke.

Another construction & demonstration of the proposition of Appollonius.



But now, as I sayd, by this you may very redily, and mechanically, without demonstration, vpon a line geuen, and to a point in it geuen describe a rectiline angle equall to a rectiline angle geuen. For in the rectiline angle geuen, you neede onely to marke the two pointes, where the circumference of the circle cutteth the lines contayning the angle geuen: as the points F and G : and likewise to marke in the line geuen as in AB , the point H , & so making the centre the point A , according to the space AH (which is put to be equal to DF) describe a peece of a circumference on that side that you wil haue the angle to be, as for example the circumference HK , and opening your compasse to the width from

How to do this proposition redily and mechanically.



K. i.

the

The first Booke

the point *F*, to the point *G*, set one foote thereof fixed in the point *H*, and marke the point where the other foote cutteth the sayde circumference, which point let be *K*. And from that point to the point *A*, draw a right line: and so shall you haue described at the point *A*, an angle equal to the angle at the point *D*. As in the figures in the end of the other side put.

Oenopides the first inuenter of this proposition.

Oenopides was the first inuenter of this proposition as witnesseth *Eudemius*.

The 15. Theoreme The 24. Proposition.

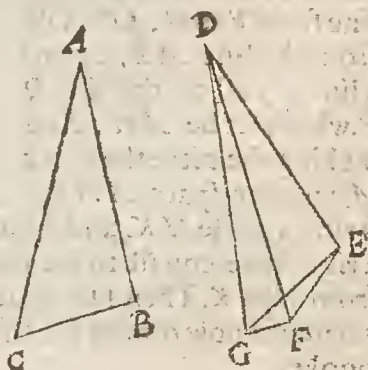
If two triangles haue two sides of the one equall to two sides of the other, ech to his correspondent side, and if the angle cotayned vnder the equall sides of the one, be greater then the angle contayned vnder the equall sides of the other: the base also of the same, shalbe greater then the base of the other.



Suppose that there be two triangles ABC , and DEF , hauing two sides of the one, that is, AB , and AC , equall to two sides of the other, that is, to DE , and DF , ech to his correspondent side: that is, the side AB , to the side DE , and the side AC to the side DF : and suppose that the angle BAC be greater then the angle EDF . Then I saye that the base BC , is greater then the base EF . For forasmuch as the angle BAC is greater then

Construction.

the angle EDF , make (by the 23. proposition) vpon the right line DE , and to the point in it geue D , an angle EDG equall to the angle geuen BAC . And to one of these lines, that is, either to AC , or DF , put an equall line DG . And (by the first peticio) draw a right line from the point G , to the point E , and an other from the point F , to the point G . And forasmuch as the line AB is equall to the line DE , and the line AC to the line DG , the one to y other, and the angle BAC is (by construction) equall to



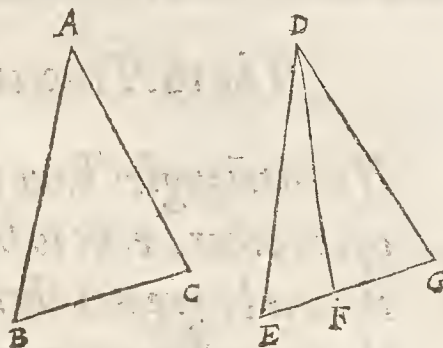
Demonstration.

the angle EDG ; therefore (by the 4. proposition) the base BC , is equall to the base EG . Agayne for as much as the line DG is equall to the line DF , ther (by the 5. proposition) the angle DGF , is equall to the angle DFG . Wherefore the angle DFG is greater then the angle EGF . Wherefore the angle EFG is much greater then the angle EGF . And forasmuch as EFG is a triangle, hauing the angle EFG greater then the angle EGF , and (by the 18. proposition) vnder the greater angle is subtended the greater side, therefore the side EG is greater then the side EF . But the side EG is equall to the side BC : Wherefore the side BC is greater then the side EF . If therefore two triangles haue

haue two sides of the one equall to two sides of the other, eche to his correspondent side, and if the angle containd vnder the equall sides of the one, be greater then the angle containd vnder the equall sides of the other: the base also of the same shalbe greater then the base of the other: which was required to be proued.

In this Theoreme may be three cases. For the angle $\angle EDG$, being put equall to the angle $\angle BAC$, and the line DG , being put equall to the line AC , and a line being drawn from E to G , the line EG shall either fall about the line GF , or vpon it, or vnder it. Euclides demonstration serueth, when the line GE falleth about the line GF , as we haue before manifestly seene.

But if it fall vpon it, as in this figure here put. Then forasmuch as the two lines AB and AC , are equal to the two lines DE and DG , the one to the other, and they containe equall angles by construction: therefore (by the 4. proposition) the base BC , is equal to the base EG : but the base EG , is greater then the base EF : wherefore also the base BC , is greater then the base EF : which was required to be proued.

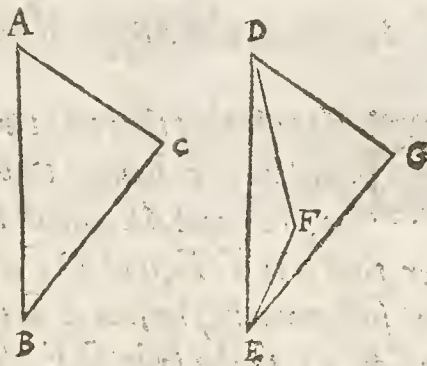


Three cases in this proposition.

The first case.

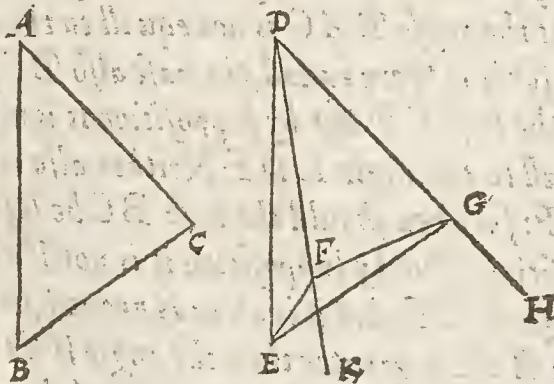
Second case.

But now let the line EG , fall vnder the line EF , as in the figure here put. And forasmuch as these two lines AB , and AC , are equal to these two lines DE and DG , the one to the other, and they containe equall angles, therefore (by the 4. proposition) the base BC , is equal to the base EG . And forasmuch as within the triangle DEG , the two lines DF and FE , are set vpon the side DE : therefore (by the 21. proposition) the lines DF and FE are lesse then the outward lines DG and GE : but the line DG is equal to the line DF . Wherefore the line GE is greater then the line FE . But GE is equall to BC . Wherefore the line BC is greater then the line EF . Which was required to be proued.



Third case.

This third case may also another way be demonstrated. produce the lines DF and DG , which are equal, vnto the points K and H : and draw a line from F to G : wherefore (by the second part of the fifth proposition) the angles KFG and FGH , which are vnder the base FG , are equall: therefore the angle EFG is greater then the angle FGH . Wherefore (by the 18 proposition) the side EG is greater then the side EF : but the base BC is equal vnto the base EG : Wherefore the base BC , is greater then the base EF : Which was required to be proued.



An other demonstration of the third case.

It may peraduenture seme, that Euclide should here in this proposition haue proued, that not onely the bases of the triangles are vnequall, but also that the areas of the same are vnequall: for so in the fourth proposition, after he had proued

The first Booke

Why Euclide
doe proueth
not the areas of
the triangles to
be vnequall.

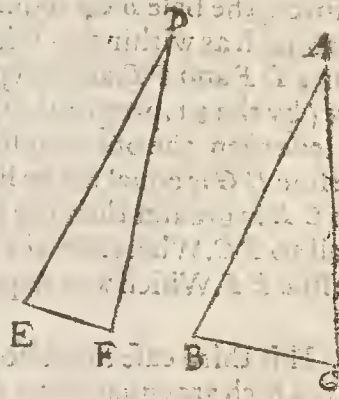
After the 37.
Proposition you
shall finde the
comparison of
triangles, whose
sides being
equall, their ba-
ses and angles
at the toppe
are vnequall.

ued the base to be equall, he proued also the areas to be equall. But hereto may be answered, that in equall angles and bases, and vnequall angles and bases, the consideration is not like. For the angles and bases being equall, the triangles also shall of necessitie be equall, but the angles and bases being vnequall, the areas shall not of necessitie be equall. For the triangles may both be equall and vnequall: and that may be the greater, whiche hath the greater angle, and the greater base, and it may also be the lesse. And for that cause Euclide made no mention of the comparison of the triangles. VVhereof this also mought be a cause, for that to the demonstration thereof are required certayne Propositions concerning parallel lines, which we are not as yet come vnto. Howbeit after the 37. proposition of this booke you shall find the comparison of the areas of triangles, which haue their sides equall, and their bases and angles at the toppe vnequall.

The 16. Theoreme. The 25. Proposition.

If two triangles haue two sides of the one equall to two sides of the other, eche to his correspondent syde, and if the base of the one be greater then the base of the other: the angle also of the same cotayned vnder the equall right lines, shall be greater then the angle of the other.

Suppose that there be two triangles, A, B, C , and D, E, F , hauing two sides of the one, that is, A, B , and A, C , equall to two sides of the other, that is to D, E , and D, F , ech to his correspondent side, namely, the side A, B to the side D, E , and the side A, C to the syde D, F . But let the base B, C be greater then the base E, F . The I say, that the angle B, A, C is greater then the angle E, D, F . For if not, then is it either equall vnto it, or lesse then it. But the angle B, A, C is not equall to the angle E, D, F : for if it were equall, the base also B, C should (by the 4. proposition) be equal to the base E, F : but by supposition it is not. VVherfore the angle B, A, C is not equall to the angle E, D, F . Neither also is the angle B, A, C lesse then the angle E, D, F : for then should the base B, C be lesse then the base E, F (by the former proposition) But by supposition it is not. VVherfore y angle B, A, C is not lesse then y angle E, D, F . And it is already proued, that it is not equall vnto it: wherfore y angle B, A, C is greater then the angle E, D, F . If therefore two triangles haue two sides of the one equall to two sides of the other, eche to his correspondent side, and if the base of the one be greater then the base of the other, the angle also of the same contayned vnder y equal right lines shall be greater then the angle of the other: which was required to be proued.



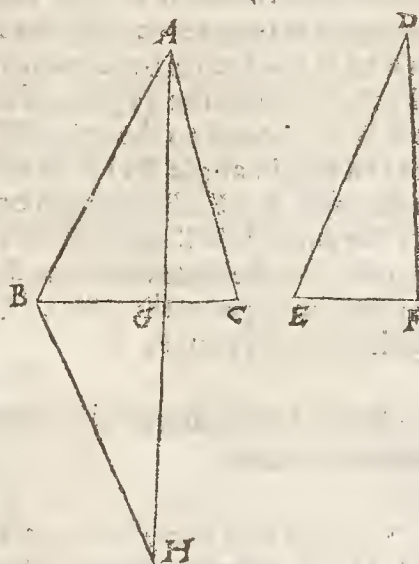
Demonstration
leading to an
absurdity.

This

This proposition is plaine opposite to the eight, & is the cōuerse of the foure and twenty which went before, and it is proued (as commonly all conuerſes are) by a reason leading to an abſurditie. But it may after Menelaus Alexandrinus be demonstrated directly, after this maner.

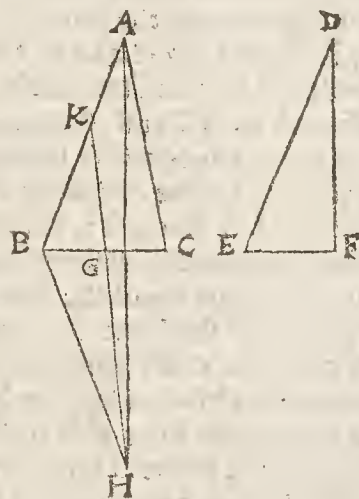
All conuerſes are commonly indirectly demonstrated. An other demonstration after Menelaus Alexandrinus.

Suppoſe that there be two triangles ABC & DEF : hauing the two ſides AB and AC equal to the two ſides DE and DF , the one to the other: and let the baſe BC be greater then the baſe EF . Then I ſay that the angle at the point A , is greater then the angle at the point D . For from the baſe BC cut off (by the thirde) a line equall to the baſe EF , and let the ſame be BG . And vpon the line GB and to the point B put (by the 23. propoſition) an angle equal to the angle DEF : which let be GBH : and let the line BH be equall to the line DE . And draw a lyne from H to G , and produce it beynd the point G : whiche being produced ſhall fall either vppon the angle A , or vpon the line AB , or vpon the line AC . Firſt let it fall vpon the angle A . And forasmuch as theſe two lines BG and BH are equall to theſe two lines EF and ED , the one to the other, and they contayne equall angles (by conſtruction) namely, the angles GBH and DEF : therefore (by the 4. propoſition) the baſe GH is equall to the DF , and the angle BHG to the angle EDF . Agayne forasmuch as the line BH is equall to the line BA (for the line AB is ſuppoſed to be equal to the line DE , vnto which line the line BH is put equal) therefore (by the 5. propoſition) the angle BHA is equall to the angle BAH : wherfore alſo the angle EDF is equal to the angle BAH . But the angle BAC is greater then the angle BAH : wherfore alſo the angle BAC is greater then the angle EDF .



Diuers caſes in this demonstration. Firſt caſe.

But now let it fall vpon the line AB in the point K , and draw a line from A to H . And forasmuch as theſe two lines BG and BH are equall to theſe two lines EF and ED , the one to the other, & they contayne equal angles (by conſtruction) namely, the angles GBH and DEF : therefore (by the 4. propoſition) the baſe GH is equall to the baſe DF , and the angle BHG to the angle EDF . Agayne forasmuch as in the triangle BAH , the ſide BA is equal to the ſide BH , therefore (by the 5. propoſition) the angle BAH is equal to the angle BHA . But the angle BHA is greater then the angle BHG : wherfore alſo the angle BAH is greater then the angle BHG . Wherfore the angle BAC is much greater then the angle BHG . But it is proued that the angle BHG is equall to the angle at the point D . Wherfore the angle BAC is greater then the angle at the point D : Which was required to be proued.



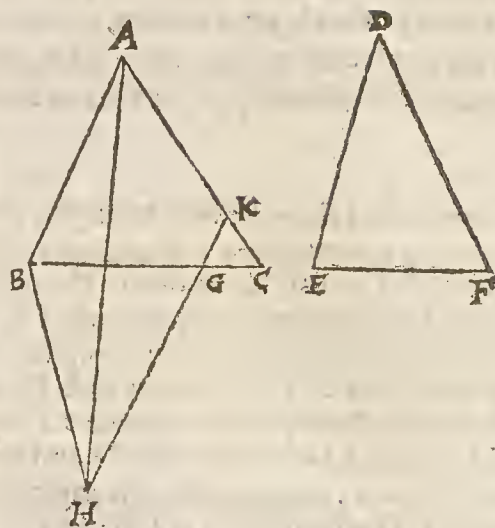
Second caſe.

But now ſuppoſe that the line HG beyng produced doo fall vppon the line AC , namely, in the point K . And agayne draw alſo a line from A to H . And forasmuch as BG is equall to EF , and BH to ED , therefore theſe two lines BG and BH are equall to theſe two lines EF and ED , the one to the other, and (by conſtruction) they contayne equall angles, namely, the angles GBH and FED . Wherfore (by the fourth propoſitiō)

Third caſe.

The first Booke

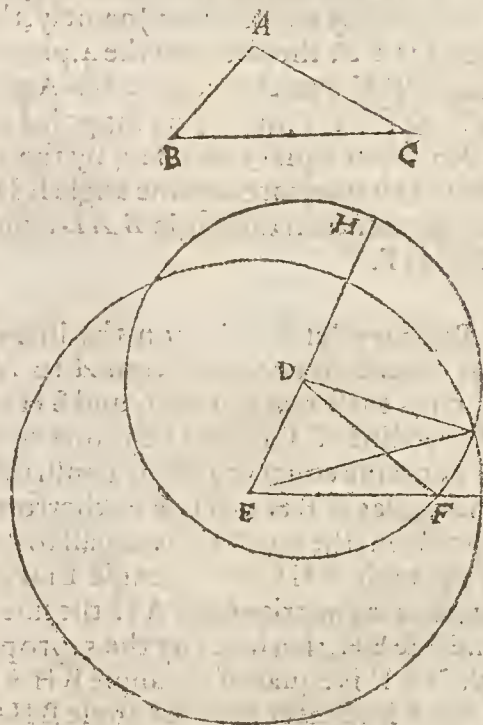
the base GH is equal to the base DF : & th'an-
gle BHG is equal to th'angle EDF . And for-
asmuch as GH is equal to DF , and DF is
equal to AC : therefore GH also is equal to A
 C . Wherefore HK is greater then AC , where-
fore HK is much greater then AK . Wherefore
(by the 18. proposition) the angle KAH is
greater then the angle KHA . Agayne foras-
much as BH is equal to AB (for BH is put
equal to DE , which is by supposition equal to
 AB) therefore (by the 5. proposition) the an-
gle BHA is equal to the angle BAH . Where-
fore the whole angle BHK is lesse then the
whole angle BAK . But it hath bene proued,
that the angle BHK is equal to the angle at
the point D , wherefore the angle BAC is grea-
ter then the angle at the point D , which was
required to be proued.



Hero Mechanicus also demonstrateth it another way, and that by a direct demonstration.

Another de-
monstration af-
ter Hero Me-
chanicus.

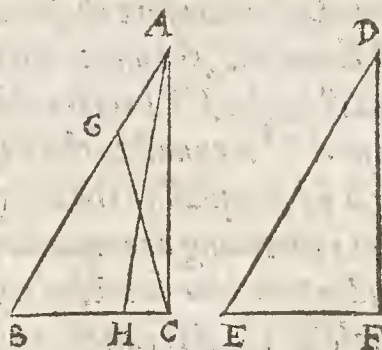
Suppose that there be two triangles ABC and DEF , hauyng the two sides AB , and AC , equall to the two sides DE , & DF , the one to the other, and let the base BC , be greater then the base EF . Then I say, that the angle at the point A , is greater then the angle at the point D . For forasmuch as BC , is greater thē EF , produce EF to the point G , and put the line EG , equall to the line BC . Likewise produce the line DE to the point H , and put the line DH , equall to the line DF . Wherefore making the centre the point D , and the space DF , describe a circle, and it shall passe also by the point H . Let the same circle be FKH . And forasmuch as AC and AB are greater thē BC (by the 20. propositiō) & the lines AB & AC , ar equal to the line EH , & the line BC is equal to the line EG . Therefore the line EH is greater then the line EG . Wherefore making the centre the point E and the space EG describe a circle, and it shall cut the line EH . Let the same circle be GK : and from the common sectiō of the circles, which let be the point K , draw these right lines KD and KE . And forasmuch as the point D is the centre of the circle HKF , therefore (by the 15. definition) the line DK , is equall to the line DH , that is vnto the line AC . Agayne forasmuch as E is the centre of the circle GK , therefore the line EK is equal to the line EG , that is, to the line BC . And forasmuch as these two lines AB and AC , are equall to these two lines DE and DK , and the base BC is equal to the base EK (for EK is equall to EG (by the 15. definition) & EG is put to be equal to BC). Wherefore (by the 4. proposition) the angle BAC is equal to the angle EDK . But the angle EDK is greater then the angle EDF : wherefore also the angle BAC , is greater then the angle EDF : which was required to be proued.



The 17. Theoreme. The 26. Proposition.

If two triangles haue two angles of the one equall to two angles of the other, ech to his correspondent angle, and haue also one side of the one equall to one side of the other, either that side which lieth betwene the equall angles, or that which is subtended vnder one of the equall angles: the other sides also of the one, shalbe equall to the other sides of the other, eche to his correspondent side, and the other angle of the one shalbe equall to the other angle of the other.

Suppose that there be two triangles ABC , and DEF , hauing two angles of the one, that is, the angles ABC , and BCA , equall to two angles of the other, that is, to the angles DEF , and EFD , ech to his correspondent angle, that is, the angle ABC , to the angle DEF , and the angle BCA to the angle EFD , and one side of the one equall to one side of the other, first that side which lieth betwene the equall angles, that is, the side BC , to the side EF . The I say that the other sides also of the one shalbe equall to the other sides of the other, ech to his correspondent side, that is, the side AB , to the side DE , and the side AC , to the side DF , and the other angle of the one, to the other angle of the other, that is, the angle BAC to the angle EDF . For if the side AB be not equall to the side DE , the one of them is greater. Let the syde AB be greater: and (by the 3. proposition) vnto the line DE , put an equall line GB , and draw a right line from the point G , to the point C . Now forasmuch as the line GB , is equall to the line DE , and the line BC to the line EF , therefore these two lines GB and BC , are equall to these two lines DE and EF , the one to the other, and the angle GBC is (by supposition) equall to the angle DEF . Wherefore (by the 4. proposition) the base GC is equall to the base DF , and the triangle GCB is equall to the triangle DEF , and the angles remayning are equall to the angles remayning vnder which are subtended equall sydes. Wherefore the angle GCB is equall to the angle DFE . But the angle DFE is supposed to be equall to the angle BCA . Wherefore (by the first common sentence) the angle BCG is equal to the angle BCA , the lesse angle to the greater: which is impossible. Wherefore the line AB is not vnequall to the line DE . Wherefore it is equall. And the line BC is equall to the line EF : now therefore there are two sydes AB and

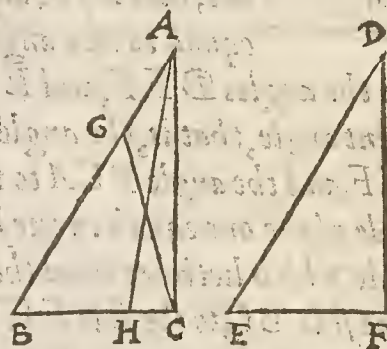


Demonstration
leading to an
absurditie.

The first Booke

BC equall to two sydes DE and EF , the one to the other, and the angle ABC , is equall to the angle DEF . Wherefore (by the 4. proposition) the base AC is equall to the base DF , and the angle remayning BAC is equall to the angle remayning EDF .

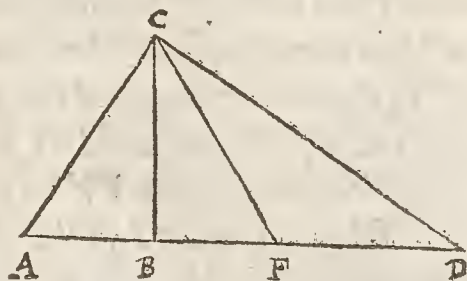
Agayne suppose that the sydes subtending the equall angles be equall the one to the other, let the syde I say AB be equall to the syde DE . Then agayne I say, that the other sydes of the one are equall to the other sydes of the other, ech to his correspondent syde, that is the syde AC to the syde DF , and the syde BC to the syde EF : and moreouer the angle remayning, namely, BAC , is equall to the angle remayning, that is, to the angle EDF . For if the syde BC be not equall to the syde EF , the one of them is greater: let the syde BC , if it be possible, be greater. And (by the third propofytion) vnto the line EF , put an equall line BH , and drawe a right line from the point A to the point H . And forasmuch as the line BH is equall to the line EF , and the line AB to the line DE , therefore these two sydes AB and BH , are equall to these two sydes DE and EF , the one to the other, and they containe equall angles. Wherefore (by the 4. proposition) the base AH is equall to the base DF , and the triangle ABH , is equall to the triangle DEF , and the angles remayning are equall to the angles remayning, vnder which ar subtended equal sydes. Wherefore the angle BHA is equall to the angle EFD . But the angle EFD is equall to the angle BCA . Wherefore the angle BHA is equal to the angle BCA . Wherefore the outward angle of $\triangle AHC$, namely, the angle BHA , is equall to the inward and opposite angle, namely, to the angle HCA , which (by the 16 proposition) is impossible. Wherefore the syde EF is not vnequall to the syde BC , wherefore it is equall. And the syde AB is equall to $\triangle$ syde DE : wherefore these two sydes AB and BC , are equall to these two sydes DE and EF , the one to the other, and they containe equall angles: Wherefore (by the 4. proposition) the base AC is equall to the base DF : and the triangle ABC , is equall to the triangle DEF , and the angle remayning, namely, the angle BAC is equall to the angle remayning, that is, to the angle EDF . If therefore two triangles haue two angles of the one equall to two angles of the other, ech to his correspondent angle, and haue also one syde of the one equall to one syde of the other, either that syde which lieth betwene the equall angles, or that which is subtended vnder one of the equall angles: the other sydes also of the one shalbe equall to the other sydes of the other, eche to his correspondent side, and the other angle of the one shalbe equall to the other angle of the other: which was required to be proued.



Whereas in this proposition it is sayde, that triangles are equall, which hauing two angles of the one equall to two angles of the other, the one to the other,

ther, haue also one side of the one equall to one side of the other, either that side which lieth betwene the equall angles, or that side which subtendeth one of the equall angles: this is to be noted that without that caution touching the equall side, the proposition shall not alwayes be true. As for example.

Suppose that there be a rectangle triangle ABC , whose right angle let be at the point B , & let the side BC be greater the the side BA : and produce the line AB , frō the point B to the point D . And vpon the right line BC & to the point in it C , make vnto the angle BAC an equal angle (by the 23. proposition), which let be BCD , & let the lines BD & CD , being produced cōcurre in the point D . Now the there are two triangles ABC , and BCD , which haue two angles of the one equall to two angles of the other, the one to the other, namely, the angle ABC to the angle DBC (for they are both right angles), & the angle BAC , to the angle BCD (by construction) and haue also one side of the one equall to one side of the other, namely, the side BC , which is cōmon to them both. And yet notwithstanding the triangles are not equall: for the triangle DBC , is greater then the triangle ABC . For vpon the right line BC , and to the point in it C , describe an angle equall to the angle ACB : which let be FCB (by the 23. proposition). And forasmuch as the side BC was supposed to be greater then the side, AB , therefore (by the 18. proposition) the angle BAC is greater then the angle BCA , wherefore also the angle BCD is greater then the angle BCF . Wherefore the triangle BCD is greater then the triangle BCF . Agayne forasmuch as there are two triangles ABC and BCF , hauing two angles of the one equal to two angles of the other, the one to the other, namely, the angle ABC to the angle FCB (for they are both right angles) and the angle ACB to the angle FCB (by construction), and one side of the one is equall to one side of the other, namely, that side which lieth betwene the equall angles, that is, the side BC which is common to both triangles. Wherefore (by this proposition) the triangles ABC and FCB are equal. But the triangle DBC is greater the the triangle FCB : Wherefore also the triangle DBC is greater then the triangle ABC . Wherefore the triangles ABC and DBC , are not equall: notwithstanding they haue two angles of the one equall to two angles of the other, the one to the other, and one side of the one equall to one side of the other.



The reason wherof is, for that the equal side in one triangle, subtēdeth one of the equall angles, and in the other lieth betwene the equal angles. So that you see that it is of necessitie that the equall side do in both triangles, either subtend one of the equall angles, or lie betwene the equall angles.

Of this proposition was Thales Milesius the inuentor, as witnesseth Eudemus in his booke of Geometricall enarrations.

*Thales Milesius
the inuentor of
this proposition.*

The 18. Theoreme. The 27. Proposition.

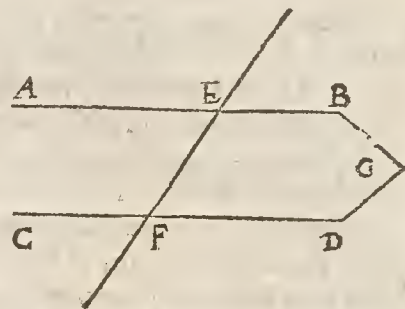
If a right line falling vpon two right lines, do make the alternate angles equall the one to the other: those two right lines are parallels the one to the other.

L.j.

Suppose

The first Booke

Suppose that the right line EF falling vpon these two right lines AB and CD , do make the alternate angles, namely, the angles AEF & EFD equall the one to the other. Then I say that AB is a parallel line to CD . For if not, then these lines produced shall mete together, either on the side of B and D , or on the syde of A & C . Let them be produced therfore, and let them mete if it be possible on the syde of B and D , in the point G . VVherfore in the triangle GEF , the outward angle AEF is equal to the inward and opposite angle EGF , which (by the 16. proposition) is impossible. VVherfore the lines AB and CD beyng produced on the side of B and D , shall not meete. In like sorte also may it be proued that they shall not mete on the syde of A and C . But lines whiche being produced on no syde meete together, are parrallell lines (by the last definition:) wherfore AB is a parrallel line to CD . If therfore a right line falling vpon two right lines, do make the alternate angles equall the one to the other: those two right lines are parrallels the one to the other: which was required to be demonstrated.



*Demonstration
leading to an
absurditie.*

*This worde al-
ternate & used in
diuers senses.*

*How it is taken
in this place.
Which angles
are called altera-
mate.*

This worde alternate is of Euclide in diuers places diuerfly taken: somtimes for a kind of situation in place, and sometime for an order in proportion, in which signification he vseth it in the v. booke, and in his booke of numbers. And in the first signification he vseth it here in this place, and generally in all hys other booke, hauing to do with lines & figures. And those two angles he calleth alternate, which beyng both containned within two parallel or equidistant lynes are neither angles in order, nor are on the one and selfe same side, but are seperated the one from the other by the line which falleth on the two lines: the one angle beyng aboue, and the other beneath.

The 19. Theoreme.

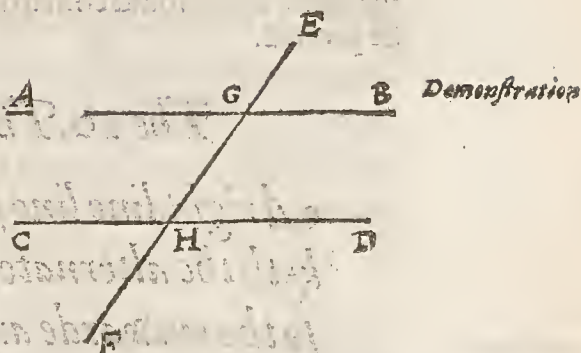
The 28. Proposition.

If a right line falling vpon two right lines, make the outward angle equall to the inward and opposite angle on one and the same syde, or the inward angles on one and the same syde, equall to two right angles: those two right lines shall be parrallels the one to the other.



Suppose that the right line EF , falling vpon these two right lines AB and CD , do make the outward angle EGB equall to the inward and opposite angle GHD , or do make the inward angles on one and the

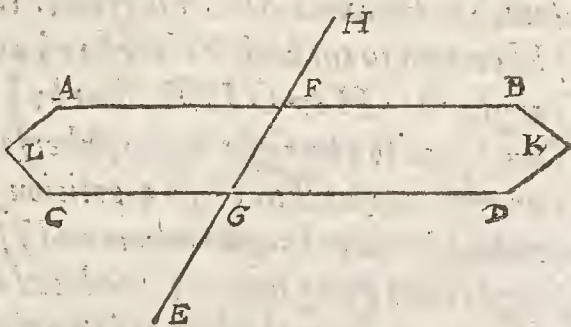
the same side, that is, the angles BGH and GHD equall to two right angles. Then I say that the lyne AB is a parallel line to the lyne CD . For forasmuch as the angle EGB is (by supposition) equall to the angle GHD , and the angle EGB is (by the 15. proposition) equall to the angle AGH : therefore the angle AGH is equall to the angle GHD : and they are alternate angles. Wherefore (by the 27. proposition) AB is a parallel line to CD .



Agayne forasmuch as the angles BGH and GHD are (by supposition) equall to two right angles, & (by the 13. proposition) the angles AGH and BGH , are also equall to two right angles, wherefore the angles AGH and BGH , are equall to the angles BGH and GHD : take away the angle BGH which is common to them both. Wherefore the angle remainyng, namely, AGH is equall to the angle remainyng, namely, to GHD . And they are alternate angles. Wherefore (by the former proposition) AB is a parallell line to CD . If therefore a right line fallyng vpon two right lines, do make the outward angle equall to the inward and opposite angle on one and the same side, or the inward angles on one and the same side, equall to two right angles, those two right lines shall be parallels the one to the other: which was required to be proued.

Ptolomeus demonstrateth the second part of this proposition, namely, that the two inward angles on one and the same side being equall, the right lines are parellels, after this manner.

Suppose that there be two right lines AB and CD , and let a certayne right line EF cutte them in suche sorte, that it make the angles BFG and FGD equall to two right angles. Then I say, that these right lines AB and CD are parallel lines, that is, they shall not concur. For if it be possible, let the lines BF and GD being produced concur in the point K . Nowe forasmuche as the right line EF standeth vpon the right line AB , therefore (by the 13. proposition) it maketh the angles AFE , and BFE equall to two right angles: likewise forasmuch as the line FG standeth vpon the line CD , therefore (by the same proposition) it maketh the angles CGF and DGF equall to two right angles. Wherefore the foure angles BFE , AFE , CGF , and DGF are equal to foure right angles: of which the two angles BFG and FGD are (by supposition) equall to two right angles, wherefore the angles remaining, namely, AFG and CGF are also equall to two right angles. If therefore the right lines FB and GD being produced (the inward angles being equall to two right angles) do concur, then shall the lynes FA and GC being produced concur. For the angles AFG and CGF are equall to two right angles. For either the right lines shall concur on either side, or els on neither side. For that on either side the angles are equall to two right angles. Wherefore let the right lines FA and GC concur in the point L . Wherefore the two right lines AFK and $LCGK$ do comprehend a space, which (by the 6. petition) is impossible.



An other demonstration of the second part of this proposition after Ptolomeus

The first Booke

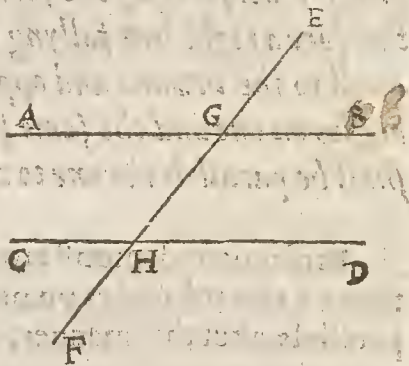
ble. Wherefore it is not possible that the inward angles being equal to two right angles, the right lines should concur. Wherefore they are parallels: which was required to be proued.

The 20. Theoreme. The 29. Proposition.

A right line line falling vppon two parallel right lines: maketh the alternate angles equall the one to the other: and also the outwarde angle equall to the inwarde and opposite angle on one and the same side: and moreouer the inwarde angles on one and the same side equall to two right angles.



Vppose that vpon these parallel lines AB and CD do fall the right line EF . Then I say that the alternate angles which it maketh, namely, the angles AGH and GHD , are equall the one to the other: and y^e the outward angle EGB is equal to the inwarde and opposite angle on the same side,



namely, to y^e angle GHD : and y^e the inward angles on one and the selfe same side, that is, the angles BGH and GHD , are equall to two right angles. For if the angle AGH be not equal to the angle GHD , the one of them is greater. Let the angle AGH be greater. And forasmuch as the angle AGH is greater then the angle GHD , put the angle BGH commō to the both. VVherfore y^e angles AGH and BGH , are greater the y^e angles BGH & GHD . But by y^e 13. proposition y^e angles AGH & BGH are equall to two right angles, wherfore y^e angles BGH & GHD are lesse the two right angles. But (by y^e 5. petition) if vpo two right lines do fall a right line, making y^e inward angles on one and y^e same side, lesse the two right angles, those right lines being infinitely produced must needes at y^e length meete on the side wherin are the angles lesse the two right angles. VVherfore the right lines AB and CD being infinitely produced will at y^e length meete. But they cannot meete, because they are parallels (by supposition): wherfore the angle AGH is not vnequall to the angle GHD : wherfore it is equall.

And the angle AGH is (by the 15. proposition) equall to the angle EGB . VVherfore (by the first common sentence) the angle EGB is equall to the angle GHD .

Put the angle BGH common to them both: wherfore the angles EGB and BGH , are equall to the angles BGH and GHD . But the angles EGB and

Demonstration
leading to an
impossibility.
First part.

Second part.

Third part.

and BGH are (by the 13. proposition) equall to two right angles. Wherefore the angles BGH and GHD are also equall to two right angles. If a lyne therefore do fall vpon two parallel right lines: it maketh the alternate angles equal the one to the other: and also the outward angle equall to the inward and opposite angle on one and the same side: and moreover the inward angles on one and the same syde equall to two right angles: whiche was required to be demonstrated.

This proposition is the conuerse of the two propositions next going before. For that which in either of them is the thing sought, or cōclusion, is in this the thing geuen, or supposition. And contrariwise the thinges which in them were geuen or suppositions, are in this proued, and are conclusions.

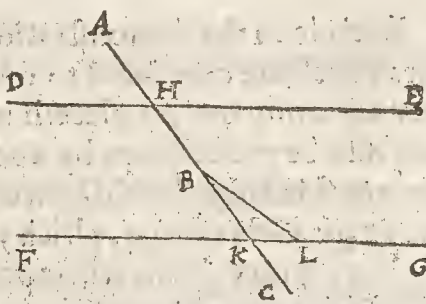
This proposition is the conuerse of the two former propositions.

Pelitarus after this proposition addeth this witty conclusion.

If two right lines which cut two parallel lines, do betwene the sayde parallel lines concurre in a point, and make the alternate angles equall, or the outward angle equall to the inward and opposite angle on the same side, or finally the two inward angles on one and the selfe same side equall to two right angles: those two right lines are drawen directly and make one right line.

An addition of Pelitarus.

Suppose that there be two right lines AB and CB , which let cut two parallel lines DE and FG : and let AB cut the line DE in the point H : and let CB cut the line FG in the point K : & let the lines AB & CB , concurre betwene the two parallel lines DE & FG in the point B : and let the angle DHB be equal to the angle BKG : or let the angle AHD be equal to the angle BKF : or finally let the angles BHD and BKF be equal to two right angles. The I say that the two lines AB and BC are drawen directly, and do make one right line. For if they be not, then produce AB vntil it cut FG in the point L , and let AL be one right line, and so shal be made the triangle BEK . Now then (by the first part of this 29. proposition) the angle DHB shalbe equal to the alternate angle GLB : but (by supposition) the angle DHB is equall to the angle BKG . Wherefore the angle BLG is equall to the angle BKL , namely, the outward angle to the inward and opposite angle: which (by the 16. proposition) is impossible.



Demonstration leading to an absurditie. First part.

Moreover (by the secōd part of this 29. proposition) the angle AHD shalbe equal to the angle BLK , namely, the outward angle to the inward and opposite angle on one and the same side. But the same angle AHD is supposed to be equal to the angle BKF : wherefore the angle BKF is equall to the angle BLK . Which (by the selfe same 16. proposition) is impossible.

Second part.

Lastly forasmuch as the angles BHD and BKF are supposed to be equal to two right angles, & the angles BHD & BLK are also by the last part of this 29 proposition equal to two right angles, therefore the angle BKF shalbe equal to the angle BLK : which agayne by the selfe same 16. proposition is impossible.

Third part.

The 21. Theoreme The 30. Proposition.

Right lines which are parallels to one and the selfe same right line: are also parrallel lines the one to the other.

L.ij.

Suppose

The first Booke

Suppose that these right lines AB and CD , be parallel lines to the right line EF . Then I

say, that the line AB is a parallel line to CD . Let there fall vpon these thre

Demonstration.

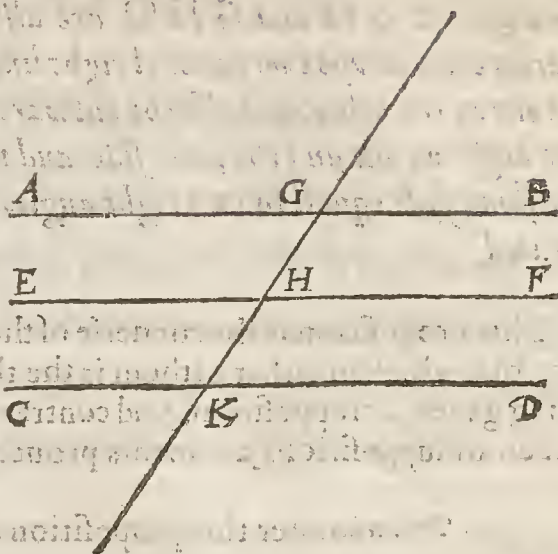
lines a right line $G HK$. And forasmuch as the right line $G HK$, falleth vpon these parallell right lines AB

*or the proposition
of the first booke
of the first booke
of the first booke*

and EF , therefore (by the proposition going before) the angle AGH is equall to the angle $G HF$. Agayne forasmuch as the right line $G K$ falleth

vpon these parallell right lines EF and CD , therefore (by the same) the angle $G HF$ is equall to the angle $G K D$. Now then it is proued that the angle AGH

is equall to the angle $G HF$, and the angle $G K D$ is equall to the angle $G HF$. Wherefore the angle AGH is equall to the angle $G K D$. And they are alternate angles: wherefore AB is a parallel line to CD . Right lines therefore which are parallels to one and the selfe same right line, are also parallel lines the one to the other: which was required to be proued.

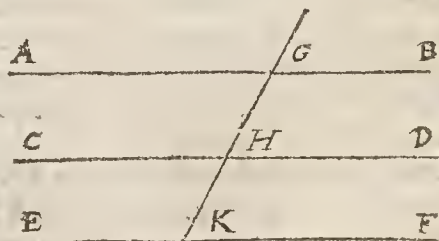


Euclide in the demonstration of this proposition, setteth the two parallel lines which are compared to one, in the extremes, and the parallel to whome they are compared, he placeth in the middle, for the easier demonstration. It may also be proued euen by a principle onely. For if they shoulde concur on any one side, they shoulde concur also with the middle line, and so shoulde they not be parallels vnto it, which yet they are supposed to be.

*As other case is
this Proposition.*

But if you will alter their position and placing, and set that line to which you will compare the other two lines, aboue, or beneath: you may vse the same demonstration which you had before. As for example.

Suppose that the lines AB and CD be parallels to the line EF : and let both the lines AB and CD , be aboue, and let the line EF be beneath, and not in the midst. Vpon which let the right line $G HK$ fall. And forasmuch as either of the angles KHD and HGB is equall to the angle HKE , (for they are alternate angles) therefore they are (by the first common sentence) equall the one to the other. Wherefore (by the 28. proposition) the right lines AB and EF , are parallels.



An objection.

Answer.

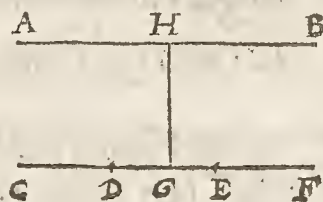
But here if a man will object that the lines $E K$ and $K F$, are parallels vnto the line CD , and therefore are parallels the one to the other. We will answer that the lines $E K$ and $K F$ are partes of one parallel line, and are not two parallel lines.

lines. For parallel lines are vnderstanded to be produced infinitely. But EK being produced falleth vpon KF . Wherefore it is one and the selfe same with it, and not an other. Wherefore all the partes of a parrallel line are parallels, both to the right line vnto which the whole parallel line is a parallel, and also to al the parts of the same right line. As the line EK is a parallel vnto HD , and the line KE to the line CH . For if they be produced infinitely, they will neuer concurre.

Parallel lines are vnderstanded to be produced infinitely.

Howbeit there are some which like not, that two distinct parrallel lines, should be taken and counted for one parallel line: for that the continuall quantity, namely, the line is cut asunder, and cesseth to be one. VVherefore they say, that there ought to be two distinct parallel lines compared to one. And therfore they adde to the proposition a correction, in this maner. *Two lines being parallels to one line: are either parallels the one the other, or els the one is set directly againste the other, so that if they be produced they should make one right line. As for example.*

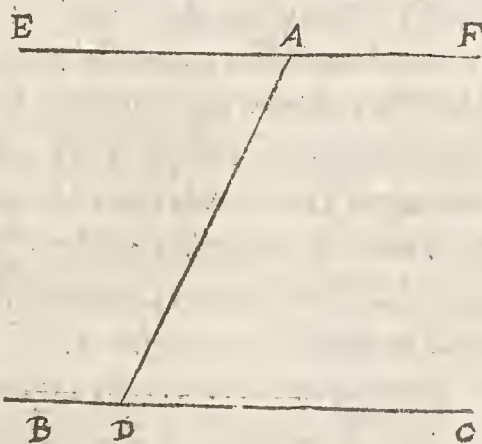
Suppose that the lines CD and EF be parallels to one and the selfe same line AB , and let them not be parallels the one to the other. Then I say, that the two lines CD & EF , are directly set the one to the other. For for as much as they are not parallel lines, let them concurre in the point G , And from the point G draw a line cutting the line AB in the point H . Now by the former proposition the angles AHG & HGC are equall to two right angles, but by the same proposition, the angle AHG , is equall to the alternate angle HGF . Wherefore the angles HGC and HGF are equal to two right angles. Wherefore (by the 14 proposition) the lines CG and FG are drawn directly and make one right line. Wherefore also the lines CD and EF are set directly the one to the other: and being produced they will make one right line.



The 10. Probleme. The 31. Proposition.

By a point geuen, to draw vnto a right line geuen, a parallel line.

Suppose that the point geuen be A , and let the right line geuen be BC . It is required by the point geuen, namely A , to draw vnto the right line BC , a parallel line. Take in the line BC a point at all aduentures, and let the same be D . and (by the first peticio) draw a right line from the point A , to the point D . And (by the 23. proposition) vpon the right line geuen AD , and to the point in it geuen A , make an angle DAE , equall to the angle geuen ADC . And (by the 14. proposition) put vnto the line AE the line AF directly, in such sorte that they



Construction.

L.iii.

both

The first Booke

both make one right line. And forasmuch as the right line AD falling vpon the right lines BC and EF , doth make the alternate angles, namely, EAD , and ADC equall $\forall$ one to the other, therfore (by the 27. proposition) EF is a parallel line to BC . VVherfore by the point geuen, namely A , is drawne to the right line geuen BC a parallel line EA : which was required to be done.

This proposition is to be vnderstanded of a point geuen without the line geuen, and in such sorte also, that the same line geuen being produced, doo not fall vppon the pointe geuen.

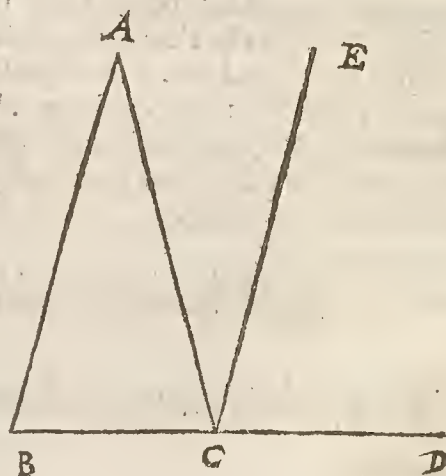
The 22. Theoreme. The 32. Proposition.

If one of the sydes of any triangle be produced: the outwarde angle that it maketh, is equal to the two inward and opposite angles. And the three inward angles of a triangle are equall to two right angles.



Suppose $\triangle ABC$ be a triangle, & produce one of $\forall$ sides thereof, namely, CB to the pointe D .

Then I say, that the outwarde angle ACD is equall to the two inward and opposite angles CAB & ABC : and $\forall$ the three inward angles of the triangle, that is, the angles ABC , BCA , and CAB are equall to two right angles. For (by the proposition going before) rayse vp frō the point C , a parallel to the right line AB , and let the same be CE . And forasmuch



Construction.

Demonstration.

as AB is a parallel to CE , and vpon them falleth the right line AC : therefore the alternate angles BAC and ACE are equall the one to the other. Agayne forasmuch as AB is a parallel vnto CE , and vpon them falleth the right line BD , therfore the outward angle ECD is (by the 29. proposition) equall to the inward and opposite angle ABC . And it is proued that the angle ACE is equal to the angle BAC : wherfore the whole outwarde angle ACD is equall to the two inward and opposite angles, that is, to the angles BAC and ABC . Put the angle ACB common to them both. VVherfore the angles ACD and ACB , are equall to these three angles ABC , BCA , and BAC . But the angles ACD & ACB are equall to two right angles (by the 13. proposition): wherfore the angles ACB , CBA , and CAB are equall to two right angles. If therfore one of the sides of any triangle be produced, the outwarde angle that it maketh, is equall to the two inward and opposite angles. And the three inward angles of a triangle

are

are equall to two right angles: which was required to be demonstrated.

Euclide demonstrateth either part of this composed Theoreme, by drawing frō one angle of the triangle a parrallel line to one of the sides of the same triangle, without the triangle. Either part thereof may also be proued without drawing of a parallel line without the triangle, only chaunging the order of the thinges required or conclusions. For Euclide first proueth that the outward angle of a triangle (one of his sides beyng produced) is equall to the two inward and opposite angles: and by that he proueth the second part: namely, that the 3. inward angles of a triangle are equall to two right angles. But here it is contrariwise. For first is proued that the three inward angles of a triangle are equall to two right angles, and by that is proued the other part of the Theoreme, namely, that one side of a triangle beyng produced, the outward angle is equal to the two inward and opposite angles. And that after this maner.

Suppose that there be a triangle ABC , and produce the side BC to the point E . And take in the line BC at a point auentures which let be F : & draw a line from A to F . And by the point F draw vnto the line AB a parallell line (by the former proposition) which let be FD . Now forasmuch as FD is a parallell vnto AB , and vpon them falleth the right line AF , and also the right line BC , therefore the alternate angles are equall, and also the outward angle is equall to the inward angle. Wherefore the whole angle AFC is equall to the angles FAB and ABF . And by the same reason (if by the point F we draw a parallell line to the line AC) may we proue that the angle AFB is equall to the angles FAC , and ACF . Wherefore the two angles AFB & AFC are equall to the three angles of the triangle ABC . But the two angles AFB & AFC are (by the 13. proposition) equall to two right angles. Wherefore also the three angles of the triangle ABC are equall to two right angles.

But the angles ACF and ACE are also (by the 13. proposition) equall to two right angles. Take away the angle ACF which is common, wherefore the angle remaining, namely, the outward angle ACE is equall to the two angles remaining, namely, to the two inward and opposite angles ABC and CAB : which was required to be proued.

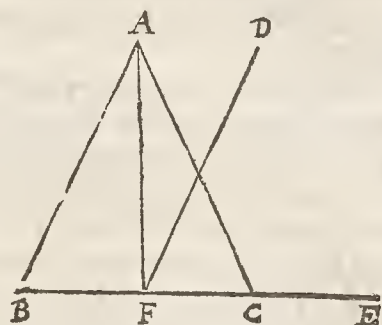
Eudemus affirmeth that the latter part of this Theoreme, *The three angles of a triangle are equall to two right angles*, was first found out by Pythagoras, whose demonstration thereof is thus.

Suppose that there be a triangle ABC : and by the point A , draw (by the former proposition) vnto the line BC , a parallell line, which let be DE . And forasmuch as the right lines BC and DE are parallels, and vpon them falleth the right lines AB and AC , therefore (by the 29. proposition) the alternate angles are equall. Wherefore the angle DAB is equall to the angle ABC , and the angle EAC to the angle ACB . Adde the angle BAC common. Wherefore the angles DAB , BAC , CAE , that is, the angles DAB and BAE , namely, two angles equal to two right angles, are equal to the thre angles of the triangle ABC . Wherefore the thre angles of a triangle are equall to two right angles: which was required to be proued.

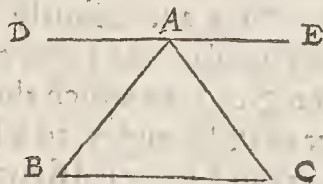
The conuerse of this proposition is thus.

M.j.

If



Another demonstration.



The latter part of this Theoreme first found out by Pythagoras.

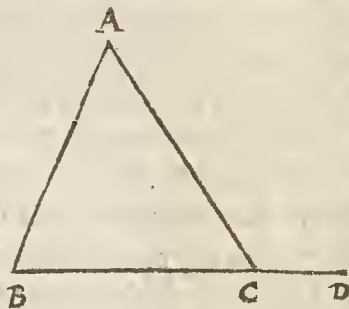
The demonstration thereof after him.

The first Booke

The conuerse of this proposition. If the outward angle of a triangle be equall to the two inward angles opposite against it: one of the sides of the triangle is produced, and the line without the triangle, is drawn directly to the side of the triangle, & maketh one right line with it. And if the three inward angles of a rectiline figure be equal to two right angles, the same rectiline figure is a triangle.

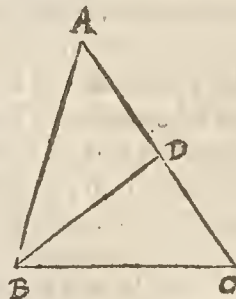
Demonstration of the first part of the conuerse.

Suppose that there be a triangle ABC : and let the outward angle ACD be equal to the two inward & opposite angles ABC and CAB . Then I say that the side BC is produced to the poynt D . And that BCD is one rightline. For forasmuch as the angle ACD is equal to the two inward & opposite angles, adde the angle ACB common. Wherefore the angles ACD and ACB are equal to the three angles of the triangle ABC . But the three angles of the triangle ABC are equall to two right angles. Wherefore also the two angles ACD and ACB are equall to two right angles. But if vnto a right line, and to a point in the same line be drawn two right lines, not both on one and the same side, making the side angles equal to two right angles: those two right lines shal be drawē directly, and make one right line (by the 14. proposition.) Wherefore the right line BC is drawn directly to the line CD , and so is BCD one right line: which was required to be proued.



Demonstration of the second part of the conuerse.

Agayne suppose that there be a certayne rectiline figure ABC , hauing onely three angles, namely, at the pointes A, B, C : which angles let be equal to two right angles. Then I say that ABC is a triangle. First AC is one right line. For draw the line BD . And forasmuch as in either of the triangles ABD and DBC , the three angles are equal to two right angles, of which the angles at the pointes A, B, C , are equal to two right angles. Wherefore the angles remayning, namely, ADB and CDB are equall to two right angles. Wherefore (by the 14. proposition) the line DC is set directly to the line DA . Wherefore the side AC is one right line. And in like sort may we proue that the side AB is one right line, and also that the side BC is one right line. Wherefore the figure ABC is a triangle: which was required to be proued.



A Corollary.

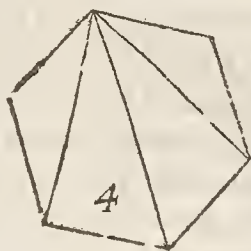
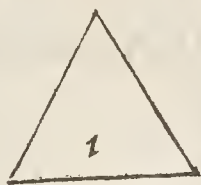
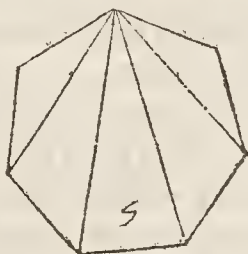
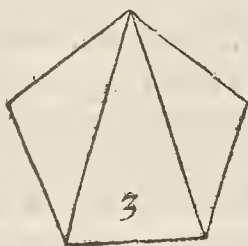
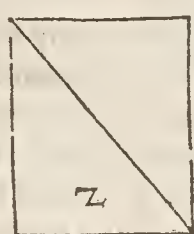
By the secōd part of this 29. proposition, namely, *three angles of a triangle are equall to two right angles*, may easely be knowen, to how many right angles, the angles within any figure hauing right lines and many angles are equall. As are figures of fower angles, of fīue angles, of sixe angles, and so consequently: and infinitely. And this is to be noted, that euery rightlined figure is resoluēd into triangle. For that a triangle is the first of all figures. For two lines accomplish no figures. VWherefore how many sides the figure hath, into so many triangles may it be resoluēd, sauing two. As if the figure haue fower sides, it is resoluēd into two triangles, if it haue fīue sides, into 3. triangles: if 6. sides into 4. triangles, and so consequently, and infinitely. And it is proued that the three angles of euery triangle are equall to two right angles. VWherefore if you multiply the number of the triangles, into which the figure is resoluēd, by two, you shall haue the number of right angles, to which the angles of the figure are equall. So the angles of euery quadrangled figure are equall to 4. right angles. For it is composed of two triangles. And the angles of a fīue angled figure are equal to 6. right angles, for it is composed of three triangles, and so forth in like order.

The readiest and aptest maner to reduce any rectiline figure into triangles, is thus

Euery right lined figure is resoluēd in triangles. A triangle is the first of all figures. Into how many triangles a figure may be resoluēd.

thus. From any one angle of the figure to euery other angle (of the same) beyng opposite vnto it, drawe a right line, so shall you haue all the triangles of that figure described.

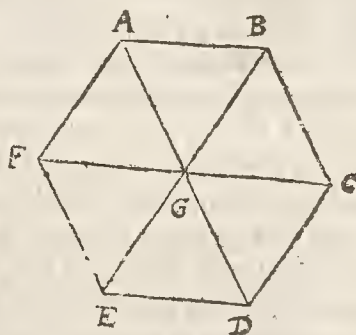
In a quadrangle, from one angle you can draw but onelyne to the opposite angle, by which it is deuided into two triangles only. In a pentagon figure, from one angle you may draw lines to two opposite angles, and so you shal



haue three triangles. In an Hexagon, you may from one angle draw lines to thre opposite angles, and so shall you haue 4. triangles. In an heptagon, from one angle may be drawne lines to foure opposite angles, and so shal there be five triangle. And so consequently of the rest, As you see in the figures here set.

This thing may also be thus exprest. In any figure of many sides, the number of the angles of the figure doubled, is the nūber of the right angles to which the angles of the figure are equall, sauing foure. As for example.

Let there be an hexagon figure A B C D E F, and within it take a point at all auentures, namely, G. And draw from the same point to euery one of the angles a right line, & so shal there be comprehended in the figure so many triangles, as there are angles in the same. VVherefore by this 32. proposition all the angles of these triangles taken together, are equall to double so many right angles, as there be angles in the figure. VVherefore forasmuch as there are fixe triangles, there are twelue right angles. But all the angles at the point G are equall to 4. right angles by the 13. proposition. VVherefore take away foure out of twelue, and there rest eight. VVherefore the fixe angles in the Hexagon figure are equall to eight right angles.



Another way to know the number of right angles vnto which the angles of any figure are equall.

By that which hath now bene declared, it foloweth that all the angles of any figure hauing many sides, takē together, are equal to twise so many right angles, as the figure is in the row or order of figures. A triagle is the first figure in order, & his angles are equal to two right angles, which are twise one. A quadrangle is the second figure in order. VVherefore his angles are equal to fower right angles, which are twise two. The order of figures is gathered of the sides. For if you take two from the number of the sides of a figure, the number of the sides remayning, is the number of the order of the figure. As if you will know, how many in order is a figure of fixe sides: from six (which is the number of his sides) take away two, and there will remaine foure. VVherefore a figure of fixe sides is the fourth figure

Another expression of the former Corollary.

A triangle the first figure in order.

A quadrangle the second, and so consequently. How the order of figures is gathered.

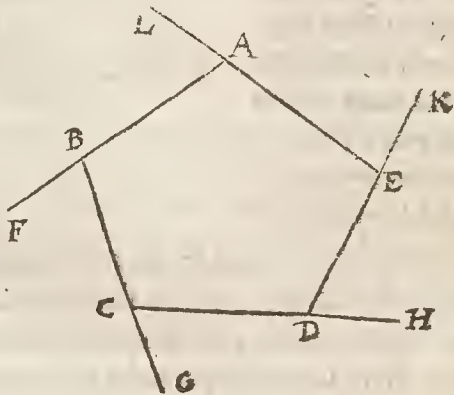
The first Booke

in the order of figures. Then double foure, so shall you haue eight. VVherefore the angles therof are equall to eight right angles. And so of all other figures.

Another Corollary.

Hereby also it is manifest, that the outward angles of any figure of many sides taken together, are equall to foure right angles. For the inward angles together with the outward angles, are equall to twise so many right angles, as there be angles in the figure (by the 13. proposition) But the inward angles are equal to twise so many right angles, as there be angles in the figure, sauyng foure: as it was before declared. VVherefore the outward angles are always equal to foure right angles. As for example.

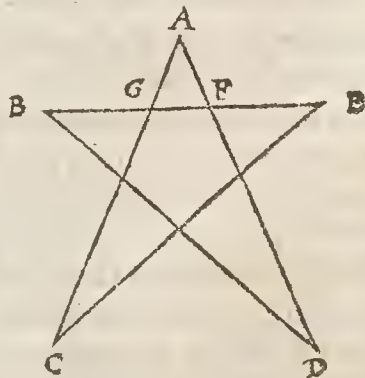
Suppose that there be a pentagon, $A B C D E$. And produce the five sides thereof to the points F, G, H, K, L . Now (by the 13. proposition) the two angles at the point A shall be equal to two right angles. And (by the same) the two angles at the point B shall be also equall to two right angles. And so taking euery two angles, they shall be in all equall to tenne right angles. VVherefore takyng away the inward angles, whiche (as hath before bene proued) are equall to sixe right angles, the outward angles shall be equall to foure right angles. And so of all other figures,



Another Corollary.

It is also manifest, that euery pentagon, which is so described, that ech side thereof deuideth two of the other sides, hath his five angles equall to two right angles.

For suppose that $A B C D E$ be such a pentagon as is there required so that let the side $A C$ cut the side $B E$ in the point G : & let the side $A D$, cut the same side $B E$ in the point F . Now the by this proposition the angle $A F G$ shall be equall to the two angles at the point B and D : namely, the outward angle to the two inward and opposite angles. And by the same reason the angle $F G A$ is equal to the angles at the points C and E which are in the triangle $C E G$. But the two angles $A F G$ and $F G A$, together with the angle at the point A , are equall to two right angles (by this proposition). Wherefore the fower angles at the pointes, B, C, D, E , together with the angle at the point A , are equal to two right angles: which was required to be proued.



Another Corollary.

By this proposition also it is manifest, that euery angle of an equilate triangle is two third partes of a right angle. And that in a triangle of two equall sides hauing a right angle at the toppe, either of the two angles at the base is the halfe of a right angle. And in a triangle called Scalenum, such a Scalenu (I say) which is made by the drawght of a perpendicular line from any one of the angles of an equilater triagle to the opposite side thereof, one angle is a right angle, an other is two third parts of a right angle, namely, that angle which was also an angle of the equilater triangle, wherefore of necessity the angle remaining is one third part of a right angle. For the three angles of a triagle must be equall to two right angles.

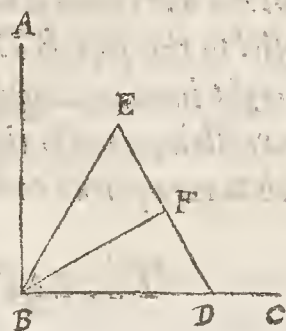
Another Corollary.

Moreover by this proposition it is manifest, that if there be two triangles, and if two angles of the one be equal to two angles of the other: the angle remaining

ning shall also be equall to the angle remayning. For forasmuch as three angles of any triangle are equal to three angles of any other triangle (for that in ech the three angles are equal to two right angles): If from ech triangle be taken away the two equall angles, the angle remayning shall (by the 3. common sentence) be equall to the angle remayning.

And here I thinke it good to shew how to deuide a right angle into three equall partes, for that the demonstration thereof dependeth of this proposition.

Suppose that there be a right angle ABC , containd of the right lines AB and BC ; & in the line BC , take a point at all aduentures, which let be D . And vpon the line BD describe (by the first) an equilater triangle BDE . And (by the 9. proposition) deuide the angle DBE into two equall partes by the right line BF . Then I say that the right angle ABC is deuided into three equal parts by the right lines BE and BF . For forasmuch as EBD is an equilater triangle, therefore as hath before bene declared, the angle EBD is two thirde partes of a right angle. But the whole angle ABC , is a right angle. Wherefore the angle remaining, namely, ABE is one third part of a right angle. Again forasmuch as the angle EBD , is two third partes of a right angle, and it is deuided into two equall parts by the right line BF , therefore either of these two angles EBF & FBD is one third part of a right angle. Wherefore the three angles ABE , EBF and FBD are equall the one to the other. Wherefore the right angle ABC is deuided into three equall partes by the right lines BE and BF : which was required to be done.



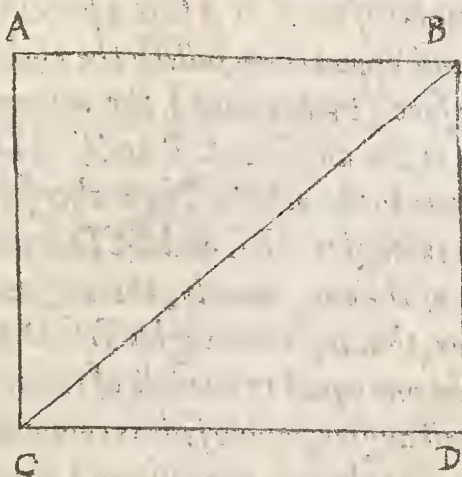
How to deuide a right angle into three equall partes.

The 23. Theoreme. The 33. Proposition.

Two right lines ioyning together on one and the same side, two equall parallel lines: are also them selues equall the one to the other, and also parallels.



Suppose that AB and CD be right lines equal, and parallels: and let these two right lines AC and BD iojne the together, the one on the one side, and the other on y other side. Then I say that the lines AC & BD are both equall, & also parallels. Draw (by the first petition) a right line from the point B to the point C . And forasmuch as AB is a parallel to CD , and vpo them falleth the right line BC , therefore the alternate angles ABC and BCD , are equall the one to the other (by the 29. proposition). And forasmuch as the line AB is equall to the line CD , and the line BC is common to them both,



Demonstratio

M.iii.

there.

The first Booke

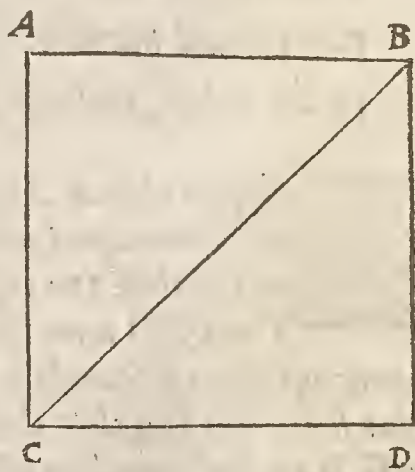
therefore these two lines AB and BC , are equall to these two lines BC and CD , and the angle ABC is equall to the angle BCD . Wherefore (by the 4. proposition) the base BD is equall to the base AC , and the triangle ABC , is equall to the triangle BCD , and the angles remayning are equall to the angles remayning the one to the other, vnder which are subtended equall sides, wherefore the angle ACB is equall to the angle CBD , and the angle BAC to the angle BDC . And forasmuch as vpon these right lines AC and BD falleth the right line BC making the alternate angles, that is the angles ACB and CBD , equall the one to the other, therefore (by the 27. proposition) the line AC is a parallel to the line BD . And it is proued that it is equall vnto it. Wherefore two right lines ioyning together on one and the same side two equall lines which are parallels, are also themselves equall the one to the other, and also parallels: which was required to be proued.

The 24. Theoreme. The 34. Proposition.

In parallelogrammes, the sides and angles which are opposite the one to the other, are equall the one to the other, and their diameter deuideth them into two equall partes.

Demonstration

Suppose that $ABCD$ be a parallelogramme and let the diameter thereof be BC . Then I say that the opposite sides and angles of the parallelogramme $ACDB$ are equall the one to the other, and y^e the diameter BC deuideth it into two equall partes. For forasmuch as AB is a parallel line vnto CD , and vpon them falleth a right line BC : therefore (by the 29. proposition) the alternate angles ABC and BCD are equall the one to the other. Agayne forasmuch as AC is a parallel line to BD , and vpon them falleth the right lyne BC : therefore (by the same) the alternate angles, that is, the angles ACB and CBD are equall the one to the other. Now therefore there are two triangles ABC and BCD , hauing two angles of the one, namely, the angles ABC and ACB equall to two angles of the other, that is, to the angles BCD and CBD , the one to the other, and one side of the one equal to one side of the other, namely, that syde that lieth betwene the equall angles, which syde is common to them both, namely, the side BC . Wherefore (by the 26. proposition) the other sides remaining are equall to the other sides remaining, the one to the other, and the angle remaining is equal to the angle remayning. Wherefore the side AB is equall to the side CD , and the side AC



to

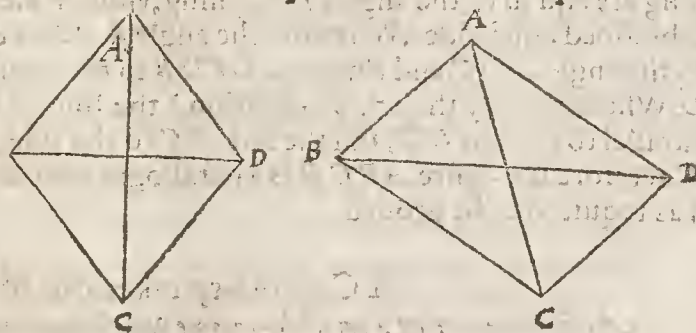
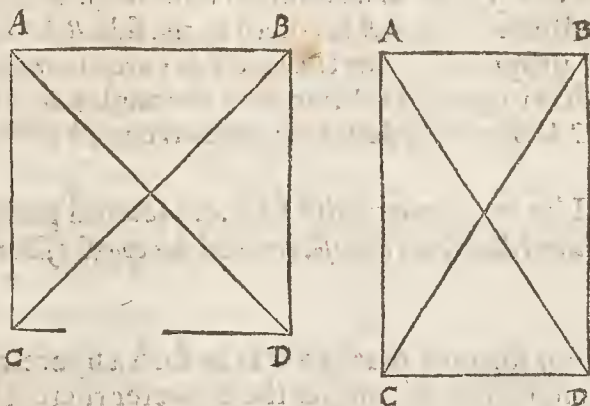
to the side BD , & the angle BAC is equal to the angle BDC . And forasmuch as the angle ABC is equal to the angle BCD , and the angle CBD to the angle ACB : therefore (by the second common sentence) the whole angle ABD is equall to the whole angle ACD . And it is proued that the angle BAC is equall to the angle CDB . Wherefore in parallelogrammes, the sides and angles which are opposite are equall the one to the other. I say also that the diameter thereof deuideth it into two equall partes. For forasmuch as AB is equall to CD , and BC is common to them both, therefore these two AB and BC are equall to these two BC and CD , and the angle ABC is equal to the angle BCD . Wherefore (by the 4. proposition) the base AC is equall to the base BD , and the triangle ABC is equall to the triangle BCD . Wherefore the diameter BC deuideth the parallelogramme $ABCD$ into two equall partes: which is all that was required to be proued.

In this Theoreme, are demonstrated three passions or properties of parallelogrammes. Namely, that their opposite sides are equal; that their opposite angles are equal: and that the diameter deuideth the parallelogramme into two equall partes. Which is true in all kindes of parallelogrammes. There are fower kindes of parallelogrammes, a square, a figure of one side longer then the other, a Rhombus, or diamond figure, and a Rhomboides or diamondlike figure. And here is to be noted, that in those parallelogrammes, all whose angles are right angles (as is a square, and a figure on the one side longer) the diameters do not only deuide the figure into two equall partes, but also they are equal the one to the other. As for example.

Suppose that $ABCD$ be a square, or a figure on the one side longer, and draw in it these diametres AD and BC . And forasmuch as the line AB is equall to the line CD (by the definition of a square, and of a figure on the also one side longer) & the line AC is common to the both: therefore two sides of the triangle ABC are equal to two sides of the triangle ACD , the one to the other, and the angles which they containe are equal, namely, the angles BAC & ACD , for they are right angles. Wherefore the bases namely, the diametres AD and BC , are (by the 4. proposition) equal.

But in those parallelogrammes whose angles are not right angles, as is a Rhombus, and a Rhomboides, the diameters be euer vnequall. As for example.

Suppose that $ABCD$ be a Rhombus, or a Rhomboides and drawe in it these diametres AC and BD . And forasmuch as AB is equall to CD , and BC is common to them both, & the angle ABC is not equall to the angle BCD (by the definition of a Rhombus and also of a Rhomboides)



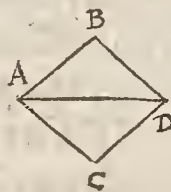
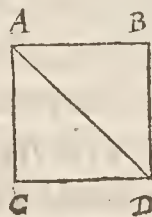
Three passions of parallelogrammes demonstrated in this Theoreme. Fower kindes of parallelogrammes.

The first Booke

therefore (by the 24. proposition) the bases also are vnequall, namely, the diameters AC and BD .

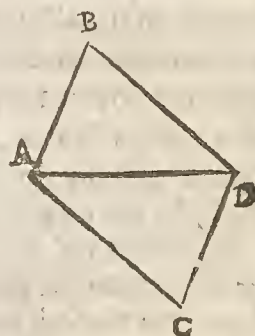
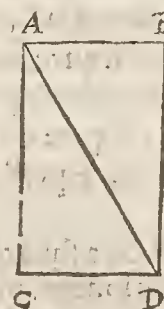
Agayne. In parallelogrammes of equall sides, as are a square, and a Rhombus, the diameters do not onely deuide the figures into two equall partes, but also they deuide the angles into two equall partes.

For suppose that there be a square or Rhombus $ABCD$, and draw the diameter AD . And forasmuch as the sides AB and BD are equall to the sides AC and CD (for the figures are equilateral) and the angles ABD and ACD are equall (for they are opposite angles) and the base AD is common to both triangles. Therefore (by the fourth proposition) the angles BAD & CAD are equall, and so also are the angles BDA and CDA equall. Wherefore the angles BAC and CDB are deuided into two equall partes.



But in parallelogrammes whose sides are not equall, such as are a figure on the one side longer, and a Rhomboides it is not so.

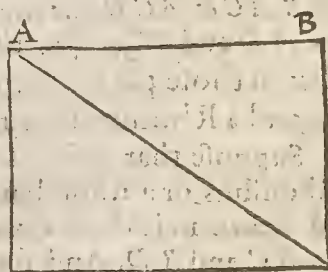
For suppose $ABCD$ to be a figure on the one side longer or a Rhomboides. And draw the diameter AD . And now if the angles BAC and CDB be deuided into two equall partes by the diameter AD , then forasmuch as the angle CAD is (by the 29. proposition) equall to the angle ADB , the angle also BAD shal be equal to the angle ADB (by the first common sentence). Wherefore also the side AB is equall to the side BD (by the 6. proposition). But the sayd sides are vnequall: which is impossible. Wherefore the angles BAC and CDB are not deuided into two equall partes.



*The conuerse of
this proposition
after Proclus.*

The conuerse of the first and second part of this proposition after Proclus.
If a rectiline figure whatsoeuer haue his opposite sides and angles equall: then is a parallelograme.

For suppose that $ABCD$ be such a figure, namely, which hath his opposite sides and angles equall. And let the diameter thereof be AD . Now forasmuch as the sides AB and BD are equall to the sides DC and AC , and the angles which they containe are equall, and the base AD is common to each triangle, therefore (by the 4. proposition) the angles remayning are equall to the angles remayning, vnder which are subtended equal sides. Wherefore the angle BAD is equal to the angle ADC , and the angle ADB to the angle CAD . Wherefore (by the 27. proposition) the line AB is a parallel to the line CD , and the line AC to the line BD . Wherefore the figure $ABCD$ is a parallelogramme: which was required to be proued.

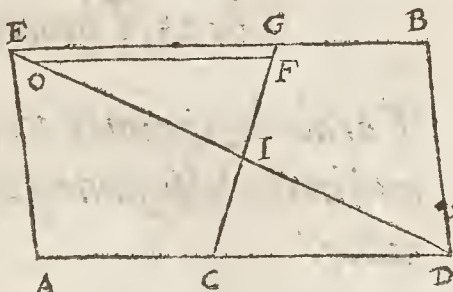


*A Corollary taken
out of Euclid.*

A Corollary taken out of Flustates.
A right line cutting a parallelogramme which way soeuer into two equall partes, shall also deuide the diameter thereof into two equall partes.

For

For if it be possible let the right line GC deuide the parallelogramme $AEBD$ into two equal partes, but let it deuide the diameter DE into two vnequall partes in the point I . And let the part IE be greater then the part ID , And vnto the line ID put the line IO equall (by th³. propositiō). And by the point O , draw vnto the lines AD and BE a parallel line OF (by the 31. proposition.) Wherefore in the triangles FOI and CDI , two angles of the one are equal to two angles of the other, namely, the angles IOF and IDC (by the 29. proposition), & the angles FIO & CID (by the 15. propositiō), & the side ID is equal to the side IO . Wherefore (by the 26. proposition) the triangles are equall. Wherefore the whole triangle EIG is greater then the triangle DIC . And forasmuch as the trapezium $GBDC$ is supposed to be the halfe of the parrallelograme, and the halfe of the same parallelograme is the triangle EBD (by this proposition). From the trapezium $GBDC$ and the triangle EBD which are equall, take away the trapezium $GBDI$ which is common to them both, and the residue namely, the triangle DIC shalbe equall to the residue, namely, to the triagle EIG : but it is also lesse (as hath before ben proued): which is impossible. Wherefore a right line deuiding a parallelogramme into two equall partes, shall not deuide the diameter thereof vnequally. Wherefore it shall deuide it equally: which was required to be proued.



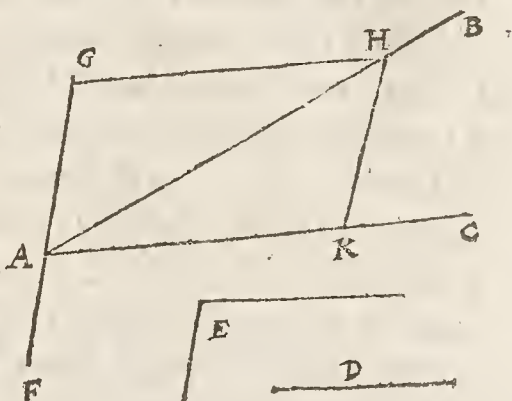
Demonstration leading to an absurditie.

An addition of Pelitarius.

Betweene two right lines being infinite and making an angle geuen: to place a line equal to a line geuen, in such sorte, that it shall make with one of those lines an angle equall to an other angle geuen. Now it becometh that the two angles geuen be lesse then two right angles.

An addition of Pelitarius.

Suppose that there be two lines AB and AC , making an angle geuen BAC : and let them be infinite on that side where they open one from the other. And let the line geuen be D , and let the other angle geuen be E . And let the two angles A and E be lesse then two right angles (otherwise there could not be made a triangle, as it is manifest by the 17 proposition). It is required between the lines AB and AC to place a line equal to the line geuen D , which with one of them as for exāple with the line AC , may make an angle equal to the angle geuen E . Now then vpon the line AC and to the point in it A , make an angle equall to the angle geue E (by the 23. proposition), which let be CAF . And produce the line FA on the other side of the point A to the point G : and let AG be equall to the line geuen D (by the 3. proposition). And by the point G , draw (by the 31. proposition) a parallel line to the line AC , which let be GH , and produce it vntil it concurse with the line AB : which concurse let be in the point H . And agayne by the point H draw the line HK parallel vnto the line GF : which let cut the line AC in the point K . Then I say that the line HK is placed between the lines AB & AC & is equall to the line D . And that the angle at the point K is equall to the angle geuen E . For forasmuch as (by construction) $AGHK$ is a parallelogramme the line KH is equall to the line AG (by this proposition). Wherefore also it is equall to the line D . And forasmuch as the line AK falleth vpon the two parallel lines, FG and KH , therefore the angle AKH is equal to the angle FAK (by the 29. propositiō) for that they are alternate angles. Wherefore also the same angle at the point K is equal to the angle geuen E . Wherefore the line HK being placed between the two lines AB and AC , and being equall to the line D , maketh the angle at the point K equall to the angle geuen E : which was required to be done.



Construction.

Demonstration

Though this addition of Pelitarius be not so muche pertayning to the
N.i. propo-

proposition: yet because it is witty and semeth somewhat difficult, I thought it good here to anexe it.

The 25. Theoreme. The 35. Proposition.

Parallelogrammes consisting vppon one and the same base, and in the selfe same parallel lines, are equall the one to the other.



Vppose that these parallelogrammes $ABCD$ and $EBCF$ do consist vpon one and the same base, that is, vppon

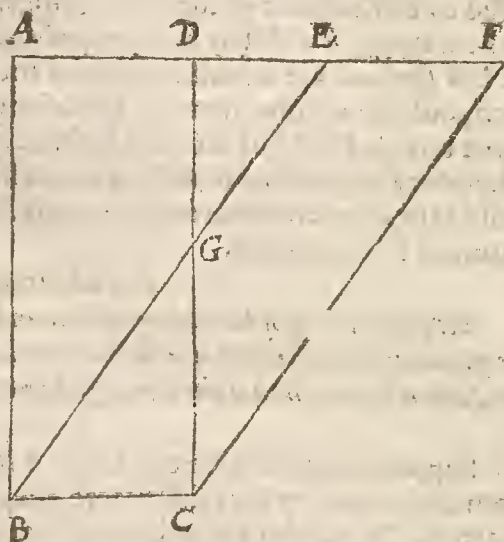
BC , and in the selfe same parallel lines, that is AF , and BC . Then I say, that the parallelograme $ABCD$ is equal to the parallelograme $EBCF$. For forasmuch as $ABCD$ is a parallelogramme, therefore (by the 34. proposition) the side AD , is equall to the side BC , and by the same reason also the side EF is equall to the side BC , wherfore AD is equall to EF : and DE is common to them both.

Wherfore the whole line AE is equall to the whole line DF . And the side AB is equall to y side DC : wherfore these two EA and AB are equall to these two FD and DC , the one to the other: and y angle FDC is equall to the angle EAB , namely, the outward angle to y inward angle (by y 29. proposition): wherfore (by y 4. proposition) the base EB is equall to the base FC , and the triangle EAB is equall to the triangle FDC . Take away the triangle DGE , which is common to them both. Wherfore the residue, namely, the trapezium $ABGD$ is equall to the residue, that is, to the trapezium $EGCF$. Put the triangle GBC commo to them both. Wherfore the whole parallelogramme $ABCD$ is equall to the whole parallelogramme $EBCF$. Wherfore parallelogrammes consisting vpon one and the same base, and in the selfe same parallel lines, are equal the one to the other: which was required to be demonstrated.

Parallelogrammes are sayde to be in the selfe same parallel lines, when their bases, and the opposite sides vnto them, are one and the selfe same lynes wyth the parallels.

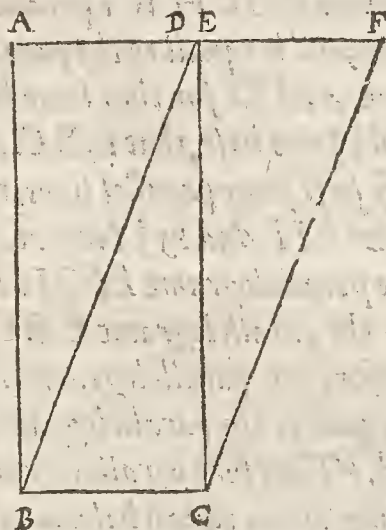
Three cases in this proposition.
The first case.

In this proposition are three cases. For the line BE may cutte the line AF , either bey ond the point D , or in the point D , or on this side the point D . When



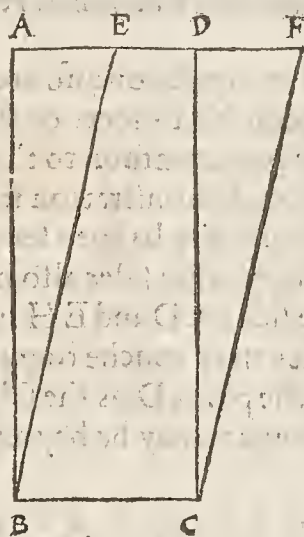
it cutteth the line AF beyond the point D the demonstration before put scr-
uerh.

But if the line BE do cutte the line AF in the point D , then forasmuch as (by the former proposition) the triangle BCD or BCE is the halfe of either of these parallelogrammes $ABCD$ and $EBCF$ (for in the parallelogramme $ABCD$ the diameter BD maketh the triangle BCD the halfe of the same parallelogramme, and in the parallelogramme $EBCF$ the diameter EC or DC maketh the selfe same triangle BCD the halfe of the parallelogramme $EBCF$) therefore (by the 7. common sentence) the parallelogrammes $ABCD$ and $EBCF$ are equall.



The second case.

But if the liue BE do cutte the lyne AF on this side the point D , then forasmuch as eyther of the lines AD and EF is equall to the line BC , therefore by the first common sentence they are equall the one to the other. Wherefore taking away ED , which is common to both, the residue AE shalbe equall to the residue DF . Agayne forasmuch as (by the 34. proposition) the line AB is equall to the line CD , and (by the 17. proposition) the angle EAB is equal to the angle fdc : therefore (by the 4. proposition) the triangles EBA and FCd are equal. Adde the trapesium $CDEB$ common to them both: and so (by the seconde common sentence) the two parallelogrammes $ABCD$ and $EBCF$ shalbe equall: which was required to be proued.



The third case.

The 26. Theoreme. The 36. Proposition.

Parallelogrammes consisting vpon equall bases, and in the selfe same parallel lines, are equall the one to the other.



Suppose that these parallelogrammes $ABCD$ and $EFGH$ do consist vpon equall bases, that is, vpon BC and FG , and in the selfe same parallel lines, that is, AH and BG . Then I say, that the parallelogramme $ABCD$ is equall to the parallelogramme $EFGH$. Draw a right line from the point B to the point E , and an other from the point C to the point H . And forasmuch as BC is equall to FG , but FG is equall to EH , therefore BC also is equall to EH , and they are parallel lines, and the lines BE and CH ioyn them together: but two right lynes ioynnyng together two equall right lines

Construction.

Demonstration

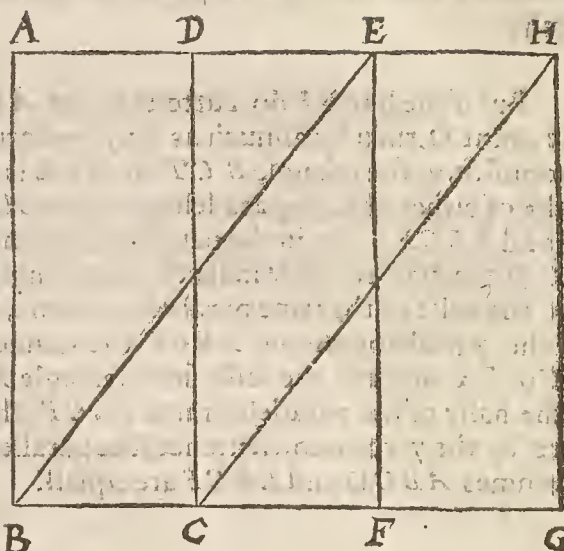
N.ij.

lines

lines being parallels, are themselves also (by the 33. proposition) equall the one to the other, and parallels.

Wherefore $EBC.H$ is a parallelogramme, and is equall to the parallelogramme $ABCD$: for they haue both one and y same base, that is, BC . And are in y selfe same parallel lines, that is, BC & EH . And by y same reason also the parallelogramme $EFGH$ is equal to the parallelogramme $EBC.H$. Wherefore the parallelogramme $ABCD$ is equal to the parallelogramme $EFGH$. Wherefore parallelogrammes.

consisting vppon equall bases, and in the selfe same parrallel lines, are equall the one to the other: which was required to be proued.



Three cases in this proposition.

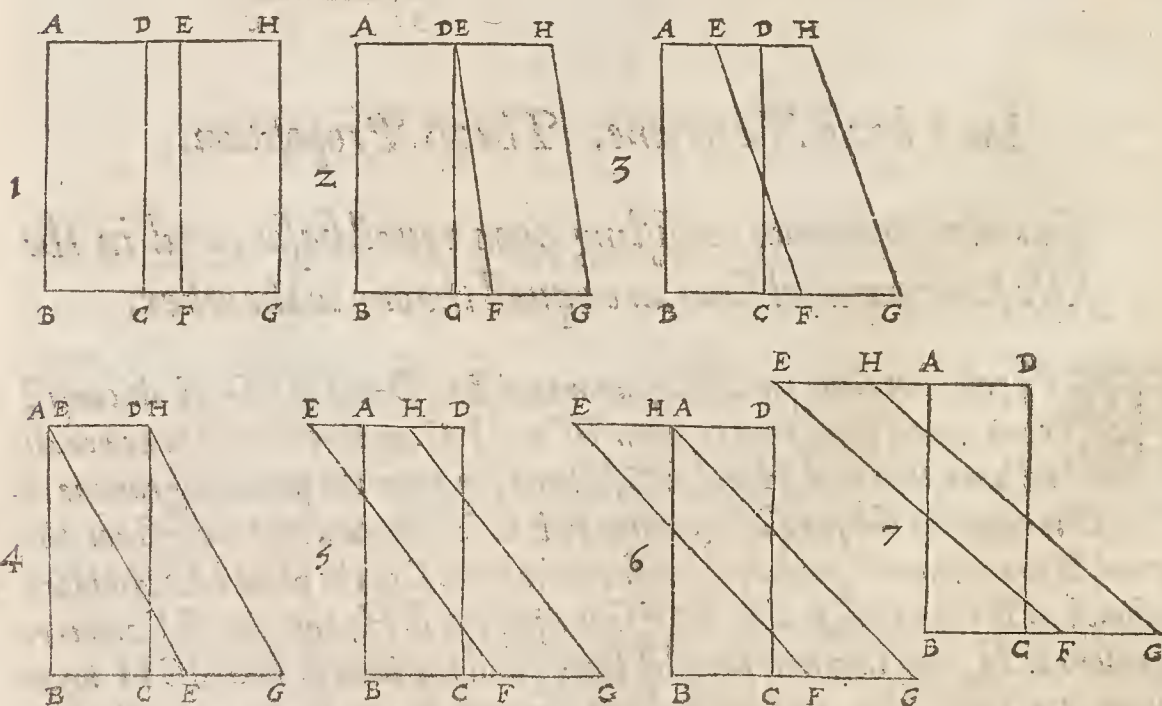
The first case. Every case may happen seuen diuers wayes.

In this proposition also are three cases. For the equall bases may either be vtterly seperated a sonder: or they may touche at one of the endes: or they may haue one part common to them both.

Euclides demonstration serueth when the bases be vtterly seperated a sonder. VVhich yet may happen seuen diuers wayes. For the bases being seperated a sonder, their opposite sides also may be vtterly seperated a sonder bey ond the point D , as the sides AD and EH in the first figure.

Or they may touche together in one of the endes, and the whole side may be bey ond the point D , as the sides AD and EH do in the second figure.

Or one part may be bey ond the point D , and an other part common to them



both, as in the third figure, the sides AD and EH haue the part ED common to them both.

Or they may iustly agree the one with the other, that is, the pointes A and D may fall vpon the pointes E and H : as in the fourth figure.

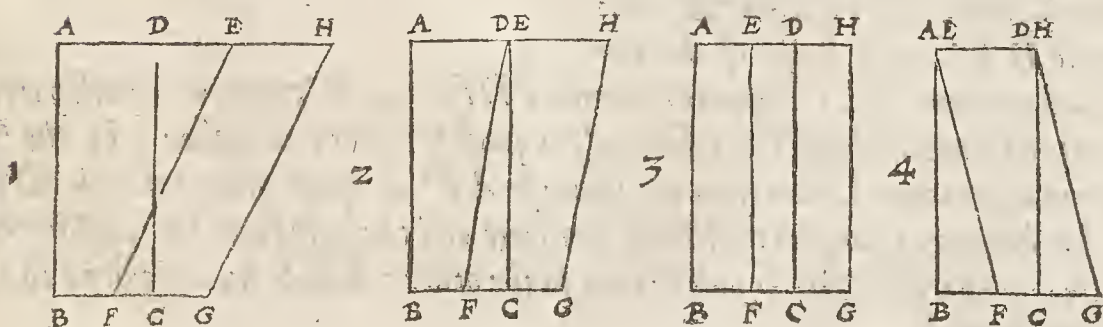
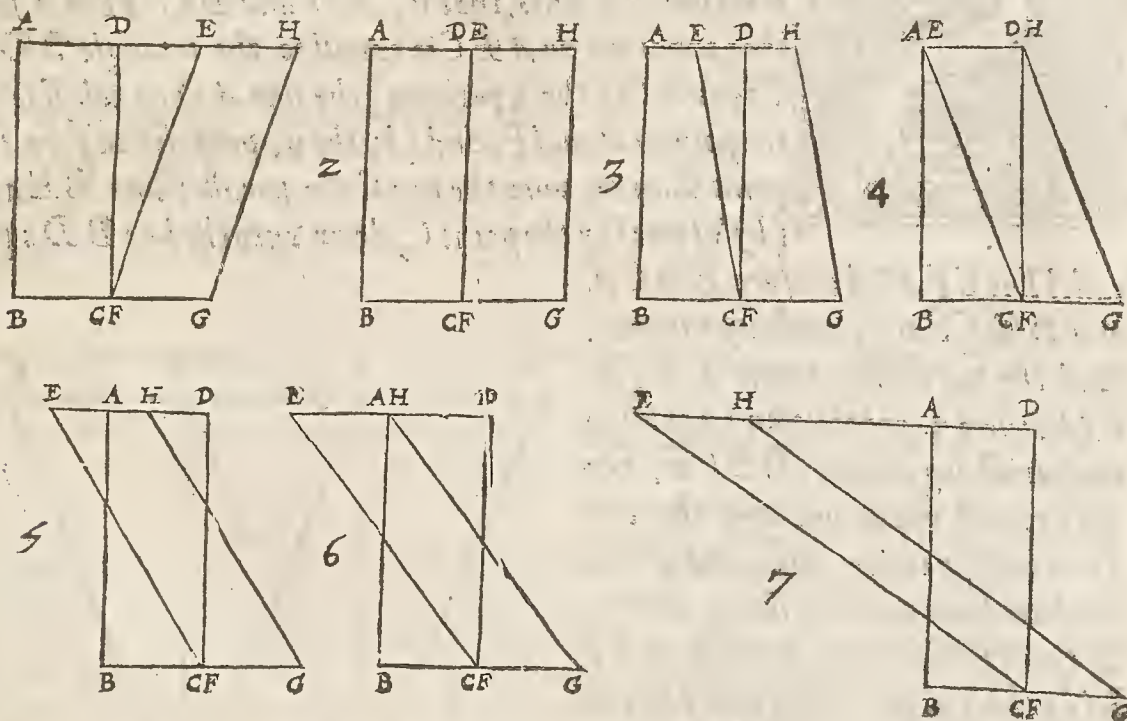
Or the side AD being produced on this side the point A , part of the opposite side vnto the base FG may be on this side the point A , and another part may be common with the line AD , as in the fifth figure.

Or one ende of the side EH may light vpon the point A , and the whole side on this side of it: As in the sixt figure.

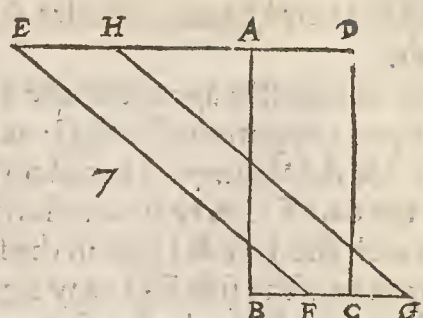
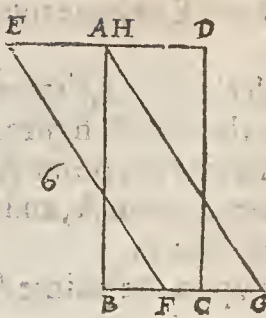
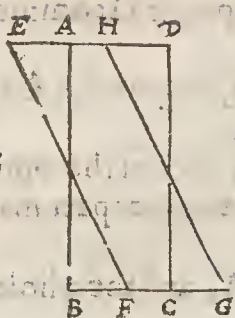
Or the said side EH may vtterly be seperated a sonder on this side the point A , as in the seuenth figure.

And the two other cases also may in like maner haue seuen varieties: as in the figures here vnderneath and on the other side of this leafe set it is manifest. And here is to be noted, that in these three cases and in all their varieties also, the construction & demonstration put by Euclide (namely, the drawing of lines frō the point B to the point E & from the point C to the point H , and so prouing it by the former proposition) will serue onely in the fourth varietie of each case, there nedeth no farther construction: for that the conclusion straight way followeth by the former proposition.

The like Variety in each of the other two cases. Euclides construction and demonstration serueth in all these cases, and in their varieties also.



The first Booke



The 27. Theoreme. The 37. Proposition.

Triangles consisting vpon one and the selfe same base, and in the selfe same parallels: are equall the one to the other.

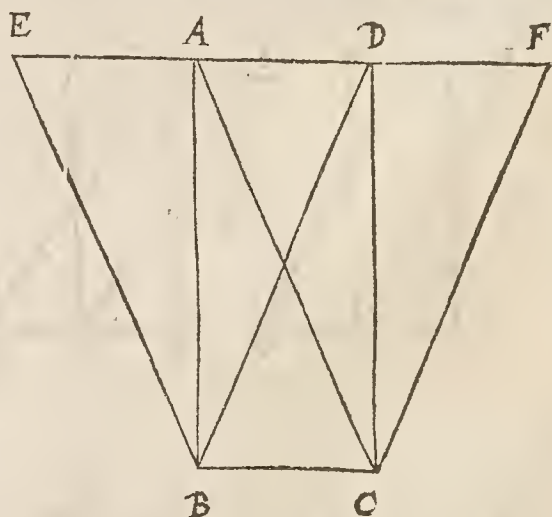


Construction.

Suppose that these triangles ABC and DBC do consist vpon one and the same base, namely, BC , and in the selfe same parallel lines, that is, AD and BC . Then I say, that the triangle ABC is equall to the triangle DBC . Produce (by the 2. petition) the line AD on ech side to the pointes E and F . And (by the 31. proposition) by the point B , draw vnto the line CA a parallel line BE and (by the same) by the point C , draw vnto the line BD a pa-

Demonstration.

rallel line CF . Wherefore $EBCA$ and $DBC F$ are parallelogrammes. And the parallelogamme $EBCA$ is (by the 35. proposition) equall to the parallelogamme $DBC F$. For they consist vpon one and the selfe same base, namely, BC , and are in the selfe same parallel lines, that is, BE (and CF) and AD (and BC). But the triangle ABC is (by the 34. proposition) the halfe of the parallelogamme $EBCA$, for the diameter AB deuideth it into two equall parts: & (by the same) the triangle DBC is the halfe of the parallelogamme $DBC F$, for the diameter DC deuideth it into two equall parts: but the halues of thinges equall are also equall the one to the other (by the 7. common sentence), wherefore the triangle ABC is equall to the triangle DBC . Wherefore triangles consisting vpon one and the selfe same base, and in the selfe same parallels: are equall the one to the other: which was required to be demonstrated.



Those

Those triangles are saide to be contayned within the selfe same parallel lines, which hauing their bases in one of the parallel lines, haue their toppes in the other.

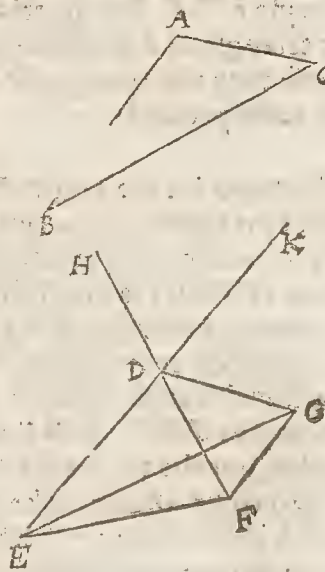
How triangles are saide to be in the selfe same parallel lines.

Here as I promised will I shew out of Proclus the comparison of two triangles, which hauing their sides equall, haue the bases and angles at the toppes vnequall. And first I say that the vnequall angles at the toppes being equall to two right angles, the triangles shalbe equall. As for example.

Comparison of two triangles whose sides being equal, their bases and angles at the toppes are vnequall.

Suppose that these two triangles ABC and DEF haue two sides of the one, namely, AB and AC , equall to two sides of the other, namely, to DE and DF , eche to his correspondent side, that is, AB to DE , and AC to DF , and let the base BC be greater then the base EF : and let the angle at the point A be greater then the angle at the point D . But let the sayde angles at the pointes A and D , be equall to two right angles. Then I say that the triangles ABC and DEF are equall. For as much as the angle BAC is greater then the angle EDF , vpon the line ED , and to the point D describe (by the 23. proposition) an angle equall to the angle BAC ; which let be EDG : and put the line DG equall to the line AC : and draw a line from E to G , and an other from F to G : and produce the lines ED & FD beyond the poynt D to the pointes H and K . Now for as much as the angle BAC is equall to the angle EDG , and the angles BAC and EDF are equall to two right angles, therefore the angles EDG and EDF are equall to two right angles. But the angles EDG and KDG , are also equal to two right angles: take away the angle FDG common to them both: wherefore the angle remayning EDF is equall to the angle remayning GDK . But the angle EDF is equall to the angle HDK (by the 15. proposition) for they are hed angles. Wherefore the angle GDK is equall to the angle HDK . And for as much as in the triangle GDF the outward angle GDH is (by the 32. proposition) equal to the two inward and opposite angles at the points G and F : which two angles also are (by the 5. proposition) equall the one to the other: for the line DG is by construction equall to the line AC , namely, to the line DF . Wherefore the angle GDH is double both to the angle at the point G , and to the angle at the point F . But the angle GDH is also double to the angle GDK (for the angle GDK is proued to be equall to the angle KDH) wherefore the angle at DGF is equall to the angle GDK : and they are alternate angles. Wherefore (by the 27. proposition) the line DE is a parallel to the line FG . Wherefore the triangles GDE and FDE are vpon one and the selfe same base, namely, DE , and in the selfe same parallel lines DE and GF . Wherefore by this proposition they are equall. But the triangle GDE is by construction equall to the triangle ABC . Wherefore also the triangle DEF is equall to the triangle ABC : which was required to be proued.

When the two angles at the toppes are equall to two right angles.

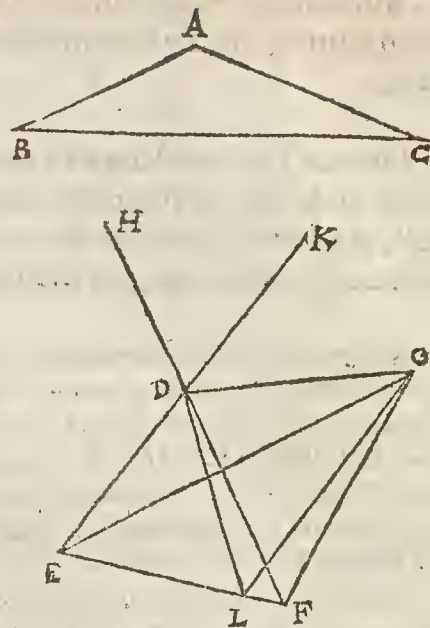


But now let the angles BAC and EDF be greater then two right angles: & let the angle at the point A be greater then the angle at the point D , as it was before. The I say that the triangle ABC is lesse then the triangle DEF . Let the same construction be here that was in the former. And for as much as the angles BAC and EDF , that is, the angles EDG and EDF are greater then two right angles; but the angles EDG and GDK are equall to two right angles: take away the angle FDG which is common to them both. Wherefore the angle remayning, namely, EDF is greater then the angle remayning, namely, then GDK : that is, the angle KDH (which by the 15. proposition is equall to the angle EDF) is greater then the angle GDK , wherefore the angle GDK is more then double to the angle GDK : but the angle GDK is double to the angle GDK .

When they are greater then two right angles.

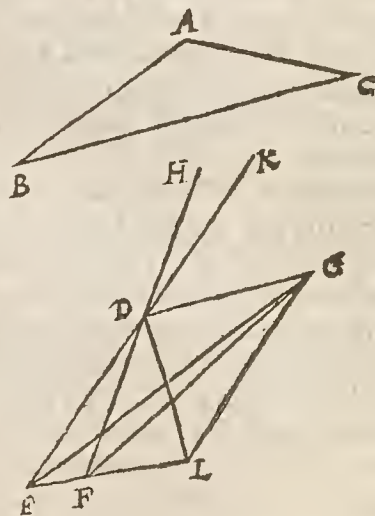
The first Booke

gle DGF , as was before proued. Wherefore the angle GDK is lesse then the angle DGF . Vnto the angle GDK put (by the 23. proposition) the angle DGL equall: and produce the line GL till it concurre with the line EF in the pointe L . And draw a line from D to L . Wherefore (by the 27. proposition) GL is a parallel line to DE , for that the alternate angles DGL and GDK are equal. Wherefore the triangles GDE and LDE are (by this proposition) equal (for they consist vpon one and the self same base, namely, DE , and are in the selfe same parallel lines, namely, ED and GL) But the triangle LDE is lesse then the triangle FDE . Wherefore also the triangle GDE is lesse then the triangle FDE . But the triangle GDE is equal to the triangle ABC . Wherefore the triangle ABC is lesse then the triangle DEF ; which was required to be proued.



*When they are
lesse then two
right angles.*

But now let the angles BAC and EDF be lesse then two right angles: and agayne let the angle at the pointe A be greater then the angle at the point D . Then I say that the triangle ABC is greater then the triangle DEF . Let the same construction be also here that was in the two former. And forasmuch as the angles BAC and EDF , that is, the angles EDG & EDF , are lesse then two right angles, but the angles EDG and GDK are equal to two right angles, take away the angle FDG which is common to them both, wherefore the angle remayning, namely, EDF is lesse then the angle remayning, namely, then GDK : that is, the angle HDK (which by the 15. proposition is equall to the angle EDF) is lesse then the angle GDK . Wherefore the whole angle GDH is lesse then double to the angle GDK . But it is double to the angle DGF (as before it was proued): wherefore the angle GDK is greater then the angle DGF . Put the angle DGL equall to the angle GDK (by the 23. proposition) and produce the line GL till it concurre with the line EF also produced, & let the concurse be in the point L . And draw a line from D to L . And for as much as the angle DGL is equall to the angle GDK , and they are alternate angles, therefore the line GL is a parallel to DE (by the 27. proposition). Wherefore (by this proposition) the triangles GDE and LDE are equal: but the triangle LDE is greater then the triangle FDE , and the triangle GDE is equall to the triangle ABC . Wherefore the triangle ABC is greater then the triangle DEF : which was required to be proued.



The 28. Theoreme. The 38. Proposition.

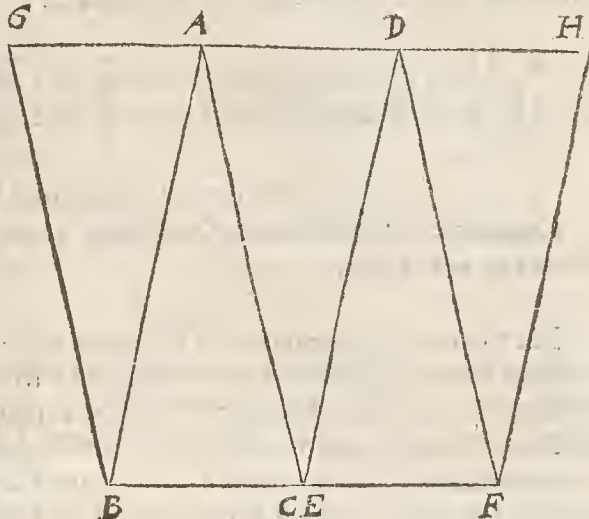
Triangles which consist vpon equall bases, and in the selfe same parallel lines, are equall the one to the other.

Suppose that these triangles ABC and DEF do consist vpon equal bases, that is, vpon BC and EF , and in the selfe same parallel lines, that is BF and AD . Then I say that the triangle ABC is equall to the triangle

gle ABC is equall to the triangle DEF . Produce (by the second petition) the line AD on eche side to the pointes G and H . And (by the 31. proposition) by the point B drawe vnto CA a parallel line BG , and (by the same) by the point F drawe vnto DE a parallel line FH . Wherefore $GBCA$ and $DEFH$ are parallelogrammes. But the parallelograme $GBCA$ is (by the 36 proposition) equal to the parallelogramme $DEFH$, for they consist vpon equall bases, that is, BC and EF , and are in the selfe same parallel lines, that is, BG and FH . But (by the 34. proposition) the triangle ABC is the halfe of the parallelogramme $GBCA$, for the diameter AB deuideth it into two equall partes: and the triangle DEF is (by the same) the halfe of the parallelogramme $DEFH$, for the diameter FD deuideth it into two equall partes. But the halues of thinges equall are (by the 7. common sentence) equall the one to the other. Wherefore the triangle ABC is equall to the triangle DEF . Wherefore triangles which consist vpon equall bases, and in the selfe same parallel lines, are equall the one to the other: which was required to be proued.

Construction.

Demonstration.



In this proposition are three cases. For the bases of the triangles either haue one part common to them both or the base of the one toucheth the base of the other onely in a point: or their bases are vtterly seuered a sunder. And ech of these cases may also be diuersly, as we before haue sene in parallelogrammes consisting on equall bases, and being in the selfe same parallel lines. So that he which diligently noteth the variety that was there put touching them, may also easely frame the same varietie to ech case in this proposition. Wherefore I thinke it nedeles here to repeat: the same agayne: for how soeuer the bases be put, or the toppes, the manner of construction and demonstration here put by Euclide will serue: namely, to draw parallel lines to the sides.

Three cases in this proposition.

Ech of these cases also may be diuersly.

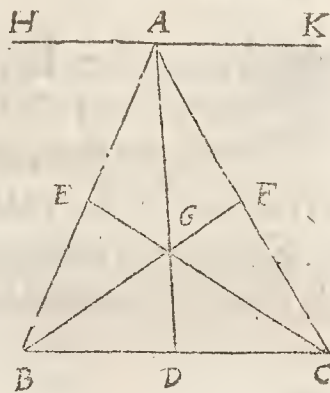
An addition of Pelitarius.

To deuide a triangle geuen into two equall partes.

Suppose that the triangle geuen to be deuided in to two equall partes be ABC . Deuide one of the sides therof, namely, BC into two equall partes (by the 10. proposition) in the point D . And draw a line from the point D to the point A . The I say that the two triangles ABD & ACD , are equal, which is easy to proue (by the 38. proposition) if by the point A we drawe vnto the line BC a parallel line (by the 31. proposition), which let be HK : for so the triangles ABD and ACD , consisting vpon equal bases BD & DC , and being in the selfe same parallel lines HK and BC are of necessitie equall. The selfe

O.i.

fame



An addition of Pelitarius, to deuide a triangle into two equall partes.

The first Booke

same thing also wil happen if the side BA be deuided into two equall partes in the point E , and so be drawen a right line from the point E , to the point C . Or if the side AC be deuided into two equall partes in the point F , and so be drawen a right line from the point F to the point B : which is in like manner proued by drawing parallel lines by the pointes B , and C , to the lines BA and AC .

Note.

And so by this you may deuide any triangle into so many partes as are signified by any number that is euenly euen: as into 14. 16. 32. 64. &c.

An other addition of Pelitarius.

An other addition of Pelitarius.

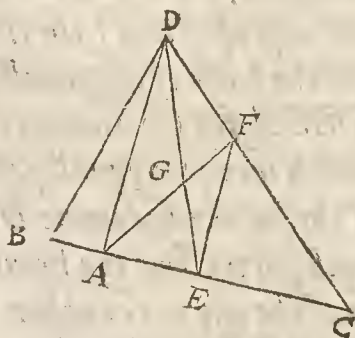
From any point geuen in one of the sides of a triangle, to draw a line which shal deuide the triangle into two equall partes.

Construction.

Let the triangle geuen be BCD : and let the point geuen in the side BC be A . It is required from the point A to draw a line which shal deuide the triangle BCD into two equall partes. Deuide the side BC into two equall partes in the point E . And drawe a

Demonstration

right line from the point A to the point D . And (by the 31. proposition) by the point E draw vnto the line AD a parallel line EF : which let cutte the side DC in the point F . And draw a line from the point A to the point F . Then I say that the line AF deuideth the triangle BCD into two equall partes: namely, the trapezium $ABFD$ is equall to the triangle ACF . For draw a line from E to D , cutting the line AF in the point G . Now then it is manifest (by the 38. proposition) that the two triangles BED and CED are equal (if we vnderstand a line to be drawen by the point D parallel to the line BC , for the bases BE and EC are equal). The two triangles also DEF and AEF are (by the 37. proposition) equall: for they consist vpon one and the selfe same base EF , and are in the selfe same parallel lines AD and EF . Wherefore taking away the triangle EFG which is comon to the both, the triangle AEG shalbe equall to the triangle DFG : wherefore vnto either of the adde the trapezium $CFGE$, and the triangle ACF shalbe equal to the triangle DEC . But the triangle DEC is the halfe part of the whole triangle BCD wherefore the triangle ACF is the halfe part of the same triangle BCD . Wherefore the residue, namely, the trapezium $ABFD$ is the other halfe of the same triangle. Wherefore the line AF deuideth the whole triangle BCD into two equall partes: which was required to be done.

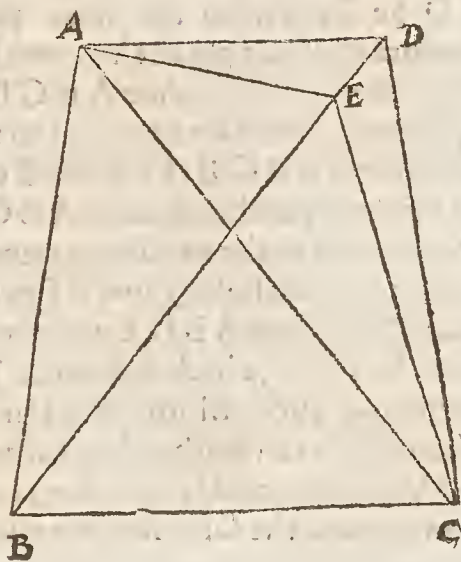


The 29. Theoreme. The 39. Proposition.

Equall triangles consisting vpon one and the same base, and on one and the same side: are also in the selfe same parallel lines.

Suppose that these two equall triangles ABC and DBC do consist vpon one and the same base, namely, BC and on one and the same side. The I say that they are in the selfe same parallel lines. Drawe a right line from the point A to the point D . Now I say that AD is a parallel line to BC . For if not, then (by the 31. proposition) by the point A drawe vnto the right line BC a parallel line AE , and draw a right line from the point E to the point

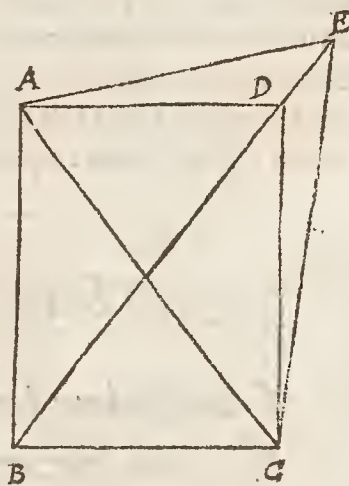
point C. *Wherefore* triangle $EB C$ is (by 37. *propositiō*) equal to the triangle $AB C$, for they consist vpon one and the selfe same base, namely, BC , and are in selfe same parallels, that is, AE and BC . But the triangle $DB C$ is (by supposition) equall to the triangle $AB C$. *Wherefore* the triangle $DB C$ is equal to the triangle $EB C$, the greater vnto the lesse: which is impossible. *Wherefore* the line AE , is not a parallel to the line BC . And in like sorte may it be proued that no other line besides AD is a parallel line to BC , wherefore AD is a parallel line to BC . *Wherefore* equall triangles consisting vpon one and the same base, and on one and the same side, are also in the selfe same parallel lines: which was required to be proued.



This proposition is the conuerse of the 37. proposition. And here is to be noted that if by the point A, you draw vnto the line BC a parallel line, the same shal of necessitie either light vpo the point D, or vnder it, or about it. If it light vpo it, then is that manifest which is required: but if it light vnder it, then foloweth that absurditie which Euclide putteth, namely, that the greater triangle is equall to the lesse: which selfe same absurditie also will follow, if it fall about the point D. As for example.

*This Theoreme
the conuerse of
the 37. proposi-
tion.*

Suppose that these equall triangles $AB C$ and $DB C$ do consist vppon one and the selfe same base BC , and on one and the same side. Then I say, that they are in the selfe same parallel lines, and that a right line ioyning together their toppes is a parallel to the base BC . Draw a right line frō A to D . Now if this be not a parallel to the base BC , let AE be a parallel vnto it, and let AE fall without the line AD . And produce the line BD till it concur with the line AE in the pointe E and draw a line from E to C . Wherefore the triangle $AB C$ is equal to the triangle $EB C$: but the triangle $AB C$ is equall to the triangle $DB C$: Wherefore the triangle $EB C$ is equall to the triangle $DB C$. Namely, the whole to the part: which is impossible. Wherefore the parallel line falleth not without the line AD . And Euclide hath proued that it falleth not within. Wherefore the line AD is a parallel vnto the line BC . Wherefore equall triangles which are on the selfe same side, and on one and the selfe same base, are also in the selfe same parallel lines: which was required to be proued.



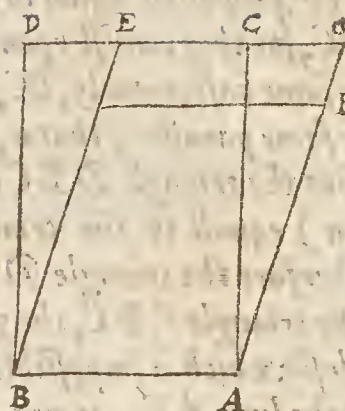
An addition of Flusses.

The selfe same also followeth in parallelogrames. For if vpon the base AB be set on one & the same side these equal parallelogrames $ABCD$ & $ABGE$, they shall of necessitie be in the selfe same parallel lines. For if not, but one of them is

*An addition of
Flusses.*

The first Booke

set cyther within or without, let the parallelograme $B F$ being equall to the parallelograme $A B C D$ be set within the same parallel lines: wherefore the same parallelograme $B F$ beyng equall to the parallelograme $A B C D$ (by the 35. proposition) shall also be equall to the other parallelograme $A B G E$. (by the first common sentence) For the parallelograme $A B G E$ is by supposition equall to the parallelogramme $A B C D$: wherefore the parallelograme $B F$ being equall to the parallelograme $A B G E$, the parte shall bee equal to the whole, which is absurde. The same inconuenience also will followe, if it fall without, VVherefore it can neither fall within nor without, VVherefore equall parallelogrames beyng vpon one and the selfe same base and on one and the same side, are also in the selfe same parallel lines.

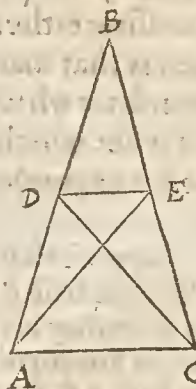


An addition of Campanus.

An addition of Campanus.

If a right line deuide two sides of a triangle into two equall partes: it shall be equidistant vnto the third side.

Suppose that there be a triangle $A B C$: and let there bee a right lyne $D E$, which let deuide the two sides $A B$ and $B C$ into two equall partes in the pointes D and E . Then I say, that the line $D E$ is a parallel to the line $A C$. Drawe these two lines $A E$ and $D C$. Now then imagining a line to be drawne by the point E parallel to the line $A B$, the triangle $B D E$ shall (by the 38. proposition) bee equall to the triangle $D A E$ (for their two bases $A D$ and $D B$ are put to be equall) And by the same reason the triangle $B D E$ is equall to the triangle $C E D$. VVherefore (by the first common sentēce) the triangles $E A D$ and $E C D$ are equall, and they are erected on one and the selfe same base, namely, $D E$, and on one and the same side. VVherefore (by the 39. proposition) they are in the selfe same parallel lines, and the line which ioyneth together their toppes is a parallel to their base. VVherefore the lynes $D E$ and $A C$ are parallels: which was required to be proued.



The 30. Theoreme. The 40. Proposition.

Equall triangles consisting vpon equall bases, and in one and the same side: are also in the selfe same parallel lines.

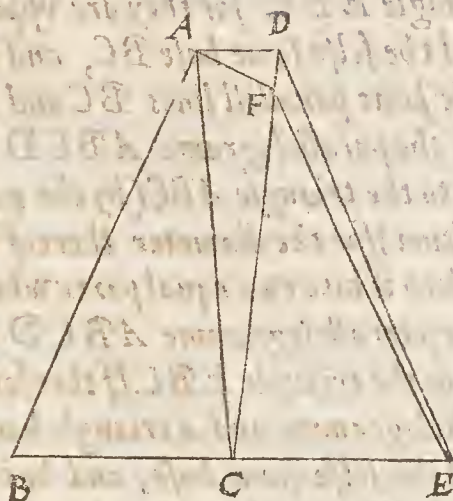


Suppose that these equall triangles $A B C$ and $C D E$ do consist vpon equall bases, that is, vpon $B C$ and $C E$, and on one and the same syde, namely, on the side of A . Then I say that they are in the selfe same parallel

lell lines. Drawe by the first petition a right line from the point A to the pointe D . Now I say that AD is a parallel line to BE . For if not, then (by the 31. proposition) by the point A draw vnto the line BE a parallell line AF . And drawe a right line from the point F to the pointe E . Wherefore (by the 38. proposition) the triangle BAC is equall to the triangle CFE : for they consist vpon equal bases, that is BC and CE , and are in the selfe same parallel lines, namely, BE and AF . But by supposition the triangle ABC is equal to the triangle CDE . Wherefore the triangle DCE is equall to the triangle FCE , namely, the greater vnto the lesse, which is impossible. Wherefore AF is not a parallel line to BE . And in like sorte may we proue that no other line besides AD is a parallell line to BE . Wherefore AD is a parallell line to BE . Equall triangles therefore consisting vpon equall bases, and in one and the same side: are also in the selfe same parallel lines: which was required to be proued.

This proposition is the conuerse of the 38. proposition. And in this as in the former proposition, if the parallel line drawn by the point A , should not passe by the point D , it must passe eyther beneath it, or aboue it. Euclide setteth forth onely the absurdity which should follow if it passe beneath it: but the selfe same absurditie also wd follow if it should passe aboue it: as it is not hard to see by the gathering thereof in the former proposition. And therefore here I omitte it.

Construction.



Demonstration leading to an absurditie.

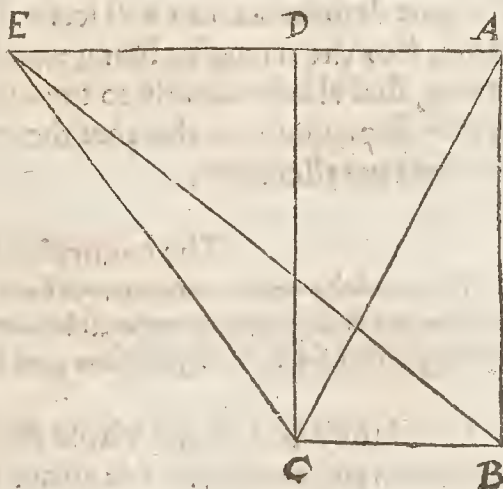
This proposition is the conuerse of the 38. proposition.

The 31. Theoreme. The 41. Proposition.

If a parallelograme & a triangle haue one & the selfe same base, and be in the selfe same parallel lines: the parallelograme shalbe double to the triangle.



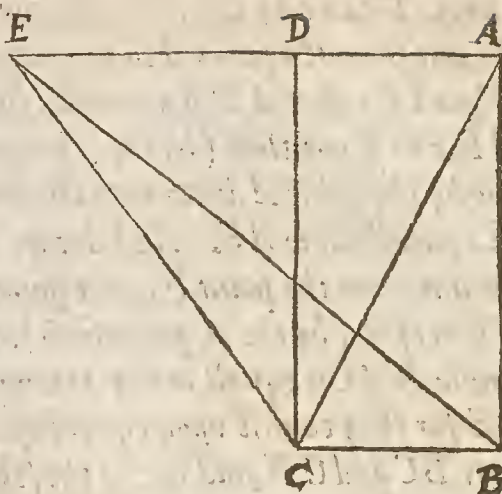
Suppose that the parallelograme $ABCD$ and the triangle EBC haue one & the same base, namely, BC , and let thē be in the selfe same parallel lines, namely, BC & AE . Thē I say, that the parallelograme $ABCD$ is double to the triangle BEC . Draw (by the first petition) a right line from the pointe A to y pointe C . Wherefore (by the 37. proposition) $O.ij.$ sition



Demonstration

The first Booke

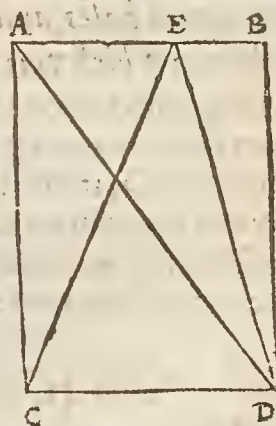
sition) the triangle ABC is equall to the triangle $EB C$: for they are vppon one and the selfe same base BC , and in the selfe same parallell lines BC and EA : but the parallelograme $ABCD$ is double to the triangle ABC (by the 34. proposition) for the diameter thereof AC deuideth it into two equal parts: wherefore the parrallelogramme $ABCD$ is double to the triangle $EB C$. If therefore a parallelogramme and a triangle haue one and the selfe same base, and be in the selfe same parallels, the parallelogramme shall be double to the triangle: which was required to be proued.



Two cases in
this proposition.

This proposition hath two cases. For the base beyng one, the triangle may haue his toppes without the parallelogramme, or within. The first case is demonstrated of the author, The second case is thus.

Suppose that there be a parallelogramme $ABCD$, and let the triangle be ECD , either of which let haue one and the selfe same base, namely, CD , and let them be in the selfe same parallel lines CD and AB , and let the toppes of the triangle ECD , namely, the point E , be within the parallelogramme $ABCD$. Then I say that the parallelogramme $ABCD$ is double to the triangle ECD . Draw a right line from the point A to the point D . Now forasmuch as the parallelogramme $ABCD$ is double to the triangle ACD (by the 34. proposition): and the triangle ADC is equall to the triangle EDC (by the 37. proposition). Therefore the parallelogramme $ABCD$ is double to the triangle ECD : which was required to be proued.



A corollary.

By this proposition it is manifest that if the base be doubled, the triangle erected vppon it shalbe equall to the parallelogramme.

The selfe same
demonstration
will serue if the
triangle & the
parallelogramme
be vpon equall
bases.

And if the bases of the triangle and of the parallelogramme be equall, the selfe same demonstration will serue if you drawe the diameter of the parallelogramme. For the triangles being equal, the parallelogramme which is double to the one, shal also be double to the other. And the triangles must nedes be equall (by the 38. proposition) for that their bases are equal, and for that they are in the selfe same parallel lines.

The conuerse of this proposition is thus.

If a parallelogramme and a triangle haue one and the selfe same base, or equall bases the one to the other, and be described on one and the same side of the base: if the parallelogramme be double to the triangle, they shalbe in the selfe same parallel lines.

The conuerse of
this proposition.

For if they be not, the whole shalbe equall to his parte. For then the toppes of the triangle must nedes fall either within the parallel lines or without. And whether

whether of both fouer it do, one and the selfe same impossibilitie will follow, if by the toppe of the triangle be drawen vnto the base a parallel line.

An other conuerse of the same proposition.

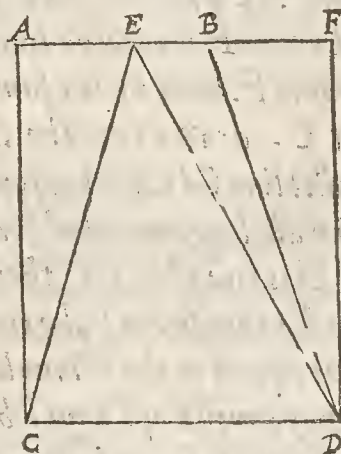
If a parallelogramme be the double of a triangle, being both within the selfe same parallel lines: then are they vpon one and the selfe same base, or vpon equall bases. For if in that case their bases should be vnequal, then might straight way be proued, that the whole is equall to his part: which is impossible.

An other conuerse of the same proposition.

A trapezium hauing two sides onely parallel lines, is eyther more then double, or lesse then double to a triangle contayned within the selfe same parallel lines, and hauing one and the selfe same base with the trapezium, or table. Iust the double it cannot be, for then it should be a parallelogramme. A trapezium hauing two sides parallels hath of necessitie the one of them longer then the other: for if they were equall then should the other two sides enclosing them be also equall (by the 33. proposition.) If the greater side of the trapezium be the base of the triangle, then shall the trapezium be lesse then the double of the triangle. And if the lesse side of the trapezium be the base of the triangle then shall the trapezium be greater then the triangle.

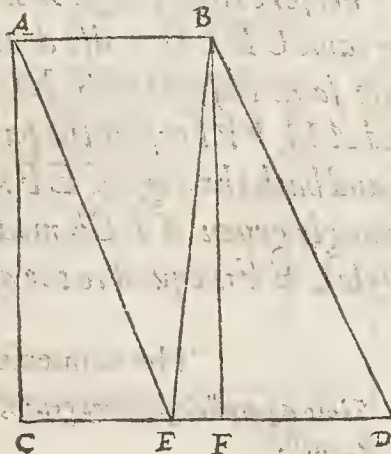
Comparison of a triangle and a trapezium being vpon one & the selfe same base, and in the selfe same parallel lines.

For suppose that $ABCD$ be a trapezium, and let two sides thereof, namely, AB and CD be parallel lines, and let the side AB be lesse then the side CD , & produce the side AB infinitlye on the side B to the point F . And let the triangle ECD haue one and the selfe same base with the trapezium, namely, the line CD . Then I say that the trapezium $ABCD$ is lesse then the double of the triangle ECD . For put the line AF equall to the line CD (by the 3. proposition) and draw a line from D to F . Wherefore $ACDF$ is a parallelogramme (by the 33. proposition). Wherefore (by the 34 proposition) it is double to the triangle ECD . But the trapezium $ABCD$ is a part of the parallelogramme $ACDF$. Wherefore the trapezium $ABCD$ is lesse then the double of the triangle ECD : which was required to be proued.



When the greater side of the trapezium is the base of the triangle.

Agayne let the triangle haue to his base the side AB . Then I say that the trapezium $ABCD$ is greater then the double of the triangle AEB . For from the side CD cut of the line CF equall to the line AB (by the 3. proposition) And draw a line from B to F . Wherefore (by the 33. proposition) $ABCF$ is a parallelogramme: and therefore is (by the 34. proposition) double to the triangle AEB . Wherefore the trapezium $ABCD$ is more then the double of the triangle AEB : which was required to be proued.



When the lesse side is the base.

The first Booke

The 11. Probleme. The 42. proposition.

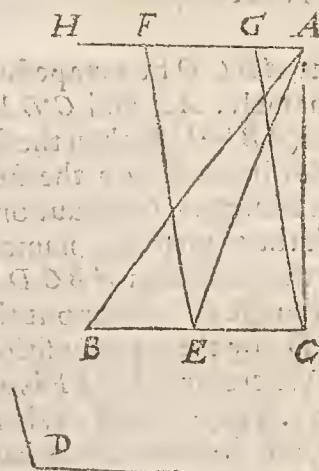
Vnto a triangle geuen, to make a parallelograme equal, whose angle shall be equall to a rectiline angle geuen.



Construction.

Suppose that the triangle geuen be ABC , and let the rectiline angle geuen be D . It is required that vnto the triangle ABC there be made a parallelograme equal, whose angle shall be equall to the rectiline angle geuen, namely, to the angle D . Deuide (by the 10. proposition) the line BC into two equall partes in the pointe E . And (by the first petition) draw a right line from the point A to the point E . And (by the 23. proposition) vpon the right

line geuen EC , and to the point in it geuen E , make the angle CEF equal to the angle D . And (by the 31. proposition) by the point A draw vnto the line EC a parallel line AH : and let the line EF cut the line AH in the point F : and (by the same) by the point C , draw vnto the line EF a parallel line CG . Wherefore $FECG$ is a parallelograme. And forasmuche as BE is equall to EC , therefore (by the 28. proposition) the triangle ABE is equall to the triangle AEC , for they consist vpon equall bases that is BE and EC , and are in the selfe same parallel lines, namely, BC and AH . Wherefore the triangle ABC is double to the triangle AEC . And the parallelograme $CEFG$ is also double to the triangle AEC : for they haue one & the selfe same base, namely, EC : and are in the selfe same parallel lines, that is, EC and AH . Wherefore the parallelograme $FECG$ is equall to the triangle ABC , and bath the angle CEF equall to the angle geuen D . Wherefore vnto the triangle geuen ABC is made an equall parallelograme, namely, $FECG$, whose angle CEF is equall to the angle geuen D : which was required to be done.



Demonstration

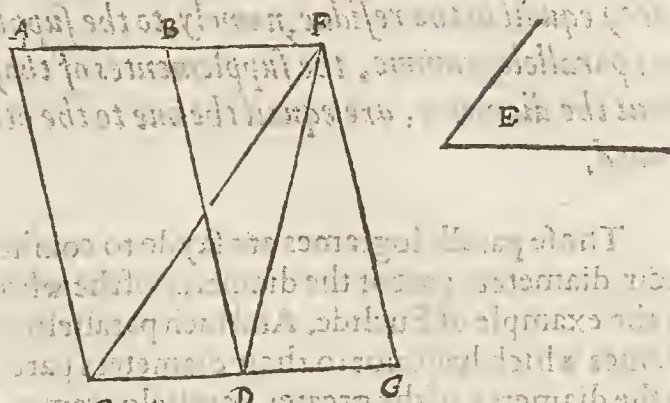
The conuerse of this proposition after Pelitarius.

The conuerse of this former proposition.

Vnto a parallelograme geuen, to make a triangle equall, hauyng an angle equall to a rectiline angle geuen.

Suppose that the parallelograme geuen be $ABCD$, and let the angle geuen be E . It is required vnto the parallelograme $ABCD$ to make a triangle equall hauyng an angle

angle equal to the angle E . Vpon the line CD and to the pointe in it C , describe (by the 23. proposition) an angle equall to the angle E : which let be DCF : and let the line CF cut the line AB being produced, in the point F : and produce the line CD (which is parallel to the line AF) to the point G . And put the line DG equall to the line CD and draw a line from F to G . Then I say that the triangle CFG is equal to the parallelograme $ABCD$. For forasmuch as (by the 38. proposition) the whole triangle CFG is double to the triangle CDF . And (by the 41. proposition) the parallelograme $ABCD$ is double to the same triangle CDF : therefore the parallelograme $ABCD$ and the triangle CFG are equall the one to the other: which was required to be done.

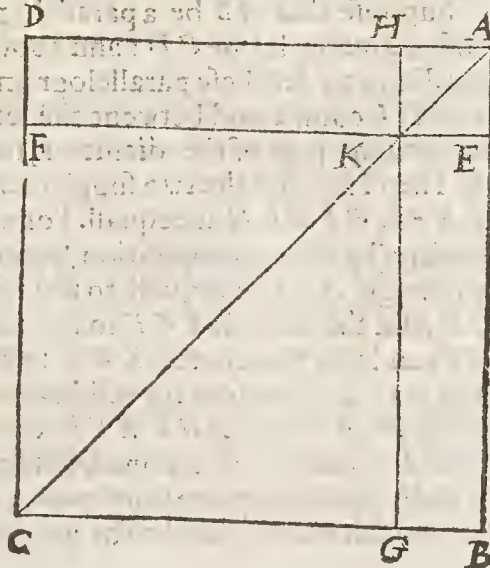


The 32. Theoreme. The 43. Proposition.

In euery parallelograme, the supplementes of those parallelogrammes which are about the diameter, are equall the one to the other.

Suppose that $ABCD$ be a parallelograme, and let the diameter thereof be AC : and about the diameter AC let these parallelogrammes EH and GF consist: and let the supplementes be BK and KD . Then I say that the supplement BK is equall to the supplement KD . For

forasmuch as $ABCD$ is a parallelograme and the diameter thereof is AC , therefore (by the 34. proposition) the triangle ABC is equall to the triangle ADC . Agayne forasmuch as $A E K H$ is a parallelograme, and the diameter thereof is AK , therefore (by the same) the triangle $A E K$ is equall to the triangle $A H K$. And by the same reason also the triangle $K F C$ is equall to the triangle $K G C$. And forasmuch as the triangle $A E K$ is equall to the triangle $A H K$, and the triangle $K F C$ to the triangle $K G C$, therefore the triangles $A E K$ and $K G C$ are equall to the triangles $A H K$ and $K F C$: and the whole triangle $A B C$ is equall to the whole triangle $A D C$.

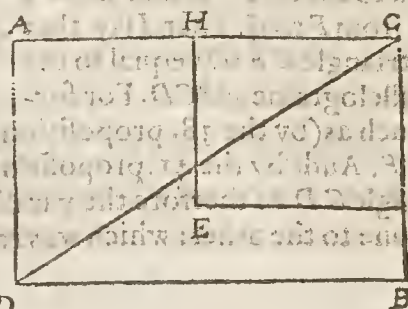


The first Booke

C: wherefore the residue, namely, the supplement BK is (by the 3. common sentence) equall to the residue, namely, to the supplement KD . Wherefore in every parallelogramme, the supplementes of those parallelogrammes whiche are about the diameter, are equall the one to the other: whiche was required to be proued.

How parallelogrammes are sayde to consist about a diameter.

Those parallelogrammes are sayde to consist about a diameter which haue to their diameters part of the diameter of the whole and great parallelogramme, as in the example of Euclide. And such parallelogrammes which haue not to their diameters part of the diameter of the greater parallelogramme, are sayde not to consist about the diameter. For the diameter of the greater parallelogramme cutteth the side of the lesse cōtayned wythin it. As in the parallelogramme AB , the diameter CD , cutteth the side EH of the parallelogramme CE . Wherefore the parallelogramme CE is not about D one and the selfe same diameter with the parallelogramme CD .



Supplementes & Complementes.

Supplementes or Complementes are those figures which with the two parallelogrammes accomplish the whole parallelogramme. Although Pelitarius for distinction sake putteth a difference betwene Supplementes and Complementes. The parallelogrammes about the diameter he calleth Complementes, the other two figures he calleth Supplementes.

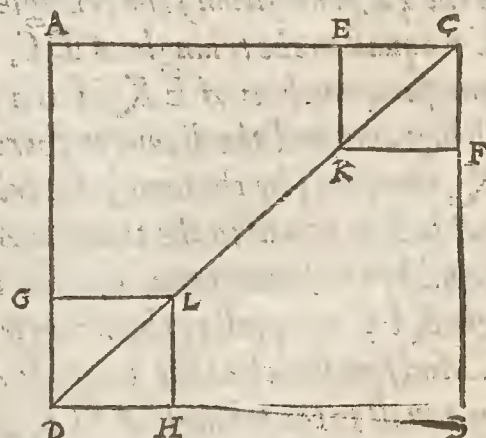
Three cases in this Theoreme.

This theoreme hath three cases onely. For the parallelogrammes which consist about the diameter, eyther touch the one the other in a point: or by a cer rayne parte of the diameter are seuered the one from the other: or els they cutte the one the other. For the first case is the example of Euclide before set. The second case is thus:

The first case.

The second case.

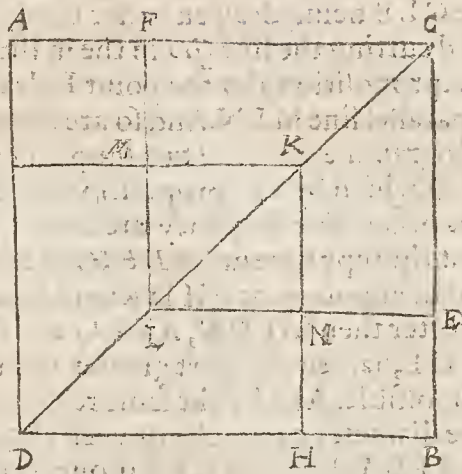
Suppose that AB be a parallelogramme: whose diameter let be CD : and aboute the same diameter let these parallelogrammes CK and DL consist: and betwene the let there be a certayne part of the diameter, namely, LK . Then I say that the two supplementes $AGLKE$ & $BFKLH$ are equall. For we may as before (by the 34. proposition) proue that the triangle ACD , is equall to the triangle BCD , and the triangle ECK to the triangle KCF , and also the triangle DGL to the triangle DHL . Wherefore the residue, namely, the five sided figure $AGLKE$ is equall to the residue, namely, to the five sided figure $BFKLH$: that is, the one supplement to the other: which was required to be proued.



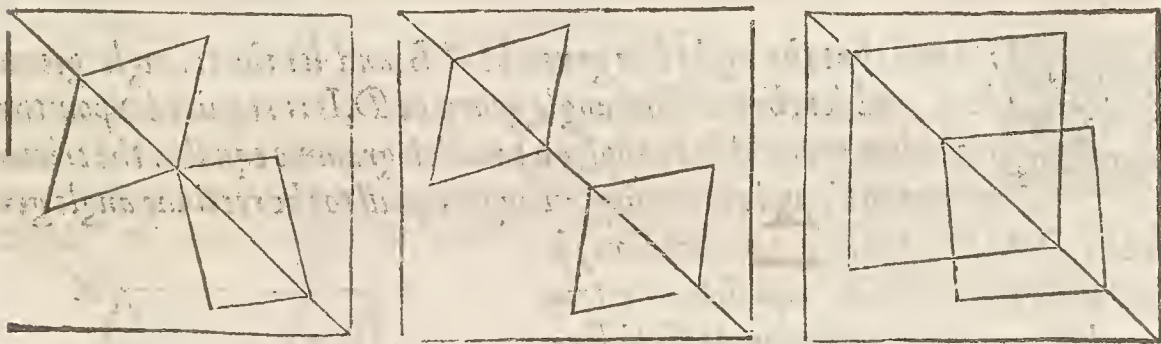
The third case.

But now suppose AB to be a parallelogramme, and let the diameter thereof be CD : and let the one of the parallelogrammes about it be $ECFL$, and let the other be $DGKH$.

$DGKH$, of which let the one cut the other. Then I say that the supplementes FG and EH are equall. For forasmuch as the whole triangle DGK is equal to the whole triangle DHK (by the 34. proposition), and part also of the one, namely, the triangle KLK is equal to part of the other, namely, to the triangle KLK (by the same), for LK is a parallelogram: therefore the residue, namely, the Trapezium $DLNH$ is equal to the residue, namely, to the trapezium $DLMG$: but the triangle ADC is equal to the triangle BCD , and in the parallelogram EF , the triangle FCL is equal to the triangle ECL , and the trapezium DGM is (as it hath bene proued) equal to the trapezium $DHNL$. Wherefore the residue, namely, the quadrilater figure GF is equal to the residue, namely, to the quadrilater figure EH , that is, the one supplement to the other: which was required to be proued.



This is to be noted that in ech of those three cases it may so happen, that the parallelogrammes about the diameter shall not haue one angle common wyth the whole parallelogramme, as they haue in the former figures. But yet though they haue not, the selfe same demonstration wil serue, as it is playne to see in the figures here vnderneath put. For alwayes, if from thinges equall be taken away thinges equall, the residue shalbe equall.



This proposition Pelitarius calleth Gnomicall, and mysticall, for that of it (sayth he) spring infinite demonstrations, and vses in geometry. And he putteth the conuerse thereof after this manner.

This proposition called Gnomical and mysticall.

If a parallelogramme be deuided into two equall supplementes, and into two complements whatsoever: the diameter of the two complements shall be set directly, and make one diameter of the whole parallelogramme.

The conuerse of this proposition.

Here is to be noted as I before aduertised that Pelitarius for distinction sake putteth a difference betwene supplementes and complementes, which difference, for that I haue before declared, I shall not neede here to repete agayne.

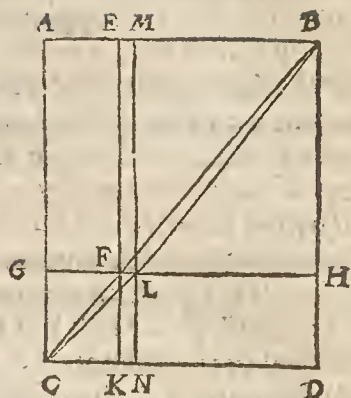
Suppose that there be a parallelogramme $ABCD$, whose two equall supplements let be $AELG$ and $FHDK$, and let the two complementes thereof be $GFCK$ and $EBFH$: whose diameters let be CF and FB . Then I say that CFB is one right line, and is the diameter of the whole parallelogramme $ABCD$: for if it be not, then is there an

P.ij.

other

The first Booke

other diameter of the whole parrallelogramme, which let be CLB being drawn vnder the diameters CF and FB , and cutting the line GH in the in the point L . And (by the 31. proposition) by the point L , draw vnto the line AC a parallel line MLN . And so are there in the whole parrallelogramme $ABCD$ two supplements $AMGL$ and $LHND$, which by this proposition shalbe equall the one to the other. For that they are about the diameter CLB . But the supplement $AEFG$ is (by supposition) equall to the supplement $FHDK$: and forasmuch as $FHDK$ is greater then $LHND$, $AEFG$ also shalbe greater then $AMGL$, namely, the part greater then the whole: which is impossible. And by the same reaso may it be proued, that the diameter cannot be drawn about the diameters CF and FB . Wherefore CFB is one diameter of the whole parrallelogramme $ABCD$: which was required to be proued.



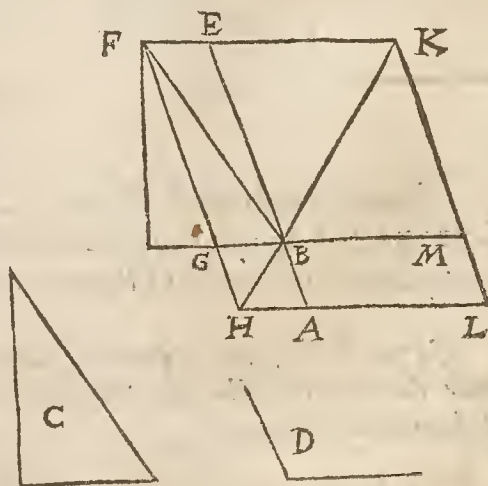
The 12. Probleme. The 44. Proposition.

Uppon a right line geuen, to applye a parallelograme equall to a triangle geuen, and contayning an angle equall to a rectiline angle geuen.



Construction.

Suppose that the right line geuen be AB , and let the triangle geuen be C , and let the rectiline angle geuen be D . It is required vpon the right line geuen AB , to applye a parallelogramme equal to the triangle geuen C , and contayning an angle equall to the rectiline angle geuen D . Describe (by the 44. proposition) y parallelograme $BGEF$ equall to the triangle C , and hauing the angle BGF equall to the angle D . And vnto the line EB ioyn the line AB in such sort that they make both one right line. And extend the line FG beyond the point G to the poynte H . And (by the 31. proposition) by the point A drawe to either of these lines BG and EF a parallelline AH . And (by the first petition) draw a right line from the point H to the point B . And forasmuch as vpon the parallel lines AH and EF falleth a certayne right line HF , therefore (by the 29. proposition) the angles AHF and HFE are equall to two right angles: wherefore the angles BHG and GFE are lesse then two right angles: but if vpon two right lines fall a right line making the inward angles on one and the same



same side lesse then two right angles, those right lines being infinitely produced shall at the length mete on that side in which are the angles lesse then two right angles (by the 5. petition). Wherefore the lines HB and FE being infinitely produced will at the length mete. Let them be produced, & let them mete in the point K . And (by the 31 proposition) by the point K draw to either of these lines EA and FH a parallel line KL . And (by the 2. petition) extend the lines HA and GB till they cōcurre with the line KL in the pointes L and M . Wherefore $HLKF$ is a parallelogramme, and the diameter thereof is HK ; and about the diameter HK are the parallelogrammes AG and ME , and the supplementes are LB and BF : wherefore (by the 43. proposition) the supplement LB is equall to the supplement BF : but by construction the parallelograme BF is equall to the triangle C : wherefore also the parallelogramme LB is equall to the triangle C . And forasmuch as the line FH is a parallel to the line KL , and vpon them lighteth the line GM , therefore (by the 27. proposition) the angle FGB is equall to the angle BML . But the angle FGB is equall to the angle D , therefore the angle BML is equal to the angle D . Wherefore vpon the right line geuen AB is applied the parrallelograme LB , equal to the triangle geuen C , and contayning the angle BML equal to the rectiline angle geuen D : which was required to be done.

Demonstration

Applications of spaces or figures to lines with excesses or wantes is (sayth Eudemus) an auncient inuention of Pithagoras.

Applications of spaces with excesses or wantes an auncient inuention of Pithagoras. How a figure is sayde to be applied to a line.

VVhen the space or figure is ioyned to the whole line, thē is the figure sayd to be applied to the line. But if the length of the space be longer then the line, thē it is sayde to exceede: and if the length of the figure be shorter then the line, so that part of the line remayneth without the figure described, then is it sayde to want.

In this probleme are three thinges geuen. A right line to which the application is made, which here must be the one side of the parallelogramme applied. A triangle whereunto the parallelogramme applied must bee equall: and an angle wherunto the angle of the parallelograme applied must be equall. And if the angle geuen be a right angle, thē shal the parallelograme applied be either a square, or a figure on the one side longer. But if the angle geuen be an obtuse or an acute angle, then shall the parallelograme applied be a Rhombus or diamond figure, or els a Rhomboides or diamondlike figure.

Three thinges geuen in this proposition.

The conuerse of this proposition after Pelitarius.

Upon a right line geuen, to applie vnto a parallelograme geuen an equall triangle hauyng an angle equall to an angle geuen.

The conuerse of this proposition.

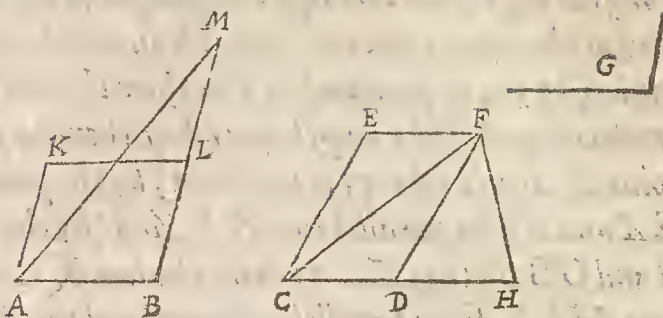
Suppose that the right line geuen be AB , and let the parallelograme geuen be $CDEF$, and let the angle geuen be G . It is required vpon the line AB to describe a tria- gle equall to the parallelograme $CDEF$, hauyng an angle equall to the angle G . Drawe the diameter CF & produce CD beyond the point D to the point H . And put the line

P. iij.

DH

The first Booke

DH equall to the line CD . And draw a line from F to H . Now the (by the 41. proposition) the triangle CHF is equall to the parallelogramme $CDEF$. And (by this proposition) vppon the line AB describe a parallelogramme $ABKL$ equall to the triangle CHF , hauing the angle ABL equal to the angle geuen G ; and produce the line BL beyonde the point L to the point M . And put the line LM equall to the line BL , and draw a line from A to M . Then I say that vpon the line AB is described the triangle ABM , which is such a triangle as is required. For (by the 41. proposition) the triangle ABM is equal to the parallelogramme $ABKL$ (for that they are betwene two parallel lines BM and AK , & the base of the triangle is double to the base of the parallelogramme): but $ABKL$ is by construction equall to the triangle CHF : and the triangle CHF is equall to the parallelogramme $CDEF$. Wherefore (by the first common sentence) the triangle ABM is equall to the parallelogramme geuen $CDEF$, and hath his angle ABM equal to the angle geuen G : which was required to be done.



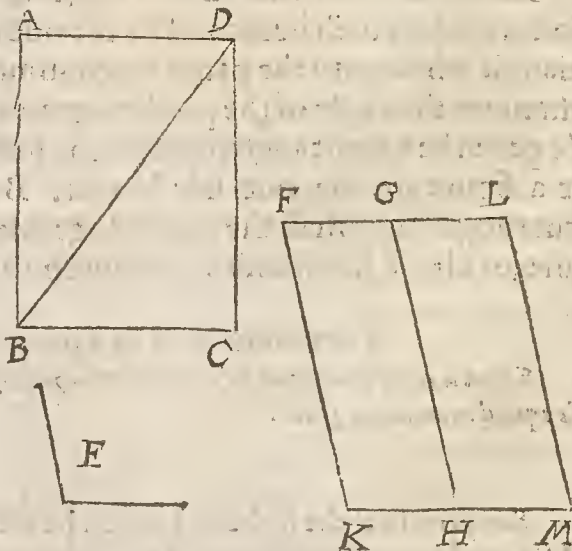
The 13. Probleme. The 45 Proposition.

To describe a parallelogramme equal to any rectiline figure geuen, and contayning an angle equall to a rectiline angle geuen.

Suppose that the rectiline figure geuen be $ABCD$, and let the rectiline angle geuen be E . It is required to describe a parallelogramme equall to the rectiline figure geuen $ABCD$, and contayning an angle equal to the rectiline angle geuen E . Draw (by the first petition) a right line from the point D to the point B . And (by the 42. proposition) vnto the triangle ABD describe an equall parallelogramme FH , hauing his angle FKH equall to the angle E . And (by the 44. of the first) vpo the right line GH apply the parallelogramme GM equal to the triangle DBC , hauing his angle GHM equall to the angle E . And forasmuch as eyther of those angles HKF and GHM is equall to the angle E : therefore the angle HKF is equall to the angle GHM : put the angle KHG common to them both, wherefore the angles FKH and KHG are equall to the angles KHG and GHM . but the angles FKH and KHG are (by the 29. proposition) equall to two right angles. Wherefore the angles KHG and GHM are equall to two right

Construction.

Demonstration.



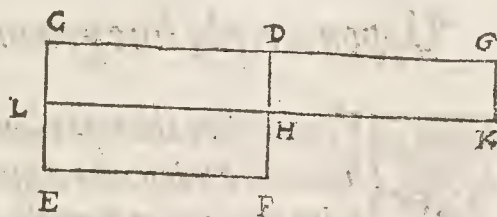
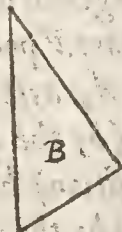
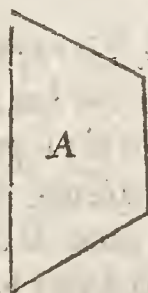
right angles. Now then vnto a right line GH , and to a point in the same H , are drawen two right lines KH and HM not both on one and the same side, making the side angles equall to two right angles. *VV*herfore (by the 14. proposition) the lines KH and HM make directly one right line. And forasmuch as vpon the parallel lines KM and FG falleth the right line HG , therefore the alternate angles MHG and HGF are (by the 29. proposition) equall the one to the other: put the angle HGL common to them both. *VV*herfore the angles MHG and HGL are equall to the angles HGF and HGL . But the angles MHG & HGL are equall to two right angles (by 29. proposition). *VV*herfore also the angles HGF and HGL are equall to two right angles. *VV*herfore (by the 14. proposition) the lines FG and GL make directly one right line. And forasmuch as the line KF is (by the 24. proposition) equal to the lyne HG , and it is also parallel vnto it: and the line HG is (by the same) equall to the line ML , therefore (by the first common sentence) the line FK is equall to the lyne ML , and also a parallel vnto it (by the 30. proposition). But the right lynes KM and FL ioyn them together. *VV*herfore (by the 33. proposition) the lines KM and FL are equall the one to the other and parallel lines. *VV*herfore $KFLM$ is a parallelograme. And forasmuch as the triangle ABD is equal to the parallelograme FH , and the triangle DBC to the parallelogramme GM : therefore the whole rectiline figure $ABCD$ is equall to the whole parallelograme $KFLM$. *VV*herfore to the rectiline figure geuen $ABCD$ is made an equall parallelograme $KFLM$, whose angle FKM is equal to the angle geuen, namely, to E : which was required to be done.

The rectiline figure geue is in the example of Euclide is a parallelograme. But if the rectiline figure be of many sides, as of 5. 6. or mo, the must you resolue the figure into his triangles, as hath bene before taught in the 32. proposition. And the apply a parallelograme equal to euey triangle vpon a line geue, as before in the example of the author. And the same kind of reasoning wil serue that was before, only by reaso of the multitude of triangles, you shall haue neede of oftener repeticio of the 29. and 14. propositions to proue that the bases of al the parallelogrames made equall to all the triangles make one right line, and so also of the toppes of the said parallelogrames. Pelitarius addeth vnto this proposition this Probleme following.

Two vnequall rectiline superficieses beyng geuen, to find out the excesse of the greater aboue the lesse.

An addision of Pelitarius.

Suppose that there be two vnequall rectiline superficieses A & B of which let A be the greater. It is required to finde out the excesse of the superficies A aboue the superficieses B . Describe (by the 44.

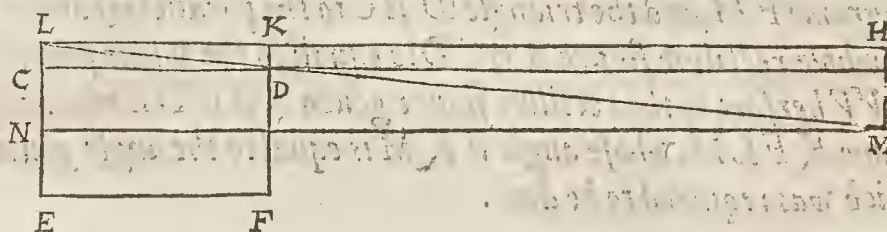


The first Booke

proposition) the parallelograme $CDEF$ equall to the rectiline figure A , containyng a right angle. And produce the line CD beyond the point D to the point G : & put the line DG equall to the line CD . And againe (by the 44. proposition) vpon the line DG describe the parallelograme $DGHK$ equall to the rectiline figure B , and hauyng the angle DGH a right angle. And produce the line KH beyond the point H , vntill it cutte the line CE in the point L . Then I say that $HLEF$, is the excesse of the rectiline figure A aboue the rectiline figure B . For first that $CGKL$ is a parallelogramme it is manifest, neither nedeth it to be demonstrated. And forasmuch as the lines CD and DG are by supposition equal and either of them is a parallel to KL , therfore (by the 36. proposition) the two parallelogrames CH and DK are equall. And forasmuch as DK is supposed to be equall to the rectiline figure B , CH also shall be equall to the same rectiline figure B . Wherefore forasmuch as the whole parallelograme CF is equall to the rectiline figure A , and LF is the excesse of CF aboue DL or DK , it followeth that LF is the excesse of the rectiline figure A aboue the rectiline figure B : whiche was required to be done.

An other more redy way.

Let the parallelograme $CDEF$ remayne equall to the rectiline figure A , & produce the line CD beyond the point D to the point G . And vpon the line DG describe the parallelograme $DGHK$ equall to the rectiline figure B . And produce the lines EC & HK beyond the points C and K till they concur in the point L . And by the point D draw the diameter LDM , which let cutte the line HG beyng produced, beyonde the point G in the point M ; & by the point M drawe vnto the line HL a paral-



lel MN cutting the line EL in the point N : and by that meanes is $HLMN$ a parallelograme. Then I say that NF is the eccess of the rectiline figure A aboue the rectiline figure B . For forasmuch as the parallelograme HD is equall to the rectiline figure B , & the supplementes HD and DN ar (by the 43. proposition) equall: therfore DN also is equall to the rectiline figure B , which rectiline figure DN being taken away frō the parallelograme CF (which is supposed to be equall to the rectiline figure A) the residue NF shall be the eccess of the rectiline figure A aboue the rectiline figure B : which was required to be done.

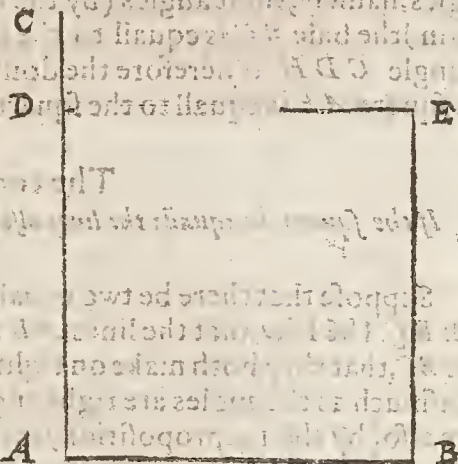
The 14. Probleme. The 46. Proposition.

Vppon a right line geuen, to describe a square.



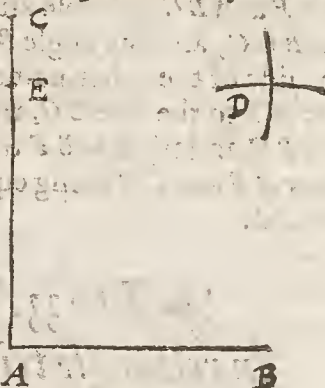
Vppose that the right line geuen be AB . It is required vpo the right line AB , to describe a square. Vpon the right line AB , and from a point in it geuen, namely, A , rayse vp (by the 11. proposition) a perpendicular line AC . And (by the 3. proposition) vnto AB put an equall line AD . And (by the 31. proposition) by the point D drawe vnto AB a parallel line DE . And (by the same) by the point B drawe vnto AD a parallel line

line BE . Wherefore $ADEB$ is a parallelogramme. Wherefore the line AB is equall to the line DE , and the line AD to the line BE : but the line AB is equall to the line AD wherefore these former lines BA, AD, DE, EB , are equall the one to the other. Wherefore the parallelogramme $ADEB$ consisteth of equall sides. I say also that it is rectangle. For forasmuch as vpon the parallel lines AB and DE falleth a right line AD : therefore (by the 29. proposition) the angle BAD and ADE are equal to two right angles: but the angle BAD is a right angle. Wherefore the angle ADE also is a right angle. But in parallelogrammes the sides and angles which are opposite are equall the one to the other (by the 34. proposition). Wherefore the two opposite angles ABE and BED are each of them a right angle. Wherefore the parallelogramme $ABED$ is rectangle: & it is also proued that it is equilateral. Wherefore it is a square, & it is described vpon the right line geuen AB : which was required to be done.



This is to be noted that if you will mechanically and readily, not regarding demonstration describe a square vpon a line geuen, as vpon the line AB , after that you haue erected the perpendicular line CA vpon the line AB , and put the line AE equall to the line AB : then open your compasse to the wydth of the line AB or AE , & set one foote thereof in the point E , and describe a peece of the circumference of a circle: and againe make the centre the point B , and describe also a peece of the circumference of a circle, namely, in such sort that the peece of the circumference of the one may cut the peece of the circumference of the other, as in the point D : and from the point of the interfection, draw vnto the points E & B right lines: & so shalbe described a square. As in this figure here put, wherein I haue not drawn the lines ED and DB , that the peeces of the circumference cutting the one the other might the plainlier be sene.

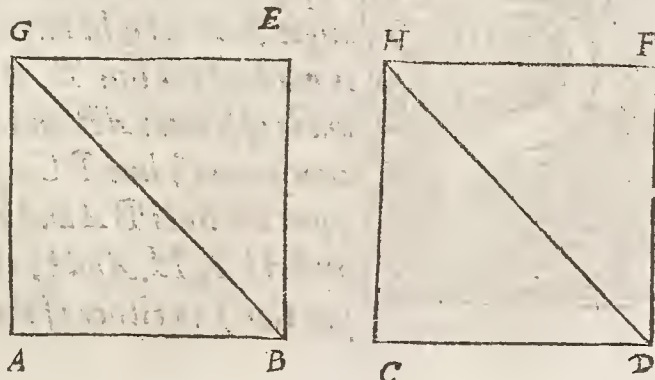
To describe a square mechanically.



An addition of Proclus.

If the lines vpon which the squares be described be equall, the squares also are equall.

Suppose that these right lines AB and CD be equall, & vpon the line AB describe a square $ABEG$: and vpon the line CD describe a square $CDHF$. Then I say that the two squares $ABEG$ & $CDHF$ are equal. For draw these right lines GB and HD . And for as much as the right lines AB and CD are equall, & the lines AG and HC are also equall, and they contayne equall



Q.j.

angles

An addition of Proclus.

The first Booke

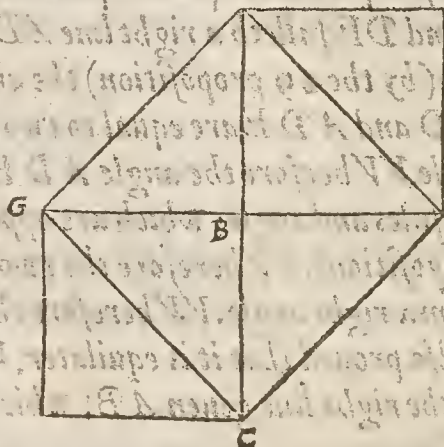
angles, namely, right angles (by the definition of a square) therefore (by the 4. proposition) the base BG is equall to the base HD . And the triangle ABG is equall to the triangle CDH . Wherefore the doubles of the saide triangles are equall. Wherefore the square AE is equall to the square CF which was required to be proued.

The conuerse thereof is thus.

The conuerse
sheweth.

If the squares be equall: the lines also vpon which they are described are equall.

Suppose that there be two equal squares AF and CG described vpon the lines AB & BC . The I say, that the lines AB and BC are equall. Put the line AB directly to the line BC , that they both make on right line. And forasmuch as the angles are right angles, therefore also (by the 14. proposition) the right line FB is set directly to the right line BG . Drawe these right lines FC , AG , AF , and CG . Now forasmuch as the square AF is equal to the square CG , the triangle also AFB shalbe equall to the triangle CBG : put the triangle BCF comon to them both. Wherefore the whole triangle ACF is equall to the whole triangle CFG . Wherefore the line AG is a parallel vnto the line CF (by the 38. proposition): for the triangles consist vpon one and the selfe same base, namely CF . Again forasmuch as either of these angles AFG & CBG is the halfe of a right angle, therefore (by the 17. proposition) the line AF is a parallel to the line CG . Wherefore the right line AF is equal to the right line CG (for the opposite sides of a parallelograme are equall). And forasmuch as there are two triangles ABF and BCG , whose alternate angles are equall, namely, the angle AFB to the angle BGC , and the angle BAF to the angle BCG , and one side of the one is equall to one side of the other, namely, the side which lieth betwene the equal angles, that is, the side AF to the side CG , therefore (by the 26. proposition) the side AB is equal to the side BC , and the side BF to the side BG . Wherefore it is proued that the squares of the lines AF and CG being equall, their sides also shalbe equall: which was required to be proued.



The 33. Theoreme. The 47. Proposition.

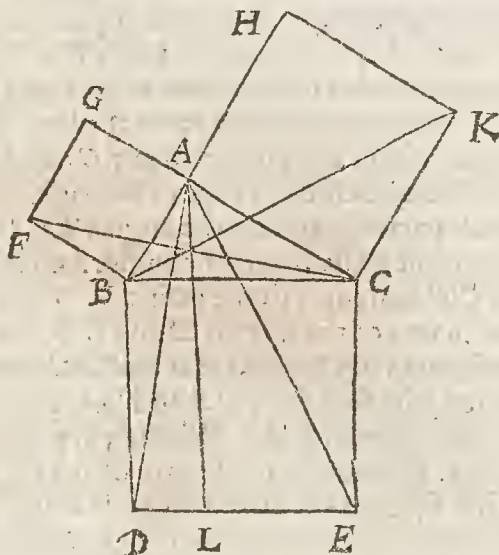
In rectangle triangles, the square whiche is made of the side that subtendeth the right angle, is equal to the squares which are made of the sides containyng the right angle.

Construction.



Suppose that ABC be a rectangle triangle, hauyng the angle BAC a right angle. The I say y^e the square which is made of the line BC is equall to the squares which are made of y^e lines AB and AC . Describe (by y^e 46. proposition) vpon y^e line BC a square $BDCE$, and (by y^e same) vpon the lines BA and AC describe the squares $ABFG$ and $ACKH$. And by the point A draw (by the 31. proposition) to either of these lynes BD and CE a parallel line

line AL . And (by the first petition) draw a right line from the point A to the point D , and an other from the point C to the point E . And forasmuch as the angles BAC and BAG are right angles, therefore vnto a right line BA , and to a point in it geuen A , are drawn two right lines AC and AG , not both on one and the same side, making the two side angles equall to two right angles: wherfore (by the 14. proposition) the lines AC and AG make directly one right line. And by the same reason the lines BA and AH make also directly one right line. And forasmuch as the angle DBC is equall to the angle FBA (for either of the is a right angle) put the angle ABC common to them both: wherfore the whole angle DBA is equall to the



Demonstration

whole angle FBC . And forasmuch as these two lines AB and BD are equal to these two lines BF and BC , the one to the other, and the angle DBA is equal to the angle FBC : therefore (by the 4. proposition) the base AD is equal to the base FC , and the triangle ABD is equal to the triangle FBC . But (by the 31. proposition) the parallelogramme BL is double to the triangle ABD , for they haue both one and the same base, namely, BD , and are in the selfe same parallel lynes, that is, BD and AL and (by the same) the square GB is double to the triangle FBC , for they haue both one and the selfe same base, that is, BF , and are in the selfe same parallel lynes, that is, FB and GC . But the doubles of thinges equall, are (by the sixte common sentence) equall the one to the other. Wherefore the parallelogramme BL is equal to the square GB . And in like sorte if (by the first petition) there be drawen a right line from the point A to the point E , and an other from the point B to the point K , we may proue y^e the parallelogramme CL is equal to the square HC . Wherefore the whole square $BDEC$ is equal to the two squares GB and HC . But the square $BDEC$ is described vpon the line BC , and the squares GB and HC are described vppon the lines BA & AC : wherefore the square of the side BC , is equal to the squares of the sides BA and AC . Wherefore in rectangle triangles, the square whiche is made of the side that subtendeth the right angle, is equal to the squares which are made of the sides contayning the right angle: which was required to be demonstrated.

This most excellent and notable Theoreme was first inuented of the greate philosopher Pithagoras, who for the exceeding ioy conceiued of the inuention therof, offered in sacrifice an Oxe, as recorde Hierone, Proclus, Lycius, & Virtruuius. And it hath bene commonly called of barbarous writers of the latter time **Dulcarnon**.

*Pythagoras the
first inuenter of
this proposition.*

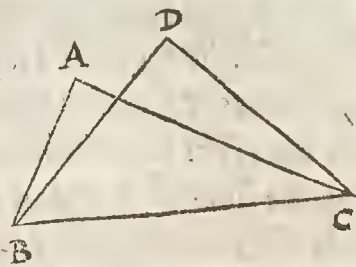
The first Booke

An addition of
Pelitarius.

An addition of Pelitarius.

To reduce two vnequall squares to two equall squares.

Suppose that the squares of the lines AB and AC be vnequall. It is required to reduce them to two equall squares. Ioyne the two lines AB and AC at their endes in such sort that they make a right angle BAC . And draw a line from B to C . Then vpon the two endes B and C make two angles eche of which may be equal to halfe a right angle (This is done by erecting vpon the line BC perpendicular lines, from the pointes B and C : and so (by the 9. proposition) deuiding eche of the right angles into two equall partes): and let the angles BCD and CBD be either of the halfe of a right angle. And let the lines BD and CD concur in the point D . Then I say that the two squares of the sides BD and CD , are equall to the two squares of the sides AB and AC . For (by the 6 proposition) the two sides DB and DC are equall, and the angle at the point D is (by the 32. proposition) a right angle. Wherefore the square of the side BC is equal to the squares of the two sides DB and DC (by the 47. proposition): but it is also equall to the squares of the two sides AB and AC (by the self same proposition) wherefore (by the common sentence) the squares of the two sides BD and DC are equall to the squares of the two sides AB and AC : which was required to be done.



An other additiō
of Pelitarius.

An other addition of Pelitarius.

If two right angled triangles haue equall bases, the squares of the two sides of the one are equall to the squares of the two sides of the other.

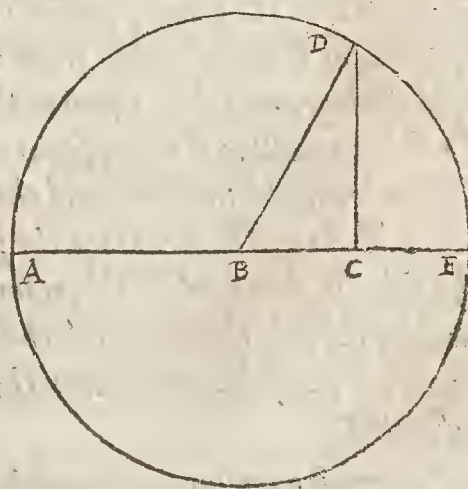
This is manifest by the former construction and demonstration.

An other additiō
of Pelitarius.

An other addition of Pelitarius.

Two vnequall lines being giuen, to know how much the square of the one is greater then the square of the other.

Suppose that there be two vnequal lines AB and BC : of which let AB be the greater. It is required to search out how much the square of AB exceedeth the square of BC . That is I wil finde out the square, which with the square of the line BC shalbe equal to the square of the line AB . Put the lines A B and BC directly, that they make both one right line: and making the centre the point B , and the space BA describe a circle ADE . And produce the line AC to the circumference, and let it concur with it in the point E . And vpon the lyne AE and frō the point C erect (by the 11. proposition) a perpendicular line CD , which produce till it concur with the circumference in the point D : & draw a line from B to D . Then I say, that the square of the line CD , is the excesse of the square of the line AB aboue the square of the line BC . For forasmuch as in the triangle BCD , the angle at the point C is a right angle, the square of the base BD is equall to the squares of the two sides BC and CD (by this 47. proposition). Wherefore also the square of the line AB is equall to the selfe same squares of the lines BC and CD . Wherefore the square of the line BC is so much lesse then the square of the line AB , as is the square of the line CD : which was required to search out.



An

An other addition of Pelitarius.

The diameter of a square being given, to gene the square thereof.

Another additiō
of Pelitarius.

This is easie to be done. For if vpon the two endes of the line be drawen two halfe right angles, and so be made perfect the triangle then shalbe described half of the square; the other halfe whereof also is after the same manner easie to be described.

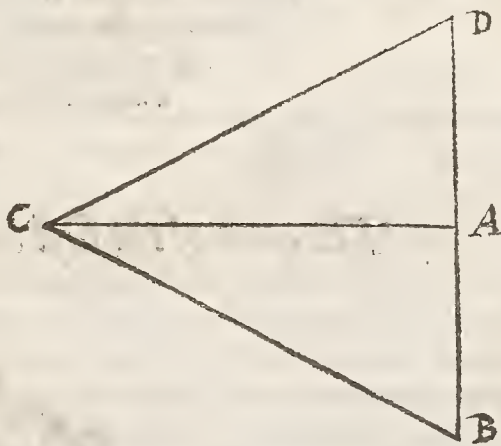
Hereby it is manifest, that the square of the diameter is double to that square whose diameter it is. *A Corollary.*

The 34. Theoreme. The 48. Proposition.

If the square which is made of one of the sides of a triangle, be equall to the squares which are made of the two other sides of the same triangle: the angle comprehended vnder those two other sides is a right angle.



Suppose that ABC be a triangle, and let the square which is made of one of the sides there, namely, of the side BC , be equall to the squares which are made of the sides BA and AC . Then I say that the angle BAC is a right angle. Rayse vp (by the 11. proposition) from the point A vnto the right line AC a perpendicular line AD . And (by the thirde proposition) vnto the line AB put an equall line AD . And by the first petition draw a right line from the point D to the point C . And forasmuch as the line DA is equall to the line AB , the square which is made of the line DA is equall to the square which is made of the line AB . Put the square of the line AC , common to them both. Wherefore the squares of the lines DA and AC are equal to the squares of the lines BA and AC . But (by the proposition going before) the square of the line DC is equal to the squares of the lines AD and AC . (For the angle DAC is a right angle) and the square of BC is (by supposition) equall to the squares of AB and AC . Wherefore the square of DC is equall to the square of BC ; wherefore the side DC is equall to the side BC . And forasmuch as AB is equall to AD and AC is common to them both, therefore these two sides DA and AC are equall to these two sides BA and AC , the one to the other, and the base DC is equall to the base BC ; wherefore (by the 8. proposition) the angle DAC is equall to the angle BAC . But the angle DAC is a right angle, wherefore also the angle BAC is a right angle. If



Q. ij.

there.

The first Booke

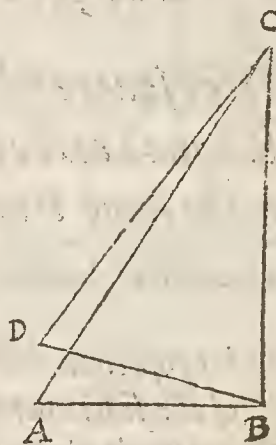
therefore the square which is made of one of the sides of a triangle, be equall to the squares which are made of the two other sides of the same triangle, the angle comprehended vnder those two other sides is a right angle: which was required to be proued.

This proposition is the conuerse of the former.

This proposition is the conuerse of the former, and is of Pelitarius demonstrated by an argument leading to an impossibilitie after this maner.

An other Demonstration after Pelitarius.

Suppose that ABC be a triangle: & let the square of the side AC , be equal to the squares of the two sides AB and BC . Then I say that the angle at the point B , which is opposite to the side AC , is a right angle. For if the angle at the point B be not a right angle, then shal it be eyther greater or lesse the a right angle. First let it be is greater. And let the angle DBC be a right angle, by erecting from the point B a perpendicular line vnto the line BC (by the 11. proposition) which let be BD : and put the line BD equall to the lyne AB (by the thirde proposition): And drawe a line from C to D . Now (by the former proposition) the square of the side CD shalbe equall to the squares of the two sides BD and BC : wherefore also to the squares of the two sides AB and BC . Wherefore the base CD shalbe equall to the base CA , when as their squares are equall: which is contrary to the 24. proposition. For forasmuch as the angle ABC is greater then the angle DBC , and the two sides AB and BC are equall to the two sides DB and BC , the one to the other, the base CA shall be greater then the base CD . It is also contrary to the 7. proposition, for from the two endes of one & the same line, namely, frō the points B & C should be drawn on one and the same side two lines BD and DC ending at the pointe D , equall to two other lines BA and AC drawn from the same endes and ending at an other point, namely, at A , which is impossible. By the same reason also may we proue that the whole angle at the point B is not lesse then a right angle. Wherefore it is a right angle: which was required to be proued.



(∴)

The ende of the first booke of Euclides Elementes.



clides Elementes.



IN this second booke Euclide sheweth, what is a Gnomon, and a right angled parallelogramme. Also in this booke are set forth the powers of lines, deuided euenly and vneuenly, and of lines added one to an other. The power of a line, is the square of the same line: that is, a square, euery side of which is equall to the line. So that here are set forth the qualities and proprieties of the squares and right lined figures, which are made of lines & of their parts. The Arithmetician also out of this booke gathereth many compendious rules of reckoning, and many rules also of Algebra, with the equatiōs therein vsed. The groundes also of those rules are for the most part by this second booke demonstrated. This booke moreouer contayneth two wonderfull propositions, one of an obtuse angled triangle, and the other of an acute: which with the ayde of the 47. proposition of the first booke of Euclide, which is of a rectangle triangle, of how great force and profite they are in matters of astronomy, they knowe which haue trauayled in that arte. VVherefore if this booke had none other profite beside, onely for these 2. propositions sake it were diligently to be embraced and studied.

The argument of the second booke.

What is the power of a line.

Many compendious rules of reckoning gathered out of this booke, and also many rules of Algebra.

Two wonderfull propositions in this booke.

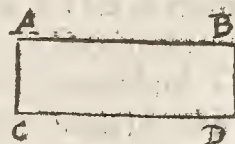
The definitions.

1. *Euery rectangled parallelogramme, is sayde to be containned vnder two right lines comprehending a right angle.*

First definition.

A parallelogramme is a figure of fower sides, whose two opposite or contrary sides are equall the one to the other. There are of parallelogrammes fower kyndes, a square, a figure of one side longer, a Rombus or diamond, and a Romboides or diamondlike figure, as before was sayde in the 33. definition of the first booke. Of these fower sortes, the square and the figure of one side longer are onely right angled Parallelogrammes: for that all their angles are right angles. And either of them is containned (according to this definition) vnder two right lyties which concur together, and cause the right angle, and containe the same. Of which two lines the one is the length of the figure, & the other the breadth. The parallelogramme is imagined to be made by the draught or motion of one of the lines into the length of the other. As if two numbers shoulde be multiplied the one into the other. As the figure A B C D is a parallelograme, and is sayde to be containned vnder the two right lines A B and A C, which containne the right angle B A C, or vnder the two right lines A C and C D, for they likewise containne the right angle A C D: of which 2. lines the one, namely, A B is the length, and the other, namely, A C is the breadth. And if we imagine the line A C to be drawen or moued directly according to the length

What a parallelogramme is. Fower kyndes of parallelogrammes.



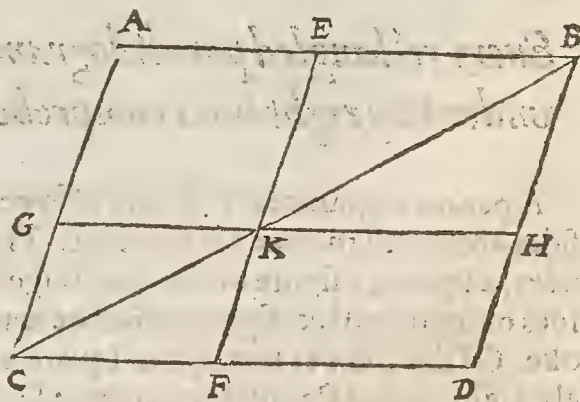
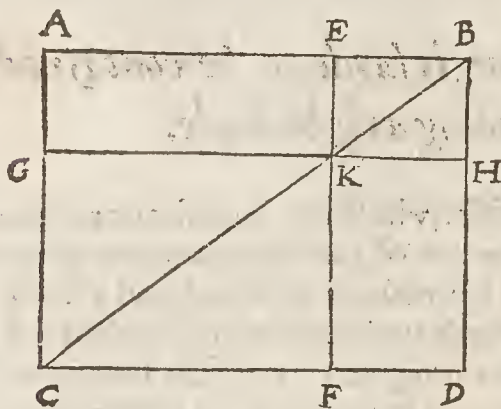
The second Booke

of the line A B, or contrary wise the line A B to be moued directly according to the length of the line A C, you shall produce the whole rectangle parallelogramme A B C D which is sayde to be contayned of them: euen as one number multiplied by an other produceth a plaine and righte angled superficiall number, as ye see in the figure here set, where the number of fixe or fixe vnities, is multiplied by the number of fixe or by fixe vnities: of which multiplication are produced 30, which number being set downe and described by his vnities representeth a playne and a right angled number. VVherefore euen as equall numbers multiplied by equal numbers produce numbers equall the one to the other: so rectangle parallelogrammes which are comprehended vnder equal lines are equal the one to the other.

Second definition.

2. *In euery parallelogramme, one of those parallelogrammes, which soeuer it be, which are about the diameter, together with the two supplementes, is called a Gnomon.*

Those perticuler parallelogrammes are sayde to be about the diameter of the parallelogramme, which haue the same diameter which the whole parallelogramme hath. And supplementes are such, which are without the diameter of the whole parallelogramme. As of the parallelogramme A B C D the partial or perticuler parallelogrammes G K C F and E B K H are parallelogrammes about the diameter, for that ech of them hath for his diameter a part of the diameter of the whole paral-



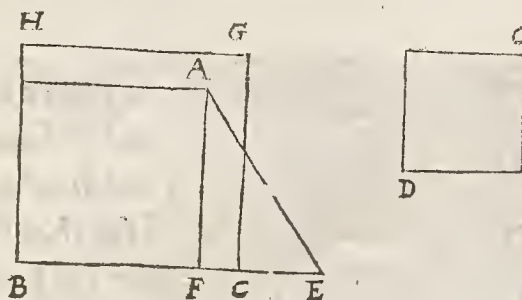
lelogramme. As C K and K B the perticuler diameters, are partes of the line C B, which is the diameter of the whole parallelogramme. And the two parallelogrammes A E G K and K H F D, are supplementes, because they are wythout the diameter of the whole parallelogramme, namely, C B. Now any one of those partiall parallelogrammes about the diameter together with the two supplementes make a gnomon. As the parallelogramme E B K H, with the two supplementes A E G K and K H F D make the gnomon F G E H. Likewise the parallelogramme G K C F with the same two supplementes make the gnomon E H F G. And this diffinition of a gnomon extendeth it selfe, and is generall to all kyndes of parallelogrammes, whether they be squares or figures of one side longer or Rhombus or Romboides. To be shorte, if you take away from the whole parallelogramme one of the partiall parallelogrammes which are about the diameter

ameter whether ye will, the rest of the figure is a gnomon.

Campane after the last proposition of the first booke addeth this proposition. Two squares being given, to adioyne to one of them a Gnomon equall to the other square: which, for that as then it was not taught what a Gnomon is, I there omitted, thinking that it might more aptly be placed here. The doing and demonstration whereof, is thus.

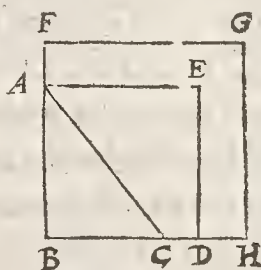
A proposition added by Campane after the last proposition of the first booke.

Suppose that there be two squares AB and CD : vnto one of which, namely, vnto AB , it is required to adde a Gnomon equall to the other square, namely, to CD . Produce the side BF of the square AB directly to the point E . and put the line FE equall to the side of the square CD . And draw a line from E to A . Now then forasmuch as EFA is a rectangle triangle, therefore (by the 47. of the first) the square of the line EA is equall to the squares of the lines EF & FA . But the square of the line EF is equall to the square CD , & the square of the side FA is the square AB . Wherefore the square of the line EA is equall to the two squares CD and AB . But the sides EF and FA are (by the 21. of the first) longer then the side AE , and the side FA is equall to the side FB . Wherefore the sides EF and FB are longer thē the side AE . Wherefore the whole line BE is longer then the line AE . From the line BE cut of a line equall to the line AE , which let be BC . And (by the 46. proposition) vpon the line BC describe a square, which let be $BCGH$; which shalbe equal to the square of the line AE , but the square of the line AE is equal to the two squares AB and DC . Wherefore the square $BCGH$ is equal to the same squares. Wherefore forasmuch as the square $BCGH$ is composed of the square AB and of the gnomon $FGAH$, the sayde gnomon shalbe equall vnto the square CD : which was required to be done.



Another more redy way after Pelitarius.

Suppose that there be two squares, whose sides let be AB and BC . It is required vnto the square of the line AB , to adde a gnomon equall to the square of the line BC . Set the lines AB and BC in such sort that they make a right angle ABC . And draw a line frō A to C . And vpon the line AB describe a square which let be $ABDE$. And produce the line BA to the point F , and put the line BF equall to the line AC . And vpon the line BF describe a square which let be $BFGH$: which shalbe equal to the square of the line AC , whē as the lines BF and AC are equal; and therefore it is equal to the squares of the two lines AB and BC . Now forasmuch as the square $BFGH$ is made complete by the square $ABDE$ and by the gnomon $FECD$, the gnomon $FECD$ shalbe equal to the square of the line BC : which was required to be done.



The second Booke

The 1. Theoreme. The 1. Proposition.

If there be two right lines, and if the one of them be deuided into partes howe many soeuer : the rectangle figure comprehended vnder the two right lines, is equall to the rectangle figures whiche are comprehended vnder the line vndeuided, and vnder euery one of the partes of the other line.



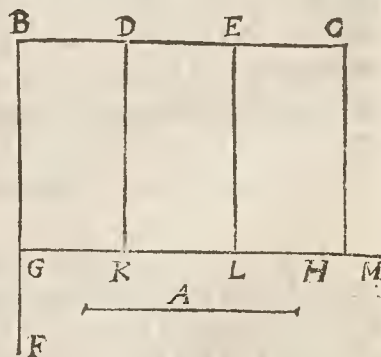
Suppose that there be two right lynes A and BC and let one of them, namely, BC be deuided at all aduentures in the pointes D and E . Then I say that the rectangle figure comprehended vnder the lines A , and BC , is equall vnto the rectangle figure comprehended vnder the lines A , and BD , & vnto the rectangle figure which is cōprehended vnder the lines A and DE , and also vnto the rectangle figure which is comprehended vnder the

Construction.

lines A and EC . For from the pointe B rayse vp (by the 11. of the first) vnto the right line BC a perpendiculer line BF , & vnto the line A (by the third of the first) put the line BG equall, and by the point G (by the 31. of the first) draw a parallel line vnto the right line BC and let the same be GM , and (by the selfe same) by $\dot{y}$ points D, E , and C , draw vnto the line BG these parallel

Demonstratiō

lines DK, EL and CH . Now then the parallelograme BH is equall to these parallelogrammes BK, DL , and EH . But the parallelograme BH is equall vnto that which is contayned vnder the lines A and BC . (For it is comprehended vnder the lines GB & BC , and the line GB is equall vnto the line A) And the parallelograme BK is equall to that which is contayned vnder the lines A and BD : (for it is comprehended vnder the line GB and BD , and GB is equall vnto A) And the parallelograme DL is equall to that which is contayned vnder the lines A and DE (for the line DK , that is, GB is equal vnto A) And moreouer likewise the parallelograme EH is equall to that which is contained vnder the lines A & EC . VVherfore that which is comprehended vnder $\dot{y}$ lines A & BC is equall to that which is comprehended vnder the lines A & BD , & vnto $\dot{y}$ which is comprehended vnder the lines A and DE , and moreouer vnto that which is comprehended vnder the lines A and EC . If therfore there be two right lines, and if the one of them be deuided into partes how many soeuer, the rectan-



gle figure comprehended vnder the two right lines, is equall to the rectangle figures which are comprehended vnder the line vndeuided and vnder euery one of the partes of the other line: which was required to be demonstrated.

Because that all the Propositions of this second booke for the most part are true both in lines and in numbers, and may be declared by both: therefore haue I haue added to euery Proposition conuenient numbers for the manifestation of the same. And to the end the studious and diligent reader may the more fully perceauē and vnderstand the agreement of this art of Geometry with the science of Arithmetique, and how nere & deare sisters they are together, so that the one cannot without great blemish be without the other, I haue here also ioyned a little booke of Arithmetique written by one *Barlaam*, a Greeke authour a man of greate knowledge. In whiche booke are by the authour demonstrated many of the selfe same proprieties and passions in number, which *Euclide* in this his second booke hath demonstrated in magnitude, namely the first ten propositions as they follow in order. V Which is vndoubtedly great pleasure to consider, also great increase & furniture of knowledge. V Whose Propositions are set orderly after the propositions of *Euclide*, euery one of *Barlaam* correspondēt to the same of *Euclide*. And doubtles it is wonderful to see how these two contrary kynds of quantity, quantity discrete or number, and quantity continual or magnitude (which are the subiectes or matters of Arithmetique and Geometry) shoulde haue in them one and the same proprieties common to them both in very many points, and affections, although not in all. For a line may in such sort be deuided, that what proportion the whole hath to the greater parte the same shall the greater part haue to the lesse. But that can not be in number. For a number cannot so be deuided, that the whole number to the greater part thereof, shall haue that proportion which the greater part hath to the lesse, as *Iordane* very playnely proueth in his booke of Arithmetike, which thyng *Campane* also (as we shall after ward in the 9. booke after the 15. proposition see) proueth. And as touching these tenne firste propositions of the seconde booke of *Euclide*, demonstrated by *Barlaam* in numbers, they are also demonstrated of *Campane* after the 15. proposition of the 9. booke, whose demonstrations I mynde by Gods helpe to set forth when I shal come to the place. They are also demonstrated of *Iordane* that excellent learned authour in the first booke of his Arithmetike. In the meane tyme I thought it not amisse here to set forth the demonstrations of *Barlaam*, for that they geue great light to the seconde booke of *Euclide*, besides the inestimable pleasure, which they bring to the studious considerer. And now to declare the first Proposition by numbers, I haue put this example following.

Take two numbers the one vndeuided as 74. the other deuided into what partes and how many you list, as 37. deuided into 20. 10. 5. and 2: which altogether make the whole 37. Then if you multiply the number vndeuided, namely, 74. into all the partes of the number deuided as into 20. 10. 5. and 2. you shall produce 1480. 740. 370. 148. which added together make 2738: which self number is also produced if you multiplie the two numbers first geuen the one into the other. As you see in the example on the other side set.

The second Booke

Multiplication of the whole
nũber vndeuided into the
partes of the whole num-
ber deuided.

Multiplication of the one
whole number into the o-
ther.

$$\begin{array}{r} 74 \\ 1480 \\ 740 \\ 370 \\ 148 \\ \hline 2738 \end{array}$$

$$2738$$

$$\begin{array}{r} 74 \\ 37 \\ \hline 518 \\ 2222 \\ \hline 2738 \end{array}$$

$$2738$$

$$\begin{array}{r} 37 \\ 20 \\ 10 \\ 5 \\ 2 \end{array}$$

the number produced of the one
whole number into the partes of
the other whole number

equall to

the number produced of the
same whole into the other whole

So by the aide of this Proposition is gotten a compendious way of multiplication by breaking of one of the numbers into his partes: which oftentimes serueth to great vse in working, chiefly in the rule of proportions. The demonstration of which proposition followeth in Barlaam. But first are put of the author these principles following.

Barlaam.

7 Principles.

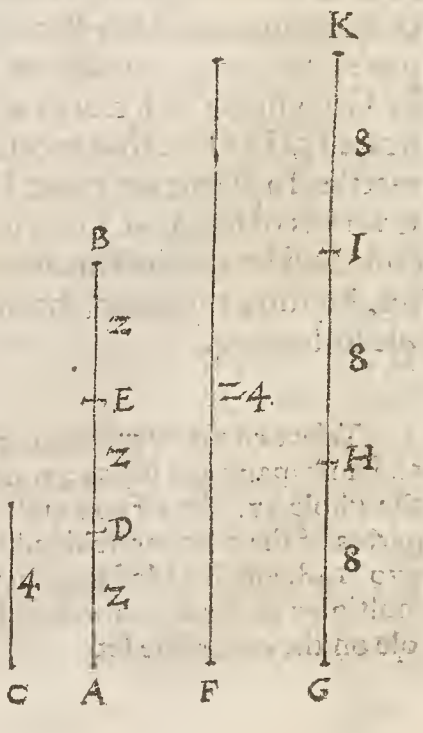
1. A number is sayd to multiply an other number: When the number multiplid is so oftentimes added to it selfe, as there be vnities in the number, which multiplieth: wherby is produced a certaine number which the number multiplied measureth by the vnities which are in the number which multiplieth.
2. And the number produced of that a multiplication is called a plaine or superficiall number.
3. A square number is that which is produced of the multiplication of any number into it selfe.
4. Eury lesse number compared to a greater is sayd to be a part of the greater, whether the lesse measure the greater, or measure it not.
5. Numbers, whome one and the selfe same number measureth equally, that is, by one and the selfe same number are equall the one to the other.
6. Numbers that are equemultiples to one and the selfe same number, that is, which contayne one and the same number equally and alike, are equall the one to the other.

The first Proposition.

Barlaam.

Two numbers beyng geuen, if the one of them be deuided into any numbers how many soeuer: the playne or superficiall number which is produced of the multiplication of the two numbers first geuen the one into the other, shall be equall to the superficiall numbers which are produced of the multiplication of the number not deuided into eury part of the number deuided.

Suppose that there be two numbers AB and C . And deuide the number AB into certayne other numbers how many soeuer, as into AD , DE , and EB . Then I say that the superficiall number which is produced of the multiplication of the number C into the number AB is equall to the superficiall numbers which are produced of the multiplication of the number C into the number AD , and of C into DE , and of C into EB . For let F be the superficiall number produced of the multiplication of the number C into the number AB , and let GH be the superficiall number produced of the multiplication of C into AD : And let HI be produced of the multiplication of C into DE : and finally of the multiplication of C into EB let there be produced the number



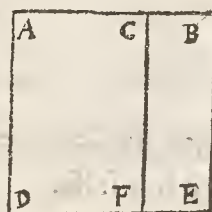
ber IK . Now foras much as AB multiplying the number C produced the number F : therefore the number C measureth the number F by the vnities which are in the number AB . And by the same reason may be proued that the number C doth also measure the number GH , by the vnities which are in the number AD , and that it doth measure the number HI by the vnities which are in the number DF and finally that it measureth the number IK by the vnities which are in the number EB . Wherefore the number C measureth the whole number GK by the vnities which are in the number AB . But it before measured the number F by the vnities which are in the number AB , wherefore either of these numbers F and GK is equemultiplex to the number C . But numbers which are equemultiples to one & the selfe same numbers are equall the one to the other (by the 6. definition). Wherefore the number F is equall to the number GK . But the number F is the superficial number produced of the multiplication of the number C into the number AB : and the number GK is composed of the superficial numbers produced of the multiplication of the number C not deuided into euery one of the numbers AD , DE , and EB . If therefore there be two numbers geuen and the one of them be deuided &c. Which was required to be proued.

The 2. Theoreme. The 7. Proposition.

If a right line be deuided by chaunce, the rectangles figures which are comprehended vnder the whole and euery one of the partes, are equall to the square whiche is made of the whole.



Suppose that the right line AB be by chaunce deuided in the point C . Then I say that the rectangle figure comprehended vnder AB and BC together with the rectangle comprehended vnder AB and AC is equall vnto the square made of AB . Describe (by the 46. of the first) vpon AB a square $ADEB$: and (by the 31. of the first) by the point C draw a line parallel vnto either of these lines AD and BE , and let the same be CF . Now is the



Construction.

parallelogramme AE equall to the parallelogrammes AF and CE , by the first of this booke. But AE is the square made of AB . And AF is the rectangle parallelogramme comprehended vnder the lines BA and AC : for it is comprehended vnder the lines DA and AC : but the line AD is equall vnto the line AB . And likewise the parallelogramme CE is equall to that which is containd vnder the lines AB and BC : for the line BE is equal vnto the line AB . Wherefore that which is containd vnder BA and AC together with that which is containd vnder the lines AB and BC , is equall to the square made of the line AB . If therefore a right line be deuided by chaunce, the rectangle figures which are comprehended vnder the whole, and euery one of the partes, are equall to the square which is made of the whole: which was required to be demonstrated.

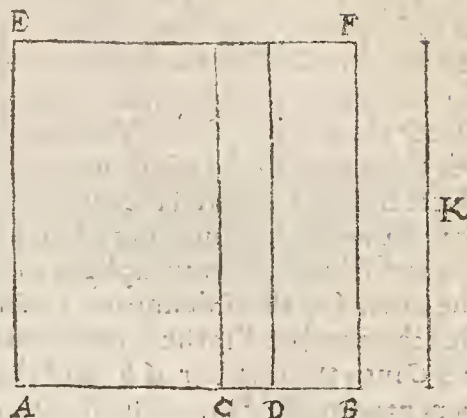
An other demonstration of Campane.

R iii.

Sup-

The second Booke

Suppose that the line AB be deuided into the lines AC , CD , and DB . Then I say that the square of the whole line AB , which let be AE BF , is equal to the rectangle figures which are containd vnder the whole and euery one of the partes : for take the line K , which let be equal to the line AB . Now then by the first proposition the rectangle figure contained vnder the lines AB and K , is equal to the rectangle figures containd vnder the line K and al the partes of the line AB . But that which is containd vnder the lines K and AB is equal to the square of the line AB , and the rectangle figures containd vnder the line K and al the partes of AB , are equal to the rectangle figures containd vnder the line AB and all the partes of the line AB : for the lines AB and K are equal: wherefore that is manifest which was required to be proued.



An example of this Proposition in numbers.

Take a number, as 11. and deuide it into two partes, namely, 7. and 4. and multiply 11. into 7. and then into 4. and there shalbe produced 77. and 44. both which numbers added together make 121. which is equal to the square number produced of the multiplication of the number 11. into himselfe, as you see in the example.

{ Multiplication of the whole into his partes. Multiplication of the whole into himselfe.	$\begin{array}{r} 11 \\ 77 \\ 44 \\ \hline 121 \end{array}$	$\left\{ \begin{array}{r} 11 \\ 7 \\ 4 \end{array} \right\}$	{ the number produced of the whole into his partes. equal to the number produced of the whole into himselfe.
	$\begin{array}{r} 11 \\ 11 \\ \hline 11 \\ 11 \\ \hline 121 \end{array}$		

The demonstration whereof followeth in Barlaam.

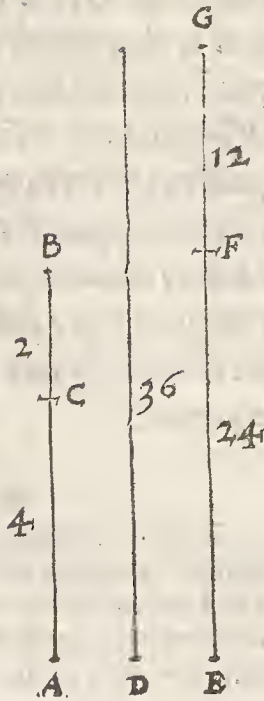
The second Proposition.

Barlaam.

If a number geuen be deuided into two other numbers: the superficial numbers, which are produced of the multiplication of the whole into either part, added together, are equal to the square number of the whole number geuen.

Suppose that the number geuen be AB : and let it be deuided into two other numbers AC and CB . Then I say that the two superficial numbers, which are produced of the multiplication of AB into AC , and of AB into BC , those two superficial numbers (I say) beyng added together, shalbe equal to the square number produced of the multiplication of the number AB into it selfe. For let the number AB multiplying it selfe produce the number D . Let the number AC also multiplying the number AB produce

produce the number EF : againe let the number CB multiplying the selfe same number AB produce the number FG . Now forasmuch as the number AC multiplying the number AB produced the number EF : therefore the number AB measureth the number EF by the vnities which are in AC . Againe forasmuch as the number CB multiplied the number AB , and produced the number FG : therefore the number AB measureth the number FG by the vnities which are in the number CB . But the same number AB before measured the number EF by the vnities which are in the number AC . Wherefore the number AB measureth the whole number EG by the vnities which are in AB . Farther forasmuch as the number AB multiplying it selfe produced the number D : therefore the number AB measureth the number D by the vnities which are in himselfe. Wherefore it measureth either of these numbers, namely, the number D , and the number EG , by the vnities which are in himselfe. Wherefore how multiplex the number D is to the number AB , so multiplex is the number EG to the same number AB . But numbers which are equimultiples to one and the selfe same number, are equal the one to the other. Wherefore the number D is equall to the number EG . And the number D is the square number made of the number AB , and the number EG is composed of the two superficial numbers produced of AB into BC , and of BA into AC . Wherefore the square number produced of the number AB is equall to the superficial numbers, produced of the number AB into the number BC , and of AB into AC , added together. If therefore a number be deuided into two other numbers &c, which was required to be proued.

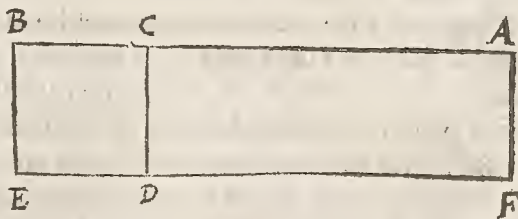


The 3. Theoreme. The 3. Proposition.

If a right line be deuided by chaunce: the rectangle figure comprehended vnder the whole and one of the partes, is equall to the rectangle figure comprehended vnder the partes, & vnto the square which is made of the foresaid part.



*V*ppose that the right line geuen AB be deuided by chaunce in the point C . Then I say that the rectangle figure comprehended vnder the lines AB and BC is equall vnto the rectangle figure comprehended vnder the lines AC and CB , and also vnto the square which is made of the line BC . Describe (by the 46. of the first) vpon the line BC a square CD *Construction.* EB : and (by the second petition) extend ED vnto F . And by the point A draw (by the 31. of the first) a line parallel vnto either of these lines CD and BE , and let the same be AF . Now the parallelograme AE is equall vnto the parallelogrammes AD and CE . And AE is the rectangle figure comprehended vnder the lines AB and BC . For it is comprehended vnder the lines AB and BE ,



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which line BE is equall vnto the line BC . And the parallelograme AD is equall to that which is contayned vnder the lines AC and CB : for the line DC is equall vnto the line CB . And DB is the square which is made of the lyne CB . VVherfore the rectangle figure comprehended vnder the lynes AB and BC is equall to the rectangle figure comprehended vnder the lines AC and CB & also vnto the square which is made of the line BC . If therfore a right line be deu-
 uided by chaunce, the rectangle figure comprehended vnder the whole and one
 of the partes, is equall to the rectangle figure comprehended vnder the partes,
 and vnto the square which is made of the foresayd part: which was required to
 be proued.

An example of this Proposition in numbers.

Suppose a number, namely, 14. to be deuided into two partes 8. and 6. The whole
 number 14. multiplied into 8. one of his partes, produceth 112: the partes 8. & 6. mul-
 tiplied the one into the other produce 48, which added to 64 (which is the square of 8.
 the former part of the number) amounteth also to 112: whiche is equall to the former
 summe. As you see in the example.

Multiplication of the whole into one of his partes.	14 8	{ 8 6 } the partes.	
	112		the number produced of the whole into one of his partes.
Multiplication of the one part into the other.	8 6		
	48	48	equal to
		64	
Multiplication of the for- mer part into it selfe.	8 8	112	the number composed of the one part into the other, and of the former part into him- selfe.
	64		

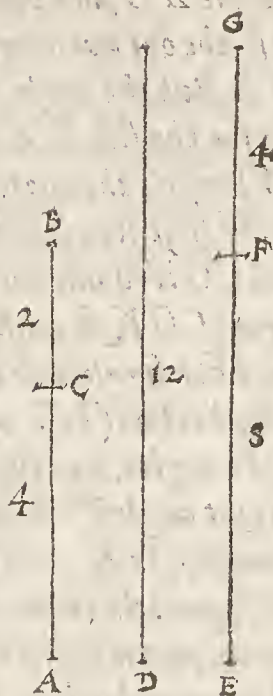
The demonstration hereof followeth in Barlaam.

The third proposition.

Barlaam. If a number geuen be deuided into two numbers: the superficial number which is produced of
 the multiplication of the whole into one of the partes, is equall to the superficial number which is pro-
 duced of the partes the one into the other, and to the square number produced of the foresayd part.

Suppose that the number geuen be AB , which let be deuided into two numbers A
 C and CB . Then I say that the superficial number whiche is produced of the multipli-
 cation of the number AB into the number BC , is equall to the superficial number
 which is produced of the multiplication of the number AC into the number CB , and
 to the square number produced of the number CB . For let the number AB multipli-
 eng the number CB produce the number D . And let the number AC multiplieng the
 number CB produce the number EF ; and finally let the number CB multiplieng him-
 selfe produce the number FG . Now forasmuch as the number AB multiplieng the
 number

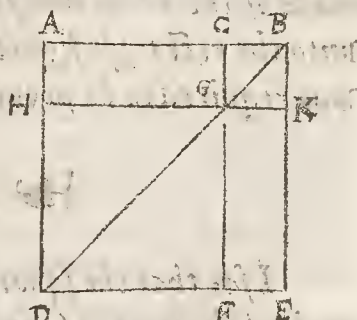
number CB produced the number D . Therefore the number C B measureth the number D by the vnities whiche are in the number AB . Agayne forasmuch as the number AC multiplied the number CB , and produced the number EF , therefore the number CB measureth the nūber EF by the vnities which are in AC . Agayne forasmuch as the number CB multiplied it selfe and produced the number FG : therefore the number CB measureth the number FG by the vnities which are in it selfe. But as we haue before proued the selfe same nūber CB measureth also the number EF by the vnities which are in the nūber AC , wherfore the number CB measureth the whole number EG by the vnities which are in the number AB . And it also measureth the number D by the vnities whiche are in the number AB . Wherfore the number CB equally measureth either number, namely, the number D , and the number EG . But those numbers whome one and the selfe same number measureth equally, are equall the one to the other. Wherfore the number D is equall to the number EG . But the number D is a superficiall number produced of the multiplication of the number AB into the number BC , and the number EG is the superficial number produced of the multiplication of the nūber AC into the number CB , and of the square of the number CB . Wherfore the superficial number produced of the multiplication of the number AB into the number CB is equal to the superficial number produced of the number AC into the number CB , and to the square of the number CB . If therfore a number be deuided into two numbers, the superficial nūber &c: which was required to be proued.



The 4. Theoreme. The 4. Proposition.

If a right line be deuided by chaunce, the square whiche is made of the whole line is equal to the squares which are made of the partes, & vnto that rectangle figure which is comprehended vnder the partes twise.

Suppose that the right lyne AB be by chaunce deuided in the pointe C . Then I say that the square made of the line AB is equall vnto y squares which are made of the lines AC and CB , and vnto the rectangle figure contained vnder the lines AC and CB twise. Describe (by y 46. of the first) vpon the line AB a square $ADEB$: and draw a line from B to D , and (by the 31. of the first) by the point C draw a line parallel vnto either of these lines AD and BE cutting the diameter BD in the point G , and let the same be CF . And (by the point G (by the selfe same) draw a line parallel vnto eyther of these lines AB and DE , and let the same be HK . And forasmuch as the line CF is a parallel vnto



Construction.

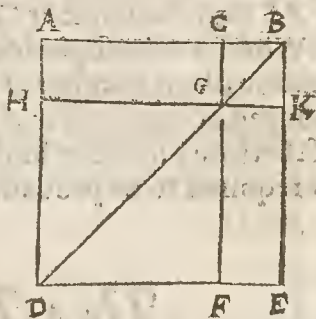
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the line AD , and vpon them falleth a right line BD : therefore (by the 29. of the first) the outward angle CGB is equall vnto the inward and opposite angle ADB . But the angle ADB is (by the 5. of the first) equall vnto the angle ABD : for the side BA is equall vnto the side AD (by the definition of a square). VVherfore the angle CGB is equall vnto the angle GBD : wherfore (by the 6. of the first) the side BC is equall vnto the side CG . But CB is equall vnto GK , and CG is equall vnto KB : wherfore GK is equall vnto KB . VVherfore the figure $CGKB$ consisteth of foure equall sides. I say also that it is a rectangle figure. For forasmuch as CG is a parallel vnto BK & vpon the falleth a right line CB , therefore (by $\S$ 9. of the 1.) the angles KBG , and GCB are equal vnto two right angles. But the angle KBG is a right angle, wherfore $\S$ angle BCG is also a right angle. VVherfore (by the 34. of the first) the angles opposite vnto them, namely, CGK , and GKB are right angles. VVherfore $CGKB$ is a rectangle figure. And it was before proued that the sides are equall. VVherfore it is a square, and it is described vpon the line BC . And by the same reason also HGF is a square, and is described vpon the line HG , that is, vpon the line AC . VVherfore the squares HGF and CKG are made of the lines AC and CB . And forasmuch as the parallelogramme AG is (by the 43. of the first) equall vnto the parallelogramme GE . And AG is that which is containd vnder AC and CB , for CG is equal vnto CB , wherfore GE is equall to that which is contained vnder AC and CB . VVherfore AG and GE are equall vnto that which is comprehended vnder AC and CB twice. And the squares HGF and CKG are made of the lines AC and CB . VVherfore these foure rectangle figures HGF , CKG , AG , and GE are equall vnto the squares whiche are made of the lines AC and CB , and to the rectangle figure which is comprehended vnder the lines AC and CB twice. But the rectangle figures HGF , CKG , AG , and GE are the whole rectangle figure $ADEB$ which is the square made of the line AB . VVherfore the square which is made of the line AB is equall to the squares which are made of the lines AC and CB , and vnto the rectangle figure which is comprehended vnder the lines AC and CB twice. If therefore a right line be deuided by chaunce, the square whiche is made of the whole line, is equall to the squares whiche are made of the partes, & vnto the rectangle figure which is comprehended vnder the partes twice: which was required to be proued.

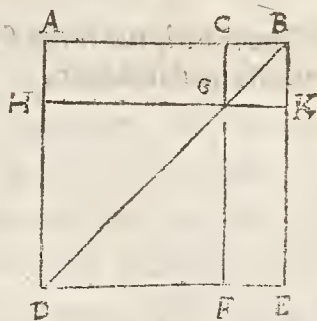


In other demonstration.

I say that the square of the line AB is equall vnto the squares whiche are made of the lines AC and CB , & vnto the rectangle figure which is comprehended vnder the lines AC and CB twice. For the selfe same description abiding forasmuch

asmuch as the line AB is equall vnto y line AD , y angle ABD is (by the 5. of the first) equall vnto the angle ADB . And forasmuch as the three angles of euery triangle are equal to two right angles (by the 32. of the first). therefore y three angles of the triangle ABD , namely, the angles ADB , DBA , and BAD , are equall to two right angles. But the angle BAD is a right angle, wherefore the angles remayning ABD , and ADB , are equall vnto one right angle: and they are equall y one to the other, wherefore either of these angles ABD , & ADB , is the halfe of a right angle.

And the angle BCG is a right angle, for it is equall vnto the opposite angle at the point A (by the 29. of the first). Wherefore the angle remayning CGB is the halfe of a right angle. Wherefore the angle CGB is equal vnto the angle CBG : wherefore also the side BC is equall vnto the side CG . But BC is equall vnto GK , and CG is equal vnto BK . Wherefore the figure CK consisteth of equall sides: and in it is a right angle CBK . Wherefore CK is a square, and is made of the line BC . And by the same reason HF is a square, and is equall vnto that square which is made of the line AC . Wherefore CK and HF are squares, and are equall to those squares which are made of the lines AC and CB . And forasmuch as AG is equall vnto EG : and AG is that which is containd vnder AC and CB , for GC is equal vnto CB : wherefore EG also is equall to that which is comprehended vnder AC and CB : wherefore AG and EG are equall vnto that rectangle figure which is comprehended vnder AC , and CB twice. And CK , and HF are equal vnto the squares which are made of AC , and CB : wherefore CK , HF , AG , and GE are equal vnto those squares which are made of AC , and CB , and vnto that rectangle figure which is comprehended vnder AC and CB twice. But CK , HF , AG , and GE are the whole square AE which is made of AB . Wherefore the square which is made of AB , is equall to the squares which are made of AC and CB , and vnto the rectangle figure which is comprehended vnder AC and CB twice: which was required to be demonstrated.



Hereby it is manifest that the parallelogrames which consist about the diameter of a square must needes be squares. A Corollary.

This proposition is of infinite vse chiefly in surde numbers. By helpe of it is made in the additiō & substractiō, also multiplicatiō in Binomials & residu- als. And by helpe hereof also is demonstrated that kinde of equation, which is, when there are three denominations in naturall order, or equally distant, and two of the greater denominations are equall to the thirde being lesse. On this proposition is grounded the extraction of square roots. And many other things are also by it demonstrated.

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An example of this Proposition in numbers.

Suppose a number namely, 17. to be devided into two partes 9. and 8. The whole number 17. multiplied into himselfe, produceth 289. The square numbers of 9. and 8. are 81. and 64. the numbers produced of the multiplication of the partes the one into the other twise are 72. and 72: which two numbers added to the square numbers of 9. and 8. namely, to 81. and 64. make also 289. which is equall to the square number of the whole number 17. As you see in the example.

The multiplication of the whole into himselfe.	17	{ 9 8 }	the partes of the whole
	17		
The multiplication of eche part into himselfe.	119		the number produced of the whole into himselfe.
	17		
	289		
The multiplication of the one part into the other twise.	9	81 64 72 72	equall to
	9		
	81		
	8		
	8	289	the number composed of eche part into himselfe, and of the one into the other twise.
	64		
	9		
	8		
	72		
	9		
	8		
	72		

The demonstration wherof followeth in Barlaam.

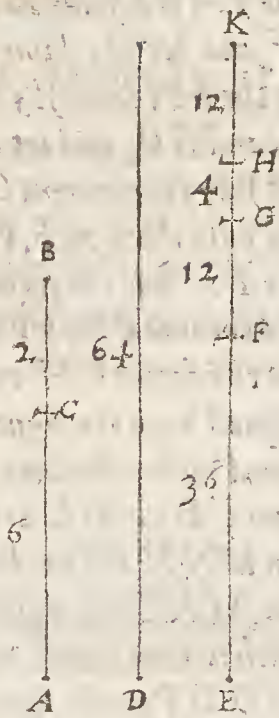
The fourth Proposition.

Barlaam.

If a number geuen be devided into two numbers: the square number of the whole, is equall to the square numbers of the partes, and to the superficial number which is produced of the multiplication of the partes the one into the other twise.

Suppose that the number geuen be AB : which let be devided into two numbers AC and CB . Then I say that the square number of the whole number AB , is equall to the squares of the partes, that is, to the squares of the numbers AC and CB , and to the superficial number produced of the multiplication of the numbers AC and CB the one into the other twise. Let the square number produced of the multiplication of the whole number AB into himselfe be D . And let CA multiplied into himselfe produce the number EF : And CB multiplied into it selfe let it produce GH : and finally of the multiplicatiō of the numbers AC and CB the one into the other twise let there be produced either of these superficial numbers FG and HK . Now forasmuche as the number AC multiplying it self produced the number EF : therefore the number AC measureth the number EF by the vnities which are in it selfe. And forasmuch as the number CB multiplied the number CA and produced the number FG : there-
fore

fore the number AC measureth the nūber FG by the vnities
whiche are in the number CB . But it before also measured
the number EF by the vnities which are in it selfe. Where-
fore the number AB multiplying the number AC produ-
ceth the number EG . And therefore the number EG is the
superficiall number produced of the multiplication of the
number BA into the number AC . And by the same rea-
son may we proue that the number GK is the superficiall
number produced of the multiplication of the number AB
into the number BC . Farther the number D is the square of
the number AB . But if a number be deuided into two num-
bers, the square of the whole number is equall to the two
superficiall numbers which are produced of the multipli-
cation of the whole into either the partes (by the 2. Theo-
reme.) Wherefore the square number D is equall to the su-
perficiall number EK . But the number EK is composed of
the squares of the numbers AC and CB , and of the superfi-
cial number which is produced of the multiplication of the
nūber AC and CB the one into the other twise: & the num-
ber D is the square of the whole number AB . Wherefore the
square number produced of the multiplication of the num-
ber AB into himselfe, is equall to the square numbers of
the partes, that is, to the square numbers of the nūbers AC
and CB , and to the superficiall number produced of the multi-
plication of the num-
bers AC and CB , the one into the other twise. If therefore a number geuen be deu-
ided into two numbers &c. Which was required to be proued.



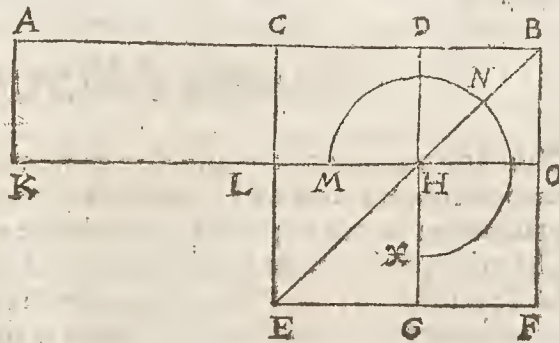
The 5. Theoreme. The 5. Proposition.

If a right line be deuided into two equall partes, & into two vnequall partes: the rectangle figures comprehended vnder the vnequa'll partes of the whole, together with the square of that which is betwene the sectiōs, is equal to the square which is made of the halfe.

Suppose that the right line AB be divided into two equal partes in the point C , and into two vnequall partes in the point D . Then I say that the rectangle figure comprehended vnder AD and DB together with the square which is made of CD , is equall to the square which is made of CB . *Dea Construction.*

scribe (by the 46. of the first) vppon CB a square, and let the same be $CEFB$. And (by the first petition) drawe a line from E to B . And by $\dot{y}$ point D drawe (by the 31. of the first) a line parallel vnto ech of these lines CE and BF cutting the diameter BE in the point H , and let $\dot{y}$ same be DG . And agayne (by the selfe

The diagram shows a horizontal line segment AB with points A, C, D, B in order. Below it, a vertical line segment EF is drawn, with E aligned with A and F aligned with B . A rectangle $CEFB$ is formed by the line CE and BF . A line DE is drawn from D to E . A line DB is drawn from D to B . A line AD is drawn from A to D . A line CE is drawn from C to E . A line BF is drawn from B to F . A line DE is drawn from D to E . A line DB is drawn from D to B . A line AD is drawn from A to D . A line CE is drawn from C to E . A line BF is drawn from B to F . A line DE is drawn from D to E . A line DB is drawn from D to B . A line AD is drawn from A to D . A line CE is drawn from C to E . A line BF is drawn from B to F . A line DE is drawn from D to E . A line DB is drawn from D to B . A line AD is drawn from A to D . A line CE is drawn from C to E . A line BF is drawn from B to F . A line DE is drawn from D to E . A line DB is drawn from D to B . A line AD is drawn from A to D . A line CE is drawn from C to E . A line BF is drawn from B to F . A line DE is drawn from D to E . A line DB is drawn from D to B . A line AD is drawn from A to D . A line CE is drawn from C to E . A line BF is drawn from B to F . A line DE is drawn from D to E . A line DB is drawn from D to B . A line AD is drawn from A to D . A line CE is drawn from C to E . A line BF is drawn from B to F . A line DE is drawn from D to E . A line DB is drawn from D to B . A line AD is drawn from A to D . A line CE is drawn from C to E . A line BF is drawn from B to F . A line DE is drawn from D to E . A line DB is drawn from D to B . A line AD is drawn from A to D . A line CE is drawn from C to E . A line BF is drawn from B to F . A line DE is drawn from D to E . A line DB is drawn from D to B . A line AD is drawn from A to D . A line CE is drawn from C to E . A line BF is drawn from B to F . A line DE is drawn from D to E . A line DB is drawn from D to B . A line AD is drawn from A to D . A line CE is drawn from C to E . A line BF is drawn from B to F . A line DE is drawn from D to E . A line DB is drawn from D to B . A line AD is drawn from A to D . A line CE is drawn from C to E . A line BF is drawn from B to F . A line DE is drawn from D to E . A line DB is drawn from D to B . A line AD is drawn from A to D . A line CE is drawn from C to E . A line BF is drawn from B to F . A line DE is drawn from D to E . A line DB is drawn from D to B . A line AD is drawn from A to D . A line CE is drawn from C to E . A line BF is drawn from B to F . A line DE is drawn from D to E . A line DB is drawn from D to B . A line AD is drawn from A to D . A line CE is drawn from C to E . A line BF is drawn from B to F . A line DE is drawn from D to E . A line DB is drawn from D to B . A line AD is drawn from A to D . A line CE is drawn from C to E . A line BF is drawn from B to F . A line DE is drawn from D to E . A line DB is drawn from D to B . A line AD is drawn from A to D . A line CE is drawn from C to E . A line BF is drawn from B to F . A line DE is drawn from D to E . A line DB is drawn from D to B . A line AD is drawn from A to D . A line CE is drawn from C to E . A line BF is drawn from B to F . A line DE is drawn from D to E . A line DB is drawn from D to B . A line AD is drawn from A to D . A line CE is drawn from C to E . A line BF is drawn from B to F . A line DE is drawn from D to E . A line DB is drawn from D to B . A line AD is drawn from A to D . A line CE is drawn from C to E . A line BF is drawn from B to F . A line DE is drawn from D to E . A line DB is drawn from D to B . A line AD is drawn from A to D . A line CE is drawn from C to E . A line BF is drawn from B to F . A line DE is drawn from D to E . A line DB is drawn from D to B . A line AD is drawn from A to D . A line CE is drawn from C to E . A line BF is drawn from B to F . A line DE is drawn from D to E . A line DB is drawn from D to B . A line AD is drawn from A to D . A line CE is drawn from C to E . A line BF is drawn from B to F . A line DE is drawn from D to E . A line DB is drawn from D to B . A line AD is drawn from A to D . A line CE is drawn from C to E . A line BF is drawn from B to F . A line DE is drawn from D to E . A line DB is drawn from D to B . A line AD is drawn from A to D . A line CE is drawn from C to E . A line BF is drawn from B to F . A line DE is drawn from D to E . A line DB is drawn from D to B . A line AD is drawn from A to D . A line CE is drawn from C to E . A line BF is drawn from B to F . A line DE is drawn from D to E . A line DB is drawn from D to B . A line AD is drawn from A to D . A line CE is drawn from C to E . A line BF is drawn from B to F . A line DE is drawn from D to E . A line DB is drawn from D to B . A line AD is drawn from A to D . A line CE is drawn from C to E . A line BF is drawn from B to F . A line DE is drawn from D to E . A line DB is drawn from D to B . A line AD is drawn from A to D . A line CE is drawn from C to E



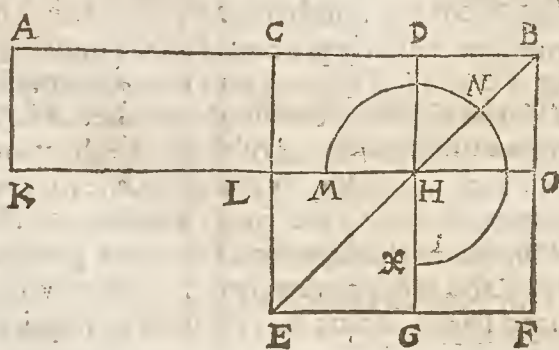
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same) by the point *H* drawe a line parallel vnto eche of these lines *AB* and *EF*, and let the same be *KO*: and let *KO* be equall vnto *AB*. And againe (by the selfe same) by the point *A* draw a line parallel vnto either of these lines *CL* and *BO*, and let the same be *AK*. And forasmuch as (by the 43. of the first) the supplement *CH* is equall to the supplement *HF*. put the figure *DO* common vnto them both. VVherefore the whole figure *CO* is equall to the whole figure *DF*. But the figure *CO* is equall vnto the figure *AL*, for y line *AC* is equall vnto the line *CB*. VVherefore the figure *AL* also is equal vnto the figure *DF*. Put the figure *CH* common vnto them both. VVherfore the whole figure *AH* is equall vnto the figures *DL* and *DF*. But *AH* is equall to that which is containned vnder the lines *AD* and *DB*, for *DH* is equall vnto *DB*. And the figures *FD* & *DL* are the Gnomon *MNX*. VVherfore y Gnomon *MNX* is equall to that which is containned vnder *AD* and *DB*. Put the figure *LG* common vnto them both, which is equal to the square which is made of *CD*. VVherefore the Gnomon *MNX* and the figure *LG* are equall to the rectangle figure comprehendend vnder *AD* and *DB* and vnto the square which is made of *CD*. But the Gnomon *MNX*, and the figure *LG* are the whole square *CEFB*, which is made of *BC*. VVherefore the rectangle figure comprehendend vnder *AD* and *DB*, together with the square which is made of *CD*, is equall to the square which is made of *CB*. If therefore a right line be deuided into two equall partes, and into two vnequall partes, the rectangle figure comprehendend vnder the vnequall partes of the whole, together with the square of that which is betwene the sections, is equall to the square which is made of the halfe: which was required to be proued.



This Proposition also is of greate vse in Algebra. By it is demonstrated that equation wherein the greatest and least karectes or numbers are equall to the middle.

An example of this proposition in numbers.

Take any number as 20: and deuide it into two equall partes 10. and 10. and then into two vnequall partes as 13. and 7. And take the differēce of the halfe to one of the vnequall partes which is 3. And multiply the vnequall partes, that is, 13 and 7. the one into the other, which make 91. take also the square of 3. which is 9. and adde it to the foresayde number 91: and so shall there be made 100. Then multiply the halfe of the whole number into himself, that is, take the square of 10. which is 100. which is equal to the number before produced of the multiplication of the vnequal parts the one into the other, & of the difference into it selfe which is also 100. As you se in the example.

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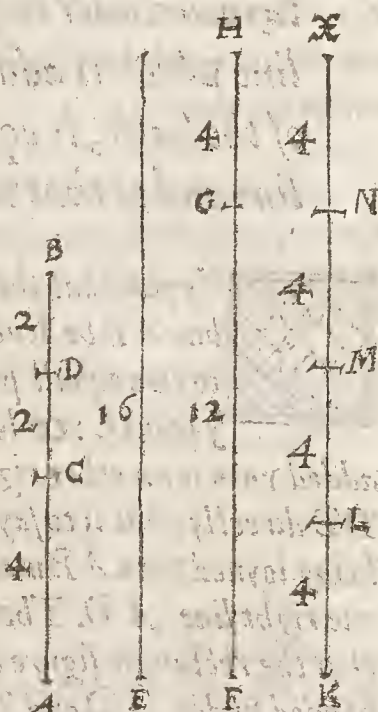
The whole euen number.	20	10 10	the equall partes
		13 7	the vnequall partes
		3	The difference of one of the vnequal partes to the halfe.
Multiplication of the vnequall partes the one into the other.	13 7	91	
		91	
		9	
Multiplication of the difference into it selfe.	3 3	100	the nūber composed of the multiplication of the vnequal partes the one into the other, & of the difference into it selfe
		9	equall to
Multiplication of the halfe into it selfe.		10 10	
		100	the number produced of the halfe into it selfe.

The demonstration wheroffolloweth in Barlaam.

The fifth proposition.

If an euen number be deuided into two equall partes, and againe also into two vnequall partes: the superficiall number which is produced of the multiplication of the vnequall partes the one into the other, together with the square of the number set betwene the parts, is equal to the square of halfe the number.

Suppose that AB be an euen number: which let be deuided into two equall numbers AC and CB , and into two vnequall numbers AD and DB . Then I say, that the square number which is produced of the multiplication of the halfe number CB into it selfe, is equall to the superficiall number produced of the multiplication of the vnequall numbers AD and DB the one into the other, and to the square number produced of the number CD which is set betwene the sayde vnequall partes. Let the square number produced of the multiplication of the halfe number CB into it selfe be E . And let the superficiall number produced of the multiplication of the vnequall nūbers AD and DB the one into the other, be the number FG : and let the square of the number DC which is set betwene the partes be GH . Now forasmuch as the number BC is deuided into the numbers BD and DC , therefore the square of the number BC , that is, the number E , is equall to the squares of the numbers BD and DC , and to the superficiall number which is composed of the multiplication of the numbers BD and DC the one into the other twise, (by the 4. proposition of this booke) Let the square of the number BD be the number KL : & let NX be the square of the number DC : and finally of



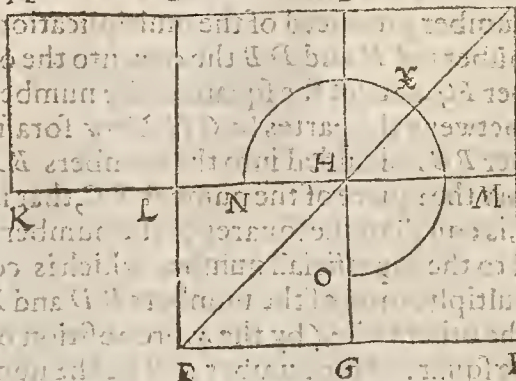
the multiplication of the numbers BD and DC the one into the other twise, let be produced either of these numbers LM and MN . Wherefore the whole number KX is equall to the number E . And forasmuch as the number BD multiplying it selfe produced the number KL , therefor it measureth it by the vnities which are in it selfe. More ouer forasmuch as the number CD multiplying the number BD produced the number LM , therefore also DB measureth LM by the vnities which are in the number CD : but it before measured the number KL by the vnities which are in it selfe. Wherefore the number DB measureth the whole number KM by the vnities which are in C . But the number CB is equall to the number CA . Wherefore the number DB measureth the number KM by the vnities which are in CA . Agayne forasmuch as the number CD multiplying the number DB produced the number MN : therefore the number DB measureth the number MN by the vnities which are in the number CD : but it before measured the number KM by the vnities which are in the number AC . Wherefore the number BD measureth the whole number KN by the vnities which are in the number AD . Wherefore the number FG is equall to the number KN . For numbers which are equemultiplices to one and the selfe same number, are equall the one to the other. But the number GH is equall to the number NX : for either of them is supposed to be the square of the number CD . Wherefore the whole number KX is equall to the whole number FH . But the number KX is equall to the number E . Wherefore also the number FH is equall to the number E . And the number FH is the superficial number produced of the multiplication of the numbers AD and DB the one into the other together with the square of the number DC . And the number E is the square of the number CB . Wherefore the superficial number produced of the multiplication of the vnequal partes AD and DB the one into the other, together with the square of the number DC which is set betwene those vnequall partes, is equall to the square of the number CB , which is the halfe of the whole number AB . If therfore an euen number be deuided into two equall partes, &c. which was required to be proued.

The 6. Theoreme. The 6. Proposition.

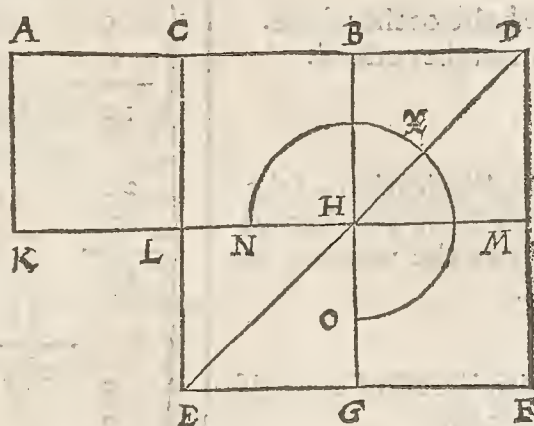
If a right line be deuided into two equal partes, and if vnto it be added an other right line directly, the rectangle figure containned vnder the whole line with that which is added, & the line which is added, together with the square which is made of the halfe, is equall to the square which is made of the halfe line and of that which is added as of one line.



Suppose that the right line AB be deuided into two equall partes in point C : & let there be added vnto it an other right line DB directly, that is to say, which being ioyned vnto AB make both one right line AD . Then I say, that the rectangle figure comprehended vnder AD and DB , together



gether with the square whiche is made of BC is equall to the square whiche is made of DC . Describe (by the 46. of the 1.) vpon CD a square $CEFD$, and (by the first petition) draw a line from D to E : and (by the 31. of the first) by γ point B draw a line parallel vnto either of these lines EC & DF , cutting the diameter DE in the point H , and let the same be BG , & (by the self same) by γ point H draw to either of these lines AD and EF a parallel line KM : and moreouer by the pointe A drawe a line parallel to either of these lines CL



and DM : and let the same be AK . And forasmuch as AC is equall vnto CB , therefore (by the 36. of the first) the figure AL is equal vnto the figure CH . But (by the 43. of the first) CH is equal vnto the figure HF , wherfore AL is equall vnto HF . Put the figure CM common to them both, wherfore the whole line AM is equal vnto the gnomon NXO . But A

M is that which is containned vnder AD and DB : for DM is equal vnto DB : wherfore the gnomon NXO , is equall vnto the rectangle figure contained vnder AD and DB . Put the figure LG common to them both, which is equall to the square which is made of CB . Wherfore the rectangle figure which is containned vnder AD and DB together with the square which is made of CB is equall to the gnomon NXO , and vnto LG . But the gnomon NXO and LG are the whole square $CEFD$ which is made of CD . Wherfore the rectangle figure containned vnder AD and DB together with the square which is made of CB is equall to the square which is made of CD . If therefore a right line be devided into two equall partes, and vnto it be added an other right line directly: the rectangle figure containned vnder the whole line with that which is added, and the line which is added, together with the square which is made of the halfe, is equall to the square which is made of the halfe line and of that which is added as of one line: which was required to be demonstrated.

By this Proposition (besides many other vses) is in Algebra demonstrated that equation wherin the two lesse numbers be equall to the number of the greatest denomination.

An example of this proposition in numbers.

Take any euen number as 18. and adde vnto it any other number as 3. which make in all 21. And multiply 21. into the number added, namely, into 3. which maketh 63. Take also the halfe of the whole euen number, that is, of 18. which is 9. And multiply 9. into it self which maketh 81. which adde vnto 63. (the number produced of the whole euen number, and the number added into the number added) and you shal make 144.

T.j.

Then

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Then adde 9. the halfe of the whole euen number vnto 3. the number added which maketh 12. And multiply 12. into it selfe, that is, take the square of 12. which is 144. which is equall to the number composed of the multiplication of the whole number and the number added into the number added, and of the square of the number added, which is also 144. As you see in the example.

The whole euen number.	18	
	3	
The number added.	21	the number composed of the whole number, & the number added.
	9	
The halfe of the whole.	3	
	12	the number composed of the halfe and of the number added.
The number added.	21	
	3	
Multiplication of the whole & the number added into the number added.	63	63
		81
Multiplicatiō of the halfe into it selfe.	9	144
	9	
Multiplicatiō of the halfe and the number added in to it selfe.	81	
	12	
	12	
	24	
	12	
	144	

The demonstration wherof followeth in Barlaam,

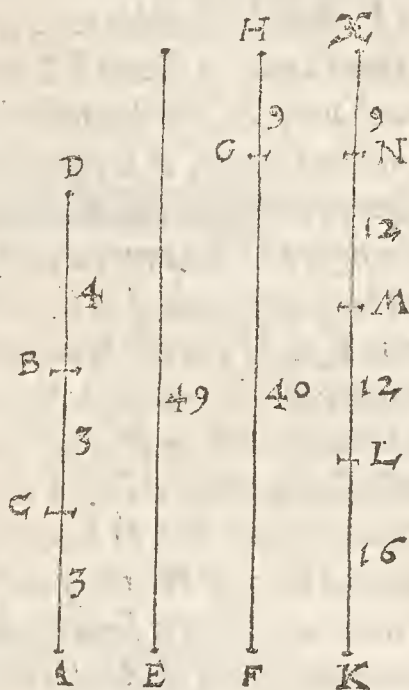
The sixth Proposition.

If an euen number be deuided into two equall numbers, and vnto it be added some other number, the superficiall number which is made of the multiplication of the number composed of the whole number and the number added, into the number added, together with the square of the halfe number, is equall to the square of the number composed of the halfe and the number added.

Suppose that AB be an euen number, and let it be deuided into two equall numbers AC and CB : and vnto it let there be added an other number BD . Then I say that the superficiall number produced of the multiplication of the number AD into the number DB is equall to the square of the number CD . For let the square number of the number CD be the number E , and let the superficial number produced of the multiplication of the number AD into the number DB be the number FG : and finally let the square number of CB be the number GH . And forasmuch as the square of the number CD is (by the 4. proposition) equall to the squares of the numbers DB and BC together with the superficial number which is produced of the multiplication of the numbers DB and BC the one into the other twise. Let the square of the number BD be the number KL : and let the superficial numbers produced of the multiplication of the numbers DB and BC the one into the other twise be either of these numbers

LM

LM and MN; and finally let the square of the number BC be the number NX . Wherefore the whole number KX shall be equall to the square of the number CD . But the square of the number CD is the number E . Wherefore the number KX is equall to the number E . And forasmuch as the number BD multiplieng it selfe produced the number KL : therefore the number BD measureth the number KL , by the vnities which are in it selfe, but it also measureth the number LM by the vnities which are in the number CB . Wherefore the number DB measureth the whole number KM by the vnities which are in the number CD . The number DB also measureth the number MN by the vnities which are in the number CB : & the number CB is equall to the number CA by supposition. Wherefore the number DB measureth the whole number KN by the vnities which are in the number AD . But the number DB doth also measure the number FG by the vnities which are in the number AD : for by supposition the number FG is the superficiall number produced of the multiplication of the numbers AD and DB the one into the other. Wherefore the number FG is equall to the number KN . But the number HG is equall to the number NX : for either of them is the square number of the number CB . Wherefore the whole number FH is equall to the number KX : and the number KX is proued to be equall to the number E . Wherefore the number FH shall also be equall to the number E . And the number FH is the superficiall number produced of the multiplication of the numbers AD and DB the one into the other, together wyth the square of the number CB : and the number E is the square of the number CD . Wherefore the superficiall number produced of the multiplication of the numbers AD and DB the one into the other, together with the square of the number CB , is equall to the square of the number CD . If therefore an euen number &c.



The 7. Theoreme. The 7. Proposition.

If a right lyne be deuided by chaunce, the square whiche is made of the whole together with the square which is made of one of the partes, is equall to the rectangle figure which is contained vnder the whole and the said parte twise, and to the square which is made of the other part.



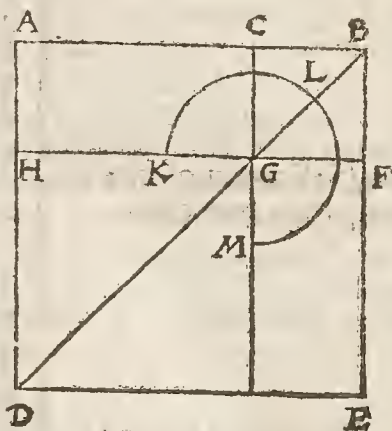
Suppose that the right line AB be deuided by chaunce in the point C . Then I say that the square which is made of AB , together with the square which is made of BC , is equal vnto the rectangle figure which is contained vnder the lines AB and BC twise, and vnto the square which is made of AC . Describe (by the 46. of the first) vppon AB a square $ADEB$, and make complete the figure. And forasmuch as (by the 43. of the first) the figure AG is equall vnto the figure GE . Put the figure CF common to the

T. y.

both;

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both: wherfore the whole figure AF is equall to the whole figure CE . VVherfore the figures AF and CE are double to the figure AF . But the figures AF and CE are the gnomon KLM , and the square CF : wherfore the gnomon KLM , and the square CF is double to the figure AF . But the double to AF is that which is containd vnder AB and BC twice, for BF is equall vnto BC . VVherfore the gnomon KLM and the square CF is equall vnto the rectangle figure containd vnder AB and BC twice. Put the figure DG common vnto them both, which is the square made of AC . VVherfore the gnomon KLM and the squares BG and GD are equal vnto the rectangle figure which is containd vnder AB & BC twice, & vnto the square which is made of AC . But the gnomon KLM , & the squares BG , & DG are y whole square $B A D E$, & y part or square CF , which squares are made of the lines AB & of BC , therfore y squares which are made of AB & BC are equall vnto the rectangle figure which is containd vnder AB and BC twice, and also vnto the square of AC . If therfore a right line be deuided by chaunce: the square which is made of the whole together with the square which is made of one of the partes, is equall to the rectangle figure which is containd vnder the whole and the sayd part twice, and to the square which is made of the other parte: whiche was required to be demonstrated.



Flussates addeth vnto this Proposition this Corollary.

The squares of two vnequall lines do exceede the rectangle figures containd vnder the said lines by the square of the excessse wherby the greater lyne exceedeth the lesse.

For if the line AB be the greater, and the line BC the lesse, it is manifest that the squares of AB and BC are equall to the rectangle figure containd vnder the lynes AB and BC twice, and moreouer to the square of the line AC , wherby the line AB exceedeth the line BC ,

By this proposition most wonderfully was found out the extraction of roote squares in irrational numbers, beside many other straunge thinges.

An example of this proposition in numbers.

Take any number as 13. and deuide it into two partes as into 4. & 9. Take the square of 13. which is 169. take also the square of 4. which is 16. and adde these two squares together which make 185. Then multiply the whole number 13. into 4. the foresayde part twice, and you shall produce 52. and 52: take also the square of the other part, that is, of 9. which is 81. And adde it to the productes of 13. into 4. twice, that is, vnto 52. and 52. and those three numbers added together shall make 185. whiche is equall to the number composed of the squares of the whole and of one of the partes, which is also 185. As you see in the example.

The

The whole	13	{ 4 } partes of the whole.
Multiplication of the whole into it selfe.	13 13 ----- 39 13 ----- 169	169 16 ----- 185
Multiplication of one of the partes into it selfe.	4 4 ----- 16	
Multiplication of the whole into the foresayde part twise.	13 4 ----- 52	
Multiplication of the other part into it selfe.	9 9 ----- 81	

The number composed of the squares of the whole, and of one of the partes

equall to

the number composed of the whole into the fore-said part twise, and of the square of the other part.

The demonstration wheroffolloweth in Barlaam.

The seventh proposition.

If a number be deuided into two numbers: the square of the whole number together with the square of one of the partes, is equall to the superficiall number produced of the multiplication of the whole number into the foresaid part twise, together with the square of the other part.

Suppose that the number AB be deuided into the numbers AC and CB . Then I say that the square numbers of the numbers BA and AC are equall to the superficiall number produced of the multiplicatiō of the number BA into the number AC twise, together with the square of the number BC . For forasmuch as (by the 4. of this booke) the square of the number AB is equall to the squares of the numbers BC and CA , and to the superficiall number produced of the multiplication of the numbers BC and CA the one into the other twise: adde the square of the number AC common to them both. Wherefore the square of the number AB together with the square of the number AC is equall to two squares of the number AC and to one square of the number CB , and also to the superficiall number produced of the multiplication of the numbers BC and CA the one into the other twise. And forasmuch as the superficial number produced of the multiplication of the numbers BA and CA the one into the other once, is equall to the superficiall nūber produced of the multiplication of BC into CA once, and to the square of the number CA (by the third of this booke): therefore the number produced of the multiplication of BA into AC twise is equall to the number produced of the multiplication of BC into CA twise, and also to two squares of the number CA . Adde the square number of BC common to them both. Wherefore two squares of the number AC and one square of the number CB together with the superficiall number

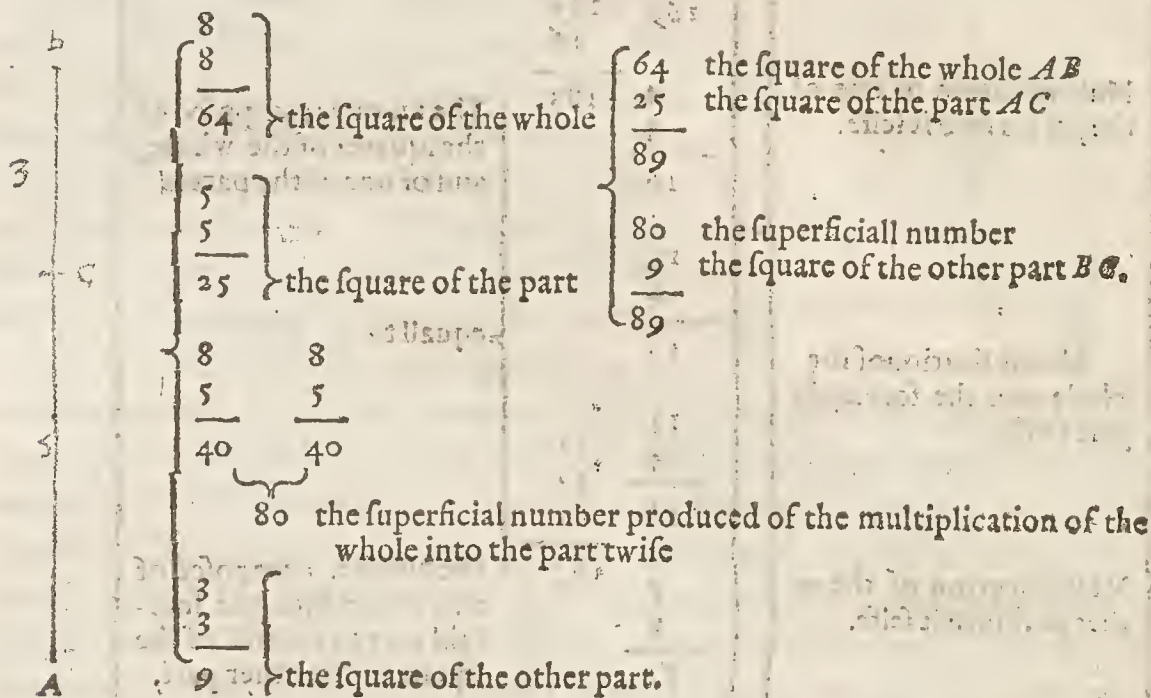
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duced of the multiplication of BC into CA twise are equall to the superficiall number produced of the multiplication of the number BA into the number AC twise together with the square of the number CB . Wherefore the square of the number AB together with the square of the nūber AC is equal to the superficial nūber produced of the multiplication of the number BA into the number AC twise, together with the square of the number CB . If therfore a number be deuided into two numbers &c. which was required to be demonstrated.



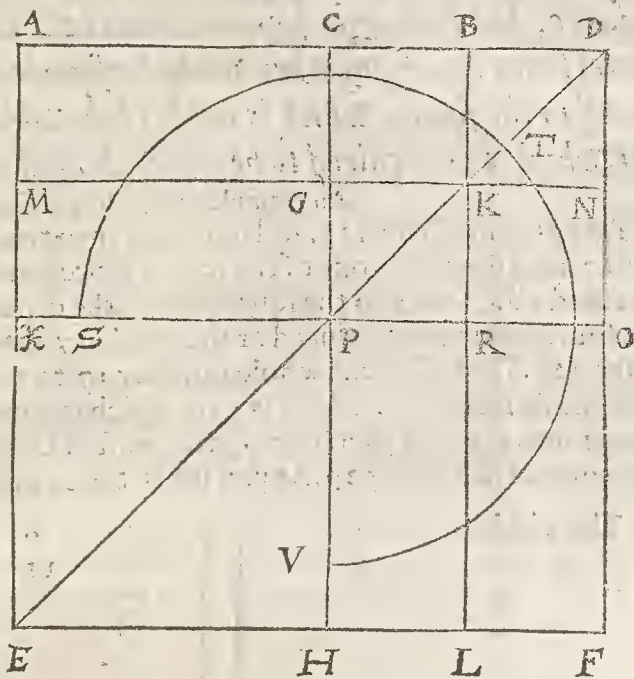
The 8. Theoreme. The 8. Proposition.

If a right line be deuided by chaūce, the rectangle figure comprehended vnder the whole and one of the partes foure times, together with the square which is made of the other parte, is equall to the square which is made of the whole and the fore-said part as of one line.



Suppose that there be a certayne right line AB , and let it be deuided by chaunce in the point C . Then I say that the rectangle figure comprehended vnder AB and BC foure tymes, together with the square which is made of AC is equall to the square made of AB and BC as of one line. Extend the line AB (by the second petition). And (by the third of the first) vnto CB put an equall lyne BD . And (by the 46. of the first) describe vppon AD a square $A E F D$. And describe a double figure. And forasmuch as CB is equall vnto BD , but CB is equall vnto GK (by the 34. of the first) and likewise BD

is equall vnto KN , wherefore GK also is equall vnto KN : and by the same reason also PR , is equall vnto RO . And forasmuch as BC is equall vnto BD , and GK vnto KN , therefore (by the 36. of the first) the figure CK is equall vnto the figure KD , and the figure GR is equall vnto the figure RN . But (by $\S$ 43. of $\S$ 1.) the figure CK is equall vnto the figure RN : for they are the supplementes of the parallelograme CO . Wherefore the figure KD also is equall vnto the figure RN . Wherefore these figures DK, CK, GR, RN , are equall the one to the other. Wherefore those foure are quadruple to the figure CK . Agayne forasmuch as CB is equal vnto BD , but BD is equall vnto BK , that is, vnto CG . And



CB is equall vnto GK that is vnto GP : therefore CG is equall vnto GP . And forasmuch as CG is equall vnto GP , and PR is equall vnto RO , therefore the figure AG is equall vnto the figure MP , and the figure PL is equall vnto the figure RF . But the figure MP is (by the 43. of the first) equall vnto the figure PL , for they are the supplementes of the parallelogramme ML : wherefore the figure also AG is equall vnto the figure RF . Wherefore these foure figures AG, MP, PL , and RF are equall the one to the other: wherefore those foure are quadruple to the figure AG . And it is proued, that these foure figures CK, KD, GR, RN , are quadruple to the figure CK . Wherefore the eight figures which containe the gnomon STV , are quadruple to the figure AK . And forasmuch as the figure AK , is that which is containd vnder the lines AB and BD , for the line BK is equall vnto the line BD : therefore that which is containd vnder the lines AB and BD foure tymes is quadruple vnto the figure AK . And it is proued that the gnomon STV is quadruple to AK . Wherefore that which is containd vnder the lines AB and BD foure tymes is equall vnto the gnomon STV . Put the figure XH which is equall to the square made of AC common vnto them both. Wherefore the rectangle figure comprehended vnder the lines AB and BD foure tymes together with the square which is made of the line AC , is equall to the gnomon STV , and vnto the figure XH . But the gnomon STV : and the figure XH are the whole square $AEFD$, which is made of AD : wherefore that which is containd vnder the lines AB and BD foure tymes together with the square which is made of AC , is equall to the square which is

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made of $A D$. But $B D$ is equall vnto $B C$. Wherefore the rectangle figure containned foure tymes vnder $A B$ and $B C$ together with the square which is made of $A C$, is equall vnto the square which is made of $A D$, that is, vnto that which is made of $A B$ and $B C$ as of one line. If therefore a right lyne be deuided by chaunce, the rectangle figure comprehended vnder the whole and one of the partes foure tymes, together with the square which is made on the other part, is equall to the square which is made of the whole and the foresaid part, as of one line, which was required to be demonstrated.

An example of this Proposition in numbers.

Take any number as 17. and deuide it into two partes, as into 6. and 11. And multiply 17. into 6. namely one of the partes foure tymes, and you shall produce 102. 102. 102. and 102. Take also the square of 11. the other part, which is 121: and adde it vnto the foure numbers produced of the whole 17. into the part 6. foure tymes, & you shall make 529. Then adde the whole number 17. to the foresaid part 6. which make 23: & take the square of 23. which is 529. which is equall to the number composed of the whole into the sayd part foure tymes, and of the square of the other part, which number composed is also 529. As you see in the example.

The whole.	17	6	} partes of the whole
	11		
	17		
	6		
	102		
	17		
	6		
	102		
	17		
	6		
	102	102	
	17	102	
	6	102	
	102	102	
	102	121	
	11	529	
	11		
	11		
	11		
	121		
	17		
	6		
	23		
	23		
	23		
	169		
	46		
	529		

the number composed of the whole into one of the partes foure tymes, & of the square of the other part

equal to

the square of the number composed of the whole & the foresaid part.

The demonstration wheroffolloweth in Barlaam.

The eight proposition.

If a number be deuided into two numbers, the superficial number produced of the multiplication of the whole into one of the partes foure tymes, together with the square of the other parte, is equall to the square of the number composed of the whole number and the foresayd part.

Suppose that the number AB be deuided into two numbers AC and CB . Then I say that the superficial number produced of the multiplication of the number AB into the number CB foure tymes together with the square of the number AC , is equall to the square of the number composed of the numbers AB & CB . For vnto the number BC let the number BD be equall. Now forasmuch as the square of the number AD is equal to the squares of the numbers AB and BD , & to the superficial number produced of the multiplication of the numbers AB & BD the one into the other twise (by the 4. of this booke) And the number BD is equall to the number BC : therefore the square of the number AD is equall to the squares of the numbers AB and BC , and to the superficial number produced of the multiplication of the numbers AB and BC the one into the other twise. But the squares of the numbers AB and BC are equall vnto the superficial number produced of the multiplication of the numbers AB and BC the one into the other twise, and to the square of AC (by the former proposition) Wherefore the square of the number AD is equall to the superficial number produced of the multiplication of the numbers AB and BC the one into the other foure tymes, and to the square of the number AC . But the square of the number AD is the square of the number composed of the numbers AB and BC : for the number BD is equal to the number BC . Wherefore the square of the number composed of the numbers AB and BC is equall to the superficial number produced of the multiplication of the numbers AB and BC the one into the other foure tymes, & to the square of the number AC . If therefore a number be deuided into two numbers, &c.

8	8	8	8
2	2	2	2
16	16	16	16
64. the superficial number produced of the multiplication of the numbers AB and BC the one into the other foure tymes.			
6			
6			
36	the square of AC .		
10			
10			
100	the square of the number composed of AB and BC .		
64	the superficial nuber produced of the multiplicatiō made 4. times		
36	the square number of AC .		
100			

The second Booke

The 9. Theoreme. The 9. Proposition.

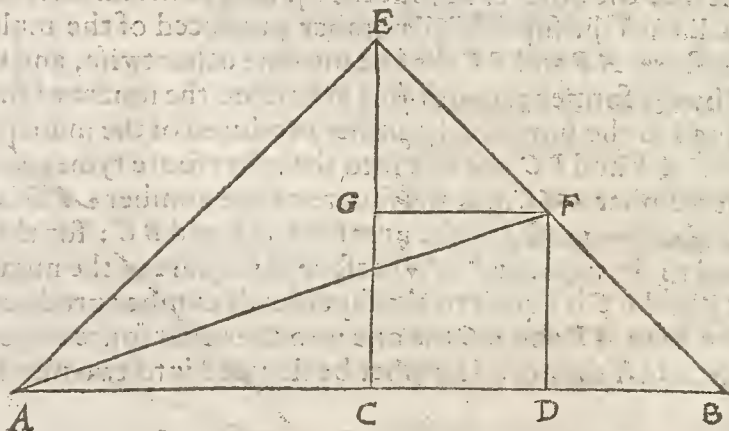
If a right line be deuided into two equall partes, and into two vnequall partes, the squares which are made of the vnequall partes of the whole, are double to the squares, which are made of the halfe lyne, and of that lyne which is betwene the sections.



*S*uppose that a certayne right line AB be deuided into two equall partes in the pointe C , and into two vnequall partes in the pointe D . Then I say that the squares which are made of the lines AD and DB , are double to the squares whiche are made of the lynes AC and CD .

Construction.

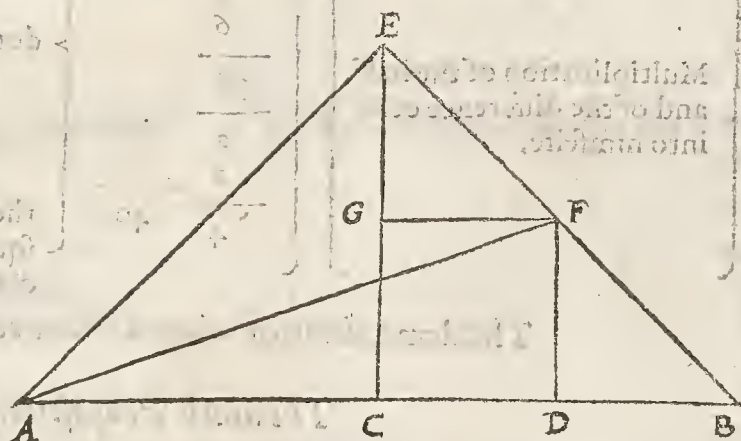
D . For (by the 11. of the first) erect from C to the right line AB a perpendiculer line CE . And let CE (by the 3. of the first) be put equall vnto either of these lines AC & CB : and (by the first peticiō) draw lines from A to E , and from E to B . And (by the 31. of the first) by the point



D draw vnto the line EC a parallel lyne, and let the same be DF : and (by the selfe same) by the point F draw vnto AB a line parallel, and let the same be FG . And (by the first petition) draw a line from A to F . And forasmuch as AC is equall vnto CE , therefore (by the 5. of the first) the angle EAC is equal vnto the angle CEA . And forasmuch as the angle at the point C is a right angle: therefore the angles remayning EAC , and AEC , are equall vnto one right angle, wherefore eche of these angles EAC and AEC is the halfe of a right angle. And by the same reason also eche of these angles EBF and CEB is the halfe of a right angle. Wherefore the whole angle AEB is a right angle. And forasmuch as the angle GFE is the halfe of a right angle, but EGF is a right angle. For (by the 29. of the first) it is equall vnto the inward and opposite angle, that is, vnto ECB : wherefore the angle remayning EFB is the halfe of a right angle. Wherefore (by the 6. common sentence) the angle GFE is equall vnto the angle EFB . Wherefore also (by the 6. of the first) the side EG is equall vnto the side FB . Againe forasmuch as the angle at the point B is the halfe of a right angle, but the angle FDB is a right angle, for it also (by the 29. of the first) is equall vnto the inward

Demonstration.

inwarde and opposite angle $E C B$. Wherefore the angle remayning $B F D$ is the halfe of a right angle. Wherefore the angle at the point B is equall vnto the angle $D F B$. Wherefore (by the 6. of the first) the side $D F$ is equall vnto the side $D B$. And forasmuch as $A C$ is equall vnto $C E$, therefore the square which is made of $A C$ is equall vnto the square which is made of $C E$. Wherefore the squares which are made of $C A$ and $C E$ are double to the square which is made of $A C$. But (by the 47. of the first) the square which is made of $E A$ is equall to the squares which are made of $A C$ and $C E$ (For the angle $A C E$ is a right angle) wherefore the square of $A E$ is double to the square of $A C$. Agayne forasmuch as $E G$ is equall vnto $G F$, the square therefore which is made of $E G$ is equal to the square which is made of $G F$. Wherefore the squares which are made of $G E$ and $G F$ are double to the square which is made of $G F$. But (by the 47. of the first) the square which is made of $E F$ is equall to the squares which are made of $E G$ and $G F$. Wherefore the square which is made of $E F$ is double to the square which is made of $G F$. But $G F$ is equall vnto $C D$. Wherefore the square which is made of $E F$ is double to the square which is made of $C D$.



And the square which is made of $A E$ is double to the square which is made of $A C$. Wherefore the squares which are made of $A E$ and $E F$ are double to the squares which are made of $A C$ and $C D$. But (by the 47. of the first) the square which is made of $A F$ is equal to the squares which are made of $A E$ and $E F$ (For the angle $A E F$ is a right angle). Wherefore the square which is made of $A F$ is double to the squares which are made of $A C$ and $C D$. But (by the 47. of the first) the squares which are made of $A D$ and $D F$ are equall to the square which is made of $A F$. For the angle at point D is a right angle. Wherefore the squares which are made of $A D$ and $D F$ are double to the squares which are made of $A C$ and $C D$. But $D F$ is equall vnto $D B$. Wherefore the squares which are made of $A D$ and $D B$, are double to the squares which are made of $A C$ and $C D$. If therefore a right line be deuided into two equall partes and into two vnequall partes, the squares which are made of the vnequall partes of the whole, are double to the squares which are made of the halfe lyne, and of that lyne which is betwene the sections: which was required to be proued.

¶ An example of this proposition in numbers.

Take any euen number as 12. And deuide it first equally as into 6. and 6. & then vnequally as into 8. & 4. And take the difference of the halfe to one of the vnequal partes

V.ij.

which

The second Booke

which is 2. And take the square numbers of the vnequall partes 8, and 4, which are 64, and 16: and adde them together, which make 80. Then take the squares of the halfe 6, and of the differēce 2: which are 36, and 4: which added together make 40. Vnto which number, the number composed of the squares of the vnequall partes, whiche is 80, is double. As you see in the example.

The whole.	12	6	} the equall partes
		6	
		8	} the vnequall partes
		4	
	8	2	the difference of the halfe to one of the partes
	8		
Multiplication of eche vnequal part into himself.	64		
	4		
	4	64	} the number composed of the squares of the vnequal partes
	16	16	
		80	
	6		} double to
	6		
Multiplication of the half and of the difference eche into himselfe.	36		
	2	36	
	2	4	
	4	40	} the number composed of the squares of the halfe, and of the difference.

The demonstration wherof followeth in Barlaam.

The ninth Proposition.

If a number be deuided into two equall numbers, and againe be deuided into two inequall partes: the square numbers of the vnequall numbers, are double to the square which is made of the multiplication of the halfe number into it selfe, together with the square whiche is made of the number set betwene them.

For let the number AB being an euen number be deuided into two equall numbers AC & CB : & into two vnequall nūbers AD and DB . Then I say that the square numbers of AD and DB , are double to the squares which are made of the multiplication of the numbers AC and CD into themselues. For forasmuch as the number AB is an euen number, and is deuided also into two equal numbers AC and CB , and afterward into two vnequall nūbers AD and DB : therefore the superficial nūber produced of the multiplicatiō of the nūbers AD & DB , th' one into the other, together with the square of the number DC , is equal to the square of the number AC (by the fift proposition) Wherefore the superficial number produced of the multiplication of the numbers AD and DB the one into the other twise, together with two squares of the number CD , is double to the square of the number AC . Forasmuch as also the number AB is deuided into two equal numbers AC and CB , therefore the square number of AB is quadruple to the square number produced of the multiplication of the number AC into it selfe (by the 4. proposition). Moreouer forasmuch as the superficial number produced of the multiplication of the numbers AD & DB the one into the other twise together, with two squares of the number DC , is double to the square number of CA . & forasmuch

forasmuch as there are two numbers, of which the one is quadruple to one and the selfe same number, and the other is double to the same number: therefore that number which is quadruple shall be double to that number which is double. Wherefore, the square of the number AB is double to the number produced of the multiplication of the numbers AD and DB the one into the other twice together with the two squares of the number DC . Wherefore the number which is produced of the multiplication of the numbers AD and DB the one into the other twice, is lesse than the halfe of the square of the number AB by the two squares of the numbers DC . And forasmuch as the number produced of the multiplication of the numbers AD & DB the one into the other twice, together with the number composed of the squares of the numbers AD and DB is (by the 4. proposition) equal to the square of the number AB : therefore the number composed of the squares of the numbers AD & DB is greater than the halfe of the square number of AB , by the two squares of the number DC . And the square of the number AB is quadruple to the square of the number AC . Wherefore the number composed of the squares of the numbers AD and DB is greater than the double of the square of the number AC by two squares of the number DC . Wherefore the said number is double to the squares of the numbers AC and CD . If therefore a number be deuised &c. which was required to be demonstrated.



8		2
8		2
64	the square of the vnequall part AD	4 the square of the vnequall part BD
5		
5		
25	the square of the halfe AC .	
3		
3		
9	the square of CD , namely, of the number set betwene.	
25		
9		
34	the squares of the halfe, and of the number set betwene.	
64		
4		
68	the squares of the vnequall partes.	

$$\begin{array}{r} 68 \\ 34 \end{array} | 2.$$

The 10. Theoreme. The 10. Proposition.

If a right line be deuised into two equal partes, & vnto it be added an other right line directly: the square which is made of the whole & that which is added as of one line, together with
 U. iij. the

The second Booke

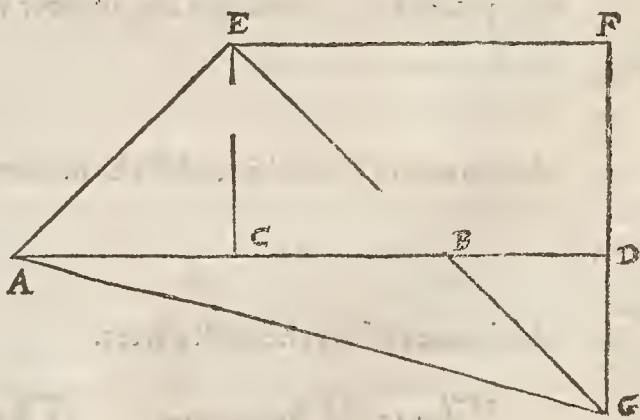
the square whiche is made of the lyne whiche is added, these two squares (I say) are double to these squares, namely, to the square which is made of the halfe line, & to the square which is made of the other halfe lyne and that whiche is added, as of one lyne.

Construction.

Suppose that a certayne right line AB be deuided into two equall partes in the point C . And vnto it let there be added an other right line directly, namely, BD . The I say, that the squares which are made of the lines AD and DB are double to the squares which are made of the lines AC and CB . Rayse vp (by the 11. of the first) from the point C vnto the right line ACD a perpendicular lyne, and let the same be CE . And let CE (by the 3. of the first) be made equall vnto either of these lines AC and CB . And (by the first petition) draw right lines from E to A , and from E to B . And (by the 31. of the first) by the point E , draw a line parallel vnto CD , and let the same be EF . And (by self same) by the point D draw a line parallel vnto CE and let the same be DF . And forasmuch as vpon these parallel lines CE & DF lighteth a certain right line EF , therefore (by the 29. of the first) the angles CEF and EFD are equal vnto two right angles. VVherfore the angles FEB , and EFD are lesse then two right angles. But lines produced from angles lesse then two right angles (by the fifth petition) at the length meete together. VVherfore the lines EB and FD beyng produced on that side that the line BD is, will at the length meete together. Produce them and let them meete together in the point G . And (by the first petition) draw a line from A to G . And forasmuch as the line AC is equall

Demonstration.

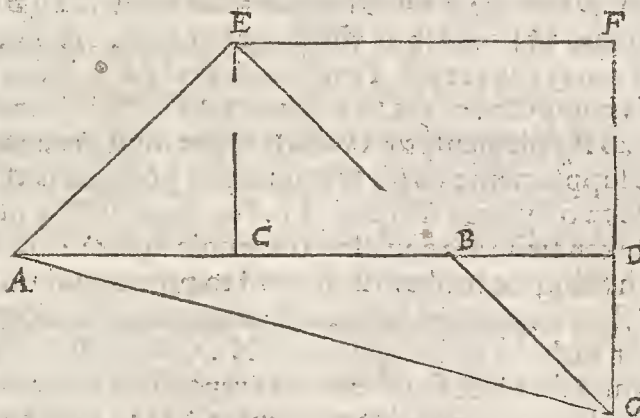
vnto the line CE , the angle also AEC is (by the 5. of the first) equall vnto the angle EAC . And the angle at y point C is a right angle. VVherfore eche of these angles EAC & AEC is the halfe of a right angle. And by the same reason eche of these angles ECB , and EBD is the halfe of a right angle. VVherfore



the angle AEB is a right angle. And forasmuch as the angle EBD is the halfe of a right angle, therefore (by the 15. of the first) the angle DBG is the half of a right angle. But y angle BDG is a right angle (for it is equal vnto the angle DCE , for they are alternate angles) VVherfore the angle remaining DGB is the halfe of a right angle. VVherfore (by the 6 common sentence of the first) the angle DGB is equall to the angle DBG . VVherfore (by the 6. of the first) the
side

BD is equall vnto the side GD . Agayne forasmuch as the angle EGF is the halfe of a right angle: and the angle at the pointe F is a right angle: for (by the 34. of the first) it is equall vnto the opposite angle ECD . Wherefore the angle remayning $FE G$ is the halfe of a right angle. Wherefore the angle EGF is equall to the angle $FE G$. Wherefore (by the 6. of the first) the side FE is equall vnto the side FG . And forasmuch as EC is equall vnto CA , the square also which is made of EC is equall to the square which is made of CA . Wherefore the squares which are made of CE and CA are double to the square which is made of AC . But the square which is made of EA is (by the 47. of the first) equall vnto the squares which are made of EC and CA . Wherefore the square which is made of EA is double to the square which is made of AC . Agayne forasmuch as GF is equall vnto EF , the square also which is made of GF is equall to the square which is made of FE . Wherefore the squares which are made of GF and EF are double to the square which is made of EF . But (by the 47. of the first) the square which is made of EG is equall to the squares which are made

of GF and EF . Wherefore the square which is made of EG is double to the square which is made of EF . But EF is equall vnto CD , wherefore the square which is made of EG is double to the square which is made of CD . And it is proued the square which is made of EA is double to the square which is made of



AC . Wherefore the squares which are made of AE and EG are double to the squares which are made of AC and CD . But (by the 47. of the first) the square which is made of AG is equall to the squares which are made of AE and EG . Wherefore the square which is made of AG is double to the squares which are made of AC and CD . But vnto the square which is made of AG are equall the squares which are made of AD and DG . Wherefore the squares which are made of AD and DG are double to the squares which are made of AC and CD . But DG is equall vnto DB . Wherefore the squares which are made of AD and DB are double to the squares which are made of AC and CD . If therefore a right line be deuided into two equall partes, and vnto it be added an other lye directly, the square which is made of the whole and that which is added, as of one line, together with the square which is made of the line which is added, these two squares (I say) are double to these squares, namely, to the square which is made of the halfe lye, and to the square which is made of the other halfe lye and that which is added, as of one lye: which was required to be proued.

¶ An other demonstration after Pelitarius.

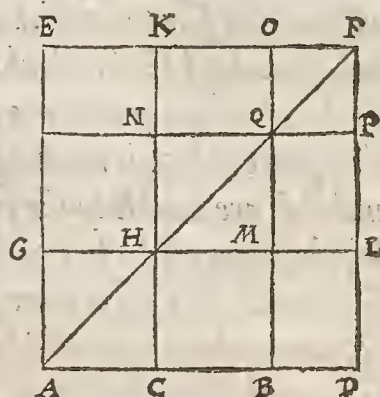
V.iiii.

Suppose

The second Booke

Suppose that the lyne AB be deuided into two equall partes in the poynte C . And vnto it let there be added an other right lyne directly, namely, BD . Then I say that the square of AD together with the square of BD is double to the squares of A C and CD .

Vpon the whole line AD describe a square $ADEF$. And vpon the halfe lyne AC describe the square $ACGH$. And produce the sides GH and CH till they cut the sides EF & DF , wherby shalbe described the figure $HLKF$, which shalbe the square of the line CD : as (by the Corollary of the 4. of this booke, & by the 34. Proposition of the 1.) it is manifest if we draw the diameter CD . For the lyne KF is equall to the line CD . And making also the lines HM and HN equall to either of these lynes AC and CB , drawe the lynes MO and NP cutting the one the other right angled wise in the point Q . Either of which lynes let cut the sides of the square AD : EF in the pointes O and P . Now it nedeth not to proue that the figure HQ is the square of the lyne AC , seying that it is the square of the line CB : as the figure QF is the square of the line BD : neither also needeth it to proue that the parallelograme HP is equall to either of the supplementes EH and HD : nor that the supplementes NO and QL are equall. For all this is manifest euē by the forme of the figure, for that all the angles about the diameter are half right angles, & the sides are equall. Wherfore if we diligently marke of what partes the square HF which is the square of CD , is composed, we may thus reaso. Forasmuch as the whole square ED is composed of the two squares AH and HF and of the two supplementes EH and HD , we must proue that these supplementes with the square QF (which is the square of the line BD) are equall to the two squares AH and HF . For then shall we proue that these two squares AH & HF taken twise are equall to the whole square DE together with the square of QF , which thing we tooke first in hand to proue. And thus do I proue it.



The Supplement EH is equall to the parallelograme HP . And the square AH together with the lesser supplement, NO , is equall to the other supplement HD . (by the first common sentence so oftentimes repeted as is neede) wherfore the two supplementes EH and HD are equall to the square AH and to the Gnomon $KHLPQQ$. If therfore vnto either of them be added the square QF : the two supplementes EH and HD together with the square of QF shal be equal to the square AH , & to the Gnomon $KHLPQQ$ and to the square QF . But these three figures do make the two squares AH and HF . Wherfore the two supplementes EH and HD together with the square QF are equall to the two squares AH and HF , which was the second thing to be proued. Wherfore the two squares AH and HF beyng taken twise are equall to the whole square DE together with the square of QF . Wherfore the square DE together with the square QF is double to the squares AH and HF : which was required to be proued.

¶ An example of this Proposition in numbers.

Take any euen number as 18: and take the halfe of it which is 9. and vnto 18. the whole, adde any other number as 3. which maketh 21. Take the square number of 21. (the whole number and the number added) which maketh 441. Take also the square of 3 (the number added) which is 9. which two squares added together make 450. Then adde the halfe number 9. to the number added 3. which maketh 12. And take the square of 9. the halfe number and of 12. the halfe number and the number added which squares are 81. and 144. and which two squares also added together make 225: vnto which summe the foresayd number 450. is double. As you see in the example.

The whole.

The number added.

Multiplication of the whole and
the number added into himself.

Multiplication of the number
added, into himfelfe.

Multiplication of the halfe in-
to himfelfe.

Multiplication of the halfe, &
the number added into it selfe.

the halfe

The number composed of
the square of the whole &
the number added and of
the square of the number
added,

double to

the number composed of
the square of the halfe and
of the square of the halfe
and the number added.

The demonstration wherof followeth in Barlaam.

The tenth Proposition.

If an even number be divided into two equal numbers, and unto it be added any other number: the square number of the whole number composed of the number and of that which is added, and the square number of the number added: these two square numbers (I say) added together, are double to these square numbers, namely, to the square of the half number, and to the square of the number composed of the half number and of the number added.

Suppose that the number AB being an even number be divided into two equal numbers AC and CB : and unto it let be added an other number BD . Then I say, that the square numbers of the numbers AD and DB are double to the square numbers of AC and CD . For forasmuch as the number AD is divided into the numbers AB and BD : therefore the square numbers of the numbers AD and DB are equal to the superficial number produced of the multiplication of the numbers AD and DB the one into the other twice, together with the square of the number AB (by the 7 proposition). But the square of the number AB is equal to four squares of either of the numbers AC or CB (for AC is equal to the number CB): wherefore also the squares of the numbers AD and DB are equal to the superficial number produced of the multiplication of the numbers AD and DB the one into the other twice, and to four squares of the number BC or CA . And forasmuch as the superficial number produced of the multiplication of the numbers AD and DB the one into the other, together with the square of the number CB , is equal to square of the number CD (by the 6 proposition): therefore the number produced of the multiplication of the numbers AD and DB the one into the other twice together with two squares of the number CB , is equal to two squares of the number CD . Wherefore the squares of the numbers AD and DB are equal to

The second Booke

two squares of the number CD , and to two squares of the number AC . Wherefore they are double to the squares of the numbers AC and CD . And the square of the number AD is the square of the whole and of the number added: And the square of DB is the square of the number added: the square also of the number CD is the square of the number composed of the halfe and of the number added: If therefore an even number be devided. &c. Which was required to be proued.



$$\left. \begin{array}{r} 8 \\ 8 \\ \hline 64 \end{array} \right\} \text{the square of } AD$$

$$\left. \begin{array}{r} 2 \\ 2 \\ \hline 4 \end{array} \right\} \text{the square of } DB$$

$$\left. \begin{array}{r} 5 \\ 5 \\ \hline 25 \end{array} \right\} \text{the square of } CD, \text{ namely, of the number composed of the halfe and of the number added.}$$

$$\left. \begin{array}{r} 3 \\ 3 \\ \hline 9 \end{array} \right\} \text{the square of the halfe } AC.$$

$$\left. \begin{array}{r} 25 \\ 9 \\ \hline 34 \end{array} \right\} \left. \begin{array}{r} 64 \\ 4 \\ \hline 68 \end{array} \right\} 34 \mid 2.$$

The I. Probleme. The II. Proposition.

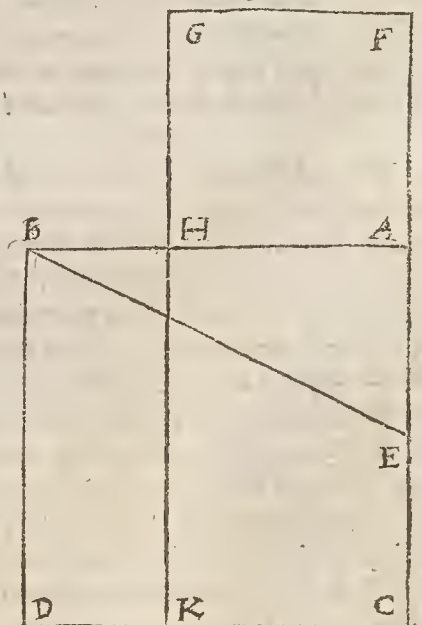
To deuide a right line geuen in such sort, that the rectangle figure comprehended vnder the whole, and one of the partes, shall be equall vnto the square made of the other part.



Suppose that the right line geuen be AB . Now it is required to deuide the line AB in such sort, that the rectangle figure containd vnder the whole and one of the partes, shall be equall vnto the square which is made of the other part. Describe (by the 46. of the first) vpon AB

Construction.

a square $ABCD$. And (by the 10. of the first) deuide the line AC into two equall partes in the point E , and draw a line from B to E . And (by the second petition) extend CA vnto the point F . And (by the 3. of the first) put the line EF equall vnto BE . And (by the 46. of the first) vpon the line AF describe a square $FGAH$. And (by the 2. petition) extend GH vnto the point K . Then I say that the line AB is devided in the point H in such sort, that the rectangle figure which is comprehended vnder AB and BH is equall to the square which is made of AH . For forasmuch as the right line AC is devided into two equall partes in the poynt E , and vnto it is added an other right line



Demonstratio

AF.

AF. Therefore (by the 6. of the second) the rectangle figure containd vnder *CF* and *FA* together with the square which is made of *AE* is equall to y^e square which is made of *EF*. But *EF* is equall vnto *EB*. VVherefore the rectangle figure containd vnder *CF*, and *FA* together with the square which is made of *EA* is equall to the square which is made of *EB*. But (by 47. of the first) vnto the square which is made of *EB* are equall the squares which are made of *BA* and *AE*. For the angle at the poynt *A* is a right angle. VVherefore that which is containd vnder *CF* and *FA*, together with the square which is made of *AE*, is equall to the squares which are made of *BA* and *AE*. Take away the square which is made of *AE* which is common, to them both: VVherefore the rectangle figure remayning containd vnder *CF* and *FA* is equall vnto the square which is made of *AB*. And that which is contained vnder the lines *CF* and *FA* is the figure *FK*. For the line *FA* is equall vnto the line *FG*. And the square which is made of *AB* is the figure *AD*. VVherefore the figure *FK* is equall vnto the figure *AD*. Take away the figure *AK* which is common, to them both. VVherefore the residue, namely, the figure *FH* is equall vnto the residue, namely, vnto the figure *HD*. But the figure *HD* is that, which is containd vnder the lines *AB* and *BH*, for *AB* is equall vnto *BD*. And the figure *FH* is the square which is made of *AH*. VVherefore the rectangle figure comprehended vnder the lines *AB* and *BH* is equall to the square which is made of the line *HA*. VVherefore the right line geuen *AB* is devided in the point *H*, in such sort that the rectangle figure containd vnder *AB* and *BH* is equall to the square which is made of *AH*: which was required to be done.

Thys proposition hath many singular vses. Vpon it dependeth the demonstration of that worthy Probleme the 10. Proposition of the 4. booke: which teacheth to describe an Isosceles triangle, in which eyther of the angles at the base shall be double to the angle at the toppe. Many and diuers vses of a line so devided shall you finde in the 13. booke of *Euclide*.

Many and singular vses of this proposition.

Thys is to be noted that thys Proposition can not as the former Propositions of thys second booke be reduced vnto numbers. For the line *EB* hath vnto the line *AE* no proportion that can be named, and therefore it can not be expressed by numbers. For forasmuch as the square of *EB* is equall to the two squares of *AB* and *AE* (by the 47. of the first) and *AE* is the halfe of *AB*, therefore the line *BE* is irrationall. For euen as two equall square numbers ioyned together can not make a square number: so also two square numbers, of which the one is the square of the halfe roote of the other, can not make a square number. As by an example. Take the square of 8. which is 64. which doubled, that is, 128. maketh not a square number. So take the halfe of 8. which is 4. And the squares of 8. and 4. which are 64. and 16. added together likewyse make not a square number. For they make 80. who hath no roote square. Which thyng must of necessity be if thys Probleme should haue place in numbers.

This proposition can not be reduced vnto numbers.

But in Irrationall numbers it is true, and may by thys example be declared.

The second Booke

Let 8. be so deuided, that that which is produced of the whole into one of his partes shall be equall to the square number produced of the other part. Multiply 8. into him selfe and there shall be produced 64. that is, the square $ABCD$. Deuide 8. into two equall partes, that is, into 4, and 4. as the line AE or EC . And multiply 4. into hym selfe, and there is produced 16, which adde vnto 64, and there shall be produced 80: whose roote is $\sqrt{80}$: which is the line EB or the line EF by the 47. of the first. And forasmuch as the line EF is $\sqrt{80}$. & the lyne EA is 4. therfore the lyne AF is $\sqrt{80}-4$. And so much shall the line AH be. And the line BH shall be $8-\sqrt{80}-4$, that is, $12-\sqrt{80}$. Now the $12-\sqrt{80}$ multiplied into 8 shal be as much as $\sqrt{80}-4$. multiplied into it selfe. For of either of them is produced $96-\sqrt{5120}$.

The 11. Theoreme. The 12. Proposition.

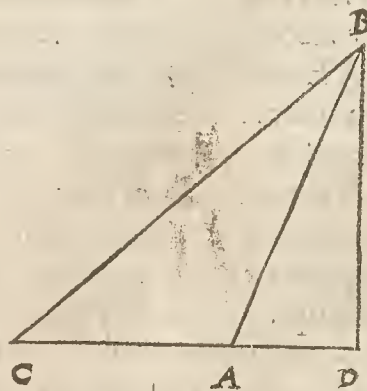
In obtuseangle triangles, the square which is made of the side subtending the obtuse angle, is greater then the squares which are made of the sides which comprehend the obtuse angle, by the rectangle figure, which is comprehended twise vnder one of those sides which are about the obtuse angle, vpon which being produced falleth a perpendicular line, and that which is outwardly taken betwene the perpendicular line and the obtuse angle.



Demonstration.

Suppose that ABC be an obtuseangle triangle hauing the angle BAC obtuse, and from the point B (by the 12. of the first) draw a perpendicular line vnto CA produced and let the same be BD . Then I say that the square which is made of the side BC , is greater then the squares which are made of the sides BA and AC , by the rectangle figure comprehended vnder the lines CA and AD

twise. For forasmuch as the right line CD is by chaunce deuided in the poynt A , therefore (by the 4. of the second) the square which is made of CD is equall to the squares which are made of CA and AD , and vnto the rectangle figure contayned vnder CA and AD twise. Put the square which is made of DB common vnto them both. Wherefore the squares which are made of CD and DB are equall to the squares which are made of the lines CA , AD , and DB , and vnto the rectangle figure contayned vnder the lines CA and AD twise. But (by the 47. of the first) the square which is made of CB is equall to the squares which are made of the lines



CD

CD , and DB . For the angle at the point D is a right angle. And vnto the squares which are made of AD and DB (by the selfe same) is equall the square which is made of AB . Wherefore the square which is made of CB , is equall to the squares which are made of CA and AB and vnto the rectangle figure containd vnder the lines CA and AD twise. Wherefore y square which is made of CB , is greater then the squares which are made of CA and AB by the rectangle figure containd vnder the lines CA and AD twise. In obtuseangle triangles therefore, the square which is made of the side subtending the obtuse angle, is greater then the squares which are made of the sides which comprehend the obtuse angle, by the rectangle figure which is comprehended twise vnder one of those sides which are about the obtuse angle, vpon which being produced falleth a perpendicular lyne, and that which is outwardly taken betwene the perpendicular lyne and the obtuse angle: which was required to be demonstrated.

Of what force thys Proposition, and the Proposition following, touching the measuring of the obtuseangle triangle and the acuteangle triangle, with the ayde of the 47. Proposition of the first booke touching the rightangle triangle, he shall well perceauce, which shall at any time neede the arte of triangles in which by three thinges knowen is euer searched out three other thinges vnknown, by helpe of the table of arkes and cordes.

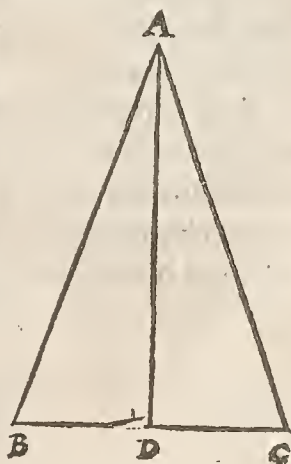
The 12. Theoreme. The 13. Proposition.

 In acuteangle triangles, the square which is made of the side that subtendeth the acute angle, is lesse then the squares which are made of the sides which comprehend the acute angle, by the rectangle figure which is cōprehended twise vnder one of those sides which are about the acuteangle, vpo which falleth a perpendicular lyne, and that which is inwardly taken betwene the perpendicular lyne and the acute angle.

 Suppose that ABC be an acuteangle triangle ha-
uyng the angle at y point B acute, & (by the 12. of
the first) from the point A draw vnto the lyne BC
 CA a perpendicular lyne AD . Then I say that the square
which is made of the lyne AC is lesse then the squares
which are made of the lyne CB and BA , by the rectangle
figure conteyned vnder the lines CB and BD twise. For
forasmuch as the right lyne BC is by chaunce denided in
the point D , therefore (by the 7. of the second) the squares

X.ij.

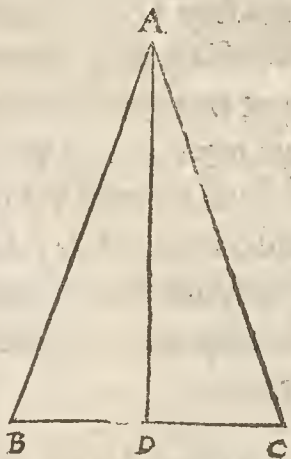
which



Demonstratio

The second Booke

which are made of the lines CB and BD are equall to the rectangle figure contained vnder the lines CB and DB twise and vnto the square whiche is made of line CD . Put the square which is made of the line DA common vnto them both. Wherefore the squares which are made of the lines CB , BD , and DA , are equall vnto the rectangle figure containd vnder the lines CB and BD twise, and vnto the squares which are made of AD and DC . But to the squares whiche are made of the lines BD and DA is equal $\hat{y}$ square which is made of the line AB : for th' angle at $\hat{y}$ point D is a right angle. And vnto the squares whiche are made of the lines AD and DC is equall the square whiche is made of the line AC (by the 47. of $\hat{y}$ first): wherefore the squares which are made of the lines CB and BA are equal to the square which is made of the line AC , and to that which is contained vnder the lines CB and BD twise. Wherefore the square which is made of the line AC beyng taken alone, is lesse then the squares which are made of the lines CB and BA by the rectangle figure, which is contained vnder the lines CB and BD twise. In rectangle triangles therfore the square which is made of the side that subtendeth the acute angle, is lesse then the squares which are made of the sides which comprehend the acute angle, by the rectangle figure which is comprehended twise vnder one of those sides which are about the acute angle, vpon which falleth a perpendicular line, and that which is inwardly taken betwene the perpendicular line and the acute angle: which was required to be proued.



¶ A Corollary added by Orontius.

A Corollary.

Hereby is easily gathered, that such a perpendicular line in rectangle triangles falleth of necessitie vpon the side of the triangle, that is, neyther within the triangle, nor without. But in obtuseangle triangles it falleth without, and in acuteangle triangles within. For the perpendicular line in obtuseangle triangles, and acuteangle triangles can not exactly agree with the side of the triangle: for then an obtuse & an acuteangle should be equal to a right angle, contrary to the eleuenth and twelfth definitions of the first booke. Likewise in obtuseangle triangles it can not fall within, nor in acuteangle triangles without: for then the outward angle of a triangle should be lesse then the inward and opposite angle, which is contrary to the 16. of the first.

This Proposition true in all kindes of triangles.

And this is to be noted, that although properly an acuteangle triangle, by the definition therof geuē in the first booke, be that triangle, whose angles be all acute: yet forasmuch as there is no triangle, but that it hath an acute angle, this proposition is to be vnderstanded, & is true generally in all kindes of triangles whatsoeuer, and may be declared by them, as you may easily proue.

The

The 2. Probleme. The 14. Proposition.

Vnto a rectiline figure geuen, to make a square equall.



Suppose that the rectiline figure geuen be A . It is required to make a square equall vnto the rectiline figure A . Make (by the 45. of y^{e} first) vnto the rectiline figure A an equall rectangle parallelogramme $BCDE$. Now if y^{e} line BE be equall vnto the line ED , then is y^{e} thyng

Construction.

done whiche was required: for vnto the rectiline figure A is made an equal square BD . But if not, one of these lines BE & ED the greater. Let BE be the greater, and let it be produced vnto y^{e} point F .

And (by the 3. of the first) put vnto ED an equall line EF . And (by the 10. of the first) deuide the line BF into two equall partes in the point G . And making the

centre the point G , and the space GB or GF describe a semicircle BHF . And (by the 2. petition) extend the line DE vnto y^{e} point H . And (by the 1. petition) draw a line from G to H . And forasmuch

as the right line FB is deuided into two equall partes in the point G , and into two vnequal partes in the point E , therefore

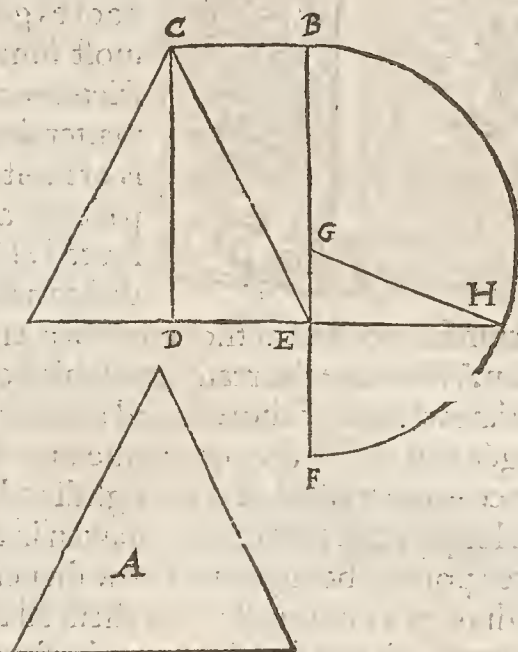
(by the 5. of the second) the rectangle figure comprehended vnder the lines BE and EF together with the square which is made of the line EG , is equall to the square which is made of the line GF . But the line GF is equall vnto the line GH . VVherfore the rectangle figure comprehended vnder the lines BE and EF

together with the square which is made of the line GE is equall to square which is made of the line GH . But vnto the square which is made of the line GH are equall the squares whiche are made of the lines HE and GE (by the 47. of the first.) VVherfore y^{e} which is contained vnder y^{e} lines BE and EF together with

y^{e} square which is made of GE is equall to y^{e} squares which are made of HE and GE . Take away the square of the line EG common to them both. VVherfore the rectangle figure contained vnder the lines BE & EF is equall to the square

which is made of the line EH . But that whiche is contained vnder the lines BE and EF is the parallelogramme BD , for the line EF is equall vnto the line ED . VVherfore the parallelogramme BD is equall to y^{e} square whiche is made of the line HE . But the parallelogramme BD is equall vnto the rectiline figure

A . VVherfore y^{e} rectiline figure A is equall to the square which is made of y^{e} line HE . VVherfore vnto the rectiline figure geuen A , is made an equall square described of the line EH : which was required to be done.



Demonstratio

¶ The ende of the second Booke
of Euclides Elementes.

The third booke of Eu- clides Elementes.

*The argument
of this booke.*



His third booke of Euclide entreateth of the most perfect figure, which is a circle. Wherefore it is much more to be esteemed then the two bookes goyng before, in which he did set forth the most simple proprieties of rightlined figures. For sciences take their dignities of the worthynes of the matter that they entreat of. But of al figures the circle is of most absolute perfection, whose proprieties and passions are here set forth, and most certainly demonstrated. Here also is entreated of right lines subtended to arkes in circles: also of angles set both at the circumference and at the centre of a circle, and of the varietie and differences of them. Wherefore the readyng of this booke, is very profitable to the attayning to the knowledge of chordes and arkes. It teacheth moreouer which are circles contingēt, and which are cutting the one the other: and also that the angle of contingence is the least of all acute rightlined angles: and that the diameter in a circle is the longest line that can be drawen in a circle. Farther in it may we learne how, three pointes beyng geuen how soeuer (so that they be not set in a right line), may be drawen a circle passing by them all three. Agayne, how in a solide body, as in a Sphere, Cube, or such lyke, may be found the two opposite pointes. Whiche is a thyng very necessary and commodious: chiefly for those that shall make instrumentes seruyng to Astronomy, and other artes.

Definitions.

The first definition.

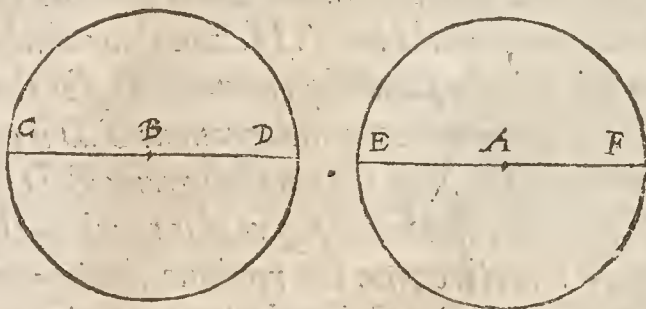
Equall circles are such, whose diameters are equall, or whose lynes drawen from the centres are equall.

The circles A and B are equal, if theyr diameters, namely, E F and C D be equall: or if their semidiameters, whiche are lynes drawen from the center to the circumference; namely A F and B D be equall.

*Why circles
take their
equality of
their diame-
ters or semi-
diameters.*

The reason why circles take theyr equalitie, of the equalitie of their diameters or semidiameters is, for that a circle is described by one reuolution or turnyng about of the semidiameter, hauing one of his endes fixed. As if you imagine the lyne A E to haue

his one point namely A fastened, and the other end namely E to moue round till
it



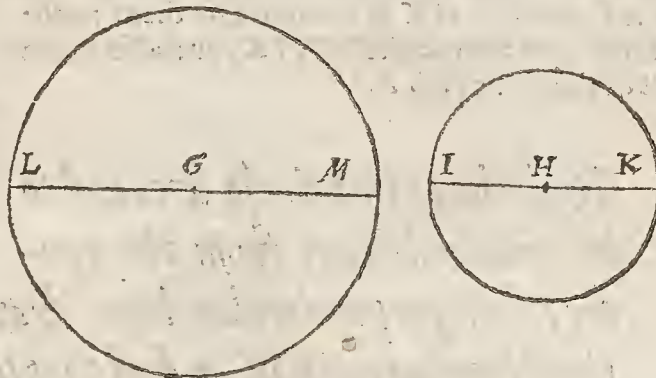
it come to the place where it begā to moue, it shal fully describe the whole circle. Wherefore if the semidiameters bee equall, the circles of necessitye must also be equall: and also the diameters.

By thys also is knowen the definition of vnequall circles.

Definition of
vnequall cir-
cles.

Circles whose diameters or semidiameters are vnequall, are also vnequall. And that circle which hath the greater diameter or semidiameter, is the greater circle: and that circle which hath the lesse diameter or semidiameter, is the lesse circle.

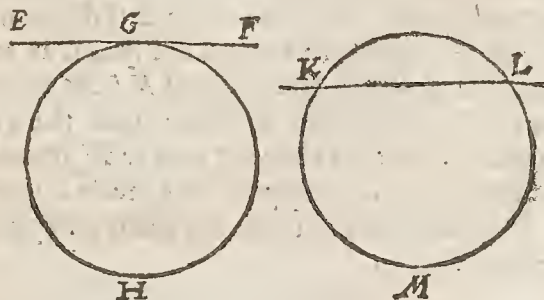
As the circle L M is greater then the circle I K, for that the diameter L M is greater then the diameter I K: or for that the semidiameter G L is greater then the semidiameter H I.



A right line is sayd to touch a circle, which touching the circle and being produced cutteth it not.

Second defini-
tion.

As the right lyne E F drawn from the point E, and passyng by a point of the circle, namely, by the point G to the point F on-ly toucheth the circle G H, and cutteth it not, nor entreth within it. For a right line entryng within a circle, cutteth and deu- ideth the circle. As the right lyne K L deu- ideth and cutteth the circle K L M, and entreth within it: and therefore toucheth it in two places. But a right lyne tou- chyng a circle, which is commonly called a cōtingent lyne, toucheth the circle on- ly in one point.

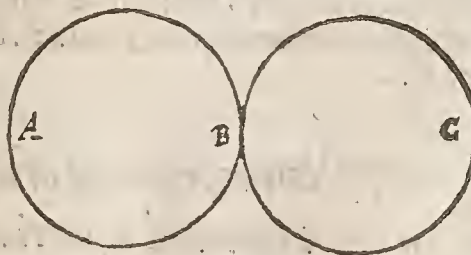


A contingent
lyne.

Circles are sayd to touch the one the other, which touching the one the other, cut not the one the other.

Third defini-
tion.

As the two circles A B and B C touch the one the other. For theyr circumferences touch together in the poynt B. But neither of them cutteth or deu ideth, the other. Neither doth any part of the one enter within the o- ther. And such a touch of circles is euer in one poynt onely: which poynt onely is common to them both. As the poynt B is in the confe- rence of the circle A B, and also is in the circu- ference of the circle B C.



The touch of
circles is euer
in one poynt
onely.

Az.j.

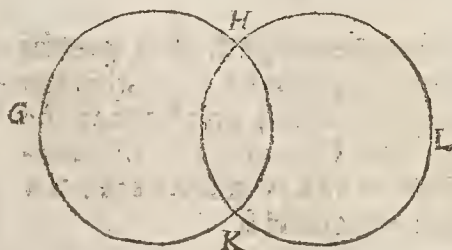
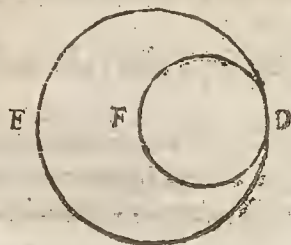
Circles

The third Booke

Circles may
touch toge-
ther two ma-
ner of wayes.

Circles may touch together two maner of wayes, either outwardly the one wholly without the other : or els the one being containd within the other.

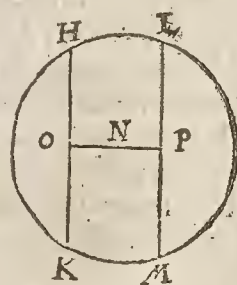
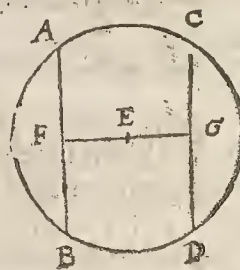
As the circles DE and DF : of which the one DE containeth the other, namely DF : and touch the one the other in the poynt D : and that onely poynt is common to them both : neither doth the one enter into the other . If any part of the one enter into any part of the other, then the one cutteth and divideth the other, and toucheth the one the other not in one poynt onely as in the other before, but in two pointes, and haue also a superficies common to them both. As the circles $G HK$ and HLK cut the one the other in two poyntes H and K : and the one entreteth into the other : Also the superficies HK is common to them both : For it is a part of the circle $G HK$, and also it is a part of the circle HLK .



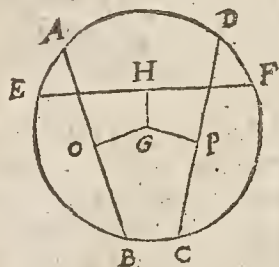
Fourth defi-
nition.

Right lines in a circle are sayd to be equally distant from the centre, when perpendicular lines drawn from the centre vnto those lines are equall. And that line is sayd to be more distant, vpon whom falleth the greater perpendicular line.

As in the circle $ABCD$ whose centre is E , the two lynes AB and CD haue equall distance from the centre E : bycause that the lyne EF drawn from the centre E perpendicularly vpon the lyne AB , and the lyne EG drawn likewise perpendicularly from the centre E vpon the lyne CD are equall the one to the other . But in the circle $HKLM$ whose centre is N the lyne HK hath greater distance from the centre N then hath the lyne LM : for that the lyne ON drawn from the centre N perpendicularly vpon the lyne HK is greater then the lyne NP which is drawn frō the centre N perpendicularly vpon the lyne LM .



So likewise in the other figure the lynes AB and DC in the circle $ABCD$ are equidistant from the centre G , bycause the lynes OG and GP perpendicularly drawn from the centre G vpon the sayd lynes AB and DC are equall . And the lyne AB hath greater distance from the centre G then hath the lyne EF , bycause the lyne OG perpendicularly drawn from the centre G to the lyne AB is greater then the lyne HG whiche is perpendicularly drawn from the centre G to the lyne EF .

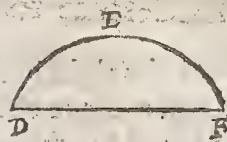
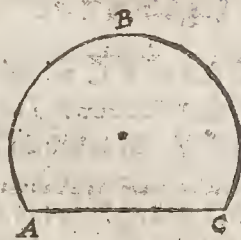


Fift defini-
tion.

A section or segment of a circle, is a figure cōprehended vnder a right line and a portion of the circumference of a circle.

As the

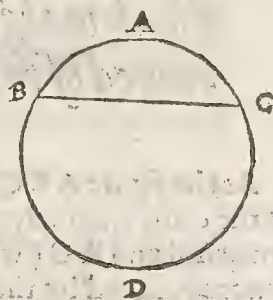
As the figure ABC is a section of a circle bycause it is comprehended vnder the right lyne AC and the circumference of a circle ABC . Likewise the figure DEF is a section of a circle, for that it is comprehended vnder the right lyne DF , and the circumference DEF . And the figure ABC for that it containeth within it the centre of the circle is called the greater section of a circle: and the figure DEF is the lesse section of a circle, bycause it is wholly without the centre of the circle as it was noted in the 16. Definition of the first booke.



An angle of a section or segment, is that angle which is containd vnder a right line and the circumference of the circle.

Sixth definition.

As the angle ABC in the section ABC is an angle of a section, bycause it is contained of the circumference BAC and the right lyne BC . Likewise the angle CBD is an angle of the section BDC bycause it is containd vnder the circumference BDC , and the right lyne BC . And these angles are commonly called mixte angles, bycause they are containd vnder a right lyne and a crooked. And these portions of circumferences are commonly called arkes, and the right lynes are called chordes, or right lynes subtended. And the greater section hath euer the greater angle, and the lesse section the lesse angle.



Mixt angles.

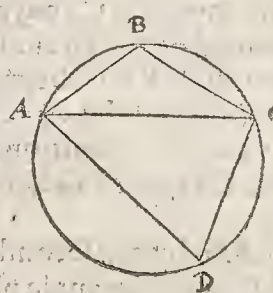
Arkes.

Chordes.

An angle is sayd to be in a section, whē in the circumference is taken any poynt, and from that poynt are drawn right lines to the endes of the right line which is the base of the segment, the angle which is containd vnder the right lines drawn from the poynt, is (I say) sayd to be an angle in a section.

Seventh definition.

As the angle ABC is an angle in the section ABC , bycause from the poynt B beyng a poynt in the circumference ABC are drawn two right lynes BC and BA to the endes of the lyne AC which is the base of the section ABC . Likewise the angle ADC is an angle in the section ADC , bycause from the poynt D beyng in the circumference ADC are drawn two right lynes, namely, DC & DA to the endes of the right line AC which is also the base to the sayd section ADC . So you see, it is not all one to say, an angle of a section, and an angle in a section. An angle of a section consisteth of the touch of a right lyne and a crooked. And an angle in a section is placed on the circumference, and is containd of two right lynes. Also the greater section hath in it the lesse angle, and the lesse section hath in it the greater angle.



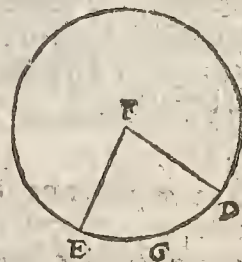
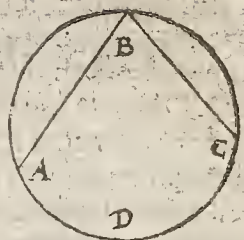
Difference of an angle of a Section, and of an angle in a Section.

But when the right lines which comprehend the angle do re- ceauē any circumference of a circle, then that angle is sayd to be correspondēt, and to pertaine to that circumference.

Eight definition.

The third Booke

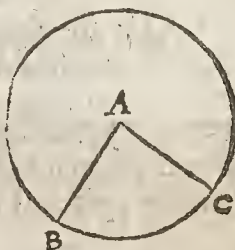
As the right lynes B A and B C which containe the angle A B C, and receave the circumference A D C therefore the angle A B C is sayd to subtend and to pertaine to the circumference A D C. And if the right lynes which cause the angle, concurre in the centre of a circle: then the angle is sayd to be in the centre of a circle. As the angle E F D is sayd to be in the centre of a circle, for that it is comprehended of two right lynes F E and F D: which concurre and touch in the centre F. And this angle likewise subtendeth the circumference E G D: which circumference also, is the measure of the greatnes of the angle E F D.



Ninth definition.

A Sector of a circle is (an angle being set at the centre of a circle) a figure containd vnder the right lines which make that angle, and the part of the circumference receaved of them.

As the figure A B C is a sector of a circle, for that it hath an angle at the centre, namely the angle B A C, & is contained of the two right lynes A B and A C (which containe that angle and the circumference receaved by them).

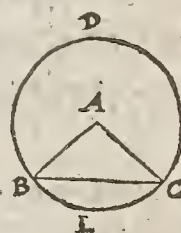


Tenth definition.

Like segmentes or sections of a circle are those, which haue equall angles, or in whom are equall angles.

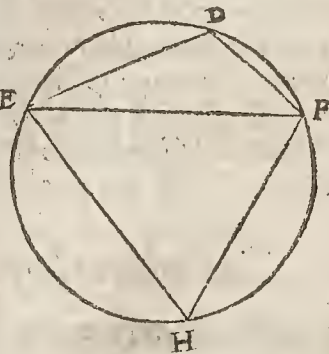
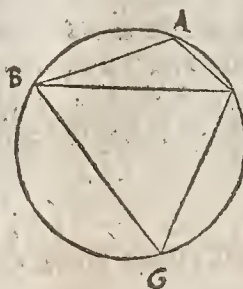
Two definitions.
First.

Here are set two definitions of like sections of a circle. The one pertaineth to the angles which are set in the centre of the circle and receave the circumference of the sayd sections: the other pertaineth to the angle in the section, which as before was sayd is euer in the circumference. As if the angle B A C, beyng in the centre A and receaved of the circumference B L C be equall to the angle F E G beyng also in the centre E and receaved of the circumference F K G, then are the two sections B C L and F G K lyke by the first definition. By the same definition also are the other two sections like, namely B C D, and F G H, for that the angle B A C is equall to the angle F E G.



Second.

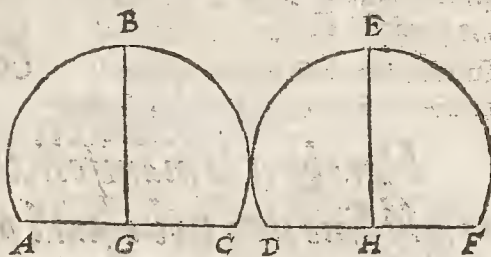
Also by the second definition if B A C beyng an angle placed in the circumference of the section B C A be equall to the angle E D F beyng an angle in the section E F D placed in the circumference, there are the two sections B C A, and E F D lyke the one to the other. Likewise also if the angle B G C beyng in the section B C G be equall to the angle E H F beyng in the section E H F the two sections B C G and E F H are lyke. And so is it of angles beyng equall in any poynt of the circumference.



Euclide

Euclide defineth not equall Sections: for they may infinite wayes be described. For there may vpon vnequall right lynes be set equall Sections (but yet in vnequall circles) For from any circle beyng the greater, may be cut of a portion equall to a portion of an other circle beyng the lesse. But when the Sections are equall, and are set vpon equall right lynes, theyr circumferences also shalbe equall. And right lynes beyng deuided into two equall partes, perpendicular lynes drawn from the poyntes of the diuision to the circumferences shalbe equall. As if the two sections ABC and DEF , beyng set vpon equall ryght lynes AC & DF , be equall: then if ech of the two lynes AC & DF be deuided into two equall partes in the poyntes G and H , & from the sayd poyntes be drawn to the circumferences two perpendicular lynes BG and EH , the sayd perpendicular lynes shalbe equall.

Why Euclide defineth not equall Sections.



The 1. Probleme. The 1. Proposition.

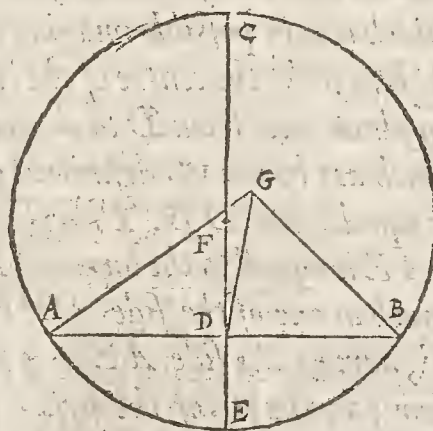
To finde out the centre of a circle geuen.



Suppose that there be a circle geuen ABC . It is required to finde out the centre of the circle ABC . Draw in it a right line at all aduentures, and let the same be AB . And (by the 10. of the first) deuide the line AB into two equall partes in the poynt D . And (by the 11. of the same) frō the poynt D raise vp vnto AB a perpendicular line DC , & (by the second petition) extend DC vnto y point

Construction.

E . And (by the 10. of the first) deuide the line CE into two equall partes in the poynt F . Then I say that the point F is the centre of the circle ABC . For if it be not, let some other point, namely G , be the centre. And (by the first petition) draw these right lines GA , GD , and GB . And for asmuch as AD is equall vnto DB , and DG is common vnto the both, therefore these two lines AD and DG are equall to these two lines GD and DB , the one to the other, and (by the 15. definition of the first) the base GA is equall to the base GB . For they are both drawn from the centre G to the circumference: therefore (by the 8. of the first) the angle ADG is equall to the angle $B DG$. But when a right line standing vpon a right line maketh the angles on eche side equall the one to the other, eyther of those angles (by the 10. definition of the first) is a right angle. Wherefore the angle $B DG$



Demonstration leading to an impossibility.

Aa. iij.

is a

The third Booke

is a right angle: but $\angle FDB$ is also a right angle by construction. Wherefore (by the 4. petition) the angle FDB is equall to the angle $B DG$, the greater to the lesse, which is impossible. Wherefore the poynt G is not the centre of the circle ABC . In like wise may we proue that no other poynt besides F is the centre of the circle ABC . Wherefore the poynt F is the centre of the circle ABC : which was required to be done.

Correlary.

Correlary.

Hereby it is manifest, that if in a circle a right line do deuide a right line into two equall partes, and make right angles on eche side: in that right line which deuideth the other line into two equall partes is the centre of the circle.

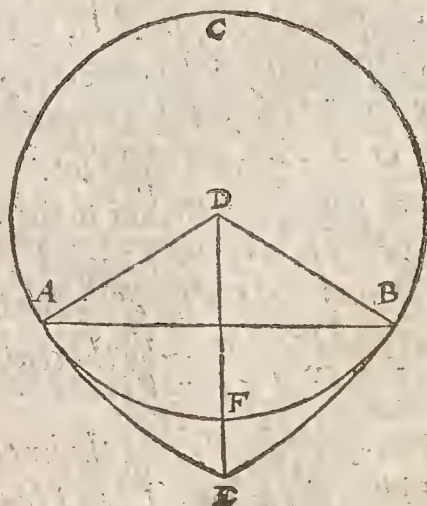
The 1. Theoreme. The 2. Proposition.

If in the circūference of a circle be take two poyntes at all aduentures: a right line drawen from the one poynt to the other shall fall within the circle.

Demonstratio leading to an impossibilitie.



Suppose that there be a circle ABC . And in the circumference thereof, let there be take at all aduentures these two poyntes A & B . Then I say that a right line drawen from A to B shall fall within the circle ABC . For if it do not, let it fall without the circle, as the line AEB doth, which if it be possible imagine to be a right line. And (by the Proposition going before) take the centre of the circle, and let the same be D . And (by the first petition) draw lines from D to A , and from D to B . And extend DF to E . And for asmuch as (by the 15. definition of γ first) DA is equall vnto DB . Therefore the angle DAE is equall to the angle DBE . And for asmuch as one of the sides of the triangle DAE , namely the side AEB is produced, therefore (by the 16. of the first) the angle DEB , is greater then the angle DAE . But the angle DAE is equall vnto the angle DBE . Wherefore the angle DEB is greater then the angle DBE . But (by the 18. of the first) vnto the greater angle is subtended the greater side. Wherefore the side DB is greater then the side DE . But (by the 15. definition of the first) the line DB is equall



equall vnto the line DF . Wherefore the line DF is greater then the line DE , namely, the lesse greater then the greater: which is impossible. Wherefore a right line drawn from A to B falleth not without the circle. In like sort also may we proue that it falleth not in the circumference: Wherefore it falleth within the circle. If therefore in the circumference of a circle be taken two poyntes at all aduentures: a right line drawn from the one poynt to the other shall fall within the circle: which was required to be proued.

The 2. Theoreme. The 3. Proposition.

If in a circle a right line passing by the centre do deuide an other right line not passing by the centre into two equall partes: it shall deuide it by right angles. And if it deuide the line by right angles, it shall also deuide the same line into two equall partes.

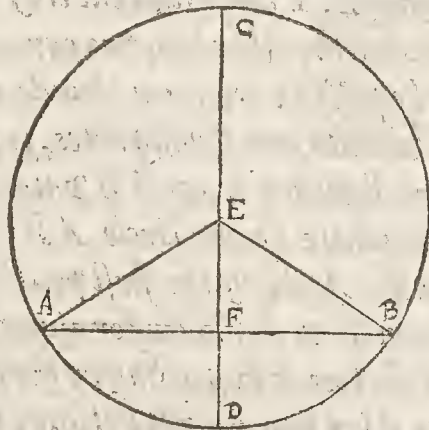


Suppose that there be a circle ABC , and let there be in it drawn a right line passing by the centre, and let the same be CD , deuiding an other right line AB not passing by the centre into two equall partes in the poynt F . Then I say that the angles at the poynt of the deuision

The first part of this Proposition.

are right angles. Take (by the first of the third) the centre of the circle ABC , and let the same be E . And (by the first petition) drawe lines from E to A & from E to B . And for asmuch as the line AF is equall vnto the line FB , and the line FE is common to them both, therefore these two lines EF and FA are equall vnto these two lines EF & FB . And the base EA is equall vnto the base EB (by the 15. definition of the first). Wherefore (by the 8. of the first) the angle AFE is equall to the angle

Construction.



Demonstration.

BFE . But when a right line standing vpon a right line doth make the angles on eche side equall the one to the other, eyther of those angles is (by the 16. definition of the first) a right angle. Wherefore either of these angles AFE , & BFE is a right angle. Wherefore the line CD passing by the centre, and deuiding the line AB not passing by the centre into two equall partes, maketh at the point of the deuision right angles.

But now suppose that the line CD do deuide the line AB in such sort that it maketh right angles. Then I say that it deuideth it into two equall partes, that is, the line AF is equall vnto the line FB . For the same order of construction remayning, for asmuch as the line EA is equall vnto the line EB (by the 15. definition,

The second part conuerse of the first.

Demonstration.

Aa.iiij. finition

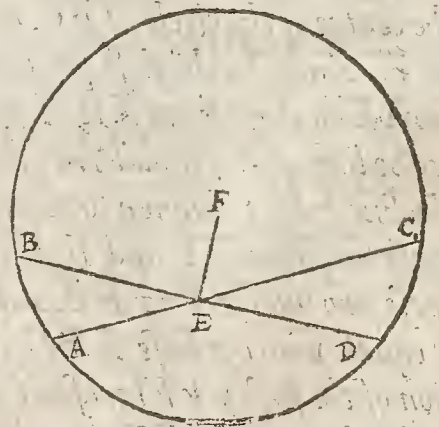
The third Booke

finition of the first). Therefore the angle EAF is equall vnto the angle EBF (by the 5. of the first). And the right angle AFE is (by the 4. petition) equall to the right angle BFE . Wherefore there are two triangles EAF , & EBF hauing two angles equall to two angles, & one side equall to one side, namely the side EF which is common to them both, and subtendeth one of the equall angles, wherefore (by the 26. of the first) the sides remayning of the one, are equall vnto the sides remayning of the other. Wherefore the line AF is equall vnto the line FB . If therefore in a circle a right line passing by the centre do deuide another right line not passing by the centre into two equall partes, it shall deuide it by right angles. And if it deuide the line by right angles it shall also deuide the same line into two equall partes: which was required to be demonstrated.

The 3. Theoreme. The 4. Proposition.

If in a circle two right lines not passing by the centre, deuide the one the other: they shall not deuide eche one the other into two equall partes.

Suppose that there be a circle $ABCD$, and let there be in it drawn two right lines not passing by the centre and deuinding the one the other, and let the same be AC and BD , which let deuide the one the other in the poynt E . Then I say that they deuide not eche the one the other into two equall partes. For if it be possible let them deuide eche the one the other into two equall partes, so that let AE be equall vnto EC , & BE vnto ED . And take the centre of the circle $ABCD$, which let be F . And (by the first petition) draw a line from F to E . Now for asmuch as a certaine right line FE passing by the centre deuiddeth another line AC not passing by the centre into two equall partes, it maketh where the deuision is right angles (by the 3. of the third). Wherefore the angle FEA is a right angle. Againe for asmuch as the right line FE , passing by the centre; deuiddeth the right line BD not passing by the centre into two equall partes, therefore (by the same) it maketh where the deuision is right angles. Wherefore the angle FEB is a right angle. And it is proued that the angle FEA is a right angle. Wherefore (by the 4. petition) the angle FEA is equall vnto the angle FEB , namely the lesse angle vnto the greater: which is impossible. Wherefore the right lines AC and BD deuide not eche one the other into two equall partes. If therefore in a circle two right lines not passing by the centre, deuide the one the other, they shall

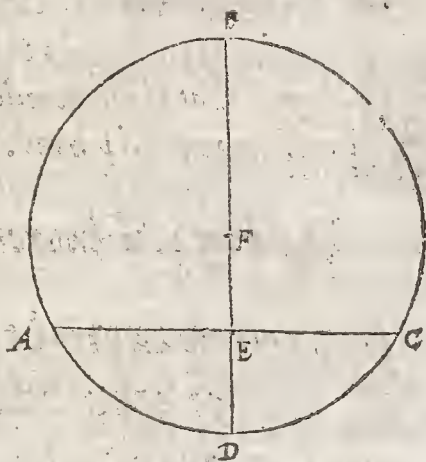


Demonstration leading to an impossibility.

shall not deuide eche one the other into two equall partes: which was required to be demonstrated.

In this Proposition are two cases. For the lines cutting the one the other, do eyther, neyther of them passe by the centre, or the one of them doth passe by the centre, & the other not. The first is declared by the author. The second is thus proued. Two cases in this Proposition.

Suppose that in the circle $ABCD$ the line BD passing by the centre doe cut the line AC not passing by the centre. Then I say that the lines AC and BD do not deuide the one the other into two equall partes. For by the former Proposition the line BD passing by the centre and deuiding the line AC into two equall partes, it shall also deuide it perpendicularly. And for asmuch as the line AC deuideth the line BD into two equall partes & right angled wise: therefore by the Correlary of the first of thys booke, the line AC passeth by the centre of the circle: which is cōtrary to the supposition. Wherefore the lines AC and BD do not deuide the one the other into two equall partes: which was required to be proued.



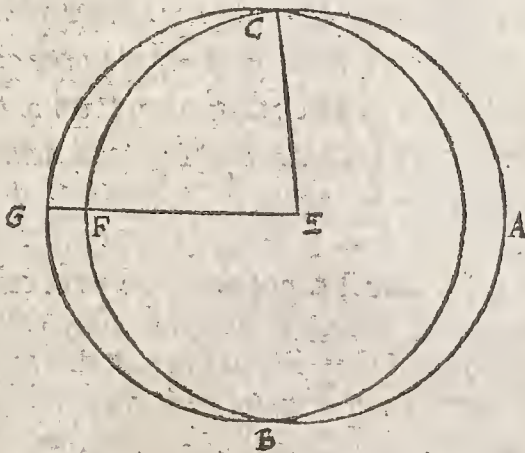
Construction for the second case.

Demonstration.

The 4. Theoreme. The 5. Proposition.

If two circles cut the one the other, they haue not one and the same centre.

Suppose that these two circles ABC , and CBG do cut the one the other in the poyntes C and B . Then I say that they haue not one & the same centre. For if it be possible let E be centre to them both. And (by the first petition) draw a line from E to C . And draw an other right line EFG at all aduentures. And for asmuch as the poynt E is the centre of the circle ABC , therefore (by the 15. definition of the first) the line EC is equall vnto the line EF . Agayne for asmuch as the poynt E is the centre of the circle CBG , therefore (by the same definition) the line EC is equall vnto the line EG . And it is proued that the line EC is equall vnto the line EF : wherefore the line EF also is equall vnto the line EG , namely the lesse vnto the greater: which is impossible. Wherefore the poynt E is not the centre of both the circles ABC , & CBG . In like sort also may we proue that no other poynt is the centre of both the sayd circles.



Construction.

Demonstration leading to an impossibility.

The third Booke

circles. If therefore two circles cut the one the other, they haue not one and the same centre: which was required to be proued.

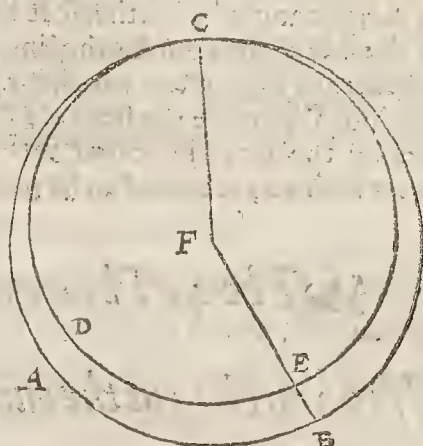
The 5. Theoreme. The 6. Proposition.

If two circles touch the one the other, they haue not one and the same centre.

Demonstration leading to an impossibility.

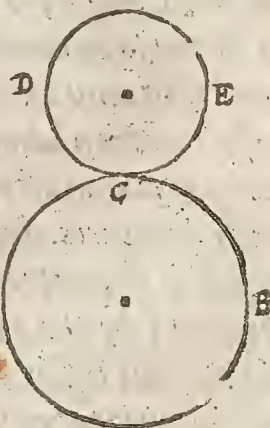


Suppose that these two circles ABC, & CDE do touch the one the other in the poynt C. Then I say that they haue not one and the same centre. For if it be possible let the point F be centre vnto them both. And (by the first petition) draw a line from F to C: and drawe the line FEB at all aduentures. And for asmuch as the poynt F is the centre of the circle ABC, therefore (by the 15. definition of the first) the line FC is equall vnto the line FB. Agayne forasmuch as the poynt F is the centre of y circle CDE, therefore (by the same definition) the line FC is equall vnto the line FE. And it is proued, that the line FC is equall vnto the line FB, wherefore the line FE also is equall vnto the line FB, namely the lesse vnto y greater: which is impossible. Wherefore the poynt F is not the centre of both the circles ABC and CDE. In like sort also may we proue that no other poynt is the centre of both the sayd circles. If therefore two circles touch the one the other: they haue not one and the same centre: which was required to be demonstrated.



Two cases in thys Proposition.

In thys Proposition are two cases: for the circles touchyng the one the other, may touch eyther within or without. If they touch the one the other within, then is it by the former demonstration manifest, that they haue not both one and the selfe same centre. It is also manifest if they touch the one the other without: for that euery centre is in the middest of hys circle.



The 6. Theoreme. The 7. Proposition.

If in the diameter of a circle be taken any poynt, which is not the

the centre of the circle, and from that poynt be drawen vnto the circumference certaine right lines: the greatest of those lines shall be that line wherein is the centre, and the lest shall be the residue of the same line. And of all the other lines, that which is nigher to the line which passeth by the centre is greater then that which is more distant. And from that point can fall within the circle on ech side of the least line onely two equall right lines.

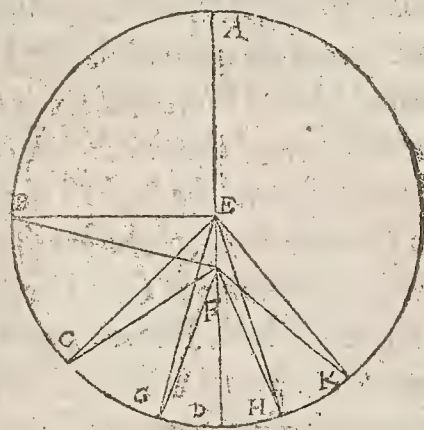


Suppose that there be a circle $ABCD$: and let the diameter thereof be AD . And take in it any poynt besides the centre of the circle, and let the same be F . And let the centre of the circle (by the 1. of γ third)

Construction.

be the poynt E . And from the poynt F let there be drawen vnto the circumference $ABCD$ these right lines FD , FC , and FG . Then I say that the line FA is the greatest: and the line FD is the lest.

And of the other lines, the line FB is greater then the line FC , and the line FC is greater then the line FG . Drawe (by the first petition) these right lines BE , CE , and GE . And for asmuch as (by the 20. of the first) in euery triangle two sides are greater then the third, therefore γ lines EB and EF are greater then the residue, namely then the line FB . But the line AE is equall vnto the line BE (by the 15. definition of the first). Wherefore the lines BE and EF are equall vnto the line AF .



The first part of this Proposition.

Demonstration.

Wherefore the line AF is greater then then the line BF . Agayne for asmuch as the line BE is equall vnto CE (by the 15. definition of the first) and the line FE is common vnto them both, therefore these two lines BE and EF are equall vnto these two CE and EF . But the angle BEF is greater then the angle CEF . Wherefore (by the 24. of the first) the base BF is greater then the base CF : and by the same reason the line CF is greater then the line FG . Agayne for asmuch as the lines GF and FE are greater then the line EG (by the 20. of the first). But (by the 15. definition of the first) the line EG is equall vnto the line ED : Wherefore the lines GF and FE are greater then the line ED , take away EF , which is comon to the both, wherfore γ residue GF is greater then the residue FD : Wherefore the line FA is the greatest, and the line FD is the lest, and the line FB is greater then the line FC , and the line FC greater

Second part.

The third Booke

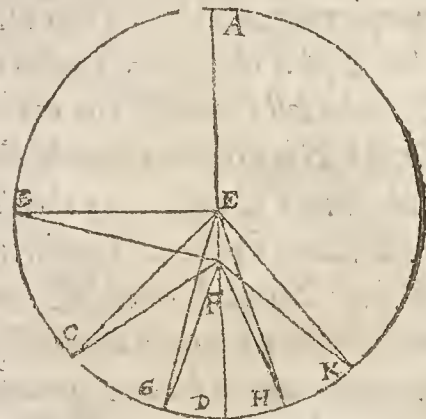
Third part.

is greater then the line FG . Now also I say that from the poynt F there can be drawen onely two equall right lines into the circle $ABCD$ on eche side of the least line, namely FD . For (by the 23. of the first) vpon the right line geuen EF and to the poynt in it, namely E , make vnto the angle GEF an equall angle FEH : and (by the first petition) draw a line from F to H . Now for asmuch as (by the 15. definition of the first) the line EG is equall vnto the line EH , and the line EF is common vnto them both, therefore these two lines GE and EH are equall vnto these two lines HE and EF , and (by construction) the angle GEF is equall vnto the angle HEF . Wherefore (by the 4. of the first) the base FG is equall vnto the base FH . I say moreover, that from the poynt F can be drawen into the circle no other right line equall vnto the line FG . For if it possible let the line FK be equall vnto the line FG . And for asmuch as FK is equall vnto FG . But the line FH is equall vnto the line FG , therefore the line FK is equall vnto the line FH . Wherefore the line which is nigher to the line which passeth by the centre is equall to that which is farther of, which we haue before proued to be impossible.

This demonstrated by an argument leading to an impossibility.

An other demonstration of the latter part of the Proposition leading also to an impossibility.

Or els it may thus be demonstrated. Draw (by the first petition) a line from E to K : and for asmuch as (by the 15. definition of the first) the line GE is equall vnto the line EK , and the line FE is common to them both, and the base GF is equall vnto the base FK , therefore (by the 8. of the first) the angle GEF is equall to the angle KEF . But the angle GEF is equall to the angle HEF . Wherefore (by the first common sentence) the angle HEF is equall to the angle KEF the lesse vnto the greater: which is impossible. Wherefore from the poynt F there can be drawen into the circle no other right line equall vnto the line GF . Wherefore but one onely. If therefore in the diameter of a circle be taken any poynt, which is not the centre of the circle, and from that poynt be drawen vnto the circumference certaine right lines: the greatest of those right lines shall be that wherein is the centre: and the least shall be the residue. And of all the other lines, that which is nigher to the line which passeth by the centre is greater then that which is more distant. And from that poynt can fall within the circle on ech side of the least line onely two equall right lines: which was required to be proued.



¶ A Corollary.

A Corollary.

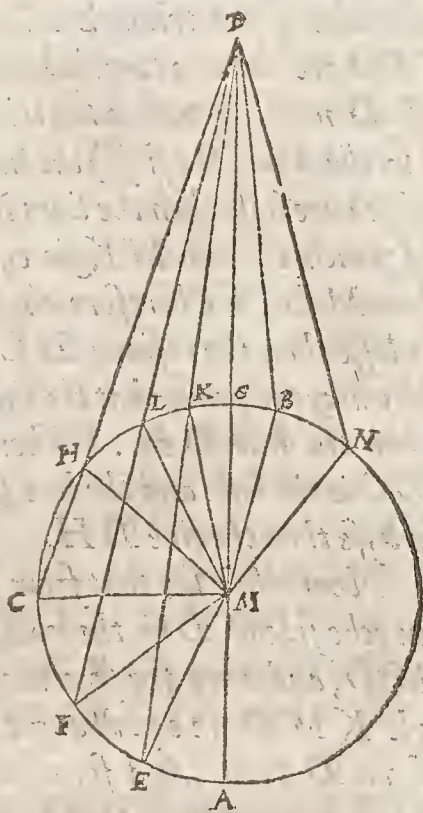
Hereby it is manifest, that two right lines being drawen frō any one poynt of the diameter, the one of one side, and the other of the other side, if with the diameter they make equall angles, the sayd two right lines are equall. As in thys place are the two lines FG and FH .

¶ The

The 7. Theoreme. The 8. Proposition.

If without a circle be taken any poynt, and from that poynt be drawn into the circle vnto the circumference certayne right lines, of which let one be drawn by the centre and let the rest be drawn at all aduentures: the greatest of those lines which fall in the concavities or hollownes of the circumference of the circle, is that which passeth by the centre: and of all the other lines that line which is nigher to the line which passeth by the centre is greater then that which is more distant. But of those right lines which end in the conuexe part of the circumference, that is the least which is drawn from the poynt to the diameter: and of the other lines that which is nigher to the least is alwaies lesse then that which is more distant. And from that poynt can be drawn vnto the circumference on ech side of the least onely two equall right lines.

Suppose y^e the circle geuen be ABC , & without y^e circle ABC , take the point D : and frō y^e same point draw certain right lines into y^e circle vnto the circumference, & let thē be DA, DE, DF, DC : & let y^e line DA passe by y^e centre. Then I say, of y^e right lines which fall in the concavities of y^e circumference AEC , y^e is, within y^e circle, y^e greatest is y^e which passeth by y^e centre, that is, DA . And of those lines which fall vpon y^e conuex part of y^e circumference, y^e lest is y^e which is drawn frō y^e point D vnto y^e end of y^e diameter AG . And of the right lines falling wⁱn the circumferēce, the line DE is greater then y^e line DF , & the line DF is greater then y^e line DC . And of the right lines which end in y^e conuex part of the circumference, y^e is, without y^e circle, that which is nigher vnto DG y^e lest, is alwayes lesse then y^e which is more distāt, that is, the line DK is lesse then the line DL , and the line DL is lesse then the line DH . Take (by the first of the third) the centre of the circle ABC , and let the



Bb.j.

same

Construētia.

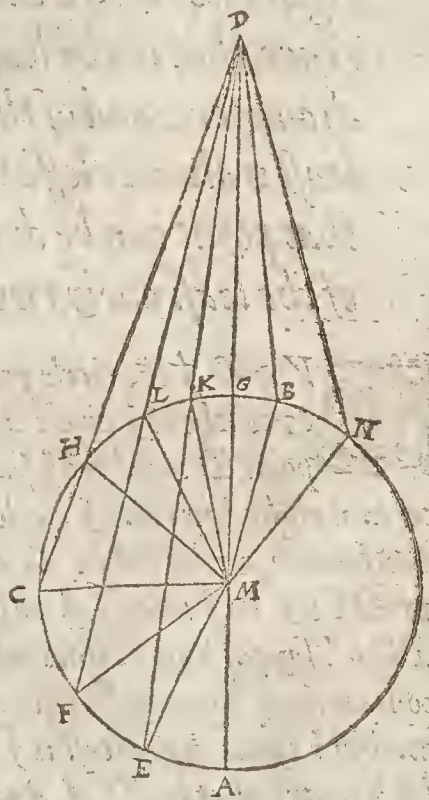
The third Booke

The first part
of this Propo-
sition.

same be M : and (by the first petition) drawe these right lines ME, MF, MC, MH, ML , and MK . And for asmuch as (by the 15. definition of the first) the line AM is equall vnto the line EM , put the line MD common to them both. VVherefore the line AD is equall vnto the lines EM and MD . But the lines EM and MD are (by the 20. of the first) greater then the line ED . VVherefore the line AD also is greater then the line ED . Agayne for asmuch as (by the 15. definition of the first) the line ME is equall vnto the line MF , put the line MD common to them both: VVherefore the lines EM and MD are equall to the lines FM and MD , and the angle EMD is greater then the angle FMD : VVherefore (by the 19. of the first) the base ED is greater then the base FD . In like sort also may we proue that the line FD is greater then the line CD . VVherefore the line DA is the greatest, and the line DE is greater then the line DF , and the line DF is greater then the line DC .

Second part.

And for asmuch as (by the 20. of the first) the lines MK and KD are greater then the line MD . But (by the 15. definition of the first) the line MG is equall vnto the line MK . VVherefore the residue KD is greater then γ residue GD . VVherefore the line GD is lesse then the line KD . And for asmuch as from the endes of one of the sides of the triangle MLD , namely, MD are drawn two right lines MK and KD meeting within the triangle, therefore (by the 21. of the first) the lines MK and KD are lesse then the lines ML & LD , of which the line MK is equall vnto the line ML . VVherefore the residue DK is lesse then the residue DL . In like sort also may we proue that the line DL is lesse then the line DH . VVherefore the line DG is the lest, and the line DK is lesse then the line DL , and the line DL is lesse then the line DH .



Third part.

Now also I say that from the poynt D can be drawen vnto the circumference on eche side of DG the least onely two equall right lines. Vpon the right line MD , and vnto the poynt in it M make (by the 23. of the first) vnto the angle KMD an equall angle DMB . And (by the first petition) drawe a line from D to B . And for asmuch as (by the 15. definition of the first) the line MB is equall vnto the line MK put the line MD common to the both, wherefore these two lines MK and MD are equall to these two lines BM and MD the one to the other, and the angle KMD is (by the 23. of the first) equall to the angle $BM D$: VVherefore (by the 4. of the first) the base DK is equall to the

to the base DB .

Now I say that from the poynt D on that side that the line DB is, can not be drawn vnto the circumference any other line besides DB equall vnto the right line DK . For if it be possible let there be drawn an other line besides DB , and let the same be DN . And for asmuch as the line DK is equall vnto the line DN . But vnto the line DK is equall the line DB . Therefore (by the first common sentence) the line DB is equall vnto the line DN . Wherefore that which is nigher vnto DG the least, is equall to y which is more distant: Which we haue before proued to be impossible.

Or it may thus be demonstrated. Draw (by the first petition) a line from M to N . And for asmuch as (by the 15. definition of the first) the line KM is equall vnto the line MN , and the line MD is common to them both. And the base KD is equall to the base DN (by supposition) therefore (by the 8. of the first) the angle KMD is equall to the angle DMN . But the angle KMD is equall to the angle $BM D$. Wherefore the angle $BM D$ is equall to the angle $NM D$, the lesse vnto the greater: which is impossible. Wherefore from the poynt D can not be drawn vnto the circumference ABC on eche side of DG the left, more then two equall right lines. If therefore without a circle be taken any poynt and from that poynt be drawn into the circle vnto the circumference certaine right lines, of which let one be drawn by the centre, and let the rest be drawn at all aduentures: the greatest of those right lines which fall in y concauitie or hollownes of the circumference of the circle is that which passeth by the centre. And of all the other lines, that line which is nigher to the line which passeth by the centre, is greater then that which is more distant. But of those right lines which end in the conuexe part of the circumference, that line is the lest which is drawn from the poynt to the dimetient: and of the other lines that which is nigher to the least is alwayes lesse then that which is more distant. And from that poynt can be drawn vnto the circumference on ech side of the left only two equall right lines: which was required to be proued.

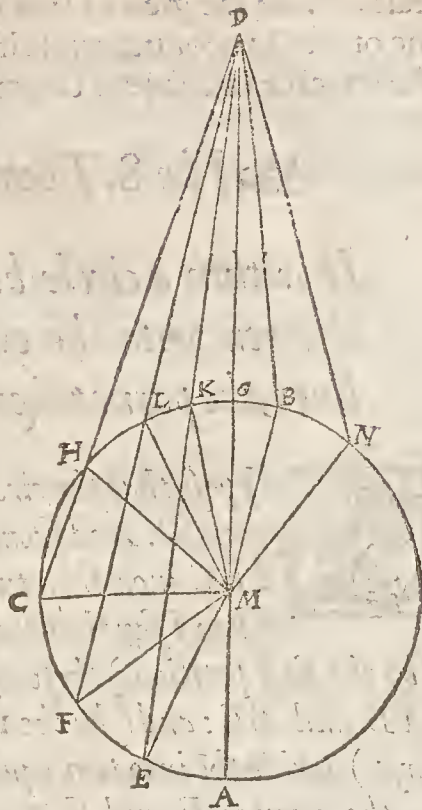
Thys Proposition is called commonly in old bookes amongst the barbarous, *Cauda Pannonis*, that is, the Peacockes taile.

¶ A Corollary.

Hereby it is manifest, that the right lines, which being drawn from the poynt

Bb.ij.

geuen



This is demonstrated by an argument leading to an absurdity.

An other demonstration of the latter part, leading also to an impossibility.

This Proposition is commonly called *Cauda Pannonis*.

A Corollary.

The third Booke

geuen without the circle, and fall within the circle, are equally distant from the least, or from the greatest (which is drawn by the centre) are equall the one to the other: but contrarywyse if they be vnequally distant, whether they light vpon the concaue or conuexe circumference of the circle, they are vnequall.

The 8. Theoreme. The 9. Proposition.

If within a circle be taken a poynt, and from that poynt be drawen vnto the circumference moe then two equall right lines, the poynt taken is the centre of the circle.

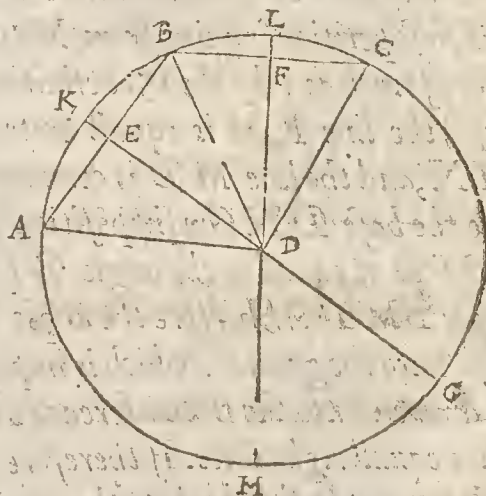
Construction.



Demonstration.

Suppose that the circle be ABC , and within it let there be taken the poynt D . And from D let there be drawen vnto the circumference ABC moe then two equall right lines, that is, DA , DB , and DC . Then I say that the poynt D is the centre of the circle ABC . Draw

(by the first petition) these right lines AB and BC : and (by the 10. of the first) deuide the into two equall partes in the poyntes E and F : namely, the line AB in the poynt E , and the line BC in the poynt F . And draw γ lines ED and FD , and (by the second petition) extend the lines ED and FD on eche side to the poyntes K , G , and H , L . And for asmuch as the line AE is equall vnto the line EB , and the line ED is common to them both, there-



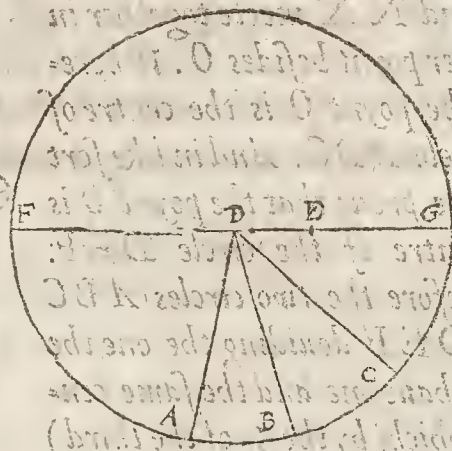
fore these two sides AE and ED are equall vnto these two sides BE , and ED : and (by supposition) the base DA is equall to the base DB . Wherefore (by the 8. of the first) the angle AED is equall to the angle BED . Wherefore eyther of these angles AED and BED is a right angle. Wherefore the line GK deuideth γ line AB into two equall partes and maketh right angles. And for asmuch as, if in a circle a right line deuide an other right line into two equall partes in such sort that it maketh also right angles, in γ line that deuideth is the centre of the circle (by the Correllary of the first of the third). Therefore (by the same Correllary) in the line GK is the centre of the circle ABC . And (by the same reason) may we proue that in γ line HL is the centre of the circle ABC , and the right lines GK , and HL haue no other poynt common to them both besides the poynt D . Wherefore the poynt D is the centre of the circle ABC . If therefore within a circle be taken a poynt, and from that point be drawen vnto the circumference more then two equall right lines, the poynt taken is the centre of the circle: which was required to be proued.

¶ An

An other demonstration.

Let there be taken within the circle ABC the poynt D . And from the poynt D let there be drawen unto the circumference more then two equall right lines, namely, DA , DB , and DC . Then I say that the poynt D is the centre of the circle. For if not, then if it be possible let the point E be the centre: and draw a line from D to E , and extend DE to the poyntes F and G . Wherefore the line FG is the diameter of the circle ABC . And for asmuch as in FG the diameter of the circle ABC is taken a poynt, namely D , which is not the centre of that circle, therefore (by the 7. of the third) the line DC is γ greatest, and the line DC is greater then the line DB , and the line DB is greater then the line DA . But the lines DC , DB , DA , are also equall (by supposition): which is impossible. Wherefore the poynt E is not the centre of the circle ABC . And in like sort may we proue that no other poynt besides D . Wherefore the poynt D is the centre of the circle ABC : which was required to be proued.

An other demonstration of the same leading also to an impossibilitie.

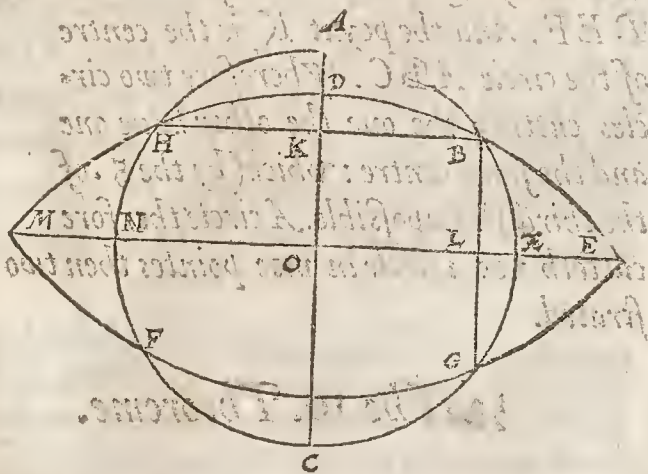


The 9. Theoreme. The 10. Proposition.

A circle cutteth not a circle in moe pointes then two.

Or if it be possible let the circle ABC cut the circle DEF in moe pointes then two, that is, in B , G , H , & F . And drawe lines fro B to G , and from B to H . And (by γ 10. of the first) deuide either of the lines BG & BH into two equall partes, in γ pointes K and L . And (by the 11. of the first) from the poynt K raise vp vnto γ line BH a perpendicular line KC , and likewise from the poynt L raise vp vnto γ line BG a perpendicular line LM , and extend the line CK to the poynt A , and LN to the poyntes X and E . And for asmuch as in the circle ABC , the right line AC deuideth the right line BH into two equal partes and maketh right angles, therefore (by the 3. of the third)

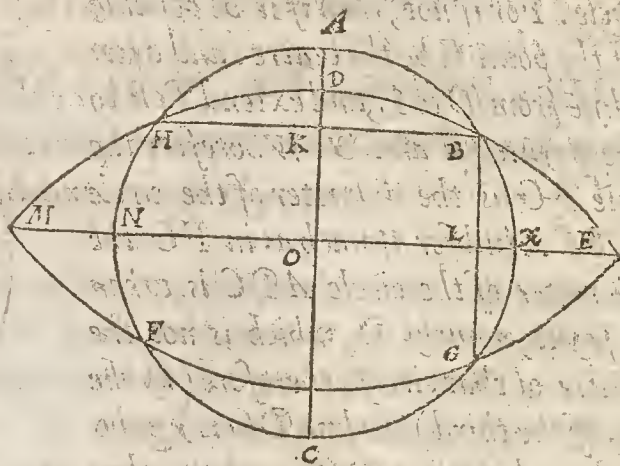
Demonstration leading to an impossibilitie.



Bb. iij. in the

The third Booke

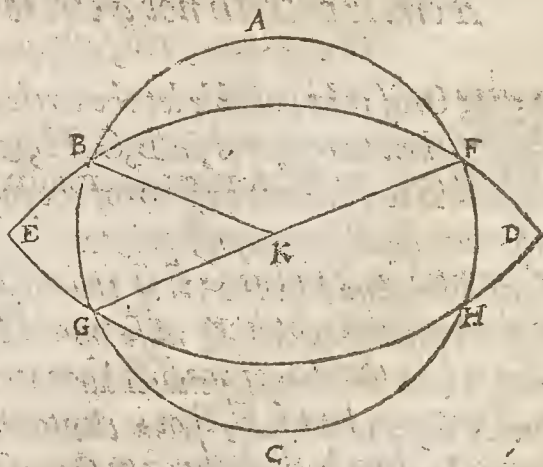
in the line AC is the centre of the circle ABC . Agayne, for asmuch as in the selfe same circle ABC the right line NX , that is, the line ME deuiderh the right line BG into two equall partes and maketh right angles, therefore (by the third of the third) in the line NX is the centre of the circle ABC . And it is proued that it is also in the line AC . And these two right lines AC and NX meete together in no other poynt besides O . Wherefore the poynt O is the centre of the circle ABC . And in like sort may we proue that the poynt O is the centre of the circle DEF . Wherefore the two circles ABC and DEF deuiding the one the other haue one and the same centre: which (by the 5. of the third) is impossible. A circle therefore cutteth not a circle in moe poyntes then two: which was required to be proued.



✱ An other demonstration to proue the same.

An other demonstration of the same leading also to an impossibility.

Suppose that the circle ABC do cut the circle DGF in moe poyntes then two, that is, in B, G, F , and H . And (by the first of the third) take the centre of the circle ABC and let the same be the poynt K . And draw these right lines KB , KG , and KF . Now for asmuch as within the circle DEF is taken a certaine poynt K , and from that poynt are drawn vnto the circumference moe then two equall right lines, namely, KB , KG , and KF : therefore (by the 9. of the third) K is the centre of the circle DEF . And the poynt K is the centre of the circle ABC . Wherefore two circles cutting the one the other haue one and the same centre: which (by the 5. of the third) is impossible. A circle therefore cutteth not a circle in moe pointes then two: which was required to be demonstrated.



✱ The 10. Theoreme. The 11. Proposition.

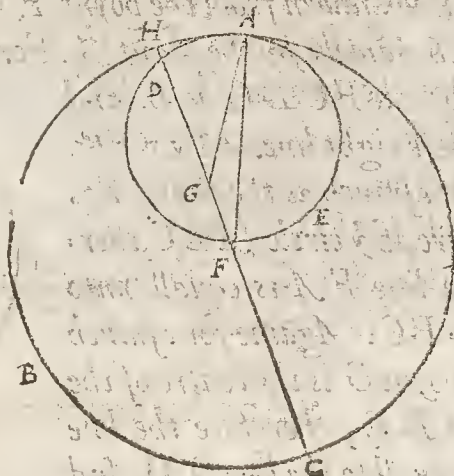
If two circles touch the one the other inwardly, their centres being

being geuen: a right line ioyning together their centres and produced, will fall vpon the touch of the circles.

Suppose that these two circles ABC , and ADE do touch the one the other in the poynt A . And (by the first of the third) take the centre of the circle ABC , and let the same be F : and likewise γ centre of the circle ADE , and let the same be G . Then I say that a right line drawn from F to G and being produced, will fall vpon the poynt A . For if not, then if it be possible let it fall as the line $FGDH$ doth. And draw these right lines AF , & AG .

Construction.

Demonstration leading to an impossibility.



Now forasmuch as the lines AG and GF are (by the 20. of the first) greater then the line FA , that is, then the line FH , take away the line GF which is common to them both. Wherefore the residue AG is greater then the residue GH . But the line DG is equall vnto the line GA (by the 15. definition of the first). Wherefore the line GD is greater then γ line GH : the lesse then the greater: which is impossible. Wherefore a right line drawn from the poynt F to the poynt G and produced, falleth not besides the poynt A , which is γ point of the touch. Wherefore it falleth vpon the touch. If therefore two circles touch the one γ other inwardly, their centres being geuen, a right line ioyning together their centres and produced, will fall vpon the touch of the circles: which was required to be proued.

*An other demonstration to proue the same.

But now let it fall as GFC falleth, and extend γ line GFC to the poynt H : and draw these right lines AG and AF . And forasmuch as the lines AG and GF are (by the 20. of the first) greater then the line AF . But the line AF is equall vnto the line CF , that is, vnto the line FH . Take away the line FG common to them both. Wherefore the residue AG is greater then γ residue GH , that is, the line GD is greater then the line GH : the lesse greater then γ greater: which is impossible.

An other demonstration of the same leading also to an impossibility.

Which thing may also be proued by the 7. Proposition of this booke. For forasmuch as the line HC is the diameter of the circle ABC , & in it is taken a poynt which is not the centre, namely, the poynt G , therefore the line GA is greater then the line GH by the sayd 7. Proposition. But the line GD is equall to the line GA (by the definition of a circle). Wherefore the line GD is greater then the line GH , namely, the part greater then the whole: which is impossible.

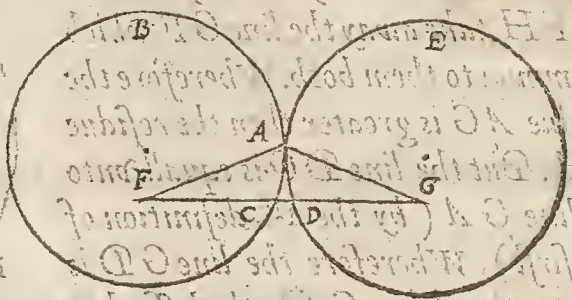
The same againe demonstrated by an argument leading to an absurditie.

The 11. Theoreme. The 12. Proposition.

If two circles touch the one the other outwardly, a right line drawn by their centres shall passe by the touch.

Demonstration leading to an impossibility.

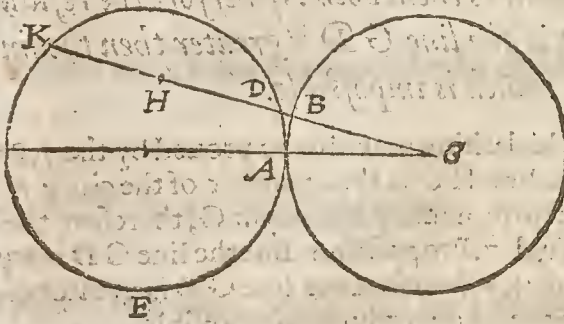
Suppose that these two circles ABC and ADE do touch the one the other outwardly in the poynt A . And (by the third of the third) take the centre of the circle ABC , and let the same be the poynt F : and likewise the centre of the circle ADE , and let the same be the poynt G . Then I say that a right line drawn from the poynt F to the poynt G shall passe by the poynt of the touch, namely, by the poynt A . For if not, then if it be possible, let it passe as the right line $FCDG$ doth. And draw these right lines AF & AG . And for asmuch as the poynt F is the centre of γ circle ABC , therefore the line FA is equall vnto the line FC . Againe for asmuch as the poynt G is the centre of the circle ADE , therefore the line GA is equall to the line GD . And And it is proued that the line FA is equall to the line FC . Wherefore the lines FA and AG are equall vnto the lines FC and GD . Wherefore the whole line FG is greater then the lines FA and AG . But it is also lesse (by the 20. of the first), which is impossible. Wherefore a right line drawn from the poynt F to the poynt G shall passe by the poynt of the touch, namely, by the poynt A . If therefore two circles touch the one the other outwardly, a right line drawn by their centres, shall passe by the touch: which was required to be demonstrated.



¶ An other demonstration after Pelitarius.

An other demonstration after Pelitarius leading also to an absurditie.

Suppose that the two circles ABC and DEF do touch the one the other outwardly in the poynt A : And let G be the centre of the circle ABC : From which poynt produce by the touch of the circles the line GA to the poynt F of the circumference DEF . Which for asmuch as it passeth not by the centre of the circle DEF (as the aduersary affirmeth) draw from the same centre G an other right line GK , which if it be possible let passe by the centre of the circle DEF , namely, by the poynt H : cutting the circumference ABC in the poynt B , & the circumference DEF in the poynt D , & let the opposite poynt thereof be in the point K . And for asmuch as frō the poynt G taken without the circle DEF is drawn the line GK passing by the centre H , and frō the same poynt is drawn also an other



other line not passing by the centre, namely, the line GF . Therefore (by the 8. of this booke) the outward part GD of the line GK shall be lesse then the outward part GA of the line GF . But the line GA is equall to the line GB . Wherefore the line GD is lesse then the line GB , namely, the whole lesse then the part: which is absurde.

The 12. Theoreme. The 13. Proposition.

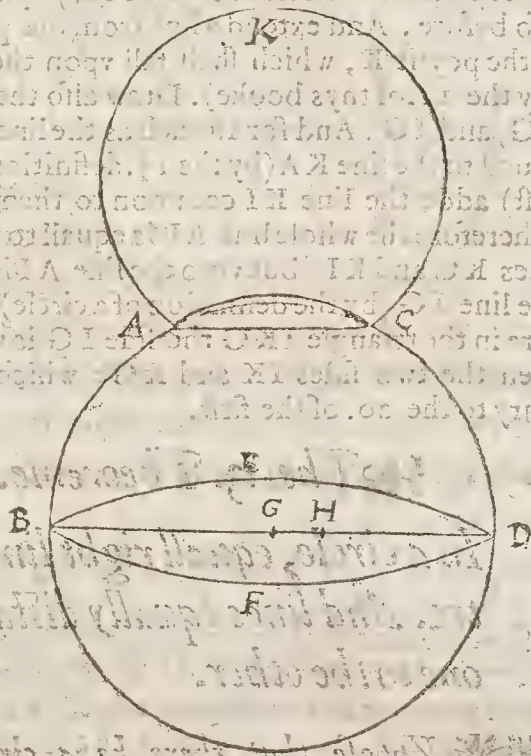
A circle can not touch an other circle in moe poyntes then one, whether they touch within or without.



Or if it be possible, let the circle $ABCD$ touch y circle $EBFD$ first inwardly in moe poyntes then one, that is, in D and B . Take (by the first of the third) the centre of the circle $ABCD$, and let the same be y point G ; and likewise y centre of the circle $EBFD$, and let the same be y poynt H . Wherefore (by the 11. of the same)

Of circles which touch the one the other inwardly.

a right line drawn from the poynt G to the poynt H and produced, will fall vpon the poyntes B and D : let it so fall as the line $BGH D$ doth. And for asmuch as the poynt G is the centre of the circle $ABCD$, therefore (by the 15. definition of the first) the line BG is equall to the line DG . Wherefore the line BG is greater then the line HD : Wherefore the line BH is much greater then the line HD . Agayne for asmuch as the poynt H is the centre of the circle $EBFD$, therefore (by the same definition) the line BH is equall to the line HD : and it is proued that it is much greater then it: which is impossible. A circle therefore can not touch a circle inwardly in moe poyntes then one.



Of circles which touch the one the other outwardly.

Now I say that neither outwardly also a circle toucheth a circle in moe poyntes then one. For if it be possible, let the circle ACK touch y circle $ABCD$ outwardly in moe poyntes the one, that is, in A and C : And (by the first petition) draw a line from the poynt A to the poynt C . Now for asmuch as in y circumference of either of the circles $ABCD$, and ACK , are taken two poyntes at all aduentures, namely, A and C , therefore (by the second of the third) a right line ioyning together those poyntes shall fall within both the circles. But it falleth within the circle $ABCD$, & without the circle ACK : which is absurde. Wherefore a circle shall not touch a circle outwardly in moe poyntes then one, and it is proued y neither also inwardly. Wherefore a circle can not touch an other circle in moe poyntes then one, whether they touch

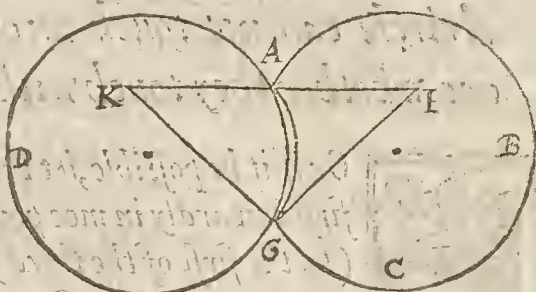
The third Booke

touch within or without: which was required to be demonstrated.

¶ *An other demonstration after Pelitarius and Flussates.*

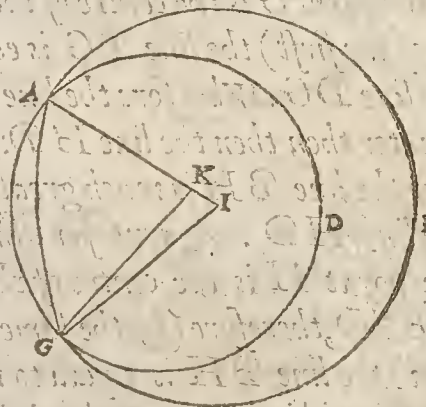
An other demonstration after Pelitarius & Flussates, of circles which touch the one the other outwardly.

Suppose that there be two circles ABG and ADG , which if it be possible, let touch the one the other outwardly in moe poyntes then one, namely, in A and G . Let the centre of the circle ABG be the poynt I , and let the centre of the circle ADG be the poynt K . And draw a right line from the poynt I to the poynt K , which (by the 12. of thys booke) shall passe both by the poynt A and by the poynt G : which is not possible: for then two right lines should include a superficies, contrary to the last common sentence. It may also be thus demonstrated. Draw a line from the centre I to the centre K , which shall passe by one of the touches, as for example by the poynt A . And draw these right lines GK and GI , and so shall be made a triangle, whose two sides GK and GI shall not be greater then the side IK : which is contrary to the 20. of the first.



Of circles which touch the one the other inwardly.

But now if it be possible, let the foresayd circle ADG touch the circle ABG inwardly in moe poyntes then one, namely, in the pointes A and G : and let the centre of the circle ABG be the poynt I , as before: and let the centre of the circle ADG be the poynt K , as also before. And extend a line from the poynt I to the poynt K , which shall fall vpon the touch (by the 11. of thys booke). Draw also these lines KG , and IG . And for asmuch as the line KG is equall to the line KA (by the 15. definition of the first) adde the line KI common to them both. Wherefore the whole line AI is equall to the two lines KG and KI : but vnto the line AI is equall the line IG (by the definition of a circle). Wherefore in the triangle IKG the side IG is not lesse then the two sides IK and KG : which is contrary to the 20. of the first.



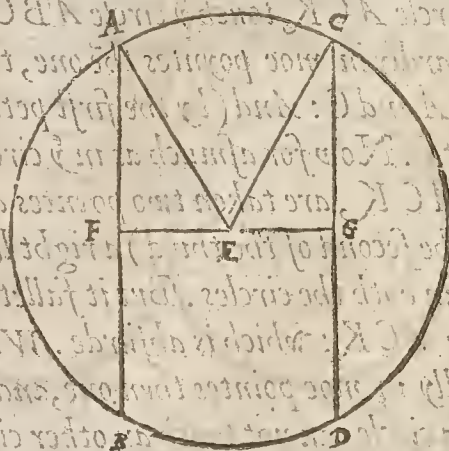
§ *The 13. Theoreme. The 14. Proposition.*

In a circle, equall right lines, are equally distant from the centre. And lines equally distant from the centre, are equall the one to the other.

The first part of this Theoreme.

Construction.

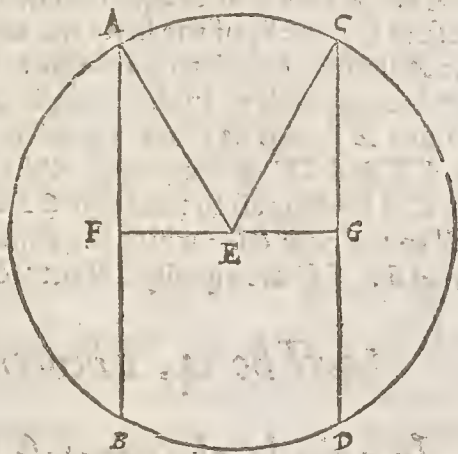
Suppose that there be a circle $ABCD$, and let there be in it drawen these equall right lines AB and CD . Then I say that they are equally distant from the centre. Take (by the first of the third) the centre of the circle $ABCD$; and let the same be y poynt E . And (by the 12. of the first) from the poynt E , draw vnto the lines AB & CD .



perpendicular

perpendicular lines EF and EG . And (by the first petition) draw these right lines AE and CE . Now for asmuch as a certaine right line EF drawn by the centre cutteth a certaine other right line AB not drawn by the centre, in such sort that it maketh right angles, therefore (by the third of the third) it devideth it into two equall partes. Wherefore the line AF is equall to the line FB . Wherefore the line AB is double to the line AF : and by the same reason also the line CD is double to the line CG . But the line AB is equall to the line CD . Wherefore the line AF is also equall to the line CG . And for asmuch as (by the 15. definition of the first) the line AE is equall to the line EC , therefore the square of the line EC is equall to the square of the line AE . But vnto the square of the line AE , are equall (by the 47. of the first) the squares of the lines AF & FE : for the angle at the poynt F is a right angle. And (by y^e selfe same) to the square of the line EC are equall the squares of the lines EG and GC : for the angle at the poynt G is a right angle. Wherefore the squares of the lines AF and FE are equall to the squares of the lines CG and GE : of which the square of the line AF is equall to the square of the line CG : for the line AF is equall to the line CG . Wherefore (by the third common sentence) the square remayning, namely, the square of the line FE , is equall to the square remayning, namely, to the square of the line EG . Wherefore the line EF is equall to the line EG . But right lines are sayd to be equally distant from y^e centre, when perpendicular lines drawn fro the centre to those lines, are equall (by the 4. definition of the third). Wherefore the lines AB and CD are equally distant from the centre.

Demonstration.



Demonstration.

But now suppose that the right lines AB and CD be equally distant from the centre, that is, let the perpendicular line EF be equall to the perpendicular line EG . Then I say that the line AB is equall to the line CD . For the same order of construction remayning, we may in like sort proue that the line AB is double to the line AF , and that the line CD is double to the line CG . And for asmuch as the line AE is equall to the line CE , for they are drawn from y^e centre to the circumference, therefore the square of the line AE is equall to y^e square of the line CE . But (by the 47. of the first) to the square of the line AE are equall the squares of the lines EF and FA . And (by the selfe same) to y^e square of the line CE are equall the squares of the lines EG and GC . Wherefore the squares of the lines EF and FA are equall to the squares of the lines EG and GC . Of which the square of the line EG is equall to the square of the line EF , for the line EF is equall to the line EG . Wherefore (by the third common sentence) the square remayning, namely, the square of the line AF , is equall to the square of the line CG . Wherefore the line AF is equall vnto the line CG . But

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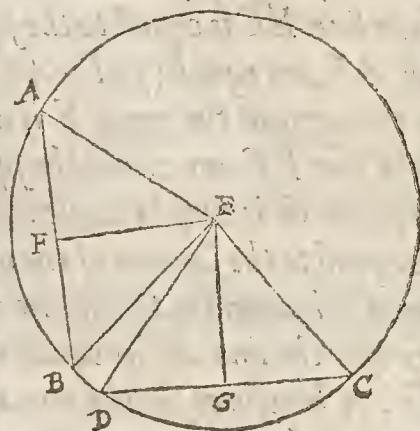
The third Booke

the line AB is double to the line AF ; and the line CD is double to the line CG . Wherefore the line AB is equall to the line CD . Wherefore in a circle equall right lines are equally distant from the centre. And lines equally distant from the centre, are equall the one to y other: which was required to be proued.

¶ An other demonstration for the first part after Campane.

An other demonstration of the first part after Campane.

Suppose that there be a circle $ABDC$, whose centre let be the poynt E . And draw in it two equall lines AB and CD . Then I say that they are equally distant from the centre. Draw from the centre vnto the lines AB and CD , these perpendicular lines EF and EG . And (by the 2. part of the 3. of this booke) the line AB shall be equally devided in the poynt F , and the line CD shall be equally devided in the poynt G . And draw these right lines EA, EB, EC , and ED . And for asmuch as in the triangle AEB the two sides AB and AE are equall to the two sides CD and CE of the triangle CED , & the base EB is equall to the base ED , therefore (by the 8. of the first) the angle at the point A shall be equall to the angle at the point C . And for asmuch as in the triangle AEF the two sides AE and AF are equall to the two sides CE and CG of the triangle CEG , and the angle EAF is equall to the angle CEG , therefore (by the 4. of the first) the base EF is equall to the base EG : which for asmuch as they are perpendicular lines, therefore the lines AB & CD are equally distant frō the centre, by the 4. definition of this booke.

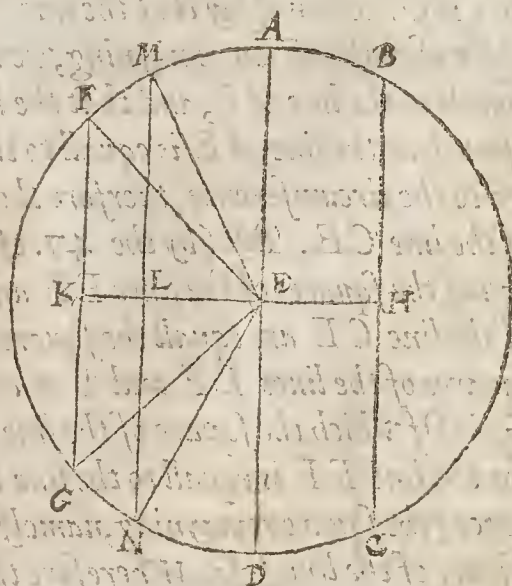


The 14. Theoreme. The 15. Proposition.

In a circle, the greatest line is the diameter, and of all other lines that line which is nigher to the centre is alwayes greater then that line which is more distant.

Suppose that there be a circle $ABCD$, and let the diameter thereof be the line AD , and let the centre thereof be the poynt E . And vnto the diameter AD let the line BC be nigher then the line FG . Then I say that the line AD is the greatest; and the line BC is greater then y line FG . Draw (by the 12. of the first) from the centre E to the lines BC and FG perpendicular lines EH and EK . And for asmuch as the line BC is nigher vnto the centre then the line FG , therefore

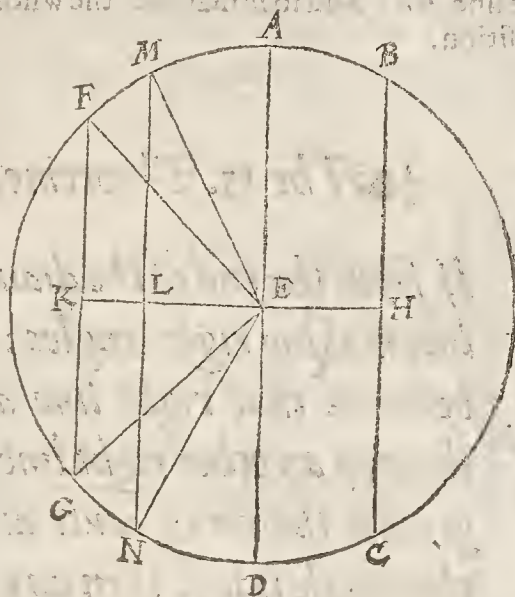
Construction.



(by the

(by the 4. definition of the third) the line $E K$ is greater then the line $E H$. And (by the third of the first) put vnto the line $E H$ an equall line $E L$. And (by the 11. of the first) from the point L raise vp vnto the line $E K$ a perpendicular line $L M$: and extend the line $L M$ to the poynt N . And (by the first petition) draw these right lines, $E M$, $E N$, $E F$, and $E G$. And for asmuch as the line $E H$ is equall to the line $E L$, therefore (by the 14. of the third, and by the 4. definition of the same) the line $B C$ is equall to the line $M N$. Again for asmuch as the line $A E$ is equall to the line $E M$, and the line $E D$ to the line $E N$, therefore the line $A D$ is equall to the lines $M E$ and $E N$. But the lines $M E$ and $E N$ are (by the 20. of the first) greater then the line $M N$. Wherefore the line $A D$ is greater then the line $M N$. And for asmuch as these two lines $M E$ and $E N$ are equall to these two lines $F E$ and $E G$ (by the 15. definition of the first) for they are drawn from the centre to the circumference, and the angle $M E N$ is greater then the angle $F E G$, therefore (by the 24. of the first) the base $M N$ is greater then the base $F G$. But it is proued that the line $M N$ is equall to the line $B C$: Wherefore the line $B C$ also is greater then the line $F G$. Wherefore the diameter $A D$ is the greatest, and the line $B C$ is greater then the line $F G$. Wherefore in a circle, the greatest line is the diameter, and of all the other lines, that line which is nigher to y^e centre is alwaies greater then that line which is more distant: which was required to be proued.

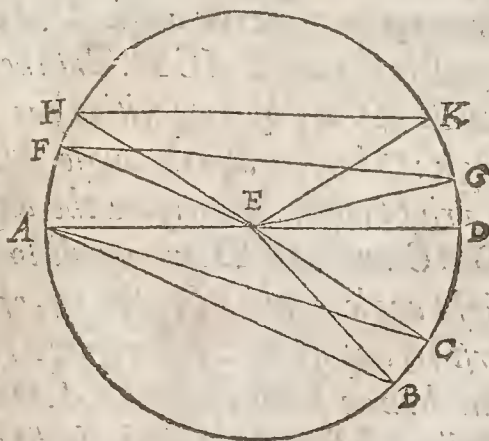
Demonstration.



¶ An other demonstration after Campane.

In the circle $A B C D$, whose centre let be the poynt E , draw these lines, $A B$, $A C$, $A D$, $F G$, and $H K$, of which let the line $A D$ be the diameter of the circle. Then I say that the line $A D$ is the greatest of all the lines. And the other lines eche of the one is so much greater then ech of the other, how much higher it is vnto the centre. Ioyne together the endes of all these lines with the centre, by drawing these right lines $E B$, $E C$, $E G$, $E K$, $E H$, and $E F$. And (by the 20. of the first) the two sides $E F$ and $E G$ of the triangle $E F G$, shall be greater then the third side $F G$. And forasmuch as the sayd sides $E F$ & $E G$ are equall to the line $A D$ (by the definition of a circle) therefore the line $A D$ is greater then the line $F G$. And by the same reason it is greater then euery one of the rest of the lines, if they be put to be bases of triangles: for that euery two sides drawn fro the

An other demonstration after Campane.

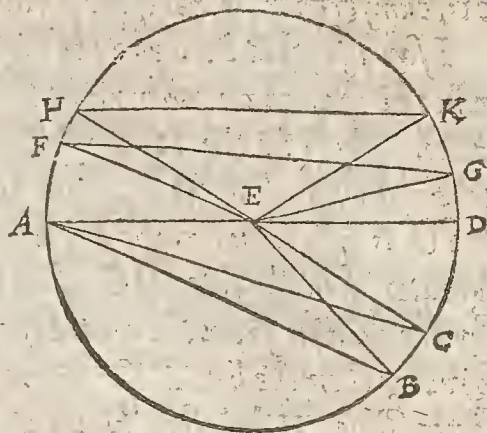


Cc.j.

centre

The third Booke

centre are equall to the line AD . Which is the first part of the Proposition. Agayne, for as much as the two sides EF and EG of the triangle EFG , are equall to the two sides EH and EK of the triangle EHK , and the angle FEG is greater then the angle HEK , therefore (by the 24. of the first) the base FG is greater then the base HK . And by the same reason may it be proued, that the line AC is greater then the line AB . And so is manifest the whole Proposition.



The 15. Theoreme. The 16. Proposition.

If from the end of the diameter of a circle be drawen a right line making right angles: it shall fall without the circle: and betwene that right line and the circumference can not be drawen an other right line: and the angle of the semicircle is greater then any acute angle made of right lines, but the other angle is lesse then any acute angle made of right lines.

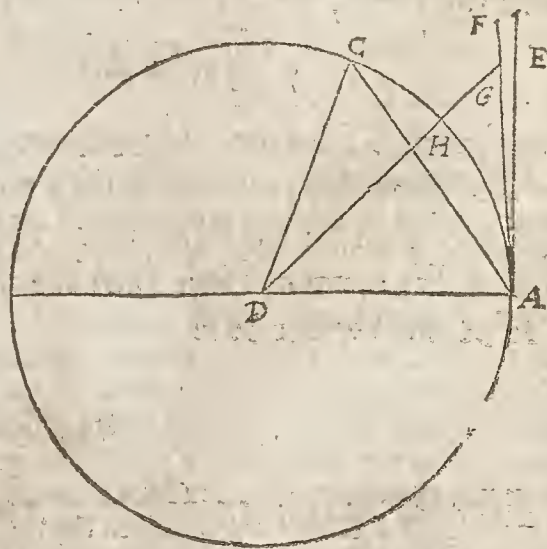


Suppose that there be a circle ABC : whose centre let be the point D , and let the diameter thereof be AB . Then I say, a right line drawen from the poynt A , making with the diameter AB right angles, shall fall without the circle. For if it do not, then if it be possible, let it fall

*The first part
of this Theo-
reme.*

*Demonstra-
tion leading
to an absurdi-
tie.*

within the circle as the line AC doth, and draw a line from the point D to the point C . And for as much as (by the 15. definition of the first) the line DA is equall to the line DC , for they are drawen from the centre to the circumference, therefore the angle DAC is equall to the angle ACD . But the angle DAC is (by supposition) a right angle: Wherefore also the angle ACD is a right angle. Wherefore the angles DAC and ACD , are equall to two right angles: which (by the 17. of the first) is impossible. Wherefore a right line drawen from the poynt A , making with the diameter AB right angles, shall not fall within the circle. In like sort also may we proue, that it falleth not in



the

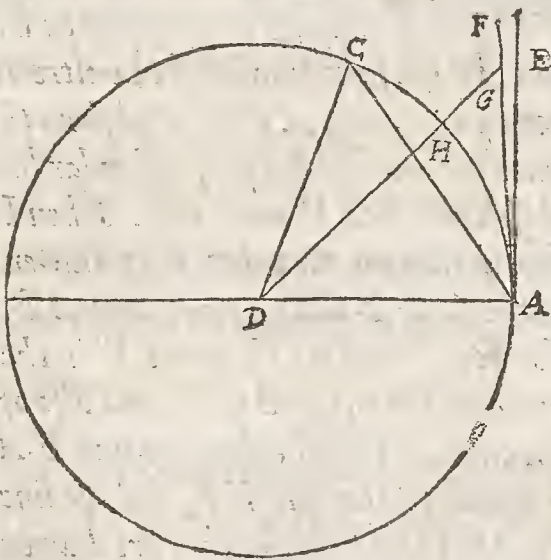
the circumference. Wherefore it falleth without, as the line AE doth.

Second part.

I say also, that betwene the right line AE , and the circumference ACB , can not be drawn an other right line. For if it be possible, let the line AF so be drawn. And (by the 12. of the first) from the poynt D draw vnto the line FA a perpendicular line DG . And for asmuch as AGD is a right angle, but DAG is lesse then a right angle, therefore (by the 19. of the first) the side AD is greater then the side DG . But the line DA is equall to the line DH , for they are drawn from the centre to the circumference. Wherefore the line DH is greater then the line DG : namely, the lesse greater then the greater: which is impossible. Wherefore betwene the right line AE and the circumference ACB , can not be drawn an other right line.

Third part.

I say moreouer, that the angle of the semicircle containd vnder y^e right line AB and the circumference CHA , is greater then any acute angle made of right lines. And the angle remayning cotayned vnder y^e circumference CHA and the right line AE , is lesse then any acute angle made of right lines. For if



there be any angle made of right lines greater then that angle which is containd vnder the right line BA and the circumference CHA , or lesse then that which is containd vnder the circumference CHA and the right line AE , then betwene the circumference CHA and the right line AE , there shall fall a right line, which maketh the angle containd vnder the right lines, greater then that angle which is containd vnder the right line BA and the circumference CHA , and lesse then the angle which is containd vnder the circumference CHA and the right line AE . But there can fall no such line, as it hath before bene proued. Wherefore no acute angle contained vnder right lines, is greater then the angle containd vnder the right line BA and the circumference CHA , nor also lesse then the angle containd vnder the circumference CHA and the line AE .

Correlary.

Hereby it is manifest that a perpendicular line drawn frō the end of the diameter of a circle toucheth the circle: and that a right line toucheth a circle in one poynt onely. For it was proued (by the 2. of the third) that a right line drawn from two pointes taken in

The third Booke

the circumference of a circle, shall fall within the circle. Which was required to be demonstrated.

The 2. Probleme. The 17. Proposition.

From a poynt geuen, to draw a right line which shall touch a circle geuen.

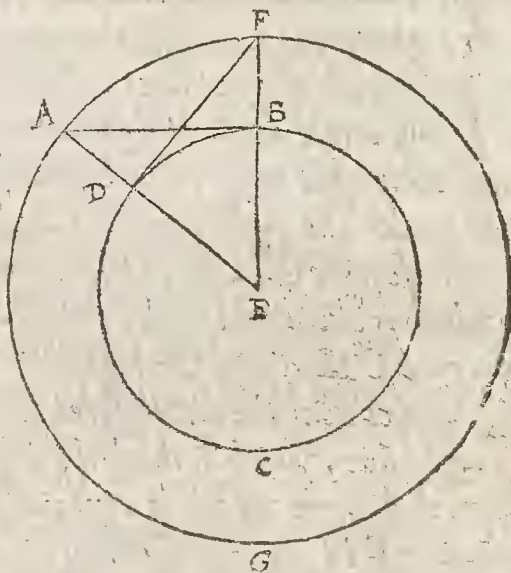
Construction.



Suppose that the poynt geuen be A , and let the circle geuen be BCD . It is required from the poynt A to draw a right line which shall touch the circle BCD . Take (by the first of the third) the centre of the circle, and let the same be E . And (by the first petition) draw the right line ADE . And making the centre E , and the space AE , describe (by the third petition) a circle AFG . And from the poynt D raise up (by the 11. of the first) vnto the line EA a perpendicular line DF . And (by the first petition) drawe these lines EBF and AB . Then I

Demonstration.

say, that from the point A is drawen to the circle BCD a touch line AB . For forasmuch as the point E is the centre of the circle BCD , and also of the circle AFG , therefore the line EA is equall to the line EF , and the line ED to the line EB , for they are drawen from the centre to the circumference. Wherefore to these two lines AE and EB , are equall these two lines EF and ED , and the angle at the poynt E is common to them both: Wherefore (by the 4. of the first) the base DF is equall to the base AB , and the triangle DEF is equall to the triangle EBA , and the rest of the angles remayning to the rest of the angles remayning. Wherefore the angle EDF is equall to the angle EBA . But the angle EDF is a right angle: Wherefore also the angle EBA is a right angle, and the line EB is drawen from the centre. But a perpendicular line drawen from the end of the diameter of a circle, toucheth the circle (by the Corollary of the 16. of the third). Wherefore the line AB toucheth the circle BCD . Wherefore from the point geuen, namely, A , is drawen vnto the circle geuen BCD , a touch line AB : which was required to be done.



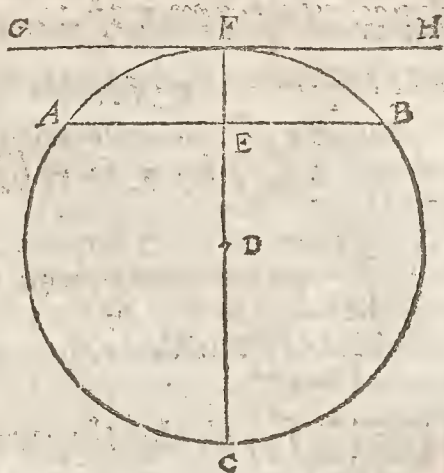
¶ *An addition of Pelitarius.*

An addition of Pelitarius.

Vnto a right lyne which cutteth a circle, to drawe a parallel line which shall touch the circle.

Suppose

Suppose that the right lyne AB do cut the circle ABC in the poyntes A and B . It is required to draw vnto the line AB a parallel lyne which shall touche the circle. Let the centre of the circle be the point D . And deuide the lyne AB into two equall partes in the point E . And by the point E and by the centre D , draw the diameter $CDEF$. And from the point F (which is the ende of the diameter) rayse vp (by the 11. of the first) vnto the diameter CF a perpendicular line GFH . Then I say that the lyne GFH (which by the corollary of the 16. of this booke toucheth the circle) is a parallel vnto the line AB . For forasmuch as the right line CF falling vpon either of these lines AB & GH maketh all the angles at the point E right angles (by the 3. of this booke) and the two angles at the point F are supposed to be right angles: therefore (by the 29. of the first) the lines AB and GH are parallels: which was required to be done. And this Probleme is very commodious for the inscribing or circumscribing of figures in or about circles.



This Probleme commodious for the inscribing and circumscribing of figures in or about circles.

The 16. Theoreme. The 18. Proposition.

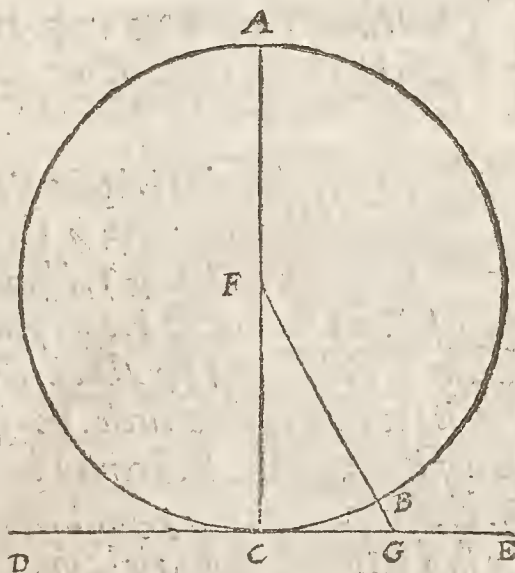
If a right lyne touch a circle, and from the centre to the touch be drawn a right line, that right line so drawn shall be a perpendicular lyne to the touche lyne.



Suppose that the right line DE do touch the circle ABC in the point C . And take the centre of the circle ABC , and let the same be F . And (by the first petition) from the poynt F to the poynt C drawe a right line FC . Then I say, that FC is a perpendicular line to DE . For if

Demonstration leading to an impossibility.

not, draw (by the 12. of the first) from the poynt F to the line DE a perpendicular line FG . And for asmuch as the angle FGC is a right angle, therefore the angle GCF is an acute angle: Wherefore the angle FGC is greater then $\angle FCG$, but vnto the greater angle is subtended the greater side (by the 19. of the first). Wherefore the line FC is greater then the line FG . But the line FC is equall to the line FB , for they are drawn from the centre to the circumference: Wherefore the line FB also is greater then the line FG , namely, the lesse then the greater: which is impossible. Wherefore the line FG is not a perpendicular line vnto the line DE . And in like sort may we proue, that no other line is a perpendicular line vnto $\angle$ line DE besides the line FC : Wherefore the line FC is a perpendicular line to DE . If therefore a right line touch



Cc.ij. a circle,

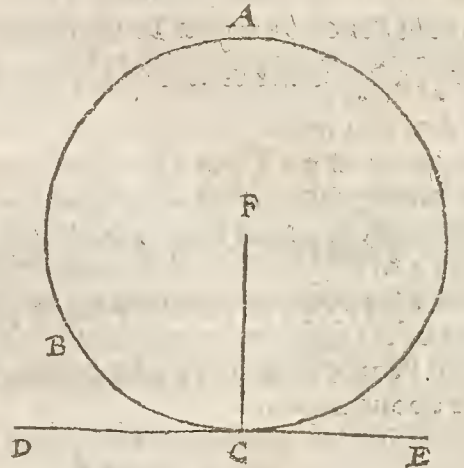
The third Booke

a circle, & from $\hat{y}$ centre to $\hat{y}$ touch be drawn a right line, $\hat{y}$ right line so drawn shall be a perpendicular line to $\hat{y}$ touch line: which was required to be proued.

¶ An other demonstration after Orontius.

An other demonstration after Orontius.

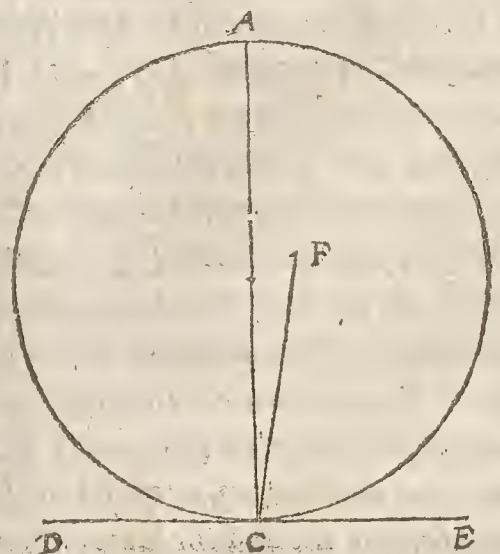
Suppose that the circle geuen be ABC , which let the right lyne DE touch in the point C . And let the centre of the circle be the point F . And draw a right line from F to C . Then I say that the line FC is perpendicular vnto the line DE . For if the line FC be not a perpendicular vnto the line DE , then, by the conuerse of the x. definition of the first booke, the angles DCF & FCE shal be vnequall: & therfore the one is greater then a right angle, and the other is lesse then a right angle. (For the angles DCF and FCE are by the 13. of the first equall to two right angles) Let the angle FCE , if it be possible, be greater then a right angle, that is, let it be an obtuse angle. Wherefore the angle DCF shal be an acute angle. And forasmuch as by suppositiō the right line DE toucheth the circle ABC , therefore it cutteth not the circle. Wherefore the circumference BC falleth betwene the right lines DC & CF : & therfore the acute and rectiline angle DCF shall be greater then the angle of the semicircle BCF which is containd vnder the circumference BC & the right line CF . And so shall there be geuē a rectiline & acute angle greater then the angle of a semicircle: which is contrary to the 16. proposition of this booke. Wherefore the angle DCF is not lesse then a right angle. In like sort also may we proue that it is not greater then a right angle. Wherefore it is a right angle, and therfore also the angle FCE is a right angle. Wherefore the right line FC is a perpendicular vnto the right line DE by the 10. definition of the first: which was required to be proued.



¶ The 17. Theoreme. The 19. Proposition.

If a right lyne doo touche a circle, and from the point of the touch be rayssed vp vnto the touch lyne a perpendicular lyne, in that lyne so rayssed vp is the centre of the circle.

Suppose that the right line DE do touch the circle ABC in the point C . And from C raise vp (by $\hat{y}$ 11. of the first) vnto the line DE a perpendicular line CA . Then I say, that in the line CA is the centre of the circle. For if not, then if it be possible, let $\hat{y}$ centre be without the line CA , as in $\hat{y}$ poynt F . And (by the first petition) draw a right line from C to F . And forasmuch as a certaine right line DE toucheth the circle ABC , and from the centre to the touch is drawn a right line CF , therefore (by the 18. of the third) FC is a perpendicular line to



Demonstration leading to an impossibility.

DE .

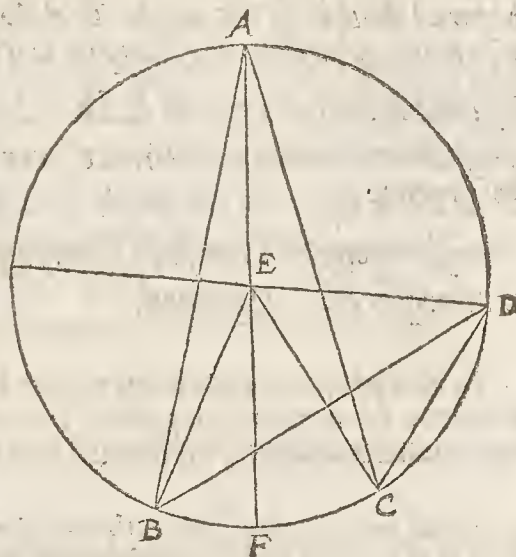
D E. Wherefore the angle FCE is a right angle. But the angle ACE is also a right angle: Wherefore the angle FCE is equall to the angle ACE , namely, the lesse vnto the greater: which is impossible. Wherefore the poynt F is not the centre of the circle ABC . And in like sort may we proue, $\dot{\gamma}$ it is no other where but in the line AC . If therefore a right line do touch a circle, and from the point of the touch be raised vp vnto the touch line a perpendicular line, in that line so raised vp is the centre of the circle: which was required to be proued.

The 18. Theoreme. The 20. Proposition.

In a circle an angle set at the centre, is double to an angle set at the circumference, so that both the angles haue to their base one and the same circumference.



Vppose that there be a circle ABC , and at the centre thereof, namely, the poynt E , let $\dot{\gamma}$ angle BEC be set, & at the circumference let there be set the angle BAC , and let them both haue one and the same base, namely, the circumference BC . Then I say, that the angle BEC is double to the angle BAC . Draw $\dot{\gamma}$ right line AE , and (by the second petition) extend it to the poynt F . Now for asmuch as the line AE is equall to the line EB , for they are drawn from the centre vnto the circumference, the angle EAB is equall to the angle EBA (by the 5. of the first). Wherefore the angles EAB and EBA are double to the angle EAB . But (by the 32. of the same) the angle BEF is equall to the angles EAB and EBA : Wherefore the angle BEF is double to the angle EAB . And by the same reason the angle FEC is double to the angle EAC . Wherefore the whole angle BEC is double to the whole angle BAC .



Two cases in thys Proposition the one when the angle set at the circumference includeth the center.

Demonstration.

Againe, suppose that there be set an other angle at the circumference, and let the same be BDC . And (by the first petition) draw a line from D to E . And (by the second petition) extend the line DE vnto the poynt G . And in like sort may we proue, that the angle GEC is double to the angle EDC . Of which the angle GEB is double to the angle EDB . Wherefore the angle remayning BEC is double to the angle remayning BDC . Wherefore in a circle an angle set at the centre, is double to an angle set at the circumference, so that both the angles haue to their base one and the same circumference: which was required to be demonstrated.

The other whē the same angle set at the circumference includeth not the center.

The third Booke

The 19. Theoreme. The 21. Proposition.

In a circle the angles which consist in one and the selfe same section or segment, are equall the one to the other.

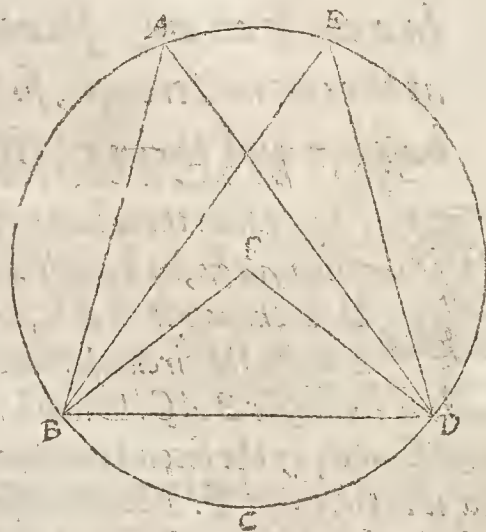
Suppose y there be a circle $ABCD$, & in the segment thereof $BAED$, let there consist these angles BAD and BED . Then I say, that the angles BAD and BED are equall the one to the other. Take (by the

first of the third) the centre of the circle $ABCD$, and let the same be the point F .

Construction.

Demonstration.

And (by the first petition) draw these lines BF and FD . Now for asmuch as the angle $BF D$ is set at the centre, and the angle BAD at the circumference, and they haue both one and y same base, namely, the circumference BCD , therefore the angle $BF D$ is (by the Proposition going before) double to the angle BAD : and by the same reason the angle $BF D$ is also double to the angle BED . Wherefore (by the 7. common sentence) the angle BAD is equall to the angle BED . Wherefore in a circle the angles which consiste in one and the selfe same segment, are equall the one to the other: which was required to be proued.

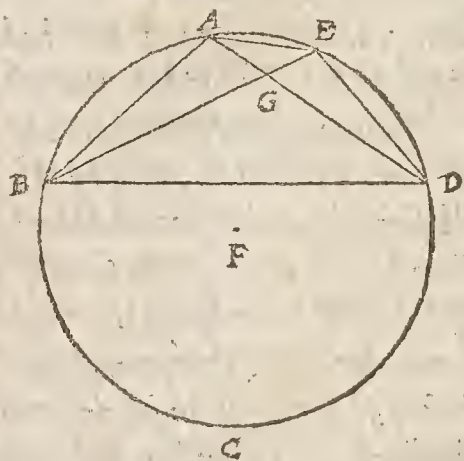


Three cases in this Proposition.

*The first case.
The second case.*

In this proposition are three cases. For the angles consisting in one and the self same segment, the segment may either be greater then a semicircle, or lesse then a semicircle, or els iust a semicircle. For the first case the demonstration before put serueth.

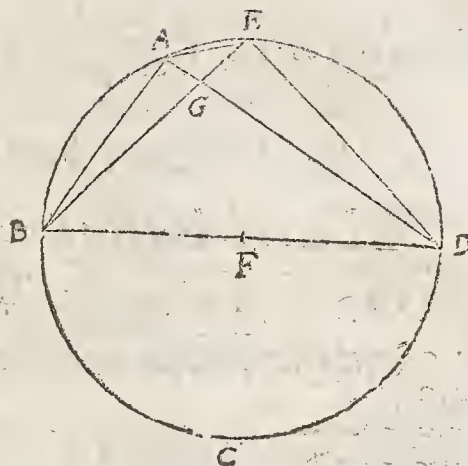
But now suppose that the angles BAD and BED do consist in the sectiō $BAED$, which let be lesse then a semicircle. Euen in this case also I say that the angles BAD and BED are equall. For draw a right line from A to E . And let the lines AD and BE cutte the one the other in the poynt G , wherefore the segment ACE is greater then a semicircle. And therefore by the first part of this proposition the angles whiche are in it, namely, the angles ABE and EDA are equall the one to the other. And forasmuch as in the triangle ABG the inward and opposite angles ABG and GAB are equall to the outwarde angle BGD , and by the same reason the two angles EDG and GED of the triangle DEG are equall to the selfe same outward angle BGD . Wherefore the two angles ABG and GAB are equall to the two angles EDG and GED



by the

by the first commo sentence. From which if there be taken equall angles, namely, ABG , and EDG , the angle remainyng BAG shall be equall to the angle remainyng DEG , that is, the angle BAD to the angle DEB (by the third common sentence) which was required to be proued.

The selfe same construction and demonstration will also serue, if the angles were set in a semi-circle as it is playne to see in the figure here set.



The third case.

The 20. Theoreme. The 22. Proposition.

If within a circle be described a figure of fower sides, the angles therof which are opposite the one to the other, are equall to two right angles.

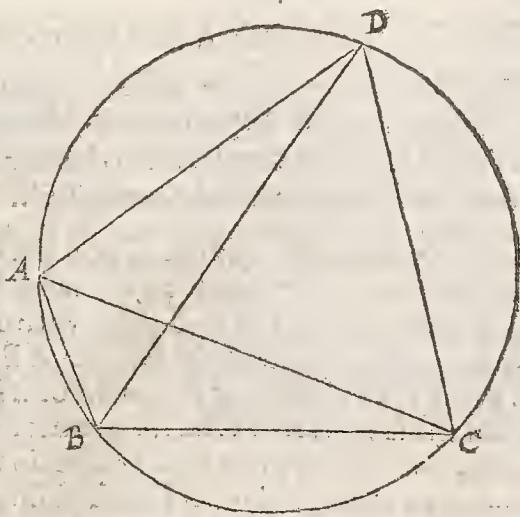


Suppose that there be a circle $ABCD$, and let there be described in it a figure of fower sides, namely, $ABCD$. Then I say, that the angles thereof which are opposite the one to the other, are equall to two right angles. Draw (by the first petition) these right lines AC and BD .

Construction.

Now for as much as (by the 32. of the first) the three angles of euery triangle are equall to two right angles: therefore y three angles of the triangle ABC , namely, CAB , ABC , and BCA , are equall to two right angles. But (by the 21. of the third) the angle CAB is equall to the angle BDC , for they consist in one and the self same segmēt, namely, $BADC$. And (by the same Proposition) the angle ACB is equall to the angle ADB , for they consist in one and the same segmēt $ADCB$. Wherefore the whole angle ADC is equall to y angles BAC and ACB : put the angle ABC common to them both. Wherefore the angles ABC , BAC , and ACB , are equall to the angles ABC and ADC . But the angles ABC , BAC , and ACB , are equall to two right angles. Wherefore the angles ABC and ADC are equall to two right angles. And in like sort also may we proue, that the angles BAD and DCB are equall to two right angles.

Demonstration.



The third Booke

angles. If therefore within a circle be described a figure of fower sides, the angles thereof which are opposite the one to y other, are equall to two right angles: which was required to be proued.

The 21. Theoreme. The 23. Proposition.

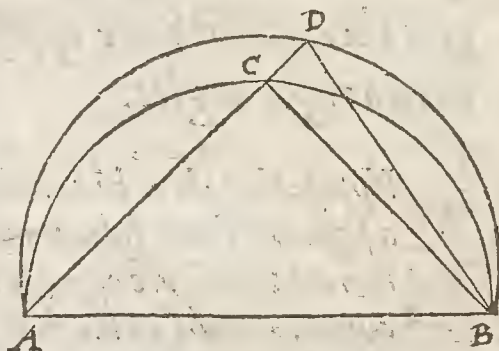
Upon one and the selfe same right line can not be described two like and vnequall segmentes of circles, falling both on one and the selfe same side of the line.

Demonstration leading to an impossibilitie.



Or if it be possible, let there be described vpon the right line AB two like & vnequall sections of circles, namely, ACB & ADB , falling both on one and the selfe same side of the line AB . And (by the first petition) drawe the right line ACD , and (by the third petition) drawe right lines from C to B , and from D to B .

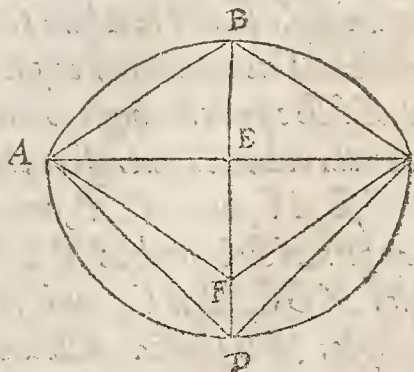
And for asmuch as the segment ACB is like to the segment ADB : and like segmentes of circles are they which haue equall angles (by the 10. definition of the third). Wherefore the angle ACB is equall to the angle ADB , namely, the outward angle of y triangle ADB to the inward angle: which (by the 16. of the first) is impossible. Wherefore vpon one and the selfe same right line can not be described two like & vnequall segmentes of circles, falling both on one & the selfe same side of the line: which was required to be demonstrated.



An addition of Campane demonstrated by Pelitarius.

Here Campane addeth that vpon one and the selfe same right lyne cannot be described two like and vnequall sections neither on one and the selfe same side of the lyne, nor on the opposite side. That they can not be described on one and the selfe same side, hath bene before demonstrated, and that neither also on the opposite side, Pelitarius thus demonstrateth.

Let the section ABC be set vpon the lyne AC , and vpon the other side let be set the section ADC vpon the selfe same lyne AC , and let the section ADC be lyke vnto the section ABC . Then I say that the sections ABC and ADC being thus set are not vnequal. For if it be possible let the section ADC be the greater. And deuide the lyne AC into two equal partes in the point E . And draw the right lyne BED deuiding the lyne AC right angled wise. And draw these right lynes AB , CB , AD and CD . And forasmuch as the section ADC is greater then the section ABC , the perpendicular lyne also ED shall be greater then the perpendicular lyne EB : as is before declared in the end of the definitions of this third booke. Wherefore



Demonstration leading to an impossibilitie.

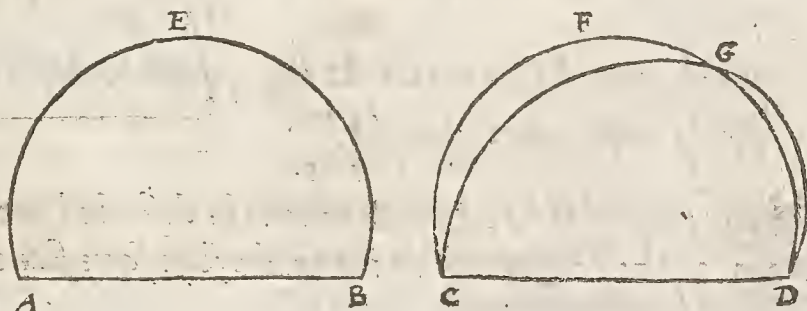
fore

fore from the lyne ED cut of a lyne equall to the lyne EB : which let be EF . And draw these right lynes AF and CF . Now then (by the 4. of the first) the triangle AEB shall be equall to the triangle AEF , and the angle EBA shall be equall to the angle EFA . And by the same reason the angle ECB shall be equall to the angle ECF . Wherefore the whole angle ABC is equall to the whole angle AFC . But by the 21. of the first, the angle AFC is greater then the angle ADC . Wherefore also the angle ABC is greater then the angle ADC . Wherefore by the definition the sections ABC and ADC are not lyke, which is contrary to the supposition. Wherefore they are not lyke and vnequall: which was required to be proued.

The 22. Theorme. The 24. Proposition.

Like segmentes of circles descriled vpon equall right lines, are equall the one to the other.

*S*uppose that vpon these equall right lines AB and CD be described these like segmentes of circles, namely, AEB and CFD . Then I say, that the segment AEB is equall to the segment CFD . For putting the segment AEB vpon the segment CFD , and the poynt A vpon $\hat{y}$ poynt C , and the right line AB vpon the right line CD , the poynt B also shall fall vpon the poynt D , for $\hat{y}$ line AB is equall to the line CD . And the right line AB exactly agreing with the right line CD , $\hat{y}$ segment also AEB shall exactly agree with the segment CFD . For if the right line AB do exactly agree with the right line CD ,



Demonstration leading to an impossibilitie.

and the segment AEB do not exactly agree with the segment CFD , but differeth as the segment CGD doth: Now (by the 13. of the third) a circle cutteth not a circle in more pointes then two, but the circle CGD cutteth $\hat{y}$ circle CFD in more pointes the two, that is, in the points C , G , and D : which is (by the same) impossible. Wherefore the right line AB exactly agreing with the right line CD , the segment AEB shall not but exactly agree with the segment CFD : Wherefore it exactly agreeth with it, and is equall vnto it. Wherefore like segmentes of circles described vpon equall right lines, are equall the one to the other: which was required to be proued.

This Proposition may also be demonstrated by the former proposition. For if the sections AEB and CFD being like and set vpon equall right lines AB and CD , should be vnequall, then the one beyng put vpon the other, the greater shall exceede the lesse: but the line AB is one line with the line CD : so that therby shal follow the contrary of the former Proposition.

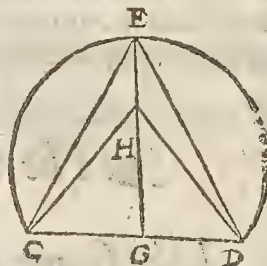
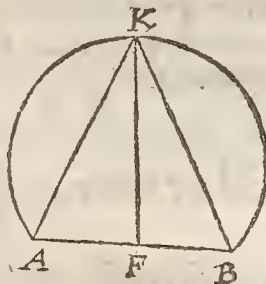
An other demonstration.

The third Booke

Pelitarus demonstrateth this Proposition an other way.

An other demonstration of Pelitarus.

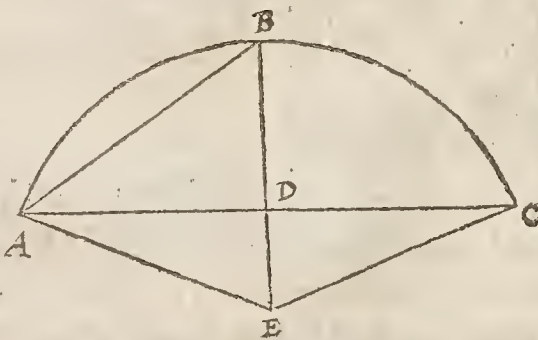
Suppose that there be two right lines AB & CD which let be equall: and vpon the let there be set these like sections ABK , and CDE . Then I say that the said sections are equall. For if not, then let CED be the greater section. And deuide the two lines AB and CD into two equall partes, the line AB in the pointe F , and the line CD in the point G . And erect two perpendicular lines FK and GE . And draw these right lines AK & KB : EC , ED . And forasmuch as the section CED is the greater, therefore the perpendicular line GE is greater then the perpendicular FK : From the lyne GE cut of a line equall to the line FK , which let be GH : and draw these right lines CH and HD . And forasmuch as in the triangle AKF the two sides AF and FK are equall to the two sides CG & GH of the triangle CHG , and the angles at the pointes F and G are equal (for that they are right angles) therefore (by the 4. of the first) the base AK is equall to the base CH , and the angle AKF to the angle CHG . And by the same reason the angle BKF is equall to the angle DHG . Wherefore the whole angle AKB is equall to the whole angle CHD . But the angle CHD is greater then the angle CED by the 21. of the first. Wherefore also the angle AKD is greater then the angle CED . Wherefore the sections are not lyke, which is contrary to the supposition.



The 3. Probleme. The 25. Proposition.

A segment of a circle beyng geuen to describe the whole circle of the same segment.

Suppose that y segment geuen be ABC . It is required to describe the whole circle of the same segment ABC . Deuide (by the 10. of the first) the line AC into two equall partes in the poynt D . And (by the 11. of the same) from the poynt D raise vp vnto the line AC a perpendicular line BD . And (by the first petition) draw a right line from A to B . Now then the angle ABD being compared to y angle BAD , is either greater then it, or equall vnto it, or lesse then it.



Construction.

Three cases in this Proposition.

The first case.

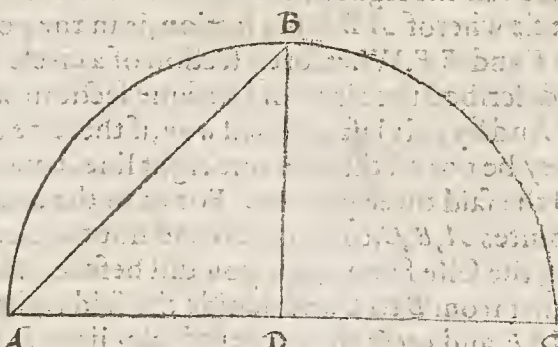
Demonstration.

First let it be greater. And (by the 23. of the same) vpon the right line BA , and vnto the poynt in it A , make vnto the angle ABD an equall angle BAE . And (by the second petition) extend the line BD vnto the poynt E . And (by the first petition) draw a line from E to C . Now for asmuch as the angle ABE is equall to the angle BAE , therefore (by the 6. of the first) the right line EB is equall to the right line AE . And for asmuch as the line AD is equall to y line DC , and the line DE is common to them both: therefore these two lines AD and

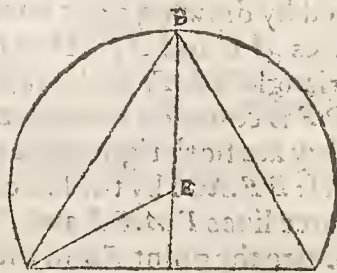
and DE , are equall to these two lines CD and DE the one to the other. And the angle ADE is (by the 4. petition) equall to the angle CDE , for either of them is a right angle. Wherefore (by the 4. of the first) the base AE is equall to the base CE . But it is proved, that the line AE is equall to y line BE . Wherefore the line BE also is equall to the line CE . Wherefore these three lines AE , EB , and EC , are equall the one to the other. Wherefore making the centre E , and the space either AE , or EB , or EC , describe (by the third petition) a circle, and it shall passe by the poyntes A, B, C . Wherefore there is described the whole circle of the segment geuen. And it is manifest, that the segment ABC is lesse then a semicircle, for the centre E falleth without it.

The like demonstration also will serue if the angle ABD be equall to the angle BAD . For the line AD being equall to either of these lines, BD , and DC , there are three lines, DA, DB , and DC , equall the one to the other. So that the point D shall be the centre of the circle being complete, and ABC shall be a semicircle.

But if the angle ABD be lesse then the angle BAD , then (by the 23. of the first) vpon the right line BA , and vnto the point in it A , make vnto the angle ABD an equall angle within y segment ABC . And so the centre of the circle shall fall in y line DB , and it shall be the point E : and the segment ABC shall be greater then a semicircle. Wherefore a segment being geuen, there is described the whole circle of the same segment: which was required to be done.



The second case.

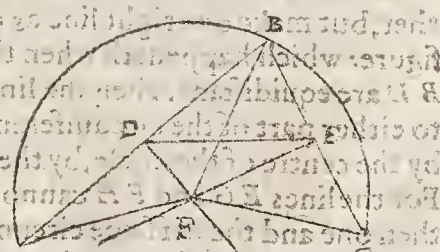


The third case.

Corollary.

Hereby it is manifest, that in a semicircle the angle BAD is equall to the angle DBA : but in a section lesse then a semicircle, it is lesse: in a section greater then a semicircle, it is greater.

There is also another generall way to finde out the foresaid centre, which will serue indifferently for any section whatfoeuer: And that is thus. Take in the circumference geuen or section ABC three poyntes at all auentures which let be A, B, C . And draw these lines AB and BC (by the first petition) And (by the 10. of the first) deuide into two equall partes either of the sayde lines, the line AB in the point D , & the line BC in the point E . And (by the 11. of the first) from the poyntes



An other more ready way.

Dd. jcc D and

The third Booke

Demonstration.

D and *E* rayse vp vnto the lines *AB* and *BC* perpendicular lynes *DF* and *EF*. Now forasmuch as either of these angles *BD F*, and *BE F* is a right angle, a right line produced from the point *D* to the point *E*, shall deuide either of the said angles: and forasmuch as it falleth vpon the right lines *DF* and *EF*, it shall make the inward angles on one and the selfe same side, namely, the angles *DEF* and *ED F* lesse then two right angles. Wherefore (by the fift petition) the lines *DF* and *EF* being produced shall concurre. Let them concurre in the point *F*. And forasmuch as a certaine right line *DF* deuideth a certaine right lyne *AB* into two equall partes and perpendicularly, therfore (by the corollary of the first of this booke) in the line *DF* is the centre of the circle, & by the same reason the centre of the selfe same circle *A* shalbe in the right line *EF*. Wherefore the centre of the circle wherof *ABC* is a section, is in the point *F*, which is commo to either of the lines *DF* and *EF*. Wherefore a section of a circle being geue, namely, the section *ABC*, there is described the circle of the same section: which was required to be done.

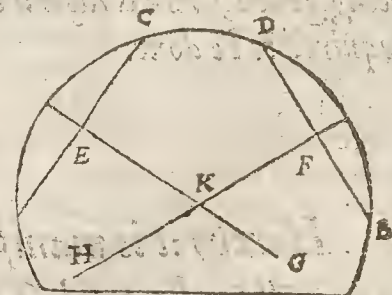


An addition.

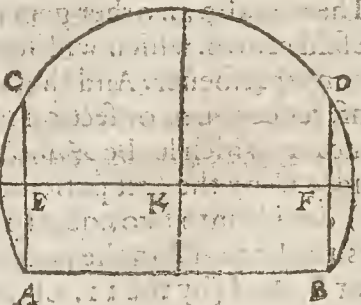
And by this last generall way, if there be geuen three pointes, set howsoever, so that they be not all three in one right line, a man may describe a circle which shall passe by all the said three pointes. For as in the example before put, if you suppose onely the 3. pointes *A, B, C*, to be geuen and not the circumference *ABC* to be drawn, yet following the selfe same order you did before, that is, draw a right line from *A* to *B* and another from *B* to *C* and deuide the said right lines into two equall parts, in the points *D* and *E*, and erect the perpendicular lines *DF* and *EF* cutting the one the other in the point *F*, and draw a right line from *F* to *B*: and making the centre the point *F*, and the space *FB* describe a circle, and it shall passe by the pointes *A* & *C*: which may be proved by drawing right lines from *A* to *F*, and from *F* to *C*. For forasmuch as the two sides *AD* and *DF* of the triangle *ADF* are equall to the two sides *BD* and *DF* of the triangle *BD F* (for by supposition the line *AD* is equall to the line *DB*, and the lyne *DF* is common to them both) and the angle *ADF* is equall to the angle *BD F* (for they are both right angles) therfore (by the 4. of the first) the base *AF* is equall to the base *BF*. And by the same reason the line *FC* is equall to the line *FB*. Wherefore these three lines *FA, FB* and *FC* are equall the one to the other. Wherefore making the centre the point *F* and the space *FB*, it shall also passe by the pointes *A* and *C*. Which was required to be done. This proposition is very necessary for many things as you shall afterward see.

An other construction and demonstration of this Proposition after Campane

Campane putteth an other way, how to describe the whole circle of a sectio geuen. Suppose that the section be *AB*. It is required to describe the whole circle of the same section. Draw in the section two lines at all aduentures *AC* and *BD*: which deuide into two equall parts *AC* in the point *E*, and *BD* in the point *F*. Then from the two pointes of the deuisions draw within the section two perpendicular lines *EG* and *FH* which let cutte the one the other in the point *K*. And the centre of the circle shall be in either of the said perpendicular lines by the corollary of the first of this booke. Wherefore the point *K* is the centre of the circle: which was required to be done.



But if the lines *EG* & *FH* do not cut the one the other, but make one right line as doth *GH* in the secod figure: which happeneth when the two lines *AC* and *BD* are equidistant. Then the line *GH*, being applyed to either part of the circumference geuen, shall passe by the centre of the circle, by the selfe same Corollary. For the lines *EG* and *FH* cannot be equidistant. For then one and the selfe same circumference should haue two centres. Wherefore the line *HG* being deuided in

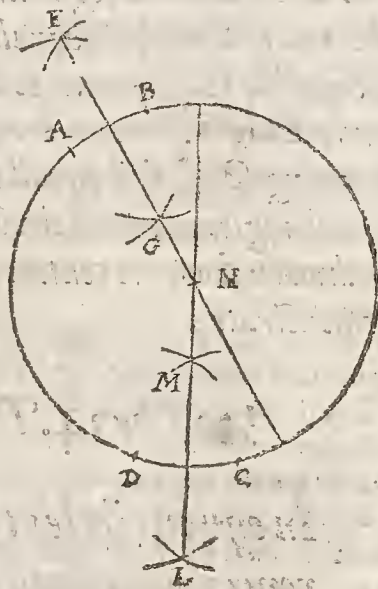
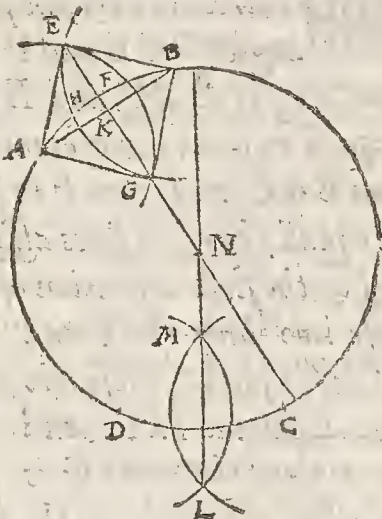


in two equall partes in the point K , the said point K shall be the centre of the section.

Pelitaris here addeth a brieve way how to finde out the centre of a circle, which is commonly vsed of Artificers.

Suppose that the circumference be $A B C D$, whose centre it is required to finde out. Take a point in the circumference geuen which let be A , vpon which describe a circle with what opening of the compasse you will, which let be $E F G$. Then take an other point in the circumference geuen which let be B , vpon which describe an other circle with the same opening of the compasse, that the circle $E F G$ was described, and let the same be $E H G$, which let cut the circle $E F G$ in the two pointes E and G . (I haue not here drawn the whole circles, but onely those partes of them which cut the one the other for auoyding of confusion) And drawe from those centres these right lines $A E$, $B E$, $A G$, and $B G$, which foure lines shall be equall, by reason they are semidiameters of equall circles. And draw a right line from A to B , and so shall there be made two Isosceles triangles $A E B$, and $A G B$ vnto whom the line $A B$ is a common base. Now then deuide the line $A B$ into two equal partes in the point K which must nedes fall betwene the two circumference $E F G$ and $E H G$, otherwise the part should be greater then his whole. Drawe a line from E to K and produce it to the point G . Now you see that there are two Isosceles triangles deuided into foure equall triangles $E A K$, $E B K$, $G A K$ and $G B K$. For the two sides $A E$ and $A K$ of the triangle $A E K$ are equall to the two sides $B E$ and $B K$ of the triangle $B E K$, and the base $E K$ is common to them both. Wherefore the two angles at the point K of the two triangles $A E K$ and $B E K$ are by the 8. of the first equall: and therefore are right angles. And by the same reason the other angles at the poynte K are right angles. Wherefore $E G$ is one right lyne by the 14. of the first. Which forasmuch as it deuideth the line $A B$ perpendicularly, therefore it passeth by the center by the corollary of the first of this booke. And so if you take two other poyntes, name C and D in the circumference geuen, and vpon the describe two circles cuttyng the one the other in the pointes L and M , and by the said poyntes produce a right line, it shall cutte the lyne $E G$ beyng produced in the pointe N , which shall be the centre of the circle by the same Corollary of the first of this booke, if you imagine the right line $C D$ to be drawn and to be deuided perpendicularly by the lyne $L M$, which it must nedes be as we haue before proued. And here note that to do this mechanically not regarding demonstration, you neede onely to marke the poyntes where the circles cut the one the other, namely, the poyntes E , G , and L , M , and by these poyntes to produce the lines $E G$ and $L M$ till they cut the one the other, and where they cut the one the other, there is the centre of the circle, as you see here in the seconde figure.

A ready way to finde out the center of a circle commonly vsed amongst artificers.



The third Booke

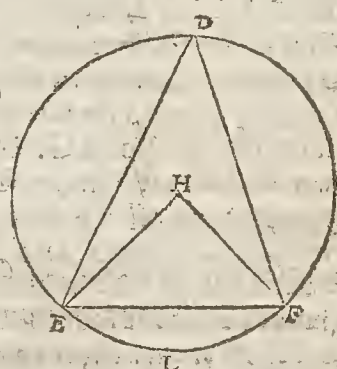
The 23. Theoreme. The 26. Proposition.

Equall angles in equall circles consist in equall circūferences, whether the angles be drawen from the centres, or from the circumferences.

Construction.

Demonstration.

Suppose that these circles ABC and DEF , be equall. And from their centres, namely, the pointes G and H , let there be drawen these equall angles BGC and EHF : and likewise from their circumferences these equall angles BAC and EDF . Then I say, that the circumference BKC is equall to the circumference ELF . Draw (by the first petition) right lines from B to C , and from E to F . And for asmuch as the circles ABC and DEF are equall, the right lines also drawen from their centres to their circumferences, are (by the first definition of the third) equall the one to the other. Wherefore these two lines BG and GC , are equall to these two lines EH and HF . And the angle at the poynt G is equall to the angle at the point H : Wherefore (by the 4. of the first) the base BC is equall to y base EF . And for asmuch as the angle at



the poynt A is equall to the angle at the point D , therefore the segment BAC is like to the segment EDF . And they are described vpon equall right lines BC and EF . But like segmentes of circles described vpon equall right lines, are (by the 24. of the third) equall the one to the other. Wherefore the segment BAC is equall to the segment EDF . And the whole circle ABC is equall to y whole circle DEF . Wherefore (by the third common sentence) the circumference remainyng BKC is equall to the circumference remainyng ELF . Wherefore equall angles in equall circles consist in equall circūferences; whether the angles be drawen from the centres or from the circumferences: which was required to be demonstrated.

The 24. Theoreme. The 27. Proposition.

In equall circles the angles which consist in equall circūferences, are equall the one to the other, whether the angles be drawen from the centres, or from the circumferences.

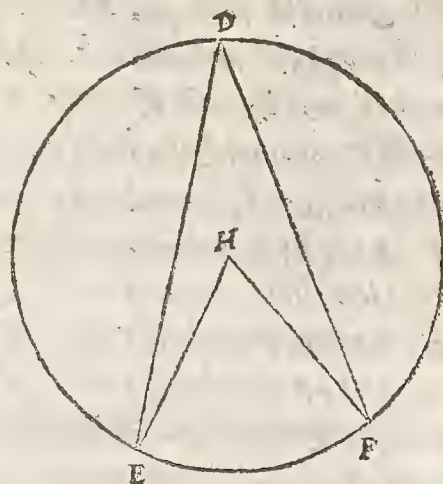
Suppose



Suppose these circles ABC , and DEF , be equall. And upon these equall circumferences of the same circles, namely, upon BC and EF , let there consist these angles BGC and EHF drawn from the centres, and also these angles BAC and EDF drawn from the circumferences. Then I say, that the angle BGC is equall to the angle EHF , and the angle BAC to the angle EDF . If the angle BGC be equall to the angle EHF , then it is manifest, that the angle BAC is equall to the angle EDF (by the 20. of the third). But if the angle BGC be not equall to the angle EHF , then is the one of them greater then the other. Let the angle BGC be greater

Demonstration leading to an impossibility.

And (by the 23. of the first) upon the right line BG , and unto the point given in it G , make unto the angle EHF an equall angle BGK . But (by the 26. of the third) equall



angles in equall circles consist upon equall circumferences, whether they be drawn from the centres or from the circumferences. Wherefore the circumference BK is equall to the circumference EF . But the circumference EF is equall to the circumference BC : Wherefore the circumference BK also is equall to the circumference BC , the lesse to the greater: which is impossible. Wherefore the angle BGC is not unequall to the angle EHF : Wherefore it is equall. And (by the 20. of the third) the angle at the point A is the halfe of the angle BGC : and (by the same) the angle at the point D is the halfe of the angle EHF . Wherefore the angle at the point A is equall to the angle at the point D . Wherefore in equall circles, the angles which consist in equall circumferences, are equall the one to the other, whether the angles be drawn from the centres or from the circumferences: which was required to be proued.

The 25. Theoreme. The 28. Proposition.

In equall circles, equall right lines do cut away equall circumferences, the greater equall to the greater, and the lesse equall to the lesse.

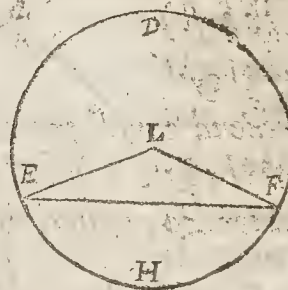
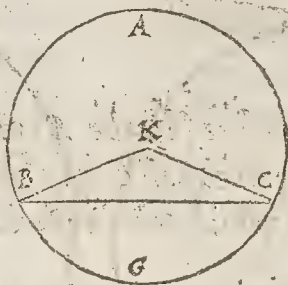
Qd. ij.

Suppose

The third Booke



Suppose that these circles ABC , and DEF , be equall. And in them let there be drawen these equall right lines BC and EF , which let cut away these circumferences BAC and DEF being the greater, & also these circumferences BGC and EHF being the lesse. Then I say, that the greater circumference BAC is equall to the greater circumference DEF : and the lesse circumference BGC is equall to the lesse circumference EHF . Take (by the first of the third) the centres of the circles, and let the same be the pointes K and L . And draw these right lines, KB , KC , LE , and LF . And for asmuch as the circles are equall, therefore (by the first definition of the third) the lines which are drawen frō the centres are equall. Wherefore these two lines BK and KC , are equall to these two lines EL and LF . And (by supposition) the base BC is equall to the base EF . Wherefore (by the 8. of γ first)



the angle BKC is equall to the angle ELF . But (by the 26. of the third) equall angles drawen from the centres, consist vpon equall circumferences. Wherefore the circumference BGC is equall to the circumference EHF : and γ whole circle ABC is equall to the whole circle DEF . Wherefore the circumference remayning BAC , is (by the third common sentence) equall to the circumference remayning EDF . Wherefore in circles, equall right lines do cut away equall circumferences, the greater equall to the greater, and the lesse equall to the lesse: which was required to be proued.

The 26. Theoreme. The 29. Proposition.

In equall circles, vnder equall circumferences are subtended equall right lines.



Suppose that these circles ABC and DEF , be equall. And in them let there be taken these equall circumferences, BGC and EHF : and drawe these right lines BC and EF . Then I say, that the right line BC is equall to the right line EF . Take (by the first of γ third) the centres of the circles, and let them be the pointes K and L , and draw these right lines KB , KC , LE , LF . And for asmuch as the circumference BGC is equall to the circumference EHF , the angle BKC is equall to γ angle ELF (by the 27. of the third). And for asmuch as the circles ABC and DEF are equall the one to the other, therefore (by the first definition of the third) the lines

The converse of the former Proposition.

Construction.

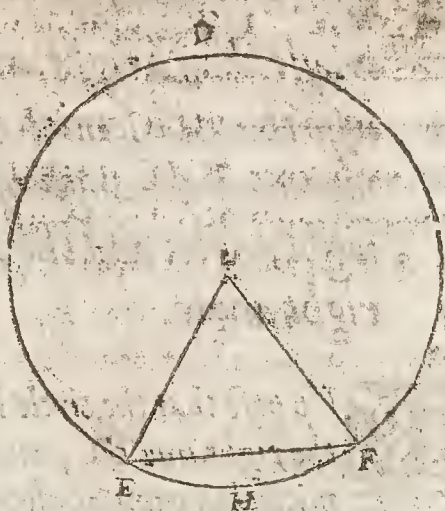
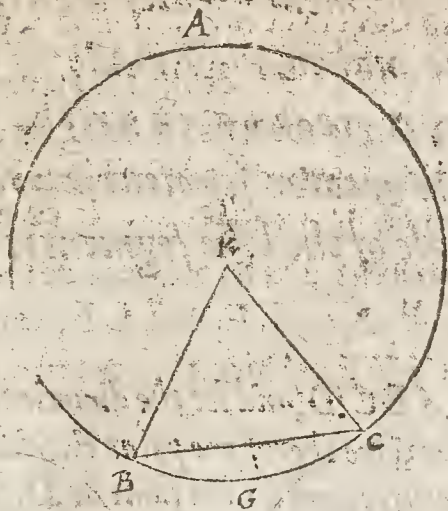
Demonstration.

which

whiche are drawe from the centres are equall.

Wherefore these y. lines BK and KC, are equall to these y. lines LE and LF, & they

comprehend equall angles. Wherefore (by the 4. of the first) the base BC is equall to the base EF. Wherefore in equall circles vnder equall circumferences, are subtended equall right lines: which was required to be demonstrated.



The 4. Probleme. The 30. Proposition.

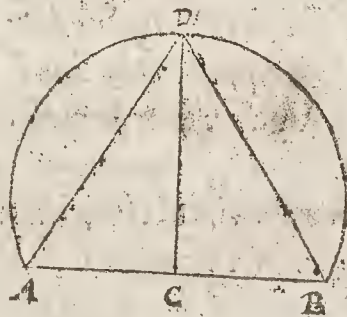
To deuide a circumference geuen into two equall partes.



Suppose that the circumference geuen be ADB . It is required to deuide the circumference ADB into two equall partes. Draw a right line from A to B . And (by the 10. of the first) deuide the line AB into two equall partes in the point C . And (by the 11. of the first) from the point C raise vp vnto AB a perpendicular line CD . And draw these right lines AD and DB . And forasmuch as the line AC is equall to the line CB , & the line CD is common to them both, therefore these two lines AC and CD are equall to these two lines BC and CD . And (by the 4. petition) the angle ACD is equall to the angle BCD , for either of them is a right angle. Wherefore (by the 4. of the first) the base AD is equall to the base DB . But equall right lines do cut away equall circumferences, the greater equall to the greater, & the lesse equall to the lesse (by the 28. of the third) And either of these circumferences AD and DB is lesse then a semicircle. Wherefore the circumference AD is equall to the circumference DB . Wherefore the circumference geuen is deuided into two equall partes: Which was required to be done.

Construction.

Demonstration.



The 27. Theoreme. The 31. Proposition.

In a circle an angle made in the semicircle is a right angle.

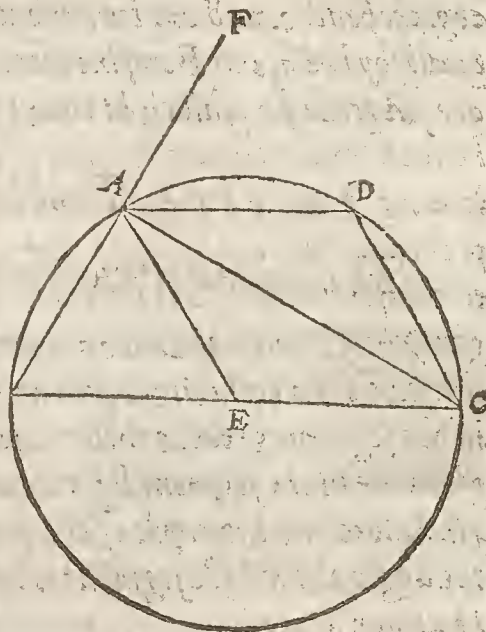
Dd. iiii. but

The third Booke

but an angle made in the segment greater then the semicircle is lesse then a right angle, and an angle made in the segment lesse then the semicircle, is greater then a right angle. And moreover the angle of the greater segment is greater then a right angle: and the angle of the lesse segment is lesse then a right angle.

The first part
of this Theoreme.

Suppose that the circle be $ABCD$, and let the diameter of the circle be the right line BC , and the centre thereof the point E . And take in the semicircle a point at all adventures, and let the same be D . And draw these right lines BA, AC, AD , and DC . Then I say that the angle in the semicircle BAC , namely, the angle BAC is a right angle. And the angle ABC which is in the segment ABC being greater then the semicircle, is lesse then a right angle. And the angle ADC which is in the segment ADC being lesse then the semicircle is greater then a right angle. Draw a line from the point A to the point E , and extend the line BA unto the point F . And forasmuch as the line BE is equall to the line EA , (for they are drawn from the centre to the circumference) therefore the angle EAB is equall to the angle EBA (by the 5. of the first). Again forasmuch as the line AE is equall to the line EC , the angle ACE is (by the same) equall to the angle CAE . Wherefore the whole angle BAC is equall to these two angles ABC and ACB . But the angle FAC which is an outward angle of the triangle ABC is (by the 32. of the first) equall to the two angles ABC & ACB . Wherefore the angle BAC is equall to the angle FAC . Wherefore either of them is a right angle. Wherefore the angle BAC which is in the semicircle BAC is a right angle.



Second part.

And forasmuch as (by the 17. of the first) the two angles of the triangle ABC , namely, ABC and BAC are lesse then two right angles, and the angle BAC is a right angle. Therefore the angle ABC is lesse then a right angle, and it is in the segment ABC which is greater then the semicircle.

Third part.

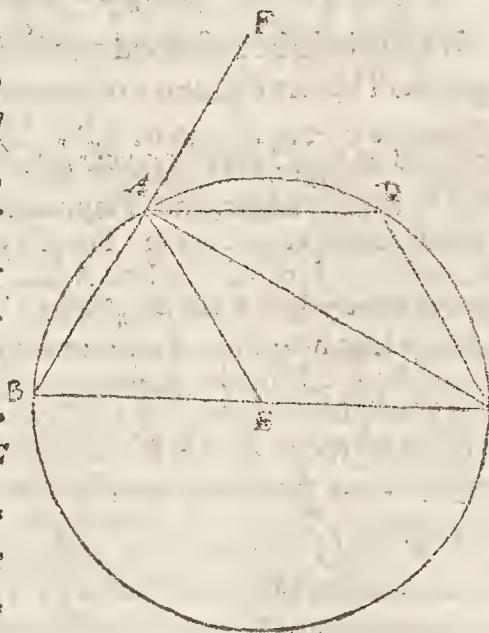
And forasmuch as in the circle there is a figure of foure sides, namely, $ABCD$. But if within a circle be described a figure of foure sides, the angles thereof which are opposite the one to the other are equall to two right angles (by the 22. of the third) Wherefore (by the same) the angles ABC and ADC are equall

to two right angles. But the angle ABC is lesse then a right angle. Wherefore the angle remayning ADC is greater then a right angle, and it is in a segment which is lesse then the semicircle.

Now also I say that the angle of the greater segment, namely, the angle which is comprehended vnder the circumference ABC and the right line AC is greater then a right angle, and the angle of the lesse segment comprehended vnder the circumference ADC , and the right line AC is lesse then a right angle, which may thus be proued. Forasmuch as the angle comprehended vnder the right lines BA and AC is a right angle, therefore the angle comprehended vnder the circumference ABC and the right line AC is greater then a right angle: for the whole is euer greater then his part (by the 9. common sentence.

The fourth part.

Againe forasmuch as the angle comprehended vnder the right lines AC and AF is a right angle, therefore the angle comprehended vnder the right line CA and the circumference ADC is lesse then a right angle. Wherefore in a circle an angle made in the semicircle is a right angle, but an angle made in the segment greater then the semicircle is lesse then a right angle, and an angle made in the segment lesse then the semicircle, is greater then a right angle. And moreover the angle of the greater segment is greater then a right angle: & the angle of the lesse segment is lesse then a right angle: which was required to be demonstrated.



The first and last part.

An other demonstration to proue that the angle BAC is a right angle. Forasmuch as the angle AEC is double to the angle BAE (by the 32. of the first) for it is equall to the two inward angles which are opposite. But the inward angles are (by the 5. of the first) equall the one to the other, and the angle AEB is double to the angle EAC . Wherefore the angles AEB and AEC are double to the angle BAC . But the angles AEB and AEC are equall to two right angles: Wherefore the angle BAC is a right angle. Which was required to be demonstrated.

An other Demonstration to proue that the angle in a semicircle is a right angle.

Correlary.

Hereby it is manifest, that if in a triangle one angle be equall to the two other angles remayning the same angle is a right angle:

A Corollary.

The third Booke

angle: for that the side angle to that one angle (namely, the angle which is made of the side produced without the triangle) is equall to the same angles, but when the side angles are equall the one to the other, they are also right angles.

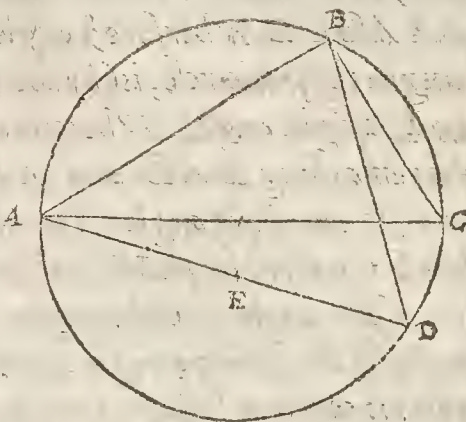
¶ An addition of Pelitarius.

If in a circle be inscribed a rectangle triangle, the side opposite vnto the right angle shall be the diameter of the circle.

*An addition
of Pelitarius.*

*Demonstra-
tion leading
to an absurdi-
tie.*

Suppose that in the circle ABC be inscribed a rectangle triangle ABC ; whose angle at the point B let be a right angle. Then I say, that the side AC is the diameter of the circle. For if not, then shall the centre be without the line AC , as in the point E . And draw a line from the poynt A to the point E , & produce it to the circumference to the point D : and let AED be the diameter: and draw a line from the point B to the point D . Now (by this 31. Proposition) the angle ABD shall be a right angle, and therefore shall be equall to the right angle ABC , namely, the part to the whole: which is absurde. Euen so may we proue, that the centre is in no other where but in the line AC . Wherefore AC is the diameter of the circle: which was required to be proued.



¶ An addition of Campane.

*An addition
of Campane.*

By thys 31. Proposition, and by the 16. Proposition of thys booke, it is manifest, that although in mixt angles, which are containd vnder a right line and the circumference of a circle, there may be geuen an angle lesse & greater then a right angle, yet can there neuer be geue an angle equall to a right angle. For euery section of a circle is eyther a semicircle, or greater then a semicircle, or lesse, but the angle of a semicircle is by the 16. of thys booke, lesse then a right angle, and so also is the angle of a lesse section by thys 31. Proposition: Likewise the angle of a greater section, is greater then a right angle, as it hath in thys Proposition bene proued.

The 28. Theoreme. The 32. Proposition.

If a right line touch a circle, and from the touch be drawn a right line cutting the circle: the angles which that line and the touch line make, are equall to the angles which consist in the alternate segmentes of the circle.

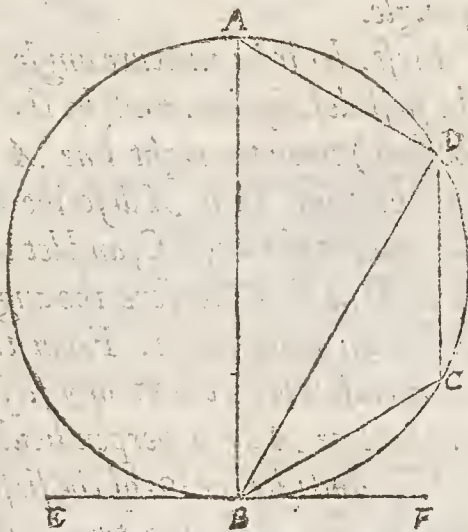


Suppose that the right line EF do touch the circle $ABCD$ in the point B : and from the point B let there be drawn into the circle $ABCD$ a right line cutting the circle, and let the same be BD . Then I say, that the angles which the line BD together with the touch

touch line EF do make, are equall to the angles which are in the alternate segmentes of the circle, that is, the angle FBD is equall to the angle which consisteth in the segment BAD , and the angle EBD is equall to the angle which consisteth in the segment BCD . Raise vp (by the 11. of the first) from y point B vnto the right line EF a perpendicular line BA . And in the circumference BD take a point at all aduentures, and let the same be C . And draw these right lines AD , DC , and CB . And for asmuch as a certaine right line EF toucheth the circle $ABCD$ in the point B , and from the point B where the touch is, is rayed vp vnto the touch line a perpendicular BA . Therefore (by the 19. of the third) in the line BA is the centre of the circle $ABCD$. Wherefore y angle ADB being in the semicircle, is (by the 31. of the third) a right angle. Wherefore the angles remayning BAD and ABD , are equall to one right angle. But the angle ABF is a right angle. Wherefore the angle ABF is equall to the angles BAD and ABD . Take away y angle ABD which is common to them both. Wherefore the angle remayning DBF , is equall to the angle remayning BAD , which is in the alternate segment of the circle. And for asmuch as in the circle is a figure of four sides, namely, $ABCD$, therefore (by the 22. of the third) the angles which are opposite the one to the other, are equall to two right angles. Wherefore the angles BAD and BCD , are equall to two right angles. But the angles DBF and DBE , are also equall to two right angles. Wherefore the angles DBF and DBE , are equall to the angles BAD and BCD . Of which we haue proued that the angle BAD is equall to the angle DBF . Wherefore the angle remayning DBE , is equall to the angle remayning DCB , which is in the alternate segment of the circle, namely, in the segment DCB . If therefore a right line touch a circle, and from the touch be drawn a right line cutting the circle: the angles which that line and the touch line make, are equall to y angles which consist in y alternate segmentes of the circle: which was required to be proued.

Construction.

Demonstration.



In thys Proposition may be two cases. For the line drawn from the touch and cutting the circle, may eyther passe by the centre or not. If it passe by the centre, then is it manifest (by the 18. of thys booke) that it falleth perpendicularly vpon the touch line, and deuidenth the circle into two equal partes, so that all the angles in eche semicircle, are by the former Proposition, right angles, and therefore equall to the alternate angles made by the sayd perpendicular line and the touch line. If it passe not by the centre, then followe the construction and demonstration before put.

Two cases in this Proposition.

The

The third Booke

The 5. Probleme.

The 33. Proposition.

Uppon a right lyne geuen to describe a segment of a circle, which shall contayne an angle equall to a rectiline angle geue.



Suppose that the right line geuen be AB , and let the rectiline angle geuen be C . It is required vpon the right line geue AB to describe a segment of a circle which shall contayne an angle equall to the angle C . Now the angle C is either an acute angle, or a right angle, or an obtuse angle.

Three cases in this Proposition.

The first case.

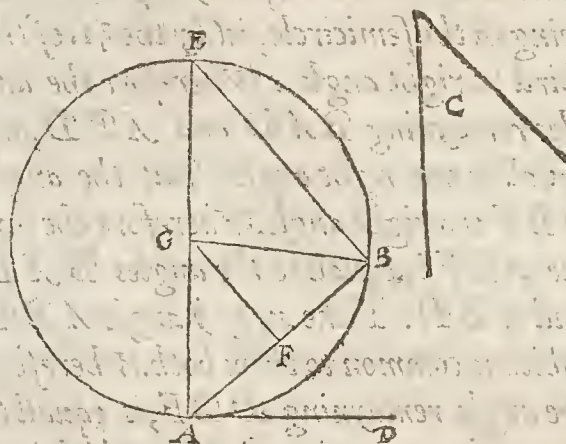
Construction.

First, let it be an acute angle as in the first description. And (by the 23 of the first) vpon the right line AB and to the point in it A describe an angle equal to the angle C , and let the same be DAB . Wherefore the angle DAB is an acute angle. From the point A raise vp (by the 11. of the first) vnto the line AD a perpendicular line AE . And (by the 10. of the first) deuide the line AB into two equall partes in the point F . And (by the 11. of the same) from the point F raise vp vnto the line AB a perpendicular lyne FG , and draw a line from G to B . And forasmuch as the line AF is equall to the line FB and the line FG is common to them both, therefore these two lines AF and FG are equall to these two lines FB and FG : and the angle AFG is (by the 4. petition) equall to the angle $GF B$. Wherefore (by the 4. of the same) the base AG is equall to the base GB . Wherefore making the centre G and the space GA describe (by the 3. petition) a circle and it shall passe by the point B : describe such a circle & let the same be ABE . And draw a line from E to B . Now forasmuch as from the ende of the diameter AE , namely, from the point A is drawen a right line AD making together with the right line AE a right angle, therefore (by the correllary of the 16. of the third) the line AD toucheth the circle ABE . And forasmuch as a certaine right line AD toucheth the circle ABE , & from the point A where the touch is, is drawn into the circle a certaine right line AB : therefore (by the 32. of the third) the angle DAB is equall to the angle AEB which is in the alternate segment of the circle. But the angle DAB is equall to the angle C , wherefore the angle C is equall to the angle AEB . Wherefore vpon the right line geuen AB is described a segment of a circle which containeth the angle AEB , which is equall to the angle geuen, namely, to C .

Demonstration.

The second case.

But now suppose that the angle C be a right angle. It is againe required vpon



The third Booke

Wherefore the angle which is in the segment AHB is equall to the angle C . Wherefore vpon the right line geuen AB , is described a segment of a circle AHB , which containeth an angle equall to the angle geuen, namely, C : which was required to be done.

The 6. Probleme. The 34. Proposition.

From a circle geuen to cut away a section which shal containe an angle equall to a rectiline angle geuen.

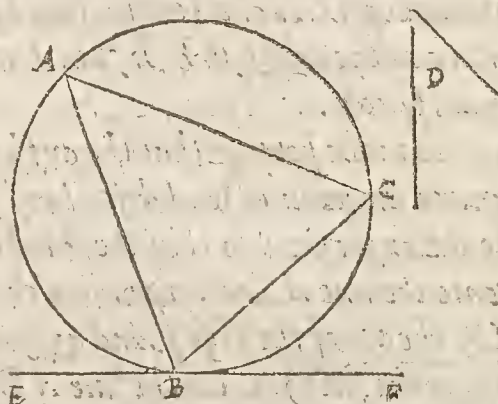
Construction.



Vppose that the circle geuen be AC and let the rectiline angle geuen be D . It is required fro the circle ABC to cut away a segment which shall containe an angle equall to the angle D . Draw (by the 17. of the third) a line touching the circle, and let the same be EF : and let it

Demonstration.

touche in the point B . And (by the 23. of the first) vpon the right line EF and to the point in it B describe the angle FBC equall to the angle D . Now forasmuch as a certayne right line EF toucheth the circle ABC in the point B : and from a point of the touche, namely, B , is drawn into the circle a certayne right line BC ; therefore (by the 32. of the third) the angle FBC is equall to the angle BAC which is in the alternate segment. But the angle FBC is equall to the angle D . Wherefore the angle BAC which consisteth in the segment BAC is equall to the angle D . Wherefore from the circle geuen ABC is cut away a segment BAC , which containeth an angle equall to the rectiline angle geuen: which was required to be done.



The 29. Theoreme. The 35. Proposition.

If in a circle two right lines do cut the one the other, the rectangle parallelograme comprehended under the segmentes or parts of the one line is equall to the rectangle parallelograme comprehended under the segment or partes of the other line.

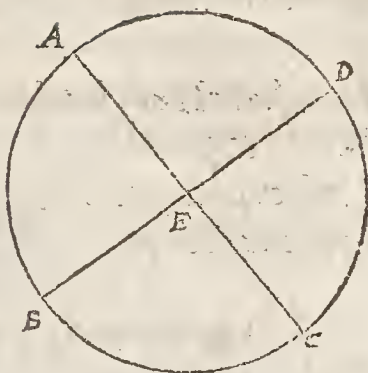


Et the circle be $ABCD$, and in it let these two right lines AC and BD cut the one the other in the point E . Then I say that the rectangle parallelogramme containd vnder the partes AE and EC is equall to the

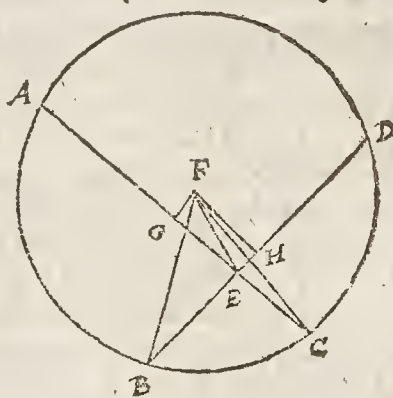
the rectangle parallelogramme contained vnder the partes DE and EB . For if the line AC and BD be drawn by the centre, then is it manifest, that for as much as the lines AE and EC are equall to the lines DE and EB by the definition of a circle, the rectangle parallelogramme also containd vnder the lines AE and EC is equall to $\hat{y}$ rectangle parallelogramme contained vnder the lines DE and EB .

But now suppose that the lines AC and DB be not extended by the centre, and take (by the 1. of the third) the centre of the circle $ABCD$, and let the same be the point F , and from the point F draw to the right lines AC and DB perpendicular lines FG and FH (by the 12. of the first) and draw these right lines FB , FC , and FE .

And forasmuch as a certaine right line FG drawn by the centre, cutteth a certaine right line AC not drawn by the centre in such sorte that it maketh right angles, it therfore deuidenth the line AC into two equall partes (by the 3. of the third). Wherefore the line AG is equall to the line GC . And forasmuch as the right line AC is deuided into two equall partes in the point G , and into two vnequall partes in the point E : therfore (by the 5. of the second) the rectangle parallelogramme contained vnder the lines AE and EC together with the square of the line EG is equall to the square of the line GC . Put the square of the line GF common to them both, wherefore that which is contained vnder the lines AE & EC together with the squares of the lines EG and GF is equall to the squares of the lines GF & GC . But vnto $\hat{y}$ squares of $\hat{y}$ lines EG & GF is equall $\hat{y}$ square of $\hat{y}$ line FE (by the 47. of the first): and to the squares of the lines GC and GF is equall the square of the line FC (by the same) Wherefore that which is contained vnder the lines AE and EC , together with the square of the line FE is equall to the square of the line FC . But the line FC is equall to the line FB . For they are drawn from the centre to the circumference. Wherefore that which is contained vnder the lines AE and EC together with the square of the line FE is equal to the square of the line FB . And by the same demonstration that which is contained vnder the lines DE and EB together with the square of the line FE is equall to the square of the line FB . Wherefore that which is contained vnder the lines AE and EC together with the square of the line FE is equall to that which is containd vnder the lines DE and EB together with the square of the line FE . Take away the square of the line FE which is common to them both. Wherefore the rectangle parallelogramme remayning which is containd vnder the lines AE and EC is equall to the rectangle parallelogramme remayning which is containd vnder the lines DE and EB . If therefore in a circle two right lines do cut the one the other: the rectangle parallelogramme compre-



Two cases in this Proposition.
First case.
Demonstration.



The second case.
Construction.

Demonstration.

The third Booke

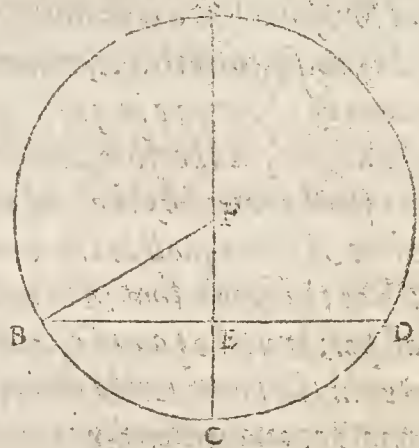
bended vnder the segmentes or parts of the one line is equall to the rectangle parallelograme comprehended vnder the segmentes or parts of the other line: which was required to be demonstrated.

Three cases in this Proposition.

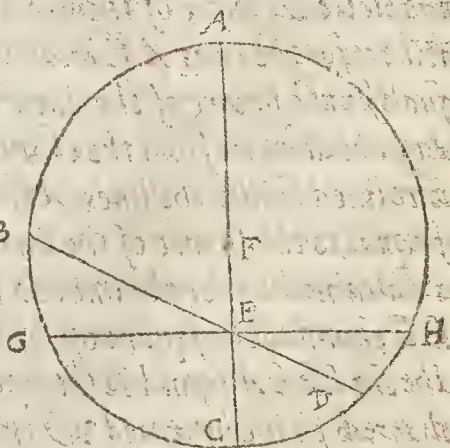
In this Proposition are three cases: For eyther both the lines passe by the centre, or neyther of them passeth by the centre: or the one passeth by the centre and the other not. The two first cases are before demonstrated.

The third case.

But now let one of the lines onely, namely, the line AC passe by the centre, which let be the poynt F , and let it cut the other line, namely, BD , in the poynt E . Now then the line AC deuiderth the line BD eyther into two equall partes, or into two vnequall partes. Fyrist let it deuide it into two equall partes: Wherefore also it deuiderth it right angled wyse by the 3. of this booke. Drawe a right line from B to F . Wherefore BEF is a right angled triangle. And for asmuch as the right line AC is deuided into two equall partes in the poynt F , & into two vnequall partes in the poynt E . Therefore the rectangle figure containd vnder the lines AE and EC together with the square of the line EF , is equall to the square of the line FC (by the 5. of the second). But vnto the square of the line FC is equall the square of the line BF (for that the lines FB and FC are equall). Therefore that which is containd vnder the lines AE and EC together with the square of the line EF , is equall to the square of the line BF . But vnto the square of the line BF , are equall the squares of the lines BE and EF (by the 47. of the first). Wherefore that which is containd vnder the lines AE and EC together with the square of the line EF , is equall to the squares of the lines BE and EF . Take away the square of the line EF which is common to them both: Wherefore that which remaineth, namely, that which is containd vnder the lines AE and EC , is equall to the residue, namely, to the square of the line BE . But the square of the line BE is that which is contained vnder the lines BE and ED : for (by supposition) the line BE is equall to the line ED . Wherefore that which is containd vnder the lines AE & EC , is equall to that which is containd vnder the lines BE and ED : which was required to be proued.



But now let the line AC passing by the centre, deuide the line BD not passing by the centre, vnequally in the poynt E . And frō the poynt E raise vp vnto the line AC a perpendicular line EH , which produce on the other side to the poynt G . Wherefore (by the 3. of this booke) the line EH is equall to the line EG . Wherefore as we haue before proued, that which is containd vnder the lines AE and EC , is equall to that which is containd vnder the lines GE & EH : but that which is containd vnder the lines BE and ED , is also equall to that which is containd vnder the lines GE and EH , by the second case of this Proposition: Wherefore that which is containd vnder the lines AE and EC , is equall to that which is containd vnder the lines BE and ED : which was againe required to be proued.



Amongest all the Propositions in this third booke, doubtles this is one of the chiefeft. For it setteth forth vnto vs the wonderfull nature of a circle. So that by it

it may be done many goodly conclusions in Geometry, as shall afterward be declared when occasion shall serue.

The 30. Theoreme. The 36. Proposition.

If without a circle be taken a certaine point, and from that point be drawen to the circle two right lines, so that the one of them do cut the circle, and the other do touch the circle: the rectangle parallelogramme which is comprehended vnder the whole right line which cutteth the circle, and that portion of the same line that lieth betwene the point and the utter circumference of the circle, is equall to the square made of the line that toucheth the circle.



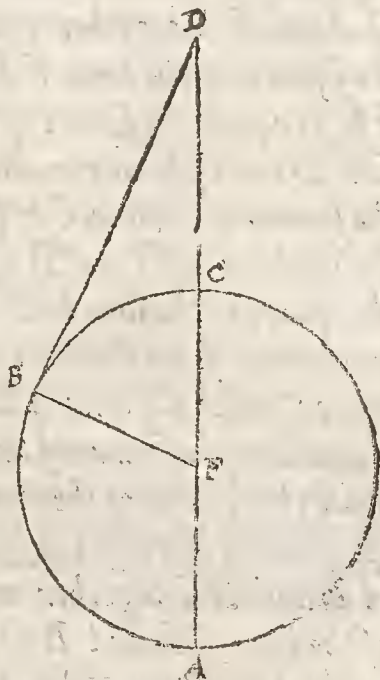
Suppose that the circle be ABC : and without the same circle take any point at all aduentures, and let the same be D . And from the point D let there be drawen to the circle two right lines DCA and DB , and let the right line DCA cut the circle ACB in the point C , and let the right line BD touch the same. Then I say, that the rectangle parallelogramme containd vnder the lines AD and DC , is equall to the square of the line BD . Now the line DCA is either drawen by the centre, or not.

Construction.

Two cases in this Proposition.

The first case.

First let it be drawen by the centre. And (by the first of the third) let the poynt F be y^e centre of the circle ABC , and drawe a line from F to B . Wherefore the angle FBD is a right angle. And forasmuch as y^e right line AC is deuided into two equall partes in the poynt F , and vnto it is added directly a right line CD , therefore (by the 6. of the second) that which is containd vnder the lines AD and DC together with the square of y^e line CF , is equall to the square of the line FD . But the line FC is equall to the line FB , for they are drawen from the centre to y^e circumference: Wherefore that which is containd vnder the lines AD and DC together with the square of the line FB , is equall to the square of the line FD . But y^e square of the line FD , is (by the 47. of the first) equall to the squares of the lines FB and BD (for the angle FBD is a right angle). Wherefore that which is containd vnder the lines AD and DC together with the square of the line FB , is equall to the



Demonstration.

Ee. ij. squares

The third Booke

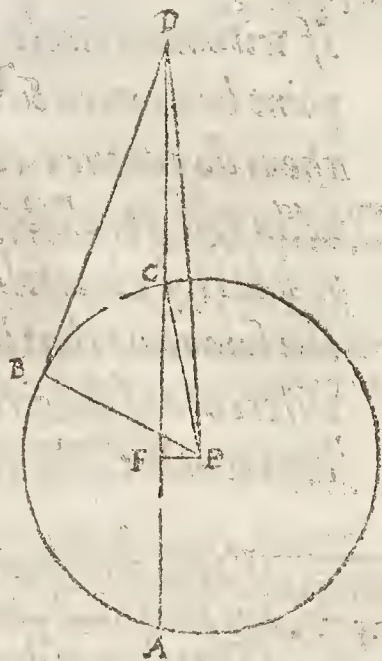
squares of the lines FB and BD . Take away the square of the line FB which is common to them both. Wherefore that which remaineth, namely, that which is contained vnder the lines AD and DC , is equall to the square made of the line DB which toucheth the circle.

The second
case.

Construction.

Demonstration.

But now suppose that the right line DC be not drawn by the centre of the circle ABC . And (by the first of the third) let the point E be $\hat{y}$ centre of the circle ABC . And from $\hat{y}$ poynt E , draw (by the 12. of the first) vnto the line AC a perpendicular line EF , and draw these right lines EB , EC , and ED . Now the angle EFD is a right angle. And forasmuch as a certaine right line EF drawn by the centre, cutteth a certayne other right line AC not drawn by the centre, in such sort that it maketh right angles, it deuideth it (by $\hat{y}$ third of the third) into two equall partes. Wherefore the line AF is equall to the line FC . And forasmuch as the right line AC is deuided into two equall partes in the poynt F , & vnto it is added directly an other right line making both one right line, therefore (by the 6. of the second)



that which is contained vnder the lines DA and DC together with the square of the line FC , is equall to the square of the line FD : put the square of the line FE common to them both. Wherefore that which is contained vnder the lines DA and DC together with the squares of the lines CF and FE , is equall to the squares of the lines FD and FE . But to the squares of the lines FD and FE , is equall the square of the line DE (by the 47. of the first) for the angle EFD is a right angle. And to the squares of the lines CF and FE , is equall the square of the line CE (by the same). Wherefore that which is contained vnder the lines AD and DC together with the square of the line EC , is equall to the square of the line ED . But the line EC is equall to the line EB : for they are drawn from the centre to the circumference. Wherefore that which is contained vnder the lines AD and DC together with the square of the line EB , is equall to the square of the line ED . But to the square of the line ED , are equall the squares of the lines EB and BD (by the 47. of the first) for the angle EBD is a right angle: Wherefore that which is contained vnder the lines AD and DC together with the square of the line EB , is equall to the squares of the lines EB and BD . Take away the square of the line EB which is common to them both: Wherefore the residue, namely, that which is contained vnder the lines AD and DC , is equall to the square of the line DB . If therfore without a circle be taken a certaine point, and from that poynt be drawn to the circle two right lines, so that the one of them do cut the circle, and the other do touch

touch the circle: the rectangle parallelogramme which is comprehended vnder the whole right line which cutteth the circle and that portion of the same line that lieth betwene the poynt and the vtter circumference of the circle, is equall to the square made of the line that toucheth the circle: which was required to be demonstrated.

¶ Two Corollaries out of Campane.

If from one and the selfe same poynt taken without a circle be drawn into the circle lines how many soeuer: the rectangle Parallelogrammes containd vnder euery one of them and hys outward part, are equall the one to the other.

First Corollary.

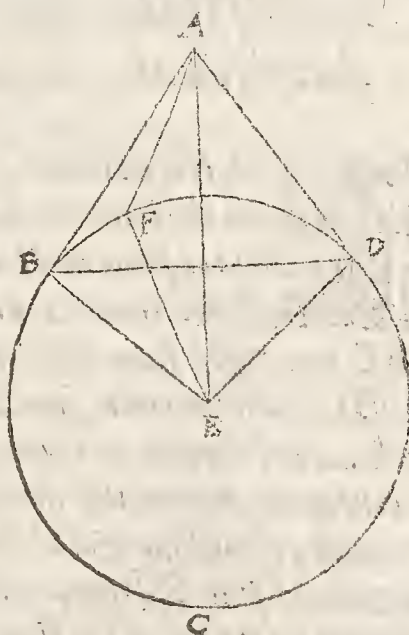
And thys is hereby manifest, for that euery one of those rectangle Parallelogrammes are equall to the square of the line which is drawn from that poynt and toucheth the circle by thys 36. Proposition. Hereunto he addeth.

If two lines drawn from one and the selfe same point do touch a circle, they are equall the one to the other.

Second Corollary.

Which although it neede no demonstration, for that the square of eyther of them is equall to that which is containd vnder the line drawn from the same poynt and hys outward part: yet he thus proueth it.

Suppose that there be a circle BCD , whose centre let be E , and without it take the point A . And from the poynt A drawe two lines AB and AD , which let touch the circle in the poyntes B and D . Then I say, that they are equall. Draw these right lines EB , ED , and AE . And by the 18. of thys booke, eyther of the angles at the poyntes B and D is a right angle. Wherefore (by the 47. of the first) the square of the line AE , is equall to the two squares of the lines AB and EB : and by the same reason, to the two squares of the lines AD and ED . Wherefore the two squares of the lines AB and EB , are equall to the two squares of the lines AD and ED . And forasmuch as the squares of the lines EB and ED are equall, therefore the two other squares of the lines AB and AD are also equall. Wherefore the line AB is equall to the line AD : which was required to be proued.



The same may be proued an other way: Draw a line from B to D . And (by the 5. of the first) the angle EBD is equall to the angle EDB . And forasmuch as the two angles ABE and ADE are equall, namely, for that they are right angles: if you take from them the equall angles EBD & EDB , the two other angles remayning, namely, the angles ABD and ADB shall be equall. Wherefore (by the 6. of the first) the line AB is equall to the line AD .

¶ Hereunto also Pelitarius addeth this Corollary.

From a poynt geuen without a circle, can be drawn unto a circle onely two touch lines.

Third Corollary.

The former description remayning, I say that from the poynt A can be drawn vnto the

Ee. iiii.

The third Booke

the circle BCD no more touch lines, but the two lines AB and AD . For if it be possible, let AF also be in the former figure a touch line, touching the circle in the point F . And draw a line from E to F . And the angle at the point F shall be a right angle, by the 18. of this booke: Wherefore it is equall to the angle EBA , which is contrary to the 20. of the first.

This may also be thus proued. For asmuch as all the lines drawn from one and the selfe same point & touching a circle are equall, as we haue before proued, but the lines AB and AF can not be equall, by the 8. Proposition of this booke, therefore the line AF can not touch the circle BCD .

The 31. Theoreme. The 37. Proposition.

If without a circle be taken a certaine point, and from that point be drawen to the circle two right lines, of which, the one doth cut the circle and the other falleth vpon the circle, and that in such sort, that the rectangle parallelogramme which is contayned vnder the whole right line which cutteth the circle, and that portion of the same line that lieth betwene the point and the vtter circumferēce of the circle, is equall to the square made of the line that falleth vpon the circle: then that line that so falleth vpon the circle shall touch the circle.

This proposition is the conuerse of the former.

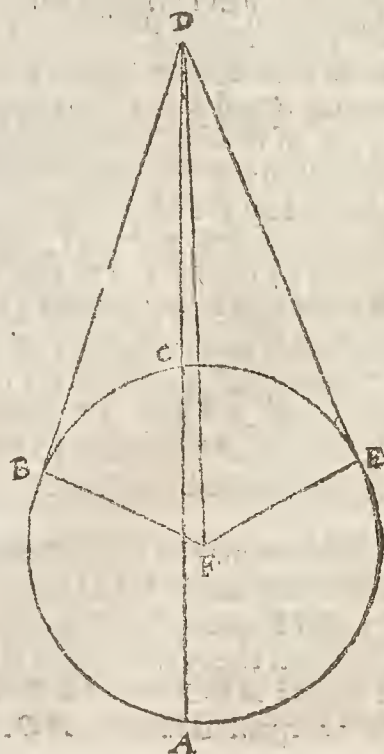
Et the circle be ABC : and without the same circle take a point, and let the same be D , & from the point D let there be drawen to the circle ABC two right lines DCA and DB : and let DCA cut the circle, and DB fall vpon the circle. And that in such sort, that that which is contayned vnder the lines AD and DC , be equall to the square of the line DB . Then I say, that $\S$ line DB toucheth the circle ABC .

Construction.

Drawe (by the 17. of the third) from the point D a right line touching the circle ABC , and let the same be DE . And (by the first of the same) let the point F be the centre of the circle ABC : and draw these right lines FE , FB , and FD . Wherefore the angle FED is a right angle.

Demonstration.

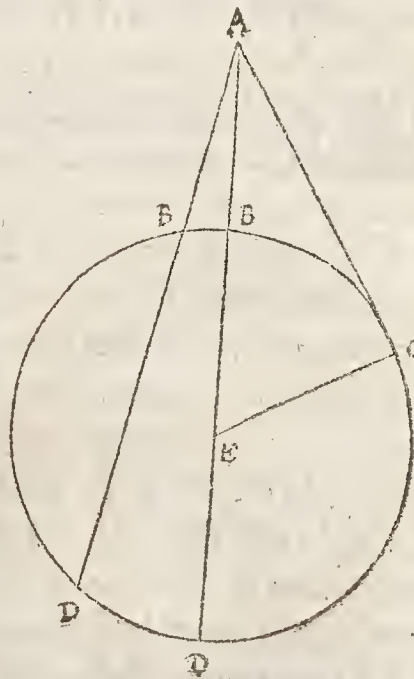
And for asmuch as the right line DE toucheth the circle ABC , and the right line DCA cutteth the same, therefore (by the Proposition going before) that which is contayned vnder the lines AD and DC , is equall to the square of the line DE . But that which is contayned vnder



under the lines AD and DC , is supposed to be equall to the square of the line DB . Wherefore the square of the line DE is equall to the square of the line DB . Wherefore also the line DE is equall to the line DB . And the line FE is equall to the line FB , for they are drawn from the centre to γ circumference. Now therefore these two lines DE and EF are equall to these two lines DB and BF , and FD is a common base to them both. Wherefore (by the 8. of the first) the angle DEF is equall to the angle DBF . But the angle DEF is a right angle. Wherefore also the angle DBF is a right angle. And γ line FB being produced, shall be the diameter of the circle. But if from the end of the diameter of a circle be drawn a right line making right angles, the right line so drawn toucheth the circle (by the Correllary of the 16. of the third). Wherefore the right line DB toucheth the circle ABC . And the like demonstration will serue if the centre be in the line AC . If therefore without a circle be taken a certaine point, and from that poynt be drawn to the circle two right lines, of which the one doth cut the circle, and the other falleth vpon the circle, and that in such sort, that the rectangle parallelogramme which is containned vnder the whole right line which cutteth the circle, and that portion of the same line that lieth betwene the poynt and the vtter circumference of the circle, is equall to the square made of the line that falleth vpon the circle: then the line that so falleth vpon the circle shall touch the circle: which was required to be proued.

¶ An other demonstration after Pelitarius.

Suppose that there be a circle BCD , whose centre let be E : and take a point without it, namely, A : And frō the poynt A drawe two right lines ABD , and AC : of which let ABD cut the circle in the poynt B , & let the other fall vpon it. And let that which is contained vnder the lines AD and AB , be equal to the square of the line AC . Then I say, that the line AC toucheth the circle. For first if the line ABD do passe by the centre, draw the right line CE . And (by the 6. of the second) that which is containned vnder the lines AD and AB together with the square of the line EB , that is, with the square of the line EC (for the lines EB and EC are equall) is equall to the square of the line AE . But that which is contained vnder the lines AD and AB , is supposed to be equall to the square of the line AC : Wherefore the square of the line AC together with the square of the line CE , is equall to the square of the line AE . Wherefore (by the last of the first) the angle at the point C is a right angle. Wherefore (by the 18. of this booke) the line AC toucheth the circle.



An other demonstration after Pelitarius.

But

The third Booke

But if the line ABD doo not passe by the centre, drawe from the point A the line AD , in which let be the centre E . And forasmuch as that which is contained vnder this whole line and his outward part, is equall to that which is contained vnder the lines AD and AB by the first Corollary before put, therefore the same is equall to the square of the line AC , wherefore the angle ECA is a right angle as hath before bene proued in the first part of this Proposition. And therefore the line AC toucheth the circle:
Which was required to be proued.

($\therefore$)

The ende of the third booke of Euclides Elementes.



The fourth booke of Eu- clides Elementes.



THIS FOVRTH BOOKE intreateth of the inscrip-
tion & circumscription of rectiline figures: how one
right lined figure may be inscribed within an other
right lined figure, and how a right lined figure may
be circumscribed about an other right lined figure, in
such as may be inscribed and circumscribed within
or about the other. For all right lined figures cannot
so be inscribed or circumscribed within or about the
other. Also it teacheth how a triangle, a square, and
certayne other rectiline figures being regular may be
inscribed within a circle. Also how they may be cir-
cumscribed about a circle. Likewise how a circle may

*The arguments
of this booke.*

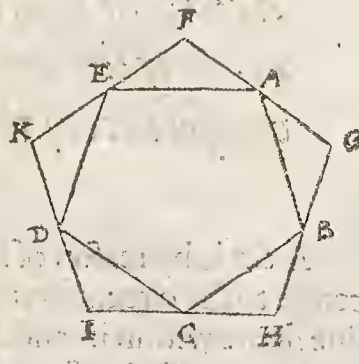
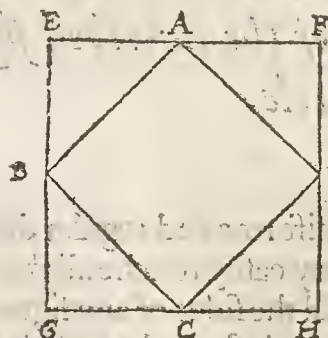
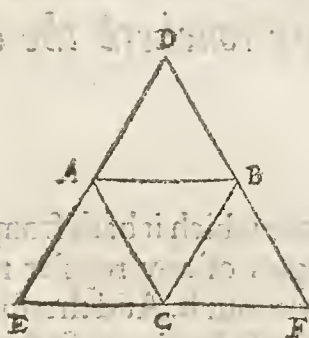
be inscribed within them. And how it may be circumscribed about them. And be-
cause the maner of entreatie in this booke is diuers from the entreaty of the for-
mer bookes, he vseth in this other wordes and termes then he vsed in them. The
definitions of which in order here after follow.

Definitions.

*A rectiline figure is sayd to be inscribed in a rectiline figure,
when euery one of the angles of the inscribed figure toucheth
euery one of the sides of the figure wherein it is inscribed.*

*First defini-
tion.*

As the triangle ABC is inscribed in the triangle DEF , because that euery angle of
the triangle inscribed, namely, the triangle ABC toucheth euery side of the triangle
within which it is described, namely, of the triangle DEF . As the angle CAB toucheth



the side ED : the angle ABC toucheth the side DF , and the angle ACB toucheth the
side EF . So likewise the square $ABCD$ is said to be inscribed within the square $EFGH$.
For euery angle of it toucheth some one side of the other. So also the Pentagon or
five angled figure $ABCDE$ is inscribed within the Pentagon or five angled figure
 $FGHIK$. As you see in the figures.

Likewise

The fourth Booke

Second de-
finition.

Likewise a rectiline figure is said to be circumscribed about a rectiline figure, when euery one of the sides of the figure circumscribed, toucheth euery one of the angles of the figure about which it is circumscribed.

As in the former descriptions the triangle DEF is said to be circumscribed about the triangle ABC , for that euery side of the figure circumscribed, namely, of the triangle DEF toucheth euery angle of the figure wherabout it is circumscribed. As the side DF of the triangle DEF circumscribed, toucheth the angle ABC of the triangle ABC about which it is circumscribed: and the side EF toucheth the angle BCA , and the side CD toucheth the angle CAB . Likewise vnderstand you of the square $EFGH$ which is circumscribed about the square $ABCD$: for euery side of the one toucheth some one side of the other. Euē so by the same reason the Pentagon $FGH IK$ is circumscribed about the Pentagon $ABCD E$, as you see in the figure on the other side. And thus may you of other rectiline figures consider.

The inscrip-
tion and cir-
cumscription
of rectiline
figures pertai-
neth only to
regular fi-
gures.

By these two definitions it is manifest, that the inscription and circumscription of rectiline figures here spoken of, pertayne to such rectiline figures onely, which haue equall sides and equall angles, which are commonly called regular. It is also to be noted that rectiline figures only of one kinde or forme can be inscribed or circumscribed the one within or about the other. As a triangle within or about a triangle: A square within or about a square: and so a Pentagon within or about a Pentagō, & likewise of others of one forme. But a triangle can not be inscribed or circumscribed within or about a square: nor a square within or about a Pentagon. And so of others of diuers kyndes. For euery playne rectiline figure hath so many angles as it hath sides. Wherefore the figure inscribed must haue so many angles as the figure in which it is inscribed hath sides: and the angles of the one (as is sayd) must touche the sides of the other. And contrariwise in circumscription of figures, the sides of the figure circumscribed must touch the angles of the figure about which it is circumscribed.

The third de-
finition.

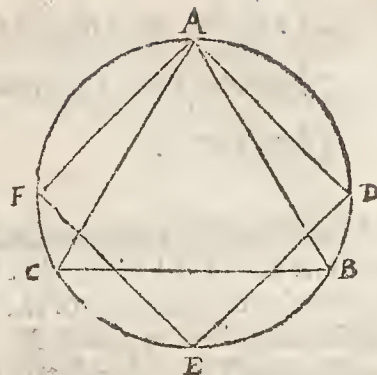
A rectiline figure is sayd to be inscribed in a circle, when euery one of the angles of the inscribed figure toucheth the circumference of the circle.

A circle by reason of his vniforme and regular distance which it hath from the centre to the circumference may easily touche all the angles of any regular rectiline figure within it: and also all the sides of any figure without it. And therefore any regular rectiline figure may be inscribed within it, and also be circumscribed about it. And agayne a circle may be both inscribed within any regular rectiline figure, and also be circumscribed about it.

As the triangle ABC is inscribed in the circle ABC : for that euery angle toucheth some one point of the circumference of the circle. As the angle CAB of the triangle ABC toucheth the point A of the circumference of the circle. And the angle ABC

of

of the triangle toucheth the pointe *B* of the circumference of the circle. And also the angle *ACB* of the triangle toucheth the pointe *C* of the circumference of the circle. In like manner the square *ADEF* is inscribed in the same circle *ABC*: for that euery angle of the square inscribed, toucheth some one poynte of the circle in which it is inscribed. And so imagine you of rectilined figures.



The fourth definition.

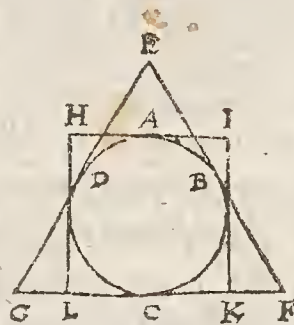
A circle is sayd to be circumscribed about a rectiline figure, whē the circumference of the circle toucheth euery one of the angles of the figure about which it is circumscribed.

As in the former example of the third definition. The circle *ADEF* is circumscribed about the triangle *ABC*, because the circumference of the circle which is circumscribed toucheth euery angle of the triangle about which it is circumscribed: namely, the angles *CAB*, *ABC*, and *BCA*. Likewise the same circle *ADEF* is circumscribed about the square *ADEF* by the same definition, as you may see.

A circle is sayd to be inscribed in a rectiline figure, when the circumference of the circle toucheth euery one of the sides of the figure within which it is inscribed.

The fifth definition.

As the circle *ABCD* is inscribed within the triangle *EFG*, because the circumference of the circle toucheth euery side of the triangle in which it is inscribed: namely the side *EF* in the point *B*, and the side *GF* in the pointe *C*, and the side *GE* in the point *D*. Likewise by the same reason the same circle is inscribed within the square *HIKL*. And so may you iudge of other rectiline figures.



A rectilined figure is said to be circumscribed about a circle, when euery one of the sides of the figure circumscribed toucheth the circumference of the circle.

The sixth definition.

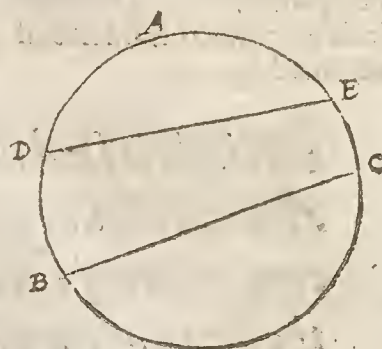
As in the former figure of the fifth definition, the triangle *EFG* is circumscribed about the circle *ABCD*, for that euery side of the same triangle beyng circumscribed toucheth the circumference of the circle, about which it is circumscribed. As the side *EG* of the triangle *EFG* toucheth the circumference of the circle in the point *D*: and the side *EF* toucheth it in the point *B*: and the side *GF* in the point *C*. Likewise also the square *HIKL* is circumscribed about the circle *ABCD*, for euery one of his sides toucheth the circumference of the circle, namely, in the pointes *A, B, C, D*. And thus consider of all other regular right lined figures (for of them onely are vnderstanded these definitions) to be circumscribed about a circle, or to be inscribed within a circle: or of a circle to be circumscribed or inscribed about or within any of them.

The fourth Booke

Seventh definition.

A right lyne is sayd to be coapted or applied in a circle, when the extremes or endes therof, fall vppon the circumference of the circle.

As the line BC is sayd to be coapted or to be applied to the circle ABC , for that both his extremes fall vpon the circumference of the circle in the pointes B and C . Likewise the line DE . This definition is very necessary, and is properly to be taken of any lyne geuen to be coapted and applied into a circle, so that it exceede not the diameter of the circle geuen.



The 1. Probleme. The 1. Proposition.

In a circle geuen, to apply a right line equall vnto a right line geuen, which exceedeth not the diameter of a circle.

Construction.



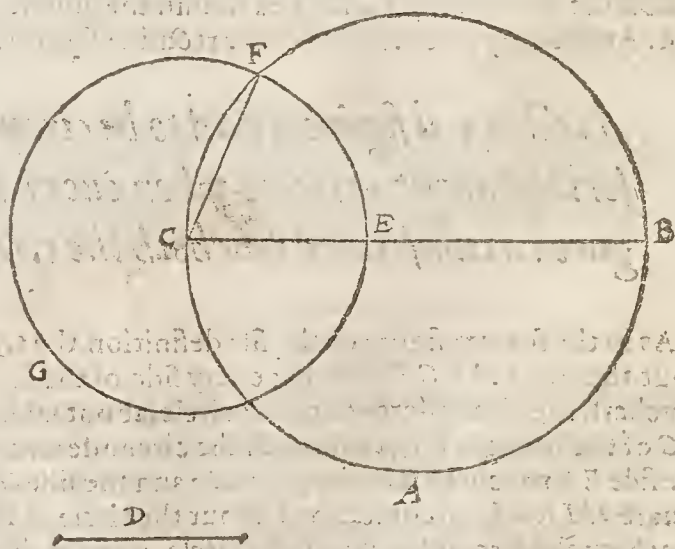
Vppose that the circle geuen be ABC , and let the right line geuen, exceeding not the diameter of the same circle, be D . Now it is required in the circle geuen ABC , to apply a right line equall vnto the right line D . Draw the diameter of the circle ABC , and let the same be BC . Now if the line BC be equall vnto the line D , then is that done which was required. For in the circle geuen

Two cases in this Proposition.

First case.

Second case.

ABC is applied a right line BC equall vnto y^e right line D . But if not, then is the line BC greater then y^e line D . And (by the third of the first) put vnto the line D an equall line CE . And making the centre C , and the space CE , describe (by the third petition) a circle EGF , cutting the circle ABC in the point F , & draw a line from C to F . And for asmuch as the point C is y^e centre of the circle EGF , therefore (by the 13. definition of the first) the line CF is equall vnto the line CE . But the line CE is equall vnto the line D . Wherefore (by the first common sentence) the line



Demonstration.

line CF also is equall vnto the line D . Wherefore in the circle geuen ABC , is applied a right line CA equall vnto the right line geuen D : which was required to be done.

 The 2. Probleme. The 2. Proposition.

In a circle geuen, to describe a triangle equiangle vnto a triangle geuen.

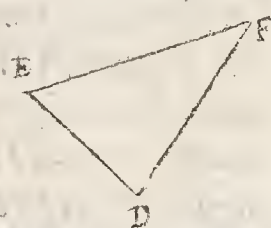
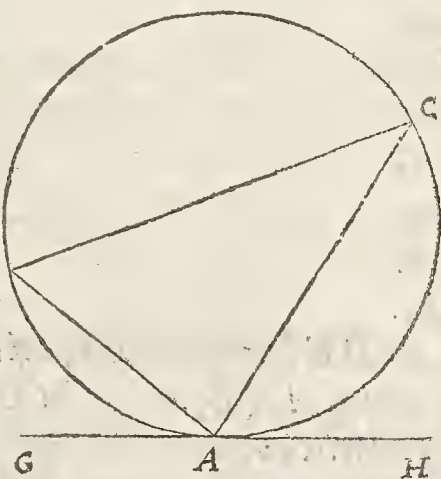


Suppose that the circle geuen be ABC : and let the triangle geuen be DEF . Now it is required in the circle geuen ABC to describe a triangle equiangle vnto the triangle geuen DEF . Draw (by the 17. of the third) a right line touching the circle ABC , and let the same be

Construction.

GAH , and let it touch in the point A . And (by the 23. of the first) vnto γ right line AH , and vnto the point in it A , describe an angle HAC equall vnto the angle DEF . And (by the selfe same) vnto the right line AG , and vnto the point in it A , make an angle GAB equall vnto the angle DFE . And draw a right line from B to C .

And for asmuch as a certaine right line GAH toucheth γ circle ABC , and from the point where it toucheth, namely, A , is drawen into the circle a right line AC , therefore (by the 31. of the third)



Demonstration.

the angle HAC is equall vnto the angle ABC which is in the alternate segment of the circle. But the angle HAC is equall to the angle DEF . Wherefore the angle ABC is equall to the angle DEF . And by the same reason, the angle ACB is equall to the angle DFE . Wherefore the angle remayning, BAC , is equall vnto the angle remayning, EDF . Wherefore the triangle ABC is equiangle vnto the triangle DEF . And it is described in the circle geuen ABC . Wherefore in a circle geuen, is described a triangle equiangle vnto a triangle geuen: which was required to be done.

 The 3. Probleme. The 3. Proposition.

About a circle geuen, to describe a triangle equiangle vnto a triangle geuen.

F.ij.

Suppose

The fourth Booke



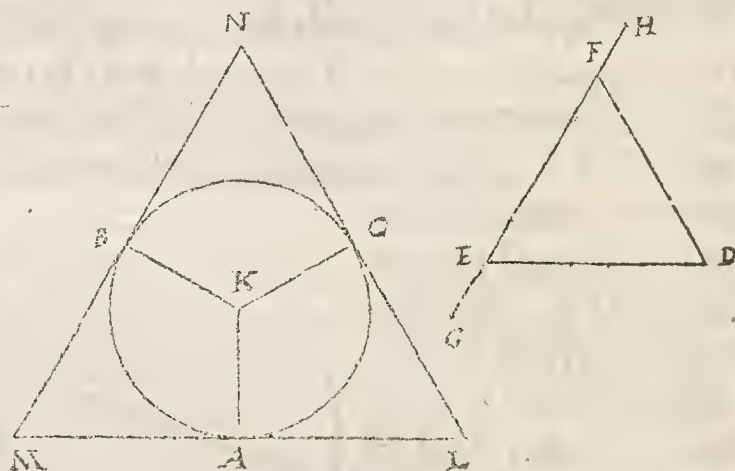
Suppose the circle geuen be ABC , and let the triangle geuen be DEF . It is required about the circle ABC to describe a triangle equiangle vnto the triangle DEF . Extend the line EF on each side to the poyntes

Construction.

G and H . And (by the first of the third) take the centre of the circle ABC , and let the same be the point K . And then draw a right line KB . And (by the 23. of the first) vnto the right line KB , and vnto the point in it K , make an angle BKA equall vnto the angle DEG , and likewise make the angle BKC equall vnto the angle DFH . And (by the 17. of the third) draw right lines touching the circle A, B, C , in the pointes A, B, C . And let the same be LAM, MBN , and NCL . And forasmuch as the right lines $LM, MN, \& NL$, do touch the circle ABC in the pointes A, B, C , and from the centre K vnto the pointes A, B, C , are drawn right lines KA, KB , and KC , therefore the angles which are at the pointes A, B, C , are right angles (by 18. of the third). And forasmuch as the fower angles of the fower sided figure $AMBK$, are equall vnto fower right angles: whose angles $KAM, \& KBM$, are two right angles: therefore the angles remayning AKB , and AMB , are equall to two right angles.

Demonstration.

And the angles $DEG, \& DEF$, are (by the 13. of the first) equall to two right angles. Wherefore the angles AKB , and AMB , are equall vnto the angles DEG , and DEF : of which two angles the angle AKB is equall vnto the angle DEG : Wherefore the angle remayning, AMB , is equall vnto the angle remayning, DEF . In like sort may it be proued, that the angle LMN , is equall to the angle DFE . Wherefore the angle remayning MLN , is equall vnto the angle remayning EDF . Wherefore the triangle LMN , is equiangle vnto the triangle DEF : and it is described about the circle ABC . Wherefore about a circle geuen is described a triangle equiangle vnto a triangle geuen: which was required to be done.



¶ *An other way after Pelitarius.*

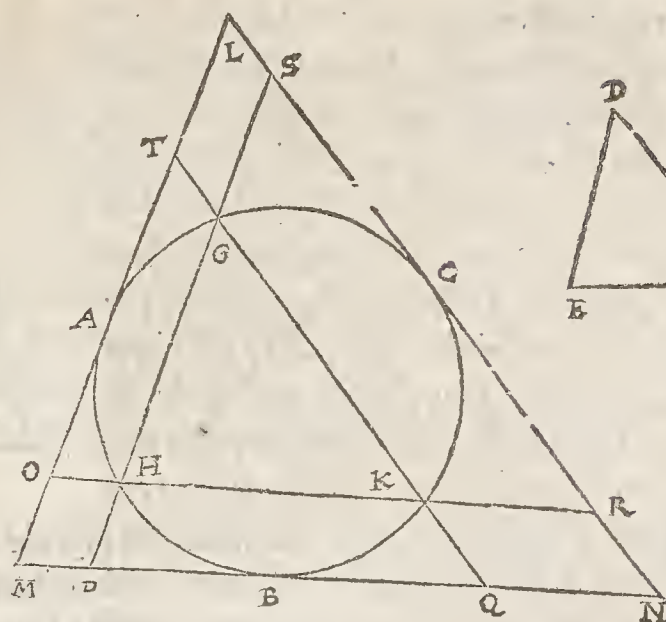
An other way after Pelitarius.

Construction.

In the circle ABC inscribe a triangle $G HK$ equiangle to the triangle EDF (by the former Proposition): so that let the angle at the poynt G be equall to the angle D , and let the angle at the point H be equall to the angle E : and let also the angle at the poynt K be equall to the angle F . Then draw the line LM parallel to the line GH , which let touch the circle in the poynt A (which may be done by the Proposition added of the sayd Pelitarius after the 17. Proposition). Draw likewise the line MN parallel vnto the line HK and touching the circle in the poynt B : And also draw the line

LN

LN parallel vnto the line GK and touching the circle in the poynt C. And these three lines shall vndoubtedly concur, as in the poyntes L, M, and N, which may easily be proued, if you produce on either side the lines GH, GK, and HK, vntill they cut the lines LM, LN, and MN, in the poyntes O, P, Q, R, S, T. Now I say, that the triangle LMN circumscribed about the circle ABC, is equiangle to the triangle DEF. For it is manifest, that it is equiangle vnto the triangle GHK,



Demonstration.

by the proprietie of parallel lines. For the angle MTQ is equall to the angle at the poynt G of the triangle GHK (by the 29. of the first) and therefore also the angle at the poynt L, is equall to the selfe same angle at the poynt G (for the angle at the point L, is by the same 29. Proposition, equall to the angle MTQ). And by the same reason the angle at the poynt M, is equall to the angle at the poynt H of the selfe same triangle: and the angle at the poynt N, to the angle at the poynt K. Wherefore the whole triangle LMN, is equiangle to the whole triangle GHK: Wherefore also it is equiangle to the triangle DEF: which was required to be done.

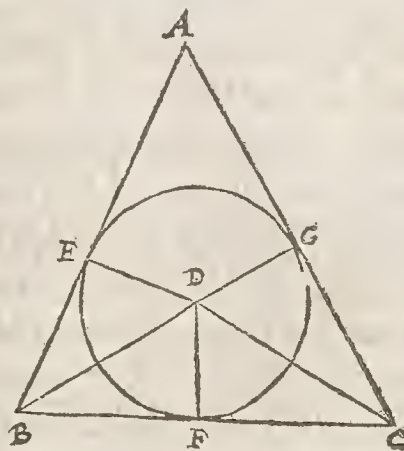
The 4. Probleme. The 4. Proposition.

In a triangle geuen, to describe a circle.

Suppose that the triangle geuen be ABC. It is required to describe a circle in the triangle ABC. Deuide (by the 9. of the first) the angles ABC, and ACB into two equall partes by two right lines BD and CD. And let these right lines meete together in the point D. And (by the 12. of the first) from the point D draw vnto the right lines AB, BC and CA perpendicular lines, namely, DE, DF, and DG. And forasmuch as the angle ABD is equall to the angle CBD, and the right angle BED is equall vnto the right angle BFD. Now then there are two triangles EBD and FBD hauing two angles equall to two angles, and one side equall to one side, namely, BD which is common to them both, and subtendeth one of the equall angles. Wherefore (by the 26. of the first) the rest of the sides are equall vnto the rest of the sides. Wherefore the line DE is equall vnto the line DF: and by the same reason also the line DG is equall vnto the line DF.

Construction.

Demonstration.

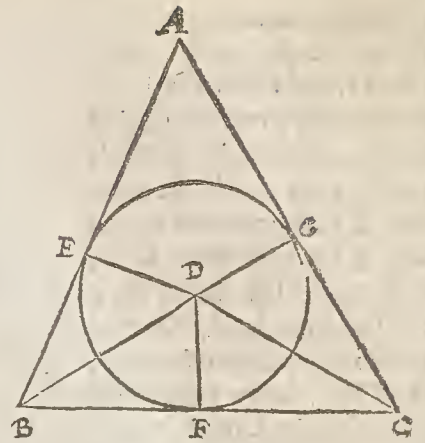


Ef. iij.

Where

The fourth Booke

Wherefore these three right lines DE , DF , & DG , are equall the one to the other (by the first common sentence). Wherefore making the centre the point D , and the space DE , or DF , or DG , describe a circle and it will passe through the pointes E , F , G , and will touch γ right lines AB , BC , and CA . For the angles made at the pointes E , F , G , are right angles. For if γ circle cut those right lines, then fro the end of the diameter of the circle shall be drawen a right line making two right angles, & falling within the circle: which is impossible, as it was manifest (by the 16. of the third). Wherefore the circle described, D being the centre thereof, and the space thereof being either DE , or DF , or DG , cutteth not these right lines AB , BC , & CA . Wherefore (by the Corollary of the same) it toucheth them, and the circle is described in the triangle ABC . Wherefore in the triangle geuen ABC , is described a circle $EF G$: which was required to be done.



Demonstration leading to an impossibility.

The 5. Probleme. The 5. Proposition.

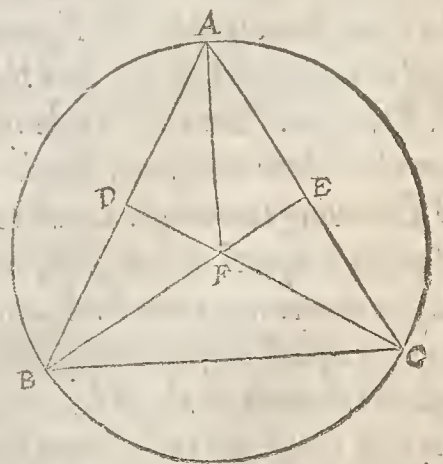
About a triangle geuen, to describe a circle.

Suppose that the triangle geuen be ABC . It is required about the triangle ABC to describe a circle. Deuide (by the 10. of the first) the right lines AB and AC into two equall partes in the pointes D and E . And from the pointes D and E (by the 11. of the first) rayse vp vnto the lines AB & AC two perpendicular lines DF and EF . Now these perpendicular lines meete together either within the triangle ABC , or in the right line BC , or els without the right line BC .

Three cases in this Proposition.

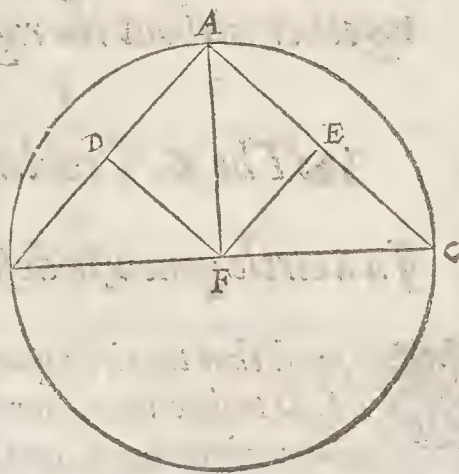
The first case.

First let them meete together within the triangle in the point F . And (by γ first petition) drawe right lines from F to B , from F to C , and from F to A . And forasmuch as the lyne AD is equall vnto the line DB , and the line DF is common vnto them both, and maketh the angles on each side of him right angles, therefore (by the 4. of the first) the base AF is equall vnto the base FB . In like sorte may we proue that the line CF is equall vnto the lyne AF . Wherefore the line FB is equall vnto the line CF . Wherefore these three right lines FA , FB , and FC are equall the one to the



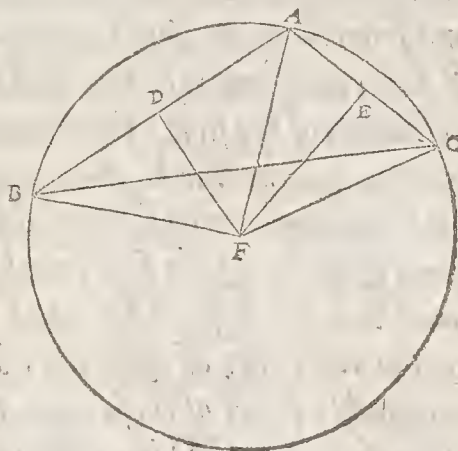
the other. Wherefore making the centre the point F , and the space FA , or FB , or FC describe a circle and it shall passe by the poyntes $A B C$. And so there is a circle described about the triangle $A B C$, as ye see in the first description.

But now suppose that the right lines DF and EF do meete together vpon the right line BC in the point F , as it is in the second description, and draw a right lyne from A to F , and in like sorte may we proue that the poynte F is the centre of the circle described about the triangle $A B C$.



The second case.

But now suppose that the right lines DF and EF do meete together without the triangle $A B C$ in the poynt F . Againe as it is in the third description draw right lines from F to A , from F to B , and from F to C . And forasmuch as the line AD is equall vnto the line DB , and the line DF is common vnto them both, and maketh a right angle on eche side of him, wherefore (by the 4. of the first) the base AF is equall vnto the base BF . And in like sort may we proue that the line CF is equal vnto the line AF . Wherefore agayne making F the centre, and the space FA , or FB , or FC , describe a circle and it shall passe by the poyntes A, B, C , and so is there a circle described about the triangle $A B C$, as ye see it is in the third description. Wherefore about a triangle geuen is described a circle: which was required to be done.



The third case.

Correlary.

Hereby it is manifest, that when the centre of the circle falleth within the triangle, the angle BAC being in a greater segment of a circle is lesse the a right angle. But when it falleth vpon the right line BC the angle BAC being in a semi-circle is a right angle. But when the centre falleth without the right line BC , the angle BAC being in a lesse segment of a circle, is greater then a right angle. Wherefore also when the angle geuen is lesse then a right angle, the right lines DF

Ff. iij. and

The fourth Booke

and EF will meete together within the sayd triangle. But when it is a right angle they will meete together vpon the line BC . But whe it is greater then a right angle, they will meete together without the right line BC .

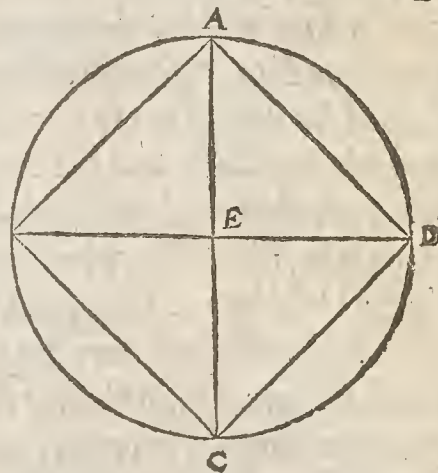
The 6. Probleme. The 6. Proposition.

In a circle geuen, to describe a square.

Construction.

Demonstration.

Suppose that the circle geuen be $ABCD$. It is required in the circle $ABCD$ to describe a square. Draw in the circle $ABCD$ two diameters making right angles, and let the same be AC and BD , and drawe right lines from A to B , from B to C , from C to D , and from D to A . And forasmuch as the line BE is equall vnto the line ED (by the 15. definition of the first) for the point E is the centre. And the line EA is common to them both, making on eche side a right angle: therefore (by the 4. of the first) the base AB is equall vnto the base AD . And by the same reason also either of these lines BC and CD is equall to either of these lines AB and AD : wherefore $ABCD$ is a figure of foure equal sides. I say also that it is a rectangle figure. For forasmuch as the right line BD is the diameter of the circle $ABCD$, therefore the angle BAD beyng in the semicircle is a right angle (by the 31. of the third) And by the same reason euery one of these angles ABC , BCD and CDA is a right angle. Wherefore the foure sided figure $ABCD$ is a rectangle figure, and it is proued that it consisteth of equall sides. Wherefore (by the 30. definition of the first) it is a square, and it is described in the circle $ABCD$: which was required to be done.



The 7. Probleme. The 7. Proposition.

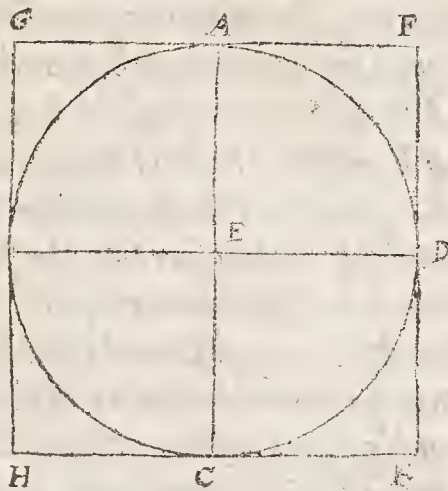
About a circle geuen, to describe a square.

Construction.



Suppose that the circle geuen be $ABCD$. It is required about the circle $ABCD$ to describe a square. Draw in the circle $ABCD$ two diameters making right angles, where they cut the one the other, and let the same be AC and BD . And by the pointes A, B, C, D , draw (by

(by the 17. of the third) right lines touching the circle $A B C D$, and let the same be $F G$, $G H$, $H K$, and $K F$. Now forasmuch as the right line $F G$ toucheth the circle $A B C D$ in the point A , and from the centre E to the point A where the touch is, is drawn a right line $E A$, therefore (by the 18. of the third) the angles at the point A are right angles, and by the same reason the angles which are at the pointes B, C, D , are also right angles. And forasmuch as the angle $A E B$ is a right angle, & the angle $E B G$ is also a right angle, therefore (by the 28. of the first) the line $G H$ is a parallel vnto the line $A C$: and by the same reason the lyne $A C$ is a parallel vnto the lyne $F K$. In like sorte also may we proue that either of these lines $G F$ and $H K$ is a parallel vnto the lyne $B E D$. Wherefore these figures $G K$, $G C$, $A K$, $F B$, and $B K$ are parallelogrames. Wherefore (by the 34. of the first) the line $G F$ is equall vnto the line $H K$, and the line $G H$ is equall vnto the line $F K$. And forasmuch as the line $A C$ is equall vnto the lyne $B D$, but the line $A C$ is equall vnto either of these lines $G H$ and $F K$: and the line $B D$ is equall to either of these lines $G F$ and $H K$. Wherefore either of these lines $G H$ and $F K$ is equall to either of these lines $G F$ and $H K$. Wherefore the figure $F G H K$ consisteth of foure equall sides. I say also that it is a rectangle figure. For forasmuch as $G B E A$ is a parallelogramme, and the angle $A E B$ is a right angle: therefore (by the 34. of the first) the angle $A G B$ is a right angle. In like sorte may we proue that the angles at the poyntes H, K , and F are right angles. Wherefore $F G H K$ is a rectangle foure sided figure, and it is proued that it consisteth of equall sides: wherefore it is a square, and it is described about the circle $A B C D$. Wherefore about a circle geuen is described a square: which was required to be done.



Demonstration.

The 8. Probleme. The 8. Proposition.

In a square geuen, to describe a circle.

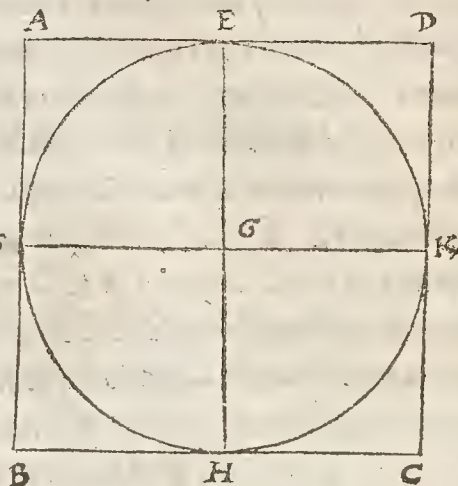
Suppose that the square geuen be $A B C D$. It is required in the square $A B C D$ to describe a circle. Deuide (by the 10. of the first) either of these lines $A B$ and $A D$ into two equall partes in the pointes E and F . And by the point E (by the 31. of the first) draw a line $E H$ parallel vnto either of these lines $A B$ and $D C$: and (by the same) by the point F draw a line $F K$ parallel vnto either of these lines $A D$ and $B C$. Wherefore euery one of these figures

Construction.

Demonstration.

The fourth Booke

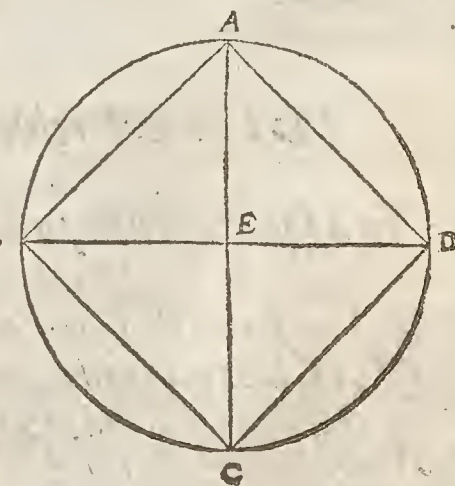
figures $AK, KB, AH, HD, AG, GC, BG$, and GD is a parallelograme, and the sides which are opposite the one to the other, are (by the 34. of the first) equall the one to the other. And forasmuch as the line AD is equall vnto the line AB , and the halfe of the line AD is the line AE , and the halfe of the line AB , is the line AF , therefore the line AE is equall vnto the line AF : wherefore (by the same) the sides which are opposite are equall. Wherefore the line FG is equal vnto the line EG . In like sort may we proue that either of these lines GH , FK and GK is equall to either of these lines FG and GE . Wherefore (by the first common sentence) these foure lines GE, GF, GH , and GK are equall the one to the other. Wherefore making the centre G , and the space either GE , or GF, GH , or GK , describe a circle and it will passe by the pointes E, F, H, K , and will touche the right lines AB, BC, CD , and DA . For the angles at the pointes E, F, H, K , are right angles. For if the circle do cut the right lines AB, BC, CD , and DA , then the line which is drawen by the ende of the diameter of the circle making right angles should fall within the circle, which is impossible (by the 16. of the third) Wherefore the centre being the poynt G and the space beyng GE , or GF , or GH , or GK if a circle be described, it shall not cut the right lines AB, BC, CD , and DA . Wherefore it shall touch them. And it is described in the square $ABCD$: wherefore in a square geuen is described a circle: which was required to be done.



The 9. Probleme. The 9. Proposition.

About a square geuen, to describe a circle.

Suppose that the square geuen be $ABCD$. It is required about the square $ABCD$ to describe a circle. Drawe right lines from A to C , and from D to B , & let them cut the one the other in the poynt E . And forasmuch as the lyne DA is equall vnto the lyne AB , and the line AC is common vnto them both, therefore these two lines DA and AC are equall vnto these two lynes BA and AC , the one to the other. And the base DC is equall vnto the base BC . Wherefore



(by the 8. of the first) the angle DAC is equall vnto the angle BAC . Wherefore the angle DAB is deuided into two equall partes by the line AC . And in like

Construction.

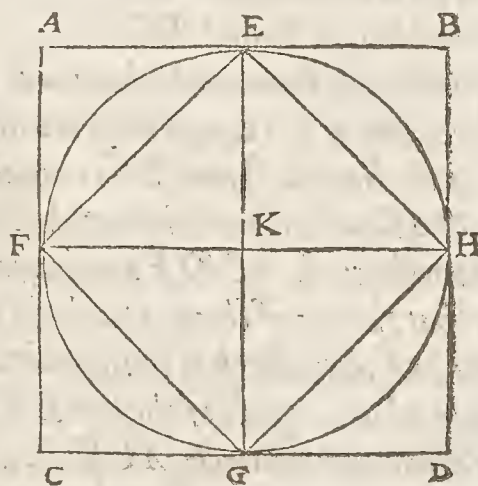
Demonstration.

like sort may we proue that euery one of these angles ABC , BCD , and CDA is deuided into two equall partes by the right lines AC and DB . And forasmuch as the angle DAB is equall vnto the angle ABC , and of the angle DAB the angle EAB is the halfe, and of the angle ABC the angle EBA is the halfe: Therefore the angle EAB is equall vnto the angle EBA : wherefore (by the 6. of the first) the side EA is equall vnto the side EB . In like sorte may we proue that either of these right lines EA and EB is equall vnto either of these lines EC and ED . Wherefore these foure lines EA , EB , EC , and ED are equall the one to the other. Wherefore making the centre E , and the space any of these lines EA , EB , EC , or ED . Describe a circle and it will passe by the pointes A , B , C , D , and shall be described about the square $ABCD$, as it is euident in the figure $ABCD$. Wherefore about a square genē is described a circle: which was required to be done.

¶ A Proposition added by Pelitarius.

A square circumscribed about a circle, is double to the square inscribed in the same circle.

Suppose that the square $ABCD$ be circumscribed about the circle $EFGH$, whose centre let be K . And let the poyntes of the touches be E , F , G , H . And drawing these two diameters EG , and FH , and these right lines EF , FG , GH , and HE , there shall be inscribed in the circle a square $EFGH$ (by the sixt of this booke). Then I say, that the square $ABCD$, is double to the square $EFGH$. For forasmuch as the side AB of the greater square, is (by the 34. of the first) equall to FH , which is the diameter of the lesse square: but the square of FH is double to the square whose diameter it is, namely, to the square $EFGH$ (by the 47. of the first). Wherefore also the square of AB which is $ABCD$, is double to the square $EFGH$: which was required to be proued.



A Proposition added by Pelitarius.

This may also be demonstrated by the equalite of the triangles and squares containd in the great squares.

✶ The 10. Probleme. The 10. Proposition.

To make a triangle of two equall sides called Isoceles, which shall haue eyther of the angles at the base double to the other angle.

Take

The fourth Booke

Construction.



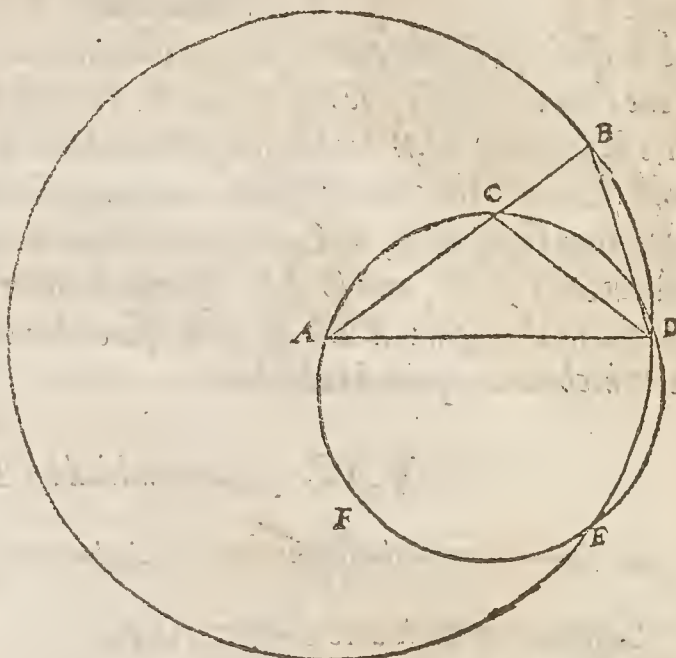
Take a right line at all adventures which let be AB , & (by $\S$ 11. of the second) let it be so devided in $\S$ pointe C , $\S$ the rectangle figure comprehended vnder the lines AB and BC be equall vnto $\S$ square which is made of the line AC . And making the centre the point A ,

& the space AB , describe (by the 3. petition) a circle BDE , and (by the 1. of the fourth) into the circle BDE apply a right line BD equall to the right lyne AC which is not greater then the diameter of the circle BDE . And draw lines from A to D and from D to C . And (by the 5. of $\S$ fourth) about the triangle ACD describe a circle $ACDF$.

Demonstration.

And forasmuch as the rectangle figure contained vnder the lines AB and BC is equall to the square which is made of the line AC : (For that is by supposition) But the line AC is equall vnto the line BD . Wherefore that which is containd vnder the lines AB and BC is equall to the square which is made of the line BD . And forasmuch as without the circle $ACDF$ is taken a poynt B , and from B vnto the circle $ACDF$ are drawn two right lines BCA , and BD , in such sort that the one of them cutteth the circle, and the other endeth at the circumference, and that which is contained vnder the lines AB and BC is equall to the square which is made of the line BD , therefore (by the 17. of the third) the line BD toucheth the circle $ACDF$. And forasmuch as the line BD toucheth in the point D , and from D where the touche is, is drawn a right line DC , therefore (by the 32. of the same) the angle BDC is equall vnto the angle DAC , which is in $\S$ alternate segment of $\S$ circle. And forasmuch as $\S$ angle BDC is equal vnto $\S$ angle DAC , put the angle CDA common vnto the both. Wherefore $\S$ whole angle BDA is equal to these two angles CDA , & DAC . But vnto $\S$ angles CDA , & DAC is equall the outward angle BCD (by the 32. of the 1.) Wherefore $\S$ angle BDA is equal vnto $\S$ angle BCD . But $\S$ angle BDA is (by $\S$ 5. of the first) equall vnto the angle CBD , for (by the 15. definition of $\S$ first) the side AD is equall vnto the side AB : wherefore (by the 1. common sentence) the angle DBA is equall vnto the angle BCD . Wherefore these three angles BDA , DBA , and BCD are equall the one to the other. And forasmuch as the angle DBC is equall vnto the angle BCD , the side therefore BD is equall vnto the side DC . But the line BD is by supposition equall vnto the lyne CA .

Where



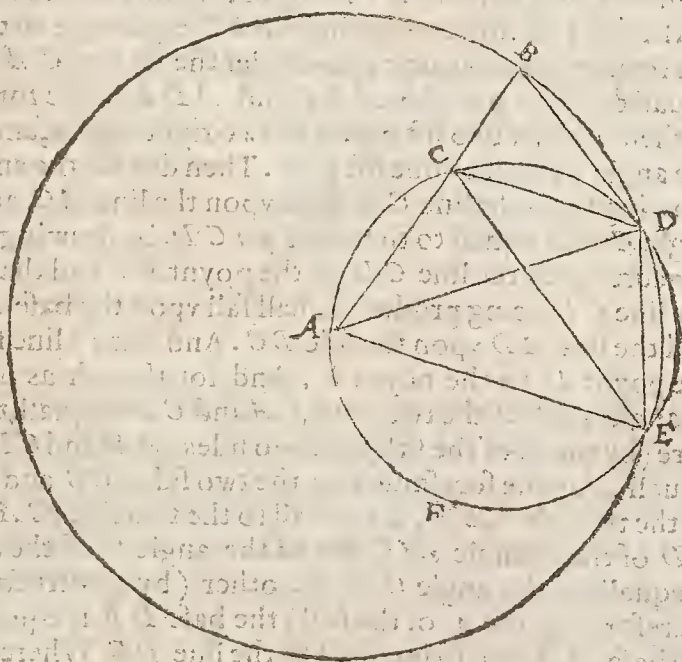
Wherefore the line AC also is equall vnto the lyne CD . Wherefore also (by the 5. of the first) the angle $CD A$ is equall vnto the angle $D A C$. Wherefore the angles $CD A$ and $D A C$ are double to the angle $C A D$. But the angle BCD is equall vnto the angles $CD A$ and $D A C$. Wherefore the angle BCD is double to the angle $C A D$. But the angle BCD is equall to either of these angles $B D A$ & $D B A$. Wherefore either of these angles $B D A$ & $D B A$ is double to the angle $D A B$. Wherefore there is made a triangle of two equall sides ABD hauing either of y angles at the base DB , double to the angle remayning: which was required to be done.

Here *Campane* addeth; that the two circles ACD and BDE , do cut the one the other: and that the circle ACD cutteth off from the circle BDE an arke equall to the arke $B'D$, and that the circle BDE cutteth off from the circle ACD an arke equall to the arke DC .

The first part is manifest. For if the lesse do not cut the greater but touch it as in the poynt D . Then (by the 11. of the third) the centre of either of them shall be in one line, namely, in the line AD : for that in it is the centre of the greater circle, and in the selfe same is the poynt of the touch. Wherefore (by the 31. of the third) the angle ACD is a right angle. And therefore (by the 13. of the first) the angle DCB is a right angle: and so shall the angle ABD be a right angle (for it is equall to the angle DCB , as it hath bene proued) which (by the 32. of the first) is impossible. Wherefore they shall cut the one the other as in the poyntes D & E . Now I say, that the arke ED of the greater circle, is equall to the arke DB : and that the arke ED of the lesse circle, is equall to the arke DC .

Draw these right lines EA , EC , & ED . Now the (by the 27. of the third) the foure angles DEC , CEA , DAC , & ADC , are equal: for that the arkcs CA and CD are equal (by the 28. of the same). Wherefore the whole angle AED is double to the angle BAD , & therefore is equall to either of the angles ABD and ADB . And forasmuch as the angle AED is equall to the angle ADE (by the 5. of the first) for that the lines AD and AE are drawn from the centre, therefore the two angles at the pointes E and D of the triangle AED , shall be equall to the two angles at the poyntes D and B of the triangle ADB : and therefore the angle remayning of the one at the poynt A , shall be equall to the angle remayning of the other at the same point A (by the 32. of the first). Wherefore (by the 26. of the third) the arke ED of the greater circle, is equal to the arke DB . And by the same the arke ED of the lesse circle, is equal to the arke DC : which was required to be proued.

Here *Pelitarus* noteth, that in euery such Isosceles triangle as in thys place is the triangle ABD (namely, eyther of whose angle at the base, is double to the angle at the toppe) the angle at the toppe, as in thys example, the angle at the poynt A is one third part of a right angle, and moreouer one fift part of a third of a right angle: that is, two fift partes of one right angle: and to be brieft, one fift part of



Certaine additions of *Campane*.

The first part demonstrated.

The second part demonstrated.

The fourth Booke

two right angles. And either of the angles at the base, is two fift partes of two right angles, or foure fift partes of one right angle. Which shall manifestly appeare, if we deuide two right angles into fife partes. For then in thys kinde of triangle, the angle at the toppe shall be one fift part, and eyther of the two angles at the base shall be two fift partes.

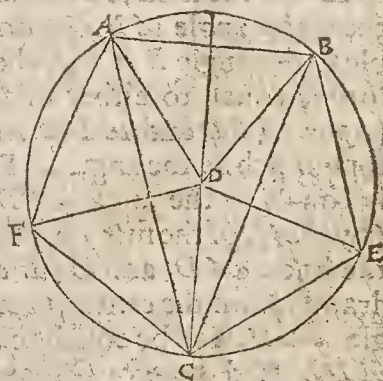
Thys also is to be noted, that the line AC is the side of an equilater Pentagon to be inscribed in the circle ACD . For by the latter construction it is manifest, that the three arkes AC , CD , and DE , of the lesse circle, are equall. And forasmuch as by the same it is manifest that the two lines AD and AE are equall, the arke also AE shall be equall to the arke AD (by the 20. of the third). Wherefore their halves also are equall. If therefore the arke AE be (by the 30. of the third) deuided into two equall partes, the whole circūference $ACDEA$ shall be deuided into fife equall arkes. And forasmuch as the lines subtending the sayd equall arkes are (by the 29. of the same) equall, therefore euery one of the sayd sides shall be the side of an equilater Pentagon: which was required to be proued. And the same line AC shall be the side of an equilater ten angled figure to be inscribed in the circle BDE : the demonstration wherof I omitte, for that it is demonstrated by Propositions following.

*A Proposition
added by Pe-
tarilius.*

¶ A Proposition added by Pelitarius.

Upon a right line geuen being finite, to describe an equilater and equiangle Pentagon figure.

Suppose that the right line geuen be AB , vpon which it is required to describe an equilater and equiangle Pentagon. Vpon the line AB describe (by the 23. and 32. of the first) an Ilosceles triangle ABC equiangle to the Ilosceles triangle described by the former Proposition: namely, let the angles CAB and CBA , at the base AB , be equall to the two angles ABD and ADB in the former construction: so that eyther of them shall be two fift partes of two right angles, and the angle at the toppe, namely, the angle C , shall be one fift part. Then deuide the angle C into two equall partes by drawing the right line CD . And vpon the line AC , and vnto the poynt A , describe the angle CAD equall to the angle ACD , by drawing the line AD , which line AD let concurre with the line CD , in the poynt D : and that within the triangle ABC , for the line CD being produced, shall fall vpon the base AB , and the line AD vpon the side BC . And draw a line from the poynt D to the poynt B . And forasmuch as in the triangle ACD the two angles A and C are equall, therefore (by the 6. of the first) the two sides AD and CD are equall. Againe forasmuch as the two sides CB and CD of the triangle $CB D$, are equall to the two sides CA and CD of the triangle ACD , and the angle C of the one, is equall to the angle C of the other (by construction) therefore (by the 4. of the first) the base DB is equall to the base DA , and so is equall to the line DC . Wherefore (by the 9. of the third) the poynt D shall be the centre of the circle described about the triangle ABC . Describe the sayd circle and let it be $ABECF$. Now then the angle ADB is double to the angle ACD (by the 20. of the third). Wherefore the angle ADB maketh two fift partes of two right angles, that is, one fift part of foure right angles. And forasmuch as the space about the centre D is equall to foure right angles, then if the sayd space be deuided into fife angles equall to the angle ADB , namely, into fife fift partes, by drawing the right lines DE & DF , which with the lines DA , DB , and DC , will cause the sayd space to be deuided into fife equall partes, and if also there be drawn these right lines AF , FC , CE , and EB , there shall be described a rectiline Pentagon figure $ABECF$, which shall be equilater, by therule of a circle and of a circumference, and helpe of the 4. Proposition of the



first;

first: and shall also be equiangle (by the 4. and 5. of the same). For the five angles A, B, E, C, F , are deuided ech into ten equall partes: which was required to be done.

If we consider well thys demonstration of Pelitarius, it will not be hard for vs, vpon a right line geuen to describe the rest of the figures whose inscriptions hereafter followe.

Note.

The 11. Probleme. The 11. Proposition.

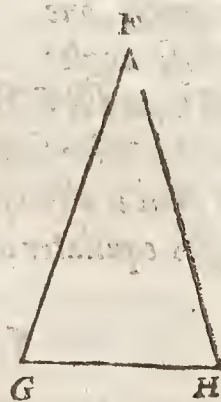
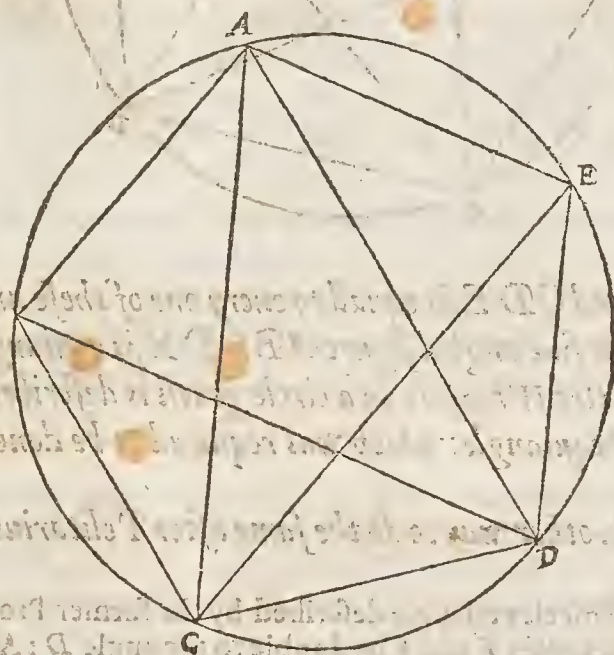
In a circle geuen to describe a pentagon figure æquilater and equiangle.



Suppose that the circle geuen be $A B C D E$. It is required in the circle $A B C D E$ to inscribe a figure of five angles of equall sides and of equall angles. Take (by the proposition going before) an Isosceles trian-

Construction.

gle $F G H$ ha-
uing eyther of
the angles at $\angle$
base $G H$ dou-
ble to the other
angle, namely,
vnto the angle B
 F . And (by the
2. of $\angle$ fourth)
in the circle A
 $B C D E$ in-
scribe a trian-
gle $A C D$ æ-
quiangle vnto
the triangle F



$G H$. So that let the angle $C A D$ be equall to the angle F , & the angle $A C D$ vnto the angle G , and likewise the angle $C D A$ to the angle H . Wherefore eyther of these angles $A C D$, and $C D A$ is double to the angle $C A D$. Deuide (by the 9. of the first) either of these angles $A C D$, and $C D A$ into two equall partes by the right lines $C E$ and $D B$: and draw right lines from A to B , from B to C , from C to D , from D to E , and from E to A . And forasmuch as either of these angles $A C D$, and $C D A$ is double to the angle $C A D$: and they are deuided into two equall partes by the right lines $C E$ and $D B$, therefore the five angles $D A C, A C E, E C D, C D B$, and $B D A$ are equall the one to the other. But equall angles (by the 26. of the third) subtend equall circumferences. Wherefore the five circumferences $A B, B C, C D, D E$, and $E A$ are equall the one to $\angle$ other. And (by $\angle$ 29. of $\angle$ same) vnto equall circumferences are subtended equal right lines: wherefore $\angle$ five right lines $A B, B C, C D, D E$, & $E A$ are equal $\angle$ one to the other. Wherefore $\angle$ figure $A B C D E$ hauing five angles is equilater.

Demonstration.

Now also I say that it is equiangle. For forasmuch as the circumference $A B$

The fourth Booke

is equall to the circumference DE , put the circumference BCD common vnto them both. Wherefore the whole circumference $ABCD$ is equall to the whole circumference $EDCB$: and vpon the circumference $ABCD$ consisteth the angle AED , and vpon the circumference $EDCB$, consisteth the angle BAE .

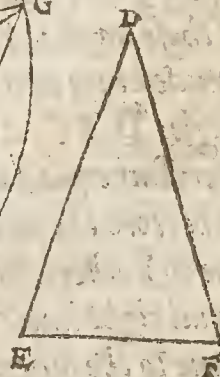
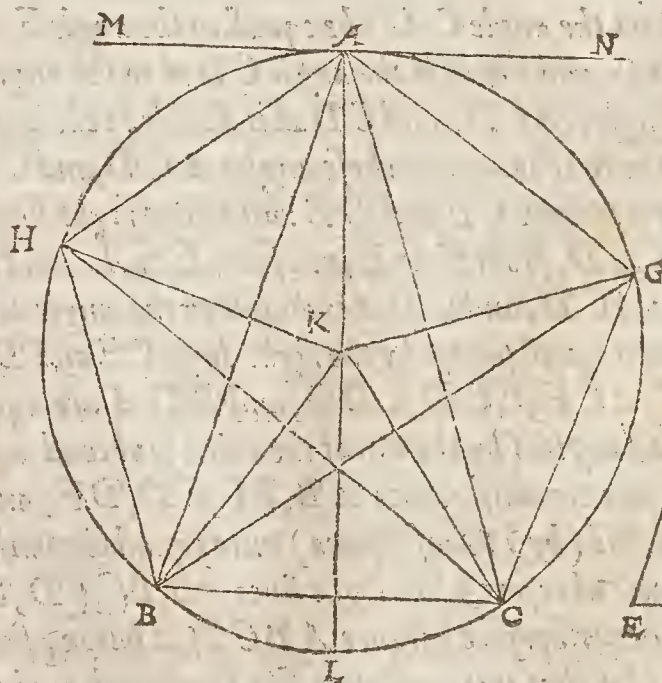
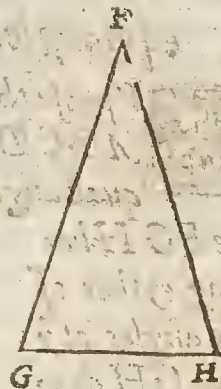
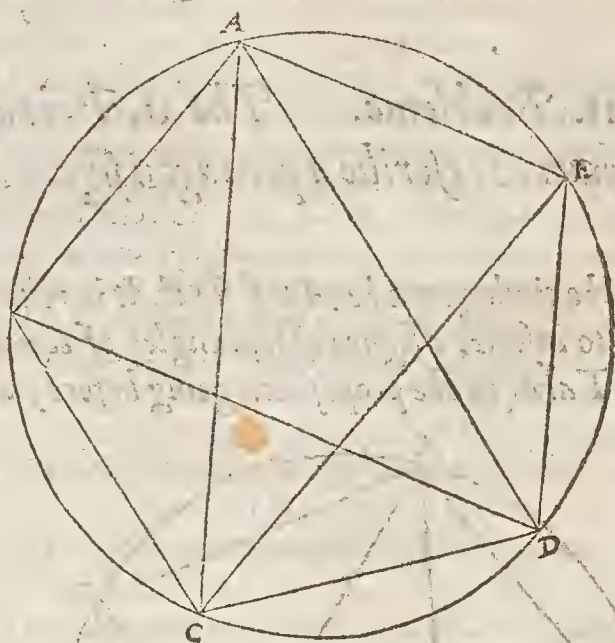
Wherefore the angle BAE is equall to $\angle AED$ (by the 27. of the third) and by the same reason euery one of these angles

ABC , and BCD , and CDE , is equall to euery one of these angles BAE and AED . Wherefore the five angled figure $ABCDE$, is equiangle, and it is proued, that is also equilater. Wherefore in a circle geuen is described a figure of five angles equilater and equiangle: which was required to be done.

¶ An other way to do the same after Pelitarius.

An other way
to do the same
after Pelita-
rius.

Suppose that the Ifosceles triangle described by the former Proposition, be DEF . so that let eyther of the angles E and F be double to the angle D : And let the circle geuen be ABC : the centre wherof let be K . And vpon the centre K describe the angle BKC equall to one of the angles E or F of the triangle DEF . And draw a right line from B to C . Then I say, that the line BC is the side of the Pentagon figure to be inscribed in the circle ABC . Deuide the angle BKC into two equall partes by drawing the diameter AKL . And draw these right lines BA & AC . Now the it is manifest (by the 20. of the third) that the angle



BKC

BKC is double to the whole angle BAC . Wherefore the whole angle BAC is equal to the angle D , vnto which angle, the angle BKC also is double. And forasmuch as in the triangle ABK the two angles A and B are (by the 5. of the first) equal (for the lines KA and KB are drawn from the centre) therefore (by the 32. of the same) the outward angle BKL is double to eyther of the inward angles KAB and KBA . And by the same reason, the angle CKL is double to eyther of the angles KAC and KCA . Wherefore forasmuch as the two angles at the poynt K are equal, the two angles A and B of the triangle ABK , are equal to the two angles A and C of the triangle ACK , the one to the other: and therefore (by the 26. of the first) the two bases AB and AC are equal. Wherefore ABC is an Ilosceles triangle. And forasmuch as the whole angle BAC is equal to the angle D , the two angles remayning ABC and ACB , shall (by the 32. of the first) be equal to the two angles remayning E & F . Wherefore the triangle ABC is equiangle to the triangle DEF . And now you may procede in the demonstration as you did in the former, imagining first the lines BG and CH to be drawn.

Here it is a pleasant thing to beholde the varietie of triangles: for in the triangle ABC either of the angles at the point A is one fift part of a right angle. Whereby is produced the side of a ten angled figure to be inscribed in the self same circle: Which is manifest if we imagine the lines BL and LC to be drawn. For the arke BC is deuided into two equal partes in the poynt L (by the 26. of the third). So then by the inscription of an equilater triangle, is knowen how to inscribe an Hexagon figure, namely, by deuiding ech of the arkcs subtended vnder the sides of the triangle into two equal partes. And so alwayes by the simple number of the side, is knowen the double thereof: as by a square is knowen an eight angled figure: and by an eight angled figure a sixtene angled figure. And so continually in the rest.

Pelitarus teacheth yet an other way how to inscribe a Pentagon. Take the same circle that was before, namely, ABC , and the same triangle also DEF . And (by the 17. of the third) draw the line MAN touching the circle in the poynt A . And vpon the line AM and to the poynt A , describe (by the 23. of the first) the angle MAB equal to one of these two angles E or F (eyther of which, as it is manifest, is lesse then a right angle) by drawing the right line AB : which let cut the circumference in the poynt B . Agayne, vpon the line AN and to the poynt in it A , describe the angle NAC equal to the angle MAB , by drawing the right line AC : which let cut the circumference in the poynt C . And draw a right line from B to C . Then I say, that BC is the side of a Pentagon figure to be inscribed in the circle ABC . Which is manifest, if we deuide the arke AB into two equal partes in the poynt H , and draw these right lines AH and BH , and if also we deuide the arke AC into two equal partes in the poynt G , and draw these right lines AG and CG . For taking the quadrangle figure $ABCG$, it is manifest (by the 32. of the third) that the angle ABC is equal to the alternate angle NAC , and therefore is equal to the angle E . Likewise taking the quadrangle figure $ACBH$, the angle ACB shall be equal to the alternate angle MAB , and therefore is equal to the angle F . Wherefore (by the 32. of the first) as before, the triangle ABC is equiangle to the triangle DEF : And now may you procede in the demonstration as you did in the former.

*An other way
also after
Pelitarus.*

The 12. Probleme. The 12. Proposition.

About a circle geuen, to describe an equilater and equiangle pentagon.

Gg.ij.

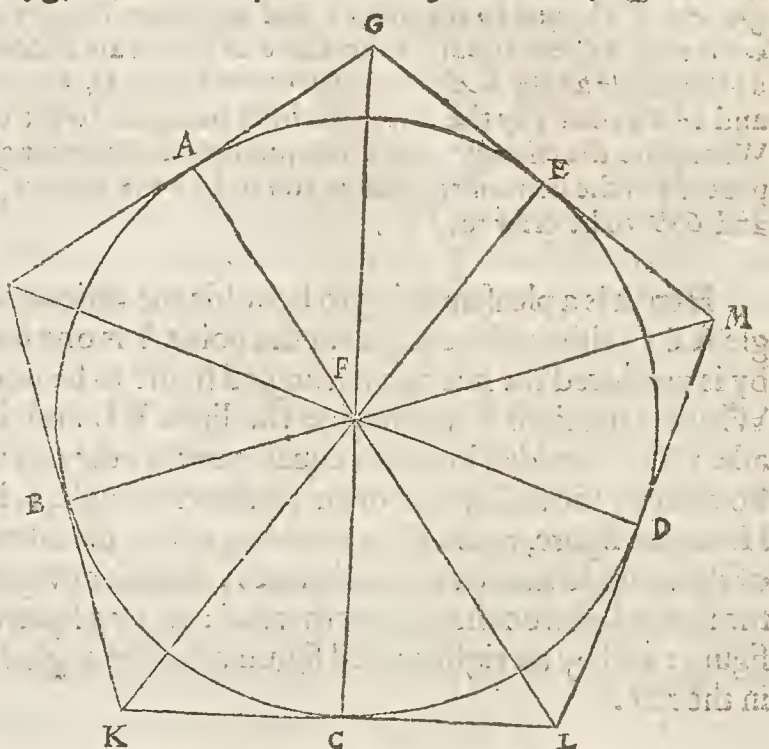
Suppose

The fourth Booke

Construction.

Suppose that the circle geuen be $A B C D E$. It is required about the circle $A B C D E$ to describe a figure of five angles consisting of equal sides and of equall angles. Take the pointes of the angles of a five angled figure described (by the 11. of the fourth) so that by the proposition goyng before, let γ circumferēces $A B, B C, C D, D E$, and $E A$ be equall the one to the other. And by the pointes A, B, C, D, E , draw (by the 17. of the third) right lines

touching the circle, and let the same be $G H, H K, K L, L M$, and $M G$. And (by the 1. of the third) take the centre of the circle & let the same be F . And drawe right lines from F to B , from F to K , from F to C , from F to L , and from F to D . And forasmuch as the right lyne $K L$, toucheth the circle $A B C D E$ in the pointe C , and from the centre F vnto the point C where the touche is, is drawn a



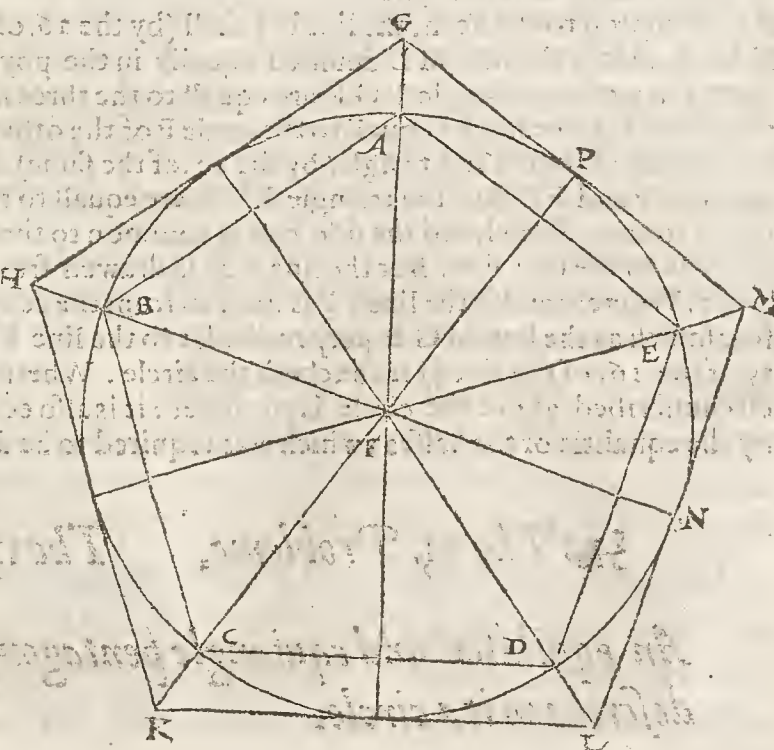
Demonstration.

right line $F C$: therefore (by the 18. of the third) $F C$ is a perpendicular line vnto $K L$. Wherefore either of the angles which are at the point C is a right angle, and by the same reason the angles which are at the pointes D and B are right angles. And forasmuch as the angle $F C K$ is a right angle, therefore the square which is made of $F K$ is (by the 47. of the first) equall to the squares which are made of $F C$ and $C K$. And by the same reason also the square which is made of $F K$ is equall to the squares which are made of $F B$ and $B K$. Wherefore the squares which are made of $F C$ and $C K$ are equall to the squares which are made of $F B$ and $B K$, of which the square which is made of $F C$ is equall to the square which is made of $F B$. Wherefore the square which is made of $C K$ is equall to the square which is made of $B K$. Wherefore the line $B K$ is equall vnto the line $C K$. And forasmuch as $F B$ is equall vnto $F C$, and $F K$ is common to them both, therefore these two $B F$ and $F K$ are equall to these two $C F$ & $F K$. And the base $B K$ is equall vnto the base $C K$. Wherefore (by the 8. of the first) the angle $B F K$ is equall vnto the angle $K F C$: and the angle $B K F$ to the angle $F K C$. Wherefore the angle $B F C$ is double to the angle $K F C$. And the angle $B K C$ is double to the angle $F K C$. And by the same reason the angle $C F D$ is double to the angle $C F L$, and the angle $D L C$ is double to the angle $F L C$. And forasmuch as the circumference $B C$ is equall vnto the circumference $C D$,

CD , therefore (by the 27. of the third) the angle $BF C$ is equall to the angle $CF D$. And the angle $BF C$ is double to the angle KFC , and the angle $DF C$ is double to the angle $LF C$, wherefore the angle KFC is equall vnto the angle $LF C$. Now then there are two triangles FKC , and FLC , hauing two angles equall to two angles, and one side equall to one side, namely, FC , which is common to them both. Wherefore (by the 26. of the third, the other sides remayning are equall vnto the sides remayning, and the angle remayning vnto the angle remayning. Wherefore the right line KC is equall to the right line CL , and the angle FKC to the angle FLC . And forasmuch as KC is equall to CL : therefore KL is double to KC , and by the same reason also may it be proued that HK is double to BK . And forasmuch as it is proued that BK is equall vnto KC , and KL is double to KC , and HK double to BK , therefore HK is equall vnto KL . In like sort may we proue y^e euery one of these lines HG , GM , & ML is equall vnto either of these lines HK and KL . Wherefore the five angled figure $G H K L M$ is of equall sides. I say also that it is of equall angles. For forasmuch as the angle FKC is equall vnto the angle FLC , and it is proued that the angle HKL is double to the angle FKC , and the angle KLM is double to the angle FLC , therefore the angle HKL is equall to the angle KLM . In like sort may it be proued that euery one of these angles KHG , HGM , and GML , is equall to either of these angles HKL , and KLM . Wherefore the five angles $G H K$, $H K L$, $K L M$, $L M G$, and $M G H$ are equall the one to the other. Wherefore the five angled figure $G H K L M$ is equiangle, and it is also proued that it is equilater, and it is described about the circle $A B C D E$: which was required to be done.

¶ An other way to do the same after Pelitarius, by parallel lines.

Suppose that the circle given be $A B C$, whose centre let be the point F : and in it (by the former Proposition) inscribe an equilateral and equiangle Pentagon $A B C D E$: by whose five angles drawe from the centre beyond the circumference, five lynes, FG , FH , FK , FL , and FM . And it is manifest, that the five angles at the centre F , are equall, when as the five sides of the triangles within are equall, and also their bases. It is manifest also, that the five angles of the Pentagon which are at the circumference, are diuided into ten equall angles (by the 4. of the first): Now then betwene the two lines FG and FH , draw the

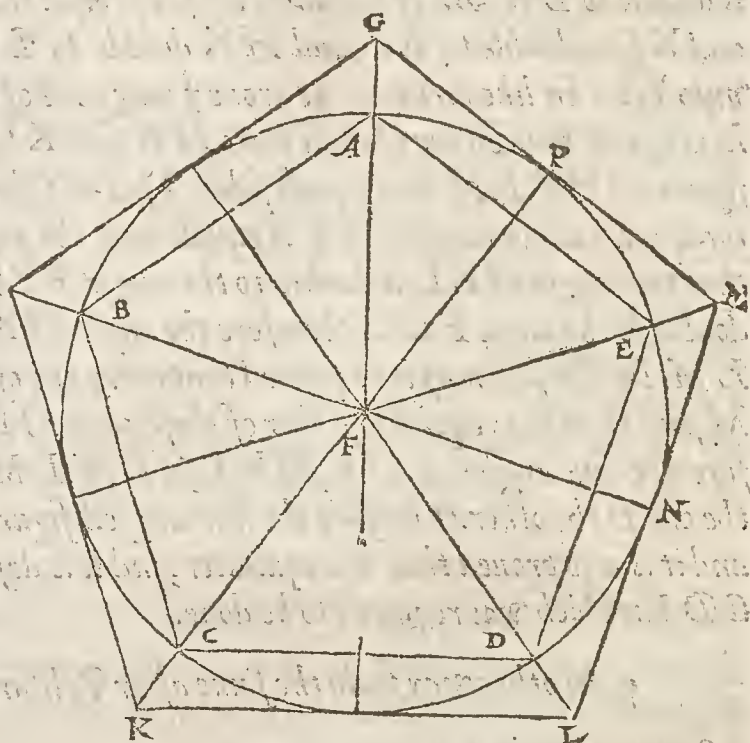


An other way
to do the same
after Pelita-
rius.

The fourth Booke

Demonstration.

line GH parallel to the side AB , and touching the circle ABC (which is done by a Proposition added by *Pelitarus* after the 17. of the third). And so likewise draw these lines HK, KL , and LM , parallel to each of these sides BC, CD , and DE , and touching the circle. And forasmuch as the lines FG and FH fall upon the two parallel lines AB and GH , the two angles $F GH$, & $F H G$, are equal to the two angles $F A B$ and $F B A$, the one to the other (by the 29. of the first). Wherefore (by the sixt of the same) the two lines FG and FH are equal. And by the same reason, the two angles $F H K$ & $F K H$, are equal to the two angles $F G H$ and $F H G$ the one to the other: and the line FK is equal to the line FH , and therefore is equal to the line FG . And forasmuch as the angles at the point F are equal, therefore (by the 4. of the first) the base HK is equal to the base GH . In like sort may we proue, that the three lines FK, FL , and FM , are equal to the two lines FG and FH . And also that the two bases KL and LM , are equal to the two bases GH and HK : and that the angles which they make with the lines FK, FL , and FM , are equal the one to the other. Now then draw the fifth line MG : which shall be equal to the four former lines (by the 4. of the first) for that as we haue proued, the two lines FG & FM , are equal, & the angle GFM is equal to euery one of the angles at the point F . Thys line also MG toucheth the circle. For H vnto the point where the line LM toucheth the circle which let be N , draw the lyne FN . And it is manifest (by the 18. of the third) that either of the angles at the point N , is a right angle. Wherefore forasmuch as the angle L of the triangle FLN , is equal to the angle M of the triangle FMN , & the angle N of the one, is equal to the angle N of the other: and the lyne FN is comon to the both, the line NL shall (by the 26. of the first) be equal to the line NM . And so is the line ML deuided equally in the point N . And forasmuch as the three sides of the triangle FGP are equal to the three sides of the triangle FMP , the angle P of the one shall be equal to the angle P of the other (by the 8. of the first). Wherefore either of the is a right angle (by the 13. of the same). And forasmuch as the two angles FMP and FMN of the triangle FMP , are equal to the two angles FMN & FNM of the triangle FMN , and the side FM is common to them both, therefore the line FP is equal to the line FN . But the line FN is drawn from the centre to the circumference. Wherefore also the line FP is drawn from the centre to the circumference. And forasmuch as the line MG is perpendicular to the line FP , therefore (by the Corollary of the 16. of the third) it toucheth the circle. Wherefore the Pentagon $G HKLM$ circumscribed about the circle is equilateral: it is also equiangle, as it is easie to proue by the equalitie of the halves: which was required to be done.



The 13. Probleme.

The 13. Proposition.

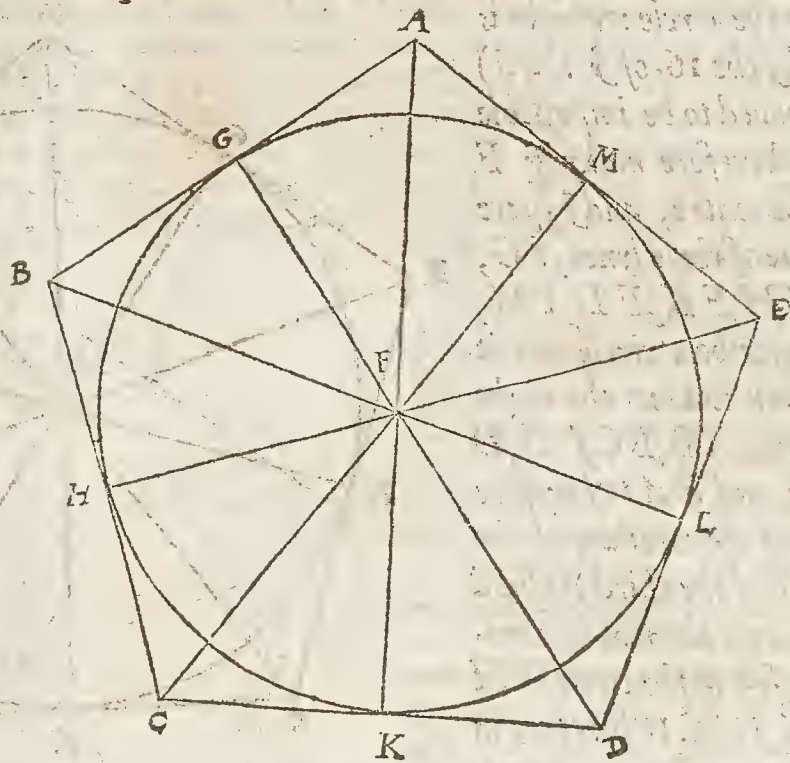
An equilater and equiangle pentagon figure beyng geuen, to describe in it a circle.

Suppose

Suppose γ γ equilateral & equiangle Penta-gon figure gen^e be $ABCDE$. It is required in the said five angled figure $ABCDE$ to describe a circle. Denide (by the 9. of the first) either of these angles BCD , and CDE into two equall partes, by these right lines CF and FD , and from the point F where these right lines CF and DF meete. Draw these right lines FB , FA , & FE . And forasmuch as BC is equal vnto CD and CF is common to them both.

Demonstration.

Wherefore these two lines BC and CF are equal to these two lines DC and CF : and the angle BCF is equall to the angle DCF . Wherefore (by the 4. of the first) the base BF is equall to the base DF , and the triangle BCF is equall to the triangle DCF , and the angles remayning are equall vnto the angles remaynyng the one to the other, vnder which are subtended e-



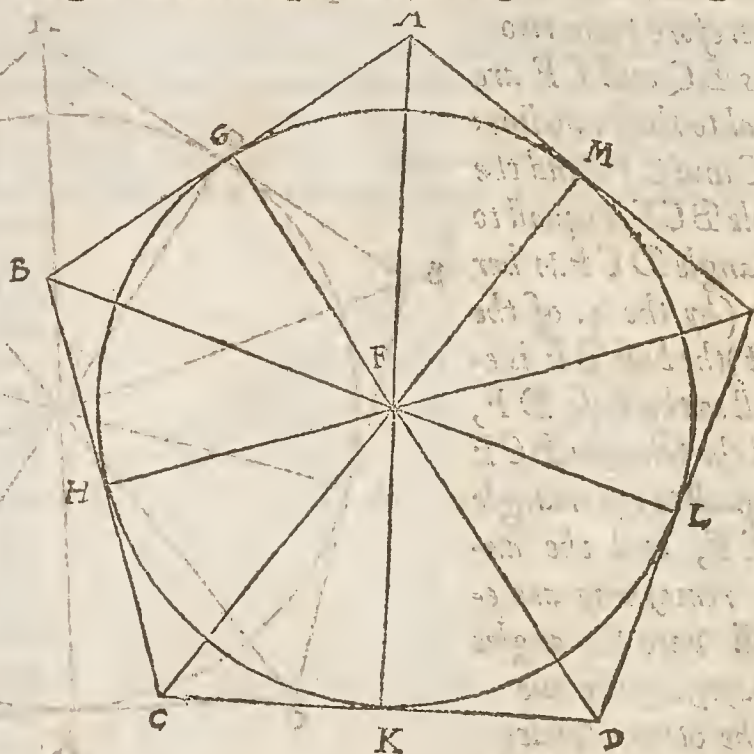
quall sides. Wherefore the angle CBF is equall to the angle CDF . And forasmuch as the angle CDE is double to the angle CDF . But the angle CDE is equall to the angle ABC , and the angle CDF is equall to the angle CBF . Wherefore the angle CBA is double to the angle CBF . Wherefore the angle ABF is equall to the angle FCB . Wherefore the angle ABC is denided into two equall partes by the right line BF . In lyke sorte also may it be proued, that either of these angles BAE , and AED are denided into two equall partes by either of these lines FA and FE . Drawe (by the 12. of the first) from the pointe E to the right lines AB , BC , CD , and EA perpendicular lines FG , FH , FK , FL , and FM . And forasmuch as the angle HCF is equall to the angle KCF , and the right angle FHC is equall to the right angle FKC : now then there are two triangles FHC , and FKC hauyng two angles equall to two angles the one to the other, and one side equall to one side: for FC is common vnto them both, and is subtended vnder one of the equal angles. Wherefore (by the 26. of the first) the sides remayning are equall vnto the sides remayning. Wherefore the perpendicular FH is equall vnto the perpendicular FK . And in like sort also may it be proued that euery one of these lines FL , FM , and FG is equall to euery one of these lines FH and FK . Wherefore these five right lines FG , FH , FK , FL , and FM are equall the one to the other.

Where-

The fourth Booke

Demonstration leading to an absurditie.

Wherefore making the centre F and the space FG , or FH , or FK , or FL , or FM . Describe a circle, and it will passe by the poyntes G, H, K, L, M . And shall touche the right lines AB, BC, CD, DE , and EA (by the corollary of the 16. of the third) for the angles which are at the pointes G, H, K, L, M , are right angles. For if it do not touche them but cut, then from the ende of the diameter of the circle, shall be drawen a right line making two right angles, and falling within the circle: whiche is (by the 16. of the third) proued to be impossible. Wherefore making F the centre, and the space one of these lynes, FG, FH, FK, FL, FM , describe a circle and it shall not cut the right lines AB, BC, CD, DE , and EA . Wherefore (by the corollary of the 16. of the third) it shall touche them as it is manifest in the circle $G H K L M$. Wherefore in the equilater and equiangle pentagon figure geuen, is described a circle: which was required to be done.



A Corollary.

By this proposition and the former, it is manifest that perpendicular lines drawn from the middle poyntes of the sides of an equilater and equiangle Pentagon figure, and produced, shall passe by the centre of the circle, in which the sayd Pentagon figure is inscribed, and shall also deuide the opposite angles equally, as here, AK is one right line, and deuideth the side CD and the angle A equally. And so of the rest which is thus proued. The space about the centre F is equall to foure right angles, which are deuided into ten equall angles by ten right lines metyng together in the point F . Wherefore the five angles AFM, MFE, EFL, LFD , and DFK are equall to two right angles. Wherefore (by the 14. of the first) the lynes AF and FK make one right lyne. The lyke prooffe also will serue touching the rest of the lynes. And this is alwayes true in all equilater figures which consist of vneuen sides.

The 14. Probleme. The 14. Proposition.

About a pentagon or figure of five angles geuen beyng equilater and equiangle, to describe a circle.

Supo



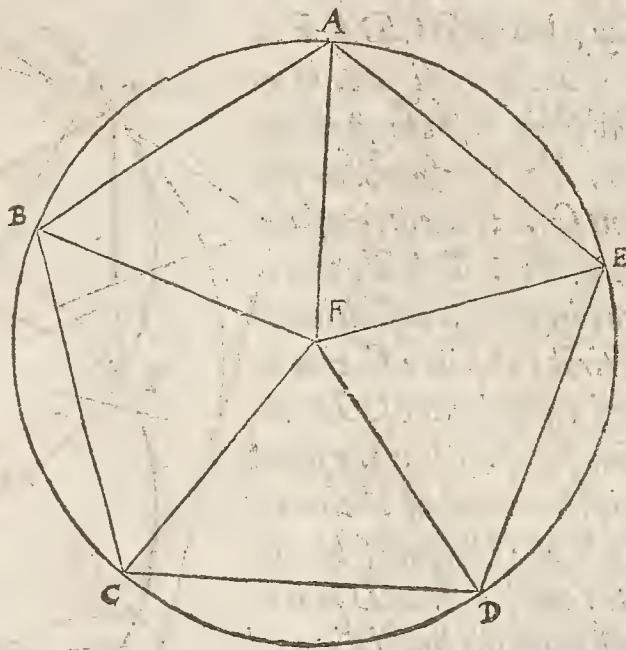
Suppose that γ Pentagon or figure of five angles geuen, being of equall sides and of equall angles, be $A B C D E$. It is required about the sayd Pentagon $A B C D E$, to describe a circle. Deuide (by the 9. of the first) eyther of these angles $B C D$, & $C D E$ into two equall partes

Construction.

by either of these right lines $C F$, and $D F$. And from the poynt F where those right lines meete, draw vnto the pointes B, A, E , these right lines $F B, F A, F E$.

Demonstration.

And in like sort (by the Proposition going before) may it be proued, that euery one of these angles $C B A$, $B A E$, and $A E D$ is deuided into two equall partes, by these right lines $F B, F A$, & $F E$. And for asmuch as the angle $B C D$ is equall to the angle $C D E$, & the halfe of the angle $B C D$ is the angle $F C D$, and likewise the halfe of the angle $C D E$ is γ angle $C D F$. Wherefore the angle $F C D$ is equall to the angle $F D C$. Wherefore γ side $F C$ is equall to the side $F D$. In like sort al-



so may it be proued, that euery one of these lines $F B, F A$, and $F E$ is equall to euery one of these lines $F C$ and $F D$. Wherefore these five right lines $F A, F B, F C, F D$, and $F E$ are equall the one to the other. Wherefore making the centre F , and the space $F A$, or $F B$, or $F C$, or $F D$, or $F E$. Describe a circle and it will passe by the pointes A, B, C, D, E , and shall be described about the five angled figure $A B C D E$ which is equiangle and equilater. Let this circle be described and let the same be $A B C D E$. Wherefore about the Pentagon geuen being both equiangle and equilater, is described a circle: which was required to be done.

The 15. Probleme.

The 15. Proposition.

In a circle geuen to describe an hexagon or figure of sixe angles equilater and equiangle.

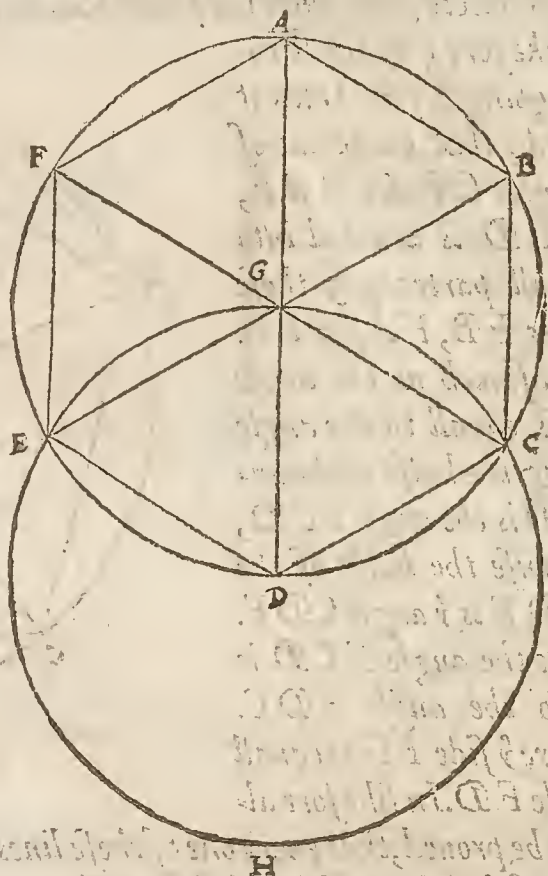


Suppose that the circle geuen be $A B C D E F$. It is required in the circle geuen $A B C D E F$ to describe a figure of sixe angles of equall sides and of equall angles. Draw the diameter of the circle $A B C D E F$, and let

let

The fourth Booke

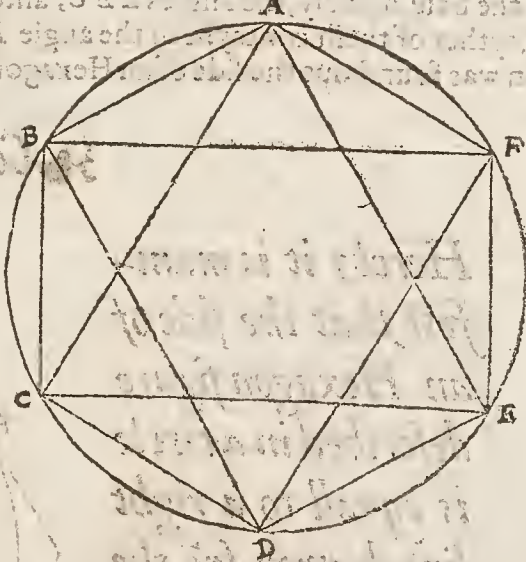
Construction. let the same be AD . And (by the first of the third) take the centre of the circle and let the same be G . And making the centre D , and the space DG , describe (by the third petition) a circle $CGEH$: and drawing right lines from E to G , and from G to C , extend them to the pointes B and F of the circumference of the circle geuen. And draw these right lines AB, BC, CD, DE, EF , and FA . Then I say, that $ABCDEF$ is an Hexagon figure of equall sides and of equall angles. For forasmuch as the point G is the centre of the circle $ABCDEF$, therefore (by the 15. definition of the first) the line GE is equall vnto the line GD . Again forasmuch as the point D is the centre of the circle $CGEH$ (therefore (by the selfe same) the line DE is equall vnto the line DG . And it is proued that the line GE is equall vnto the lyne GD . Wherefore the line GE is equall vnto the line ED (by the first common sentence) wherefore the triangle EGD is equilateral, and his three angles, namely, EGD, GDE, DEG , are equall the one to the other. And forasmuch as (by the 5. of the first) in triangles of two equal sides commonly called Isosceles, the angles at the base are equall the one to the other, and the three angles of a triangle are (by the 32. of the first) equall vnto two right angles, therefore the angle EGD is the third part of two right angles. And in lyke sorte may it be proued, that the angle DGC is the third parte of two right angles. And forasmuch as the right line CG standing vpon the right line EB doth (by the 13. of the first) make the two side angles EGC , and ECB equall to two right angles, therefore the angle remayning ECB is the third part of two right angles. Wherefore the angles EGD, DGC , and ECB are equall the one to the other. Wherefore their hed angles, that is, BGA, AGF , and FGE are (by the 15. of the first) equall to these angles EGD, DGC , and ECB . Wherefore these sixe angles EGD, DGC, ECB, BGA, AGF , and FGE are equall the one to the other. But equall angles consist vpon equall circumferences (by the 26. of the third. Therefore these sixe circumferences AB, BC, CD, DE, EF , and FA are equall the one to the other. But vnder equall circumferences are subtended



ded equall right lines (by the 29. of the same.) Wherefore these sixe right lynes AB, BC, CD, DE, EF , and FA are equall the one to the other. Wherefore the Hexagon $ABCDEF$ is equilater. I say also that it is equiangle. For forasmuch as the circumference AF is equall vnto the circumference ED adde the circumference $ABCD$ common to them both. Wherefore the whole circumference $FABCD$ is equall to the whole circumference $EDCBA$. And vppon the circumference $FABCD$ consisteth the angle FED : and vppon the circumference $EDCBA$ consisteth the angle AFE . Wherefore the angle AFE is equall to the angle DEF . In like sort also may it be proued that the rest of the angles of the Hexagon $ABCDEF$, that is euery one of these angles FAB, ABC, BCD , and CDE is equall to euery one of these angles AFE , and FED . Wherefore the Hexagon figure $ABCDEF$ is equiangle, and it is proued that it is also equilater, and it is described in the circle $ABCDEF$. Wherefore in the circle geuen $ABCDEF$ is described a figure of sixe angles of equal sides and of equall angles: Which was required to be done.

¶ An other way to do the same after Orontius.

Suppose that the circle geuen be $ABCDEF$ in which first let there be described an equilater and equiangle triangle ACE (by the second of thys booke). Wherefore the arkes ABC, CDE, EFA are (by the 28. of the third) equall the one to the other. Deuide euery one of those three arkes into two equall partes (by the 30. of the same) in the pointes $B, D, \& F$. And draw these right lines AB, BC, CD, DE, EF , and FA . Now then by the 3. definition of this booke there shall be described in the circle geue an Hexagon figure $ABCDEF$, which must nedes be equilater: for that euery one of the arkes which subtend the sides thereof are equall the one to the other. I say also that it is equiangle. For euery angle of the Hexagon figure is set vpon equall arkes, namely, vpon foure such partes of the circumference wherof the whole circumference cotayneth sixe. Wherefore the angles of the Hexagon figure are equall the one to the other (by the 27. of the third). Wherefore in the circle geuen $ABCDEF$ is inscribed an equilater and equiangle Hexagon figure: which was required to be done.



¶ An other way to do the same after Pelitarins.

Suppose that the circle in which is to be inscribed an equilater & equiangle Hexagon figure be $ABCDE$, whose centre let be F . And from the centre draw the semidiameter FA . And from the poynt A apply (by the first of thys booke) the line AB equall to the semidiameter. Which I say is the side of an equilater and equiangle Hexagon figure to be inscribed in the circle $ABCDE$. Draw a right line from F to B . And forasmuch as the line AB is equall to the line FA , & it is also equall to the line FB , there-

Hh.j. fore

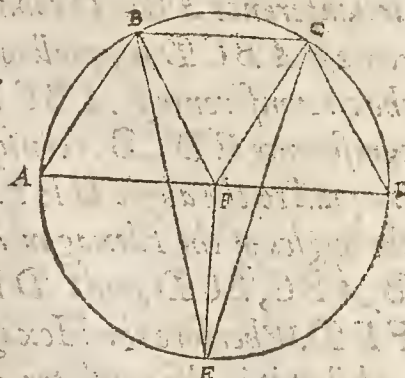
An other way
to do the same
after Orontius.

An other
way after
Pelitarins.

The fourth Booke

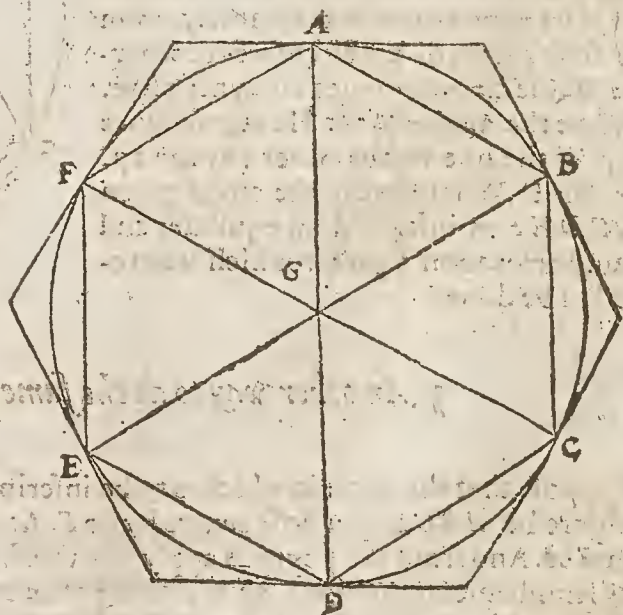
fore the triangle $\triangle AFB$ is equilateral, and by the 5. of the first equiangle. Now then vpon the centre F describe the angle $\angle BEC$ equall to the angle $\angle AFB$, or to the angle $\angle FBA$ (which is all one) by drawing the right line FC . And draw a line from B to C . And forasmuch as the angle $\angle AFB$ is the third part of two right angles by the 5. and 32. of the first, the angle $\angle BFC$ also shall be the third part of two right angles. Wherefore either of the two angles remaining $\angle FBC$ and $\angle FCB$, forasmuch as they are equall by the 5. of the first, shall be two third partes of two right angles (by the 32. of the same). Or (by the 4. of the first) forasmuch as the angle $\angle BFC$ is equall to the angle $\angle FBA$, and the two sides FB and FC are equall to the two sides AB and BF , the base BC shall be equall to the base BF , and therefore is equall to the line FC . Wherefore the triangle $\triangle FBC$ is equilateral and equiangle. Lastly make the angle $\angle CFD$ equall to eyther of the angles at the poynt F , by drawing the line FD . And draw a line from C to D . Now then by the former reason the triangle $\triangle FCD$ shall be equilateral and equiangle. And forasmuch as the three angles at the point F are equall to two right angles (for ech of them is the third part of two right angles) therefore (by the 14. of the first) AD is one right line: and for that cause is the diameter of the circle. Wherefore if the other semicircle AFD be denided into so many equall partes as the semicircle $ABCD$ is denided into, it shall comprehend so many equall lines subtended vnto it. Wherefore the line AB is the side of an equilateral Hexagon figure to be inscribed in the circle: which Hexagon figure also shall be equiangle: For the halfe of the whole angle B is equall to the halfe of the whole angle C : which was required to be done.

Now then if we draw from the centre F a perpendicular line vnto AD , which let be FE , and draw also these right lines BE and CE : there shall be described a triangle $\triangle BEC$ whose angle E which is at the toppe shall be the 6. part of two right angles by the 20. of the third. For the angle $\angle BFC$ is double vnto it. And either of the two angles at the base, namely, the angles $\angle EBC$, and $\angle ECB$ is dupla sesquialter to the angle E : that is, eyther of them containeth the angle E twise and halfe the angle E . And by this reason was found out the side of an Hexagon figure.



Correlary.

Hereby it is manifest, that the side of an Hexagon figure described in a circle is equall to a right line drawn frō the cētre of the said circle vnto the circumference. And if by the pointes A, B, C, D, E, F, be



drawen

drawen right lines touching the circle, then shall there be described about the circle an Hexagon figure equilater and equiangle, which may be demonstrated by that which hath bene spoken of the describing of a Pentagon about a circle. And moreover by those things which haue ben spokē of Pentagons we may in a Hexagon geuen either describe or circumscribe a circle: which was required to be done.

☞ The 16. Probleme. The 16. Proposition.

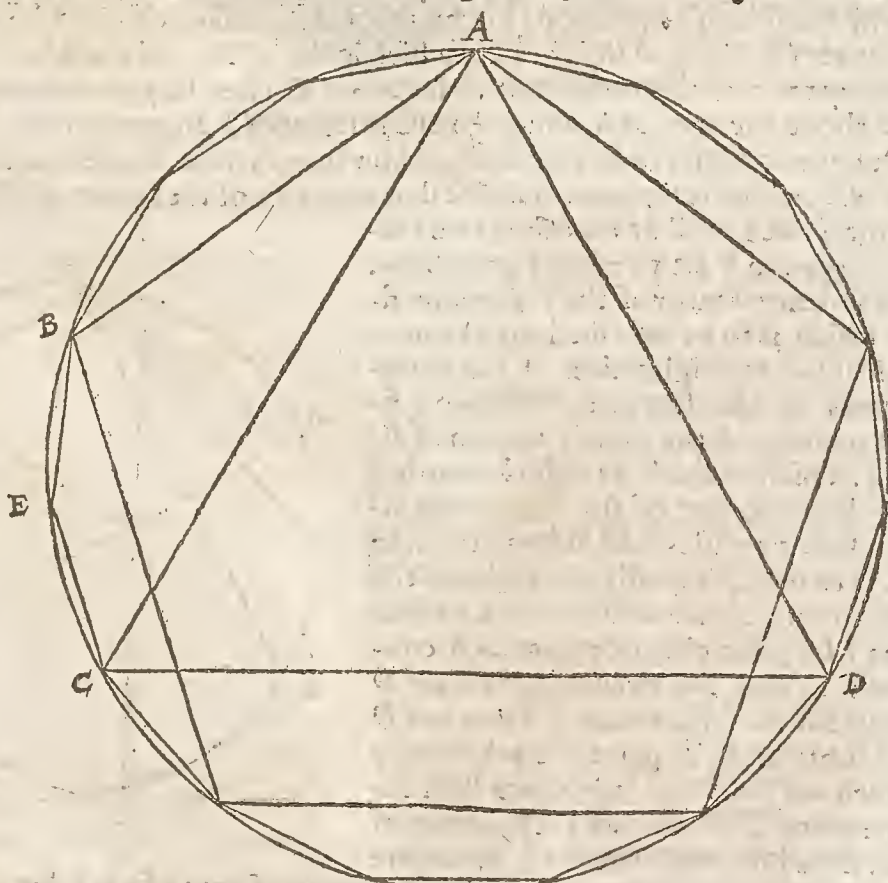
In a circle geuen to describe a quindecagon or figure of fiftene angles, equilater and equiangle.



Suppose that the circle geuen be $ABCD$. It is required in the circle $ABCD$ to describe a figure of fiftene angles consisting of equall sides and of equall angles. Describe in the circle $ABCD$ the sides of an equilater triangle, and let the same be AC , and in γ arke AC describe the side of an equilater pentagon and let the same be AB . Now then of such equall partes wherof the whole circle $ABCD$ containeth fiftene, of such partes I say, the circumference ABC being the third parte of the circle shall contayne five. And the

Construction.

circumference AB being the fift part of a circle shall contain thre, wherefore the residue BC shall containe two. Deuide (by the 30. of the first) the arke BC into two equall partes in the point E . Wherefore either of these circumferences BE



Demonstration.

EC is the fiftene part of the circle $ABCD$. If therefore there be drawn right lines.

Hh.ij

The fourth Booke

lines from B to E , and from E to C , and then beginning at the point B or at the point C there be applied into the circle $ABCD$ right lines equall vnto EB or EC , and so continuing till ye come to the point C if you began at B , or to B point if you began at C , and there shall be described in the circle $ABCD$ a figure of fiftene angles equilater and equiangle: which was required to be done. And in like sort as in a pentagon, if by the pointes where the circle is deuided, be drawen right lines touching the circle in the said pointes, there shall be described about γ circle a figure of fiftene angles equilater & equiangle. And in like sort by γ selfe same obseruations that were in Pentagons, we may in a figure of fiftene angles geuen being equilater and equiangle either inscribe, or circumscribe a circle.

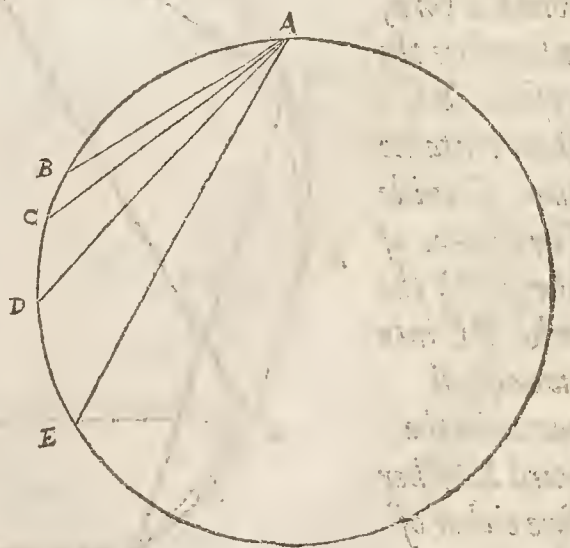
*An addition
of Flussates.*

¶ *An addition of Flussates to finde out infinite figures of
many angles.*

* *A Polygonon
figure is a fi-
gure consisting
of many sides.*

*If into a circle from one poynt be applyed the sides of two * Polygonon figures: the ex-
cesse of the greater arke aboue the lesse, shall comprehend an arke containning so many
sides of the Polygonon figure to be inscribed by how many vnities the denomination of
the Polygonon figure of the lesse side exceedeth the denomination of the Polygonon figure
of the greater side: and the number of the sides of the Polygonon figure to be inscribed
is produced of the multiplication of the denominations of the foresayd Polygonon figures
the one into the other.*

As for example. Suppose that into the circle ABE be applyed the side of an equilater and equiangle Hexagon figure (by the 15. of thys booke) which let be AB : and likewise the side of a Pentagon (by the 11. of this booke) which let be AC : and the side of a square (by the 6. of thys booke) which let be AD : and the side of an equilater triangle (by the 2. of this booke) which let be AE . Then I say, that the excesse of the arke AD aboue the arke AB , which excesse is the arke BD , containeth so many sides of the Polygonon figure to be inscribed, of how many vnities the denominator of the Hexagon AB , which is sixe, exceedeth the denominator of the square AD , which is foure. And forasmuch as that excesse is two vnities, therefore in BD there shall be two sides. And the denominator of the Polygonon figure which is to be inscribed shall be produced of the multiplication of the denominators of the foresayd Polygonon figures, namely, of the multiplication of 6. into 4. which maketh 24. which number is the denominator of the Polygonon figure, whose two sides shall subtend the arke BD . For of such equall partes wherof the whole circumference cōtayneth 24. of such partes I say, the circumference AB containeth 4, and the circumference AD containeth 6. Wherefore if from AD which subtendeth 6. partes be taken away 4. which AB subtendeth, there shall remaine vnto BD two of such partes of which the whole containeth 24. Wherefore of an Hexagon and a square is made a Polygonon figure of 24. sides. Likewyse of the Hexagon AB and of the Pentagon AC shall be made a Polygonon figure of 30. sides,



sides, one of whose sides shall subtend the arke BC . For the denomination of AB which is 6. exceedeth the denomination of AC which is 5. onely by vnitie. So also forasmuch as the denomination of AB which is 6. exceedeth the denomination of AE which is 3. by 3. therefore the arke BE shall containe 3. sides of a Polygonon figure of 18. sides. And obseruing thys selfe same methode and order, a man may finde out infinite sides of a Polygonon figure.

(:.)

**The end of the fourth booke
of Euclides Elementes.**

Hh.iii.

The



The fifth booke of Euclides Elementes.

The argument
of this fift
booke.



HIS FIFTH BOOKE of *Euclide* is of very great commoditie and vse in all Geometry, and much diligence ought to be bestowed therin. It ought of all other to be thoroughly and most perfectly and readily knowne. For nothyng in the bookes followyng can be vnderstand without it: the knowledge of them all depende of it. And not onely they and other writings of Geometry, but all other Sciences also and artes: as *Musike*, *Astronomy*, *Perspectiue*, *Arithmetique*, the arte of accomptes and reckoning, with other such like. This booke therefore is as it were a chiefe treasure, and a peculiar iuell much to be accompted of.

It entreateth of proportion and Analogie, or proportionalitie, which pertayneth not onely vnto lines, figures, and bodies in Geometry: but also vnto soundes & voyces, of which *Musike* entreateth, as witnesseth *Boetius* and others which write of *Musike*. Also the whole arte of *Astronomy* teacheth to measure proportions of tymes and mouinges. *Archimides* and *Iordan* with other, writing of waightes, affirme, that there is proportion betwene waight and waight, and also betwene place & place. Ye see therefore how large is the vse of this fift booke. Wherefore the definitions also thereof are common, although hereof *Euclide* they be accomodate and applied onely to Geometry. The first author of this booke was as it is affirmed of many, one *Eudoxus* who was *Platos* scholer, but it was afterward framed and put in order by *Euclide*.

The first author of this
booke *Eudoxus*.

Definitions.

A parte is a lesse magnitude in respect of a greater magnitude, when the lesse measureth the greater.

As in the other bookes before, so in this, the author first setteth orderly the definitions and declarations of such termes and wordes which are necessarily required to the entreatie of the subiect and matter therof, which is proportion and comparison of proportions or proportionalitie. And first he sheweth what a parte is. Here is to be considered that all the definitions of this fift booke be general to Geometry and *Arithmetique*, and are true in both artes, euen as proportion and proportionalitie are common to them both, and chiefly appertayne to number, neither can they aptly be applied to matter of Geometry, but in respect of number and by number. Yet in this booke, and in these definitions here set, *Euclide* semeth to speake of them onely Geometrically, as they are applied to quantitie continuall, as to lines, superficiences, and bodies: for that he yet continueth in Geometry, I wil notwithstanding for facilitie and farther helpe of the reader, declare the both by example in number, and also in lynes.

For the clearer vnderstandyng of a parte, it is to be noted, that a part is taken in the Mathematicall Sciences two maner of wayes. One way a part is a lesse quantitie

A part taken
two maner of
wayes.
The first way.

tie in respect of a greater, whether it measure the greater or no. The second way, a part is onely that lesse quantitie in respect of the greater, which measureth the greater. A lesse quantitie is sayd to measure or number a greater quantitie, when it, beyng oftentimes taken, maketh precisely the greater quantitie without more or lesse, or beyng as oftentimes taken from the greater as it may, there remaineth nothyng. As suppose the line A B to contayne 3. and the lyne C D to contayne 9. the doth the line A B measure the line

C D: for that if it be take certayne tymes, namely, 3. tymes, it maketh precisely the lyne C D, that is 9. without more or lesse. Agayne if the sayd lesse lyne A B be taken from the greater C D, as often as it may be, namely, 3. tymes, there shall remaine nothing of the greater. So the number 3. is sayde to measure 12. for that beyng taken certayne tymes, namely, foure tymes, it maketh iust 12. the greater quantitie: and also beyng taken from 12. as often as it may, namely, 4. tymes, there shall remaine nothyng. And in this meaning and signification doth *Euclide* vndoubtedly here in this define a part: saying, that it is a lesse magnitude in comparison of a greater, when the lesse measureth the greater. As the lyne A B before set, contayning 3. is a lesse quantitie in comparison of the lyne C D which containeth 9. and also measureth it. For it beyng certayne tymes taken, namely, 3. tymes, precisely maketh it, or taken from it as often as it may, there remaineth nothyng. Wherefore by this definition the lyne A B is a part of the lyne C D. Likewise in numbers, the number 5. is a part of the number 15. for it is a lesse number or quantitie compared to the greater, and also it measureth the greater: for beyng taken certayne tymes, namely, 3. tymes, it maketh 15. And this kynde of part is called commonly *pars metiens* or *mensurans*, that is, a measuring part: some call it *pars multiplicatiua*: and of the barbarous it is called *pars aliquota*, that is an aliquote part. And this kynde of parte is commonly vsed in Arithmetique.

The other kinde of a part, is any lesse quantitie in comparison of a greater, whether it be in number or magnitude, and whether it measure or no. As suppose the line A B to be 17. and let it be deuided into two partes in the poynt C, namely, into the line A C, & the line C B, and let the

lyne A C the greater part contayne 12. and let the line B C the lesse part contayne 5. Now eyther of these lines by this definition is a part of the whole lyne A B. For eyther of them is a lesse magnitude or quãtity in cõparisõ of the whole lyne A B: but neither of the measureth the whole line A B: for the lesse lyne C B contayning 5. taken as oftẽ as ye list, will neuer make precisely A B which contayneth 17. If ye take it 3. tymes it maketh only 15. so lacketh it 2. of 17. which is to litle. If ye take it 4. tymes, so maketh it 20. the are there thre to much, so it neuer maketh precisely 17. but either to much or to litle. Likewise the other part A C measureth not the whole lyne A B: for take once, it maketh but 12. which is lesse then 17. and taken twise, it maketh 22. which are more then 17. by 5. So it neuer precisely maketh by takyng therof the whole A B, but either more or lesse. And this kynde of part they commonly call *pars constituens*, or *componens*: Because that it with some other part or partes, maketh the whole. As the lyne C B together with the line A C maketh the whole lyne A B. Of the barbarous it is called *pars aliquanta*. In this signification it is taken in *Barlaam* in the beginnyng of his booke, in the definition of a part, when he saith: Every

The second way.

How a lesse quantity is sayd to measure a greater.

In what signification Euclide here taketh a part.

Pars metiens or *mensurans*.
Pars multiplicatiua.
Pars aliquota.
This kinde of part cõmonly vsed in Arithmetique.
The other kinde of part.

Pars constituens, or *componens*.

Pars aliquanta.

The fifth Booke

lesse number compared to a greater, is sayd to be a part of the greater, whether the lesse measure the greater, or measure it not.

The second definition.

Multiplex is a greater magnitude in respect of the lesse, when the lesse measureth the greater.

As the line C D before set in the first example, is multiplex to the lyne A B. For that C D a lyne contayning 9. is the greater magnitude, and is compared to the lesse, namely, to the lyne A B contayning 3. and also the lesse lyne A B measureth the greater line C D: for taken 3. tymes, it maketh it, as was aboue sayde. So in numbers 12. is multiplex to 3: for 12 is the greater number, and is compared to the lesse, namely, to 3. which 3. also measureth it: for 3 taken 4 tymes maketh 12. By this worde *multiplex* which is a terme proper to Arithmetike and number, it is easy to consider that there can be no exact knowledge of proportion and proportionalitie, and so of this fifth booke wyth all the other bookes followyng, without the ayde and knowledge of numbers.

Numbers very necessary for the understanding of this booke and the other bookes following.

The third definition.

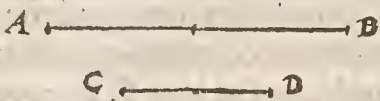
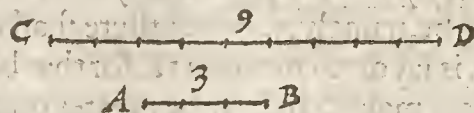
Proportion is a certaine respecte of two magnitudes of one kinde, according to quantitie.

Euclide as in the first definition, so in this & the other following, and likewise in all his Propositions of this booke, mentioneth onely magnitudes, and geueth his examples and demonstrations of lines: for that hetherto in the 4. bookes before he hath entreated of lines & figures, and so cōtinueth in his sixth booke following after this, comparing figure to figure, and sides of figures to sides of figures, without mention of number at all. Notwithstanding as it is sayd they are generall to all kinde of quantitie, both discrete and continuall, namely, number and magnitude: and neede for the young reader and student in these artes to be declared in both. For, the opening of them in numbers (in which they are first and naturally founde) geueth a great and marueilous light to their declaration in magnitudes. Proportion (sayth he) is a certaine behaviour, that is, a certaine respect or comparison of two quantities of one kinde: as of one line to an other, and one figure generally to an other, and one number to an other, as touching quantitie, that is to say, that the quantitie compared, is to that wherunto it is compared, eyther equall, or greater, or lesse then it. For after these three maners may thinges be compared the one to the other. But quantities of diuers kindes can not be compared together. A superficies can not be compared to a line: nor number to a body: nor a body to a line or number: for that they are not of one kinde. For example of this definition, take two quantities, namely, two lines A B and C D, and compare the one to the other, namely, A B to C D according to some certaine respect of greatnes, or lessenes, or equalitie, namely, in this example, let A B be greater then C D, & containe it twise. Now this comparison, relation, or respect of A B to C D, and generally of any one quantitie to any other, is called proportion. Likewise is it of

what proportion is.

Things compared together this wayes.

Example of this definition in magnitudes.



it, if AB be equall to CD , as in the second example : or if AB be lesse then CD , as in the third example. The like is it also in numbers comparing 5. to 5. equall to equall, or 6. to 3. the greater to the lesse, or 4. to 8. the lesse to the greater. So to the accomplishing of any Proportion there are required two quantities, and also a comparing or respect of the one to the other. The quantities compared together are commonly called the *Termes* of the Proportion. And in this booke of *Euclide*, and also in other writers of *Geometrie*, the first *Terme*, namely, that which is compared is called the antecedent, whether it be equall, greater, or lesse then the other : And the second *Terme*, namely, that wherunto the comparison is made, is called the consequent. As in the former example, the line AB compared to the line CD is antecedent : and the line CD is consequent : And contrariwise if the line CD be compared to the line AB , then is the line CD antecedent, and the line AB consequent. Albeit in Arithmetike *Boetius* and others call the terme compared *Dux*, and the terme to whom the cōparison is made they call *Comes*. This booke hath bene accompted of all men one of the hardest and most intricate of all *Euclides* bookes. And proportion is a generall knowledge to all learnings, chiefly to the Mathematicallis. Wherefore it shall be very necessary some litle briefe instruction and induction to be here added in the beginning hereof : of the knowledge and nature of proportions and what they are, and of how many kindes : which things are here of *Euclide* supposed to be before known, and therefore maketh no mention so distinctly of them.

Ye must vnderstand that there are of proportions two generall kindes, the one is called rationally, certaine, and known, and the other irrationally, vncertaine and vnknown. Such magnitudes or quantities, which may be expressed by numbres, are called *rationall* magnitudes or quantities. As suppose a line, namely, the line AB to containe 5. inches, & compare it to the line CD , contayning 3. inches : these quantities ye see may be expressed by numbers, namely, by these numbers 5. and 3 : and therefore are *rationall*, and haue the same proportion, that number hath to number, namely, that the number 5. hath to the number 3 : and therefore the proportion of the one to the other, is a rationall, certaine, and known proportion. And generally when soeuer one number is compared to an other, or two lines or other magnitudes, both which may be expressed by number, the proportion betwene them is euer rationall, and onely the proportion of such quantities is rationall. So that in Arithmetike all proportions are rationall, for that therein euer one number is compared to an other.

There are certaine lines magnitudes or quantities which cā not be named and expressed by number, and therefore commonly are called *Surd* lines or magnitudes. As suppose the square $ABCD$ to containe 16, then the side or roote thereof, namely, the line AB containeth 4, and the diameter of the same square, namely, the line BC shall be $\sqrt{32}$, which is a surd number, and can not be expressed by any determinate and certaine number, but onely by this maner of circumlocution Roote square of 32. Now if ye compare the line AB to the line BC , or contrariwise the line BC to the line AB , for that one of them is a surde quantitie, neither can ech of them be expressed by number (and therefore can not haue that proportion



Example thereof in numbers.



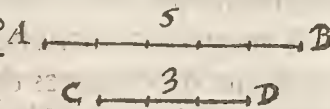
In proportions two quantities required, which are called termes.

The i. terme called the antecedent.
The ii. terme called the consequent.

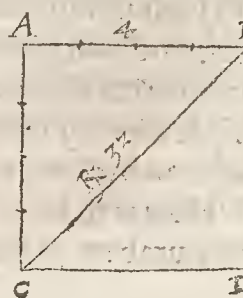
Dux.

Comes.

This booke counted the hardest of all *Euclides* bookes.



Proportion of two kindes, rational and irrational.
Proportion rational.



In Arithmetique all proportions are rational.
Proportion irrational.

The fifth Booke

proportion that number hath to number) the proportion betwene them is irrational, confused, vnknownen, vncertaine, and furd. And this kinde of proportion is found onely in magnitudes, as in lines and figures (and not in numbers) of which he of purpose entreateth in his tenth booke. Wherefore I wil here omit to speake of it, and remit it to his due place. And somewhat will I now say for the elucidation of the first kinde.

Rational proportion deuided into two kindes. Proportion of equality.

Proportion of inequality.

Proportion of the greater to the lesse.

Multiplex.

Duple proportion.

Triple, quadruple, Quintuple.

Superperticular.

Sesquialtera.

Sesquitercia.

Sesquiquarta.

Superpartiens.

Superbipartiens.

Supertripartiens.

Superquadrupartiens.

Superquintupartiens.

Multiplex superperticular.

Dupla Sesquialtera.

Dupla sesquitercia.

Tripla sesquialtera.

Multiplex superpartient.

Dupla superbipartiens.

Dupla supertripartiens.

Tripla superbipartiens.

Tripla superquadrupartiens.

Proportion rationall is deuided into two kindes, into proportion of equalitie, and into proportion of inequality. Proportion of equalitie is, when one quantitie is referred to an other equall vnto it selfe: as if ye compare 5 to 5, or 7 to 7, & so of other. And this proportion hath great vse in the rule of *Cosse*. For in it all the rules of equations tende to none other ende but to finde out and bring forth a number equall to the number supposed, which is to put the proportion of equalitie.

Proportion of inequality is, when one vnequall quantity is compared to an other, as the greater to the lesse, as 8. to 4: or 9. to 3: or the lesse to the greater as 4. to 8: or 3. to 9.

Proportion of the greater to the lesse hath fiue kindes, namely, *Multiplex*, *Superparticular*, *Superpartiens*, *Multiplex superperticular*, and *Multiplex superpartiens*.

Multiplex, is when the antecedent containeth in it selfe the consequent certayne times without more or lesse: as twice, thrice, foure tymes, and so farther. And this proportion hath vnder it infinite kindes. For if the antecedent contayne the consequent iustly twise, it is called duple proportion, as 4 to 2. If thrice tripla, as 9. to 3. If 4. tymes quadrupla as 12. to 3. If 5. tymes quintupla as 15. to 3. And so infinitely after the same maner.

Superperticular is, whē the antecedēt containeth the consequent only once, & moreouer some one part therof as an halfe, a third, or fourth, &c. This kinde also hath vnder it infinite kindes. For if the antecedent contayne the consequent once and an halfe, therof it is called *Sesquialtera*, as 6. to 4: if once and a third part *Sesquitercia*, as 4. to 3: if once and a fourth part *Sesquiquarta*, as 5. to 4. And so in like maner infinitely.

Superpartiens is, whē the antecedent cōtaineth the consequent onely once, & moreouer more partes then one of the same, as two thirdes, three fourthes, foure fifthes and so forth. This also hath infinite kindes vnder it. For if the antecedent contayne aboue the consequent two partes, it is called *Superbipartiens*, as 7. to 5. If 3. partes *Supertripartiens* as 7. to 4. If 4. partes *Superquadrupartiens*, as 9. to 5. If 5. partes *Superquintupartiens* as 11. to 6. And so forth infinitely.

Multiplex Superperticular is when the antecedent containeth the consequent more then once, and moreouer onely one parte of the same. This kinde likewise hath infinite kindes vnder it. For if the antecedent contayne the consequent twise and halfe therof, it is called duple *Sesquialtera*, as 5. to 2. If twise and a third *Dupla Sesquitercia* as 7. to 3. If thrice and an halfe *Tripla sesquialtera* as 7. to 2. If foure times and an halfe *Quadrupla Sesquialtera*, as 9. to 2. And so goyng on infinitely.

Multiplex Superpartient, is when the antecedent contayneth the consequent more then once, and also more partes then one of the consequent. And this kinde also hath infinite kindes vnder it. For if the antecedent contayne the consequent twise, and two partes ouer, it is called duple *Superbipartiens* as 8. to 3. If twice and three partes, duple *Supertripartiens* as 11. to 4. If thrice and two partes, it is named *Tripla Superbipartiens* as 11. to 3. If three tymes and foure partes, *Treble Superquadrupartiens* as 31. to 9. And so forth infinitely.

Here is to be noted that the denomination of the proportion betwene any two numbers, is had by deuiding of the greater by the lesse. For the quotient or number

number produced of that diuision is euer the denomination of the proportion. Which in the first kinde of proportion, namely, multiplex, is euer a whole number, and in all other kindes of proportion it is a broken number.

How to know
the denomi-
nation of any
proportion.

As if ye will know the denomination of the proportion betwene 9 and 3. Deuide 9. by 3. so shall ye haue in the quotient 3. which is a whole number, and is the denomination of the proportion : and sheweth that the proportion betwene 9. & 3. is Tripla. So the proportion betwene 12. and 3. is quadrupla, for that 12. beyng deuided by 3. the quotient is 4. and so of others in the kinde of multiplex. And although in this kinde the quotient be euer a whole number, yet properly it is referred to vnitie, and so is represented in maner of a broken number as $\frac{1}{1}$ and $\frac{4}{1}$ for vnitie is the denominaton to a whole number.

Likewise the denomination of the proportion betwene 4 and 3 is $1\frac{1}{3}$ for that 4 deuided by 3. hath in the quotient $1\frac{1}{3}$ one and a third part, of which third part, it is called *sesquitercia*: so the proportion betwene 7 and 6. is $1\frac{1}{6}$ one and a sixt, of which sixt part it is called *sesquisepta*, and so of other of that kinde. Also betwene 7 and 5 the denomination of the proportion is $1\frac{2}{5}$ one and two fiftes, which denomination consisteth of two parts, namely, of the munerator and denominator of the quotient of 2. and 5: of which two fiftes it is called *superbipartiens quintas*: for 2 the numerator sheweth the denomination of the number of the partes, and 5. the denominator, sheweth the denominatiō, what parts they are, & so of others. Also the denomination betwene 5 and 2. is $2\frac{1}{2}$ two and a halfe, which consisteth of a whole number and a broken, of 2. the whole number it is dupla, and of the halfe, it is called *sesquialtera*, so is the proportion *dupla sesquialtera*.

Agayne the denomination of the proportion betwene 11. and 3. is $3\frac{2}{3}$ three and two thirdes, consisting also of a whole number and a broken, of 3. the whole numbre it is called tripla, and of $\frac{2}{3}$ the broken number, it is called *Superbipartiens tertias*, so the proportion is *tripla superbipartiens tertias*. Thus much hetherto touching proportion of the greater quantitie to the lesse.

Proportion of the lesse quantitie to the greater hath as many kindes, as that of the greater to the lesse, which kindes are in the same order : and haue also the selfe same names, but that to the names afore put ye must adde here this word *sub*. As comparing the greater to the lesse, it was called *multiplex*, *superparticular*, *superpartient*, *multiplex superparticular*, and *multiplex superpartient*, now comparing the lesse quantitie to the greater, it is called *submultiplex*, *subsuperparticular*, *subsuperpartient*, *submultiplex superparticular*, and *submultiplex superpartient*. And so in like maner to all the inferior kindes of all sortes of proportion ye shall adde that worde *sub*. The examples of the former serue also here, onely transposing the termes of the proportion making the antecedent consequent, and the consequent the antecedent. As 4. to 2. is dupla proportion : so 2. to 4. is subdupla. As 9. to 3. is tripla: so is 3. to 9. subtripla. And as 9. to 6. is sesquialtera, so 6. to 9. is subsesquialtera. As 7. to 5. is superbipartiens quintas, so is 5. to 7. subsuperbipartiens quintas. As 5. to 2. is dupla sesquialtera, so is 2. to 5. subdupla sesquialtera. And also as 8. to 3. is dupla superbipartiens tertias, so is 3. to 8. subdupla superbipartiens tertias. And so may ye procede infinitely in all others. Thus much thought I good in this place for the ease of the beginner to be added touching proportion.

Proportion of
the lesse in the
greater.

Submulti-
plex.
Subsuperpar-
ticular.
Subsuperper-
tient. &c.

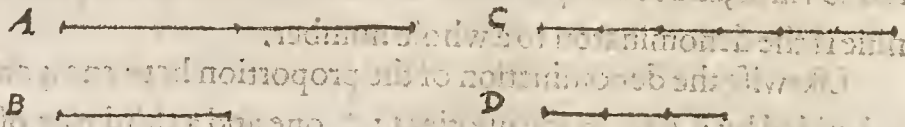
Pro

The fourth
definition.

Proportionallitie, is a similitude of proportions.

Example of
this definition
in magni-
tudes.

As in proportion are compared together two quantities, and proportion is nothing els but the respect and comparison of the one to the other, and these quantities are the termes of the proportion: so in proportionallitie are compared together two proportions. And proportionallitie is nothing els, but the respect & comparison of the one of them to the other. And these two proportions are the termes of this proportionallitie. He calleth it the *similitude*, that is, the likenes or idemptitie of proportions: As, if ye will cōpare the proportion of the line A contaynyng 2. to the line B contayning 1, to the proportion of the line C contayning 6. to the line D contayning 3, either proportion is dupla. This likenes, idemptitie, or equallitie of proportion is called proportionallitie. So in number 9. to 3. and 21. to 7. either proportion is tripla. Where note that proportions compared together, are sayd to be like the one to the other: but magnitudes compared together, are said to be equall the one to the other.



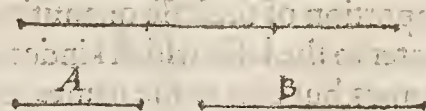
Example
thereof in
numbers.
Note.

The fifth defi-
nition.

Those magnitudes are sayd to haue proportion the one to the other, which being multiplied may exceede the one the other.

An example
of this defini-
tion in mag-
nitudes.

Before he shewed and defined, what proportion was, now by this definition he declareth betwene what magnitudes proportion falleth, saying: That those quantities are said to haue proportion the one to the other, which being multiplyed, may exceede the one the other. As for that the line A being multiplied by what soeuer multiplication or nūber, as taken twise, thrise, or foure, fise, or more times, or once and halfe, or once and a third, & so of any other part, or partes, may exceede and become greater then the line B: or contrariwise, then these two lines are said to haue proportion the one to the other. And so ye may see that betwene any two quāties of one kinde, there is a proportion. For the one remayning vnmultiplied, & the other being certaine times multiplied, shall be greater then it. As 3. to 24. hath a proportion, for leauing 24. vnmultiplied, and multiplying 3. by 9, ye shall produce 27: which is greater then 24, and exceedeth it. Here is to be noted, that *Euclide* in defining what quantities haue proportion, was compelled to vse multiplication, or els should not his definition be generall to either kinde of proportion: namely, to ratiōall and irratiōall: to such proportion I say which may be expressed by number, and to such as cannot be expressed by any determinate number, but remaineth surd and innominable. In ratiōall quantities which haue one common measure, the excesse of the one aboue the other is knowen, and by it is knowen the proportion, which may be expressed by some determinate number. But in irratiōall quantities which haue no cōmon measure, it is not so. For in them the excesse of the one to the other is euer vnknown,



Why *Euclide*
in defining of
Proportion
vsed multi-
plication.

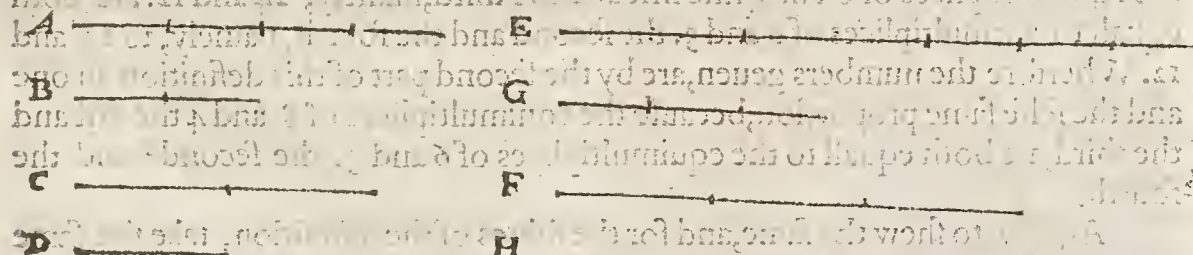
known, & therefore is surd, and innominable. As betwene the side of a square and the diameter thereof, there is vndoubtedly a proportion, for that the side certaine times multiplied may excede the diameter. Likewise betwene the diameter of a circle and the circumference thereof there is certaintie, by this definition, a proportion, for that the diameter certaine times multiplied may excede the circumference of the circle: although neither of these proportions can be named & expressed by number. For this cause therefore vsed Euclide this maner of defining by multiplication.

Magnitudes are sayd to be in one or the selfe same proportion, the first to the second, and the third to the fourth, when the equimultiples of the first and of the third beyng compared with the equimultiples of the second and of the fourth, according to any multiplication: either together excede the one the other, or together are equall the one to the other, or together are lesse the one then other.

The sixth definition.

In the definition last going before, he shewed what magnitudes haue proportion the one to the other, & now this diffinition sheweth what magnitudes are in one and the selfe same proportion, and how to know whether they be in one and the selfe same proportion, or not. It is plaine that euery proportio hath two termes, so that when ye compare proportion to proportion, ye must of necessitie haue 4. termes, that is, an antecedent and a consequent, to either of the proportiōs. As suppose A, B, C, D, to be foure magnitudes, A the first, B the second, C the third, and

An example of this definition in magnitudes.



D the fourth: now if ye take the equimultiples of A and C the first & the third, that is, if ye multiply A and C by one and the selfe same number, as let the multiplex of A be E, and let the equimultiplex of C be F. Likewise also if ye take the equimultiples of B and D, the second and the fourth, that is if ye multiply them by any one number, whether it be by that number wherby ye multiplied A & C, or by any other number greater or lesse, as let the multiplex of B be G, and the equimultiplex of D be H: now if the equimultiples of A and C be both greater then the eqnimultiples of B and D, that is if the multiplex of A be greater then the multiplex of B, and the multiplex of C be greater then the multiplex of D, or if they be both lesse then they: or both equall to them, then are the magnitudes A, B and C, D in one and the selfe same proportion.

An example in numbers.

Likewise in numbers 8. to 6. hath a proportion, also 4. to 3. hath a proportion: li. j. now

how to see whether they be in one and the selfe same proportion or not, set them in order as in the example here written, 8 the first, 6 the second, 4 the third, and 3 the fourth. Now take the equimultiples of 8 and 4. the first, and the third, that is, multiply them by one and the selfe same number, suppose it be by 3. so the triple of 8 is 24. & the triple of 4 is 12. likewise take the equimultiples of 6 and 3. the second and the fourth, multiplyeng them likewise by one and the selfe same number, suppose it be also by 3 as before ye did, the triple of 6 is 18. and the triple of 3 is 9. Now ye see that the triple of 8 the first, namely, 24. exceedeth the triple of 6, the second, namely, 18 : likewise the triple of 4 the third number, namely, 12. exceedeth the triple of 3. the fourth, namely, 9. Wherefore by the first part of this definition, the numbers 8 to 6. and 4 to 3. are in one and the selfe same proportio, because that the equimultiples of 8 and 4. the first & the third, do both exceede the equimultiples of 6 and 3. the second and the fourth.

First	8	Third	4
	<u>24</u>		<u>12</u>
Second	6	Fourth	3
	<u>18</u>		<u>9</u>

An other example in numbers.

Againe, take the same numbers and try the same after this maner. Take the equimultiples of 8. and 4. the first and the third, multiplieng eche by 3, as before ye did, so shall ye haue 24 for the triple of 8. and 12 for the triple of 4. as ye had before. Then take the equimultiples of 6 and 3. the second and the fourth, multiplyng them by some one number, but not by 3 as before ye did: but by 4. so for the quadruple of 6 the second number, shall ye haue 24. and for the quadruple of 3 the fourth number, ye shall haue 12. And now ye see that the equimultiples of 8 and 4. the first and the third, namely, 24 and 12. are both equall to the multiplies of 6 and 3. the second and the fourth, namely, to 24 and 12. Wherefore the numbers geuen, are by the second part of this definition in one and the selfe same proportion, because the equimultiples of 8 and 4 the first and the third, are both equall to the equimultiples of 6 and 3. the seconde and the fourth.

First	8	Third	4
	<u>24</u>		<u>12</u>
Second	6	Fourth	3
	<u>24</u>		<u>12</u>

An other example in numbers.

Agayne to shew the same, and for the fulnes of the diffinition, take the same numbers 8, 6, 4, 3. and take the equimultiples of 8 and 4. the first and the thirde, multiplieng eche by 2. so haue ye 16 for the duple of 8, the first number, and 8 for the duple of 4 the third number : then take also the equimultiples of 6 and 3, the second and the fourth, multiplieng eche by 3. so haue ye 18 for the triple of 6, the second, and 9 for the triple of 3. the fourth number. And now ye see that the equimultiples of 8 and 4. the first and the third, namely, 16. and 8 are both lesse then the equimultiples of 6 and 3 the second & the fourth namely 18 and 9. For 16 are lesse then 18, and 8 are lesse then 9. Wherefore by the third part of this diffinition, the numbers proposed are in one and the selfe same proportion, for that the equimultiples of 8 and 4 the first and the third are both lesse then the equimultiples of 4 and 3 the second and the fourth.

First	8	Third	4
	<u>16</u>		<u>8</u>
Second	6	Fourth	3
	<u>18</u>		<u>9</u>

Further

Farther in this diffinition, this particle (according to any multiplication) is most diligently to be considered, which signifieth by any multiplication indifferently whatsoever. For whensoever the quantities be in one and the selfe same proportion, then by any multiplication whatsoever, the equimultiples of the first and the third, shall exceede the equimultiples of the second and the fourth, or shall be equall vnto them, or lesse then them. Yet it may so happen by some one multiplication, that the equimultiples of the first and the third, do exceede the equimultiples of the second and the fourth, and yet the quantities geuen shal not be in one and the selfe same proportion. As in this example here set, where the equimultiples of 6 and 5, the first and the thirde, namely, 18. and 15. doo both

Note this particle according to any multiplication.

exceede the equimultiples of 4 and 3. the second and the fourth, namely, 8 and 6. yet are not the numbers geuen in one and the selfe same proportion. For 6 hath not that proportion to 4. which 5. hath to 3. In this example 6 and 5 the first and the third were multiplied by 3: which made their equimultiples 18 and 15. which exceede the equimultiples of 4 and 3, the second and the fourth beyng multiplied by 2. namely, 8 and 6: but if ye shall multiply 6 and 5 the first and the thirde by 2. ye shall produce 12 and 10 for their equimultiples, and then if ye multiply 4 and 3. the second and the fourth by 3. so shall ye produce for their equimultiples 12 and 9. Now ye see that by this multiplication the equimultiples of the first and the thirde doo not

$$\left\{ \begin{array}{l} \text{First} \\ \text{Second} \end{array} \right\} \begin{array}{r} 18 \\ 6 \\ 4 \\ 8 \end{array} \left\{ \begin{array}{l} \text{Third} \\ \text{Fourth} \end{array} \right\} \begin{array}{r} 15 \\ 5 \\ 3 \\ 6 \end{array}$$

An example where the equimultiples of the first and third exceede the equimultiples of the second and fourth, and yet the quantities geuen are not in one and the selfe same proportion.

both exceede the equimultiples of the second and fourth: for 12 the multiplex of 6 doth not exceede 12 the multiplex of 4. and therefore the numbers or quantities are not in one and the selfe same proportion, for that it holdeth not in all multiplications whatsoever.

$$\left\{ \begin{array}{l} \text{First} \\ \text{Second} \end{array} \right\} \begin{array}{r} 12 \\ 6 \\ 4 \\ 12 \end{array} \left\{ \begin{array}{l} \text{Third} \\ \text{Fourth} \end{array} \right\} \begin{array}{r} 10 \\ 5 \\ 3 \\ 9 \end{array}$$

And because this diffinition requireth all maner of multiplicatiōs to bring forth the excesses, equalities, and wantes of the antecedents aboue, to, or vnder the consequents, to auoide the tediousnes and infinite labour therof, I haue set forth a rule much to be made of and esteemed, wherby ye may in any rationall proportion produce equimultiples of the first and the third equall to the equimultiples of the second and the fourth.

The rule is this, take two numbers whatsoever in that proportion in which your quantities are, & by the number which is antecedent multiply the consequents of your proportions, namely, the second and the fourth: and by the number which is the consequent multiply the antecedentes of your proportions, namely, the first and the third: then necessarily shalbe produced the equimultiples of the first and the third equall to the equimultiples of the second & the fourth. As by example, take 6 to 2. and 3 to 1. which are in one & the selfe same proportiō, & taking these two nūbers 9 & 3. which are in the same proportiō, now by 9 the antecedent multiply the consequēts 2 & 1. and so shal ye haue 18 & 9 for the equimultiples of the second & the fourth, then by 3 the consequent multiply the antedēts 6 & 3, so shal ye haue 18 & 9 for the equimultiples of the first & the third, which are equal to the former equimultiples of the secōd & fourth. Wherof it foloweth that if ye multiply 18 & 9 the equimultiples of the first and the third by any nūber greater thē 3. wherby they were now multiplied, they shal both euer exceede the equimultiples of the second & the fourth: & if ye multiply thē

A rule to produce equimultiples of the first and third equall to the equimultiples of the seconde and fourth.

Example thereof.

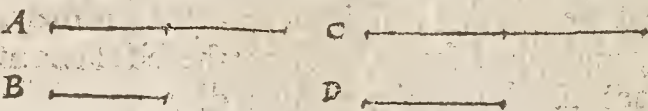
The fifth Booke

by any number less than 3. they shall euer both want of them. So that whatsoever multiplication it be, they shall euer both exceede, be equal, or want aboue, to, or from 18. and 9. the equimultiplices of the second and fourth.

The seventh
definition

*Magnitudes which are in one and the selfe same proportion,
are called Proportionall.*

As if the lyne A, haue the same proportion to the line B, that the lyne C hath to the lyne D, then are the said foure magnitudes A, B, C, D, called proportionall.



9 12
3 4

Also in numbers for that 9. to 3. hath that same proportiō that 12 hath to 4: therefore these foure nūbers 9. 3. 12. 4.

Proportionality of two
sortes, continuall and discontinuall.

An example
of continuall
proportionality in numbers.

16. 8. 4. 2. 1.

In continuall proportionality the quantities cannot be of one kinde.

Discontinuall proportionality.

Example of discontinuall proportionality in numbers. In discontinuall proportionality the proportions may be of diuers kindes.

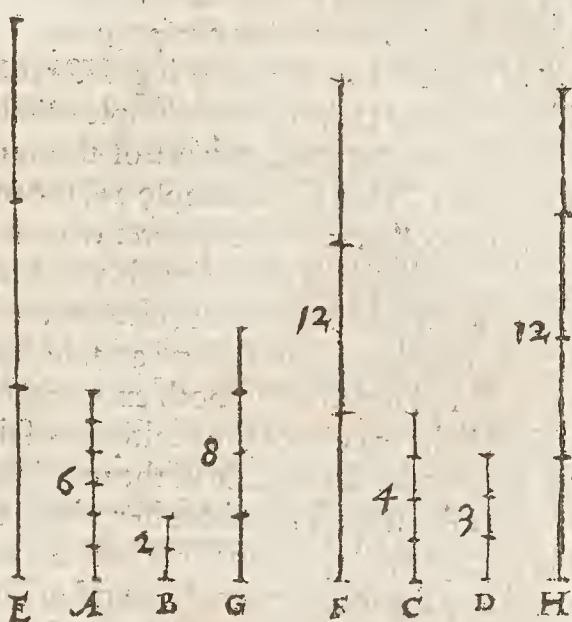
are said to be proportionall. Here is to be noted that this likenes or idemptitie of proportiō which is called, as before was said proportionality, is of two sortes: the one is continuall, the other is discontinuall. Continuall proportionality is, when the magnitudes set in lyke proportion, are so ioyned together, that the second which is consequent to the first, is antecedent to the third, and the fourth which is consequent to the third, is antecedent to the fifth, and so continually forth. So euerie quantitie or terme in this proportionality, is both antecedent and consequent (consequent in respect of that which went before, & antecedent in respect of that which followeth) except the first, which is onely antecedent to that which followeth, and the last which is onely consequent to that which went before. Take an example in these numbers, 16. 8. 4. 2. 1. In what proportion 16. is to 8, in the same is 8. to 4, in the same also is 4. to 2, and likewise 2. to 1. For they all are in duple proportion: 16. the first is antecedent to 8, and 8. is consequent vnto it: and the selfe same 8. is antecedent to 4: which 4 beyng consequent to 8. is antecedent to 2, which 2 likewise is consequent to 4. and antecedent to 1: which because he is the last, is onely consequent, and antecedent to none, as 16. because it was the first, was antecedent onely, and consequent to none. Also in this proportionality all the magnitudes must of necessitie be of one kynde, by reason of the continuation of the proportions in this proportionality, because there is no proportion betwene quantities of diuers kyndes. Discontinuall proportionality is, when the magnitudes which are set in lyke proportion, are not continually set, as before they were, hauyng one terme referred both to that which went before, and to that which followeth, but haue their termes distinct and seuered asonder: as the first is antecedent to the second, so is the third antecedent to the fourth. Example in numbers, as 8 is to 4. so is 6. to 3. for either proportion is duple. Where ye see, how ech proportion hath hys owne antecedent and consequent distinct from the antecedent and consequent of the other, and no one number is antecedent and consequent in diuers respectes. And by reason of the discontinuance of the proportions in this proportionality, the quantities compared, may be of diuers kyndes, because the consequent in the first proportion is not the antecedent in the second proportion. So that ye may compare superficies to superficies, or body to body in the selfe same proportion that ye do lyne to lyne.

When

When the equemultiplices being taken, the multiplex of the first exceedeth the multiplex of the second, & the multiplex of the third, exceedeth not the multiplex of the fourth: then hath the first to the second a greater proportion, then hath the third to the fourth. The eighth definition.

In the sixth definition was declared what magnitudes are said to be in one and the same proportion: now he sheweth in this definition what magnitudes are said to be in a greater proportion. And here is supposed the same order of multiplication, that there in that definition was used: namely, that the first and the third be equally multiplied, that is, by one & the selfe same nūber: and also that the second and the fourth be equally multiplied by the same or some other number: and

then if the multiplex of the first, exceede the multiplex of the second: & the multiplex of the third, exceede not the multiplex of the fourth, the first hath a greater proportion to the second, then hath the third to the fourth. As suppose that there be foure quantities, A, B, C, D: of which let A be the first, B the second, C the third, & D the fourth. And let A the first cōtaine 6. and let B the second containe 2. & C the third 4. & D the fourth 3: Now take the equimultiplices of A and C, the first & the third, which let be E and F, so that how multiplex E is to A so multiplex let F be to C: namely, for example sake let either of them be triple: so haue you 18. for the multiplex of A, and 12. for the multiplex of C. Likewise take the equimultiplices of B & D, the second & the fourth, multiplying them also by one and the selfe same number, as by 4: so haue ye for the multiplex of B the second 8, namely, the line G, and for the multiplex of D the fourth 12, namely, the line H. Now because the line E multiplex to the first, namely, 18, exceedeth the line G multiplex to the second, namely, 8: And the line F multiplex to the third, namely, 12, exceedeth not the line H multiplex to the fourth, namely, 12 (for that they are equall) the proportiō of A to B the first to the second, is greater then the proportiō of C to D the third to the fourth. So likewise in nūbers: take 11. to 2. & 7. to 3. and multiply 11. & 7. (the first, and the third) by 2, so shall ye haue 22. for the multiplex of the first, and 14. for the multiplex of the third: and multiply 2. and 3. the second and the fourth by 6: so shall the multiplex of the second be 12. and the multiplex of the fourth be 18:



An example of this definition in magnitudes.

Likewise in nūbers: take 11. to 2. & 7. to 3. and multiply 11. & 7. (the first, and the third) by 2, so shall ye haue 22. for the multiplex of the first, and 14. for the multiplex of the third: and multiply 2. and 3. the second and the fourth by 6: so shall the multiplex of the second be 12. and the multiplex of the fourth be 18:

First	22	Third	14
	11		7
Second	2	Fourth	3
	12		18

An example in numbers.

liiij.

Now

The fifth Booke

Now ye see 22. the multiplex of the first, excedeth 12. the multiplex of the second. But 14. the multiplex of the third, excedeth not 18. the multiplex of the fourth. Wherefore the proportion of 11. to 2. the first to the second, is greater then the proportion of 7. to 3, the third to the fourth. And so of all other quantities and numbers, which are not in one and the selfe same proportion, ye may know when the first to the second hath a greater proportion then the third to the fourth.

¶ An other example.

This example haue I set to declare that although the proportion of the first to the second be greater then the proportion of the third to the fourth, yet the multiplex of the first excedeth not the multiplex of the second. Wherefore it is diligently to be noted, that it is sufficient to shew that the proportion

{	<i>First</i>	$\frac{16}{8}$	{	<i>Third</i>	$\frac{18}{9}$
	<i>Second</i>	$\frac{4}{20}$		<i>Fourth</i>	$\frac{9}{45}$

Note.

on of the first to the second is greater then the proportion of the third to the fourth, if the want or lacke of the multiplex of the first from the multiplex of the second, be lesse then the want or lacke of the multiplex of the third to the multiplex of the fourth. As in this example 16. the multiplex of 8. the first, wanteth of 20. the multiplex of 4. the second, foure: whereas 18. the multiplex of 9, the third, wanteth of 45, the multiplex of 9 the fourth, 27. And so of all others whereas (the proportions being diuers) the equimultiples of the first and the third are both lesse, then the equimultiples of the second and the fourth. Likewise if the equimultiples of the first and the third do both excede the equimultiples of the second & the first, then shall the excesse of the multiplex of the first aboue the multiplex of the second, be greater then the excesse of the multiplex of the third, aboue the multiplex of the fourth. As in these numbers here set, the equimultiples of 6. and 4. the first and the third, namely, 12. and 8. do both excede the equimultiples of 2. and 3. the second and the fourth, namely, 4. and 6. But 12. the multiplex of the first excedeth 4. the multiplex of the second by 4. and 8. the multiplex of the third excedeth 6. the multiplex of the fourth by 2. but 8. is more then 2. Howbeit this is generally certaine that when soeuer the proportion of the first to the second is greater then the proportion of the third to the fourth, there may be found some multiplication, that when the equimultiples of the first and the third shall be compared to the equimultiples of the second and the fourth, the multiplex of the first shall excede the multiplex of the second, & the multiplex of the third shall not excede the multiplex of the fourth, according to the plaine wordes of the definition.

{	<i>First</i>	$\frac{12}{6}$	{	<i>Third</i>	$\frac{8}{4}$
	<i>Second</i>	$\frac{2}{4}$		<i>Fourth</i>	$\frac{3}{6}$

In like maner when you haue taken the equimultiples of the first & the third, and also the equimultiples of the second and the fourth, if the multiplex of the first excede not the multiplex of the second, and the multiplex of the third excede the multiplex of the fourth: then hath the first to the second a lesse proportion, then hath the third to the fourth. As in the example before, if ye chaunge the termes, and make C the first, D the second, A the third, and B the fourth: then shall F, namely,

E, namely, 12. the multiplex of the first not excede H, namely, 12. the multiplex of the second: but E, namely, 18. the multiplex of the third excedeth G, namely, 8. the multiplex of the fourth. Wherefore the proportion of C to D, the first to the second, is lesse then the proportion of A to B, the third to the fourth.

Euen so in numbers. As in this example, 5. to 4. and 7. to 3. If ye multiply 5. and 7. the first and the third eche by 3, ye shall for the multiplex of 5. the first haue 15. and for the multiplex of 7. the third shall ye haue 21: againe if ye multiply 4. and 3. the second & the fourth by 6, for the multiplex of 4. the second ye shall haue 24, and for the multiplex of 3. the fourth, ye shall haue 18. So ye see that 15. the multiplex of the first, is lesse then 24, the multiplex of the second: and 21. the multiplex of the third is greater then 18. the multiplex of the fourth. Wherefore the proportion of 5. to 4. the first to the second is lesse then the proportion of 7. to 3. the third to the fourth.

First	15	Third	21
	5		7
Second	4	Fourth	3
	24		18

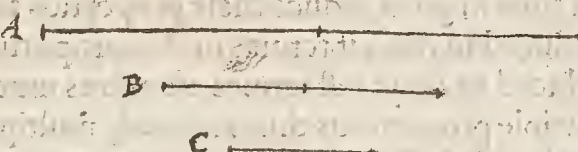
Proportionallitie consisteth at the left in three termes.

The ninth definition.

Before it was sayd, that proportionallitie is a likenesse or an idemptitie of proportions. Wherefore of necessitie in proportionallitie, there must be two proportions, and euery proportion hath two termes, namely, his antecedent and consequent. Therefore in euery proportionallitie there are foure termes. But for that somtyme, one terme supplieth by diuers relations, the roume of two, for in respect to the first it is consequent, and in respect to that which followeth, it is antecedent: therefore three termes at least and not vnder may suffice in proportionallitie, which three are in power foure, and occupy the roume of foure, as is sayd. As suppose that A hath to B that proportion, that B hath to C: then are these three quantities A, B, C, set in the left number of proportionality. Likewise in numbers, as 8. 4. 2. and 9. 6. 4.

An example of this definition in magnitudes.

Example in numbers.



When there are three magnitudes in proportion, the first shall be vnto the third in double proportion that it is to the second. But when there are foure magnitudes in proportion the first shall be vnto the fourth in treble proportion that it is to the second. And so alwaies in order one more, as the proportion shall be extended.

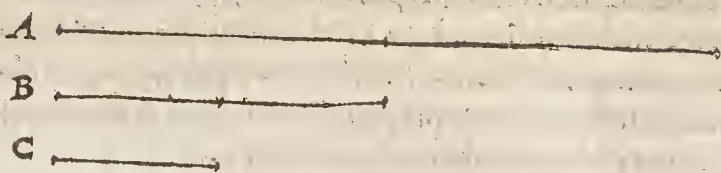
The tenth definition.

This definition is also vnderstand in continuall proportionallitie. As if the three magnitudes A, B, C, bee proportionall: then shall the proportion of A the first

Example of this definition in magnitudes.

The fifth Booke

first to C the thirde, bee double to the proportion which is betwene A & B the first and the seconde, that is the proportiō of A to B taken twise, or added to itself (which is all one)



shall make the proportiō of A to C. For the easier vnderstādyng of this & the practise therof, it shall be much necessary somewhat to instruct the rude beginner how proportions may be added one to an other. Which is done by this rule.

A rule to
adde proporti-
ons to propor-
tions.

Multiply the antecedent of the one proportion by the antecedēt of the other, and the number produced shall be the antecedent of the proportion which con-
tayneth them both. Likewyse multiply the consequent of the one proportion by
the consequent of the other, and the number produced shall be consequent of the
proportion which shall contayne them both.

Example
therof.

An example. If ye will adde the proportion which is betwene 4 and 2. (which is dupla) to the proportion which is betwene 9 and 3. (which is tripla) multiply 9. the antecedent of the first proportion by 4. the antecedent of the second proportion, and ye shall produce 36. which reserue and kepe for the antecedent of the proportion which ye seeke for. Likewise multiply 3 the consequent of the first proportion, by 2 the consequent of the second, so shall ye haue 6. which 6. shall be consequent to the former antecedēt, namely, to 36: so shal the proportiō which is betwene 36 and 6. namely, sextupla, contayne in it the two proportions geuen, namely, tripla, and dupla. And by this meanes are they added together, & brought into one. And by this may ye adde all other kyndes of proportions whatsoever they be. Now for that the diffinition sayth, that if there be three quantities in proportiō, that is, what proportiō the first hath to the secōd, the same hath the second to the third, which for example let be triple, as in these nūbers, 27. 9. 3. adde triple to triple by the rule abouesaid. And forasmuch as it is easier to worke in small nūbers then in great, reduce these proportiōs to theyr least denomination: So 27. to 9. reduced to the left termes in that proportion, is as much as 3. to 1. Likewise 9 to 3 reduced to theyr left termes are also as much as 3 to 1. now adde together these two triple proportions thus reduced, multipling 3 by 3. the one antecedent by the other, so shall ye produce 9 for a new antecedent, then multiply 1 by 1. the one consequent by the other, so shall you produce 1. which let be consequent to 9. your antecedent, so the proportion betwene 9 and 1. which is noncuple contayneth both the two triple proportions. And because they were equal the one to the other, it is. duple to eche of them. Ye see also that the proportion of 27 to 3. the first to the third, is also noncuple. Wherefore according to the definition, the proportion of the first to the third, is double to the proportion of the first to the second, as 9 to 1. beyng noncuple, is double 3 to 1. which is triple, because it contayneth it twise.

27. 9. 3.

9
3 — 3
1 — 1

8. 4. 2. 1.

2 2 2
1 1 1

So if there be 4. quantities in continuall proportion, the proportion of the first to the fourth, shall be triple to the proportion which is betwene the first and the second, that is, it shall contayne it three tymes. As for example, Take 4. numbers in continual proportion 8. 4. 2. 1. Ye see that the proportiō of 8 to 1. the first to the fourth, is octupla: the proportion of 8 to 4. the first to the second is dupla, now treble dupla proportion, that is, adde 3. dupla proportions together, by the rule before geuen, as ye see in the example. Multiply all the antecedentes together 2. the

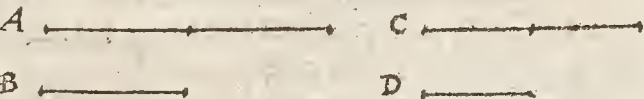
the antecedent of the first proportion, by 2. the antecedent of the second, so haue ye 4: which 4. multiply by 2 the antecedent of the third proportiō, so shal ye haue 8 for a new antecedent. In lyke maner multiply all the consequentes together, 1. the consequent of the first proportion by 1. the consequent of the second proportiō, so shal ye haue 1, which 1. multiply agayne by 1. the cōsequent of the third proportion, so shall ye haue agayne 1: which 1. let be consequent to your former antecedent 8: so haue ye 8 to 1. which is octupla, which was also the proportion of the first to the fourth, which octupla is also brought fourth of the addition of thre dupla proportions together, and contayneth it three tymes, wherefore octupla is tripla to dupla, and therefore as the diffinition sayth: the proportion of the first to the fourth is tripla to the proportion of the first to the second. And so consequently forth as long as the proportionalitie continueth accordyng to the sentence of the diffinition, the termes of the proportions exceding the number of proportions by one. As if ye haue 5. termes in proportion, the proportiō of the first to the fifth shal be quadrupla to the proportion of the first to the second, and if there be 6. termes, it shall be quintupla and so in order.

Magnitudes of like proportion, are sayd to be antecedents to antecedentes, and consequentes to consequentes.

The eleuenth definition.

For that before it was sayd, that proportion was a relation or a respect of one quantitie to an other, now sheweth he what magnitudes are sayd to be of like proportion, namely, these whose antecedents haue like respect to their consequentes, and whose consequents receyue like respectes of their antecedēts.

As putting 4. magnitudes A, B, C



D. If A antecedent to B, be dou-

ble to B, and C antecedent to D,

be double also to D, the haue the

two antecedentes like respectes to their consequents. Likewise if B the consequent be halfe of A, and also D the consequent be halfe of C, then the two consequentes B and D receiue of their antecedentes like respectes and relations. And by this diffinition, are these magnitudes A, B, C, D, of like proportion.

Example of this definition in magnituds.

Also in numbers, 9. 3. 6. 2: because 9 the antecedent is triple to 3. his consequent, and the antecedent 6. is also triple to 2 his consequent: the two antecedēts 9 and 6 haue like respectes to their consequentes, and because that 3 the consequent is the subtriple or third part of 9. his antecedent, and likewise 2 the consequent is the subtriple or third part of 6. his antecedent, the two consequentes 3 and 2 receiue also lyke respectes of their antecedentes, and therefore are numbers of like proportion.

9. 6.
3. 2.

Example in numbers.

Proportion alternate, or proportion by permutation is, when the antecedent is compared to the antecedent, and the consequent to the consequent.

The twelfth definition.

The vnderstanding of this definition & of all the definitions following, dependeth of the definition going before, and vse it for a generall supposition, namely, to haue

The fifth Booke

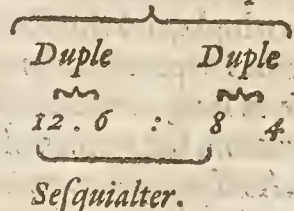
Example of
this definition
in magnitudes.

haue foure quantities in proportion. Suppose foure magnitudes A, B, C, D, to be in proportion, namely, as A is to B, so let C be to D. Now if ye compare A the antecedent of the first proportion to C the antecedent of the second as to his consequent, & likewise if ye compare

B the cōsequent of the first proportion as an antecedent to D the consequent of the second as

Example in
numbers.

to his consequent : then shall ye haue the magnitudes in this sort : as A to C, antecedent to antecedent, so B to D, consequent to consequent, & this is called permutation or alternate. In numbers as 12. to 6, so 8. to 4. either is dupla. Wherefore by permutation of proportion, as 12. to 8. antecedent to antecedent, so is 6. to 4. consequent to consequent, for either is sesquialtera.

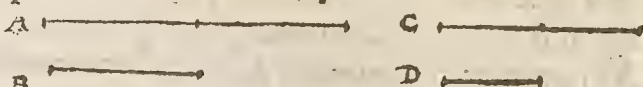


The thirteenth
definition.

Conuerse proportion, or proportion by conuersion is, when the consequent is taken as the antecedent, and so is compared to the antecedent as to the consequent.

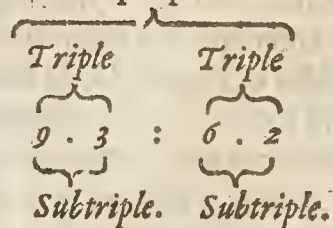
Example of
this definition
in magnitudes.

Suppose as before foure magnitudes in proportion, A, B, C, D, as A to B, so C to D : if ye referre B the consequent of the first proportion, as antecedent, to A the antecedent of the first, as to his consequent : and likewise if ye referre D the consequent of the second proportion as antecedent to C the antecedent of the second proportion, as to his cōsequent:



Example in
numbers.

the shall ye haue the magnitudes in thys order. As B to A cōsequent to antecedent: so D to C consequent to antecedent. And thys is called cōuerse proportion. So also in numbers, 9. to 3, as 6. to 2, eyther is tripla, wherefore comparing 3. to 9, the consequent of the first to hys antecedent 9, and also 2. the consequent of the second to hys antecedent 6, by conuerse proportion it commeth to passe as 3. to 9, so 2. to 6 : For either is subtripla.

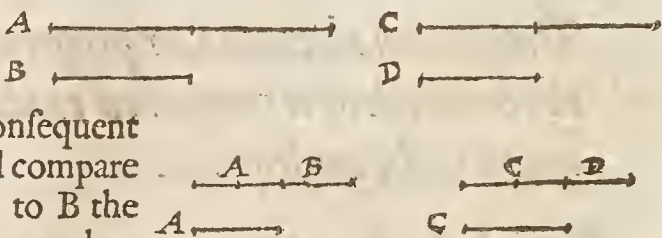


The fourteenth
definition.

Proportion composed, or composition of proportion is, when the antecedent and the consequent are both as one compared vnto the consequent.

Example of
this definition
in magnitudes.

Suppose that in the former foure magnitudes in proportion, A, B, C, D, as A is to B, so is C to D: if ye adde A and B the antecedent and the consequent of the first proportion together, and compare them so added as one antecedent to B the consequent of the first proportion as to hys consequent: and likewise if ye adde together



C and D,

C and D the antecedent and the consequent of the second proportion, and so added, compare them as one antecedent to D the consequent of the second proportion, as to his consequent: then shall ye haue the magnitudes in this order. As A B to B, so C D to D, for either of them is tripla. And this is called composed proportion, or composition of proportion. And so also in numbers. As 8. to 4, so 6. to 3: 8. and 4, the antecedent and consequent of the first proportion added together, make 12: which 12. as antecedent compare to 4. the consequent of the first proportion as to his consequent: so adde together 6. and 3, the antecedent and consequent of the second proportion, they make 9: which 9. as antecedent compare to 3. the consequent of the second proportion, as to his consequent: so shall ye haue by composition of proportion, as 12. to 4, so 9. to 3, for either of them is tripla.

Example in numbers.
8 . 4 :: 6 . 3
12 . 4 :: 9 . 3

*Proportion deuided, or diuision of proportio is, when the ex-
cesse wherein the antecedent exceedeth the consequent, is com-
pared to the consequent.*

The fiftene
definition.

This definition is the conuerse of the definition going next before: in it was vsed composition, and in this is vsed diuision. As before so now suppose foure magnitudes in proportion A B the first, B the second, C D the third, and D the fourth: as A B to B: so C D to D: A B, the antecedent of the first proportion exceedeth B the consequent of the first proportion by the magnitude A, wherefore A is the excesse of the antecedent A B aboue the consequent B: so likewise C D the antecedent of the second proportion, exceedeth D the consequent of the same proportion, by the quantitie C, wherefore C is the excesse of the antecedent C D aboue the consequent D. Now if ye compare A the excesse of A B the first antecedent, aboue the consequent B, as antecedent to B the consequent, as to his consequent: also if ye compare D the excesse of the second antecedent C D, aboue the consequent D, as antecedent to D the consequent, as to his consequent: then shall your magnitudes be in this order. As A to B, so is C to D: which is called diuision of proportion, or proportion deuided.

This is the co-
uerse of the
former defini-
tion.

Example in
magnitudes.

And so in numbers, as 9. to 6, so 12. to 8, either proportion is sesquialtera: the excesse of 9. the antecedent of the first proportion aboue 6. the consequent of the same is 3: the excesse of 12. the antecedent of the second proportion aboue 8, the consequent of the same, is 4: then if ye compare 3. the excesse of 9. the first antecedent aboue the consequent, as antecedent to 6. the consequent, as to hys consequent: and also if ye compare 4. the excesse of 12. the second antecedent aboue the consequent, as antecedent, to 8. the consequent, as to hys consequent, ye shall haue your numbers after this maner by diuision of proportion, as 3. to 6: so 4. to 8: for either proportion is subdupla.

Example in
numbers.

$$9 . 6 :: 12 . 8$$

$$3 . 6 :: 4 . 8$$

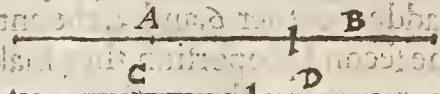
*Conuersio of proportion (which of the elders is commonly cal-
led euerse proportion, or euerfio of proportion) is, whe the an-
tecedent*

The sixtene
definition.

Antecedent is compared to the excesse, wherein the antecedent exceedeth the consequent.

An example of this definition in magnitudes.

Four magnitudes supposed as before, A B the first, B the second, C D the third, and D the fourth. As A B to A, so C D to C: A B the antecedent of the first proportion exceedeth B the consequent of the same by the magnitude A, wherefore A is the excesse of the antecedent A B about the consequent B:



so also the magnitude C is the excesse of C D the antecedent of the second proportion about D the consequent of the same: now if ye referre A B the antecedent of the first proportion, as antecedent, to A the excesse thereof about the consequent B, as to his consequent: if ye compare also C D the antecedent of the second proportion as antecedent to C the excesse thereof about the consequent D, as to his consequent: then shall your magnitudes come to thys order. As A B to A, so C D to C, and thys is called conuersion of proportion, and of some euerfion of proportion. Likewyse in numbers, as 9. to 6, so 12. to 8. eyther proportion is sesquialtera:

An example in numbers.

the excesse of 9. the antecedent of the first proportion about 6. the consequent of the same is 3: the excesse of 12. the antecedent of the second proportion about 8. the consequent of the same, is 4: now compare the antecedent of the first proportion 9. as antecedent to 3. the excesse thereof about 6. the consequent, as to his consequent, likewise compare 12. the antecedent of the second proportion as antecedent to 4. the excesse thereof about 8. the consequent, as to his consequent: so shall your numbers be in thys order by conuersion of proportion: as 9. to 3: so 12. to 4: for either proportion is triple.

9 . 6 : 12 . 8
2 . 3 : 12 . 4

The sentence definition.

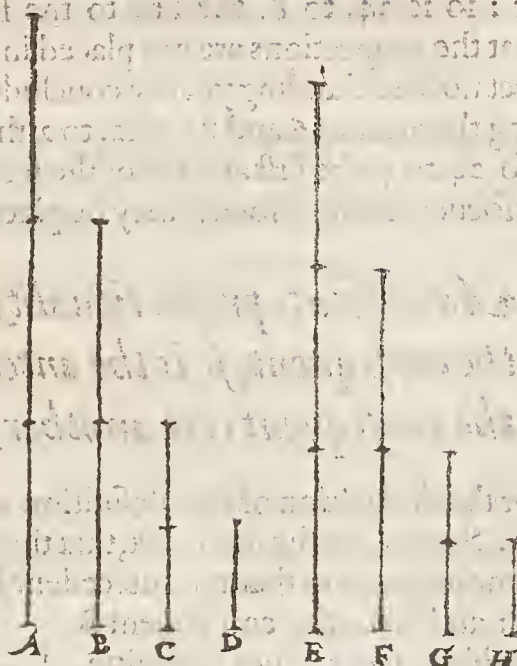
Proportion of equalitie is, when there are taken a number of magnitudes in one order, and also as many other magnitudes in an other order, comparing two to two beyng in the same proportion, it commeth to passe, that as in the first order of magnitudes, the first is to the last, so in the second order of magnitudes is the first to the last. Or otherwise it is a comparison of extremes together, the middle magnitudes being taken away.

An example of this definition in magnitudes.

To the declaration of thys definition are required two orders of magnitudes equall in number, and in lyke proportion: As if there be taken in some determinate number certayne magnitudes, namely, foure, A, B, C, D. And also in the same number be taken other quantities, namely, foure, E, F, G, H: then take the equall proportions by two and two: as A to B, so E to F: as B to C, so F to

so F to G : and as C to D, so G to H. Now according to the first definition, if A the first magnitude of the first order be to D the last magnitude of the same order, as E the first magnitude of the second order is to H the last magnitude the same, then it is called proportion of equalitie, or equall proportion.

By the second definition, which is all one in substance with the first, ye leaue the meane magnitudes in eyther order, namely, B, C, on the one side, and F, G on the other side, and onely compare the extremes of ech side together, which by thys definition shall be in lyke proportion, as A is to D, so is E to H.



Euen so in numbers, take for example these two orders, 27. 9. 12. 24. 15. and 9. 3. 4. 8. 5. there are in eche order as ye see, fve numbers, then see that all the proportions taken by two & two be like : betwene

27. & 9, numbers of the first order, and betwene 9. and 3, numbers of the second order, there is one and the self same proportion, namely, tripla : also betwene

27	9
9	3
12	4
24	8
15	5

9. and 12, numbers of the first

order, and 3. and 4, numbers of the second order, is like proportion, namely, subsesquitercia proportion : so betwene 12. and 24, numbers of the first order, and 4. and 8, numbers of the second order, is also lyke proportion, namely, subdupla :

Last of all, betwene 24. and 15, numbers of the first rowe, and betwene 8. and 5, numbers of the second rowe, the proportion is one, namely, supertripartiens quintas. Wherefore by this definition, leauing out all the meane numbers of eche side, ye may compare together onely the extremes, and conclude that as 27. of the first rowe is to 15. the last of the same rowe, so is 9. the first of the second rowe to 5, the last of the same rowe : for the proportion of ech is superquadrupartiens quintas.

Here is to be considered, that it is not of necessitie that all the proportions in eche rowe of numbers be set in like order, as in the one so in the other : but it shall be sufficient that the proportions be the same and in equall number in eche rowe. Whether it be in the selfe same order, or in contrary, or inuerterd order, it maketh no matter. As in these numbers, 12. 6. 2. in the first row, and 24. 8. 4. in the second. As 12. is to 6, the first to the second of the first row, so is 8. to 4. the second to the third of the second row : either is duple proportion. And as 6. to 2. the second to the third in the first

12 24
6 8
2 4
order;

An example
in numbers.

Note.

Kk.j.

The fifth Booke

order : so is 24. to 8. the first to the second in the second order : Where ye see that the proportions are not placed in one and the selfe same order, and course, and yet notwithstanding ye may conclude by equalitie of proportion, leaving the meanes 6. and 8: as 12. to 2. the first to the last of the first order, so 24. to 4. the first and last of the second order . And so of others what soeuer and how soeuer they be placed.

The eighteenth definition.

An ordinate proportionality is, when as the antecedent is to the consequent, so is the antecedent to the consequent, and as the consequent is to an other, so is the consequent to an other.

An example of this definition in magnitudes.

For the declaration of this definition are also required two orders of magnitudes . Suppose in the first order, that the antecedent A, to his consequent B, have the same proportion that the antecedent D, hath to his consequent E in the second order : and make the consequent B antecedent to some other quantitie, as to C . Also make the consequent E antecedent to an other quantitie, as to F, so that there be the same proportion of B to C, which is of E to F . And thys disposition of proportions is called ordinate proportionality . Likewise in numbers, 18.9.3. and 6.3.1. As 18. to 9. antecedent to consequent, so is 6. to 3. antecedent to consequent : either is dupla proportion : and as 9. the consequent is to an other, namely, to the number 3, so is the consequent 3. to an other, namely, to vnity. And this ordinate proportionality may be extended as farre as ye list, as ye may see in the example of numbers in the definition next before.

Example in numbers.

The nineteenth definition.

An inordinate proportionality is, when as the antecedent is to the consequent, so is the antecedent to the consequent : and as the consequent is to an other, so is an other to the antecedent.

An example of this definition in magnitudes.

This definition also as the other before, requireth two orders of magnitudes. Suppose in the first order that the antecedent A be to the consequent B, as the antecedent C, in the second order is to the consequent D, & let B the consequent of the first proportion be to some other, namely, to the magnitude E, as some other, namely, the magnitude F, is to the antecedent C of the second proportion: this kinde of proportionality is called inordinate or perturbate. Take also an example in numbers, as 9 to 6. the antecedent to the consequent, so is 3 to 2 the antecedent to the consequent, either proportion is sesquialtera, and as 6 the consequent of the first

Example in numbers.

proportion,

proportion, is to an other, namely, to the number 3. so is another namely, the number 6. to 3. the antecedent of the second proportion, for eyther is dupla proportion.

An extended proportionality is, when as the antecedent is to the consequent, so is the antecedent to the consequent, and as the consequent is to an other, so is the consequent to an other.

The 20.
definition.

A pertinbate proportionality is, when, thre magnitudes being compared to three other magnitudes, it cometh to passe, that as in the first magnitudes the antecedent is to the consequent, so in the second is the antecedent to the consequent, & as in the first magnitudes the consequent is to an other magnitude, so in the second magnitudes is an other magnitude to the antecedent.

The 21. definition.

These two last definitions here put by Zamberte seeme all one with the other two last before set. Wherefore it is not lyke that they were written and set here by Euclide, for that they seeme not necessary, but rather superfluous, neither are they found in the Greeke examples commonly set forth in print, nor mentioned of any that hath written commentaries vpon Euclide, olde or new: Not of Campane, Sceubelius, Pellitarius, Orontius, nor Flussates: wherefore it is not of necessitie to adde vnto them any explanation or example either in magnitudes or in numbers. The examples of the two last definitions set before, may likewise serue for them also.

These two last
definitions
not found in
the greeke ex-
amplers.

The 1. Theoreme.

The 1. Proposition.

If there be a number of magnitudes how many soeuer equimultiples to a like number of magnitudes ech to ech: how multiplex on magnitude is to one, so multiples are all the magnitudes to all.

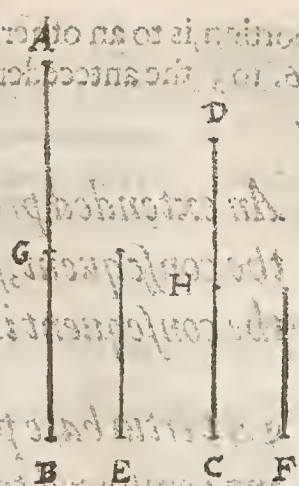


Vppose that there be a number of magnitudes, namely, AB and DC equimultiples to a like number of magnitudes E and F ech to ech. Then I say, that how multiplex AB is to E , so multiples are AB and DC to E and F . For forasmuch as how multiplex AB is to E , so multiplex is DC to F , therefore how many magnitudes there are in AB equall vnto E , so many are there in DC

$Kk.y.$ equall

Construction.

equall vnto F . Deuide AB into the magnitudes which are equall vnto E , that is, into AG and GB , and likewise DC into the magnitudes which are equall vnto F , that is, into DH and HC . Now then the multitude of these DH & HC , is equall vnto the multitude of these AG & GB . And forasmuch as AG is equall vnto E , and DH vnto F : therefore AG and DH , are equall vnto E and F : and by the same reason forasmuch as GB is equall vnto E , and HC vnto F , GB also and HC are equall vnto E and F . Wherefore how many magnitudes there are in AB equall vnto E , so many are there in AB and DC equall vnto E & F . Wherefore how multiplex AB is to E , so multiplies are AB and DC to E and F . If therefore there be a number of magnitudes how many soeuer equemultiplies to a like number of magnitudes ech to ech, how multiplex one magnitude is to one, so multiplies are all the magnitudes to all: which was required to be proued.



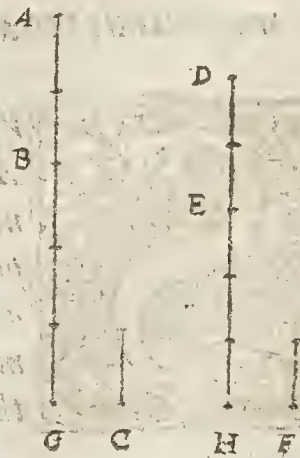
The 2. Theoreme. The 2. Proposition.

If the first be equemultiplex to the second as the third is to the fourth, and if the fifth also be equemultiplex to the second as the sixth is to the fourth: then shall the first and the fifth composed together be equemultiplex to the second, as the third and the sixth composed together is to the fourth.



Suppose that there be sixe quantities, of which let AB be the first, C the second, DE the third, F the fourth, BG the fifth, & EH the sixth: and suppose that the first, AB , be equemultiplex vnto the second, C , as the third, DE , is to the fourth, F : and let the fifth, BG , be equemultiplex vnto the second, C , as the sixth, EH , is to the fourth, F . Then I say, that the first and the fifth composed together, which let be AG , is equemultiplex vnto the second, C , as the third and sixth composed together, which let be DH , is to the fourth, F . For forasmuch as AB is equemultiplex to C , as DE is to F , therefore how many magnitudes there are in AB equall vnto C , so many magnitudes are there in DE equall vnto F : and by the same reason how many there are in BG equall vnto C , so many also are there in EH equall vnto F . Wherefore how many there are in the whole AG equall vnto C , so many are there in the whole DH equall vnto F . Wherefore how multiplex AG is vnto C , so multiplex is DH vnto F . Wherefore the first and the fifth composed together,

Demonstration.



together, namely, AG is equemultiplex vnto the second C , as the third and the sixte composed together, namely, DH , is to the fourth F . If therfore the first be equemultiplex to the second as the third is to the fourth, and if the fifth also be equemultiplex to the second as the sixt is to the fourth: then shall the first & the fifth cōposed together, be equemultiplex to the second, as the third and the sixt composed together, is to the fourth: which was required to be proued.

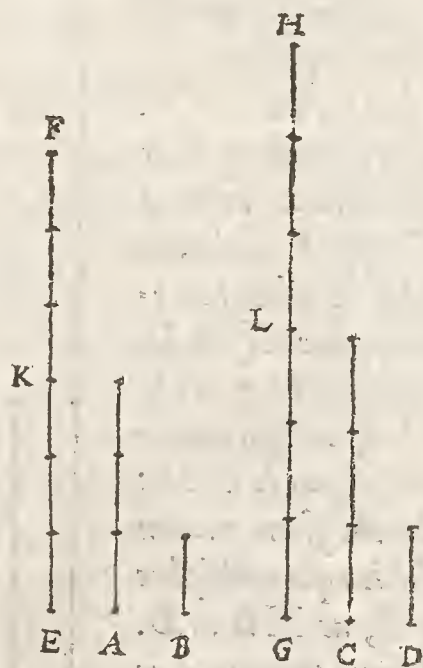
The 3. Theoreme.

The 3. Proposition.

If the first be equemultiplex to the second, as the third is to the fourth, and if there be taken equemultiplices to the first & to the third: they shall be equemultiplices to them which were first taken, the one to the second, the other to the fourth.

Suppose that there be foure magnitudes, of which let A be the first, B the second, C the third, and D the fourth. And let the first, A , be equemultiplex to the second, B , as y^e third, C , is to the fourth, D . And vnto A and C take equemultiplices, which let be EF and GH , so that how multiplex EF is to A , so multiplex let HG be to C . Then I say, that EF is equemultiplex vnto B , as GH is vnto D . For forasmuch as EF is equemultiplex vnto A , as GH is vnto C , therefore how many magnitudes there are in EF equall vnto A , so many magnitudes also are there in GH equall vnto C . Let EF be deuided into the magnitudes that are equall vnto A , that is, into EK and KF . And likewise GH into the magnitudes equall vnto C , that is, into GL and LH . Now then the multitude of these magnitudes EK and KF , is equall vnto y^e multitude of these magnitudes GL and LH . And forasmuch as A is equemultiplex to B , as C is to D : but EK is equall vnto A , and GL vnto C , therefore EK is equemultiplex vnto B , as GL is vnto D . And by the same reason KF is equemultiplex vnto B , as LH is to D . Now then

Construction.



Demonstration.

there are sixe magnitudes whereof EK is the first: B y^e second: GL y^e third: D y^e fourth: KF the fifth: LH the sixth. And forasmuch as the first EK is equemultiplex to the second B , as the third GL is to the fourth D : and the fift KF is equemultiplex to the second B , as the sixt LH is to the fourth D : therefore (by

Kk.ij. the

The fifth Booke

the second of the fifth) the first & the fifth composed together, namely, EF is equemultiplex vnto the second B , as the third and the sixth composed together, namely, GH is to the fourth D . If therefore the first be equemultiplex to the second, as the third is to the fourth, and if there be taken the equemultiples to the first and to the third, they shall be equemultiples to them which were first taken, the one to the second, the other to the fourth: which was required to be proued.

The 4. Theoreme. The 4. Proposition.

If the first be vnto the second in the same proportion that the third is to the fourth: then also the equemultiples of the first and of the third, vnto the equemultiples of the second and of the fourth, accordyng to any multiplication, shall haue the same proportion beyng compared together.

Suppose that there be foure magnitudes, of which let A be the first, B the second, C the third, and D the fourth. And let the first A be vnto the second B in the same proportion, that the third C is vnto the fourth D .

Construction. And to A and C take equemultiples E , and F , and likewise to B and D , any other equimultiples G & H . The I say, as E is to G , so is F to H . Vnto E & F take equemultiples K

Demonstration.

& L , & vnto G and H any other equemultiples, that is, M and N .

And forasmuch as E is equemultiplex vnto A as F is vnto C , and vnto E and F be taken the equemultiples K & L , therefore (by the 3. of the fifth) K is equemultiplex to A , as L is to C : and by the same reason also M is equemultiplex vnto B , as N is to D .

And seing that as A is to B , so is C to D , and of A and C are taken equemultiples K and L , and likewise of B & D

are taken other equemultiples, namely, M and N , therefore if K excede M , L also



also exceedeth N : and if it be equall, it is equall, and if it be lesse, it is lesse (by the conuerse of the 6. definition of the fifth). And K and L are equemultiples to E and F : and M and N are other equemultiples to G and H . Wherefore as E is to G , so is F to H by the said sixth definition. If therefore the first be vnto the second in the same proportion that the third is to the fourth: then also the equemultiples of the first and of the third, vnto the equemultiples of the second and of the fourth, according to any multiplication, shall haue the same proportion being compared together: which was required to be proued.

An Assumpt. Wherefore seing it hath bene proued that if K exceede M , L also exceedeth N , and if it be equall it is equall: and if it be lesse, it is lesse: it is manifest that if M exceede K , N also exceedeth L : and if it be equall it is equall: and if it be lesse it is lesse: and by this reason as G is to E , so is H to F .

A Lemma, or an assumpt.

A Corollary.

Hereby it is manifest that if there be foure magnitudes proportional, they shal also by conuerse be proportionall: that is, if the first be vnto the second, as the thide is to the fourth: then by conuerse as the seconde is to the first, so is the fourth to the third.

A Corollary. Conuerse proportion.

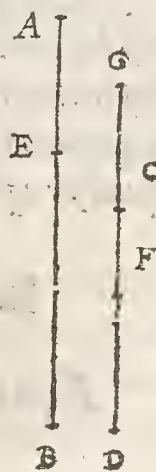
The 5. Theoreme. The 5. Proposition.

If a magnitude be equemultiplex to a magnitude, as a parte taken away of the one, is to a part taken away from the other: the residue also of the one, to the residue of the other, shal be equemultiplex, as the whole is to the whole.

Suppose that the whole magnitude AB be vnto the whole magnitude CD equemultiplex, as the part taken away of the one, namely, AE , is to the part taken away of the other, namely, CF . Then I say that the residue of the one, namely, EB , is to the residue of the other, namely, FD equemultiplex as the whole AB is to the whole CD . How multiplex AE is to CF , so multiplex make EB to CG . And forasmuch as (by the first of the fifth) AE is to CF equemultiplex, as AB is to GF : but AE is to CF equemultiplex, as AB is to CD . Wherefore AB is equemultiplex to either of these GF and CD . Wherefore GF is equall vnto CD . Take away CF which is common to them both. Wherefore that which remaineth namely, GC , is equall vnto that which remaineth namely, FD . And forasmuch as AE is to CF equemultiplex as EB is to GC , but GC is equall vnto FD , therefore AE is to CF equemultiplex as EB is to FD . But AE is put to be equemultiplex to CF , as AB is to CD , wherefore EB is to FD equemultiplex, as AB is to CD . Wherefore the residue EB is to the residue FD equemultiplex, as the whole AB is to the whole CD . If therefore a

Construction.

Demonstration.



Kk.iiij. magni-

The fifth Booke

magnitude be equemultiplex to a magnitude, as a part taken away of the one is to a parte taken away of the other: the residue of the one also to the residue of the other, shalbe equemultiplex as the whole is to the whole: which was required to be proued.

The 6. Theoreme. The 6. Proposition.

If two magnitudes be æquemultiplices to two magnitudes, & any partes taken away of them also, be æquemultiplices to the same magnitudes: the residues also of them shal vnto the same magnitudes be either equall, or equemultiplices.



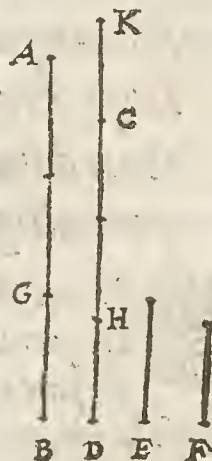
*V*ppose that there be two magnitudes AB and CD equemultiplices to two magnitudes E and F , and let the partes takē away of the magnitudes AB and CD , namely, AG and CH be equemultiplices to the same magnitudes E and F . Then I say that the residues GB and HD , are vnto the selfe same magnitudes E and F either equall, or els equemultiplices.

Two cases in
this Propo-
sition.

The first.

Construction.
Demonstra-
tion.

Suppose first that GB be equall vnto E . Then I say that HD is equall vnto F . Vnto F put an equall magnitude CK . And forasmuch as AG is equemultiplex vnto E , as CH is vnto F : but GB is equall vnto E , & KC vnto F : therefore AB is equemultiplex to E , as KH is to F . But AB is put equemultiplex vnto E , as CD is to F . Wherefore KH is equimultiplex vnto F , as CD is to F . And forasmuch as either of these KH and CD are equimultiplices vnto F , therefore (by the common sentence) KH is equall vnto CD . Take away CH which is common to them both. Wherefore the residue KC is equal vnto the residue HD . But KC is equal vnto F , wherefore HD is equall vnto F . Wherefore if GB be equal vnto E , HD also shall be equall vnto F .



The second.

And in like sort may we proue, if GB be multiplex to E , HD also shal be so multiplex vnto F . If therefore there be two magnitudes equemultiplices to two magnitudes, and any partes taken away of them be also equemultiplices to the same magnitudes: the residues also of them shall vnto the same magnitudes be either equall, or equemultiplices: which was required to be proued.

The 7. Theoreme. The 7. Proposition.

Equall magnitudes haue to one & the selfe same magnitude,
one

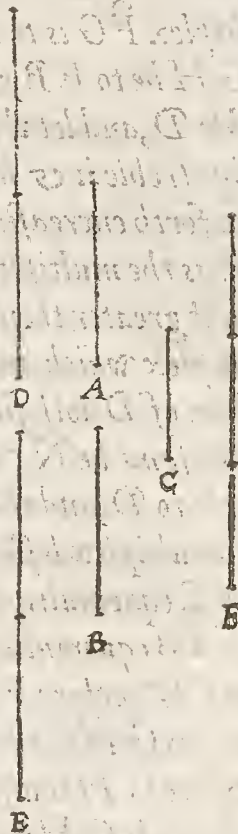
one and the same proportion. And one and the same magnitude hath to equall magnitudes one and the selfe same proportion.



Suppose that A and B be equall magnitudes, and take any other magnitude, namely, C . Then I say, that either of these A and B haue vnto C one and the same proportion: and that C also hath to either of these A and B one and the same proportion.

Take the equemultiples of A and B , and let the same be D and E : and likewise of C , take any other multiplex, and let the same be F . Now forasmuch as D is vnto A equemultiplex as E is to B , but A is equall vnto B , therefore (by the first common sentence) D is equall vnto E . And of C there is taken any other multiplex F . Wherefore if D exceede F , E also exceedeth F : and if it be equall it is equal, and if it be lesse it is lesse. But D and E are the equemultiples of A & B , and F is of C an other multiplex. Wherefore as A is to C , so is B to C .

I say moreouer, that C hath to either of these A and B one and the same proportion. For the same order of constructiō remaining, we may in like sort proue, y^t D is equal vnto E , & there is taken an other multiplex to C , namely, F . Wherefore if F exceede D , it also exceedeth E : and if it be equall it is equall: and if it be lesse it is lesse. But F is multiplex to C : and D & E are other equemultiples to A and B . Wherefore as C is to A , so is C to B . Wherefore equall magnitudes haue to one and the same magnitude, one and the same proportion: and one and the same magnitude hath to equall magnitudes one and the selfe same proportion: which was required to be demonstrated.



The first part of this Proposition demonstrated.

The second part demonstrated.

The 8. Theoreme. The 8. Proposition.

Unequall magnitudes being taken, the greater hath to one and the same magnitude a greater proportion then hath the lesse. And that one and the same magnitude hath to the lesse a greater proportion then it hath to the greater.



Suppose that AB and C be vnequall magnitudes, of which let AB be the greater, and C y^e lesse. And let there be an other magnitude whatsoever, namely, D . Then I say that AB hath vnto D a greater proportion then hath C to D : and also that D hath to C a greater proportion,

The fifth Booke

tion, then it hath to AB . For forasmuch as AB is greater then C , let there be taken a magnitude equall vnto C , namely, BE .

The first part
of this Propo-
sition demon-
strated.

Now then the lesse of these two magnitudes AE and EB being multiplied will at the length be greater then D .

The first case
of the same.

First let AE be lesse then EB :

and let AE be so oft multiplied vntill that which is produced be greater then D , and let that multiplex be FG which is greater then D . And how

multiplex FG is to AE , so multiplex let GH be to EB , & K to C . Then

double D , and let the same be L . And

again treble it & let the same be M :

& so forth encreasing by one till such

tyme as the multiplex of D so taken

be first greater then K , that is, that

multiplex which amongst al the mul-

tiplices of D doth first excede K , &

let the same be N , which here is qua-

druple to D , and the first multiplex greater then K . Now forasmuch as K is γ

first multiplex lesse then N , therefore K is not lesse then M . And for that FG is

to AE equemultiplex as GH is to EB , therefore (by the first of the fifth) FH

is to AB equemultiplex as FG is to AE : but FG is equemultiplex to AE as K

is to C . Wherefore FH and K are equemultiplices to AB and C . Again foras-

much as GH is equemultiplex to EB as K is to C , but EB is equall vnto C ,

therefore GH is also equall vnto K . But K is not lesse then M : wherefore ney-

ther also is GH lesse then M . But FG is greater then D . Wherefore the whole

FH is greater then both these D and M . But both these D and M are equall

vnto N , for M is triple to D , and M and D together are quadruple to D , and

N also is quadruple to D : wherefore both these M and D are equall vnto N ,

but FH is greater then M and D , wherefore FH exceedeth N , (that is, γ mul-

tiplex of the first, namely, of AB exceedeth the multiplex of the second, namely,

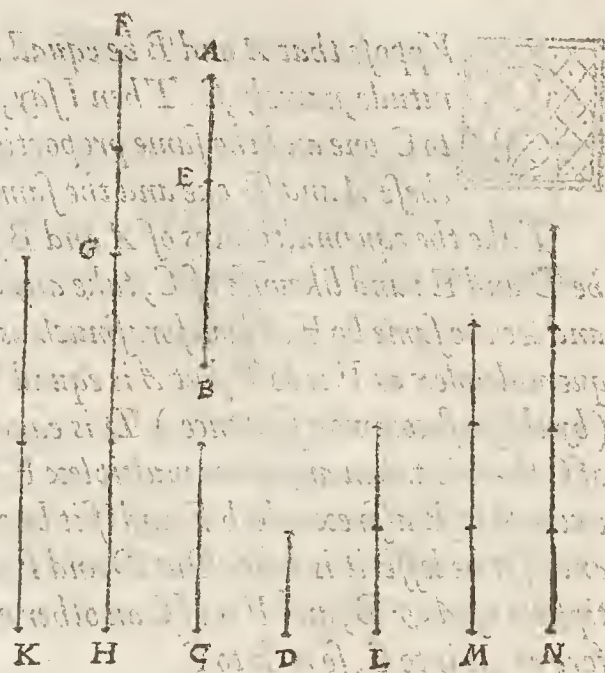
of D .) But K exceedeth not N (that is, the multiplex of the third, namely, of C ,

exceedeth not the multiplex of the fourth, namely, of the same D ;) and FH and

K are equemultiplices to AB and C , and N is a certayne other multiplex to

D . Wherefore (by the 8. definition) AB hath to D a greater proportion then

hath C to D .



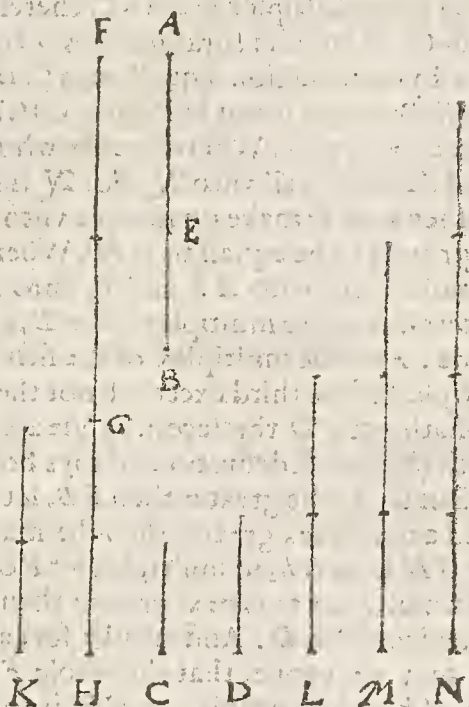
The second
part of the
proposition de-
monstrated.

I say moreouer that D hath to C a greater proportion then D hath to AB . For the same order of construction still remayning, we may in like sort proue that N is greater then K , and that it is not greater then FH . And N is multiplex to D , and FH and K are certayne other equemultiplices to AB and C . Wherefore D hath to C a greater proportion then D hath to AB .

But

But now suppose that AE be greater then EB . Now then EB beyng the lesse, and being oftentimes multiplied, will at the length be greater then D . Let it be so multiplied, and let the multiplex of EB , namely, GH , be greater then D . And how multiplex GH is to EB so multiplex let FG be to AE and K to C . And by the former reason may we proue, that FH and K are equemultiples to AB and C . Likewise also let N be multiplex of D , & also let it be y first multiplex which is greater then FG : wherefore agayn FG is not lesse then M . But GH is greater then D : wherefore the whole FH exceedeth D & M , that is, N : but K exceedeth not N . For FG which is greater then GH , that is, then K exceedeth not N . And so followyng the same order we did before, we shall performe the demonstration. Wherefore vnequall magnitudes beyng taken, the greater hath to one and the same magnitude a greater proportion then hath the lesse: and that one and the same magnitude hath to the lesse a greater proportion then it hath to the greater: which was required to be proued.

The second case of the first part.



¶ For that Orontius seemeth to demonstrate this more plainly therefore I thought it not amisse here to set it.

Suppose that there be two vnequall magnitudes, of which let AB be the greater, and C the lesse: and let there be a certaine other magnitude, namely, D . Then I say first, that AB hath to D a greater proportion then hath C to D . For forasmuch as by supposition AB is greater then the magnitude C : therefore the magnitude AB containeth the same magnitude C , and an other magnitude besides. Let EB be equall vnto C : and let AE be the part remayning of the same magnitude. Now AE and EB are eyther vnequall or equall the one to the other. First let them be vnequall: and let AE be lesse then EB . And vnto AE the lesse take any multiplex whatsoeuer, so that it be greater thē the magnitude D : and let the same be FG . And how multiplex FG is to AE , so multiplex let GH be to EB , and K to C . Agayne take the duple of D , which let be L , and then the triple, and let the same be M . And so forward, alwayes adding one: vntill there be produced such a multiplex to D which shall be next greater then GH (that is, which amongst the multiples of D , by the continuall addition of

First difference of the first part.

Demonstratio
of the same
first difference.

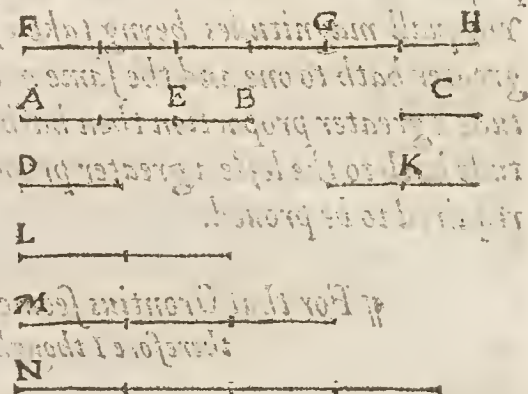
on of one, doth first beginne to exceede GH) and let the same be N , which let be quadruple to D . Now then the multiplex GH is the next multiplex lesse then N , and therefore is not lesse then M ; that is, is either equall vnto it or greater then it. And forasmuch as FG is equemultiplex to AE , as GH is to EB , therefore how multiplex FG is to AE , so multiplex is FH to AB (by the first of the fifth). But how multiplex FG is to AE , so multiplex is K to C , therefore how multiplex FH is to AB , so multiplex is K to C . Moreouer forasmuch as GH and K are equemultiples vnto EB and C : and EB is by construction equall vnto C , therefore (by the common sentence) GH is equall vnto K . But GH is not lesse then M , as hath before bene shewed, and FG was put to be greater then D . Wherefore the whole FH is greater then these two D and M . But D and M are equall vnto N . For N is quadruple to D . And M being triple to D , doth together with D make quadruple vnto D . Wherefore FH is greater then N . Farther, K is proued to be equall to GH . Wherefore K is lesse then N . But FH and K are equemultiples vnto AB and C , vnto the first magnitude, I say, and the third: and N is a certaine other multiplex vnto D , which representeth the second & the fourth magnitude. And the multiplex of the first exceedeth the multiplex of the second: but the multiplex of the third exceedeth not the multiplex of the fourth. Wherefore AB the first hath vnto D the second a greater proportion, then hath C the third to D the fourth (by the 8. definition of thys booke).

Second difference.

But if AE be greater then EB , let EB the lesse be multiplied vntill there be produced a multiplex greater then the magnitude D : which let be GH . And how multiplex GH is to EB , so multiplex let FG be to AE , and K also to C . Then take vnto D such a multiplex as is next greater then FG : and againe let the same be N , which let be quadruple to D . And in like sort as before may we proue, that the whole FH is vnto AB equemultiplex as GH is to EB : and also that FH & K are equemultiples vnto AB & C : and finally that GH is equall vnto K . And forasmuch as the multiplex N is next greater then FG : therefore FG is not lesse then M . But GH is greater then D by construction. Wherefore the whole FH is greater then D and M : and so consequently is greater then N . But K exceedeth not N : for K is equall to GH : for how multiplex K is to EB the lesse, so multiplex is FG to AE the greater. But those magnitudes which are equemultiples vnto vnequall magnitudes are according to the same proportion vnequall. Wherefore K is lesse then FG , and therefore is much lesse then N . Wherefore againe, the multiplex of the first exceedeth the multiplex of the second: but the multiplex of the third exceedeth not the multiplex of the fourth. Wherefore (by the 8. definition of the fifth) AB the first, hath to D the second, a greater proportion, then hath C the third to D the fourth.

Third difference.

But now if AE be equall vnto EB , eyther of them shall be equall vnto C . Wherefore vnto either of those three magnitudes take equemultiples greater then D . So that let FG be multiplex to AE , and GH vnto EB , and K agayne to C : which (by the 6. comon sentence) shall be equall the one to the other. Let N also be multiplex to D , and be next greater then euery one of them, namely, let it be quadruple to D . This construction finished, we may again proue that FH and K are equemultiples to AB and C : and that FH the multiplex of the first magnitude exceedeth N the multiplex of the second magnitude: and that K the multiplex



plex of the third exceedeth not the multiplex of the fourth. Wherefore we may conclude that AB hath vnto D a greater proportion, then hath C to D .

Now also I say, that the self same magnitude D hath vnto the lesse magnitude C a greater proportion, then it hath to the greater AB . And this may plainly be gathered by the foresayd discourse, without chaunging the order of the magnitudes & of the equemultiplices. For seeing that euery way it is before proued, that FH exceedeth N , and K is exceeded of the self same N : therefore conuersely N exceedeth K , but doth not exceed FH . But N is multiplex to D that is to the first and third magnitude: and K is multiplex to the second, namely, to C : and FH is multiplex to the fourth, namely, to AB : Wherefore the multiplex of the first exceedeth the multiplex of the second: but the multiplex of the third exceedeth not the multiplex of the fourth. Wherefore (by the 8. definiton of this fifth booke) D the first hath vnto C the second a greater proportion, then hath D the third to AB the fourth: which was required to be proued.

The second part of this proposition.

The 9. Theoreme. The 9. Proposition.

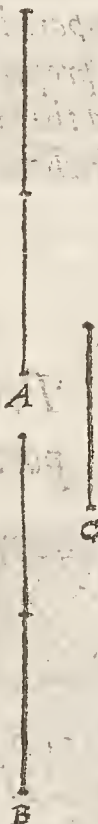
Magnitudes which haue to one and the same magnitude one and the same proportion: are equall the one to the other. And those magnitudes vnto whome one and the same magnitude hath one and the same proportion: are also equall.

Suppose that either of these two magnitudes A and B haue to C one and the same proportion. Then I say that A is equall vnto B . For if it be not, then either of these A and B should not haue to C one & the same proportiō (by the 8. of 5. fifth) but by supposition they haue, wherefore A is equall vnto B .

Againe, suppose that the magnitude C haue to either of these magnitudes A and B one and the same proportion. Then I say that A is equall vnto B . For if it be not, C should not haue to either of these A and B one and the same proportion (by the former proposition): but by supposition it hath, wherefore A is equall vnto B . Wherefore magnitudes which haue to one and the same magnitude one and the same proportion, are equall the one to the other. And those magnitudes vnto whome one and the same magnitude hath one and the same proportion, are also equall: which was required to be proued.

The first part of this Proposition demonstrated.

The second part proued.



The 10. Theoreme. The 10. Proposition.

Of magnitudes compared to one and the same magnitude, that which hath the greater proportion, is the greater. And that magnitude wherunto one and the same magnitude hath the greater proportion, is the lesse.

Ll.j.

Suppose

The fifth Booke

The first part
of this propo-
sition proued.

Suppose that A haue to C a greater proportion then B hath to C . Then I say that A is greater then B . For if it be not, then either A is equall vnto B or lesse then it. But A cannot be equal vnto B , for then either of these A and B should haue vnto C one and the same proportion (by the 7 of the fifth): but by supposition they haue not, wherfore A is not equall vnto B . Neither also is A lesse then B , for then should A haue to C a lesse proportion, then hath B to C (by the 8. of the fifth) but by supposition it hath not. Wherfore A is not lesse then B . And it is also proued that it is not equall, wherfore A is greater then B .

The second
part demon-
strated.

Agayne suppose that C haue to B a greater proportion then C hath to A . Then I say that B is lesse then A . For if it be not, then is it either equall vnto it or els greater, but B cannot be equall vnto A , for then should C haue to either of these A and B one and the same proportion (by the 7. of the fifth) but by supposition it hath not, wherfore B is not equall vnto A . Neither also is B greater then A , for then should C haue to B a lesse proportion then it hath to A (by the 8. of the fifth) but by supposition it hath not: wherfore B is not greater then A . And it was proued that it is not equall vnto A , wherfore B is lesse then A . Wherfore of magnitudes compared to one and the same magnitude, that which hath y greater proportion, is the greater. And that magnitude wherunto one and the same magnitude hath the greater proportion, is the lesse. Which was required to be proued.

The 11. Theoreme. The 11. Proposition.

Proportions which are one and the selfe same to any one proportion, are also the selfe same the one to the other.

Construction.



Demonstration.

Suppose that as A is to B , so is C to D , and as C is to D , so is E to F . Then I say that as A is to B , so is E to F . Take equemultiples to A , C and E , which let be G , H , K . And likewise to B , D and F take any other equemultiples, which let be L , M , and N . And because as A is to B , so is C to D : and to A and C are taken equemultiples G & H , & to B and D are take certaine other equemultiples L & M . If therfore G excede L , then also H excedeth M , and if it be equall it is equall, and if it be lesse it is lesse (by the conuerse of the 6. definition of the fifth). Agayne because that as C is to D , so is E to F : and to C and E are taken equemultiples H and K : and likewise to D & F are take certaine other equemultiples M & N . If therfore H excede M , then also K excedeth N : and if it be equall, it is equall, and if it be lesse, it is lesse (by the same conuerse) But if K excede M , then also G excedeth

deth L: and
if it be equal
it is equal,
and if it be
lesse, it is
lesse (by the
same con-
uerse) Where-
fore if G ex-
cede L, then
K also ex-
cedeth N,
and if it be
equal it is e-
quall, and if
it be lesse, it
is lesse. But

G & K are
equemultipli-
ces of A & E. And L & N are certaine other equemultiplices of B & F. Where-
fore (by the 6. definition) as A is to B, so is E to F. Proportions therfore which
are one and the selfe same to any one proportion, are also the selfe same one to the
other: which was required to be proued.

The 12. Theoreme. The 12. Proposition.

If there be a number of magnitudes how many soeuer propor-
tionall: as one of the antecedentes is to one of the cōsequentes,
so are all the antecedentes to all the consequentes.

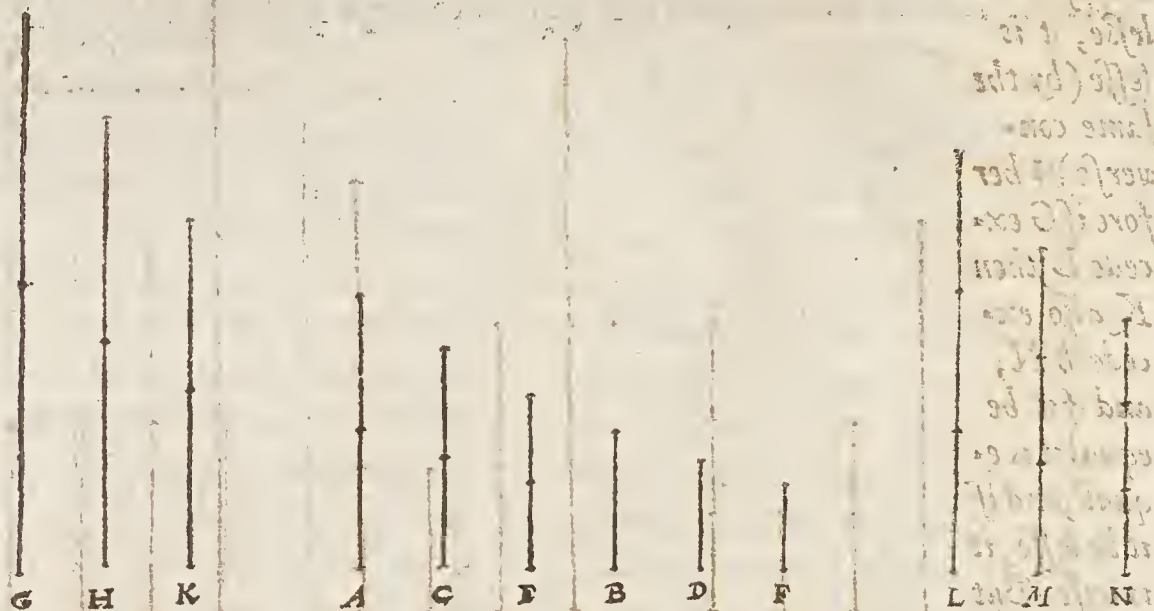
Suppose that there be a number of magnitudes how many soeuer, namely,
A, B, C, D, E, F, in proportion: so that as A is to B, so let C be to D,
and E to F. Then I say, that as A is to B, so is A, C, E, to B, D, F. Take
equemultiplices to A, C, and E. And let the same be G, H, K. And likewise to
B, D, and F, take any other equemultiplices, which let be L, M, N. And because
that as A is to B, so is C to D, and E to F. And to A, C, E, are taken equemulti-
plices G, H, K, and likewise to B, D, F, are taken certaine other equemultiplices
L, M, N. If therefore G excede L, H also exceedeth M, and K, N: and if it
be equal, it is equal, and if it be lesse, it is lesse (by the conuerse of the sixth defini-

Construction.

Demonstra-
tion.

The fifth Booke

tion of the fifth). Wherefore if G exceede L , then G, H, K , also exceede L, M, N : and if they be equall, they are equall: and if they be lesse, they are lesse (by the same). But G , and G, H, K , are equemultiples to the magnitude A , and to the



magnitudes A, C, E . For (by the first of the fifth) (if there be a number of magnitudes equemultiples to a like number of magnitudes ech to ech, how multiplex one magnitude, is to one, so multiples are all the magnitudes, to all). And by the same reason also L , and L, M, N , are equemultiples to the magnitude B , and to the magnitudes B, D, F : Wherefore as A is to B , so is A, C, E , to B, D, F (by the sixth definition of the fifth). If therefore there be a number of magnitudes how many soeuer, proportionall: as one of the antecedentes is to one of the consequentes, so are all the antecedentes to all the consequentes: which was required to be proued.

The 13. Theoreme. The 13. Proposition.

If the first haue vnto the second the self same proportion that the third hath to the fourth, and if the third haue vnto the fourth a greater proportiō thē the fifth hath to the sixth: thē shall the first also haue vnto the second a greater proportion then hath the fifth to the sixth.

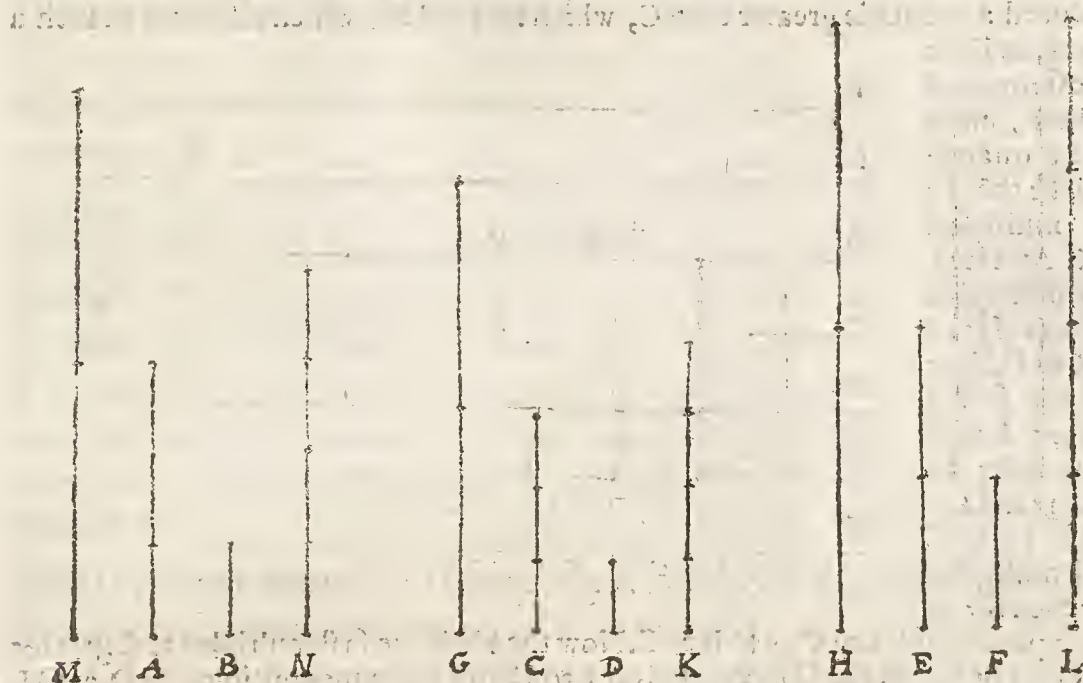


Suppose that there be sixe magnitudes, of which let A be the first, B the second, C the third, D the fourth, E the fifth, and F the sixth. Suppose that A the first haue vnto B the second, the self same proportion that C the third hath to D the fourth. And let C the third haue vnto D the fourth, a greater proportion then hath E the fifth to F the sixth.

Then

Then I say that *A* the first hath to *B* the second a greater proportion, then hath *E* the fifth to *F* the sixth. For forasmuch as *C* hath to *D* a greater proportion then hath *E* to *F*, therefore there are certaine equemultiplices to *C* and *E*, and likewise any other equemultiplices whatsoeuer to *D* and *F*, which being compared together, the multiplex to *C* shall exceede the multiplex to *D*, but the multiplex to *E* shall not exceede the multiplex to *F* (by the conuerse of the eight definition of

Constructions.



this booke). Let those multiples be taken, and suppose that the equemultiplices to *C* and *E*, be *G* and *H*: and likewise to *D* and *F* take any other equemultiplices whatsoeuer, and let the same be *K* and *L*, so that let *G* exceede *K*, but let not *H* exceede *L*. And how multiplex *G* is to *C*, so multiplex let *M* be to *A*. And how multiplex *K* is to *D*, so multiplex also let *N* be to *B*. And because that as *A* is to *B*, so is *C* to *D*: and to *A* and *C* are taken equemultiplices *M* and *G*. And likewise to *B* and *D* are taken certayne other equemultiplices *N* & *K*: if therefore *M* exceede *N*, *G* also exceedeth *K*: and if it be equall it is equall, and if it be lesse it is lesse (by the conuerse of the sixt definition of the fifth.) But by construction *G* exceedeth *K*, wherfore *M* also exceedeth *N*, but *H* exceedeth not *L*. But *M* & *H* are equemultiplices to *A* & *E*: and *N* & *L* are certaine other equemultiplices whatsoeuer, to *B* and *F*. Wherfore *A* hath vnto *B* a greater proportion then *E* hath to *F* (by the 8. definition.) If therefore the first haue vnto the second the selfe same proportion that the third hath to the fourth, and if the third haue vnto the fourth a greater proportion then the fifth hath to the sixth, then shall the first also haue vnto the second a greater proportion then hath the fifth to the sixth: Which was required to be proued.

Demonstration.

¶ An addition of Campane.

The fifth Booke

An addition
of Campane

If there be foure quantities, and if the first haue vnto the second a greater proportion the hath the third to the fourth: then shall there be some equemultiplices of the first and the third, which being compared to some equemultiplices of the second and the fourth, the multiplex of the first shall be greater then the multiplex of the second, but the multiplex of the third shall not be greater then the multiplex of the fourth.

Which is thus proued. Suppose that A B haue vnto C a greater proportion the hath D to E. And let A F be to C as D is to E. Now then by this proposition & the tenth, A F is lesse then A B. Let it be lesse then A B by the quantitie F B. And multiply F B vntil there be produced a quantitie greater then C, which let be G H: which also must be such a

multiplex, as D be yng so oftentimes multiplied, maye produce a quantitie not lesse the E: whiche multiplex let be K. And let L G be so multiplex to A F, as G H is to F B, or K to D. Now then by the first of this booke L H is equemultiplex to A B as K is to D.

And let M be to E the first multiplex greater then K: &

let N be equemultiplex to C as M is to E. Now then N is the first multiplex to C greater then L G: For for that as D is to E, so is A F to C, and K is equemultiplex to D as G L is to A F, also M is equemultiplex to E, as N is to C: therefore (by the 4. of this booke) as K is to M, so is G L to N: but K is to M the first multiplex lesse then M: wherefore also G L is the first multiplex lesse then N: and G L by supposition is not lesse the C. Wherefore take the greatest multiplex of C vnder N: or a multiplex equall to N, if peraduenture N be the first of the multiplices of C, which let be O. Now then N shall consist of O and C. Wherefore forasmuch as L G is not lesse then O, and G H is greater then C, therefore L H shall be greater then N. And forasmuch as K is lesse then M, therefore that which was required to be proued, is manifest.

Although this proposition here put by Campane nedeth no demonstration for that it is but the conuerse of the 8. definition of this booke, yet thought I it not worthy to be omitted for that it teacheth the way to finde our such equemultiplices, that the multiplex of the first shall excede the multiplex of the second, but the multiplex of the third shall not excede the multiplex of the fourth.

The 14. Theoreme. The 14. Proposition.

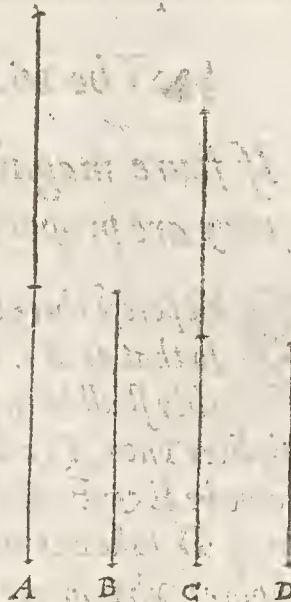
If the first haue vnto the second the selfe same proportion that the third hath vnto the fourth: and if the first be greater then the third, the second also is greater then the fourth: and if it be equall it is equall: and if it be lesse it is lesse.

Suppose



Suppose that there be foure magnitudes, of which let *A* be the first, *B* the second, *C* the third, and *D* the fourth: and let *A* the first, haue vnto *B* the second, the self same proportion that *C* the third hath vnto *D* the fourth. And let *A* be greater then *C*. Then I say, that *B* also is greater then *D*. For forasmuch as *A* is greater then *C*, and there is a certaine other magnitude, namely, *B*, therefore (by the 8. of the fift) *A* hath vnto *B* a greater proportion then *C* hath to *B*. And as *A* is to *B*, so is *C* to *D*. Wherefore *C* also hath vnto *D* a greater proportion then *C* hath to *B*. But that magnitude wherunto one and the same magnitude hath γ greater proportion, is the lesse (by the 10. of the fift). Wherefore *D* is lesse then *B*, & therefore *B* is greater then *D*. And in like sort may we proue, that if *A* be equall vnto *C*, *B* shall also be equall vnto *D*: and if *A* be lesse then *C*, *B* shall also be lesse then *D*. If therefore the first haue vnto the second the self same proportion that the third hath vnto the fourth, and if the first be greater then the third, the second also is greater then the fourth, and if it be equall it is equall, and if it be lesse it is lesse: which was required to be proued.

Demonstration.



The 15. Theoreme.

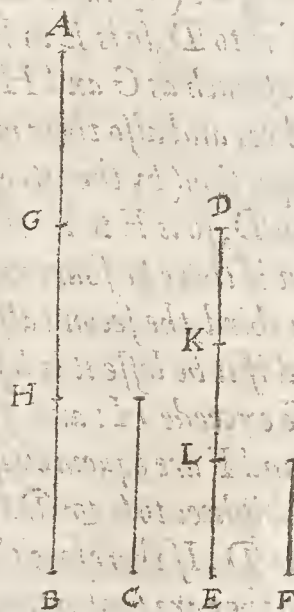
The 15. Proposition.

Like partes of multiplices, and also their multiplices compared together, haue one and the same proportion.



Suppose that *AB* be equemultiplex to *C*, as *DE* is to *F*. Then I say, that as *C* is to *F*, so is *AB* to *DE*. For forasmuch as how multiplex *AB* is to *C*, so multiplex is *DE* to *F*: therefore how many magnitudes there are in *AB* equall vnto *C*, so many are there in *DE* equall vnto *F*. Deuide *AB* into the magnitudes equall vnto *C*, that is, into *AG*, *GH*, and *HB*: and likewise *DE* into the magnitudes equall vnto *F*, that is, into *DK*, *KL*, and *LE*. Now then the multitude of these *AG*, *GH*, and *HB*, is equall to γ multitude of these *DK*, *KL*, and *LE*. And forasmuch as *AG*, *GH*, and *HB*, are equall the one to γ other: and likewise *DK*, *KL*, and *LE*, are also equall the one to γ other: therefore as *AG* is to *DK*, so is *GH* to *KL*, and *HB*

Construction.



Demonstration.

Ll.iiij. to L E.

The fifth Booke

to L E. Wherefore (by the 12. of the fifth) as one of antecedentes is to one of the consequentes , so are all the antecedentes to all the consequentes . Wherefore as A G is to D K , so is A B to D E . But A G is equall vnto C , and likewise D K to F . Wherefore as C is to F , so is A B to D E . Like partes therefore of multiples and also their multiples compared together, haue one and the same proportion that their equemultiples haue: which was required to be demonstrated.

The 16. Theoreme. The 16. Proposition.

If foure magnitudes be proportionall: then alternately also they are proportionall.

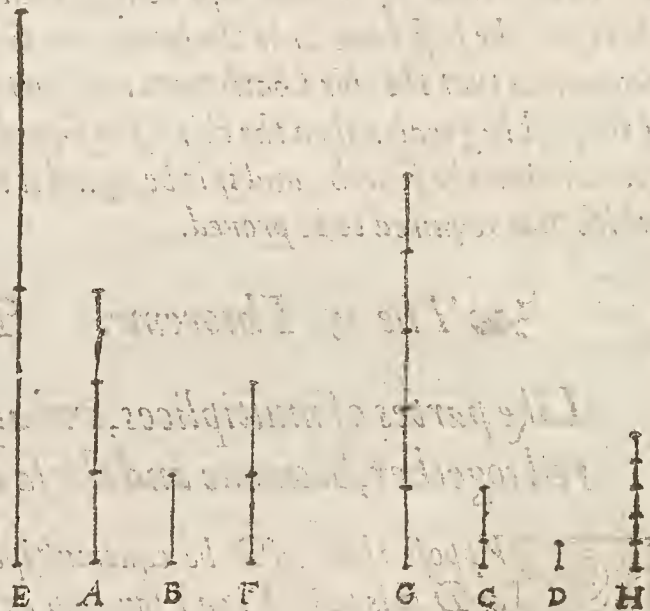
Demonstration
of alternate
proportion.

Construction:

Demonstration.



Suppose there be foure magnitudes proportionall, namely, A, B, C, D , so that as A is to B , so let C be to D . Then I say, that alternately also they shall be in proportion, that is, as A is to C , so is B to D . Take equemultiples vnto A & B , and let the same be E & F . And likewise to C and D take any other equemultiples what soeuer, and let the same be G and H . And forasmuch as how multiplex E is to A , so multiplex is F to B , but like partes of multiples & also their multiples haue one and the selfe same proportion the one to the other (by the former Proposition). Wherefore as A is to B , so is E to F . But as A is to B , so is C to D , wherefore (by the 11. of the fifth) as C is to D so is E to F . Againe, forasmuch as G and H are equemultiples to C and D , but like partes of multiples and also their multiples haue the one to the other one and the selfe same proportion (by the 15. of the fifth). Wherefore as C is to D , so is G to H . But as C is to D , so is E to F . Wherefore as E is to F , so is G to H (by the 11. of the fifth). But if there be foure magnitudes in proportion, and if the first be greater then the third, the second also is greater then the fourth: and if it be equall it is equall: and if it be lesse it is lesse (by the 14. of the fifth). If therefore E excede G , F shall also excede H : and if it be equall, it is equall: and if it be lesse, it is lesse. But E and F are equemultiples to A and B : and G and H are certaine other equemultiples to C & D . Wherefore (by the 6. definition of the fifth) as A is to C , so is B to D . If therefore there be foure magnitudes proportionall, then alternately also they are proportionall: which was required to be proued.



The

The 17. Theoreme. The 17. Proposition.

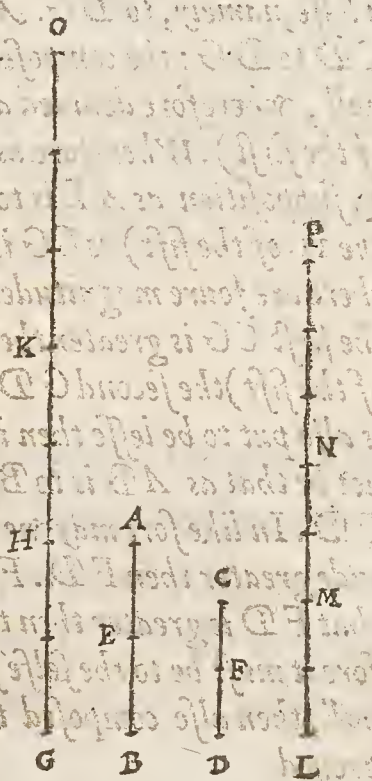
If magnitudes composed be proportionall, then also devided they shall be proportionall.

Suppose that the magnitudes composed being proportionall, be AB, BE, CD, DE , so that as AB is to BE , so is CD to DE . Then I say, that devided also they shall be proportionall, as AE is to BE , so is CF to DE . Take equemultiples vnto AE, EB, CF, FD , and let γ same be GH, HK, LM , and MN . And likewise to EB , and FD , take any other equemultiples what soeuer, and let the same be KO , and NP . And forasmuch as how multiplex GH is to AE , so multiplex is KH to EB , therefore how multiplex GH is to AE , so multiplex is GK to AB (by the first of the fifth). But how multiplex GH is to AE , so multiplex is LM to CF : Wherefore how multiplex GK is to AB , so multiplex is LM to CF (by the 11. of γ same). Againe forasmuch as how multiplex LM is to CF , so multiplex is MN to DE , therefore how multiplex LM is to CF , so multiplex is LN to CD (by the first of the self same). But how multiplex LM is to CF , so multiplex is GK to AB . Wherefore how multiplex GK is to AB , so multiplex is LN to CD . Wherefore GK and LN , are equemultiples to AB & CD . Againe forasmuch as how multiplex HK the first is to EB the second, so multiplex is MN the third to FD the fourth. And how multiplex KO the fift is to EB the second, so multiplex is NP the sixt to FD the fourth. Wherefore (by the second of the same) how multiplex HO composed of the first and fift is to EB , so multiplex is MP composed of the third and sixt to FD . And for that as AB is to BE , so is CD to DE : and to AB & CD are taken equemultiples GK and LN : and likewise to EB and FD are taken certaine other equemultiples, that is, HO , and MP . If therefore GK exceede HO , then LN also exceedeth MP : and if it be equall, it is equall: and if it be lesse, it is lesse (by the conuersion of the sixt definition of the fift). Let GK exceede HO : Wherefore KH common to them both being taken away, the residue GH , shall exceede the residue KO . But if GK exceede HO , then doth LN exceede MP : Wherefore let LN exceede MP , and MN which is comon to the both being take away, γ residue LM shall exceede the residue NP . Wherefore if GH exceede KO , then shall LM exceede NP . And in like sort may we proue, that if GH be equall vnto KO , then LM shall be equall vnto NP : and if it be lesse, it shall be lesse: but GH

Demonstratio
of proportion
by diuision.

Construccion:

Demonstration.



and

The fifth Booke

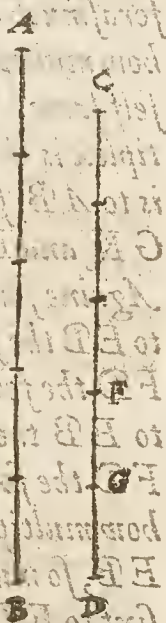
and LM are equemultiples to AE and CF : and likewise KO and NP are certayne other equemultiples to EB , and FD . Wherefore as AE is to EB , so is CF to FD (by the sixt definition of the fift). If composed magnitudes therefore be proportionall, then also deuided they shall be proportionall: which was required to be demonstrated.

The 18. Theoreme. The 18. Proposition.

If magnitudes deuided be proportionall: then also composed they shall be proportionall.

Demonstration of proportion by composition. This proposition is the conuerse of the former. Demonstratio leading to an impossibility.

Suppose that the magnitudes deuided being proportionall, be AE , EB , CF , & FD , so that as AE is to EB , so let CF be to FD . Then I say, that composed also they shall be proportionall, that is, as AB is to BE , so is CD to DF . For if AB be not vnto BE , as CD is to FD , then shall AB be vnto BE , as CD is either vnto a magnitude lesse then FD , or vnto a magnitude greater. Let it first be vnto a lesse, namely, to DG . And forasmuch as, as AB is to BE , so is CD to DG : the composed magnitudes therefore are proportionall, wherefore deuided also they shall be proportionall (by 17. of the first). Wherefore as AE is to EB , so is CG to GD . But by supposition as AE is to EB , so is CF to FD . Wherefore (by the 11. of the fift) as CG is to GD , so is CF to FD . Now then there are foure magnitudes, CG , GD , CF , and FD : of which the first CG is greater then the third CF . Wherefore (by the 14. of the fift) the second GD is greater then the fourth FD . But it is also put to be lesse then it: which is impossible. Wherefore it can not be that as AB is to BE , so is CD to a magnitude lesse then FD . In like sort may we proue, that it can not be so to a magnitude greater then FD . For by the same order of demonstration, it would follow that FD is greater then the sayd greater magnitude: which is impossible. Wherefore it must be to the selfe same. If therefore magnitudes deuided be proportionall, then also composed they shall be proportionall: which was required to be proued.



The 19. Theoreme. The 19. Proposition.

If the whole be to the whole, as the part taken away is to the part taken away: then shall the residue be vnto the residue, as the whole is to the whole.

Suppose

Suppose that as the whole AB is to the whole CD , so is the part taken away AE to the part taken away CF . Then I say, that the residue EB shall be vnto the residue FD , as the whole AB is to the whole CD . For for that as the whole AB is to the whole CD , so is AE to CF , therefore alternately also (by the 16. of the fifth) as AB is to AE , so is CD to CF . And for that when magnitudes composed are proportionall, the same deuided also are proportionall (by the 17. of the fifth): therefore as BE is to EA , so is DF to FC . Wherefore alternately also (by the 16. of the fifth) as BE is to DF , so is EA to FC . But as AE is to CF , so (by supposition) is the whole AB to the whole CD . Wherefore the residue EB shall be vnto the residue FD , as the whole AB is to the whole CD . If therefore the whole be to the whole, as the part taken away is to the part taken away, then shall the residue be vnto the residue, as the whole is to the whole: which was required to be proued.

¶ A Lemma or Assumpt.

And forasmuch as by supposition as AB is to CD , so is AE to CF : and alternately as AB is to AE , so is CD to CF . And now it is proued, that as AB is to CD , so is EB to FD . Wherefore againe alternately, as AB is to EB , so is CD to FD . Wherefore it followeth, that as AB is to AE , so is CD to CF : and againe, as the same AB is to EB , so is the same CD to FD .

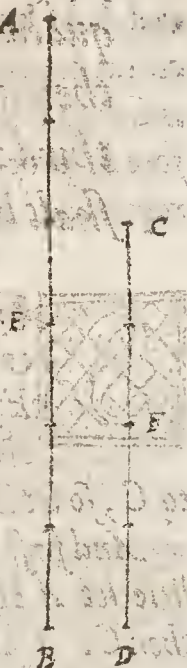
¶ Corollary.

And hereby it is manifest, that if magnitudes composed be proportionall, then also by conuersion of Proportion (which of some is called Proportion by Euerjion, and which is, as before it was defined, whē the antecedent is compared to the excesse, wherein the antecedent exceedeth the consequent) they shall be proportionall.

¶ The 20. Theoreme. The 20. Proposition.

If there be three magnitudes in one order, and as many other magnitudes in an other order, which being taken two and two in eche order, are in one and the same proportion, and if of equalitie

That which the fift of this booke proued only touching multiplies, this proueth generally of all magnitudes.



A Lemma.

A Corollary.

Conuersion of proportion.

The fifth Booke

qualitie in the first order the first be greater then the third, then in the second order the first also shall be greater then the third: and if it be equall, it shall be equall: and if it be lesse, it shall be lesse.



Suppose that there be three magnitudes in one order, namely, A, B, C , & let there be as many magnitudes in an other order, which let be D, E, F , which being taken two and two in ech order, let be in one and the same proportion, that is, as A is to B , so let D be to E , and as B is to C , so let E be to F .

And first of equalitie let A be greater

then C . Then I say, that D also is greater then F : and if it be equall, it is equall, and if it be lesse, it is lesse.

For forasmuch as A is greater then C , and there is a certaine other magnitude, namely, B , but the greater hath to one and the same magnitude a greater pro-

portion then hath the lesse (by the 8. of the first). Wherefore A hath vnto B a greater proportion then C hath to B . But as A is to B , so is D to E : and as C is to B , so is F to E (by the Corollary of the fourth of the first). Wherefore D hath vnto E a greater proportion, then F hath to E . But of magnitudes compared to one and the same magnitude that which hath the greater proportion is the greater (by the 10. of the first). Wherefore D is greater then F .

But now if A be equall vnto C , D also shall be equall vnto F . For then A and C haue vnto B one and the same proportion (by the first part of the seventh of this booke). And for that as A is to B , so is D to E , and as C is to B , so is F to E : therefore D and F haue vnto E one and the same proportion. Wherefore by the first part of the 9. of this booke D is equall vnto F .

But now suppose that A be lesse then C . Then also shall D be lesse then F . For by the 8. of this booke C shall haue vnto B a greater proportion then hath A to B . But as A is to B , so is D to E by supposition, and as C is to B , so haue we proued is F to E . Wherefore F hath vnto E a greater proportion then hath D to E . Wherefore by the first part of the 10. of this booke F is greater then D .

If therefore

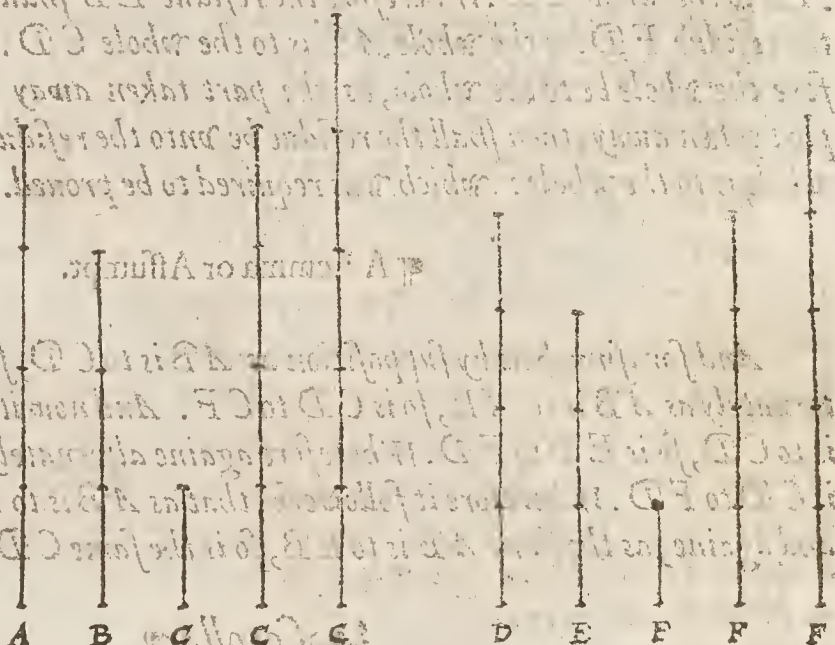
This proposition pertai-
neth to Pro-
portion of e-
quality inor-
dinate propor-
tionality.

The first dif-
ference.

Demonstra-
tion.

The second
difference.

The third
difference.



If therefore there be three magnitudes in one order, and as many other magnitudes in an other order, which being taken two and two in ech order, are in one and the same proportion, and if of equalitie in the first order the first be greater then the third, then in the second order also the first shall be greater then y third: and if it be equall, it shall be equall: and if it be lesse, it shall be lesse: which was required to be proued.

The 21. Theoreme. The 21. Proposition.

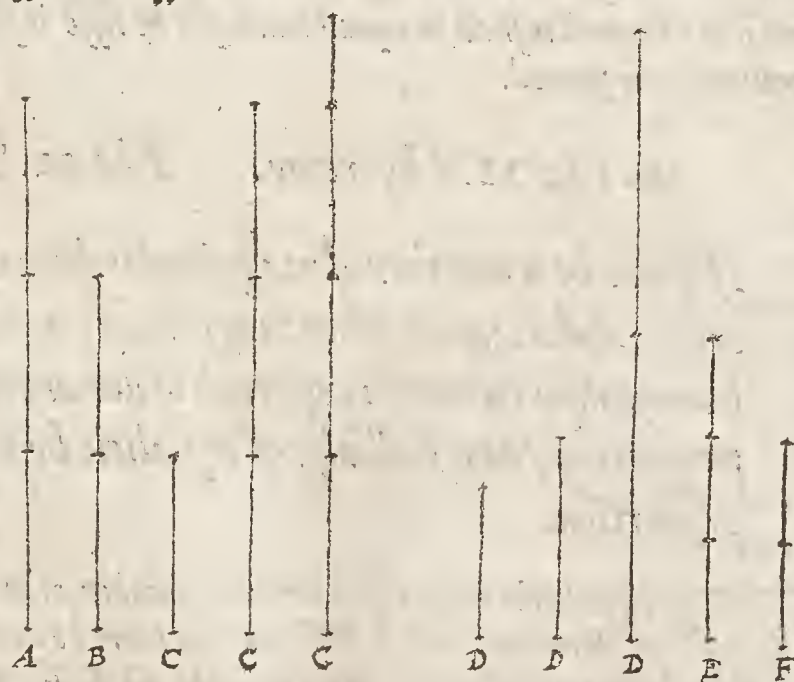
If there be three magnitudes in one order, and as many other magnitudes in an other order, which being taken two and two in eche order are in one and the same proportion, and their proportion is perturbate: if of equalitie in the first order the first be greater then the third, thē in the second order the first also shall be greater then the third: and if it be equall it shall be equall: and if it be lesse it shall be lesse.



Suppose y there be three magnitudes in one order, namely, A, B, C, and let there be as many other magnitudes in an other order, which let be D, E, F: which being taken two & two in ech order, let be in one and the same proportion, and let their proportion be perturbate. So that as A is to B, so let E be to F, & as B is to C, so let D be to E, and of equalitie let A be greater then C. Then I say, that D also is greater then F: and if it be equall it is equall: and if it be lesse it is lesse.

This proposition pertai-
neth to Pro-
portion of
equality in
perturbate
proportion-
ality.

First let A be greater then C. And for as much as A is greater then C, & there is a certaine other magnitude, namely, B: therefore (by the 8. of y fift) A hath a greater proportio vn- to B, then C hath to B. But as A is to B, so is E to F: and againe as C is to B, so is E to D (by the Corellary of y fourth of the fift). Wherefore E hath vnto F a greater



The first dif-
ference.

proportion, then E hath to D. But that magnitude wherunto one and the same magnitude hath the greater proportion, is the lesse (by the 10. of the fift). Wherefore

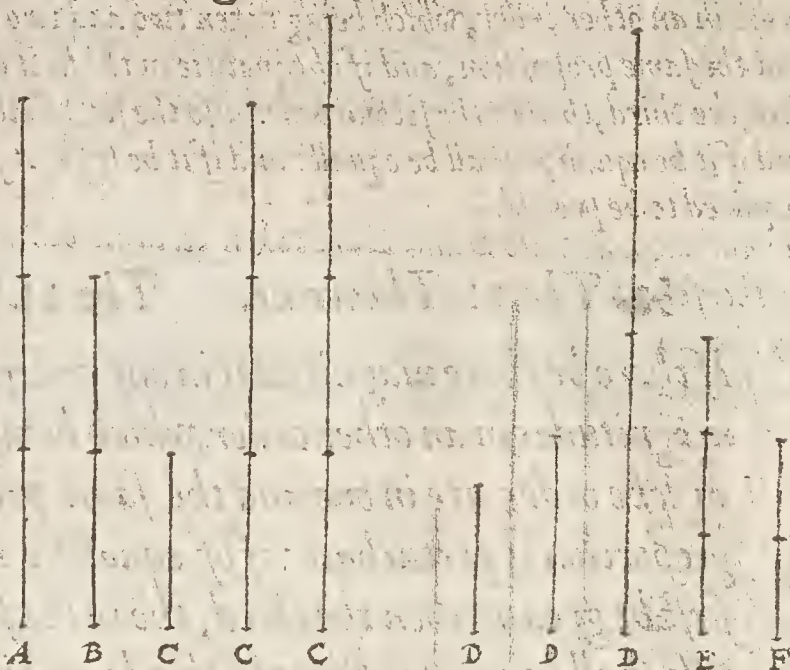
Mm.j. fore

The fifth Booke

fore F is lesse then D . Wherefore D is greater then F .

The second
difference.

But now if A be equall vnto C , we may also proue that D is equall vnto F . For then (by the first part of the 7. of this booke) A and C shall haue vnto B one and the same proportion. And for that as A is to B , so is E to F , & as C is to B , so is E to D : therefore E hath to either of these D and F , one and y same proportion. Wherefore by the



second part of the 9. of the fift D is equall vnto F .

The third
difference.

Likewise, if A be lesse then C , D also is lesse then F . For then C shall haue vnto B a greater proportion, then hath A to B . (by the 8. of the fift. Wherefore E also hath vnto D a greater proportion then it hath to F . Wherefore by the second part of the 10. of this booke D is lesse then F . If therefore there be three magnitudes in one order, & as many other magnitudes in an other order, which being taken two and two in ech order are in one & the same proportion, & their proportion is perturbate, and if of equalitie in the first order the first be greater then the third, then in the second order the first also shall be greater then y third: and if it be equall it shall be equall: and if it be lesse it shall be lesse.: which was required to be proued.

The 22. Theoreme. The 22. Proposition.

If there be a number of magnitudes, how many soeuer in one order, and as many other magnitudes in an other order, which being taken two and two in ech order are in one and the same proportion, they shall also of equalitie be in one and the same proportion.

Proportion of
equality in or-
dinate propor-
tionality.



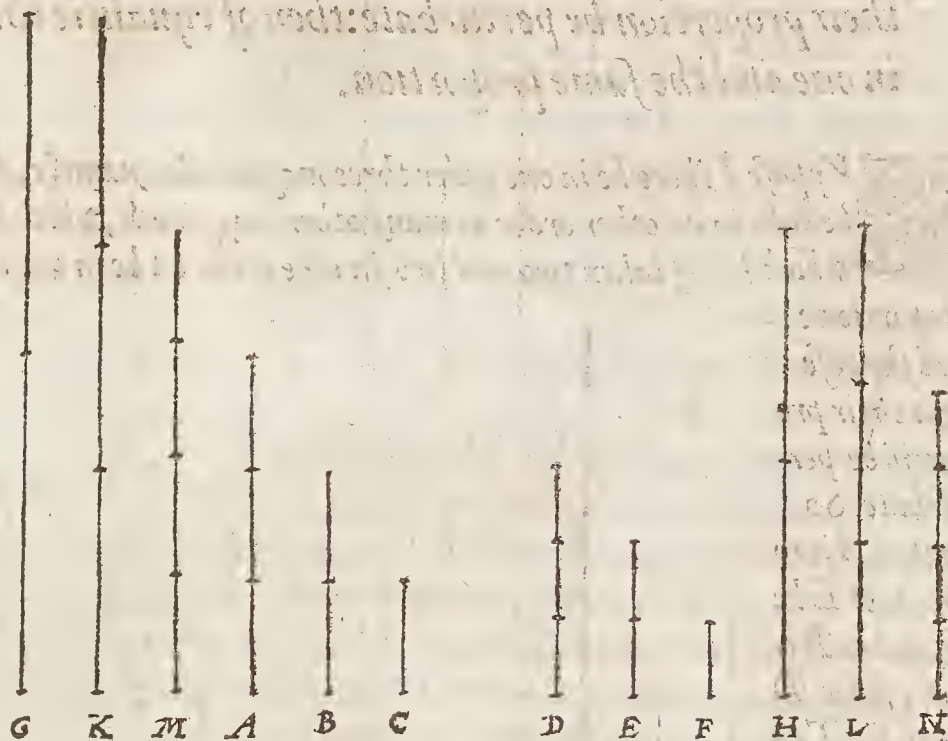
Suppose that there be a certaine number of magnitudes in one order. As for example: A, B, C , and let there be as many other magnitudes in an other order, which let be D, E, F , which being taken two and two let be in one and the same proportion. So that as A is to B , so let D be to E , and as B is to C , so let E be to F . Then I say, that of equalitie they

they shall be in the same proportion, that is, as A is to C , so is D to F . Take vnto A and D equemultiples G & H , and likewise to B & E take any other equemultiples whatsoeuer, namely, K and L , and moreouer vnto C and F take any other equemultiples also whatsoeuer, namely, M and N . And forasmuch as,

Construction.

Demonstration.

(by the 4. of the fift) as G is to K , so is H to L . And (by the same reason) as K is to M , so is L to N . Seing therefore y there be in order three magnitudes G, K, M , & as many



other magnitudes in an other order, namely, H, L, N , which being compared two to two are in one and the same proportion, therefore of equalitie (by the 20. of the fift) if N exceede M , then shall H exceede G : and if it be equall it shall be equall: and if it be lesse it shall be lesse. But G and H are equemultiples vnto A and D , and M and N are certaine other equemultiples vnto C and F : therefore (by the 6. definition of the fift) as A is to C , so is D to F .

So also if there be more magnitudes then three in either order, the first of the one order shall be to the last, as the first of the other order is to the last. As if there were foure in one order, namely, $A B C D$, and other foure in the other order, namely, $E F G H$, we may with three magnitudes A, B, C , and E, F, G , proue that as A is to C , so is E to G : And then leauing out in either order the second and taking the fourth, as leauing out B and F , and taking D and H , we may proue by these three and three A, C, D , and E, G, H , that as A is to D , so is E to H . And obseruing this order, thys demonstration will serue how many soeuer the magnitudes be in either order. If therefore there be a number of magnitudes how many soeuer in one order, and as many other magnitudes in an other order, which being taken two and two in eche order are in one and the same proportion, they shall also of equalitie be in one and the same proportion: which was required to be demonstrated.

When there are more then three magnitudes in either order.
 $A B C D$ $E F G H$

The fifth Booke

The 23. Theoreme. The 23. Proposition.

If there be three magnitudes in one order, and as many other magnitudes in an other order, which being taken two & two in eche order are in one and the same proportion, and if also their proportion be perturbate: then of equalitie they shall be in one and the same proportion.

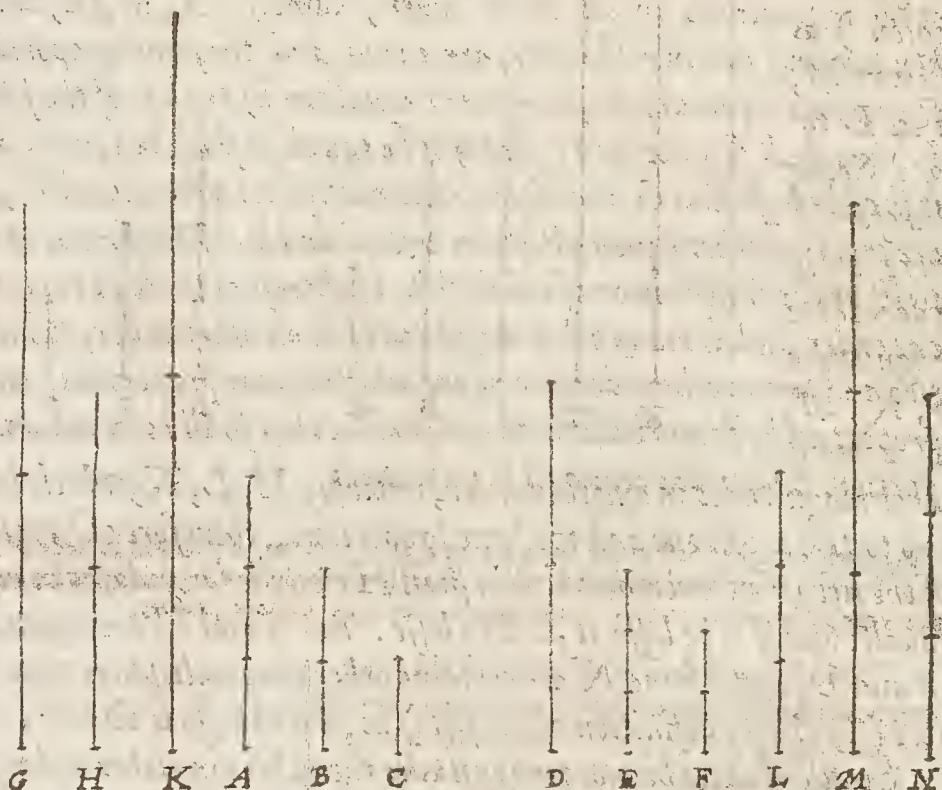
Proportion of equality in perturbate proportionality.

Suppose $\dot{y}$ there be in one order three magnitudes, namely, A, B, C , & let be take in an other order as many other magnitudes, which let be D, E, F which being taken two and two, in eche order let be in one and the same

proportion: and suppose that their proportion be perturbate. So that as A is to B , so let E be to F , and as B is to C , so let D be to E . The I say that as A is to C , so is D to F .

Construction.

Take vnto A, B, D , equemultiples, and let the same be G, H, K : and



Demonstration.

likewise vnto C, E, F , take any other equemultiples whatsoeuer, and let $\dot{y}$ same be L, M, N . And forasmuch as G and H are equemultiples vnto A and B , but the partes of equemultiples are in the same proportion that their equemultiples are (by $\dot{y}$ 15. of the fift) wherfore as A is to B , so is G to H . * And by $\dot{y}$ same reason also as E is to F , so is M to N . But as A is to B , so is E to F . Wherfore (by $\dot{y}$ 11. of $\dot{y}$ fift) as G is to H , so is M to N . And forasmuch as, as B is to C , so is D to E , and vnto B & D are taken equemultiples H & K : and likewise vnto C and E are taken certayne other equemultiples L and M : therfore (by the 4. of the fift) as H is to L , so is K to M , and alternately also (by the 16. of the fift) as B is to D , so is C to E . And forasmuch as H and K are the equemultiples of B and D , but the partes of equemultiples are in the same proportion that their

their equemultiples are (by the 15. of the fifth) therefore as B is to D, so is H to K. But as B is to D, so is C to E, therefore (by the 12. of the first) as H is to K, so is C to E. Agayne forasmuch as L and M are the equemultiples of C and E, therefore as C is to E, so is L to M. But as C is to E, so is H to K: therefore as H to K, so is L to M, and alternately (by the 16. of the fifth) as H is to L, so is K to M.*

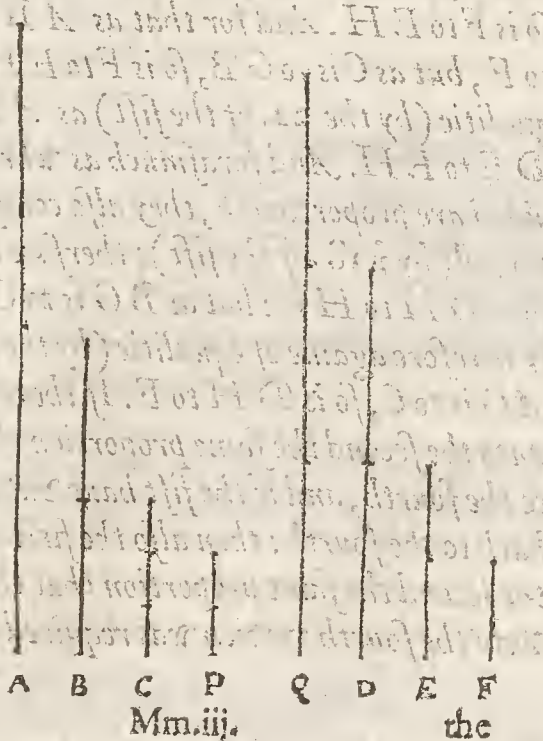
(But as A is to B so is E to F (by supposition) wherefore as G is to H, so is E to F, (by the 11. of the fifth.) Agayne for as much as M and N are equemultiples vnto E and F, therefore agayne (by the 15. of the fifth) as E is to F, so is M to N. But as E is to F, so haue we proued is G to H, wherefore as G is to H, so is M to N (by the 11. of the fifth) And for that as B is to C, so is D to E (by supposition). And vnto B and D are taken equemultiples H and K: and vnto G and E are taken certayne other equemultiples L and M, therefore as H is to L so is K to M, by the 4. of this booke).

But it is proued that as G is to H, so is M to N. Seyng therefore that there are in one orde three magnitudes, namely, G, H, L, and as many other magnitudes in an other order, namely, K, M, N, whiche being taken two and two in eche order, are in one and the same proportion, and their proportion is perturbate, therefore of equalitie (by the 21. of the fifth) if G excede E, then shall K excede N, and if it be equall, it shall be equall: and if it be lesse, it shall be lesse. But G and K are equemultiples vnto A and D, and L and N are certayne other equemultiples vnto C and F. Wherefore as A is to C, so is D to F (by the 6. definition of the fifth). If therefore there be three magnitudes in one order, and as many other magnitudes in an other order, which beyng taken two and two in eche order, are in one and the same proportion, and if also their proportion be perturbate: then of equalitie they shall be in one and the same proportion: which was required to be proued.

From this marke * first to the same marke agayne, you may if you will in stede of *Theore* argumentes which seeme somewhat intricate, read those argumentes following printed with an other letter, which are very perspicuous and brieve, and followed of the most interpreters.

This Proposition is also true if there be more then thre magnitudes in either order. As for example, let there be foure, So that as A is to B, so let E be to F, and as B is to C, so let D be to E, & as C is to P, so let Q be to D. Then I say that as A is to P, so is Q to F. For forasmuch as it is before proued, that A is to C, as D is to F taking away B & E there shall be three magnitudes A, C, P, in one order, and as many other in an other order, namely, Q, D, F, which being take in ech order two and two, are in one and the same proportion, and their proportio is perturbate, wherefore by the former propositio A is to P as Q is to F, which was required to be demonstrated.

And euen as by the demonstration in three magnitudes is taken the prooue in foure magnitudes by leauing out one of



An addition
of Campano.

Note.

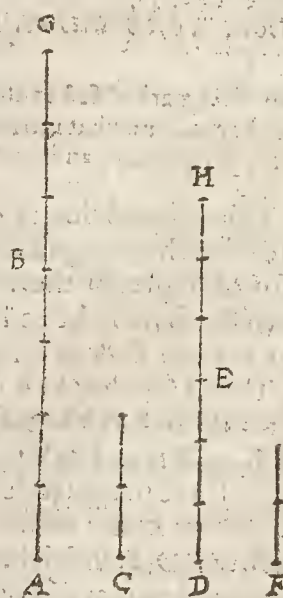
the meanes: so by the demonstration in foure magnitudes is taken the prooffe in five magnitudes by leauyng out two of the meanes: and by the demonstration in five, the prooffe in fixe, by leauing out three meanes. And so forward continually, which is also to be vnderstanded in the former kynde of proportion of equalitie, which is in ordinate proportion.

The 24. Theoreme. The 24. Proposition.

If the first haue vnto the second the same proportion that the third hath to the fourth, and if the fift haue vnto the second the same proportion that the sixt hath to the fourth: then also the first and fift composed together shall haue vnto the second the same proportion that the third and sixt composed together haue vnto the fourth.



Suppose that there be sixe magnitudes AB, C, DE, F, BG, & EH: of which let AB be the first, C the second, DE the third, F the fourth, BG the fift, and EH the sixt. And suppose that AB the first, haue vnto C the second, the same proportion that DE the third hath to F the fourth, and let BG the fift haue vnto C the second the same proportion that EH the sixt hath vnto F the fourth. Then I say, that the first and fift composed together, namely, AG, hath vnto C the second the same proportion, that the third and sixt composed together, namely, DH, hath vnto F the fourth. For, for that as BG is to C, so is EH to F: then also by conuersion (by the Corollary of the 4. of the fift) as C is to BG, so is F to EH. And for that as AB is to C, so is DE to F, but as C is to BG, so is F to EH: therefore of equalitie (by the 22. of the fift) as AB is to BG, so is DE to EH. And forasmuch as when magnitudes deuised are proportionall, they also composed are proportionall (by the 18. of the fift): therefore as AG is to GB, so is DH to HE: but as BG is to C, so is EH to F: Wherefore againe of equalitie (by the 22. of the fift) as AG is to C, so is DH to F. If therefore the first haue vnto the second the same proportion that the third hath to the fourth, and if the fift haue vnto the second the same proportion that the sixt hath to the fourth: then also the first and fift composed together shall haue vnto the second the same proportion that the third and sixt composed together haue vnto the fourth: which was required to be proued.

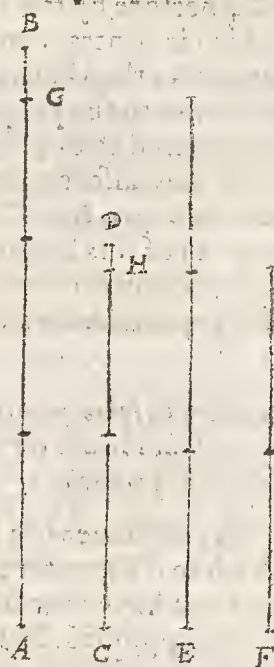


That which the second proposition of this booke proued only touching multiples, is here proued generally touching magnitudes.

The 25. Theoreme. The 25. Proposition.

If there be foure magnitudes proportionall: the greatest and the least of them, shall be greater then the other remayning.

Suppose that there be foure magnitudes proportionall AB, CD, E and F . So that as AB is to CD , so let E be to F . And let the greatest of them be AB , & the lest of them be F . Then I say, that these two magnitudes AB and F , are greater then the two magnitudes CD & E . Forasmuch as AB is supposed to be the greatest of all foure, therefore it is greater then E . Therefore from the greater AB cut of (by the 3. of the first) vnto E an equall magnitude AG . and likewise (by the same) from CD cut of vnto F an equall magnitude CH . (Which may be done, for that the magnitude CD is greater then the magnitude E . for that as AB is to CD , so is E to F , therefore alternately as AB is to E , so is CD to F (by the 16. of the fift). But AB is greater then E . Wherefore also CD is greater then F . Which thing may also be proued by 14. of the same.) Now for that as AB is to CD , so is E to F : but E is equall vnto AG , and F is equall vnto CH : therefore as AB is to CD , so is AG to CH : and forasmuch, as the whole AB is to the whole CD , so is the part taken away AG , to the part taken away CH : therefore the residue GB (by the 19. of the fift) is vnto the residue HD , as the whole AB is to the whole CD . But AB the first is greater then CD the third: Wherefore GB the second is greater then HD the fourth (by the 14. of the fift). And forasmuch as AG is equall vnto E , & CH is equall vnto F : therefore AG and F are equall vnto CH and E . And forasmuch as if vnto thinges vnequall be added thinges equall, all shall be vnequall (by the fourth common sentence): therefore seing that GB and HD are vnequall, and GB is the greater, if vnto GB be added AG and F : and likewise if vnto HD be added CH & E , there shall be produced AB and F greater then CD & E . If therefore there be foure magnitudes proportionall, the greatest and the least of them, shall be greater then the other remayning: which was required to be demonstrated.



Here follow certayne propositions added by Campane, which are not to be condemned, and are cited euen of the best learned, namely, of Iohannes Regio montanus, in the Epitome which he writeth vpon Ptolome.

The first Proposition.

If there be foure quantities, and if the proportion of the first to the second, be greater then

The fifth Booke

then the proportion of the third to the fourth : then contrariwise by conuersion, the proportion of the second to the first, shall be lesse then the proportion of the fourth to the third.

*Demonstratio
leading to an
impossibilitie.*

Suppose that the proportion of A to B be greater then the proportion of C to D . Then I say contrariwise by conuersion, the proportion of B to A is lesse then the proportion of D to C . For if the proportion of B to A be one and the same with the proportion of D to C , then conuersedly the proportion of A to B is one and the same with the proportion of C to D , which is contrary to supposition. But if the proportion of B to A be greater then the proportion of D to C , Then let E be vnto A , as D is to C . And by the 13. of this booke, the proportion of E to A shall be lesse then the proportion of B to A . Wherefore (by the first part of the tenth of the same) E is lesse the B . And therefore by the secod part of the 8. of the same, A hath vnto E a greater proportion then A hath to B . And forasmuch as by conuerse proportionalitie A is vnto E as C is to D , therefore by the 13. of the same, C hath to D a greater proportion then hath A to B , which is contrary to the supposition. For A was supposed to haue vnto B a greater proportion then hath C to D . Wherefore the proportion of B to A is neither one and the same with the proportion of C to D , nor greater then it. Wherefore it is lesse : which was required to be proued.

*An other demonstration of
the same
affirmatiuely.*

It may also be demonstrated directly. For let E be vnto B as C is to D . Then conuersedly B is to E as D is to C . And forasmuch as A is greater then E by the first part of the tenth of this booke, therefore by the second part of the 8 of the same B hath vnto A a lesse proportion then hath B to E . Wherefore by the 13. of the same B hath vnto A a lesse proportion then hath D to C : which was required to be proued.

¶ The second Proposition.

If there be foure quantities, and if the proportion of the first to the second be greater then the proportion of the third to the fourth, then alternately the proportion of the first to the third, shall be greater then the proportion of the second to the fourth.

*Demonstratio
leading to an
impossibilitie.*

Let A haue vnto B a greater proportion then hath C to D . Then I say alternately A hath to C a greater proportion then hath B to D . For one and the same proportion it can not haue : for then alternately A should be to B as C is to D , which is contrary to the suppositio. But if it haue a lesse proportion, let E be vnto C as B is to D . Now the by the 13. of this booke E hath vnto C a greater proportion then hath A to C . Wherefore (by the first part of the tenth of the same) E is greater the A . Wherefore by the first part of the 8. of the same, E hath to B a greater proportion then hath A to B . And forasmuch as by suppositio E is vnto C , as B is to D , therefore alternately E is to B as C is to D . Wherefore by the 13. of the same, C hath to D a greater proportion then hath A to B , which is contrary to the suppositio. Wherefore the proportion of A to C is neither one and the same with the proportion of B to D , nor lesse then it, wherefore it is greater. Which was required to be proued.

*An other demonstration
of the same
affirmatiuely.*

This may also be demonstrated affirmatiuely, let E be vnto B as C is to D . Now the by the first part of the tenth of this booke, E is lesse then A : wherefore by the first parte of the 8. of the same, the proportion of A to C is greater then the proportion of E to C . But alternately E is to C as B is to D . Wherefore (by the 13. of the same) A hath to C a greater

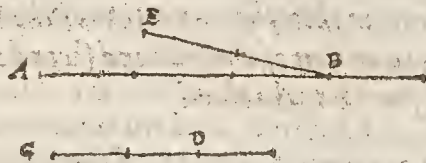
greater proportion then hath B to D : which was required to be proued.

¶ The third Proposition.

If there be foure quantities, and if the proportion of the first to the second be greater then the proportion of the third to the fourth : then by composition also the proportiō of the first and second to the second, shall be greater then the proportion of the third and fourth to the fourth.

Suppose that the proportion of A to B be greater then the proportion of C to D . Then I say that the proportion of the whole AB to B is greater then the proportion of the whole CD to D . For the proportion of AB to B can not be one and the same with the proportion of CD to D : for then by diuision also A should be vnto B as C is to D : which is contrary to the supposition. Neither also can it be lesse. For if it be possible, let it be : & let EB be vnto B as CD is to D . Now then (by the 12. of this booke EB hath vnto B a greater proportion then hath AB to B . Wherefore by the first part of the 10. of the same EB is greater then the whole AB . And by the common sentence E is greater then A . Wherefore by the first part of the 8. of this booke E hath to B a greater proportion then hath A to B . But E is to B as C is to D by diuisiō of proportion: for EB is to B as CD is to D . Wherefore (by the 12. of the same) C hath to D a greater proportion the hath A to B , which is cōtrary to the supposition. Wherefore the proportion of AB to B is not one and the same with the proportion of CD to D , neither also is it lesse then it. Wherefore it is greater : which was required to be proued.

Demonstratiō leading to an impossibilitie.



This may also be demonstrated affirmatinely. Forasmuch as the proportion of A to B is greater then the proportion of C to D : let E be vnto B as C is to D . And so by the first part of the 10. of this booke, E shall be lesse then A . And therefore by the common sentence EB shall be lesse then AB . Wherefore by the first part of the 8. of the same AB hath vnto B a greater proportion then hath EB to B . But by composition EB is to B , as CD is to D . For by supposition E is vnto B as C is to D . Wherefore (by the 12. of this booke) AB hath to B a greater proportion then hath CD to D : which was required to be proued.

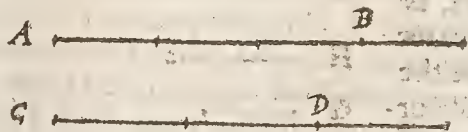
An other demonstration of the same.

¶ The fourth Proposition.

If there be foure quantities, and if the proportion of the first and the second to the second be greater then the proportion of the third and fourth to the fourth : then by diuision also the proportion of the first to the second, shall be greater then the proportion of the thirde to the fourth.

Suppose that the proportion of AB to B be greater then the proportion of CD to D . Then I say that by diuision also the proportion of A to B is greater then the proportion of C to D . For it cannot be the same. For then by composition AB should be to B as CD is to D . Neither also can it be lesse: for if the proportion of C to D be greater then the proportion of A to B , then by the former proposition, the proportion of CD to D should be greater then the proportion of AB to B : which is contrary also to the suppositiō. Wherefore the proportion of A to B is neither one and the same with the proportion of C to D , nor is lesse then it. Wherefore it is greater then it : which was required to be proued.

Demonstratiō leading to an impossibilitie.

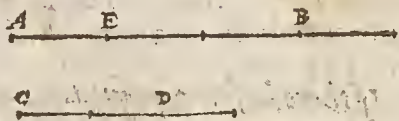


The

The fifth Booke

An other demonstration of the same affirmatively.

The same may also be proued affirmatiuely. Suppose that EB be vnto B as CD is to D . Now then (by the first part of the 10. of the fifth) EB shall be lesse then AB : and therefore by the common sentence, E is lesse then A , wherefore by the first part of the 8. of this booke, the proportion of E to B , is lesse then the proportion of A to B , but as E is to B , so is C to D : wherefore the proportion of C to D , is lesse then the proportion of A to B . Wherefore the proportion of A to B is greater then the proportion of C to D : which was required to be proued.

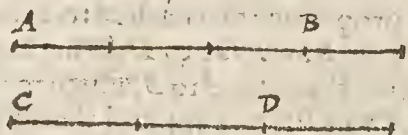


¶ The fifth Proposition.

If there be foure quantities, and if the proportion of the first and the second to the second be greater then the proportion of the third and the fourth to the fourth: then by euersion the proportion of the first and second to the first, shall be lesse then the proportion of the third and fourth to the third.

Demonstration.

Suppose that the proportion of AB to B be greater then the proportion of CD to D . Then I say that by euersion the proportion of AB to A is lesse then the proportion of CD to C . For by diuision by the former proposition the proportion of A to B is greater then the proportion of C to D . Wherefore by the first of these propositions conuerfedly, B hath vnto A a lesse proportiō thē hath D to C . Wherefore by the 3. of the same by composition, the proportion of AB to A is lesse thē the proportion of CD to C : which was required to be proued.

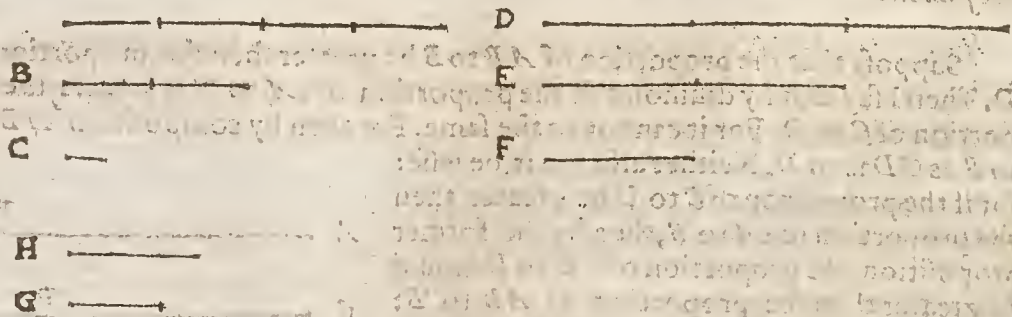


¶ The sixt Proposition.

If there be taken three quantities in one order, and as many in an other order, and if the proportion of the first to the second in the first order, be greater then the proportion of the first to the second in the latter order: then also the proportion of the first to the third in the first order, shall be greater then the proportion of the first to the third in the latter order.

Suppose that there be three quāties in one order A, B, C , & as many other quāties in an other order D, E, F . And let the proportion of A to B in the first order be greater then the proportion of D to E in the second order, and let also the proportion of B to C in the first order, be greater then the proportion of E to F in the second order. Then I say that

the proportion of A to C in the first order, is greater thē the proportion of D to F



in the second order. For let G be vnto C as E is to F . Now then by the first part of the 10. of this booke G shall be lesse then B . And therefore by the second parte of the 8. of the same,

Demonstration.

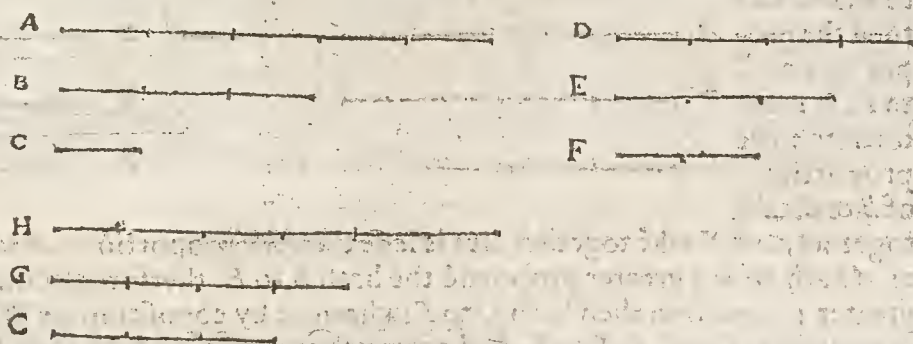
same, the proportion of A to G is greater then the proportion of A to B . Wherefore the proportion of A to G is muche greater then the proportion of D to E . Now then let H be vnto G as D is to E . Wherefore by the first part of the 10. of the same, A is greater the H . And therefore by the first part of the 8. of the same, the proportion of A to C is greater then the proportion of H to C . But by proportion of equality H is to C as D is to F (for H is to G as D is to E , and G is to C as E is to F . Wherefore by the 12. of the same A hath to C a greater proportion then hath D to F : which was required to be proued.

¶ The seuenth Proposition.

If there be taken three quantities in one order, and as many other in an other order, and if the proportion of the second to the third in the first order be greater then the proportion of the first to the second in the latter order, if also the proportion of the first to the second in the first order be greater then the proportion of the second to the third in the latter order: then shall the proportion of the first to the third in the first order be greater, then the proportion of the first to the third in the latter order.

Suppose that there be three quatities in one order A, B, C , and as many other in an other order D, E, F . And let the proportion of B to C in the first order, be greater then the proportion of D to E in the second order, and let also the proportion of A to B in the first order, be greater then the proportion of E to F in the second order. Then I say that A hath to C

a greater proportion then hath D to F . This pertaineth to proportion of equalitie. For let G be vnto C , as D is to E . And by the first part of the 10. of this booke, G shall be lesse the

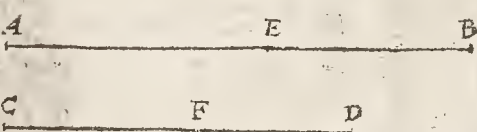


B . And therefore by the second part of the 8. of the same, the proportion of A to G is greater then the proportion of A to B . Wherefore A hath vnto G a much greater proportion then hath E to F . Now then let H be vnto G as E is to F . And by the first part of the 10. of the same, A shall be greater then H . And by the first part of the 8. of the same, the proportion of A to C is greater then the proportion of H to C . But by the 23. of the same the proportion of H to C is as the proportion of D to F (for G is to C as D is to E , and H is to G as E is to F .) Wherefore (by the 12. of the same) the proportion of A to C is greater then the proportion of D to F , which was required to be proued.

¶ The eight Proposition.

If the proportion of the whole to the whole, be greater then the proportion of a part taken away, to a part taken away: the shall the proportion of the residue vnto the residue be greater then the proportion of the whole to the whole.

Suppose that there be two quantities AB & CD : from which let there be cutte of these magnitudes AE and CF : and let the residue be EB and FD . And let the proportion of AB to CD be greater then the proportion of AE to CF . Then I say that the proportion of EB to FD is greater then the proportion of AB to CD . For (by the second



The fifth Booke

of these propositions now added) alternately the proportion of AB to AE is greater then the proportion of CD to CE . And therefore by euersion of proportion (by the 5. of the same) the proportion of AB to EB is lesse then the proportion of CD to ED . Wherefore agayne alternately the proportion of AB to CD is lesse then the proportion of EB to ED : which was required to be proued.

¶ The ninth Proposition.

If quantities how many soeuer in one order be compared to as many other in an other order, and if there be a greater proportion of euery one that goeth before to that wherunto it is referred, then of any that followeth to that wherunto it is referred: the proportion of them all taken together vnto all the other taken together, shall be greater, then the proportion of any that followeth to that wherunto it is compared, and also then the proportion of all them taken together to all the other taken together, but shall be lesse then the proportion of the first to the first.

Suppose that there be three quantities in one order, A, B, C , & as many other in an other order D, E, F . And let the proportion of A to D be greater then the proportion of B to E , let also the proportion of B to E , be greater then the proportion of C to F . Then I say that the proportion of ABC take al together, to DEF take altogether, is greater then the proportion of B

to E , and also

then the proportion of C

to F , & more

ouer then the

proportion

of B & C take

together, to EF take together, but is lesse then the proportion of A to D . For forasmuch

as A hath to D a greater proportion then hath B to E , therefore alternately A hath to B a

greater proportion then hath D to E : wherefore by composition AB hath to B a greater

proportion then hath DE to E . And agayne alternately AB hath to DE a greater proportion

then hath B to E . Wherefore by the former proposition A hath to D a greater

proportion then hath AB to DE . And by the same reason may it be proued that

hath to E a greater proportion then hath BC to EF . Wherefore A hath to D a

greater proportion then hath BC to EF . Wherefore alternately A hath to BC a greater

proportion then hath D to EF , wherefore by composition ABC hath to BC a

greater proportion then hath DEF to EF . Wherefore agayne alternately A

BC hath to DEF a greater proportion then hath BC to EF . Wherefore

(by the former proposition) the proportion of A to D is greater

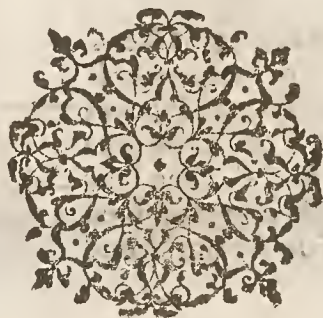
then the proportion of ABC to DEF : Which was required to be proued.

red to be proued.

Demonstration.

The end of the fifth booke

of Euclides Elementes.



The sixth booke of Eu- clides Elementes.



THIS SIXTH BOOKE, is, for vse and practife, a most speciall booke. In it are taught the proportions of one figure to an other figure, & of their sides the one to the other, and of the sides of one to the sides of an other, likewise of the angles of the one to the angles of the other. Moreouer it teacheth the description of figures like to figures geuen, and marueilous applications of figures to lines, euenly, or with decrease or excesse, with many other Theoremes, not onely of the proportions of right lined figures, but also of sectors of circles, with their angles.

*The argument
of this sixth
booke.*

On the Theoremes and Problemes of this Booke depend for the most part, the compositions of all instrumentes of measuring length, breadth, or deepenes, and also the reason of the vse of the same instrumentes, as of the Geometricall square, the Scale of the Astrolabe, the quadrant, the staffe, and such other. The vse of which instrumentes, besides all other mechanically instrumentes of rayeing vp, of mouing, and drawing huge things incredible to the ignorant, and infinite other giunnes (which likewise haue their groundes out of this Booke) are of wonderfull and vnspeakeable profite, besides the inestimable pleasure which is in them.

*This booke
necessary for
the vse of in-
strumentes of
Geometry.*

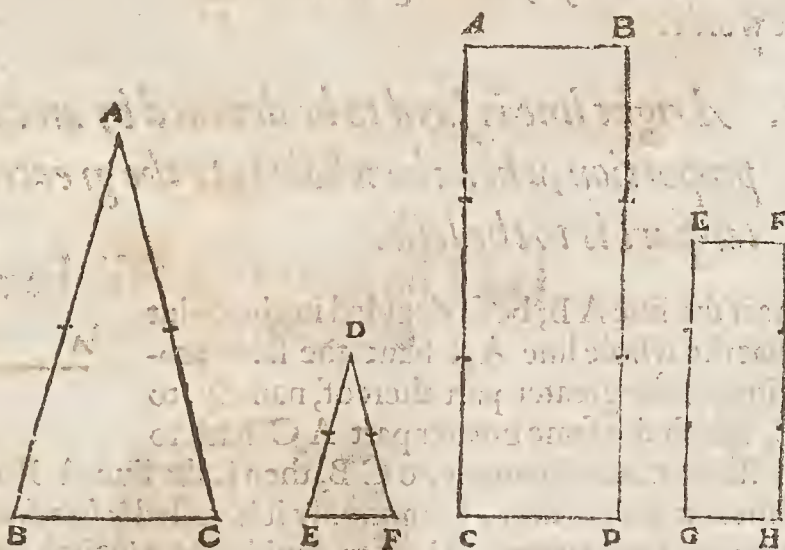
Definitions.

1. *Like rectiline figures are such, whose angles are equall the one to the other, and whose sides about the equall angles are proportionall.*

*The first de-
finition.*

As if ye take any two rectiline figures.

As for example, two triangles ABC , and DEF : if the angles of the one triangle be equall to the angles of the other, namely, if the angle A be equall to the angle D , and the angle B equall to the angle E , & also the angle C equall to the angle F . And moreouer,



if the sides which containe the equall angles be proportionall. As if the side AB haue that proportion to

Nu.j. the

The sixth Booke

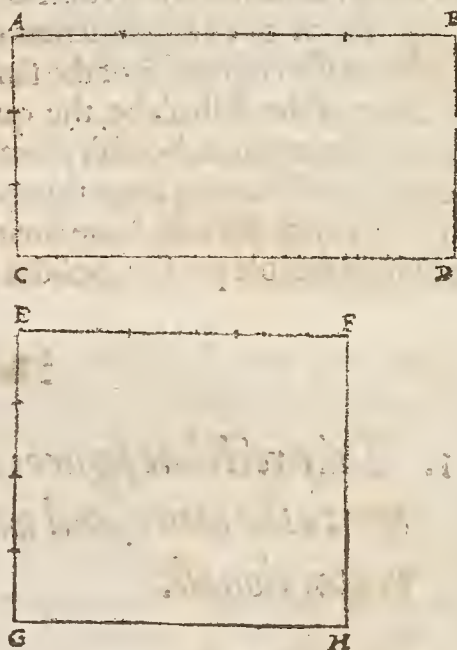
the side BC , which the side DE hath to the side EF , and also if the side BC be vnto the side CA , as the side EF is to the side FD , and moreouer, if the side CA be to the side AB , as the side FD is to the side DE , then are these two triangles sayd to be like: and so iudge ye of any other kinde of figures. As if in the parallelogrammes $ABCD$ and $EFGH$, the angle A be equall to the angle E , and the angle B equall to the angle F , and the angle C equall to the angle G , and the angle D equall to the angle H . And farthermore, if the side AC haue that proportion to the side CD which the side EG hath to the side GH , and if also the side CD be to the side DB as the side GH is to the side HF , and moreouer, if the side DB be to the side BA as the side HF is to the side FE , and finally, if the side BA be to the side AC as the side FE is to the side EG , then are these parallelogrammes like.

The second
definition.

2. *Reciprocall figures are those, when the termes of proportion are both antecedentes and consequentes in either figure.*

As if ye haue two parallelogrammes $ABCD$ and $EFGH$. If the side AB to the side EF , an antecedent of the first figure to a consequent of the second figure, haue mutuall the same proportion, which the side EG hath to the side AC an antecedent of the second figure to a consequent of the first figure: then are these two figures Reciprocall. They are called of some, figures of mutuall sides, and that vndoubtedly not amisse nor vnaptly. And to make thys definition more plaine, Campane and Pellitarius, and others, thus put it: *Reciprocall figures are, when the sides of either be mutuallly proportionall, as in the example and declaration before geuen.* Among the barbarous they are called *Mutekesia*, reseruing still the Arabike worde.

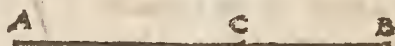
Reciprocall
figures called
mutuall fi-
gures.



The third de-
finition.

3. *A right line is sayd to be deuided by an extreme and meane proportion, when the whole is to the greater part, as the greater part is to the lesse.*

As if the line AB , be so deuided in the point C , that the whole line AB haue the same proportion, to the greater part thereof, namely, to AC , which the same greater part AC hath to the lesse part thereof, namely, to CB , then is the line AB deuided by an extreme and meane proportion. Commonly it is called *a line deuided by proportion hauing a meane and two extremes*. How to deuide a line in such sort was taught in the 11. Proposition of the second Booke, but not vnder this forme of proportion.



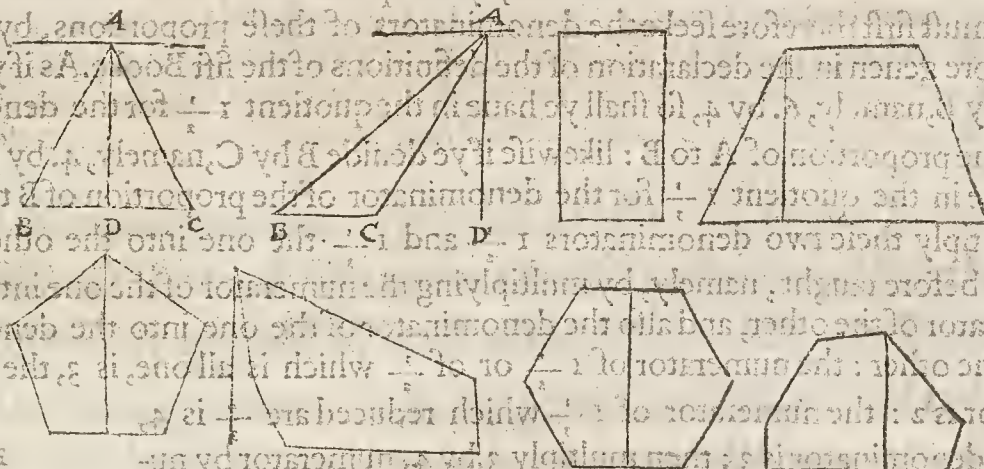
4. The

4. *The altitude of a figure is a perpendicular line drawn from the toppe to the base.*

The fourth definition.

As the altitude or hight of the triangle ABC , is the line AD being drawn perpendicularly from the poynt A , being the toppe or highest part of the triangle, to the base therof BC . So likewise in other figures as ye see in the examples here set.

which here hee calleth the altitude or height of a figure, in the first booke in the 35.



Proposition and certaine other following, he taught to be containd within two equidistant lines: so that figures to haue one altitude and to be containd within two equidistant lines, is all one. So in all these examples, if from the highest point of the figure ye draw an equidistant line to the base therof, and then fro that poynt draw a perpendicular to the same base, that perpendicular is the altitude of the figure.

5. *A Proportion is said to be made of two proportions or more, when the quantities of the proportions multiplied the one into the other, produce an other quantitie.*

The fifth definition.

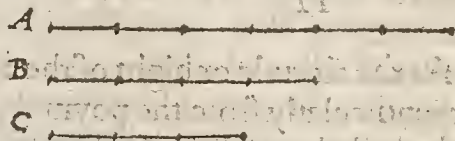
Of addition of proportions, hath bene somewhat sayd in the declaration of the 10. definition of the fift booke: which in substance is all one with that which is here taught by *Euclide*. By the name of quantities of proportions, he vnderstandeth the denominations of proportions. So that to adde two proportions together, or more, and to make one of them all, is nothyng els, but to multiply their quantities together, that is to multiply euer the denominator of the one by the denominator of the other. Thys is true in all kindes of proportion, whether it be of equalitie, or of the greater inequality, when the greater quantitie is referred to the lesse: or of the lesse inequality, when the lesse quantitie is referred to the greater: or of them mixed together. If the proportions be like, to adde two together is to double the one, to adde 3. like is to triple the one, and so forth in like proportions, as was sufficiently declared in the declaration of the 10. and 11. definitions of the fift Booke. Where it was shewed, that if there be 3. quantities in like proportion, the proportion of the first to the thyrd, is the proportion of the first to the second doubled: and if there be foure quantities in like proportion, the proportion of the first to the fourth shall be the proportion of the first to the second tripled: which thing how to do was there taught. Likewise in proportions vnlike, the proportion of the first extreme to the last is made of all the meane proportions set

By the name of quantities is vnderstanded the denominations of proportions.

The sixth Booke

Example of
this definitiō.

betwene them . Suppose three quantities A, B, C, so that let A haue to B sesquialtera proportion, namely, 6. to 4. And let B to C haue sesquitercia proportion, namely, 4. to 3. Now the proportion of A to C, the first to the thyrd, is made of the proportion of A to B, and of the proportion of B to C added together. If ye will adde them together, ye must by this definition multiply the quantitie or denominator of the one, by the quantitie or denominator of the other.



Ye must first therefore seeke the denominators of these proportions, by the rule before geuen in the declaration of the definitions of the fift Booke. As if ye deuide A by B, namely, 6. by 4, so shall ye haue in the quotient $1\frac{1}{2}$ for the denominator of the proportion of A to B: likewise if ye deuide B by C, namely, 4. by 3. ye shall haue in the quotient $1\frac{1}{3}$ for the denominator of the proportion of B to C, now multiply these two denominators $1\frac{1}{2}$ and $1\frac{1}{3}$ the one into the other, by the rule before taught, namely, by multiplying the numerator of the one into the numerator of the other, and also the denominator of the one into the denominator of the other: the numerator of $1\frac{1}{2}$ or of $\frac{1}{2}$ which is all one, is 3, the denominator is 2: the numerator of $1\frac{1}{3}$ which reduced are $\frac{1}{3}$ is 4,

the denominator is 3: then multiply 3. by 4, numerator by numerator, so haue ye 12. for a new numerator: likewise multiply 2. by 3. denominator by denominator, ye shall produce 6. for a new denominator: so haue you produced 12. and 6, betwene which there is dupla proportion. Which proportion is also betwene A and C, namely, 6. to 3, the first quantitie to the third. Wherefore the proportion of A to C is sayd to be made of the proportion of A to B and of the proportion of B to C, for that it is produced of the multiplication of the quantitie or denominator of the one, into the quantitie or denominator of the other. And so of all others be they neuer so many. As in these examples in numbers here set

Example in
numbers.

2. 3. 15. 18.

2. 3. 15. 18. In this example the lesse numbers are compared to the greater, as in the former the greater were compared to the lesse: the denominator of the proportion of 2. to 3. is $\frac{2}{3}$ that is, subsesquialtera, the denomination betwene 3. and 15. is $\frac{5}{3}$ or $\frac{5}{3}$ which is all one, that is, subquintupla, betwene 15. and 18. the denomination of the proportion is $\frac{3}{2}$ that is, subsesquiquinta, multiply all these denominations together: first the numerators: 2. into 1. produce 2, then 2. into 5. produce 10: which shall be a new numerator. Then the denominators: 3. into 5. produce 15: and 15. into 6. produce 90: which shall be a new denominator.

So haue you brought forth $\frac{10}{90}$ or $\frac{1}{9}$ which is proportion subnovecupla: which is also the proportion of 2. to 18. Wherefore the proportion of 2. to 18.

that is, of the extremes, namely, subnovecupla, is made of the proportions of 2. to 3: of 3. to 15: and of 15. to 18: namely, of subsesquialtera, subquintupla, and subsesquiquinta.

An other example, where the greater inequality and the lesse inequality are mixed together 6. 4. 2. 3. the denomination of the proportion of 6. to 4, is $1\frac{1}{2}$, of 4. to 2, is $\frac{2}{1}$, and of 2. to 3, is $\frac{2}{3}$: now if ye multiply as you ought, all these denominations together, ye shall produce 12. to 6, namely, dupla proportion.

Foras much as so much hath hetherto bene spoken of addition of proportions it shall

Another
example.

Foras much as so much hath hetherto bene spoken of addition of proportions it shall

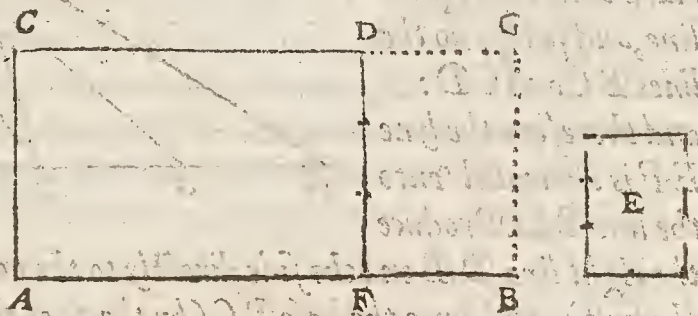
it shall not be unnecessary somewhat also to say of subtraction of them. Where it is to be noted, that as addition of them, is made by multiplicatiō of their denominations the one into the other: so is the subtraction of the one from the other done, by diuision of the denomination of the one by the denomination of the other. As if ye will from sextupla proportion subtrahe dupla proportion, take the denominations of them both. The denomination of sextupla proportion, is 6, the denomination of dupla proportion, is 2. Now deuide 6. the denomination of the one by 2. the denomination of the other: the quotient shall be 3: which is the denomination of a new proportion, namely, tripla: so that when dupla proportion is subtrahed from sextupla, there shall remayne tripla proportion. And thus may ye do in all others.

Of subtraction of proportion.

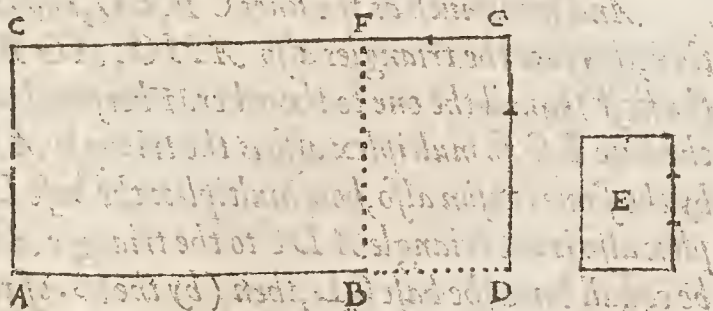
6. A Parallelogramme applied to a right line, is sayd to want in forme by a parallelogramme like to one geuen: whē the parallelogramme applied wanteth to the filling of the whole line, by a parallelogramme like to one geuen: and then is it sayd to exceede, when it exceedeth the line by a parallelogramme like to that which was geuen.

The sixth definition.

As let E be a Parallelograme geuen, and let AB be a right line, to whom is applied the parallelogramme ACDF. Now if it want of the filling of the line AB, by the parallelograme DFG B being like to the parallelogramme geuen E, then is the parallelogramme sayd to want in forme by a parallelogramme like vnto a parallelogramme geuen.



Likewise if it exceede, as the parallelogramme ACGD applied to the line AB, if it exceede it by the parallelogramme FGBD being like to the parallelogramme E which was geuen, then is the parallelogramme ABGD, sayd to exceede in forme by a parallelogramme like to a parallelogramme geuen.



This definition is added by *Flusates* as it seemeth, it is not in any cōmon Greke booke abroad, nor in any Commentary. It is for many Theoremes following very necessary.

Nn. iij.

The

The sixth Booke

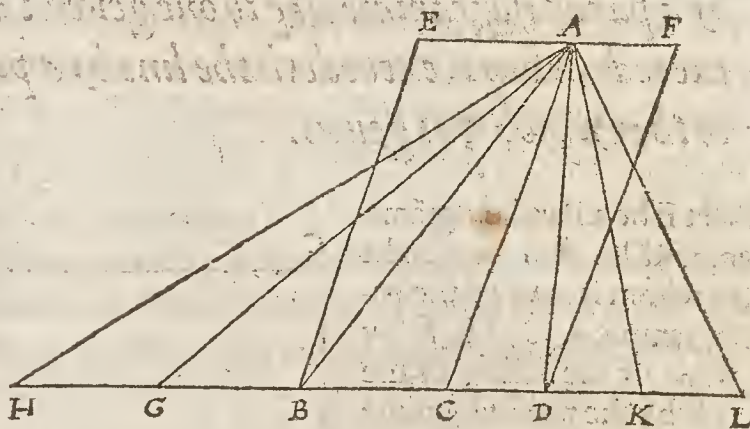
The 1. Theoreme. The 1. Proposition.

Triangles & parallelogrammes which are vnder one & the self same altitude: are in proportion as the base of the one is to the base of the other.



Suppose that there be two triangles ABC and ACD , & two parallelogrammes EC and CF . Which let be set vnder one and the self same altitude, or perpendicular line drawn from the toppe A to the base BD . Then I say that as the base BC is to the base CD so is the triangle ABC to the triangle ACD : and the parallelogramme EC to the parallelogramme CF . For forasmuch as the

Construction. parallelogrammes EC and CF are vnder one and the self same altitude, therefore the lines EA and AF make both one right line, and so also do the lines BC and CD : and therefore the lyne EF is a parallel vnto the line BD . Produce



the right line DB on eche side directly to the pointes H, L (by the 2. petition of the first) And vnto the base BC (by the 2. of the first) put as many equall lines as you will, as for example, two, namely, BG , and GH and vnto the base CD on the other side put as many equall as you did to the other base, which let be DK , and KL . Then draw these right lines AG, AH, AK , and AL .

Demonstration of the first part.

And forasmuch as the lines CB, BG , and GH are equall the one to the other, therefore the triangles also AHG, AGB and ABC , are (by the 38. of the first) equall the one to the other. Wherefore how multiplex the base HC is to the base BC , so multiplex also is the triangle AHC to the triangle ABC . And by the same reason also, how multiplex the base LC is to the base DC , so multiplex also is the triangle ALC to the triangle ADC . Wherefore if the base HC be equall vnto the base LC , then (by the 38. of the first) the triangle AHC is equall vnto the triangle ALC . And if the base HC exceede the base LC , then also the triangle AHC exceedeth the triangle ALC , and if the base be lesse, the triangle also shall be lesse. Now then there are foure magnitudes, namely, the two bases BC and CD , and the two triangles ABC , and ACD , and to the base BC , and to the triangle ABC , namely, to the first and the third, are taken eque-

mul

multiplixes, namely, the base HC and the triangle AHC , and likewise to γ base CD and to the triangle ADC , namely, to the second and the fourth, are taken certaine other equemultiplixes, that is, the base CL , and the triangle ALC . And it hath bene proued that if the multiplex of the first magnitude, that is, the base HC , do exceede the multiplex of the second, that is, the base CL , the multiplex also of the third, that is, the triangle AHC exceedeth the multiplex of γ fourth, that is, the triangle ALC , and if the said base HC be equall to the said base CL , the triangle also AHC is equall to the triangle ALC , and if it be lesse it is lesse. Wherefore by the sixth definition of γ fifth, as the first of the foresaid magnitudes is to the second, so is the third to the fourth. Wherefore as the base BC is to the base CD , so is the triangle ABC to the triangle ACD .

And because (by the 41. of the first) the parallelogramme EC is double to the triangle ABC , and (by the same) the parallelogramme FC is double to the triangle ACD , therefore the parallelogrammes EC and FC are equemultiplixes vnto the triangles ABC and ACD . But the partes of equemultiplixes (by the 15. of the fifth) haue one and the same proportion with their equemultiplixes. Wherefore as the triangle ABC is to the triangle ACD , so is the parallelogramme EC to the parallelogramme FC . And forasmuch as it hath bene demonstrated, that as the base BC is to the base CD , so is the triangle ABC , to the triangle ACD , and as the triangle ABC is to the triangle ACD so is the parallelogramme EC to the parallelogramme FC . Wherefore (by the 11. of the fifth) as the base BC is to the base CD , so is the parallelogramme EC to the parallelogramme FC . The parallelogrammes may also be demonstrated a part by themselves as the triangles are, if we describe vpon the bases BG , GH , and DK & KL parallelogrammes vnder the self same altitude that the parallelogrammes geuen are. Wherefore triangles and parallelogrammes which are vnder one and the selfe same altitude, are in proportion, as the base of the one is to γ base of the other: which was required to be demonstrated.

Demonstration of the second part.

Here Flusates addeth this Corollary.

If two right lines being geuen, the one of them be deuided how so euer: the rectangle figures containd vnder the whole line vndeuided, and eche of the segmentes of the line deuided, are in proportion the one to the other, as the segmentes are the one to the other. For imaginynge the figures BA and AD in the former description, to be rectangled, the rectangle figures containd vnder the whole right lyne AC , and the segmentes of the right line BD , which is cut in the poynt C , namely, the parallelogrammes BA and AD , are in proportion the one to the other, as the segmētes BC and CD are.

A Corollary added by Flusates.

 The 2. Theoreme. The 2. Proposition.

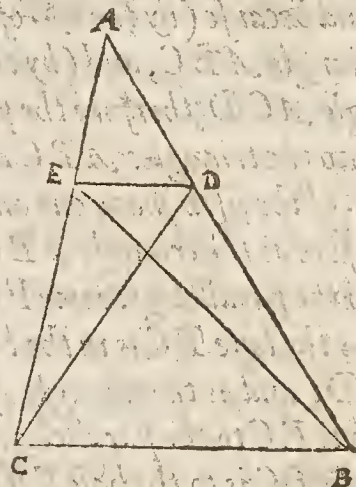
If to any one of the sides of a triangle be drawen a parallel
Nn. iiii. right

The sixth Booke

right line, it shall cut the sides of the same triangle proportionally. And if the sides of a triangle be cut proportionally, a right lyne drawn from section to section is a parallel to the other side of the triangle.

The first part
of this Theo-
reme.

Suppose that there be a triangle ABC , vnto one of the sides whereof, namely, vnto BC , let there be drawn a parallel line DE cutting the sides AC and AB in the pointes E and D . Then I say first that as BD is to DA , so is CE to EA . Draw a line from B to E , & also from C to D . Wherefore (by the 37. of the first) the triangle BDE is equall vnto the triangle CDE : for they are set vpon one and the same base DE , and are contained within the selfe same parallels DE and BC . Consider also a certaine other triangle ADE . Now thinges equall (by the 7. of the fifth) haue to one selfe thing one and the same proportion. Wherefore as $\hat{y}$ triangle BDE is to $\hat{y}$ triangle ADE , so is $\hat{y}$ triangle CDE to the triangle ADE . But as $\hat{y}$ triangle BDE is to $\hat{y}$ triangle ADE , so is $\hat{y}$ base BD to $\hat{y}$ base DA (by the first of this booke.) For they are vnder one and the selfe same toppes, namely, E , and therefore are vnder one and the same altitude. And by the same reason as the triangle CDE is to the triangle ADE , so is the lyne CE to the lyne EA . Wherefore (by the 11. of the fifth) as the line BD is to the line DA , so is the line CE to the line EA .



Demonstra-
tion of the
second part.

But now suppose that in $\hat{y}$ triangle ABC the sides AB & AC be cut proportionally, so $\hat{y}$ as BD is to DA , so let CE be to EA , & draw a line from D to E . Then secondly I say $\hat{y}$ the line DE is a parallel to $\hat{y}$ lyne BC . For the same order of construction being kept, for $\hat{y}$ as BD is to DA , so is CE to EA , but as BD is to DA , so is $\hat{y}$ triangle BDE to $\hat{y}$ triangle ADE (by the 1. of the sixth) & as CE is to EA , so (by $\hat{y}$ same) is the triangle CDE to $\hat{y}$ triangle ADE : therefore (by the 11. of the fifth) as the triangle BDE is to the triangle ADE , so is the triangle CDE to the triangle ADE . Wherefore either of these triangles BDE and CDE haue to the triangle ADE one and the same proportion. Wherefore (by the 9. of the fifth) the triangle BDE is equall vnto the triangle CDE , and they are vpon one and the selfe base, namely, DE . But triangles equall and set vpon one base, are also contained within the same parallel lines (by the 39. of the first.) Wherefore the line DE is vnto the line BC a parallel. If therefore to any one of the sides of a triangle be drawn a parallel line, it cutteth the other sides of the same triangle proportionally. And if the sides of a triangle be cut proportionally, a right lyne drawn from section to section, is parallel to the other side of the triangle: which thing was required to be demonstrated.

Here

¶ Here also Flussates addeth a Corollary.

If a line parallel to one of the sides of a triangle do cut the triangle, it shall cut off from the whole triangle a triangle like to the whole triangle. For as it hath bene proued it deuideth the sides proportionally. So that as EC is to EA , so is BD to DA , wherefore by the 18. of the fifth, as AC is to AE , so is AB to AD . Wherefore alternately by the 16. of the fifth as AC is to AB , so is AE to AD : wherefore in the two triangles EAD and CAB the sides about the common angle A are proportional. The sayd triangles also are equiangle. For forasmuch as the right lynes AEC and ADB do fall vpon the parallel lynes ED and CB , therefore by the 29. of the first they make the angles AED and ADE in the triangle ADE equall to the angles ACB and ABC in the triangle ACB . Wherefore by the first definition of this booke the whole triangle ABC is like vnto the triangle cut off ADE .

A Corollary
added by
Flussates.

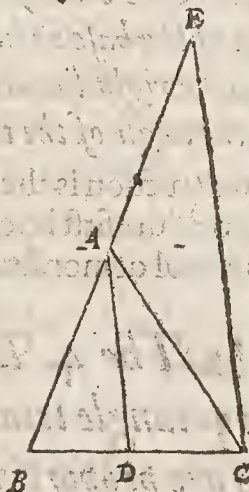
The 3. Theoreme.

The 3. Proposition.

If an angle of a triangle be deuided into two equall partes, and if the right line which deuideth the angle deuide also the base: the segmentes of the base shall be in the same proportion the one to the other, that the other sides of the triangle are. And if the segmētes of the base be in the same proportion that the other sides of the sayd triangle are: a right drawen from the toppe of the triangle vnto the section, shall deuide the angle of the triangle into two equall partes.



Suppose that there be a triangle ABC , and (by the 9. of the first) let the angle BAC be deuided into two equall partes by the right lyne AD , which let cut also the base BC in the point D . Then I say that as the segment BD is to the segment DC , so is the side BA to the side AC . For by the point C (by the 31. of the first) draw vnto the line DA a parallel line CE and extende the line BA till it concurre with the line CE in the point E , and do make the triangle BEC . But the lyne BA shall concurre with the line CE (by the 5. petition) for that the angles ECB , and BCE are lesse then two right angles. For the angle ECB is equall to the outwarde and opposite angle ADB (by the 29. of the first.) And the two angles ADB and DBA of the triangle BAD are lesse then two right angles (by the 17. of the first) Now forasmuch as vpon the parallels AD and EC falleth the right line AC , therefore



Construction.

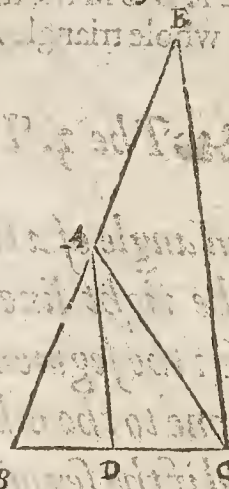
Demonstration of the first part.

(by

The sixth Booke

Demonstration of the second part, which is the converse of the first.

by the 29. of the first) the angle ACE is equall vnto the angle CAD . But vnto the angle CAD is the angle BAD supposed to be equall. Wherefore the angle BAD is also equall vnto the angle ACE . Againe because vpon the parallels AD and EC falleth the right line BAE , the outward angle BAD (by the 28. of the first) is equall vnto the inward angle AEC . But before it was proved that $\angle ACE$ is equall vnto $\angle BAD$, wherefore $\angle ACE$ is equall vnto $\angle AEC$. Wherefore (by 6. of 1. first) $\angle$ side AE is equall vnto the side AC . And because to one of $\angle$ sides of $\angle$ triangle BCE , namely, to EC is drawen a parallel line AD , therefore (by 2. of 1. sixt) as BD is to DC , so is BA to AE . But AE is equall vnto AC , therefore as BD is to DC so is BA to AC . But now suppose that as the segment BD is to the segment DC , so is the side BA to the side AC , & draw a line from A to D . Then I say that the angle BAC is by $\angle$ right line AD deuided into two equall partes. For the same order of construction remaining, for that as BD is to DC , so is BA to AC , but as BD is to DC , so is BA to AE (by the 2. of the sixt) for vnto one of the sides of the triangle BCE , namely, vnto the side EC is drawn a parallel line AD . Wherefore also as BA is to AC , so is BA to AE (by the 11. of the fifth) Wherefore (by the 9. of the fifth) AC is equall vnto AE . Wherefore also (by the 5. of the first) the angle AEC is equall vnto the angle ACE , but the angle AEC (by the 29. of the first) is equall vnto the outward angle BAD : and the angle ACE is equall vnto the angle CAD which is alternate vnto him: wherefore the angle BAD is equall vnto the angle CAD . Wherefore the angle BAC is by the right line AD deuided into two equall partes. Wherefore if an angle of a triangle be deuided into two equall partes, and if the right line which deuideth the angle cut also the base, the segmentes of the base shall be in the same proportion the one to the other, that the other sides of the said triangle are. And if the segmentes of the base be in the same proportion that the other sides of the sayd triangle are, a right line drawen from the toppe of the triangle vnto the section deuideth the angle of the triangle into two equall partes.



This construction is the halfe part of that Gnomical figure described in the 43. proposition of the first booke; which Gnomical figure is of great vse in a maner in all Geometrical demonstrations.

The 4. Theoreme. The 4. Proposition.

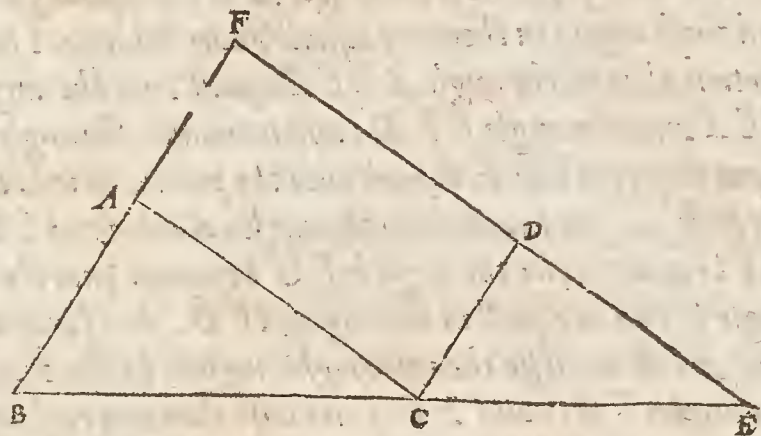
In equiangle triangles, the sides which cōtaine the equall angles are proportionall, and the sides which are subtended vnder the equall angles are of like proportion.

Suppose



*S*uppose that there be two equiangle triangles ABC and DCE : and let the angle ABC of the one triangle, be equall vnto $\angle DCE$ of the other triangle, and the angle BAC equall vnto $\angle CDE$, and moreover, the angle ACB equall vnto the angle DEC . Then I say, that those sides of $\angle$ triangles ABC , & DCE , which include the equall angles, are proportionall, and the sides which are subtended vnder the equall angles are of like proportion. For let two sides of the sayd triangles, namely, two of those sides which are subtended vnder equall angles: as for example the sides BC and CE , be so set that they both make one right line. And because the angles ABC & ACB are lesse then two right angles (by the 17. of the first): but the angle ACB is equall vnto the angle DEC : therefore $\angle$ angles ABC & DEC are lesse the two right angles. Wherefore the lines BA & ED being produced, will at the length meete together. Let them meete and ioine together in the poynt F . And because by supposition the angle DCE is equall vnto the angle ABC , therefore the line BF is (by the 28. of the first) a parallell vnto the line CD . And forasmuch as by supposition the angle ACB is equall vnto the angle DEC , therefore againe (by the 28. of the first) the line AC is a parallell vnto the line FE . Wherefore $FADC$ is a parallelogramme. Wherefore the side FA is equall vnto the side DC : and the side AC vnto the side FD (by the 34. of the first). And because vnto one of the sides of the triangle BFE , namely, to FE is drawn a parallell line AC , therefore as BA is to AF , so is BC to CE (by the 2. of the sixth). But AF is equall vnto CD . Wherefore (by the 11. of the fifth) as BA is to CD , so is BC to CE , which are sides subtended vnder equall angles. Wherefore alternately (by the 16. of the fifth) as AB is to BC , so is DC to CE . Againe forasmuch as CD is a parallell vnto BF , therefore againe (by the 2. of the sixth) as BC is to CE , so is FD to DE . But FD is equall vnto AC . Wherefore as BC is to CE , so is AC to DE , which are also sides subtended vnder equall angles. Wherefore alternately (by the 16. of the fifth) as BC is to CA , so is CE to ED . Wherefore forasmuch as it hath bene demonstrated, that as AB is vnto BC , so is DC vnto CE : but as BC is vnto CA , so is CE vnto ED : it followeth of equalitie (by the 22. of the fifth) that as BA is vnto AC , so is CD vnto DE . Wherefore in equiangle triangles, $\angle$ sides which include $\angle$ equall angles are proportionall: and $\angle$ sides which are subtended vnder the equall angles are of like proportion: which was required to be demonstrated.

Construction.



Demonstration.

The

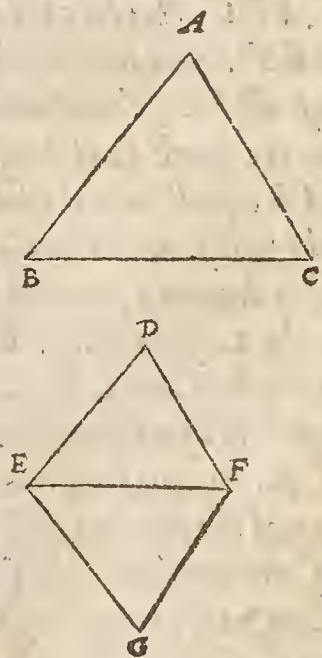
The 5. Theoreme. The 5. Proposition.

If two triangles haue their sides proportionall, the triangles are equiangle, and those angles in the are equall, vnder which are subtended sides of like proportion.

This is the
conuerse of
the former
proposition.

Suppose that there be two triangles ABC & DEF , hauing their sides proportionall, as AB is to BC , so let DE be to EF : & as BC is to AC , so let EF be to DF : and moreouer, as BA is to AC , so let ED be to DF . Then I say, that the triangle ABC is equiangle vnto the triangle DEF : and those angles in them are equall vnder which are subtended sides of like proportion, that is, the angle ABC is equall vnto the angle DEF : and the angle BCA vnto the angle EFD : and moreouer, the angle BAC to y angle EDF .

Construction. Vpon the right line EF , and vnto the pointes in it E & F , describe (by the 23. of the first) angles equall vnto the angles ABC & ACB , which let be FEG and EFG , namely, let the angle FEG be equall vnto the angle ABC , and let the angle EFG be equall to the angle ACB . And forasmuch as the angles ABC and ACB are lesse then two right angles (by the 17. of the first): therefore also the angles FEG and EFG are lesse then two right angles. Wherefore (by the 5. petition of y first) y right lines EG & FG shall at y length concurre. Let the concurre in the poynt G . Wherefore EFG is a triangle. Wherefore the angle remayning BAC is equall vnto the angle remayning EGF . (by the first Corollary of the 32. of the first). Wherefore the triangle ABC is equiangle vnto the triangle GEF . Wherefore in the triangles ABC and GEF the sides, which include the equall angles (by the 4. of the first) are proportionall, and the sides which are subtended vnder the equall angles are of like proportion. Wherefore as AB is to BC , so is GE to EF . But as AB is to BC , so by supposition is DE to EF . Wherefore as DE is to EF , so is GE to EF (by the 11. of the first). Wherefore either of these DE and EG haue to EF one and the same proportion. Wherefore (by the 9. of the first) DE is equall vnto EG . And by the same reason also DF is equall vnto FG . Now forasmuch as DE is equall to EG and EF is common vnto them both, therefore these two sides DE & EF , are equall vnto these two sides GE and EF , and y base DF is equall vnto the base FG . Wherefore the angle DEF (by the 8. of the first) is equall vnto the angle GEF : and the triangle DEF (by the 4. of the first) is equall vnto the triangle GEF : and the rest of the angles of the one triangle are

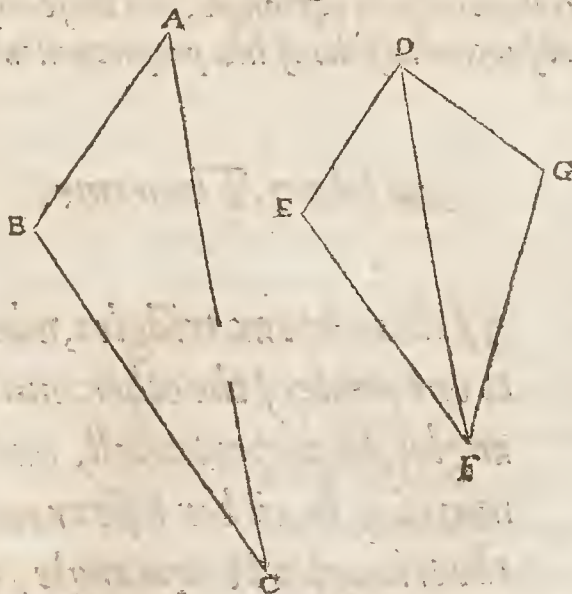


are equall vnto the rest of the angles of the other triangle the one to y other, vnder which are subtended equall sides. Wherefore the angle $\angle DFE$ is equall vnto the angle $\angle GFE$: and the angle $\angle EDF$ vnto the angle $\angle EGF$. And because the angle $\angle FED$ is equall vnto the angle $\angle GEF$: but the angle $\angle GEF$ is equall vnto the angle $\angle ABC$: therefore the angle $\angle ABC$ is also equall vnto the angle $\angle FED$. And by the same reason the angle $\angle ACB$ is equall vnto y angle $\angle DFE$: and moreouer, the angle $\angle BAC$ vnto the angle $\angle EDF$. Wherefore the triangle ABC is equiangle vnto the triangle DEF . If two triangles therefore haue their sides proportionall, the triangles shall be equiangle, & those angles in them shall be equall, vnder which are subtended sides of like proportion: which was required to be demonstrated.

 The 6. Theoreme. The 6. Proposition.

If there be two triangles wherof the one hath one angle equall to one angle of the other, & the sides including the equall angles be proportionall: the triangles shall be equiangle, and those angles in them shall be equall, vnder which are subtended sides of like proportion.

 Suppose that there be two triangles ABC , and DEF , which let haue the angle $\angle BAC$ of the one triangle equall vnto the angle $\angle EDF$ of the other triangle, and let the sides including the equall angles be proportionall, that is, as BA is to AC , so let ED be to DF . Then I say, that y triangle ABC is equiangle vnto the triangle DEF : and the angle $\angle ABC$ is equall vnto the angle $\angle DEF$, and the angle $\angle ACB$ equall vnto the angle $\angle DFE$, which angles are subtended to sides of like proportion. Vnto the right line DF , and to the poynt in it D (by the 23. of the first) describe vnto either of y angles $\angle BAC$ and $\angle EDF$, an equall angle $\angle FDG$. And vnto the right line DF , and vnto the point in it F (by y same) describe vnto y angle $\angle ACB$ an equall angle $\angle DFG$. And forasmuch as the two angles $\angle BAC$ and $\angle ACB$, are (by the 17. of the first) lesse then two right angles: therefore also the two angles $\angle FDG$ and $\angle DFG$, are lesse then two right angles. Wherefore y lines DG & FG being produced, shall cōcurre (by the 1. petition). Let the concurre in the point G . Wherefore DFG is a triangle.



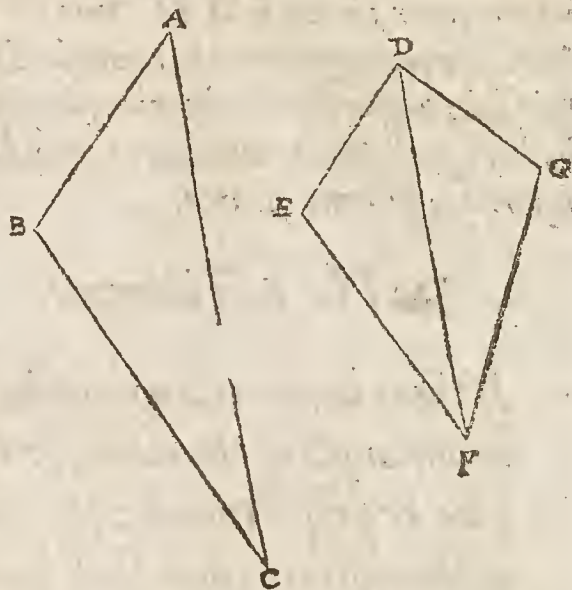
Construction.

Qo.j.

Wherefore

The sixth Booke

Wherefore the angle remaining ABC is equall vnto the angle remaining DGF (by the 32. of the first). Wherefore the triangle ABC is equiangle vnto the triangle DGF . Wherefore as BA is in proportion to AC , so is GD to DF (by the 4. of the sixth). But it is supposed, that as BA is to AC , so is ED to DF . Wherefore (by the 11. of the fifth) as ED is to DF , so is GD to DF . Wherefore (by the 9. of the fifth) ED is equall vnto GD . And DF is common vnto them both. Now then there are two sides ED and DF equall vnto two sides GD and DF : and the angle EDF (by supposition) is equall vnto the angle GDF . Wherefore (by the 4. of the first) the base EF is equall vnto the base GF , and the triangle DEF is (by the same) equall vnto the triangle GDF , and the other angles remainyng in them are equall the one to the other, vnder which are subtended equall sides. Wherefore the angle DFG is equall vnto the angle DFE : and the angle DGF vnto the angle DEF . But the angle DFG is (by construction) equall vnto $\angle ACB$. And the angle DGF is as it hath bene proued, equall to $\angle ABC$. Wherefore also $\angle ACB$ is equall vnto the angle DFE . And $\angle ABC$ is equall to the angle DEF . But by supposition the angle BAC is equall vnto the angle EDF . Wherefore the triangle ABC is equiangle vnto the triangle DEF . If therefore there be two triangles, whereof the one hath one angle equall to one angle of the other, and if also the sides including the equall angles be proportionall: then shall the triangles also be equiangle, and those angles in them shalbe equall, vnder which are subtended sides of like proportion: which was required to be proued.



The 7. Theoreme. The 7. Proposition.

If there be two triagles, wherof the one hath one angle equal to one angle of the other, and the sides which include the other angles, be proportionall, and if either of the other angles remainyng be either lesse or not lesse then a right angle: the shal the triangles be equiangle, and those angles in them shall be equall, which are containyd vnder the sides proportionall.

Suppose



Suppose that there be two triangles ABC and DEF , which let have one angle of the one, equall to one angle of the other, namely, the angle BAC equall vnto the angle EDF . And let the sides which include the other angles, namely, the angles ABC and DEF be proportionall, so that as AB is to BC , so let DE be to EF . And let the other angles remayning, namely, ACB and DFE be first either of them lesse then a right angle. Then I say that the triangle ABC is equiangle vnto the triangle DEF . And that the angle ABC is equall vnto the angle DEF , namely, the angles which are contained vnder the sides proportionall, and that the angle remayning, namely, $\angle C$ is equall vnto the angle remayning, namely, to $\angle F$. For first the angle ABC is either equall to the angle DEF , or els vnequall. If the angle ABC be equall to the angle DEF , then the angle remayning, namely, ACB , shall be equall to the angle remayning DFE (by the corollary of the 32. of the first) And therefore the triangles ABC and DEF are equiangle. But if the angle ABC be vnequall vnto the angle DEF , then is the one of them greater then the other. Let the angle ABC be the greater, and vnto the right line AB and vnto the point in it B (by the 23. of the first) describe vnto the angle DEF an equall angle ABG . And forasmuch as the angle A is equall vnto the angle D , and the angle ABG is equall vnto the angle DEF , therefore the angle remayning AGB is equall vnto the angle remayning DFE (by the corollary of the 32. of the first). Wherefore the triangle ABG is equiangle vnto the triangle DEF . Wherefore (by the 4. of the sixth) as the side AB is to the side BG , so is the side DE to the side EF . But by supposition the side DE is to the side EF , as the side AB is to the side BC . Wherefore (by the 11. of the fifth) as the side AB is to the side BC so is the same side AB to the side BG . Wherefore AB hath to either of these BC and BG one and the same proportion, and therefore (by the 9. of the fifth) BC is equall vnto BG . Wherefore (by the 5. of the first) $\angle BGC$ is equall vnto $\angle BCG$: but by supposition $\angle BCG$ is lesse then a right angle. Wherefore the angle BGC is also lesse then a right angle. Wherefore (by the 13. of the first) the side angle vnto it, namely, AGB is greater then a right angle, and it is already proued that the same angle is equall vnto the angle F . Wherefore the angle F is also greater then a right angle. But it is supposed to be lesse which is absurde. Wherefore the angle ABC is not vnequall vnto the angle DEF , wherefore it is equall vnto it. And the angle A is equall vnto the angle D by supposition. Wherefore the angle remayning, namely, C , is equall vnto the angle remayning, namely, to F (by the corollary of the 32. of the first) Wherefore the triangle ABC is equiangle vnto the triangle DEF .

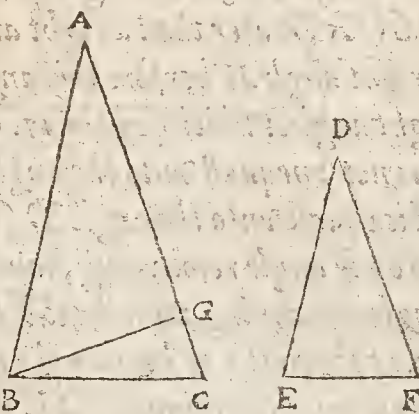
But now suppose that either of the angles ACB and DFE be not lesse then

O. ij.

a right

The first
part of this
proposition.

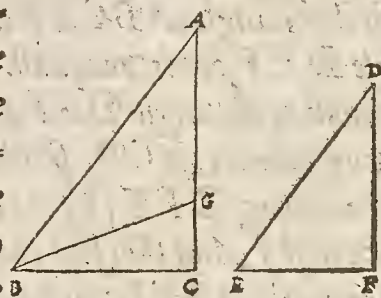
Demonstra-
tion leading to
an impossibi-
litie.



The second
part of this
proposition.

The sixth Booke

a right angle. That is, let either of them be a right angle, or either of them greater then a right angle. Then I say againe that in that case also the triangle ABC is equiangle vnto the triangle DEF . For if either of them be a right angle, forasmuch as all right angles are (by the 4. petition) equall the one to the other, straight way will follow the intent of the proposition. But if either of them be greater then a right angle, then the same order of construction that was before being kept, we may in like sort proue that the side BC is equall vnto the side EG . Wherefore also the angle BCG is equall vnto the angle BGC . But the angle BCG is greater then a right angle. Wherefore also the angle BGC is greater then a right angle. Wherefore two angles of the triangle BGC are greater then two right angles: which (by the 17. of the first) is impossible. Wherefore the angle ABC is not vnequall vnto the angle DEF . And therefore is it equall: but the angle A is equall vnto the angle D (by supposition) Wherefore the angle remayning, namely, C is equal vnto the angle remayning, namely, to F (by the corollary of the 32. of the first). Wherefore the triangle ABC is equiangle vnto the triangle DEF . If therefore there be two triangles wherof the one hath one angle equall to one angle of the other, and the sides which include the other angle be proportionall, and if either of the other angles remaining be either lesse or not lesse then a right angle, the triangle shall be equiangle, and those angles in them shall be equall which are contained vnder sides proportionall: which was required to be proued.



The 8. Theoreme. The 8. Proposition.

If in a rectangle triangle be drawen from the right angle vnto the base a perpendicular line, the perpendicular line shall deuide the triangle into two triangles like vnto the whole, and also like the one to the other.

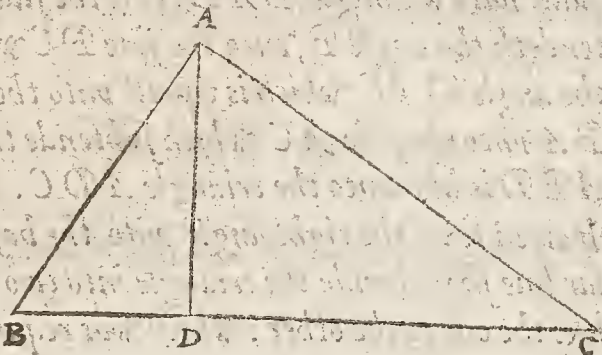
Construction.



Suppose that there be a rectangle triangle ABC , whose right angle let be BAC : and (by the 12. of the first) from the point A to the line BC let there be drawen a perpendicular line AD , which perpendicular line let deuide the whole triangle ABC into these two triangles ABD and ADC . (Note that this perpendicular line AD , drawen from the right angle to the base, must needs fall within the triangle ABC , & so deuide the triangle into two triangles. For if it should fall without, then producing the side BC vnto the perpendicular line, there should be made a triangle, whose outward angle being an acute angle, should be lesse then the inward and opposite angle which is a right angle: which is contrary to the 16. of the first. Neither

ther can it fall vpon any of the sides AB or AC : for then two angles of one and the selfe same triangle should not be lesse then two right angles, contrary to the selfe same 17. of the first. Wherefore it falleth within the triangle ABC .)

Then I say, that either of these triangles ABD and ADC , are like vnto the whole triangle ABC : and moreover, that they are like the one to the other. First that the triangle ABD is like vnto the whole triangle ABC is thus proued. Forasmuch as (by the 4. petition) the angle BAC is equall vnto the angle ADB , for either of them



Demonstration.

is a right angle. And in the two triangles ABC and ABD the angle B is common. Wherefore the angle remayning, namely, ACB is (by the Corollary of the 32. of the first) equall vnto the angle remayning, namely, to BAD . Wherefore the triangle ABC is equiangle vnto the triangle ABD . Wherefore the sides which containe the equall angles, are (by the 4. of the sixt) proportionall. Wherefore as the side CB which subtendeth $\angle$ right angle of the triangle ABC , is vnto the side BA which subtendeth the right angle of the triangle ABD , so is the same side AB which subtendeth the angle C of the triangle ABC , vnto the side BD which subtendeth $\angle$ BAD of the triangle ABD , which is equall vnto the angle C : and moreover, the side AC vnto the side AD which subtend the angle B common to both the triangles. Wherefore the triangle ABC is like vnto the triangle ABD (by the 1. definition of the sixt). In like maner also may we proue, that the triangle ADC is like vnto the triangle ABC . For the right angle ADC is equall to the right angle BAC , & the angle at the point C is common to either of those triangles. Wherefore the angle remayning, namely, DAC is equall to the angle remayning, namely, to ABC (by $\angle$ Corollary of the 32. of the first). Wherefore the triangles ABC & ADC are equiangle. And therefore (by the 4. of the sixt) the sides which are about the equall angles are proportionall. Wherefore as in the triangle ABC the side BC is to the side CA , so in the triangle ADC is the side AC to the side DC : and againe, as in the triangle ABC the side CA is to the side AB , so in the triangle ADC is the side CD to the side DA . And moreover, as in the triangle ABC the side CB is to the side BA , so in the triangle ADC is the side CA to the side AD . Wherefore the triangle ADC is like vnto the whole triangle ABC . Wherefore either of these triangles ABD , & ADC is like vnto $\angle$ whole triangle ABC .

I say also, that the triangles ABD and ADC are like the one to the other. For forasmuch as the right angle BDA is equall vnto $\angle$ right angle ADC (by the 4. petition) and as it hath already bene proued, the angle BAD is equall vnto the angle C : therefore the angle remayning, namely, B is equall vnto the angle remayning, namely, to DAC (by the Corollary of the 32. of the first). Wherefore

Oo. iij. fore

The sixth Booke

fore the triangle ABD is equiangle vnto the triangle ADC . Wherefore as the side BD which in the triangle ABD subtendeth the angle BAD is vnto the side DA which in the triangle ADC subtendeth the angle C which is equall vnto the angle BAD , so is the side AD which in γ triangle ABD subtendeth the angle B , vnto the side DC which in the triangle ADC subtendeth the angle DAC which is equall vnto the angle B : and moreouer, so is the side BA vnto the side AC which subtende the right angles. Wherefore the triangle ABD is like vnto the triangle ADC . If therefore in a rectangle triangle be drawen from the right angle vnto the base a perpendicular line, the perpendicular line shall deuide the triangle into two triangles like vnto the whole, and also like the one to the other: which was required to be proued.

Corollary.

Hereby it is manifest, that if in a rectangle triangle be drawen from the right angle vnto the base a perpendicular line, the same line drawen is a meane proportionall betwene the sections of the base: and moreouer, betwene the whole base and either of the sections, the side annex to the sayd section is the meane proportionall. For is was proued, that as CD is to DA , so is DA to DB : and moreouer, as CB is to BA , so is BA to BD : and finally, as BC is to CA , so is CA to CD .

The 1. Probleme. The 9. Proposition.

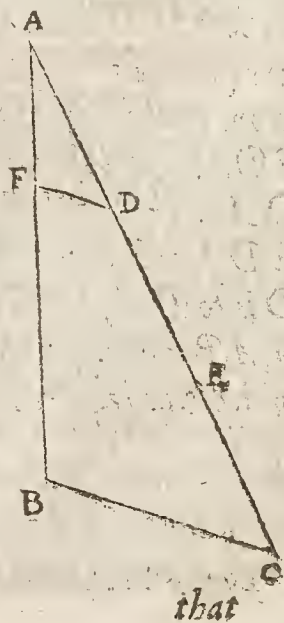
A right line being geuen, to cut off frō it any part appoynted.

Construction.



Et the right line geuen be AB . It is required that from the same line AB be cut off any part appoynted. Suppose that a thyrd part be appoynted to be cut off. From γ point A draw a right line AC making with the line AB an angle: and in the line AC take a poynt at all aduentures, which let be D . And beginning at D put vnto AD two equall lines DE & EC (by the 2. of the first). And draw a right line from B to C : and by the point D (by the 31. of the first) draw vnto BC a parallell line DF . Now forasmuch as vnto one of γ sides of the triangle ABC , namely, vnto γ side BC is drawen a parallell line FD , it followeth by γ 2. of this booke,

Demonstration.



that

that as CD is in proportion vnto DA , so is BF to FA . But (by construction) CD is double to DA . Wherefore $\therefore$ line BF is also double to the line FA . Wherefore the line BA is treble vnto the line AF . Wherefore from the right line geuen AB , is cut of a third part appoynted, namely, AF : which was required to be done.

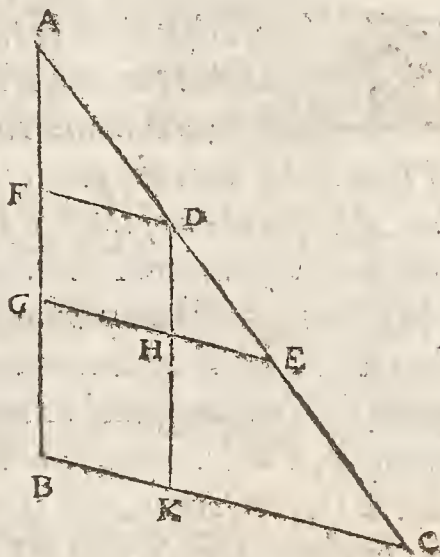
The 2. Probleme. The 10. Proposition.

To deuide a right line geuen not deuided, like vnto a right line geuen beyng deuided.

Suppose that the right line geuen not deuided be AB , and the right lyne geuen being deuided, let be AC . It is required to deuide $\therefore$ line AB which is not deuided like vnto the line AC which is deuided. Suppose the lyne AC be deuided in the pointes D and E , & let $\therefore$ lines AB & AC so be put, that they make an angle at all aduentures, and draw a line from B to C , and by the pointes D and E draw vnto the line BC (by the 31. of the first) two parallel lines DF and EG : and by the point D vnto the line AB (by the same) draw a parallel line DK . Wherefore either of these figures FH and HKB are parallelogrammes. Wherefore the line DH is equall vnto the line FG , and the line HK is equall vnto the line GB . And because to one of the sides of the triangle DKC , namely, to the side KC is drawn a parallel line HE , therefore the line CE (by the 2. of the sixt) is in proportion vnto the line ED as the line KH is to the line HD : but the line KH is equall vnto the line BG , and the line HD is equal vnto the line GF . Wherefore (by the 11. of the fift) as CE is vnto ED , so is BG to GF . Agayne because to one of the sides of the triangle AGE , namely, to GE is drawn a parallel lyne FD , therefore the line ED (by the 2. of the sixth) is in proportion vnto the lyne DA , as the line GF is to the line FA . And it is already proued that as CE is to ED , so is BG to GF . Wherefore as CE is to ED , so is BG to GF , and as ED is to DA , so is GF to FA . Wherefore the right line geuen not deuided, namely, AB is deuided like vnto the right line geuen being deuided, which is AC : which was required to be done.

Construction.

Demonstration.



A Corollary out of Fluxates.

By this Proposition we may deuide any right line geuen, according to the proportion

A Corollary out of Fluxates.

portion of any right lynes geuen. For let those right lynes hauyng proportion be ioyned together directly, that they may make all one right lyne, and then ioyne them to the lyne geuen anglewise. And so proceede as in the proposition, where you see that the right line geuen AB is deuided into the right lynes AF , FG and GB which haue the selfe same proportion that the right lines AD , DE , and EC haue.

By this and the former proposition may a right line be deuided into what partes soeuer you will.

By this and the former proposition also may a right line geuen be easily deuided into what partes so euer you will name. As if you will deuide the line AB into three equall partes, let the lyne DE be made equall to the lyne AD , and the lyne EC made equall to the same by the third of the first. And then vsing the selfe same maner of construction that was before: the lyne AB shall be deuided into three equall partes. And so of any kynde of partes whatsoever.

The 3. Probleme. The 11. Proposition.

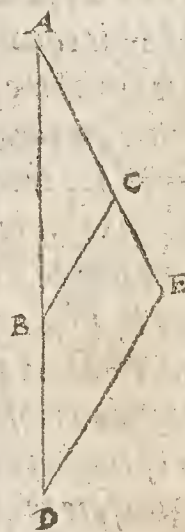
Unto two right lines geuen, to finde a third in proportion with them.

Suppose that there be two right lines geuen BA and AC , and let them be so put that they comprehend an angle howsoeuer it be. It is required to finde vnto BA and vnto AC a third line in proportion. Produce y lynes

Construction.

AB and AC vnto the pointes D and E . And vnto the line AC (by the 2. of the first) put an equall line BD , and draw a lyne from B to C . And by the pointe D (by the 31. of the first) draw vnto the lyne BC a parallel lyne DE , which let concurre with the line AC in the point E . Now forasmuch as vnto one of the sides of the triangle ADE , namely, to DE is drawne a parallel line BC ; therefore as AB is in proportion vnto BD , so (by the 2. of the sixth) is AC vnto CE . But the lyne BD is equall vnto the line AC . Wherefore as the lyne AB is to the line AC , so is the line AC to the line CE . Wherefore vnto the two right lines geuen AB and AC is found a third line CE in proportio with them: which was required to be done.

Demonstration.

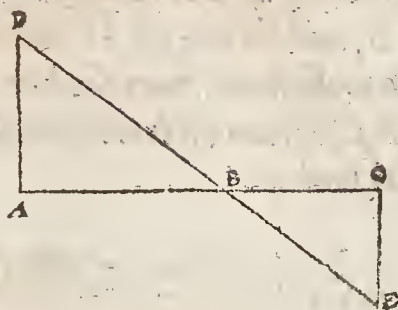


¶ An other way after Pelitarins.

An other way after Pelitarins.

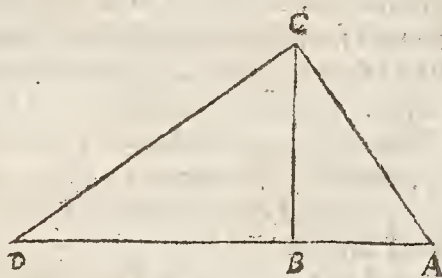
Let the lines AB and BC be set directly in such sort that they both make one right line. Then frō the point A erect the lyne AD making with the lyne AB an angle at all adventures. And put the lyne AD equall to the lyne BC . And draw a right line from D to B which produce beyond the poynt B vnto the point E . And by the point C draw vnto the lyne DA a parallel lyne CE concurring with the lyne DE in the point E . Then I say that this line CE is the third line proportionall with the lines AB and BC . For forasmuch

asmuch as by the 15. of the first, the angle B of the triangle A B D is equall to the angle B of the triangle C B E, and by the 29. of the same, the angle A is equall to the angle C, and the angle D to the angle E: therefore by the 4. of this booke A B is to D A, as B C is to C E. Wherefore (by the 11. of the fifth) A B is to B C as B C is to C E: which was required to be done.



¶ An other way also after Pelitarinus.

Let the lines A B and B C be so ioyned together, that they may make a right angle, namely, A B C. And draw a line from A to C, and from the point C draw vnto the line A C a perpendicular C D (by the 11. of the first) And produce the lyne C D till it concurre with the line A B produced vnto the pointe D. Then I say that the line B D is a third lyne proportionall with the lines A B and B C: which thing is manifest by the corollary of the 8. of this booke.



An other way after Pelitarinus.

The 4. Probleme. The 12. Proposition.

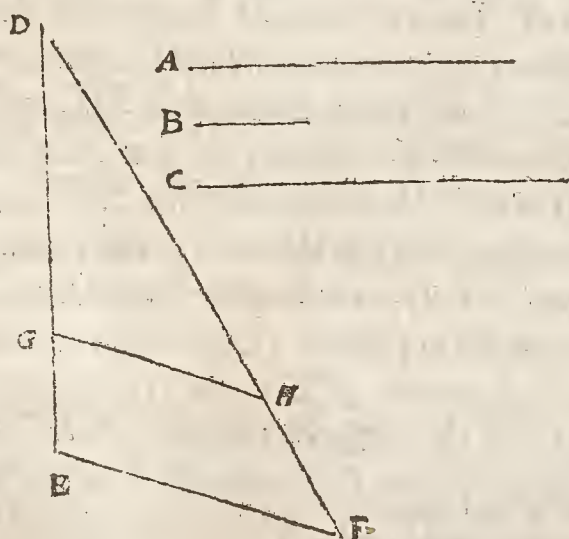
Unto three right lines geuen to finde a fourth in proportion with them.



Suppose that the three right lines geuen be A, B, C. It is required to finde vnto A, B, C, a fourth line in proportiō with them. Let there be taken two right lines D E & D F comprehending an angle as it shall happen, namely, E D F. And (by the 2. of the first) vnto the line A put

Construction.

an equall line D G. And vnto the line B (by the same) put an equall line G E. And moreouer, vnto y line C put an equall line D H. Then draw a line frō G to H. And by the poynt E (by the 31. of the first) draw vnto the line G H a parallell line E F. Now forasmuch as vnto one of the sides of the triangle D E F, namely, vnto y side E F is drawen a parallell line G H: therefore (by the 2. of the sixt) as the line D G is to the line G E, so is the line D H to the line H F.



Demonstration.

But

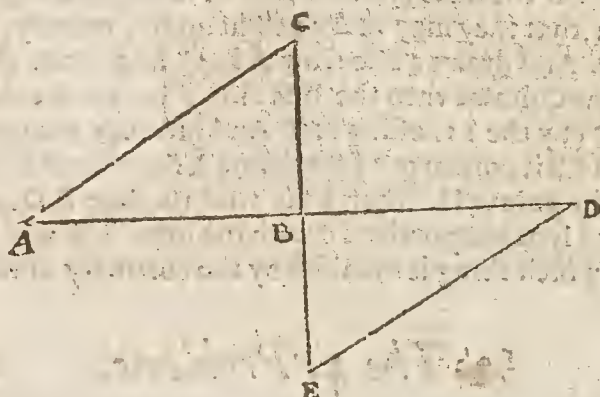
The sixth Booke

But the line DG is equall vnto the line A , and the line GE is equall vnto the line B , and the line DH vnto the line C . Wherefore as the line A is vnto the line B , so is the line C vnto the line HF . Wherefore vnto the three right lines geuen A, B, C , is found a fourth line HF in proportion with them: which was required to be done.

¶ An other way after Campane.

An other
way after
Campane.

Suppose that there be three right lines AB, BC , and BD . It is required to adde vnto them a fourth line in proportion with them. Ioyne AB the first, with BD the third, in such sort that they both make one right line, namely, AD . And vpon the said lyne AB erect from the point B the second line BC making an angle at all aduentures. And draw a line from A to C . Then by the point D draw the lyne DE parallel to the line AC , which produce vntill it concurre in the point E , with the line CB being likewise produced to the point E . Then I say that the line BE is the fourth line in proportion with the lines AB, BC , and BD : so that as AB is to BC , so is BD to BE . For forasmuch as by the 15 and 29. of the first the two triangles ABC and DBE are equiangle, therefore (by the 4. of this booke) AB is to BC , as BD is to BE : which was required to be done.



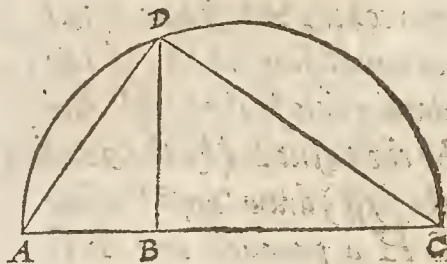
The 5. Probleme. The 13. Proposition.

Vnto two right lines geuen, to finde out a meane proportionall.

Suppose the two right lines geuen to be AB and BC . It is required betwene these two lines AB and BC to finde out a meane line proportionall. Let the lines AB and BC be so ioyned together that they

Construction.

both make one right line, namely, AC . And vpon the line AC describe a semicircle ADC , and from the poynt B raise vp vnto the line AC (by the 11. of the first) a perpendicular line BD , cutting the circumference in the point D : and draw a line from A to D , and an other from D to C .



Demonstration.

Now forasmuch as (by the 31. of the third) the angle in the semicircle ADC is a right angle, and for that in the rectangle triangle ADC is drawn from the right angle vnto the base a perpendicular line DB : therefore (by the Corollary of the 8. of the sixth) the line DB is a meane proportionall betwene the segmētes of the base AB & BC . Wherefore betwene the two right lines geuen, AB & BC , is found a meane proportionall DB : which was required to be done.

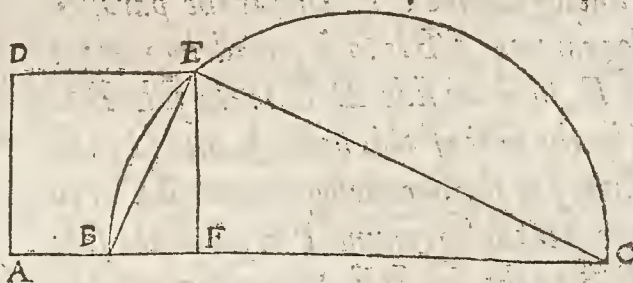
¶ A Pro-

¶ A Proposition added by Pelitarius.

A meane proportionall beyng geuen, to finde out in a line geuen the two extremes. Now it behoueth that the meane geuen be not greater then the halfe of the lyne geuen.

A proposition added by Pelitarius.

Suppose that the meane geuen be AB , and let the right line geuen be BC . It is required in the line BC to finde out two extremes, betwene which AB shal be the meane proportionall. So that yet the lyne AB be not greater then the halfe part of the line BC . For so could it not be a meane. Ioyne the lines AB and BC directly in such sort, that they both make one right line, namely, AC . Then vppon the line BC describe the semicircle BEC . And from the point A erect vnto the lyne AC a perpendicular line AD : which lyne AD put equal vnto the line AB . And by the point D draw vnto the line AC a parallel line DE , which vndoubtedly shall either cut or touch the semicircle, as in the point E , for that the line AD is not greater then the semidiameter. Then from the point E draw vnto the line BC a perpendicular line EF (by the 12. of the first) Then I say that the line BC is so devided in the point F , that the lyne AB is a meane proportionall betwene the lines BF and FC . Which thing is manifest (by the 31. of the third) & corollary of the 8. of this booke. For the line FE is equal to the line AD by the 34. of the first, and so is equall to the line AB : then if we draw the ryght lines BE and CE , there shall be made a rectangle triangle BEC . And so by the sayd corollary, the line BF shall be to the lyne FE (and therfore to the line AB) as the line FE is to the lyne FC : which was required to be done.



Flussates putteth this Proposition added by Pelitarius as a corollary following of this 13. proposition.

The 9. Theoreme. The 14. Proposition.

In equall parallelogrammes which haue one angle of the one equall vnto one angle of the other, the sides shall be reciprocall, namely, those sides which containe the equal angles. And if parallelogrammes which hauing one angle of the one, equal vnto one angle of the other, haue also their sides reciprokal, namely, those which containe the equall angles, they shall also be equall.

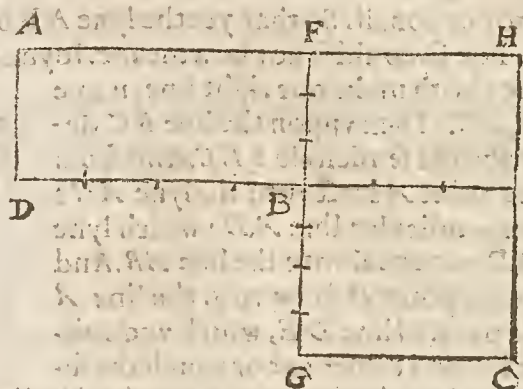
¶ Suppose that there be two equall Parallelogrammes AB and BC , hauing the angle B of the one equall vnto the angle B of the other. And let the lines DB and BE be set directly in such sort that they both make one right line, namely, DE . And then (by the 14. of the first) shall the lines FB and BG be so set that they shall make also one right line, namely, GF . Then I say, that the sides of the parallelogrammes AB and BC , which containe the equall angles,

The first part of this proposition.

The sixth Booke

*Demonstration of the
of the same.*

angles, are reciprocally proportionall: that is, as BD is to BE , so is GB to BF . Make complete the parallelogramme FE by producing the sides AF and CE , till they concur in the poynt H . Now forasmuch as the parallelogramme AB is (by supposition) equall vnto the parallelogramme BC , and there is a certaine other parallelogramme FE : therefore (by the 7. of the fift) as the parallelogramme AB is to the parallelogramme FE , so is the parallelogramme BC to the parallelogramme FE . But as the parallelogramme AB is to $\hat{y}$ parallelogramme FE , so is the side DB to the side BE (by the first of this booke). And (by the same) as the parallelogramme BC is to the parallelogramme FE , so is the side GB to the side BF . Wherefore also (by the 11. of the fift) as the side DB is to the side BE , so is the side GB to the side BF . Wherefore in the parallelogrammes AB and BC the sides which containe the equall angles, are reciprokally proportionall: which was first required to be proued.



*The second
part which is
the conuerse of
the first.*

But now suppose that the sides about the equall angles be reciprokally proportionall, so that as the side DB is to the side BE , so let the side GB be to the side BF . Then I say, $\hat{y}$ the parallelogramme AB is equall vnto $\hat{y}$ parallelogramme BC . For, for that as the side DB is to the side BE , so is the side GB to the side BF : but as the side DB is to the side BE , so (by the 1. of the sixt) is the parallelogramme AB to the parallelogramme FE : and as the side GB is to the side BF , so is the parallelogramme BC to the parallelogramme FE . Wherefore also (by the 11. of the fift) as the parallelogramme AB is to the parallelogramme FE , so is the parallelogramme BC to the same parallelogramme FE . Wherefore the parallelogramme AB is equall vnto the parallelogramme BC (by the 9. of the fift). Wherefore in equall and equiangle parallelogrammes the sides which containe the equall angles are reciprokall: and if in equiangle parallelogrammes the sides which containe the equall angles be reciprokall, the parallelogrammes also shall be equall: which was required to be proued.

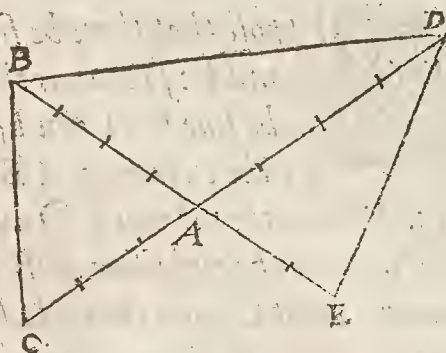
The 10. Theoreme. The 15. Proposition.

In equal triangles which haue one angle of the one equall vnto one angle of the other, those sides are reciprokall, which include the equall angles. And those triagles which hauing one angle of the one equall vnto one angle of the other, haue also their

their sides which include the equall angles reciprokal, are also equall.

Suppose that there be two equall triangles ABC , and ADE hauing one angle of the one equall vnto one angle of the other, namely, the angle BAC equall vnto the angle DAE . Then I say that in those triangles ABC and ADE , the sides which include $\hat{y}$ equal angles, are reciprocallie proportionall, that is, as the side CA is to the side AD , so is the side EA to the side AB . For let $\hat{y}$ lines CA and AD be so put, $\hat{y}$ they both make directly one right line. And so also the lines EA and AB shal both make one right line (by the 14. of the first) And draw a line from B to D . Now forasmuch as (by supposition) the triangle ABC is equall vnto the triangle ADE . And there is a certaine other triangle BAD , vnto which the two equall triangles being compared, it will follow by the 7. of the fifth, that as the triangle ABC is vnto the triangle BAD , so is $\hat{y}$ triangle EAD to the same triangle BAD . But as the triangle ABC is to the triangle BAD , so by the 1. of the sixth, is the base CA to the base AD : and as the triangle EAD is to the triangle BAD , so (by the same) is the base EA to the base AB . Wherefore (by the 11. of the fifth) as the side CA is to the side AD , so is the side EA to the side AB . Wherefore in the triangles ABC and ADE the sides which include the equall angles are reciprocallie proportionall.

The first part of this proposition.



Demonstration of the same.

But now suppose that in the triangles ABC and ADE , the sides which include the equall angles, be reciprocallie proportionall, so that as the side CA is to the side AD , so let the side EA be to the side AB . Then I say that the triangle ABC is equall vnto the triangle ADE . For agayne draw a line from B to D . And for that as the line CA is to the line AD , so is the line EA to the line AB , but as the line CA is to the line AD , so is the triangle ABC to the triangle BAD , and as the line EA is to the line AB so is the triangle EAD to the triangle BAD . Wherefore as the triangle ABC is to the triangle BAD so is the triangle EAD to $\hat{y}$ same triangle BAD . Wherefore either of these triangles ABC and EAD haue vnto $\hat{y}$ triangle BAD one and $\hat{y}$ selfe same proportion. Wherefore (by the 9. of the fifth) the triangle ABC is equal vnto the triangle EAD . If therefore there be taken equall triangles hauing one angle of the one equal vnto one angle of the other, those sides in them shal be reciprokal, which include the equal angles: and those triangles which hauing one angle of the one equall vnto one angle of the other, haue also their sides which include the equall angles reciprokal, shal also be equall: which was required to be proued.

The second part which is the conuerse of the first.

The sixth Booke

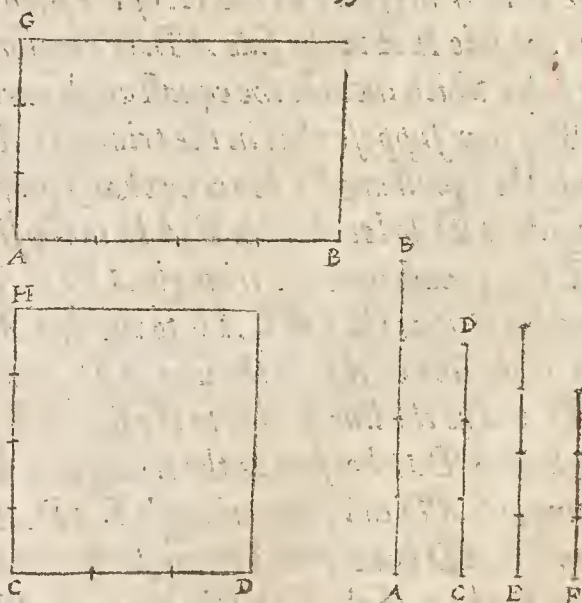
The 11. Theoreme. The 16. Proposition.

If there be foure right lines in proportion, the rectangle figure comprehended vnder the extremes: is equall to the rectangle figure containned vnder the meanes. And if the rectangle figure which is contained vnder the extremes, be equall vnto the rectangle figure which is containned vnder the meanes: then are those foure lines in proportion.



Demonstration of the first part.

Suppose that there be foure right lines in proportion, namely, AB , CD , E , and F : so that as the line AB is to the line CD , so let the line E be to the line F . Then I say, that the rectangle figure comprehended vnder the extremes AB and F , is equall vnto the rectangle figure containned vnder the meanes CD and E . From the poynt A (by the 11. of the first) raise vp vnto the right line AB a perpendicular line AG . And (by the same) from the point C vnto the right line CD raise vp a perpendicular line CH . And (by the 2. of the first) put the line AG equall vnto the line F , and put also y line CH equall vnto the line E , and make complete the parallelogrammes GB and HD . Now for that by supposition as the line AB is to the line CD , so is the line E to the line F . But the line E is equall vnto the line CH , & the line F vnto the line AG , therefore as the line AB is to the line CD , so is the line CH to the line AG . Wherefore in the parallelogrammes BG and DH the sides which include y equall angles, are reciprokally proportionall. But equiangle parallelogrammes whose sides which include the equall angles, are reciprokall, are also equall (by the 14. of the sixth). Wherefore the parallelogramme BG is equall vnto the parallelogramme DH . But the parallelogramme BG is that which is containned vnder the lines AB and F , for the line AG is put equall vnto the line F . And the parallelogramme DH is that which is contained vnder the lines CD and E , for the line CH is put equall vnto the line E . Wherefore the rectangle figure contained vnder the lines AB and F , is equall vnto the rectangle figure containned vnder the lines CD and E .



The second part which is the conuerse of the first.

But now suppose that the rectangle figure comprehended vnder the lines AB and F , be equall vnto the rectangle figure cōprehended vnder y lines CD & E .

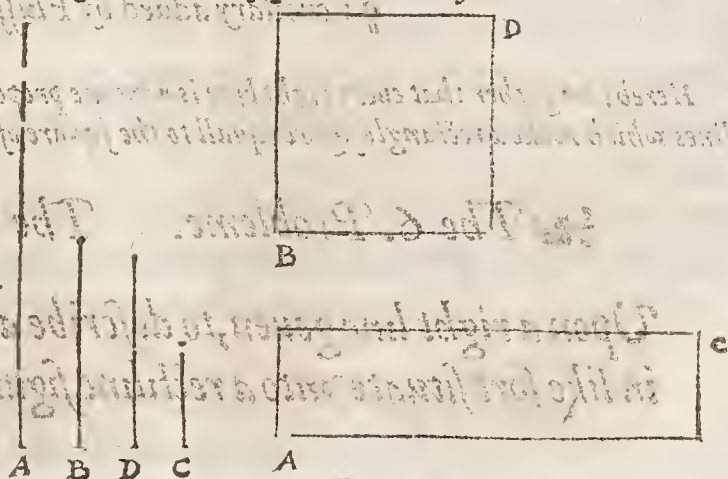
Then

Then I say, that the foure right lines AB, CD, E and F , are proportionall, that is, as the line AB is to the line CD , so is the line E to the line F . The same order of construction that was before being kept, forasmuch as that which is contained vnder the lines AB and F is equall vnto that which is contained vnder the lines CD and E , but that which is containd vnder the lines AB and F is the parallelogramme BG , for the line AG is equall vnto the line F . And that also which is contained vnder the lines CD & E is the parallelogramme DH , for the line CH is equall vnto the line E . Wherefore the parallelogramme BG is equall vnto the parallelogramme DH , & they are also equiangle. But in parallelogrammes equall & equiangle the sides which include the equall angles are reciprokall (by the 14. of the sixt). Wherefore as the line AB is to the line CD , so is the line CH to the line AG , but the line CH is equall vnto the line E , and the line AG is equall vnto $\hat{y}$ line F . Wherefore as the line AB is to the line CD , so is the line E to the line F . If therefore there be foure right lines in proportion, the rectangle figure comprehended vnder the extremes, is equall to the rectangle figure containd vnder the meanes. And if the rectangle figure which is contained vnder the extremes, be equall vnto the rectangle figure which is contained vnder the meanes, then are those foure lines in proportion: which was required to be proued.

The 12 Theoreme. The 17. Proposition.

If there be three right lines in proportion, the rectangle figure comprehended vnder the extremes, is equall vnto the square that is made of the meane. And if the rectangle figure which is made of the extremes, be equal vnto the square made of the meane, then are those three right lines proportional.

Suppose that there be three lines in proportion A, B, C , so that as A is to B , so let B be to C . Then I say that the rectangle figure comprehended vnder $\hat{y}$ lines A and C is equall vnto $\hat{y}$ square made of the line B . Vnto the line B (by the 2. of the first) put an equall line D . And because by supposition) as A is to B , so is B to C , but B is equall vnto D , wherefore (by the 7. of the fifth) as A is to B , so is D to C , but if there be foure right lines proportionall, the rectangle figure comprehended vnder



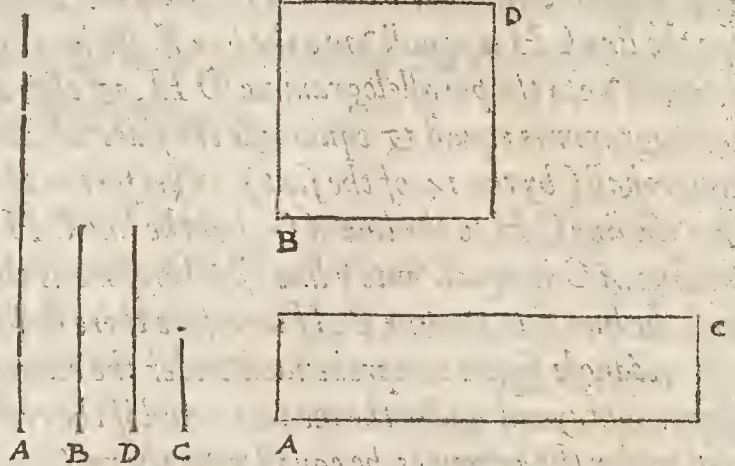
The first part of this Theoreme.

The sixth Booke

the extremes is equall vnto the rectangle figure comprehended vnder the meanes (by the 16. of the sixth). Wherefore that which is contained vnder the lines *A* and *C* is equall vnto that which is comprehended vnder the lines *B* and *D*. But that which is contained vnder the lines *B* and *D*, is the square of the line *B*, for the line *B* is equall vnto the line *D*. Wherefore the rectangle figure comprehended vnder the lines *A* and *C* is equall vnto the square made of the lyne *B*.

The second part which is the conuerse of the first.

But now suppose that that which is comprehended vnder the lines *A* & *C* be equal vnto the square made of the line *B*. Then also I say, that as the line *A* is to the line *B*, so is the line *B* to the lyne *C*. The same order of construction that was before, beyng kept, forasmuch as γ which is contained vnder the lynes



A and *C* is equall vnto the square which is made of the line *B*. But the square which is made of the line *B* is that which is contained vnder γ lines *B* & *D*, for the line *B* is put equall vnto the line *D*. Wherefore that which is containd vnder the lines *A* and *C* is equall vnto that which is containd vnder the lines *B* and *D*. But if the rectangle figure comprehended vnder the extremes, be equall vnto the rectangle figure comprehended vnder the meane lynes, the foure right lines shall be proportionall (by the 16. of the sixth) Wherefore as the line *A* is to the line *B*, so is the line *D* to the line *C*. But the line *B* is equall vnto the lyne *D*. Wherefore as the line *A* is to the lyne *B*, so is *B* to the line *C*. If therefore there be three right lynes in proportion, the rectangle figure comprehended vnder the extremes, is equall vnto the square that is made of the meane. And if the rectangle figure which is containd vnder the extremes, be equall vnto the square made of the meane, then are those three right lines proportional: which was required to be demonstrated.

A Corollary.

¶ Corollary added by Flussates.

Hereby we gather that euery right lyne is a meane proportionall betwene euery two right lines which make a rectangle figure equall to the square of the same right lyne.

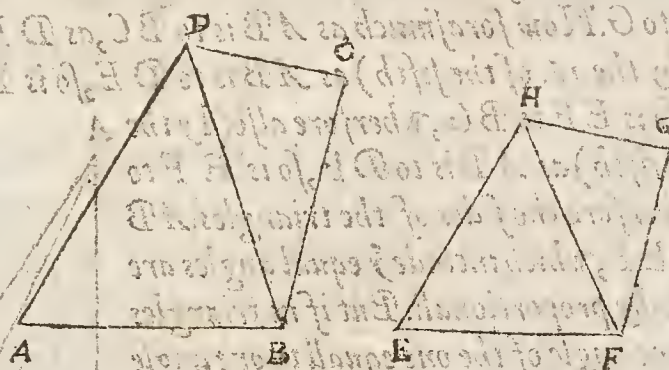
The 6. Probleme. The 18. Proposition.

Upon a right line geuen, to describe a rectiline figure like, and in like sort situate vnto a rectiline figure geuen.

Suppose

Suppose that the right line geuen be AB , and let the rectiline figure geuen be EG . It is required vpon the right line geuen AB to describe a rectiline figure like, and in like sort situate vnto the rectiline figure geuen EG . Drawe a line from H to F , and vnto the right line AB and to y point in it A , make vnto the angle E an equall angle DAB (by the 23. of the first) and vnto the right line AB and vnto the point in it B (by the same) make vnto the angle EFH an equall angle ABD . Wherefore y angle remayning EHF is equall vnto the angle remayning ADB . Wherefore the triangle HEF is equiangle vnto the triangle DAB . Wherefore (by the 4. of the sixt) as the side HF is in proportion to the side DB , so is the side HE to the side DA , and y side EF to the side AB . Againe (by the 23. of the first) vnto the right line BD and vnto y point in it D , make vnto the angle FHG an equall angle BDC , and (by the same) vnto the right line BD and vnto the point in it B , make vnto the angle HFG an equall angle DBC . Wherefore the angle remayning, namely, G , is equall vnto the angle remayning, namely, to C . Wherefore the triangle HFG is equiangle vnto the triangle DBC . Wherefore (by the 4. of the sixt) as the side HF is in proportion to the side DB , so is the side HG to the side DC , and the side GF to the side CB . And it is already proued that as HF is to DB , so is HE to DA , and EF to AB . Wherefore (by the 11. of the fift) as EH is to AD , so is EF to AB , and HG to DC , and moreover, GF to CB . And forasmuch as the angle EHF is equall vnto the angle ADB , and the angle FHG is equall vnto the angle BDC ; therefore the whole angle EHG is equall vnto the whole angle ADC , and by the same reason the angle EFG is equall vnto the angle ABC . But (by construction) the angle E is equall vnto the angle A , and the angle G is proued equall vnto the angle C . Wherefore the figure AC is equiangle vnto the figure EG , and those sides which in it include the equall angles are proportionall, as we haue before proued. Wherefore the rectiline figure AC is (by the first definition of the sixt) like vnto the rectiline figure geuen EG . Wherefore vpon the right line geuen AB is described a rectiline figure AC like G in like sort situate vnto the rectiline figure geuen EG : which was required to be done.

Description of the rectiline figure required.



Demonstration.

The 13. Theoreme. The 19. Proposition.

Pp. iij.

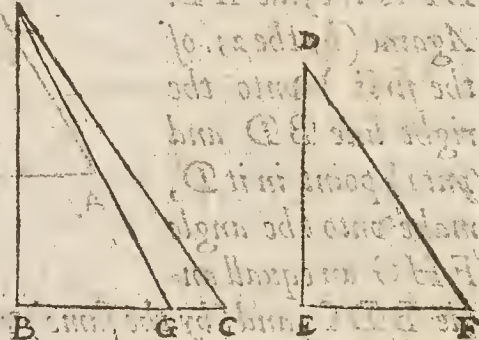
Like

The sixth Booke

Like triangles are one to the other in double proportion that the sides of lyke proportion are.



Suppose the triangles like to be ABC and DEF , having the angle B of the one triangle equal vnto the angle E of the other triangle, & as AB is to BC so let DE be to EF , so that let BC & EF be sides of like proportion. Then I say that the proportion of the triangle ABC vnto the triangle DEF is double to the proportion of the side BC to the side EF . Vnto the two lines BC and EF (by the 10. of the sixth) make a third lyne in proportion BG , so that as BC is to EF , so let EF be to BG , and draw a lyne from A to G . Now forasmuch as AB is to BC , as DE is to EF , therefore alternately (by the 16. of the fifth) as AB is to DE , so is BC to EF . But as BC is to EF , so is EF to BG , wherfore also (by the 11. of the fifth) as AB is to DE , so is EF to BG . Wherfore the sides of the triangles ABG & DEF , which include γ equal angles are reciprokally proportionall. But if in triangles having one angle of the one equall to one angle of γ other, the sides which include γ equall angles be recipokal, the triangles also (by γ 15. the sixth) shall be equall. Wherfore the triangle ABG is equall vnto γ triangle DEF . And for γ as γ line BC is to γ line EF , so is the line EF to γ line BG : but if there be three lines in proportion, the first shall haue to the third double proportion that it hath to the second (by the 10. definition of the fifth) therefore the line BC hath vnto the line BG double proportion that it hath to the line EF . But as BC is to BG , so (by the 1. of the sixth) is the triangle ABC to the triangle ABG . Wherfore the triangle ABC is vnto the triangle ABG in double proportion that the side BC is to the side EF . But the triangle ABG is equall to the triangle DEF . Wherfore also the triangle ABC is vnto the triangle DEF in double proportion that the side BC is to the side EF . Wherfore lyke triangles are one to the other in double proportion that the sides of like proportion are: which was required to be proued.



Demonstration.

Corollary.

A Corollary.

Hereby it is manifest that if there be three right lines in proportion, as the first is to the third, so is the triangle described vpon the first, vnto the triangle described vpon the second, so that the sayd triangles be like, and in lyke sort described, for it hath bene proued that as the lyne CB is to the line BG , so

is the triangle ABC to the triangle DEF : which was required to be demonstrated.

 The 14. Theoreme. The 20. Proposition.

Like Polygonon figures, are deuided into like triangles and equall in number, and of like proportion to the whole. And the one Polygonon figure is to the other Polygonon figure in double proportion that one of the sides of like proportion is to one of the sides of like proportion.

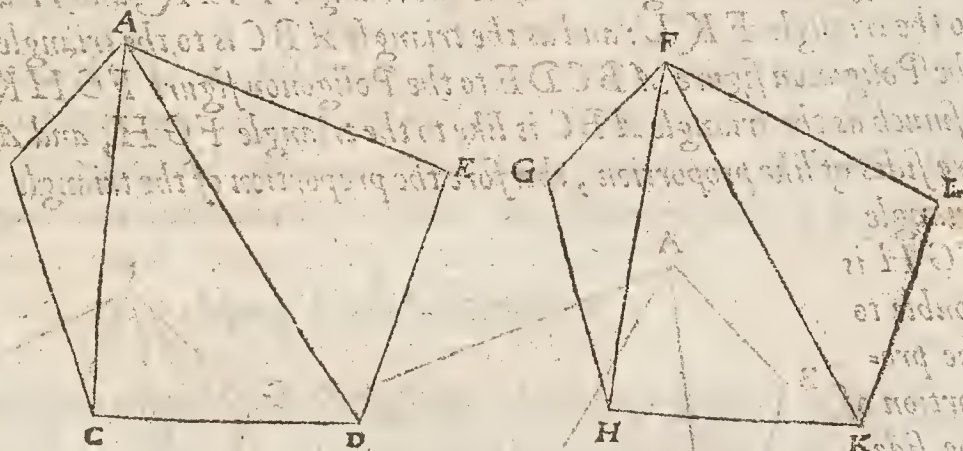


Suppose $\therefore$ the like Polygonon figures be $ABCDE$, & $FGHKL$, hauing the angle at the point F equall to the angle at the point A , and the angle at the point G equall to the angle at the point B , and the angle at the point H equall to $\therefore$ angle at the point C : and so of the rest.

And moreouer, as the side AB is to the side BC , so let the side FG be to the side GH , and as the side BC is to the side CD , so let the side GH be to the side HK and so forth. And let the sides AB & FG be sides of like proportion.

Then I say first, that these Polygonon figures

$ABCDE$,
 E & FG ,
 HKL ,
are deu-
ided into
like trian-



The first
part of this
Theoreme.

gles and equall in number. For draw these right lines, AC , AD , FH , & FK . And forasmuch as (by supposition, that is, by reason the figure $ABCDE$ is like vnto the figure $FGHKL$) the angle B is equall vnto the angle G , and as the side AB is to the side BC , so is the side FG to the side GH , it followeth that the two triangles ABC and FGH haue one angle of the one equall to one angle of the other, and haue also the sides about the equall angles proportionall. Wherefore (by the 6. of the sixt) the triangle ABC is equiangle vnto the triangle FGH . And those angles in the are equall, vnder which are subtended sides of like proportion: namely, the angle BAC is equall to the angle $G FH$, and the angle BCA to the angle $G HF$. Wherefore (by the 4. of the sixt) the sides which are about the equall angles are proportionall: and the sides which are sub-

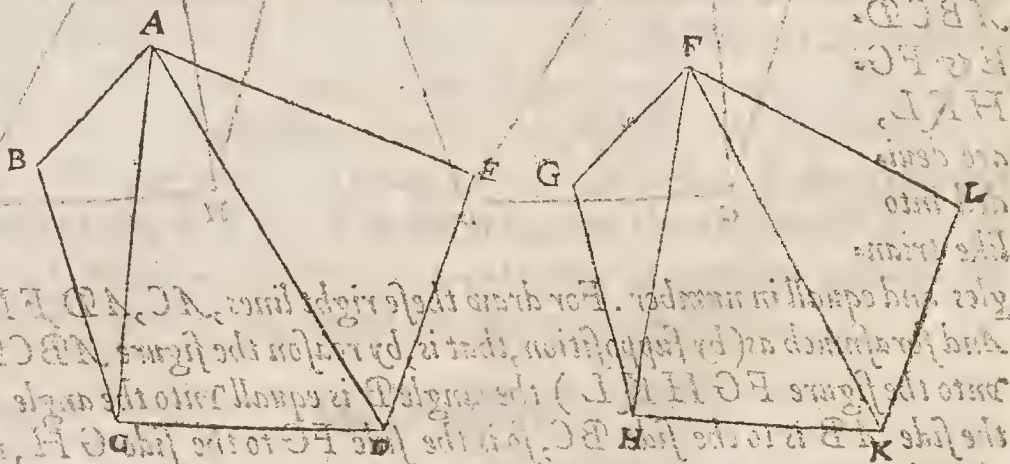
The sixth Booke

tended vnder the equall angles are of like proportion. Wherefore as AC is to BC , so is FH to GH . But by supposition as BC is to CD , so is GH to HK . Wherefore of equalitie (by the 22. of the fift) as AC is to CD , so is FH to HK . And forasmuch as by supposition the whole angle BCD is equall to the whole angle $G HK$, and it is proued that the angle BCA is equall to the angle $G HF$: therefore the angle remayning ACD is equall to the angle remayning FHK (by the 3. common sentence). Wherefore the triangles ACD , and FHK , haue againe one angle of the one equall to one angle of the other, and the sides which are about the equall sides are proportionall. Wherefore (by the same sixt of this booke) the triangles ACD & FHK are equiangle. And (by the 4. of this booke) the sides which are about the equall angles are proportionall. And by the same reason may we proue that the triangle ADE is equiangle vnto the triangle FKL . And that the sides which are about the equall angles are proportionall. Wherefore the triangle ABC is like to γ triangle FGH , and the triangle ACD to the triangle FHK , and also the triangle ADE to the triangle FKL (by the first definition of this sixt booke). Wherefore the Polygonon figures geuen $ABCDE$, and $FGHKL$, are deuided into triangles like and equall in number.

The second
part demon-
strated.

I say moreover, that the triangles are the one to the other, and to the whole Polygonon figures proportionall: that is, as the triangle ABC is to the triangle FGH , so is the triangle ACD to the triangle FHK , and γ triangle ADE to the triangle FKL : and as the triangle ABC is to the triangle FGH , so is the Polygonon figure $ABCDE$ to the Polygonon figure $FGHKL$. For forasmuch as the triangle ABC is like to the triangle FGH , and AC and FH are sides of like proportion, therefore the proportion of the triangle ABC to the triangle

FGH is double to the proportion of the side AC to the side FH (by γ former Proposition).



And therefore also the proportion of the triangle ACD to the triangle FHK , is double to the proportion that the same side AC hath to the side FH . Wherefore (by the 11. of the fift) as the triangle ABC is to the triangle FGH , so is the triangle ACD to the triangle FHK . Againe, forasmuch as the triangle ACD is like to the triangle FHK , and the sides AD & FK are of like proportion: therefore the proportion of the triangle ACD to the triangle FHK is double

is double to the proportion of the side AD to the side FK (by the foresayd 19. of the sixth). And by the same reason the proportion of the triangle ADE to γ triangle FKL , is double to the proportion of the same side AD to the side FK . Wherefore (by the 11. of the fifth) as the triangle ACD is to the triangle FHK , so is the triangle ADE to the triangle FKL . But as the triangle ACD is to the triangle FHK , so is it proued that the triangle ABC is to the triangle FGH . Wherefore also (by the 11. of the fifth) as the triangle ABC is to the triangle FGH , so is the triangle ADE to the triangle FKL . Wherefore the foresayd triangles are proportionall: namely, as ABC is to FGH , so is ACD to FHK , and ADE to FKL . Wherefore (by the 12. of the fifth) as one of the antecedentes is to one of the consequentes, so are all the antecedentes to all the consequentes. Wherefore as the triangle ABC is to the triangle FGH , so is the Polygonon figure $ABCDE$ to the Polygonon figure $FGHKL$. Wherefore the triangles are proportionall both the one to the other, & also to the whole Polygonon figures.

Lastly I say, that the Polygonon figure $ABCDE$ hath to the Polygonon figure $FGHKL$ a double proportion to that which the side AB hath to the side FG : which are sides of like proportion. For it is proued, that as the triangle ABC is to the triangle FGH , so is the Polygonon figure $ABCDE$ to the Polygonon figure $FGHKL$. But the triangle ABC hath to the triangle FGH a double proportion to that which the side AB hath to the side FG (by the former 19. Proposition of this booke): for it is proued, that γ triangle ABC is like to the triangle FGH . Wherefore the proportion of the Polygonon figure $ABCDE$ to the Polygonon figure $FGHKL$ is double to the proportion of the side AB to the side FG : which are sides of like proportion. Wherefore like Polygonon figures are denuded. &c. as before: which was required to be proued.

The third part.

The first Corollary.

Hereby it is manifest, that all like rectiline figures whatsoeuer, are the one to the other in double proportion that the sides of like proportion are. For any like rectiline figures whatsoeuer are by this Proposition denuded into like triangles and equall in number.

The first Corollary.

The second Corollary.

Hereby also it is manifest, that if there be three right lines proportionall, as the first is to the third, so is the figure described vpon the first to the figure described vpon the second, so that the sayd figures be like and in like sort described.

The second Corollary.

For

The sixth Booke

For it is proued, that the proportion of the Polygon figure $A B C D E$ to the Polygon figure $F G H K L$ is double to y proportion of the side $A B$ to the side $F G$. And if (by the 11. of the sixth) vnto the lines $A B$ and $F G$ we take a third line in proportion, namely, $M N$, the first line, namely, $A B$ shall haue vnto the third line, namely, to $M N$, double proportion that it hath to the second line, namely, to $F G$ (by the 10. definition of the fift). Wherefore as y line $A B$ is to the line $M N$, so is the rectiline figure $A B C$ to the rectiline figure $F G H$, the sayd rectiline figures being like & in like sort described.

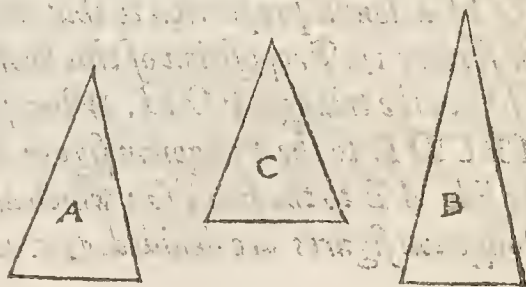
The 15. Theoreme. The 21. Proposition.

Rectiline figures which are like vnto one and the same rectiline figure, are also like the one to the other.

Demonstration.



Suppose there be two rectiline figures A and B like vnto the rectiline figure C . Then I say that the figure A is also like vnto the figure B . For forasmuch as the figure A is like vnto the figure C , it is also equiangle vnto it (by the conuersion of the first definition of the sixth) & the sides including the equall angles shall be proportionall. Agayne forasmuch as the figure B is like vnto the figure C , it is also (by the same definition) equiangle vnto it, and the sides about the equall angles are proportionall. Wherefore both these figures A and B are equiangle vnto the figure C , and the sides about the equall angles are proportionall. Wherefore (by the first common sentence) the figure A is equiangle vnto the figure B , and the sides about the equall angles are proportionall, wherefore the figure B is like vnto the figure A , which was required to be proued.



The 16. Theoreme. The 22. Proposition.

If there be foure right lines proportionall, the rectiline figures also described vpon them beyng lyke, and in like sorte situate, shall be proportional. And if the rectiline figures vpon them described be proportional, those right lynes also shall be proportionall.

Suppose

Suppose there be foure right lines AB, CD, EF , and GH , and as AB is to CD , so let EF be to GH . And vpon the lines AB and CD (by the 18. of the sixth) let there be described two rectiline figures KAB , and LCD like the one to the other, and in like sort situate. And vpon the lynes EF and GH (by the same) let there be described also two rectiline figures MEF and NH like the one to the other, and in like sorte situate. Then I say that as the figure KAB is

to the figure LCD , so is the figure MEF to the figure NH .

Vnto the lines AB and CD (by the 11. of the sixth) make a

third lyne in proportion, namely, O : and vnto the lines EF and GH in like sort make a third lyne in

a line proportion, namely, P . And for that as the line AB is to the line CD , so

is the line EF to the

line GH , but as the line CD is to the line O , so is the line GH to the lyne P .

Wherfore of equality (by the 22. of the fifth) as the lyne AB is vnto the line O , so is the lyne EF to the line P . But as the line AB is to the line O , so is the figure

KAB to the figure LCD (by the second corollary of the 20. of the sixth). And as the line EF is to the lyne P , so is the figure MEF to the figure NH . Wherefore (by the 11. of the fifth) as the figure KAB is to the figure LCD , so is the figure

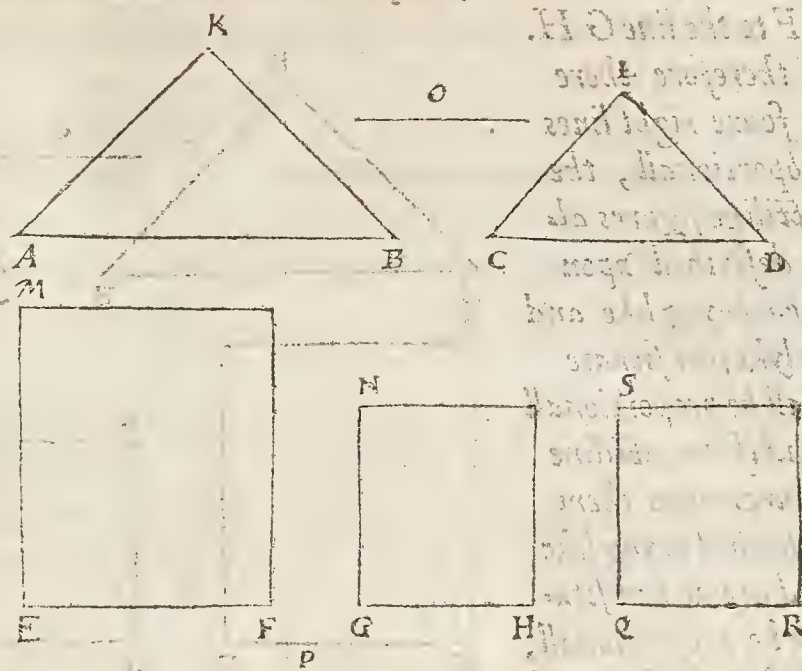
MEF to the figure NH .

But now suppose that as the figure KAB is to the figure LCD , so is the figure MEF to the figure NH , then I say that as the line AB is to the line CD , so is the line EF to the line GH . As the line AB is to the lyne CD , so (by the 12. of the sixth) let the lyne EF be to the lyne QR , and vpon the lyne QR (by the 18. of the sixth) describe vnto either of these figures MEF and NH a like figure, and in like sort situate SR . Now forasmuch as the lyne AB is to the lyne

CD , so is the lyne EF to the line QR , and vpon the lines AB and CD are described two figures lyke, and in like sort situate KAB and LCD , and vpon the lines EF and QR are described also two figures like, and in like sort situate MEF and SR , therfore as the figure KAB is to the figure LCD , so is the figure

MEF to the figure SR : wherfore also (by the 11. of the fifth) as the figure MEF is to the figure SR , so is the figure MEF to the figure NH , wherfore the figure

MEF



The first
part of this
proposition.

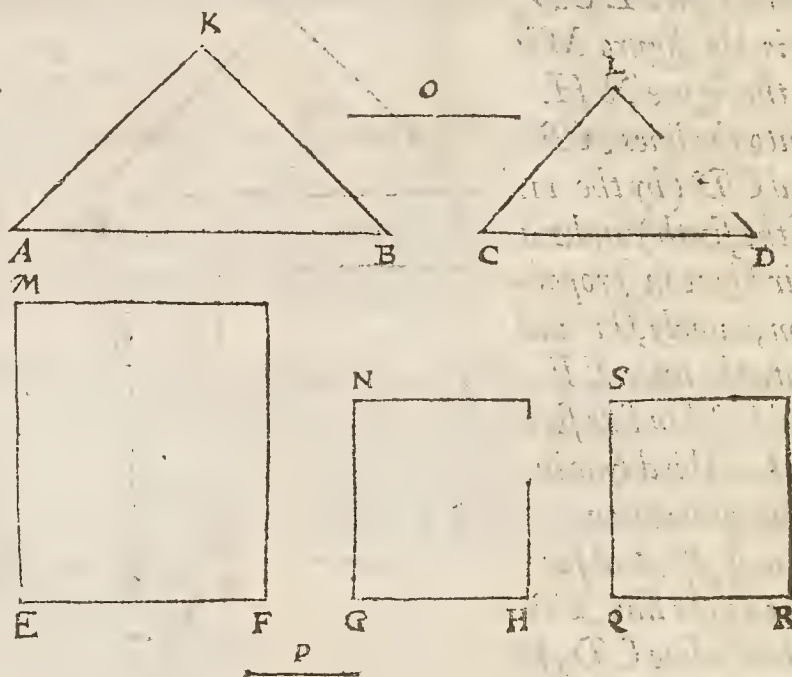
The second
part which is
the conuerse
of the first.

The sixth Booke

* Note that
this is proued
in the assumpt
following.

MF hath to either of these figures NH , and SR one and the same proportion, wherefore by the 9. of the fifth, the figure NH is equal vnto the figure SR . And it is vnto it like, and in like sort situate. * But in like and equall rectiline figures beyng in like sort situate, the sides of like proportion on which they are described are equall. Wherefore γ line GH is equall vnto the line QR . And because as the lyne AB is to the line CD , so is the line EF to the line QR , but the line QR is equall vnto the line GH , therfore as the line AB is to the line CD , so is the line EF to the line GH .

If therefore there be foure right lines proportionall, the rectiline figures also described vpon them beyng like and in lyke sort situate shall be proportionall. And if the rectiline figures vpon them described beyng like and in like sort situate be proportionall, those right lines also shall be proportional: which was required to be proued.



An Assumpt.

An assumpt.

And now that in like and equall figures, being in like sort situate, the sides of like proportion are also equall (which thing was before in this proposition taken as graunted) may thus be proued. Suppose γ the rectiline figures NH and SR be equall and like, and as HG is to GN , so let RQ be to QS , and let GH and QR be sides of like proportion. Then I say that the side RQ is equall vnto the side GH . For if they be vnequall, the one of them is greater then the other, let the side RQ be greater then the side HG . And for that as the line RQ is to the line QS , so is the line HG to the line GN , and alternately also (by the 16. of the fifth) as the line RQ is to the line HG , so is the line QS , to the lyne GN , but the line RQ is greater then the line HG . Wherefore also the line QS is greater then γ line GN . Wherefore also γ figure RS is greater then the figure HN but (by supposition) it is equall vnto it, which is impossible. Wherefore γ line QR is not greater then γ line GH . In like sorte also may we proue that it is not lesse then it, wherefore it is equall vnto it: which was required to be proued.

The

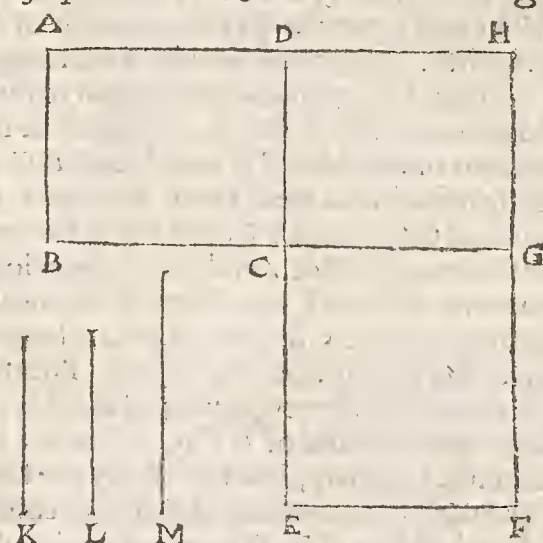
Fluſſates demonſtrath this ſecond part more briefly, by the firſt corollary of the 20. of this boke, thus. Forasmuch as the rectiline figures are by ſuppoſition in one and the ſame proportion, and the ſame proportion is double to the proportion of the ſides AB to CD , and EF to GH (by the foreſaid corollary) the proportion alſo of the ſides ſhall be one and the ſelfe ſame (by the 7. common ſentence) namely, the line AB ſhall be vnto the line CD as the line EF is to the line GH .

An other demonſtration of the ſecond part after Fluſſates.

The 17. Theoreme. The 23. Propoſition.

Equiangle Parallelogrammes haue the one to the other that proportion which is composed of the ſides.

Suppoſe the equiangle Parallelogrammes to be AC and CF , hauing the angle BCD of the one equall to the angle ECG of the other. Then I ſay, that the parallelogramme AC is vnto the parallelogramme CF in that proportion which is composed of the proportion of their ſides, that is, of that which the ſide BC hath to the ſide CG , and of that which the ſide DC hath to the ſide CE . Let the lines BC and CG be ſo put that they both make one right line (by the 14. of the firſt). Wherefore (by the ſame) the lines DC and CE ſhall make alſo one right line. Make complete the parallelogramme DG by producing the ſides AD and FG till they concur in the point H , and let there be put a certaine right line K . And as the line BC is to the line CG , ſo (by the 12. of the ſixt.) put vnto the line K a line in the ſame proportion which let be L : and as DC is to CE , ſo vnto L put a line in the ſame proportion, namely M . Wherefore the proportions of the lines K to L , and L to M , are one and ſame with the proportions of the ſides BC to CG , and DC to CE : but the proportion of K to M , is composed of the proportions of K to L , & L to M : Wherefore the proportion of K to M , is composed of the proportions of the ſides BC to CG , & EC to CD . And for that as the line BC is to the line CG , ſo is the parallelogramme AC to the parallelogramme CH (by the 1. of the ſixt). But as the line BC is to the line CG , ſo is the line K to the line L . Wherefore alſo (by the 11. of the fiſt) as the line K is to the line L , ſo is the parallelogramme AC to the parallelogramme CH . Againe, for that as the line DC is to the line CE , ſo is the parallelogramme CH to the parallelogramme CF : but as the line DC is to the line CE , ſo is the line L to the line M : Wherefore alſo (by ſame) as the line L is to the line M , ſo is the parallelogramme CH to the parallelogramme CF . And forasmuch as it is proued, that as the line K is to the line L , ſo is the parallelogramme AC to the parallelogramme CH , and as the line L is to the line M , ſo is the parallelogramme CH to the parallelogramme CF : therefore of equalitie (by the 22. of the fiſt) as the line K is to the line M , ſo is the parallelogramme AC to the parallelogramme CF . But as it hath before bene proued, the proportion of the line K to the line M , is composed of the proportions of



Demonstration.

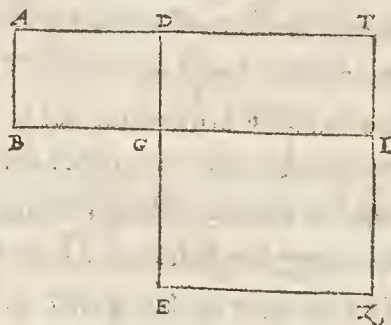
The sixth Booke

the sides BC to CG , and DC to CE . Wherefore also the proportion of the parallelogramme AC to the parallelogramme CF , is composed of the proportions of the sides BC to CG , and DC to CE . Wherefore equiangle parallelogrammes haue the one to the other that proportion which is composed of the proportions of the sides : which was required to be proued.

Flussates demonstrateth this Theoreme without taking of these three lines, K, L, M , after this maner.

An other demonstration after Flussates.

Forasmuch as (sayth he) it hath bene declared vpon the 10. definition of the fift booke, and fift definition of this booke, that the proportions of the extremes consist of the proportions of the meanes, let vs suppose two equiangle parallelogrammes $ABGD$, and $GEZI$, and let the angles at the poynt G in eyther be equall. And let the lines BG and GI be set directly that they both make one right line, namely, BGI . Wherefore EGD also shall be one right line by the conuerse of the 15. of the first. Make complete the parallelogramme GT . Then I say, that the proportion of the parallelogrammes AG & GZ is composed of the proportions of the sides BG to GI , and DG to GE . For forasmuch as that there are three magnitudes, AG, GT , and GZ , and GT is the meane of the sayd magnitudes : and the proportion of the extremes AG to GZ consisteth of the meane proportions (by the 5. definition of this booke) namely, of the proportion of AG to GT , and of the proportion GT to GZ . But the proportion of AG to GT is one and the selfe same with the proportion of the sides BG to GI (by the first of this booke). And the proportion also of GT to GZ is one and the selfe same with the proportion of the other sides, namely, DG to GE (by the same Proposition). Wherefore the proportion of the parallelogrammes AG to GZ consisteth of the proportions of the sides BG to GI , and DC to GE . Wherefore equiangle parallelogrammes are the one to the other in that proportion which is composed of theyr sides : which was required to be proued.



The 18. Theoreme. The 24. Proposition.

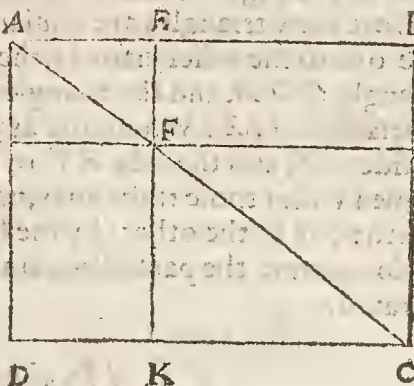
In euery parallelogramme, the parallelogrammes about the dimecient are lyke vnto the whole, and also lyke the one to the other.

Demonstration of this proposition wherein is first proued that the parallelogramme EG is lyke to the whole parallelogramme ABCD.



Suppose $\therefore$ there be a parallelogramme $ABCD$, and let the dimecient therof be AC : and let the parallelogrammes about the dimecient AC , be EG and HK . Then I say that either of these parallelogrammes EG and HK is lyke vnto the whole parallelogramme $ABCD$, and also are lyke the one to the other. For forasmuch as to one of the sides of the triangle ABC , namely, to BC is drawen a parallel lyne EF , therefore as BE is to EA , so (by the 2. of the sixt) is CF to FA . Agayne forasmuch as to one of $\therefore$ sides of the

of the triangle ADC , namely, to CD is drawn a parallel lyne FG , therefore (by the same) as CF is to FA , so is DG to GA . But as CF is to FA , so is it proved that BE is to EA . Wherefore as BE is to EA , so (by the 11. of the fifth) is DG to GA . Wherefore by composition (by the 18. of the fifth) as BA is to AE , so is DA to AG . And alternately (by the 16. of the fifth) as BA is to AD , so is EA to AG . Wherefore in the parallelogrammes $ABCD$ and EG sides which are about the common angle BAD are proportionall. And because γ line GF is a parallel vnto the lyne DC , therefore the angle AGF (by the 29. of the first) is equall vnto γ angle ADC , & γ angle GFA equall vnto γ angle DCA and the angle DAC is common to the two triangles ADC and AGF . Wherefore the triangle DAC is equiangle vnto the triangle AGF . And by the same reason the triangle ABC is equiangle vnto the triangle AEF . Wherefore the whole parallelogramme $ABCD$ is equiangle vnto the parallelogramme EG . Wherefore as AD is in proportion to DC , so (by the 4. of the sixth) is AG to GF , and as DC is to CA , so is GF to FA . And as AC is to CB , so is AF to FE . And moreover as CB is to BA , so is FE to EA . And forasmuch as it is proved that as DC is to CA , so is GF to FA ; but as AC is to CB , so is AF to FE . Wherefore of equalitie (by the 22. of the fifth) as DC is to CB , so is GF to FE . Wherefore in the parallelogrammes $ABCD$ and EG , the sides which include the equall angles are proportionall. Wherefore the parallelogramme $ABCD$ is (by the first definition of the sixth) like vnto the parallelogramme EG .



And by the same reason also the parallelogramme $ABCD$ is like to the parallelogramme KH : wherefore either of these parallelogrammes EG and KH is like vnto the parallelogramme $ABCD$. But rectiline figures which are like to one and the same rectiline figure are also (by the 21. of the sixth) like the one to the other. Wherefore the parallelogramme EG is like to the parallelogramme HK . Wherefore in euery parallelogramme, the parallelogrammes about the diemecient are like vnto the whole, and also like the one to the other. Which was required to be proued.

That the parallelogramme KH is like to the whole parallelogramme $ABCD$.

That the parallelogrammes EG and KH are like the one to the other.

¶ An other more briefe demonstration after Flussates.

Suppose that there be a parallelogramme $ABGD$, whose diemecient let be AG about which let consist these parallelogrammes EK and TI , hauing the angles at the pointes A and G common with the whole parallelogramme $ABGD$. Then I say, that those parallelogrammes EK and TI are like to the whole parallelogramme $ABGD$ and also are

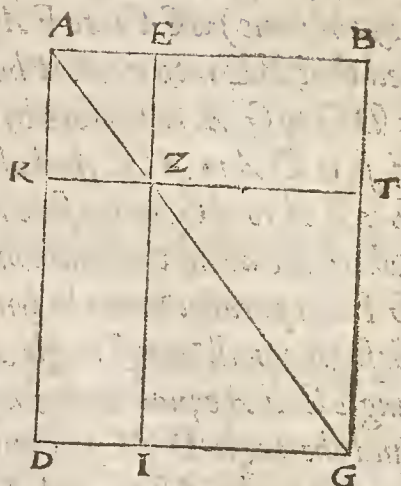
An other Demonstration after Flussates.

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like

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like the one to the other. For forasmuch as BD , EK , and TI are parallelogrammes, therefore the right line AZG falling vpon these parallell lines AEB , KZT , and DIG , or vpon these parallell lines AKD , EZI , and BTG , maketh these angles equall the one to the other, namely, the angle EAZ to the angle KZA , & the angle EZA to the angle KAZ , and the angle TZG to the angle ZGI , and the angle TGZ to the angle IZG , and the angle BAG to the angle AGD : and finally, the angle BGA to the angle DAG . Wherefore (by the first Corollary of the 32. of the first, and by the 34. of the first) the angles remayning are equall the one to the other, namely, the angle B to the angle D , and the angle E to the angle K , and the angle T to the angle I . Wherefore these triangles are equiangle and therefore like the one to the other, namely, the triangle ABG to the triangle GDA , and the triangle AEZ to the triangle ZKA , & the triangle ZTG to the triangle GIZ . Wherefore as the side AB is to the side BG , so is the side AE to the side EZ , and the side ZT to the side TG . Wherefore the parallelogrammes containd vnder those right lines, namely, the parallelogrammes $ABGD$, EK , & TI , are like the one to the other (by the first definition of this booke). Wherefore in euery parallelogramme the parallelogrammes. &c. as before: which was required to be demonstrated.

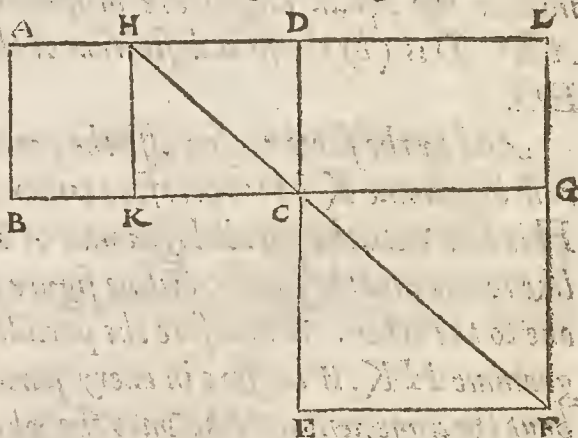


¶ A Probleme added by Pelitarius.

Two equiangle Parallelogrammes being geuen, so that they be not like, to cut of from one of them a parallelogramme like vnto the other.

An addition of Pelitarius.

Suppose that the two equiangle parallelogrammes be $ABCD$ and $CEFG$, which let not be like the one to the other. It is required from the Parallelogramme $ABCD$, to cut of a parallelogramme like vnto the parallelogramme $CEFG$. Let the angle C of the one be equall to the angle C of the other. And let the two parallelogrammes be so set, that the lines BC & CG may make both one right line, namely, BG . Wherefore also the right lines DC and CE shall both make one right line, namely, DE . And draw a line from the poynt F to the poynt C , and produce the line FC till it cōcurre with the line AD in the poynt H . And draw the line BK parallell to the line CD (by the 31. of the first). Then I say, that from the parallelogramme AC is cut of the parallelogramme $CDHK$, like vnto the parallelogramme EG . Which thing is manifest by thys 24. Proposition. For that both the sayd parallelogrammes are described about one & the selfe same dimerient. And to the end it might the more plainly be seene, I haue made complete the Parallelogramme $ABGL$.



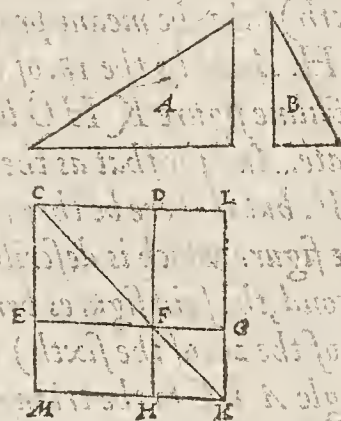
¶ An other Probleme added by Pelitarius.

Betweene two rectiline Superficiées, to finde out a meane superficies proportionall.

Another addition of Pelitarius.

Suppose that the two superficies be A and B ; betweene which it is required to place a meane superficies proportionall. Reduce the sayd two rectiline figures A and B vnto

into two like parallelogrames (by the 18. of this booke) or if you thinke good reduce eyther of them to a square, (by the last of the second). And let the said two parallelogrames like the one to the other and equall to the superficieses A and B , be $CDEF$ and $FGHK$. And let the angles F in either of them be equall, which two angles let be placed in such sort, that the two parallelogrames ED and HG may be about one and the selfe same dimetient CK (which is done by putting the right lines EF and FG in such sort that they both make one right line, namely, EG). And make copiete the parallelograme $CLKM$. Then I say, that either of the supplements FL & FM is a meane proportionall betwene the superficieses CF & FK , that is, betwene the superficieses A and B : namely, as the superficies HG is to the superficies FL , so is the same superficies FL to the superficies ED . For by this 24. Proposition the line HF is to the line FD , as the line GF is to the line FE . But (by the first of this booke) as the line HF is to the line FD , so is the superficies HG to the superficies FL ; and as the line GF is to the line FE , so also (by the same) is the superficies FL to the superficies ED . Wherefore (by the 11. of the fifth) as the superficies HG is to the superficies FL , so is the same superficies FL to the superficies ED : which was required to be done.

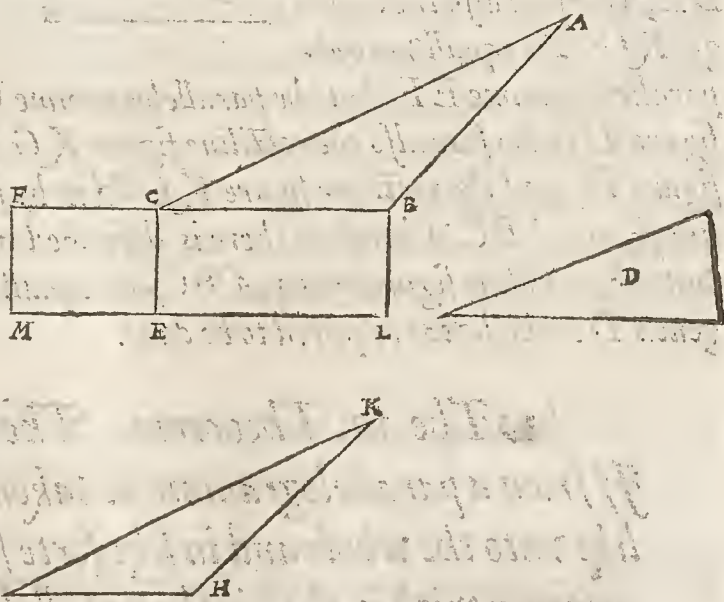


The 7. Probleme. The 25. Proposition.

Unto a rectiline figure geuen to describe an other figure lyke, which shal also be equall vnto an other rectiline figure geuen.

Suppose γ the rectiline figure geue, wherunto is required an other to be made like be ABC , and let the other rectiline figure whereunto the same is required to be made, equal be D . Now it is required to describe a rectiline figure like vnto the figure ABC , and equall vnto the figure D .

Vppon the line BC describe (by the 44. of the first) a parallelogamme BE equall vnto the triangle ABC , and by the same vpon the line CE , describe the parallelogamme CM equall vnto the rectiline figure D , and in the said parallelogamme let the angle FCE , be equall vnto the angle CBL . And forasmuch as the angle FCE is by construction equall to the angle CBL , adde the angle BCE common to them both. Wherefore the angles LCB and BCE are e-



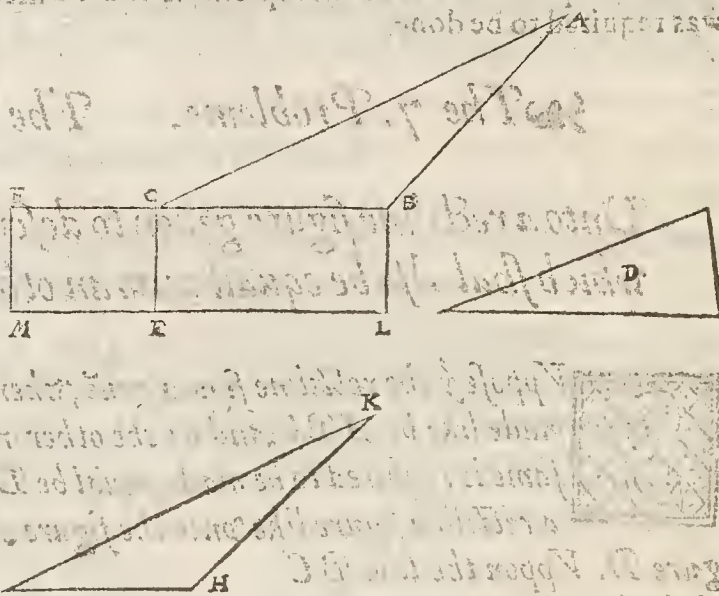
Construction.

Demonstration.

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Demonstration.

quall vnto the angles BCE and ECF , but the angles LCB and BCE are equall to two right angles (by the 29. of the first) wherfore also the angles BCE and ECF are equall to two right angles. Wherfore the lines BC and CF (by the 14. of the first) make both one right line, namely BF , and in like sort do the lines LE and EM make both one right line, namely LM . Then (by the 13. of the sixth) take the meane proportionall betwene the lines BC and CF , which let be GH . And (by the 18. of the sixth) vpon the line GH , let there be described a rectiline figure KHG like vnto the rectiline figure ABC , and in like sorte situate. And for that as the line BC is to the line GH , so is the line GH to the line CF : but if there be thre right lines proportional, as the first is to the third, so is the figure which is described of the first vnto the figure which is described of the third, the said figures being like and in like sorte situate (by the second corollary of the 20. of the sixth) wherfore as the line BC is to the line CF , so is the triangle ABC to the triangle KGH . But as the line BC is to the line CF , so is the parallelogramme BE to the parallelogramme EF (by the 1. of the sixth). Wherfore as the triangle ABC is to the triangle KGH , so is the parallelogramme BE to the parallelogramme EF . Wherfore alternately also (by the 16. of the fifth) as the triangle ABC is to the parallelogramme BE , so is the triangle KGH to the parallelogramme EF : but the triangle ABC is equall vnto the parallelogramme BE , wherfore also the triangle KGH is equall vnto the parallelogramme EF : but the parallelogramme FE is equall vnto the rectiline figure D . Wherfore also the rectiline figure KGH is equall vnto the rectiline figure D , and the rectiline figure KGH is by supposition like vnto the rectiline figure ABC . Wherfore there is described a rectiline figure KGH like vnto the rectiline figure geuen ABC , and equall vnto the other rectiline figure geuen D : which was required to be done.



The 19. Theoreme. The 26. Proposition.

If from a parallelogramme be taken away a parallelogramme like vnto the whole and in like sorte set, hauing also an angle common with it, then is the parallelogramme about one and the selfe same dimecient with the whole.

Suppose

Suppose that there be a parallelogramme $ABCD$, and from the parallelogramme $ABCD$, take away a parallelogramme $AEFG$ like unto the parallelogramme $ABCD$, and in like sort situate, having also the angle DAB common with it. Then I say, that the parallelogrammes $ABCD$ and $AEFG$ are both about one and the self same dimecient AC , that is, that the dimecient AC of the whole parallelogramme $ABCD$ passeth by the angle F of the parallelogramme $AEFG$, and is common to either of the parallelogrammes. For if AC do not passe by the point F , then if it be possible let it passe by some other point, as AHC doth. Now then the dimecient AHC shall cut eyther the side GF or the side EF of γ parallelogramme $AEFG$. Let it cut γ side GF in the point H . And (by the 31. of the first) by the point H let there be drawen to either of these lines AD and BC a parallell line HK , wherefore GK is a parallelogramme, and is about one and the self same dimecient with γ parallelogramme $ABCD$.

And forasmuch as γ parallelogrammes $ABCD$ and GK are about one and the self same dimecient, therefore (by the 24. of the sixth) the parallelogramme $ABCD$ is like unto the parallelogramme GK . Wherefore as the line DA is to the line AB , so is the line GA to the line AK (by the conuersion of the first definition of the sixth). And for that the parallelogrammes $ABCD$, and EG are (by supposition) like, therefore as the line DA is to the line AB so is the line GA to the line AE . Wherefore the line GA hath one and the selfe proportion to either of these lines AK and AE . Wherefore (by the 9. of the fifth) the line AK is equall unto γ line AE , namely, γ lesse to γ greater, which is impossible. The selfe same inconuenience also will follow, if you put the dimecient AC to cut the side FE . Wherefore AC the dimecient of the whole parallelogramme $ABCD$ passeth by the angle and poynt F . And therefore the parallelogramme $AEFG$ is about one and the selfe same dimecient with the whole parallelogramme $ABCD$. Wherefore if from a parallelogramme be taken away a parallelogramme lyke unto the whole, and in lyke sorte situate, having also an angle common with it, then is that parallelogramme about one and the selfe same dimecient with the whole: which was required to be proued.

¶ An other demonstration after Flussates, which proueth this proposition affirmatiuely.

From the parallelogramme $ABGD$ let there be taken away the parallelogramme AZK like and in like sorte situate with the whole parallelogramme $ABGD$, and having also the angle A common with the whole parallelogramme. Then I say, that both their diameters, namely, AG and AZ do make one and the selfe same right line. Divide the sides AB and BG into two equall partes in the pointes C and F (by the 10. of

Q. 11.

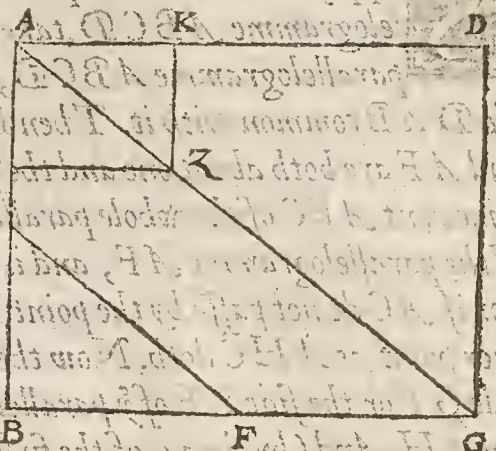
the

* By the dimecient is vnderstand here the dimecient which is drawen from the angle which is common to them both to the opposite angle. Demonstration leading to an absurditie.

An other way after Flussates.

The sixth Booke

the first.) And drawe a line from C to F. Wherefore the line CF is a parallel to the right line AG (by the corollary added by Campane after the 29. of the first) Wherefore the angles BAG and BCF are equall (by the 29. of the first): but the angle EAZ is equall vnto the angle BAG (by reason the parallelogrammes are supposed to be like) wherefore the same angle EAZ is equall to the angle BCF, namely, the outward angle to the inward and opposite angle. Wherefore (by the 28. of the first) the lines AZ and CF are parallel lynes. Now then the lines AZ and AG being parallels to one and the selfe same lynes, namely, to CF do concur in the point A. Wherefore they are set directly the one to the other, so that they both make one right line (by that which was added in the end of the 30. proposition of the first) wherefore the parallelogrammes ABGD, and AEZK are about one and the selfe same dimetient: which was required to be proued.

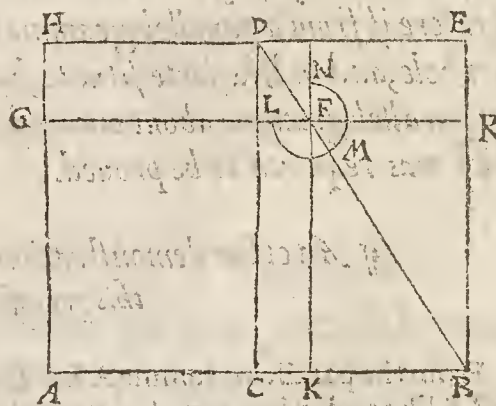


The 20. Theoreme. The 27. Proposition.

Of all parallelogrammes applied to a right line wanting in figure by parallelogrammes like and in like sort situate to that parallelograme which is described of the halfe line: the greatest parallelogramme is that which is described of the halfe line being like vnto the want.

In this proposition are two cases, in the first the parallelogramme compared to the parallelogramme described of the halfe line is described vpon a line greater then the halfe line: In the second vpon a line lesse. The first case where the parallelogramme compared namely AF is described vpon the line AK which is greater then the halfe line AC.

Et there be a right line AB, and (by the 10. of the first) deuide it in two equall partes in the point C. And vnto the right line AB apply a parallelogramme AD wanting in figure by the parallelogramme DB, which let be like and in like sort described vnto the parallelogramme described of halfe the line AB, which is BC. Then I say, that of all the parallelogrammes which may be applied vnto the line AB and which wāt in figure by parallelogrammes like and in like sort situate vnto the parallelogramme DB the greatest is the parallelogramme AD. For vnto y right line AB let there be applied a parallelogramme AF wanting in figure by the parallelogramme FB, which let be like and in like sort situate vnto the parallelogramme DB. Then I say, that the parallelogramme AD is greater then the parallelogramme AF. For forasmuch

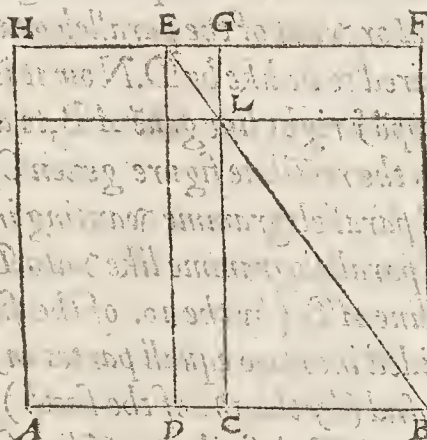


as the parallelogramme DB is like vnto the parallelogramme FB ; therefore (by the 26. of the sixt) they are about one and the selfe same dimetient. Let their dimetient be DB , and make complete the figure. Now forasmuch as (by the 43. of γ first) the supplement EC is equall vnto the supplement FE : adde the figure FB common to them both. Wherefore the whole figure CR is equall vnto the whole figure KE . But the figure CR is equall vnto the figure CG (by the 36. of the first) for that the base AC is equall vnto the base CB . Wherefore the figure GC is equall vnto the figure KE . Adde the figure CF common vnto them both. Wherefore the whole figure AF is equall vnto the whole Gnomon LMN . But the whole parallelogramme DB is greater then the Gnomon LMN (by the 3. common sentence). Wherefore also it is greater then the parallelogramme AF . But the parallelogramme AD is equall vnto the parallelogramme DB (by the 36. of the first). Wherefore the parallelogramme AD is greater then the parallelogramme AF . Wherefore of all parallelogrammes applied to a right line wanting in figure by parallelogrammes like and in like sort situate, to that parallelogramme which is described of γ halfe line, the greatest parallelogramme is that which is described of the halfe of the line, being like vnto the want: which was required to be proued.

Demonstration of this case.

Againe, let AB be deuided into two equall partes in the point C , and let the parallelogramme applied vpon the halfe line be AL , wanting in figure by the parallelogramme LB , which let be like and in like sort situate vnto the parallelogramme AL . Againe vnto the line AB let there be applied an other parallelogramme AE wanting in figure by γ parallelogramme EB being like and in like sort situate vnto the parallelogramme LB which is described vpon halfe of the line AB . The I say, that the parallelogramme AL applied vnto halfe the line is greater then the parallelogramme AE . For forasmuch as the parallelogramme EB is like vnto the parallelogramme LB , they are (by the 26. of the sixt) about one and the same dimetient. Let their dimetient be EB , and make complete the whole figure, and for that the figure LF is equall vnto the figure LH (by the 36. of the first) for the base FG is equall vnto the base GH , therefore the figure LF is greater then the figure KE . But the figure LF is equall vnto the figure DL (by the 43. of the first). Wherefore the figure DL is greater then the figure KE : put the figure KD common to them both. Wherefore the whole parallelogramme AL is greater then the whole parallelogramme AE : which was required to be proued.

The second case where the parallelogramme compared namely AE is described vpon the line AD which is lesse then the line AC . Demonstration of the second case.



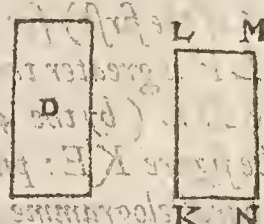
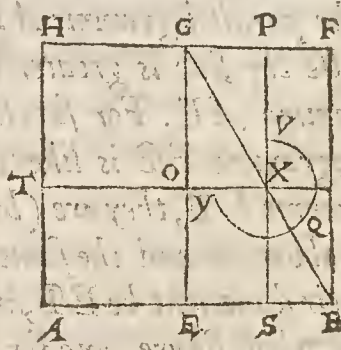
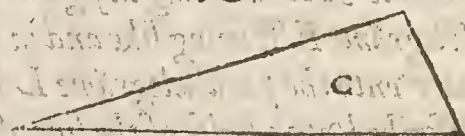
The

The 8. Probleme. The 28. Proposition.

Upon a right line geuen, to apply a parallelogramme equall to a rectiline figure geuen, & wanting in figure by a parallelogramme like vnto a parallelograme geuen. Now it beho- ueth that the rectiline figure geuen, whereunto the parallelo- grame applied must be equall, be not greater the that paralle- logramme, which so is applied vpon the halfe lyne, that the defectes shall be like, namely, the defect of the parallelograme applied vpon the halfe line, and the defect of the parallelo- gramme to be applied (whose defect is required to be like vn- to the parallelogramme geuen).



Suppose the right line geuen to be AB , and let the rectiline figure ge- uen wherunto is required to apply vpon the right line AB an equall rectiline figure be C , which figure C , let not be greater then that pa- rallelograme which is so applied vpon the halfe line, that the defectes shall be like, namely, the defect of the parallelogramme applied vpon the halfe line, and the defect of the parallelogramme to be applied (whose defect is requi- red to be like vnto the parallelogramme geue). And let the figure whereunto the defect or want of the parallelogramme is required to be like be D . Now it is requi- red vpon y^e right line geue AB , to describe vnto the rectiline figure geuen C , an e- qual parallelogramme wanting in figure by a parallelogramme like vnto D . Let the line AB (by the 10. of the first) be deuided into two equall partes in y^e point E . And (by the 18. of the sixth) vppon the line EB describe a rectiline figure $E B F G$ like vnto the parallelogramme D and in like sort situate, which shall also be a parallelograme. And make complete the parallelogramme AG . Now then the parallelogramme AG is either equal vnto the rectiline figure C , or greater then it by supposition. If the parallelo- gramme AG be equal vnto the rectiline figure C , then is that done which we



Construction.

Two cases in this Proposi- tion.

The first case. For then vpon the right line AB is described vnto the rectiline figure geuen

geuen C an equal parallelogramme AG wanting in figure by the parallelogramme GB , which is like vnto the parallelogramme D . But if AG be not equal vnto C , then is AG greater then C , but AG is equall vnto GB (by the first of the sixt). Wherefore also GB is greater then C . Take the excesse of the rectiline figure BG aboue the rectiline figure C (by that which Pelitarius addeth after the 45. of the first) And vnto that excesse (by the 15. of the sixt) describe an equall rectiline figure $KLMN$ like and in like sort situate vnto the rectiline figure D . But the rectiline figure D is like vnto the rectiline GB , wherefore also the rectiline figure $KLMN$ is like vnto the rectiline figure GB (by the 25. of the sixt) Now then let the sides KL and GE be sides of like proportion, let also LM and GF be sides of like proportion. And forasmuch as the parallelogramme GB is equal vnto the figures C and KM , therefore the parallelogramme GB is greater then the parallelogramme KM . Wherefore also the side GE is greater then the side KL , and the side GF is greater then the side LM , vnto the side KL put an equall line GO (by the 2. of the first) and likewise vnto the side LM put an equall line GP . And make perfect the parallelogramme $OGPX$. Wherefore the parallelogramme GX is equal & like vnto the parallelogramme KM . But the parallelogramme KM is lyke vnto the parallelogramme GB . Wherefore also the parallelogramme GX is like vnto the parallelogramme GB . Wherefore the parallelogrammes GX and GB are (by the 26. of the sixt) about one and the self same dimecient. Let their dimecient be GB , and make complete the figure. Now forasmuch as the parallelogramme BG is equall vnto the rectiline figure C , and vnto the parallelogramme KM , and the parallelogramme GX , which is part of the parallelogramme GB , is equal vnto KM . Wherefore the Gnomon remayning YQV is equall vnto the rectiline figure remayning, namely, to C . And forasmuch as the supplement PR is equall vnto the supplement OS , put the parallelogramme XB common vnto them both. Wherefore the whole parallelogramme PB is equall vnto the whole parallelogramme OB . But the parallelogramme OB is equal vnto the parallelogramme TE by the 1. of the sixt, (for the side AE is equal vnto the side EB) Wherefore the parallelogramme TE is equal vnto the parallelogramme PB . Put the parallelogramme OS common to them both. Wherefore the whole parallelogramme TS is equall vnto the whole gnomon YQV . But it is proued that the gnomon YQV is equal vnto the rectiline figure C . Wherefore also the parallelogramme TS is equal vnto the rectiline figure C . Wherefore vpon the right line geuen AB is applied a parallelogramme TS equal vnto the rectiline figure geuen C , and wanting in figure by a parallelogramme XB which is like vnto the parallelogramme geuen D , for the parallelogramme XB is like vnto the parallelogramme GX : which was required to be done.

The second case.

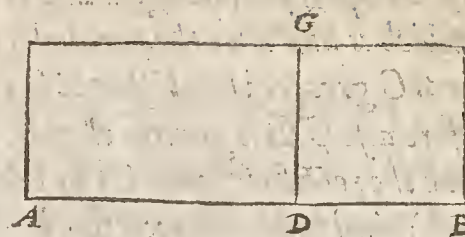
¶ A Corollary added by Flussates.

Hereby it is manifest, that if vpon a right line be applied a parallelogramme wantyng

The sixth Booke

A Corollary added by Flus-
ses, and is put
of Theon as an
assumpt before
the 17. propo-
sition of the tēth
booke: which
for that it follo-
weth of this
proposition I
thought it not
amisse here to
place.

in figure by a square, the parallelogramme ap-
plied shall be equall to the rectiline figure
which is containd vnder the segments of the
line geuen which are made by the application.
For the rest of the line is equall to the o-
ther side of the parallelogramme applied.
For that they are sides of one & the selfe
same square: as the parallelogramme A G
is contained vnder the lines A D and D B, or D G which is equall to D B.

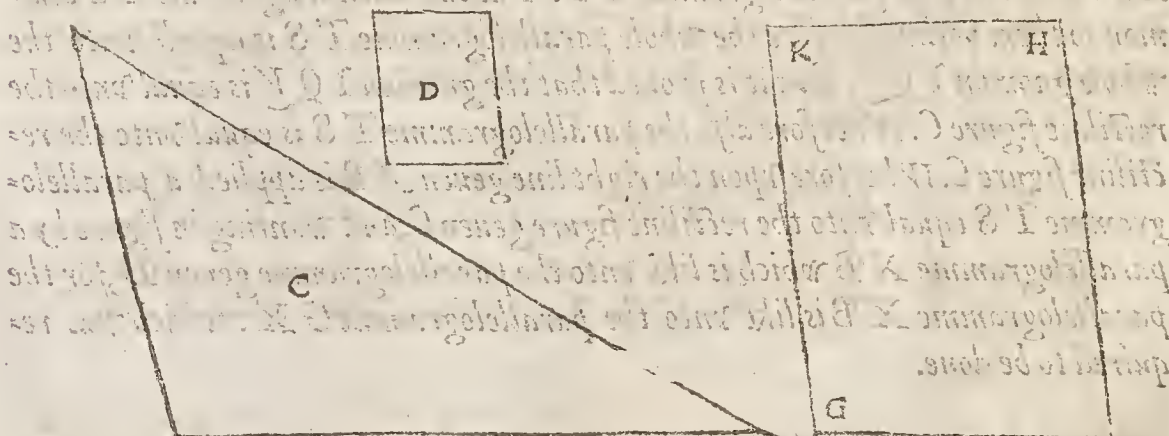
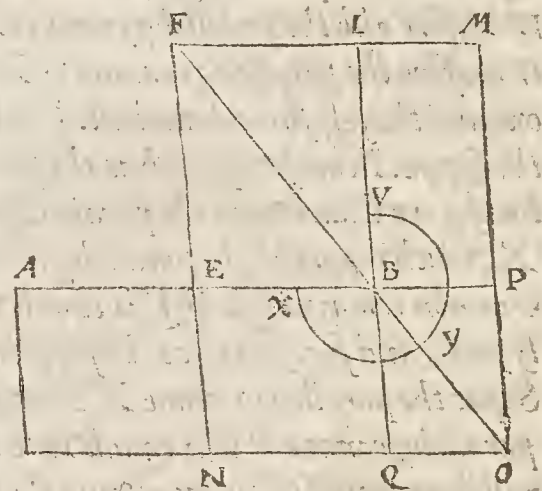


The 9. Probleme. The 29. Proposition.

Upon a right line geuen to apply a parallelogramme equall
vnto a rectiline figure geuen, and exceeding in figure by a pa-
rallelogramme like vnto a parallelogramme geuen.

Suppose the right line geuen to be AB , and let the rectiline figure geuen
whereunto is required vpon the line AB to apply an equall parallelo-
gramme be C : let also the parallelogramme wherunto y excesse is requi-
red to be like, be D . Now it is requi-
red vpon the right line AB to apply
a parallelogramme equall vnto the
rectiline figure C , and exceeding in fi-
gure by a parallelogramme like vnto
the parallelogramme D . Let the line
 AB be (by the 10. of the first) deui-
ded into two equall partes in the point
 E . And vpon the line EB (by the
18. of the sixt) describe a parallelo-
gramme BF like vnto the figure D ,
and in like sort situate. And vnto both

Construction.



these

these figures BF and C , describe an equall rectiline figure GH like vnto the figure D and in like sort situate (by the 25. of the sixt). Wherefore the parallelogramme GH is (by the 21. of the sixt) like vnto the parallelogramme BF . Let the sides KH and FL be sides of like proportion, and so also let the sides KG and FE be. And forasmuch as the parallelogramme GH is (by construction) greater then the parallelogramme FB , therefore the line KH is greater then the line FL , and the line KG is greater then the line FE . Extend the lines FL and FE to the pointes M & N , and vnto the line KH put an equall line FLM , and likewise vnto the line KG put an equall line $FE N$: and make perfect the figure MN . Wherefore the parallelogramme MN is equall and like vnto the parallelogramme GH . But the parallelogramme GH is like vnto the parallelogramme EL . Wherefore also the parallelogramme MN is like vnto the parallelogramme EL . Wherefore the parallelogrammes EL and MN are (by the 26. of the sixt) about one and the same dimetient. Let the sayd dimetient be FO , and make perfect the figure. Now forasmuch as the parallelogramme GH is equall vnto the figures EL and C . But by construction the parallelogramme GH is equall vnto the parallelogramme MN . Wherefore the parallelogramme MN is equall vnto the figures EL and C . Take away the figure EL which is common to them both. Wherefore the Gnomon remaying, namely, VYX , is equall vnto the rectiline figure C . And forasmuch as the line AE is equall vnto the line EB , therefore the parallelogramme AN is (by the 36. of the first) equall vnto the parallelogramme NB , that is, vnto the parallelogramme LP , which (by the 43. of the first) is equall vnto the parallelogramme NB . Adde the parallelogramme BO common to them both. Wherefore the whole parallelogramme AO is equall vnto the Gnomon VYX . But the Gnomon VYX is equall vnto the rectiline figure C . Wherefore the parallelogramme AO is equall vnto the rectiline figure C . Wherefore vpon the right line geuen AB is applied the parallelogramme AO equall vnto the rectiline figure geuen C , and exceeding in figure by the parallelogramme QP which is like vnto the parallelogramme geuen D . For the parallelogramme D is like vnto the parallelogramme BF : and the parallelogramme BF is like vnto the parallelogramme PQ : for they are about one and the selfe same dimetient: which was required to be done.

Demonstration.

☞ The 10. Probleme. The 30. Proposition.

To deuide a right line geuen by an extreme and meane proportion.



Suppose the right line geuen to be AB . It is required to deuide the line AB by an extreme and meane proportion. Vpon the line AB describe (by the 46. of the first) a square BC . And vpon the line AC (by the

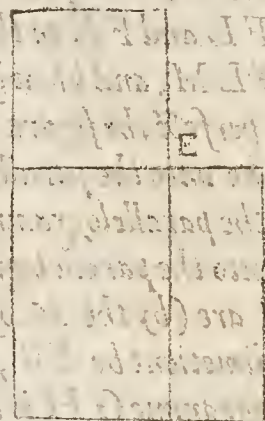
Construction.

Rr.j.

29. of

Demonstration.

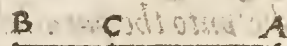
29. of the sixt) applie a parallelogramme $C D$ equall vnto the square $B C$, and exceeding in figure by the figure $A D$ like vnto the figure $B C$. Now $B C$ is a square. Wherefore also $A D$ is a square. And forasmuch as $B C$ is equall vnto $C D$, take away the figure $C E$ which is common to them both. Wherefore the figure remayning, namely, $B F$, is equall to the figure remayning, namely, to $A D$, and the angle E of the one is equall vnto the angle E of the other. Wherefore (by the 2. definition of the sixt, and by the 14. of the sixt) the sides of the figures $B F$ and $D A$, which containe the equall angles, are reciprokall. Wherefore as the side $F E$ is to the side $E D$, so is the side $A E$ to the side $E B$. But the side $F E$ is equall vnto the line $A C$, that is, vnto the line $A B$, and the side $E D$ is equall vnto the line $A E$ (by the 34. of the first). Wherefore as the line $B A$ is to the line $A E$, so is the line $A E$ to the line $E B$. But the line $A B$ is greater then the line $A E$. Wherefore also the line $A E$ is greater then the line $E B$. Wherefore the right line $A B$ is denided by an extreme and meane proportion in the point E : and the greater segment thereof is $A E$: which was required to be done.



* An other way.

An other way.

Suppose the right line geuen to be $A B$. It is required to denide the line $A B$ by an extreme and meane proportion. Denide the line $A B$ in the point C (by the 11. of the second) in such sort that the rectangle figure comprehended vnder the lines $A B$ and $B C$ may be equall vnto the square described of the line $C A$. And forasmuch as that which is comprehended vnder the lines $A B$ & $B C$, is equall vnto the square made of the line $A C$, therefore as the line $B A$ is to the line $A C$, so (by the 17. of the sixt) is the line $A C$ to the line $C B$. Wherefore the line $A B$ is denided by an extreme & meane proportion in the point C . which was required to be done.



The 21. Theoreme. The 31. Proposition.

In rectangle triangles the figure made of the side subtending the right angle, is equal vnto the figures made of the sides comprehending the right angle, so that the sayd three figures be like

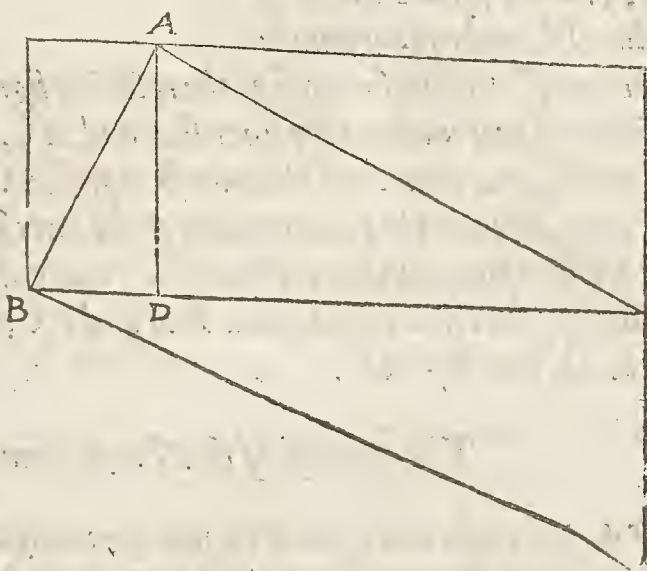
be like and in like sort described.



*S*uppose that there be a triangle ABC , whose angle BAC let be a right angle. The I say that the figure which is described of the lyne BC is equall vnto the two figures which are described of the lines BA & AC , the said thre figures being like the one to the other, and in like sort described. From the point A (by the 12. of the first) let there be drawne vnto the line BC a perpendiculer line AD . Now forasmuch as in y rectangle triangle ABC is drawn from the right angle A vnto the base BC a perpendicular line AD , therefore the triangles ABD , and ADC set vpon the perpendicular line, are like vnto the whole triangle ABC , and also like the one to the other (by the 8. of the sixth). And forasmuch as the triangle ABC is like vnto the triangle ABD , therefore as the line CB is to the line BA , so is the line AB to the lyne BD . Now for that there are three right lines proportional, therefore (by the 2. corollary of the 20. of the sixth) as the first is to the third, so is the figure made of the first, to the figure made of the second, the said figures being like and in like sorte described. Wherefore as the lyne BC is to the line BD , so is the figure made of the line BC to the figure made of the line BA , they being like and in lyke sort described. And by y same reason as the line BC is to the lyne CD , so is the figure made of the line BC , to the figure made of the line CA , they being like and in like sort described. Wherefore as the line BC is to the lines BD and DC , so is the figure made of the line BC to the figures made of the lines BA and AC , they being like and in like sort described. But the line BC is equall vnto the lines BD and DC , wherefore the figure made of the lyne BC is equall vnto the figures made of the lines BA and AC , they being like and in like sort described. Wherefore in rectangle triangles the figure made of the side subtending the right angle is equall vnto the figures made of the sides comprehending the right angle, so that the said thre figures be like and in like sort described: which was required to be proued.

Construction.

Demonstration.



An other way.

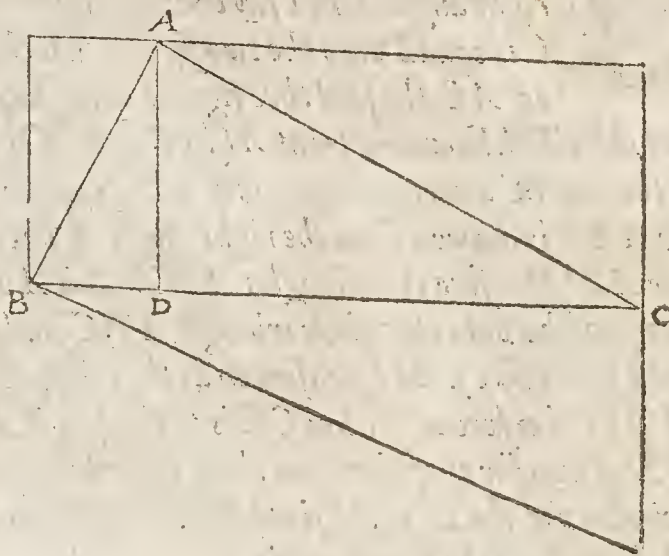
Forasmuch as (by the first corollary of the 20. of the sixth) like rectiline figures are in double proportion to that that the sides of like proportion are, there

Rr. ij.

fore

The sixth Booke

fore the rectiline figure made of the line BC is vnto the rectiline figure made of the line BA in double proportion to that that the line CB is to the line BA , and (by the same) the square also made of the line BC is vnto the square made of the line BA in double proportion to that that the line CB is vnto the line BA . Wherefore also as the rectiline figure made of the line CB is to the rectiline figure made of the line BA , so is the square made of the line CB to the square made of the line BA . And by the same reason also, as the rectiline figure made of the line BC is to the rectiline figure made of $\hat{y}$ line CA , so is the square made of the line BC to the square made of the line CA . Wherefore also as the rectiline figure made of the line BC is to the rectiline figures made of the lines BA and AC , so is the square made of the line BC to the squares made of the lines BA and AC . But the square made of $\hat{y}$ line BC is equall vnto the squares made of the lines BA and AC (by the 47. of the first) Wherefore also the rectiline figure made of the line BC is equall vnto the rectiline figures made of the lines BA and AC , the said three figures beynge lyke and in like sort described.

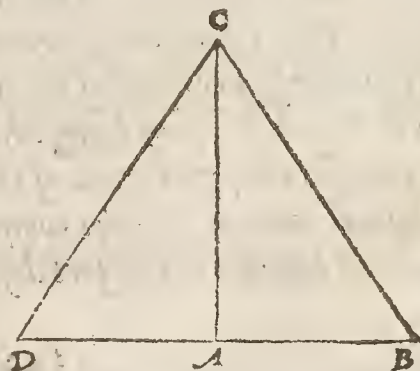


The conuerse of this Proposition after Campane.

*The conuerse
of the former
proposition.*

If the figure described of one of the sides of a triangle be equall to the figures which are described of the two other sides, the sayd figures being lyke and in like sort described, the triangle shall be a rectangle triangle.

Suppose that ABC be a triangle, and let the figure described of the side BC be equall to the two figures described of the sides AB and AC , the said figures being lyke, and in like sort described. Then I say that the angle A is a right angle. Let the angle CAD be a right angle, and put the line AD equall to the line AB , and drawe a lynes from D to C . Now then by this 31. proposition, the figure made of the line CD is equall to the two figures made of the lines AC and AD , the said figures being lyke and in like sort described. Wherefore also it is equall vnto the figure made of the line BC , which is by supposition equal to the two figures made of the lines AC and AD (for the line AD is put equall to the line AB) wherefore the line DC is equall to the line BC . Wherefore (by the 8. of the first) the angle BAC is a right angle, which was required to be proued.



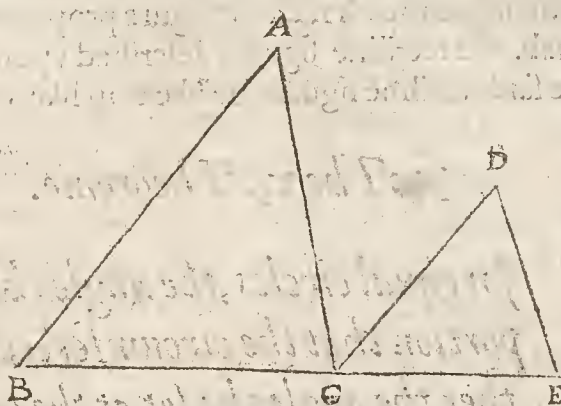
The

The 22. Theoreme. The 32. Proposition.

If two triangles be set together at one angle, hauing two sides of the one proportionall to two sides of the other, so that their sides of like proportion be also parallels: then the other sides remayning of those triangles shall be in one right line.

Suppose the two triangles to be ABC , and DCE , and let two of their sides AC & DC make an angle ACD , and let the said triangles haue two sides of the one, namely, BA and AC proportionall to two sides of the other, namely, to DC and DE , so y^t as AB is to AC , so let DC be to DE . And let AB be a parallell vnto DC , and AC a parallell vnto DE . Then I say, that the lines BC and CE are in one right line. For forasmuch as the line AB is a parallell vnto the line DC , and vpon the lighteth a right line AC : therefore (by the 29. of the first) the alternate angles BAC and ACD are equall the one to the other. And by the same reason the angle CDE is equall vnto y^e same angle ACD . Wherefore the angle BAC is equall vnto the angle CDE . And forasmuch as there are two triangles ABC and DCE , hauing the angle A of the one equall to the angle D of the other, and the sides about the equall angles are (by supposition) proportionall, that is, as the line BA is to the line AC , so is the line CD to the line DE , therefore the triangle ABC is (by the 6. of the sixth) equiangle vnto the triangle DCE . Wherefore the angle ABC is equall vnto the angle DCE . And it is proued, that the angle ACD is equall vnto the angle BAC . Wherefore the whole angle ACE is equall vnto the two angles ABC and BAC . Put the angle ACB common to them both. Wherefore the angles ACE and ACB are equall vnto the angles CAB , ACB , & CBA . But the angles CAB , ACB , and CBA , are (by the 32. of the first) equall vnto two right angles. Wherefore also the angles ACE and ACB are equall to two right angles. Now then vnto the right line AC , and vnto the point in it C , are drawen two right lines BC and CE , not on one and the same side, making the side angles ACE & ACB equall to two right angles. Wherefore the lines BC and CE (by the 14. of the first) are set directly and do make one right line. If therefore two triangles be set together at one angle hauing two sides of the one proportionall to two sides of y^e other, so y^t their sides of like proportion be also parallels: then y^e sides remayning of those triangles shall be in one right line: which was required to be proued.

Demonstration.



Although *Euclide* doth not distinctly set forth the maner of proportion of like rectiline figures, as he did of lines in the 10. Proposition of this Booke, and in the 3. following it, yet as *Flussates* noteth, is that not hard to be done by the 22. of this Booke. For two like rectiline figures being geuen to finde out a third proportionall: also betwene two rectiline superfiicies geuen to finde out a meane proportionall (which we before taught to do by *Pelitarus* after the 24. Proposition of this booke): and moreover three like rectiline figures being geuen to finde out a fourth proportionall like and in like sort described, and such kinde of proportions, are easie to be found out by the proportions of lines. As thus. If vnto two sides of like proportion we should finde out a third proportionall by the 11. of this booke: the rectiline figure described vpon that line shall be the third rectiline figure proportionall with the two first figures geuen by the 22. of this booke. And if betwene two sides of like proportion be taken a meane proportionall by the 13. of this Booke: the rectiline figure described vpon the sayd meane shall likewise be a meane proportionall betwene the two rectiline figures geue by the same 22. of the sixt. And so if vnto three sides geuen be found out the fourth side proportionall (by the 12. of this booke) the rectiline figure described vpon the sayd fourth line shall be the fourth rectiline figure proportionall. For if the right lines be proportionall, the rectiline figures described vpon them shall also be proportionall, so that the said rectiline figures be like & in like sort described by the said 22. of the sixt.

✶ The 22. Theoreme. The 33. Proposition.

In equal circles, the angles haue one and the selfe same proportion that the circumferēces haue, wherein they cōsist, whether the angles be set at the centres or at the circumferences. And in like sort are the sectors which are described vppon the

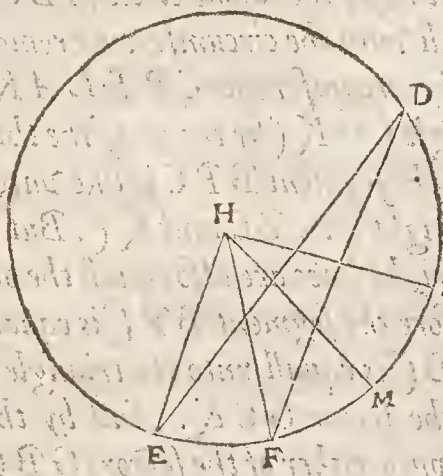
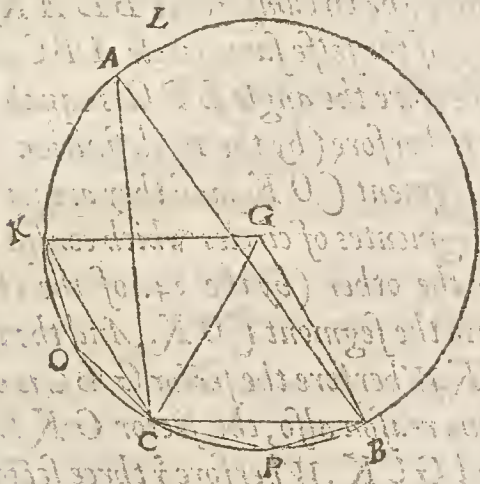
centres.

Suppose the equall circles to be ABC , and DEF , whose centres let be G and H , and let the angles set at their centres G and H , be BGC , and EHF : and let the angles set at their circumferences be BAC and EDF . Then I say that as the circumference BC is to the circumference EF so is the angle BGC to the angle EHF : and the angle BAC to the angle EDF , and moreover the sector GBC to the sector HEF . Vnto the circumference BC (by the 28. of the third) put as many equall circumferences in order as you will, namely, CK and KL , and vnto the circumference EF put also as many equall circumferences in number as you will, namely, FM , and MN . And drawe these right lines GK , GL , HM , and HN . Now forasmuch as the circumferences BC , CK and KL are equall the one to the other, the angles also BGC , and CGK , and KGL are (by the 27. of the third) equall the one to the other. Therefore how multiplex the circumference BL is to the circumference BC , so multiplex is the angle BGL to the angle BGC . And by the same reason also, how multiplex the circumference NE is to the circumference EF , so multiplex is the

That the angles at the centres are in proportion the one to the other, as the circumferences wheron they are.

angle

angle NHE to the angle EHF . Wherefore if the circumference BL be equal vnto the circumference EN , the angle BGL is equall vnto the angle EHN , and if the circumference BL be greater then the circumference EN , the angle BGL is greater then the angle NHE , and if the circumference be lesse, the angle is lesse. Now then there are foure magnitudes, namely, the two circumferences BC and EF , and the two angles that is, BGC , and EHF , and to the circumference BC and to the angle BGC , that is, to the first and third are take equemultiplices, namely, the circumference BL , and the angle BGL , and likewise to the circumference EF , and to the angle EHF , that is, to the second and fourth, are taken certayne other equemultiplices, namely, the circumference EN and the angle EHN . And it is proued, that if the circumference BL exceede the circumference EN , the angle also BGL exceedeth the angle EHN . And if the circumference be equall, the angle is equall, and if the circumference be lesse, the angle also is lesse. Wherefore (by the 6. definition of the fifth) as the circumference BC is to the circumference EF , so is the angle BGC to the angle EHF .



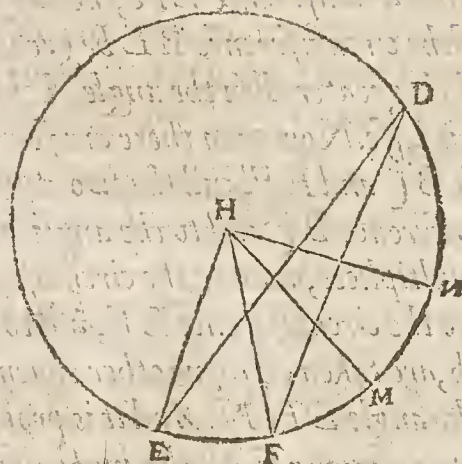
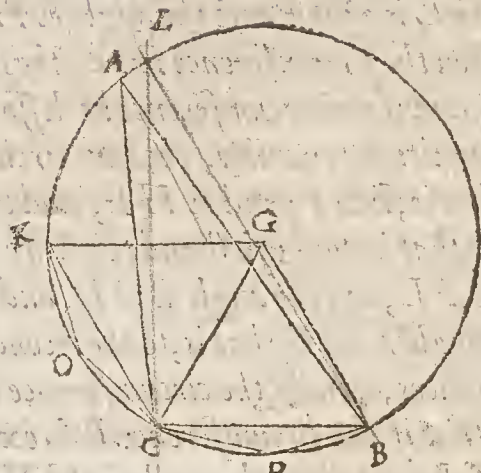
But as the angle BGC is to the angle EHF , so is the angle BAC to the angle EDF , for the angle BGC is double to the angle BAC , and the angle EHF is also double to the angle EDF (by the 20. of the third). Wherefore as the circumference BC is to the circumference EF , so is the angle BGC to the angle EHF , and the angle BAC to the angle EDF . Wherefore in equall circles the angles are in one and the selfe same proportion that their circumferences are, whether the angles be set at the centres, or at the circumferences: which was required to be proued.

That the angles at the circumferences are so also.

I say moreouer that as the circumference BC is to the circumference EF , so is the sector GBC to the sector HEF . Draw these lines BC and CK . And in the circumferences BC and CK take pointes at all aduentures, namely, P and O . And draw lines from B to P , and from P to C , from C to O , and from O to K . And forasmuch as (by the 15. definition of the first) the two lines BG and GC are equall vnto the two lines CG and GK , and they also comprehend equall angles

That the sectors are so also.

The sixth Booke



gles, therefore (by the 4. of the first) the base BC is equall vnto the base CK , & the triangle GBC is equall vnto the triangle GCK . And seing that the circumference BC is equall vnto the circumference CK , therefore the circumference remayning of the whole circle ABC , namely, the circumference $BLAKOC$, is equall vnto the circumference remayning of the selfe same circle ABC , namely, to the circumference $CPBLAK$. Wherefore the angle BPC is equall vnto the angle COK (by the 27. of the third) Wherefore (by the 10. definition of the third) the segment BPC is like vnto the segment COK , and they are set vpon equall right lines BC and CK . But like segmentes of circles which consist vpon equall right lines are also equall the one to the other (by the 24. of the third). Wherefore the segment BPC is equall vnto the segment COK . And the triangle GBC is equall vnto the triangle GCK . Wherefore the sector GBC is equall vnto the sector GCK . And by the same reason also, the sector GKL is equall vnto either of the sectors GBC and GCK . Wherefore y^e three sectors GBC and GCK , and GKL , are equall y^e one to y^e other. And by the same reason also the sectors HEF and HFM , and HMN are equall the one to the other. Wherefore how multiplex the circumference BL is to the circumference BC so multiplex is the sector GLB to the sector GBC . And by the same reason how multiplex the circumference NE is to the circumference EF , so multiplex is y^e sector HEN to the sector HEF . If therefore the circumference BL be equall vnto y^e circumference EN , y^e sector also BGL is equall vnto y^e sector HEN . And if the circumference BL excede y^e circumference EN , y^e sector also BGL exceedeth the sector HEN . And if the circumference be lesse, the sector also is lesse. Now the there are foure magnitudes, namely, the two circumferences BC and EF , and the two sectors GBC & HEF , and to the circumference BC , & to the sector GBC , namely, to the first and the third, are taken equemultiples, that is, the circumference BL , and the sector BGL , and likewise to the circumference EF , and to the sector HEF , namely, to the second and fourth, are taken certayne other equemultiples, namely, the circumference EN and the sector

tor $H E N$. And it is proued that if the circumference $B L$ excede the circumference $E N$, the sector also $B G L$ excedeth the sector $E H N$. And if the circumference be equall, the segment also is equall, and if the circumference be lesse, the segment also is lesse. Wherefore (by the conuersion of the sixt definition of the fifth) as the circumference $B C$ is to the circumference $E F$, so is $\frac{1}{2}$ sector $G B C$ vnto the sector $H E F$: which is all that was required to be proued.

 Corollary.

And hereby it is manifest, that as the sector is to the sector,
so is angle to angle by the 11. of the fifth.

Flussates here addeth fise Propositions wherof one is a Probleme hauing three Corollaries following of it, and the rest are Theoremes: which for that they are both witty, & also serue to great vse, as we shall afterward see, I thought not good to omitte, but haue here placed them: but onely that I haue not put them to folowe in order with the Propositions of *Euclide* as he hath done.

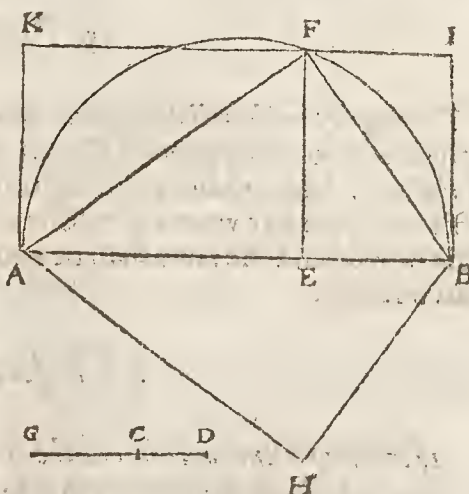
¶ The first Proposition added by Flussates.

To describe two rectiline figures equall and like vnto a rectiline figure geuen and in like sort situate, which shall haue also a proportion geuen.

Suppose that the rectiline figure geuen be $A B H$. And let the proportion geuen be the proportion of the lines $G C$ and $C D$. And (by the 10. of this booke) diuide the line $A B$ like vnto the line $G D$ in the poynt E (so that as the line $G C$ is to the line $C D$, so let the line $A E$ be to the line $E B$). And vpon the line $A B$ describe a semicircle $A F B$. And from the poynt E erect (by the 11. of the first) vnto the line $A B$ a perpendicular line $E F$ cutting the circumference in the poynt F . And draw these lines $A F$ and $F B$. And vpon either of these lines describe rectiline figures like vnto the rectiline figure $A H B$ and in like sort situate (by the 18. of the sixt): which let be $A K F$, & $F I B$. Then I say, that the rectiline figures $A K F$, and $F I B$,

Construction
of the Pro-
bleme.

haue the proportion geue (namely, the proportion of the line $G C$ to the line $C D$) and are equall to the rectiline figure geuen $A B H$ vnto which they are described like and in like sort situate. For forasmuch as $A F B$ is a semicircle, therefore the angle $A F B$ is a right angle (by the 31. of the third) and $F E$ is a perpendicular line. Wherefore (by the 8. of this booke) the triangles $A F E$ and $F B E$ are like both to the whole triangle $A F B$ and also the one to the other. Wherefore (by the 4. of this booke) as the line $A F$ is to the line $F B$, so is the line $A E$ to the line $E F$, and the line $E F$ to the line $E B$, which are sides cotayning equall angles. Wherefore (by the 22. of this booke) as the rectiline figure described of the line $A F$ is to the rectiline figure described of the line $F B$, so is the rectiline figure described of the line $A E$ to the rectiline figure described of the line $E F$, the sayd rectiline figures being like and in like sort



Demonstration
of the
same.

situate.

The sixth Booke

situate. But as the rectiline figure described of the line AE being the first, is to the rectiline figure described of the line EF being the second, so is the line AE the first, to the line EB the third (by the 2. Corollary of the 20. of thys booke). Wherefore the rectiline figure described of the line AF is to the rectiline figure described of the line FB , as the line AE is to the line EB . But the line AE is to the line EB (by construction) as the line GC is to the line CD . Wherefore (by the 11. of the fifth) as the line GC is to the line CD , so is the rectiline figure described of the line AF to the rectiline figure described of the line FB , the sayd rectiline figures being like and in like sort described. But the rectiline figures described of the lines AF and FB , are equall to the rectiline figure described of the line AB , vnto which they are (by construction) described lyke and in like sort situate. Wherefore there are described two rectiline figures AKF and FIB equall and like vnto the rectiline figure geuen ABH and in like sort situate, and they haue also the one to the other the proportion geuen, namely, the proportion of the line GC to the line CD : which was required to be done.

¶ The first Corollary.

The first Corollary.

To resolve a rectiline figure geue into two like rectiline figures which shall haue also a proportiō geue. For if there be put three right lines in the proportiō geue, and if the line AB be cut in the same proportion that the first line is to the third, the rectiline figures described of the lines AF and FB (which figures haue the same proportion that the lines AE and EB haue) shall be in double proportion to that which the lines AF and FB are (by the first Corollary of the 20. of this booke). Wherefore the right lines AF and FB are the one to the other in the same proportion that the first of the three lines put is to the second. For the first line to the third, namely, the line AE to the line EB is in double proportion that it is to the second, by the 10. definition of the fifth.

¶ The second Corollary.

The second Corollary.

Hereby may we learne, how from a rectiline figure geuen to take away a part appointed, leauing the rest of the rectiline figure like vnto the whole. For if frō the right line AB be cut of a part appoynted, namely, EB (by the 9. of this booke) as the line AE is to the line EB , so is the rectiline figure described of the line AF to the rectiline figure described of the line FB (the sayd figures being supposed to be like both the one to the other and also to the rectiline figure described of the line AB , and being also in like sort situate). Wherefore taking away from the rectiline figure described of the line AB , the rectiline figure described of the line FB , the residue, namely, the rectiline figure described of the line AF shall be both like vnto the whole rectiline figure geuen described of the line AB , and in like sort situate.

¶ The third Corollary.

The third Corollary.

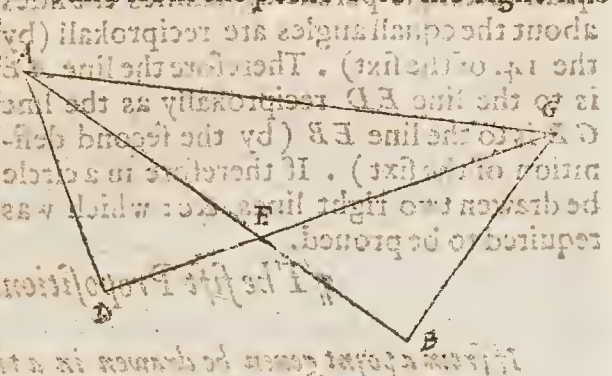
To compose two like rectiline figures into one rectiline figure like and equall to the same figures. Let their sides of like proportiō be set so that they make a right angle, as the lines AF and FB are. And vpo the line subtending the said angle, namely, the line AB , describe a rectiline figure like vnto the rectiline figures geuen and in like sort situate (by the 18. of this booke) and the same shall be equall to the two rectiline figures geuen (by the 31. of this booke).

¶ The second Proposition.

If two right lines cut the one the other obtuseangled wise, and from the endes of the lines which cut the one the other be drawen perpendicular lines to either line: the lines which are betwene the endes and the perpendicular lines are cut reciprokally.

Suppose

Suppose that there be two right lines AB and GD cutting the one the other in the point E , and making an obtuse angle in the section E . And from the endes of the lines, namely, A and G , let there be drawn to either line perpendicular lines, namely, from the point A to the line GD , which let be AD , and from the point G to the right line AB : which let be GB . Then I say, that the right lines AB and GD do, betwene the end A and the perpendicular B , and the end G and the perpendicular D , cut the one the other reciprokally in the point E : so that as the line AE is to the line ED , so is the line GE to the line EB . For forasmuch as the angles ADE and GBE , are right angles, therefore they are equall. But the angles AED and GEB are also equall (by the 15. of the first). Wherefore the angles remayning, namely, EAD , & EGB , are equall (by the Corollary of the 31. of the first). Wherefore the triangles AED and GEB , are equiangle. Wherefore the sides about the equall angles shall be proportionall (by the 4. of the sixt). Wherefore as the line AE is to the line ED , so is the line GE to the line EB . If therefore two right lines cut the one the other obtuseangled wise, &c: which was required to be proued.

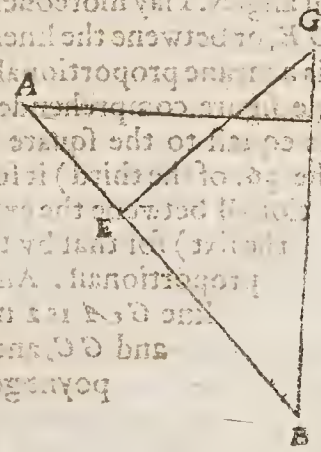


Demonstration of this proposition.

The third Proposition.

If two right lines make an acute angle, and from their endes be drawn to ech line perpendicular lines cutting them: the two right lines geuen shall be reciprokally proportionall as the segmentes which are about the angle.

Suppose that there be two right lines AB and GB , making an acute angle ABG . And from the poyntes A and G let there be drawn vnto the lines AB and GB perpendicular lines AC and GE , cutting the lines AB and GB in the poyntes E and C . Then I say, that the lines, namely, AB to GB , are reciprokally proportionall, as the segmentes, namely, CB to EB which are about the acute angle B . For forasmuch as the right angles ACB and GEB are equall, and the angle ABG is common to the triangles ABC , and GBE , therefore the angles remayning BAC and EBG are equall (by the Corollary of the 32. of the first). Wherefore the triangles ABC and GBE , are equiangle. Wherefore the sides about the equall angles are proportionall (by the 4. of the sixt): so that, as the line AB is to the line BC , so is the line GB to the line BE . Wherefore alternately as the line AB is to the line GB , so is the line CB to the line BE . If therefore two right lines make an acute angle, &c: which was required to be proued.



Demonstration of this proposition.

The fourth Proposition.

If in a circle be drawn two right lines cutting the one the other, the sections of the one to the sections of the other shall be reciprokally proportionall.

In the circle AGB let these two right lines AB and GD cut the one the other in the poynt E . Then I say, that reciprokally as the line AE is to the line ED , so is the line GE to the line EB . For forasmuch as (by the 35. of the third) the rectangle figure

Demonstration of this proposition.

The sixth Booke

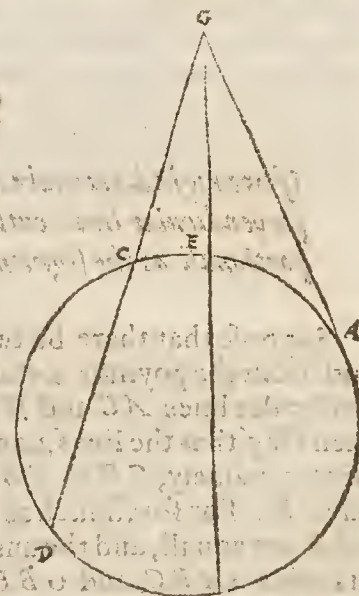
figure containned vnder the lines AE and EB is equall to the rectangle figure containned vnder the lines GE and ED , but in equall rectangle parallelogrammes the sides about the equall angles are reciprokall (by the 14. of the sixth) . Therefore the line AE is to the line ED reciprokally as the line GE is to the line EB (by the second definition of the sixth) . If therefore in a circle be drawn two right lines. &c : which was required to be proued.

¶ The fift Proposition.

If from a poynt geuen be drawn in a plaine superficies two right lines to the concaue circumference of a circle : they shall be reciprokally proportionall with their partes take without the circle. And moreover a right line drawn from the sayd poynt & touching the circle, shall be a meane proportionall betwene the whole line and the vtter segment.



Suppose that there be a circle ABD , and without it take a certayne poynt, namely, G . And from the point G drawe vnto the concaue circumference two right lines GB and GD , cutting the circle in the poyntes C and E . And let the line GA touch the circle in the point A . The I say, that the lines, namely, GB to GD are reciprokally as their partes taken without the circle, namely, as GC to GE . For forasmuch as (by the Corollary of the 36. of the third) the rectangle figure containned vnder the lines GB and GE is equall to the rectangle figure containned vnder the lines GD and GC , therefore (by the 14. of the sixth) reciprokally, as the line GB is to the line GD , so is the line GC to the line GE , for they are sides containning equall angles. I say moreover, that betwene the lines GB and GE , or betwene the lines GD and GC the touch line GA is a meane proportionall. For forasmuch as the rectangle figure comprehend vnder the lines GB and GE is equall to the square made of the line AG (by the 36. of the third) it followeth that the touch line GA is a meane proportionall betwene the extremes GB and GE (by the second part of the 17. of the sixth) for that by that Proposition the lines GB , GA , and GE are proportionall. And by the same reason may it be proued that the line GA is a meane proportionall betwene the lines GD and GC , and so of all others. If therefore from a poynt geuen, &c. which was required to be demonstrated.



Demonstration of the first part of this proposition.

Demonstration of the second part.

The end of the sixth booke of Euclides Elementes.



The seventh booke of Eu- clides Elementes.



HERETO IN THE SIXE bookes before hath Euclide passed through, and entreated of the Elementes of Geometrie without the ayde and succor of number. But the matters which remayne to be taught and to be spoken of in these his Geometrical bookes which follow as in the tenth, eleventh, and so forth, he could by no meanes fully and clearly make plaine & demonstrate, without the helpe and ayd of numbers. In the tenth is entreated of lines irrational and vncertaine, and that of many & sondry kindes: and in the eleventh & the other following he teacheth the natures of bodyes, and compareth theyr sides and lines together. All which for the most part are also irrational. And as rational quantites, and the comparisons and proportions of them, cannot be knowen, nor exactly tried, but by the meane of number, in which they are first sene and perceiued: euen so likewise cannot irrational quantities

be knowen and found out without number. As straightnes is the triall of crookednes, and inequalitye is tried by equalitie: so are quantities irrational perceiued and knowen by quantities rational: which are first and chiefly found among numbers. Wherefore in these three bookes following; being as it were in the middest of his Elementes, he is compelled of necessitie to entreate of numbers, although not so fully, as the nature of numbers requireth, yet so much as shall seme to be fit, and sufficiently to serue for his purpose. Wherby is sene the necessitie, that the one Arte, namely, Geometrie, hath of the other, namely, of Arithmetike. And also of what excellency and worthines Arithmetike is aboue Geometrie: in that, Geometrie boroweth of it principles, ayd, and succor, and is as it were maymed without it. Whereas Arithmetike is of it selfe, sufficient and needeth not at all any ayde of Geometrie, but is absolute and perfect in it selfe, and may well be taught and attained vnto without it. Agayne the matter or subiect where about Geometrie is occupied, which are lines, figures, and bodyes, are such as offer them selues to the senses, as triangles, squares, circles, cubes, and other are sene & iudged to be such as they are, by the sight: but number, which is the subiect and matter of Arithmetike, falleth vnder no sense, nor is represented by any shape, forme, or figure: and therefore cannot be iudged by any sense, but only by consideration of the minde, and vnderstanding. Now thinges sensible are farre vnder in degree then are thinges intellectuall: and are of nature much more grosse then they. Wherefore number, as being only intellectuall, is more pure, more immateriall, and more subtile, farre then is magnitude: and extendeth it selfe farther. For Arithmetike, not onely aydeth Geometrie: but ministrerth principles, and groundes to many other, nay rather to all other sciences and artes. As to musicke, Astronomy, naturall philosophy, perspectiue, with others. What other thing is in musicke entreated of; then number contracted to sound and voyce? In Astronomie, who without the knowledge of number can doo any thing, either in searching out of the motions of the heauens, and their courses, either in iudging and foretelling the effectes of them? In naturall philosophie, it is of no small force. The wisest and best learned philosophers that haue bene, as *Pythagoras*, *Timaeus*, *Plato*, and their followers, found out & taught most pithely and purely, the secret and hidden knowledge of the nature and condicion of all thinges, by numbers, and by the proprieties and passions of them. Of what force number is in perspectiue, let him declare and iudge, who hath any thing traueled therein. Yea to be short, what can be worthely and with prayse practised in common life of any man of any condition, without the knowledge of number. Yea it hath bene taught of the chiefest amongst philosophers, that all naturall thinges are framed, and haue their constitution of number. *Boetius* sayth *Hoc fuit principale in animo conditoris exemplar*: Number (sayth he) was the principall example and patron in the minde of the creator of the world. Doth not that great philosopher *Timaeus* in his booke, & also *Plato* in his *Timaeus*, following him, shew how the soule is composed of harmonically numbers, and consonantes of musicke. Number compaseth all thinges, and is (after these men) the being and very essence of all thinges. And ministrerth ayde and helpe, as to all other knowledges, so also no small to Geometrie. Which thing causeth *Euclide* in the middest of his booke of Geometrie, to inserte and place these three bookes of Arithmetike: as without the ayde of which, he could not well passe any farther.

In this seventh booke, he first placeth the generall principles, and first groundes of Arithmetike, and setteth the definitions of number or multitude and the kinds therof: as in the first booke, he did of magnitude and the kinds and partes thereof. After that he entreateth of numbers generally, and of theyr partes: and searcheth and demonstrateth, in generall the most common passions and proprieties of the same, and chiefly of numbers prime or incommensurable, and of numbers composed or commensurable and of their proprieties and partly also of the comparison or proportion of one number to an other.

Why Euclide in the middest of his workes was compelled to adde these three bookes of numbers.

Arithmetike of more excellency then Geometry.

Things intellectuall of more worthines then thinges sensible. Arithmetike ministrerth principles and groundes in a manner to all sciences.

Boetius: Cap. 2. Lib. prim. Arithmeti. Timaeus.

The argument of the seventh booke.

The seventh Booke

Definitions.

1 Vnity is that, whereby euery thing that is, is sayd to be on.

The first definition.

Without Vnity should be confusion of thinges.

Boetius in his booke de Vnitate & Vno.

An other definition of Vnity.

The second definition.

Difference betwene a point and Vnity. Boetius.

An other definition of number.

Iordane.

An other definition of numbers.

Vnity hath in it the vertue and power of all numbers. Number considered three maner of wayes.

As a point in magnitude, is the least thing in magnitude, and no magnitude at all, & yet the ground and beginning of all magnitudes: euen so is vnity in multitude or number, the least thing in number, and no number at all, and yet the ground and beginning of all numbers. And therefore it is here in this place, of *Euclide* first defined: as in the first booke, for the like reason and cause was a point first defined. *Vnity*, sayth *Euclide*, is that whereby euery thing is sayd to be one: that is, vnity is that, whereby euery thing is deuided and seperated from an other, and remayneth on in it selfe pure and distinct frō all others. Otherwise, were not this vnity, whereby all thinges are seiouyned the on from the other, all thinges should suffer mixtion and be in confusion. And where confusion is, there is no order, nor any thing can be exactly knowen, either what it is, or what is the nature, and what are the properties thereof. Vnity therefore is that which maketh euery thing to be that which it is. *Boetius* sayth very aptly: *Vnum quoddam, id est, quia Vnum numero est*, that is euery thing therefore is (that is, therefore hath his being in nature, and is that it is) for, that it is on in number. According whereunto *Iordane* (in that most excellent and absolute worke of Arithmetike which he wrote) defineth vnity after this maner.

Vnitas, est rei per se discretio: that is, vnity is properly, and of it selfe the difference of any thing. That is, vnity is that whereby euery thing doth properly and essentially differ, and is an other thing from all others. Certainly a very apt definition and it maketh playne the definition here set of *Euclide*.

2 Number is a multitude composed of vnities.

As the number of three, is a multitude composed and made of three vnities. Likewise the number of fīue is nothing ell, but the composition & putting together of fīue vnities. Although as was before sayde, betwene a poynt in magnitude, and vnity in multitude, there is great agreement and many thinges are common to them both, (for as a poynt is the beginning of magnitude, so is vnity the beginning of number. And as a poynt in magnitude is indiuisible, so is also vnity in number indiuisible) yet in this they differ and disagree. There is no line or magnitude made of pointes, as of his partes. So that although a point be the beginning of a lyne, yet is it no part therof. But vnity, as it is the beginning of number, so is it also a part therof, which is somewhat more manifestly set of *Boetius* in an other definition of number which he geueth in his Arithmetike, which is thus.

Numerus, est quantitatis aceruus ex Vnitatibus profusus, that is. Number is a masse or heape of quantities produced of vnities: which diffinition in substance is all one with the first, wherein is said most plainly, that the heape or masse, that is, the whole substance of the quantitie of number is produced & made of vnities. So that vnity is as it were the very matter of number. As foure vnities added together are the matter wherof the number 4. is made, & eche of these vnities is a part of the number foure, namely, a fourth part, or a quarter. Vnto this diffinition agreeth also the definition geuen of *Iordane*, which is thus. *Number is a quantitie which gathereth together thinges seuered a sonder*. As fīue men beyng in themselves seuered and distincte, are by the number fīue brought together, as it were into one masse, and so of others. And although vnity be no number, yet it contayneth in it the vertue and power of all numbers, and is set and taken for them.

In this place (for the farther elucidation of thinges, partly before set, and chiefly hereafter to be set, because *Euclide* here doth make mention of diuers kyndes of numbers, and also defineth the same) is to be noted, that number may be considered three maner of wayes. First, number may be considered absolutely, without comparyng it to any other number, or without applyng it to any other thing, onely vewing and payng what it is in it selfe, and in his owne nature onely, and what partes it hath, and what proprieties and passions. As this number fīue, may be considered absolutely in his owne nature, that it is an euen number, and that it is a perfect number, and hath many mo conditions and proprieties. And so conceiue ye of all other numbers, whatsoeuer, of 9. 12. and so forth.

An other way, number may be considered by way of cōparison, and in respect of some other number either as equall to it selfe, or as greater thē it selfe, or as lesse thē it selfe. As 12. may be considered, as compared to 12. which is equall vnto it, or as to 24. which is greater then it, for 12 is the halfe thereof, or as to 6. which is lesse then it, as beyng the double therof. And of this consideration of numbers ariseth and springeth all kyndes and varieties of proportiō: as hath before bene declared in the explanation of the principles of the fīft booke, so that of that matter it is needelesse any more to be sayd in this place.

The third way may numbers be considered as they are applied to formes and figures of Geometry. And numbers so considered are not represented by figures or charactes of number commonly vsed in Arithmetique, but are signified by certayne pointes or prickes, which represent the vnities which they contayne: which, accordyng to the diuersitie of the disposition and placing of them, may represent diuers formes and figures of Geometry: and accordyng to the nature of the figure which it representeth, it taketh his name, and is called a triangular number, a square number, a cube number or after any other figure. As if the figure of 10. be so described by his vni-

tics

ties that they be orderly set in a straight course, so that they represent the forme of a line, then is the number 10 called a lineall number. And if the same number 10 be so described by his vnities, that it shew forth the forme of a triangle, then it is called a triangler nuber: as ye here see. Likewise if 12 be in such sort described by his vnities, that it representeth that forme of figure which in Geometry is called a figure on the one side longer: then shall the number 12 be called a number hauyng the one side longer, and so may you conceaue of all others.

Thus much of this for the declaration of the thinges following.

3 A part is a lesse number in comparison to the greater when the lesse measureth the greater.

As the number 3 compared to the number 12. is a part. For 3 is a lesse number then is 12. and moreouer it measureth 12 the greater number. For 3 taken (or added to it selfe) certayne times (namely, 4 tymes) maketh 12. For 3 foure tymes is 12. Likewise is 2 a part of 8: 2 is lesse then 8, and taken 4 tymes it maketh 8. For the better vnderstanding of this diffinition, and how this worde Parte, is diuersly taken in Arithmetique and in Geometry, read the declaration of the first diffinition of the 5. booke.

The third definition.

4 Partes are a lesse number in respect of the greater, when the lesse measureth not the greater.

As the number 3 compared to 5, is partes of 5 and not a part. For the number 3 is lesse then the number 5, and doth not measure 5. For taken once it maketh but 3. once 3 is 3, which is lesse then 5. and 3 taken twise maketh 6, which is more then 5. Wherefore it is no part of 5 but partes, namely, three fifth partes of 5. For in the number 5 there are 5 vnities, and euery vnitie is the fifth part of 5. Wherefore 3 is three fifth partes of 5, and so of others.

The fourth definition.

5 Multiplex is a greater number in comparison of the lesse, when the lesse measureth the greater.

As 9 compared to 3 is multiplex, the number 9 is greater then the number 3. And moreouer 3 the lesse number measureth 9 the greater number. For 3 taken certayne tymes, namely, 3 tymes maketh 9. three tymes three is 9. For the more ample and full knowledge of this definition, read what is sayd in the explanation of the second definition of the 5 booke, where multiplex is sufficiently entreated of with all his kyndes.

The fifth definition.

6 An euen number is that, which may be deuided into two equal partes.

As the number 6 may be deuided into 3 and 3 which are his partes, and they are equall, the one not exceeding the other. This definition of Euclide is to be vnderstand of two such equall partes, which ioyned together, make the whole number: as 3 and 3 (the equall partes of 6) ioyned together, make 6, for otherwise many numbers both euen and odde may be deuided into many equall partes, as into 4. 5. 6. or 10, and therefore into 2. As 9 may be deuided into 3 and 3 which are his partes, and are also equall, for the one of them exceedeth not the other: yet is not therefore this number 9 an euen number, for that 3 and 3 (these equall partes of 9) added together make not 9 but onely 6. Likewise taking the definition so generally, euery number whatsoever should be an euen number: for in that sort of vnderstanding there is no number, but that it may be deuided into two equall partes: as this number 7 may be deuided into 3 partes, namely, 3. 1. and 3. of which two, namely, 3 and 3 are equal, yet is not 7 an euen number, because 3 and 3 added together, make not 7. Boetius therefore in the first booke of his Arithmetike, for the more playnes, after this maner defineth an euen number: *Numerus par est, qui potest in equalia dea diuidi, sine medio non intercedente*, that is, a number which may be deuided into two equal partes, without an vnitie comming betwene them. As 8 is deuided into 4 and 4 two equall partes without an vnitie comming betwene them, which added together, make 8, so that the sence of this definition is, that an euen number is that which is deuided into two such equall partes, which are his two halfe partes.

The sixth definition.

Boetius.

An other definition of an euen number.

Nero.

Pithagoras.

Here is also to be noted, that a part is taken in this definition, and in certayne definitions following, not in that signification as it was before defined, namely, for such a part as measureth the whole number, but for any part which helpeth to the making of the whole, and into which the whole may be resolved, so are 3 and 5 partes of 8 in this sence but not in the other sence. For neyther 3 nor 5 measureth 8. Pithagoras and his scholers gaue an other definition of an euen number (which definition Boetius also hath) after this maner.

The seventh Booke

An other definition.

An even number is that which in one and the same division is divided into the greatest, and into the least : in to the greatest, as touching space, and into the least as touching quantitie. As 10. is divided into 5 & 5. which are his greatest partes (which greatnes of partes he calleth space) and in the same division the number 10 is divided but into two partes : but into lesse the two partes nothing can be divided, which thing he calleth quantitie, so that 10 divided into 5 and 5. in that one division, is divided into the greatest, namely, into his halues, and into the least, namely, into two partes, and no mo.

An other definition.

There is also another definition more auncient, which is thus.
An even number is that which may be divided into two equal partes, and into two vnequall partes, but in neither division (to the constitution of the whole) to the even part is added the odde, neither to the odde is added the even. As 8 may be divided diuersly, partly into even partes, as into 4 and 4. likewise into 6 and 2. and partly into odde partes, as into 5 and 3. also into 7 and 1. In all which divisions, ye see no odde parte ioyned to an even, nor an even part ioyned to an odde : but if the one be even, the other is even, and if the one be odde, the other is odde. In the two first divisions, both partes were even, and in the two last, both partes were odde. It is to be considered that the two partes added together must make the whole.

An other definition.

This diffinition is generall and common to all even numbers, except to the number 2. which can not be divided into two vnequall partes, but onely into two vnities, which are equal.

There is yet geue an other diffinition of an even number, namely, thus. An even number is that which onely by an vnitie either above it or vnder it, differeth from an odde. As 8 being an even number differeth from 9 an odde number, being above it but by one. And also from 7. vnder it, it differeth likewise by one, and so of others.

The seventh definition.

7 An odde number is that which cannot be divided into two equal partes : or that which onely by an vnitie differeth from an even number.

As the number 5 can be by no meanes divided into two equal partes, namely, such two which added together, shall make 5. Or by the second definition, 5 an odde number, differeth from 6 an even number above it, by 1. And the same 5 differeth from 4 an even number vnder it likewise by 1.

Boetius defineth an odde number after this maner.

An other definition of an od number.

An odde number is that which cannot be divided into two equal partes, but that an vnitie shall be betwene them. As if ye divide 5 into 2 and 2. which are two equal partes, there remaineth one or an vnitie betwene them to make the whole number 5.

An other definition.

There is yet an other definition of an odde number. An odde number is that, which being divided into two vnequall partes, howsoever the one is ever even, and the other odde. As if 9 be divided into two partes, which added together, maketh the whole, namely, into 4 and 5. which are vnequall : ye see the one is even, namely, 4. and the other is odde, namely, 5. so if ye divide 9 into 6 and 3, or into 8 and 1. the one part is ever even, and the other odde.

The eight definition.

8 A number evenly even (called in latine pariter par) is that number, which an even number measureth by an even number.

As 8 is a number evenly even. For 4 an even number measureth 8 by 2, which is also an even number. This definition hath much troubled many, and seemeth not a true definition; for that there are many numbers which even numbers do measure, and that by even numbers, which yet are not evenly even numbers, after most mens minds: as 24. which an even number doth measure by foure, which is also an even number, and yet as they thinke is not 24 an evenly even number, for that 8 an even number doth measure also 24 by 3, an odde number. Wherefore Campane to make his sentence plaine, after this maner setteth forth this definition. Pariter par est quem cuncti pares eum numerant, paribus vicibus numerant, that is:

Campane.

An other definition of an evenly even number.

An evenly even number is, when all the even numbers which measure it, doo measure it by even times, that is, by even numbers, as 16. All the even numbers which measure 16, as are 2, 4, and 8, do measure it by even numbers. As 8 by 2, twice 8 is 16: 4 by 4, foure times 4 is 16: and 2 by 8, 8 times 2 is 16. Which particle (all even numbers) added by Campane maketh 24 to be no evenly even number. For that some one even number measureth it by an odde number as 8 by 3. Flusates also is plainly of this minde, that Euclide gaue not this definition in such maner as it is by Theon written, for the largenes & generalitie therof, for that it extendeth it to infinite numbers which are not evenly even as he thinketh, for which cause in place therof, he geueth this definition.

Flusates.

An other definition.

A number evenly even, is that which onely even numbers do measure. As 16 is measured of none but of even numbers, and therefore is evenly even. There is also of Boetius geuen an other definition of more facilitie, including in it no doubt at all, which is most commonly vsed of all writers, and is thus.

Boetius.

An other definition.

A number evenly even is, that which may be divided into two even partes, and that part againe into two even partes, and so continually dividing without stay till ye come to vnitie. As by example, 64 may be divided into

into 32 and 32. And either of these partes may be deuided into two euen partes, for 32 may be deuided into 16 and 16. Againe, 16 may be deuided into 8 and 8 which are euen partes, and 8 into 4 and 4. Againe 4 into 2 and 2, and last of all may 2 be deuided into one and one.

9 A number euenly odde (called in latine pariter impar) is that which an euen number measureth by an odde number.

The ninth definition.

As the number 6 which 2 an euen number measureth by 3 an odde number, three times 2 is 6. Likewise 10. which 2 an euen number measureth by 5 an odde number. In this diffinition also is found by all the expositors of *Euclide*, the same want that was found in the diffinition next before. And for that it extendeth it selfe to large, for there are infinite numbers which euen numbers do measure by odde numbers, which yet after their mindes are not euely odde numbers, as for example 12. For 4 an euen number, measureth 12 by 3 an odde number: three times 4 is 12. yet is not 12 as they thinke an euenly odde number. Wherefore *Campane* amendeth it after his thinking, by adding of this worde *all*, as he did in the first, and defineth it after this maner.

Campane.

A number euenly odde is, when all the euen numbers which measure it, do measure it by an euen tymes, that is, by an odde number.

An other definition.

As 10 is a number euenly odde, for no euen number but onely 2 measureth 10. and that is, by 5 an odde number. But not all the euen numbers which measure 12. do measure it by odde numbers. For 6 an euen number measureth 12 by 2 which is also euen. Wherefore 12 is not by this definition, a number euenly odde. *Flussates* also offended with the ouer large generalitie of this definition to make the definition agree with the thing defined putterh it after this maner.

Flussates.

A number euenly odde, is that which an odde number doth measure onely by an euen number.

An other definition.

As 14. which 7 an odde number doth measure onely by 2. which is an euen number. There is also an other definition of this kinde of number commonly geuen of more plaines, which is this.

An other definition.

A number euenly odde is that which may be deuided into two equall partes, but that part cannot agayne be deuided into two equall partes: as 6. may be deuided into two equall partes into 3. and 3. but neither of them can be deuided into two equall partes: for that 3. is an odde number and suffereth no such diuision.

The tenth definition.

10 A number oddly euen (called in latten im pariter par) is that which an odde number measureth by an euen number.

As the number 12: for 3 an odde number measureth 12, by 4. which is an euen number: three times 4 is 12.

This definition is not founde in the greeke neither was it doubtles euer in this maner written by *Euclide*: which thing the slendernes and the imperfection thereof and the absurdities following also of the same declare most manifestly. The definition next before geuen is in substance all one with this. For what number soeuer an euen number doth measure by an odde, the selfe same number doth an odde number measure by an euen. As 2. an euen number measureth 6. by 3. an odde number. Wherefore 3. an odde number doth also measure the same number 6. by 2. an euen number. Now if these two definitions be definitions of two distinct kindes of numbers, then is this number 6. both euenly euen, and also euenly odde and so is containd vnder two diuers kindes of numbers. Which is directly agaynst the authoritie of *Euclide* who playnely proueth here after in the 9. booke, that euery number whose halfe is an odde number, is a number euenly odde onely. *Flussates* hath here very well noted, that these two euenly odde, and oddly euen, were taken of *Euclide* for one and the selfe same kinde of number. But the number which here ought to haue bene placed is called of the best interpreters of *Euclide*, *numerus pariter par* & *impar*, that is a number euely euen, and euely odde. Yea and it is so called of *Euclide* him selfe in the 34. proposition of his 9. booke: which kinde of number *Campanus* and *Flussates* in steade of the insufficient and vnapt definition before geuen, assigne this definition.

This definition not found in the Greeke.

A number euenly euen, and euenly odde, is that which an euen number doth measure sometime by an euen number, and sometime by an odde.

An other definition.

As the number 12: for 2 an euen number, measureth 12. by 6. an euen number: two times 6. is 12. Also for 4 an euen number measureth the same number 12. by 3. an odde number. And therefore is 12. a number euenly euen, and euenly odde, and so of such others.

The cause why that *Campanus* and *Flussates* were so scrupulous in amending (as they supposed) the two definitions before, namely, of a number euely euen and of a number euenly odde, the one by adding this word *all*, and the other by adding this word *onely*, was for that they were offended with the largenes and generalitie of them. For that by them, one and the selfe same number might be comprehended vnder either definition. And so the selfe number should be both euenly euen, and also euenly odde: which they tooke for an absurditie. For that they are two distinct and diuers kindes of numbers. But all things well and iustly conceived, it shall not be hard nor amisse to thinke, that these definitions were set and written by *Euclide* in such forme and maner, as they are deliuered vnto vs by *Theon*: and

Campane and Flussates with other in an error.

The seventh Booke

One number in diuers respectes may be of diuers kindes of numbers.

that they neede not these corrections and amendementes by adding these wordes *all* and *onely*, for admit that they be distinct kinds of numbers, may not contraries be attributed in diuerse respectes to one thing? May not one line be sayd to be great and little, compared to diuers? Great in comparison of a lesse, and lesse in comparison of a greater? Euen so one number in diuers respects may be of diuers and contrary kindes of numbers: What are more diuers then a square number and a cube number. And yet is 64. in diuers respectes a number both square and cube. In respect of 8. to be his roote, it is a square number: for 8. times 8. is 64. and in respect of 4. to be his roote, it is a cube number for 4. times 4. fower times is 64: so in diuers respectes it is both, without any absurditie at all. Likewise this number 6. in diuers respectes, is a number on the one side longer, and also a triangular number: which yet are diuers and distinct kindes of numbers. For 6. described by his vnities resembling a figure of lengthe and breadthe hauing two sides, namely 2. and 3. is a plaine or superficiall number of one side longer. And if the same 6. be so described by his vnities, that it representeth the figure of a triangle, then is it and beareth it the name of a triangular figure: as here ye may see the forme of either. And if ye extend in description therof all his vnities in length onely, so is 6 also a lineall number. So you see 6 in diuers respectes is a lineall number, a number on the one side longer, and also a trigonall or triangular number, and yet therby no inconuenience at all. And why may not likewise one and the selfe number in diuers respectes be accompted a number both euenly euen, and euenly odde? Yea *Euclide* him selfe doth most manifestly proue the same, and in the same wordes, if it be diligently wayed, in his ninth booke. For he sayth, that all numbers being double in continuall course from the number 2. be euenly euen numbers only: and agayne all numbers whose halues are odde, are euenly odde numbers only: and that number which neither is duple from the number two: nor hath to his halfe an odde number is a number euenly euen and a number euenly odde. What in this can be spokē more playnely? So that by *Euclide* it is no inconuenience that on number, as 12, for example, in diuers respectes should be both a number euenly euen, and also a number euenly odde. In respect that 6. an euen number measureth 12. by 2. an euen number, 12 is a number euenly euen: and in respect that 4. an euen number measureth 12. by 3. an odde number, 12. is a number euenly odde. And thus iudge ye of all others like.

There is also an other definition geuen of this kinde of number by *Boetius* and others commonly which is thus.

Boetius definition of a number euenly euen, and euenly od.

A number euenly euen, and euenly odde is that, which may be deuided into two equall partes, and eche of them may agayne be deuided into two equall partes: and so forth. But this diuision is at length stayd, and continueth not till it come to vnitie. As for example 48: which may be deuided into two equall partes, namely, into 24. and 24. Agayne 24. which is one of the partes may be deuided into two equall partes 12. and 12. Agayne 12. into 6. and 6. And agayne 6 may be deuided into two equall partes, into 3. and 3: but 3. cannot be deuided into two equall partes. Wherefore the deuision there stayeth: and continueth not till it come to vnitie as it did in these numbers which are euenly euen only.

The eleuenth definition.

11 A number odly odde is that, which an odde number doth measure by an odde number.

As 25, which 5. an odde number, measureth by an odde number, namely, by 5. Five times five is 25: Likewise, 21. whom 7. an odde number doth measure by 3, which is likewise an odde number. Three times 7. is 21.

Flussatus.

Flussatus geueth this definition following of this kinde of number, which is all one in substance with the former definition.

An other definition.

A number odly odde, is that, which onely an odde number doth measure.

As 15. for no number measureth 15. but onely 5. and 3: also 25: none measureth it but onely 5. which is an odde number, and so of others.

The twelfth definition.

12 A prime (or first) number is that, which onely vnitie doth measure.

Prime numbers called incompounded numbers.

As 5. 7. 11. 13. For no number measureth 5, but onely vnitie. For v. vnities make the number 5. So no number measureth 7, but onely vnitie. 2. taken 3. times maketh 6. which is lesse then 7: and 2. taken 4. times is 8, which is more then 7. And so of 11. 13. and such others. So that all prime numbers, which also are called first numbers, and numbers vncompounded, haue no part to measure the, but onely vnitie.

The thirteenth definition.

13 Numbers prime the one to the other are they, which onely vnitie doth measure, being a common measure to them.

As 15. and 25. be numbers prime the one to the other. 15. of it selfe is no prime number, for not onely

by vnitie doth measure it, but also the numbers 5. and 3. for 3. times 5. is 15. Likewise 22. is of it selfe no prime number : for it is measured by 2. and 11. besides vnitie. For 11. twise, or 2. eleuen times, make 22. So that although neither of these two numbers 15. and 22. be a prime or incōposed number, but eyther haue partes of his owne, whereby it may be measured beside vnitie : yet compared together, they are prime the one to the other : for no one number doth as a common measure, measure both of them but onely vnitie, which is a common measure to all numbers. The numbers 5. and 3. which measure 15. will not measure 22 : againe, the numbers 2. and 11. which measure 22, do not measure 15.

14 A number composed, is that which some one number measureth.

The fourteenth definition.

A number composed is not measured onely by vnitie, as was a prime number, but hath some number which measureth it. As 15: the number 3. measureth 15, namely, taken 5. times. Also the number 5. measureth 15, namely, taken 3. times. 5. times 3. and 3. times 5. is 15. Likewise 18. is a composed number, it is measured by these numbers 6. 3. 9. 2. and so of others. These numbers are also called commonly second numbers, as contrary to prime or first numbers.

15 Numbers composed the one to the other, are they, which some one number, being a common measure to them both, doth measure.

The fifteenth definition.

As 12. and 8. are two composed numbers the one to the other. For the number 4. is a common measure to them both : 4. taken three times maketh 12 : and the same 4. taken two tymes maketh 8. So are 9. and 15 : 3. measureth them both. Also 10. and 25 : for 5. measureth both of them : and so infinitely of others. In thys do numbers composed the one to the other or second numbers, differre from numbers prime the one to the other : for that two numbers being composed the one to the other, ech of them seuerally is of necessitie a composed number. As in the examples before 8. and 12. are composed numbers : likewise 9. and 15 : also 10. and 25 : but if they be two numbers prime the one to the other, it is not of necessitie, that ech of them seuerally be a prime number. As 9. and 22. are two numbers prime the one to the other : no one number measureth both of them : and yet neither of them in it selfe and in his owne nature is a prime number, but ech of them is a composed number. For 3. measureth 9, and 11. and 2. measure 22.

16 A number is sayd to multiply a number, when the number multiplyed, is so oftentimes added to it selfe, as there are in the number multiplying vnities : and an other number is produced.

The sixteenth definition.

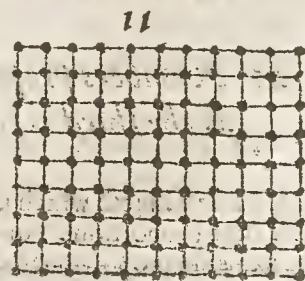
In multiplication are euer required two numbers, the one is whereby ye multiply, commonly called the multiplier or multiplicand, the other is that which is multiplied. The number by which an other is multiplied, namely, the multiplier, is sayd to multiply. As if ye will multiply 4. by 3, then is three sayd to multiply 4 : therefore according to this definition, because in 3. there are three vnities : adde 4. 3. times to it selfe, saying 3. times 4 : so shall ye bring forth an other number, namely, 12, which is the summe produced of that multiplication : and so of all other multiplications.

Two numbers required in multiplication.

17 When two numbers multiplying them selues the one the other, produce an other : the number produced is called a plaine or superficiall number. And the numbers which multiply them selues the one by the other, are the sides of that number.

The seventeenth definition.

As let these two numbers 3. and 6, multiply the one the other, saying, 3. times 6, or six tymes 3, they shall produce 18. Thys number 18. thus produced, is called a plaine number, or a superficiall number. And the two multiplying numbers which produced 12, namely, 3. and 6, are the sides of the same superficiall or plaine number, that is, the length and breadth thereof. Likewise if 9. multiply 11. or eleuen nine, there shall be produced 99. a plaine number, whose sides are the two numbers 9. and 11 : as the length and breadth of the same. They are called plaine and superficiall numbers, because being described by their vnities on a plaine superficies, they represent some superficiall forme or figure Geometricall, hauing length and breadth. As ye see of this example : and so of others. And all such plaine or superficiall numbers do euer represent right angled figures as appeareth in the example.



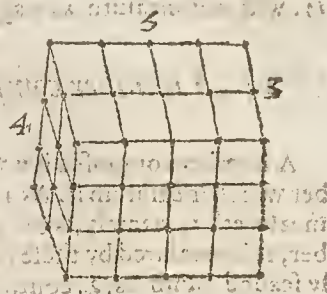
Why they are called superficiall numbers.

The seventh Booke

The eighteenth
definition.

18 When three numbers multiplyed together y one into the other, produce any number, the number produced, is called a solide number: and the numbers multiplying them selues the one into y other, are the sides therof.

As taking these three nũbers 3. 4. 5. multiply the one into the other. First 4. into 5. saying, foure times 5. is 20: then multiply that number produced, namely, 20. into 3: which is the third number, so shall ye produce 60. which is a solide number: and the three numbers which produced the number, namely, 3. 4. and 5. are the sides of the same. And they are called solide numbers, because being described by their vnities, they represent solide and bodylike figures of Geometry, which haue length, breadth, and thicknes. As ye see this number 60. expressed here by hys vnities. Whose length is hys side 5, his breadth is 3, and thicknes 4. And thus may ye do of all other three nũbers multiplying the one the other.

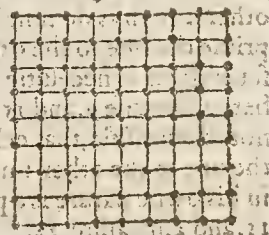


Why they are
called solide
numbers.

The nineteenth
definition.

19 A square number is that which is equally equall: or that which is contayned vnder two equall numbers.

As multiply two equall numbers the one into the other. As 9. by 9. ye shall produce 81, which is a square number. Euclide calleth it a number equally equall, because it is produced of the multiplication of two equall numbers the one into the other. Which numbers are also sayd in the second definition to contayne a square number. As in the definitions of the second booke two lines are sayd to contayne a square or a parallelogramme figure. It is called a square number, because being described by his vnities it representeth the figure of a square in Geometry. As ye here see doth the number 81, whose sides, that is to say, whose length and breadth, are 9. and 9, equall numbers: which also are sayd to contayne the square number 81: and so of others.

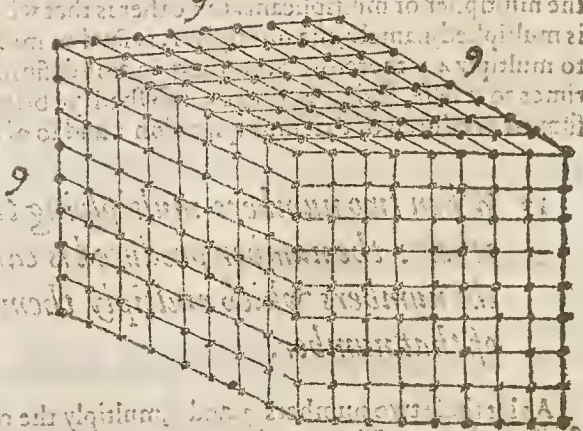


Why it is called
a square num-
ber.

The twentieth
definition.

20 A cube number is that which is equally equall equally: or that which is contayned vnder three equall numbers.

As multiply three equall numbers the one into the other, as 9, 9, and 9: first 9. by 9, so shall ye haue 81: which agayne multiply by 9, so shall ye produce 729, which is a cube number. And Euclide calleth it a number equally equall equally, because it is produced of the multiplication of three equall numbers the one into the other: which three numbers are sayd in the second definition (wherein he speaketh more applying to Geometry) to contayne the cube number. It is therefore called a cube number, because being described by hys vnities, it representeth the forme of a cube in Geometry, whose sides, that is to say, whose length, breadth, and thicknes, are the three equall numbers 9, 9, and 9, of which he was produced: which three sides also are sayd to contayne the cube number 729: beholde here the description therof.



Why it is called
a cube number.

The twenty one
definition.

21 Numbers proportionall are, when the first is to the second equemultiplex, as the third is to the fourth, or the selfe same part, or the selfe same partes.

Here he defineth which numbers are called proportionall, that is, what numbers haue one and the selfe same proportion. For example 6. to 3: and 4. to 2, are numbers proportionall, and haue one and the selfe same proportion: for 6. the first is to 3. the second equemultiplex, as 4. the third is to 2. the fourth:

6 is double to 3: and so is 4 double to 2. Likewise these foure numbers are in like proportion 3. 9. 4. 12. for what part 3 is of 9, such part is 4 of 12: 3 of 9 is a third part, so is also 4 of 12 a third part. So are these foure numbers also in proportion 2. 5: 4. 10: what partes 2 are of 5, such partes are 4 of 10: 2 of 5 are two fift partes, likewise 4 of 10 are two fift partes. Moreover, these numbers 8. 6: 12. 9: be in proportion, for what and how many parts 8 are of 6, such & so many parts are 12 of 9: 8 of 6 is foure third partes, for one third part of 6 is 2, which taken foure times maketh 8: so 12 of 9 is also foure thyrd partes: for one third part of 9 is 3, which taken foure times make 12. And so conceaue ye of all other proportionall numbers.

In the sixt definition of the v. booke, Euclide gaue a farre other definition of magnitudes proportionall, and much vnlike to this which he here geueth of numbers proportionall: the reason is as there also was partly noted, for that there he gaue a definition common to all quantities discrete and continuall, rationall, and irrational: and therefore was constrained to geue the definition by the excesse, equallitie or want of their equemultiplices, and that generally onely: for that irrational quantities haue no certayne part or common measure to be measured by or knowen, neyther can they be expressed by any certayne numbers. But here in this place because in numbers there are no irrational quantities, but all are certaynly knowen, so that both they and the proportions betwene them may be expressed by numbers certayne and knowen, by reason of their partes certayne, and for that they haue some common measure to measure them (at the left vnitie which is a common measure to all numbers) he geueth here this definition of proportionall numbers, by that the one is like equemultiplex to the other, or the same part, or the same partes: which definition is much easier then was the other: and is not so large, as is the other, neither extendeth it self generally to all kinde of quantities rationall and irrational, but containeth it selfe within the limites and bondes of rationall quantitie and numbers.

Why the definition of proportionall magnitudes is vnlike to the definition of proportionall numbers.

22 Like plaine numbers, and like solide numbers, are such, which haue their sides proportionall.

The twenty two definition.

Before he shewed that a plaine number hath two sides, and a solide number three sides, Now he sheweth by this definition which plaine numbers, and which solide numbers are like the one to the other. The likenes of which numbers dependeth altogether of the proportions of the sides of these numbers. So that if the two sides of one plaine number haue the same proportion the one to the other, that the two sides of the other plaine number haue the one to the other, then are such two plaine numbers like. For an example 6 and 24 be two plaine numbers, the sides of 6 be 2 and 3, two tymes 3 make 6: the sides of 24 be 4 and 6, foure tymes 6 makes 24. Againe the same proportion that is betwene 3 and 2 the sides of 6, is also betwene 6 and 4 the sides of 24. Wherefore 24 and 6 be two like plaine and superficiall numbers. And so of other plaine numbers. After the same manner is it in solide numbers. If three sides of the one be in like proportion together as are the three sides of the other, then is the one solide number like to the other. As 24 and 192 be solide numbers, the sides of 24 are 2, 3, and 4, two tymes three taken 4 times are 24, the sides of 192 are 4, 6, and 8, for foure tymes 6, 8 times make 192. Againe the proportion of 4 to 3 is sesquitercia, the proportion of 3 to 2 is sesquialtera, which are the proportions of the sides of the one solide number, namely, of 24: the proportion betwene 8 and 6 is sesquitercia, the proportion betwene 6 and 4 is sesquialtera, which are the proportions of the sides of the other solide number, namely, of 192: And they are one and the same with the proportions of the sides of the other, wherefore these two solide numbers 24 and 192 be like, and so of other solide numbers.

Why the definition of proportionall magnitudes is vnlike to the definition of proportionall numbers.

23 A perfect number is that, which is equall to all his partes.

The twenty three definition.

As the partes of 6 are 1, 2, 3, three is the halfe of 6, two the third part, and 1, the sixth part, and mo partes hath not: which three partes 1, 2, 3, added together, make 6 the whole number, whose partes they are. Wherefore 6 is a perfect number. So likewise is 28 a perfect number, the partes whereof are these numbers 14, 7, 2 and 1: 14 is the halfe therof, 7 is the quarter, 4 is the seventh part, 2 is a fourtenth part, and 1 an 28 part, and these are all the partes of 28, all which, namely, 1, 2, 4, 7 and 14 added together, make iustly without more or lesse 28. Wherefore 28 is a perfect number, and so of others the like. This kinde of numbers is very rare and seldome found. From 1 to 10, there is but one perfect number, namely, 6. From 10 to an 100, there is also but one, that is, 28. Also from 100 to 1000 there is but one which is 496. From 1000 to 10000 likewise but one. So that betwene euery stay in numbring, which is ener in the tenth place, there is found but one perfect number. And for their rarenes and great perfection, they are of maruelous vse in magike, and in the secret part of philosophy.

Why the definition of proportionall magnitudes is vnlike to the definition of proportionall numbers.

Perfect numbers rare & of great vse in magike & in secret philosophy.

This kinde of number is called perfect, in respect of other numbers which are imperfect. For as the nature of a perfect number standeth in this, that all his partes added together are equall to the whole: and make the whole: so in an imperfect nuber all the parts added together are not equal to the whole, nor make the whole, but make either more or lesse. Wherefore of imperfect numbers there are two kinds, the one is called abundans or abounding, the other diminutus, or wanting.

In what respect a number is perfect.

Two kinds of imperfect numbers,

A number abounding is that whose partes being all added together make more then the whole number whose partes they are, as 12 is an abundant number. For all the partes of 12, namely, 6, 4, 3, 2,

and

The seventh Booke

and 1. added together make 16: which are more then 12. Likewise 18. is a number abunding, all his partes namely 7, 9, 6, 3, 2. and 1. added together make 20. which are more then 18: and so of others.

A number wanting-

A number diminute, or wanting is that whose partes being all added together, make lesse then the whole, or number whose partes they are.

As 9. is a diminute, or wanting number, for all his partes, namely, 3. and 1. (more partes he hath not) added together make onely 4: which are lesse then 9. Also 26. is a diminute number, all his partes, namely, 13, 2, 1. added together make onely 16: which is a number much lesse then 26. And so of such like.

Common sentences.

Compane and Flusates here adde certayne common sentences, some of which, for that they are in these three bookes following somtimes alledged, I thought good here to annexe.

First common sentence.

1 The lesse part is that which hath the greater denomination: and the greater part is that, which hath the lesse denomination.

As the numbers 6. and 8. are either of them a part of the number 24: 6. is a fourth part, 4. times 6. is 24: and 8. is a third part, 3. times 8. is 24. Now forasmuch as 4. (which denominateth what part 6. is of 24) is greater then 3. (which denominateth what part 8. is of 24.) therefore is 6. a lesse part of 24. then is 8. and so is 8. a greater part of 24. the 6. is. And so in others.

Second common sentence.

2 Whatsoeuer numbers are equemultiples to one & the selfe same number, or to equall numbers, are also equall the one to the other.

As if vnto the number 3. be taken two numbers containing the same number foure times, that is, being equemultiples to the same number three: the sayd two numbers shalbe equall. For 4. times 3. will euer be 12. So also will it be if vnto the two equal numbers 3. & 3. be taken two numbers, the one cotaining the one number 3. foure times, the other containing the other number 3. also foure times, that is, being equemultiples to the equall numbers 3. and 3.

Third common sentence.

3 Those numbers to whome one and the selfe same number is equemultiplex, or whose equemultiples are equall: are also equall the one to the other.

As if the number 18. be equemultiplex to any two numbers, that is, contayne any two numbers twise, thrixe, fower times &c. As for example 3. times: then are the sayd two numbers equall. For 18. deuided by 3. will euer bring forth 6. So that that diuision made twise will bring forth 6. and 6. two equall numbers. So also would it follow if the two numbers had equall equemultiples, namely, if 18. and 18. which are equall numbers contayned any two numbers 3. times.

Fourth common sentence.

4 If a number measure the whole, and a part taken away: it shall also measure the residue.

As if from 24. be taken away 9. there remaineth 15. And for as much as the number 3. measureth the whole number 24, & also the number take away, namely, 9. it shall also measure the residue, which is 15. For 3. measureth 15 by five, five times 3. is 15. And so of others.

Fifth common sentence.

5 If a number measure any number: it also measureth euery number that the sayd number measureth.

As the number 6. measuring the number 12. shall also measure all the numbers that 12. measureth: as the numbers 24, 36, 48, 60. and so forth: which the number 12. doth measure

measure by the numbers 2, 3, 4, and 5. And for as much as the number 12, doth measure the numbers 2, 3, 4, 6, 8, and 12. And the number 6, doth measure the number 12. (namely by 2.) It followeth by this common sentence, that the number 6, measureth eche of these numbers 2, 3, 4, 6, 8, and 12. And so of others.

6 If a number measure two numbers, it shall also measure any number composed of them.

Sixth common sentence.

As the number 3 measureth these two numbers 6, and 9: it measureth 6, by 2, and 9, by 3. And therefore by this common sentence it measureth the number 15. which is composed of the numbers 6, and 9: namely it measureth it by 3.

7 If in numbers there be proportions how many soever equall or the selfe same to one proportion: they shall also be equall or the selfe same the one to the other.

Seuenth common sentence.

As yf the proportion of the number 6, to the number 3, be as the proportion of the number 8, to the number 4, if also, the proportion of the number 10, to the number 5, be as the proportion of the number 8, to the number 4: then shall the proportion of the number 5, to the number 3, be as the proportion of the number 10, to the number 5: namely, eche proportion is duple. And so of others. Euclide in his 5. booke the 11. proposition demonstrated this also in continuall quantitie: which although as touching that kinde of quantitie it might haue bene put also as a principle (as in numbers he taketh it) yet for that in all magnitudes theyr proportion can not be expressed, (as hath before bene noted & shalbe afterward in the tenth booke more at large made manifest:) therefore he demonstrateth it there in that place, and proueth that it is true as touching all proportions generally whither they be rational or irrational.

¶ The first Proposition. The first Theoreme.

If there be geuen two vnequall numbers, and if in taking the lesse continually from the greater, the number remayning do not measure the number going before, vntill it shall come to vnitie: then are those numbers which were at the beginning geuen, prime the one to the other.



Suppose that there be two vnequal numbers A B the greater, and C D the lesse, and from A B the greater, take away C D the lesse as often as you can leauing F A, and from C D take away F A as often as you can, leauing the number G C. And from F A take away G C as often as you can, and so do continually till there remayne onely vnitie, which let be H A. Then I say that no number measureth the numbers A B and

Construction.

E . . .
D . . . G . . C
B F . . H . A

Demonstration leading to an absurditie.

C D. For if it be possible let some number measure them, and let the same be E. Now C D measuring A B leaueth a lesse number then it selfe, which let be F A. And F A measuring D C leaueth also a lesse then it selfe, namely, G C. And G C measuring F A leaueth vnitie H A. And forasmuch as the number E measureth D C, and the number C D measureth the number B F, therefore the number E also measureth B F, and it measureth the whole

whole number B A, wherfore it also measureth that which remaineth, namely, the number F A (by the 4. common sentence of the seuenth). But the number A F measureth the number D G, wherfore E also measureth D G. And it measureth also the whole D C, wherfore it also measureth that which remaineth, namely, the number G C (by the same common sentence): but G C measureth the number F H, wherfore also E measureth F H, and it measureth the whole number F A, wherfore (by the former common sentence) it also measureth that which remaineth H A, which is vnitie, it selfe being a number, which is impossible. Wherfore no number doth measure the numbers A B and C D, wherfore the numbers A B and C D are prime numbers the one to the other: which was required to be proued.

The conuerse of this proposition after Campane.

The conuerse of this proposition.

And if the two numbers, namely A B and C D be prime the one to the other. Then the lesse being continually taken from the greater there shalbe no stay of that subtraction, till that you come to vnitie. For if in the continuall subtraction there be a stay before you come to vnitie.

Suppose that H A be the number whereat the stay is made, which also being subtrahed out of G C leaueth nothing. Wherfore H A measureth G C wherfore also it measureth F H by the 5. common sentence of the seuenth. And for as much as it also measureth it selfe, therefore it also measureth the whole A F by the sixth common sentence of the seuenth, wherfore also it measureth D G by the 5. common sentence. But it is before proued that it measureth G C, wherfore it measureth the whole C D, by the sixth common sentence of the seuenth: wherfore also it measureth B F by the 5. common sentence of the seuenth. And it is also proued that it measureth F A, wherfore also it measureth the whole number A B by the sixth common sentence of the seuenth. Now for as much as the number H A measureth the numbers A B and C D, therefore the numbers A B and C D are numbers composed: wherfore they are not prime the one to the other: which is contrary to the supposition.

How to know whether two numbers geuen be prime the one to the other.

And by this proposition if there be two numbers geuen. It is easy to finde out, whether they be prime the one to the other or no. For if by such continual subtraction of the lesse from the greater, you come at the length to vnitie. Then are those numbers geuen prime the one to the other. But if there be a stay before you come to vnitie, then are the numbers geuen, numbers composed the one to the other.

¶ The 1. Probleme.

The 2. Proposition.

Two numbers being geuen not prime the one to the other, to finde out their greatest common measure.

Two cases in this probleme. The first case.



Suppose the two numbers geuen not prime the one to the other to be A B and C D. It is required to finde out the greatest common measure of the said numbers A B and C D. Now the number C D either measureth the number A B or not. If C D measure A B it also measureth it selfe. Wherfore C D is a common measure to the numbers C D and A B. And it is manifest also that it is the greatest common measure: for there is no number greater then C D that will measure C D.

The second case.

But if C D do not measure A B, then if of the numbers A B and C D, the lesse be continually taken away from the greater, there will before you come to vnitie, be left a number, which will measure the number going before (by the 1. of the seuenth). For if there should not, then should the numbers A B and C D be prime the one to the other, which is contrary to the supposition. Let the sayd number left by the continuall subtraction of the lesse number out of the greater be F C. So that let the number C D measuring A B, and subtrahed out of it as often as you can leue a lesse number then it selfe, namely A E. And let A E measuring C D, and subtrahed out of it

as often as you can leaue a lesse then it selfe namely, C F. And suppose that C F do so measure A E that there remayne nothing. Then I say that C F is a common measure to the numbers A B and C D. For forasmuch as C F measureth A E, and A E measureth D F, therefore C F also measureth D F (by the fifth common sentence of the seuenth) and it likewise measureth it selfe, wherefore it also measureth the whole C D (by the sixth common sentence of the seuenth) but C D measureth B E, wherefore C F also measureth B E (by the fift common sentence of the seuenth). And it measureth also E A: wherefore it also measureth the whole B A (by the sixth common sentence of the seuenth): and it also measureth C D as we haue before proued: wherefore the number C F measureth the numbers A B & C D: wherefore the number C F is a common measure to the numbers A B & C D.

Demonstratiō
of the second
case.

That C F is a
common mea-
sure to the
numbers A B
and C D.

I say also that it is the greatest common measure. For if C F be not the greatest common measure to A B and C D, let there be a number greater then C F, which measureth A B and C D: which let be G. And forasmuch as G measureth C D, and C D measureth B E, therefore G also measureth B E (by the fift common sentence of the seuenth). And it measureth the whole A B, wherefore also it measureth the residue, namely, A E (by the 4. common sentence of the seuenth). But A E measureth D F, wherefore G also measureth D F (by the foresayd 5. common sentence of the seuenth). And it measureth the whole C D. Wherefore it also measureth the residue F C: namely, the greater number the lesse: which is impossible. No number therefore greater then C F shall measure those numbers A B and C D: wherefore C F is the greatest common measure to A B and C D: which was required to be done.

That C F is
the greatest
common mea-
sure to A B
and C D.

Corrolary.

Hereby it is manifest, that if a number measure two numbers it shall also measure their greatest common measure. For if it measure the whole & the part taken away, it shall alwayes measure the residue also, which residue is at the length, the greatest common measure of the two numbers geuen.

¶ The 2. Probleme.

Th 3. Proposition.

Thre numbers being geue, not prime the one to the other: to finde out their greatest common measure.

Suppose the three numbers geuen not prime the one to the other to be A, B, C. Now it is required vnto the sayd numbers A, B, C to finde out the greatest common measure. Take the greatest common measure of the two numbers A and B (by the 2 of the seuenth) which let be D: which number D either measureth the number C or not.

First let D measure C. And it also measureth the numbers A and B, wherefore D measureth the numbers A, B, C. Wherefore D is a common measure vnto the numbers A, B, C. Then I say also, that it is the greatest common measure vnto them. For if D be not the greatest common measure vnto the numbers A, B, C, let some number greater then D measure the numbers A, B, C. And let the same number be E. Now forasmuch as E measureth the numbers A, B, C, it measureth also the numbers A, B. Wherefore it measureth also

Two cases in
this Proposi-
tion.
The first case.

Tt.i.

the

The seventh Booke

the greatest common measure of the numbers A, B (by the Corollary of the second of the seventh). But the greatest common measure of the numbers A, B , is the number D (by construction). Wherefore the number E measureth the number D , namely, the greater the lesse: which is impossible. Wherefore no number greater then D measureth the numbers A, B, C . Wherefore D is the greatest common measure to the numbers A, B, C .

The second case.

But now suppose that D do not measure C . First I say that D & C are not prime numbers the one to the other. For forasmuch as the numbers A, B, C , are not prime the one to the other (by supposition) some one number will measure them: but that number that measureth the numbers A, B, C , shall also measure the numbers A, B , and shall likewise measure the greatest common measure of A, B , namely, D (by the Corollary of the second of the seventh). And the sayd number measureth also C . Wherefore some one number measureth the numbers D and C . Wherefore D and C are not prime the one to the other.

Now then let there be taken (by the 2. of the seventh) the greatest common measure unto the numbers D and C , which let be the number E . And forasmuch as E measureth D , and D measureth the numbers A, B , therefore E also measureth the numbers A, B (by the sixth common sentence): and it measureth also C . Wherefore E measureth the numbers A, B, C . Wherefore E is a common measure unto the numbers A, B, C . I say also that it is the greatest. For if E be not the greatest common measure unto the numbers A, B, C , let there be some number greater then E , which measureth the numbers A, B, C . And let the same number be F . And forasmuch as F measureth the numbers A, B, C : it measureth also the numbers A, B . Wherefore also it measureth the greatest common measure of the numbers A, B (by the Corollary of the 2. of the seventh). But the greatest common measure of the numbers A, B , is D . Wherefore E measureth D . And it measureth also the number C . Wherefore F measureth the numbers D, C . Wherefore also (by the same Corollary) it measureth the greatest common measure of the numbers D, C . But the greatest common measure of the numbers D, C , is E . Wherefore F measureth E , namely, the greater number the lesse: which is impossible. Wherefore no number greater then E shall measure the numbers A, B, C . Wherefore E is the greatest common measure to the numbers A, B, C : which was required to be done.

¶ Corollary.

Wherefore it is manifest, that if a number measure three numbers, it shall also measure their greatest common measure. And in like sort more numbers being given not prime the one to the other may be found out their greatest common measure, and the Corollary will followe.

¶ The 2. Theoreme.

The 4. Proposition.

Every lesse number is of every greater number, either a part, or partes.

Two cases in this Proposition.

The first case.

Suppose there be two numbers A and BC . Of which let BC be the lesse. Then I say, that BC is either a part or partes of A . For the numbers A and BC are either prime the one to the other, or not. First let A and BC be prime the one to the other. And divide the number BC into those vnities which are in it. Now every one of the

the unities which are in B C is some certaine part of A. Wherefore B C are partes of A.

But now suppose that the numbers A and B C be not prime the one to the other. Then B C either measureth A or not measureth it. If B C measure A, then is B C a part of A. But if not, take (by the 2. of the seventh) the greatest common measure of A and B C, and let the same be D. And let B C be deuided into as many partes as it hath equall vnto D, that is, into B E, E F, and F C. And forasmuch as D measureth A, therefore D is a part of A. But D is equall vnto euery one of these partes B E, E F, and F C. Wherefore also euery one of these partes B E, E F, and F C, is a part of A. Wherefore the number B C is partes of A. Wherefore euery lesse number is of euery greater nuber, either a part or partes, which was required to be proued.

The second case.

A
B C
B C
B E F C
D

The 3. Theoreme The 5. Proposition.

If a number be a part of a number, and an other nuber the selfe same part of an other number, then both the numbers added together shall be the selfe same part of both the numbers added together, which one number was of one number.



Suppose the number A to be a part of the number B C, and let an other number, namely, D, be the selfe same part of an other number, namely, of E F. The I say, that the numbers A and D added together, are the selfe same part of the numbers B C and E F added together, that A is of B C. For forasmuch as what part the number A is of the number B C, the selfe same part is the number D of the number E F, therefore how many numbers there are in B C equall vnto A, so many numbers are there in E F equall vnto D. Deuide I say, B C into the numbers that are equall vnto A, that is, into B G, and G C, and likewise deuide E F into the numbers that are equall vnto D, that is, into E H and H F. Now then the multitude of these B G and G C, is equall vnto the multitude of these E H and H F. And forasmuch as B G is equall vnto A, and E H vnto D, therefore B G and E H are equall vnto A & D. And by the same reason forasmuch as G C is equall vnto A, and H F vnto D: therefore G C and H F are also equall vnto A and D. Wherefore how many numbers there are in B C equall vnto A, so many are there in B C and E F equall vnto A and D. Wherefore how multiplex B C is to A, so multiplex are both the numbers B C and E F, to both the numbers A and D. Wherefore what part A is of B C, the selfe part also are A & D added together, of B C and E F added together: which was required to be proued.

This proposition is the 6. proposition in discrete quantity, answer to the first of the fifth in continual quantity.

Demonstration.

The 4. Theoreme The 6. Proposition.

If a number be partes of a number, and an other number the selfe same partes of an other number: then both numbers added together shall be of both numbers added together the selfe same partes, that one number was of one number.

Tt. ij. Suppose

The seventh Booke

Construction.

Demonstration.

Suppose that the number AB be partes of the number C, and let an other number, namely, DE be the selfe same partes of an other number, namely, of F. Then I say, that the numbers AB and DE added together, are of the numbers C and F added together the selfe same partes, that AB is of C. For forasmuch as what partes AB is of C, the selfe same partes is DE of F: $A \dots G \dots B$ therefore how many partes of C there are in AB, so many partes of F are there in DE. Deuide AB into the partes of C, that is, into AG and GB, and likewise DE into the partes of F, that is, into DH and HE. Now then the multitude of these AG and GB is equall to the multitude of these DH and HE. And forasmuch as what part AG is of C, the selfe same part is DH of F: therefore what part AG is of C, the selfe same part is AG and DH added together of C and F added together. And by the same reason also what part GB is of C, the selfe same part is GB and HE added together of C and F added together. Wherefore what partes AB is of C, the selfe same partes are AB and DE added together of C and F added together: which was required to be demonstrated.

¶ The 5. Theoreme.

The 7. Proposition.

If a number be the selfe same part of a number, that a part taken away is of a part taken away: then shall the residue be the selfe same part of the residue, that the whole was of the whole.

This proposition and the next following in discrete quantities answereth to the fifth proposition of the fifth booke in continuall quantity. Construction. Demonstration.

Suppose that the number AB be of the number CD the selfe same part, that the part taken away AE is of the part taken away CF. Then I say, that the residue EB is of the residue FD the selfe same part that the whole AB is of the whole CD. What part AE is of CF, the selfe same part let EB be of GC. And for that what part AE is of CF, the same part is EB of CG, therefore what part AE is of CF, the same part (by the 5. of the seventh) is AB of FG. But what part AE is of CF, the same part (by supposition) is AB of CD. Wherefore what part AB is of FG, the selfe same part is AB of CD. Wherefore AB is one & the selfe same part of both these numbers GF and CD. Wherefore GF is equall vnto CD (by the second common sentence of the seventh). Take away CF which is common to them both. Wherefore the residue GC is equall vnto the residue FD. And forasmuch as what part AE is of CF, the same part is EB of GC: but GC is equall vnto FD: therefore what part AE is of FC, the selfe same part is EB of FD. But what part AE is of CF, the same part is AB of CD. Wherefore what part EB is of FD, the same part is AB of CD. Wherefore the residue EB is of the residue FD the selfe same part that the whole AB is of the whole CD: which was required to be demonstrated.

¶ The 6. Theoreme.

The 8. Proposition.

If a number be of a number the selfe same partes, that a part taken away is of a part taken away, the residue also shall be of the residue the selfe same partes that the whole is of the whole.

Suppose



Suppose that the number AB be of the number CD the selfe same partes that the part taken away AE is of the part taken CF . Then I say, that the residue EB is of the residue FD the selfe same partes that the whole AB is of the whole CD . Vnto AB put an equall nūber GH . Wherefore what partes GH is of CD , the selfe same partes is AE of CF . Denide GH into the partes of CD , that is, GK , and KH , and likewise AE into the partes of CF , that is, into AL and LE . Now then the multitude of these GK and KH , is equall vnto the multitude of these AL and LE . And forasmuch as what part GK is of CD , the selfe same part is AL of CF : but CD is greater then CF . Wherefore GK is greater then AL . Put vnto AL an equall number MG . Wherefore what part GK is of CD , the same part is GM of CF . Wherefore the residue MK is (by the 7. of the seuenth.) of the residue FD , the selfe same part that the whole GK is of the whole CD . Againe, forasmuch as what part KH is of CD , the selfe same part is EL of CF : but CD is greater then CF . Wherefore HK is greater then EL . Put vnto EL an equall number KN . Wherefore what part KH is of CD , the selfe same part is KN of CF . Wherefore the residue also NH is (by the 7. of the seuenth) of the residue FD , the selfe same part that the whole KH is of the whole DC . Wherefore both these MK and NH added together are (by the 5. of the seuenth) of DF the self same partes that the whole HG is of the whole CD . But both these MK and NH added together, are equall vnto EB . And HG is equall vnto BA . Wherefore the residue EB is of the residue FD the selfe same partes that the whole AB is of the whole CD : which was required to be proued.

Construction.

Demonstration.

$A \dots L \dots E \dots B$
 $C \dots \dots \dots F \dots D$
 $G \dots M \dots K \dots N \dots H$

¶ An other demonstration after Flusates.

Suppose that the number AB be of the number CD the selfe same partes that the part taken away AE is of the part taken away CF . Then I say, that the residue EB is of the residue FD the selfe same partes that the whole AB is of the whole CD . Let EB be of CI the self same partes that AB is of CD , or AE of CF . Now forasmuch as EB is of CI the selfe same partes that AE is of CF : therefore both these AE and EB added together are of both these CF and CI added together (that is, the whole AB is of the whole FI) the selfe same partes that AE is of CF (by the sixt of this booke). But what partes AE is of CF , the selfe same partes is the number AB of the number CD (by supposition). Wherefore what partes the number AB is of the nūber FI , the selfe same partes is the same number AB of the number CD . Wherefore the numbers FI and CD are equall. Take away the number CF which is common to them both. Wherefore the numbers remayning CI and FD are equall. Wherefore what partes the number EB is of the number CI , the selfe same partes is the same number EB of the number FD . But what partes EB is of CI , the selfe same partes (by construction) is AB of CD . Wherefore what partes the residue EB is of the residue FD , the selfe same partes is the whole AB of the whole CD : which was required to be proued.

An other demonstration after Flusates.

$B \dots E \dots \dots A$
 $D \dots F \dots \dots C \dots I$

¶ The 7. Theoreme.

The 9. Proposition.

If a number be a part of a number, and if an other number be the self same part of an other nūber: then alternately what part or partes the first is of the third, the self same part or partes shall the second be of the fourth.



Suppose that the number A be of the number BC the selfe same part, that an other number D is of an other number EF . And let A be lesse then D . Then I say, that alternately what part or partes A is of D , the selfe same part or partes is BC of EF . For forasmuch as what part A is of BC , the selfe same part is D of EF , therefore how many

$A \dots$
 $B \dots G \dots C$
 $D \dots$
 $E \dots H \dots F$

Construction.

Try numbers

The seventh Booke

Demonstration.

numbers there are in BC equall vnto A , so many are there in EF equall vnto D . Deuide BC into the numbers equall vnto A , that is, into BG & GC : and likewise EF into the numbers equall vnto D , that is, into EH and HF . Now then the multitude of these BG and GC , is equall vnto the multitude of these EH & HF . And forasmuch as the numbers BG and GC are equall the one to the other, the numbers also EH and HF are equall the one to the other: and the multitude of these BG & GC is equall vnto the multitude of these EH and HF . Wherefore what part or partes BG is of EH , the self same part or partes is GC of HF . Wherefore what part or partes BG is of EH , the self same part or partes (by the fift & sixt of the seuenth) are BG and GC added together, of EH and HF added together. But BG is equall vnto A , and EH vnto D . Wherefore what part or partes A is of D , the self same part or partes is BC of EF : which was required to be demonstrated.

$A \dots$
 $B \dots G \dots C$
 $D \dots$
 $E \dots H \dots F$

¶ The 8. Theoreme.

The 10. Proposition.

If a number be partes of a number, and an other nūber the self same partes of an other number, then alternately what partes or part the first is of the third, the self same partes or part is the second of the fourth.



Construction.

Demonstration.

Suppose that the number AB be of the number C the self same partes, that an other number DE is of an other nūber F , and let AB be lesse then DE . Then I say, that alternately also what part or partes AB is of DE , the self same partes or part is C of F . Forasmuch as what partes AB is of C , the self same partes is DE of F : therefore how many partes of C there are in AB , so many partes of F also are there in DE . Deuide AB into the partes of C , that is, into AG and GB . And likewise DE into the partes of F , that is, DH and HE . Now then the multitude of these AG and GB , is equall vnto the multitude of these DH and HE . And forasmuch as what part AG is of C , the self same part is DH of F , therefore alternately also (by the former) what part or partes AG is of DH , the self same part or partes is C of F . And by the same reason also what part or partes GB is of HE , the same part or partes is C of F . Wherefore what part or partes AG is of DH , the self same part or partes is AB of DE . (by the 6. of the seuenth). But what part or partes AG is of DH , the self same part or partes is it proued that C is of F . Wherefore what partes or part AB is of DE , the self same partes or part is C of F : which was required to be proued.

$C \dots$
 $A \dots G \dots B$
 $F \dots$
 $D \dots H \dots E$

¶ The 9. Theoreme.

The 11. Proposition.

If the whole be to the whole, as a part taken away is to a part taken away: then shall the residue be vnto the residue, as the whole is to the whole.

This proposition in discret quantitie answereth to the ninth proposition of the fifth booke in continual quantitie.



Suppose that the whole number AB be vnto the whole number CD , as the part take away AE , is to the part take away CF . The I say that the residue EB , is to the residue FD , as the whole AB is to the whole CD . For forasmuch as, AB is to CD , as AE is to CF : therefore what part or partes AB is of CD , the self same part or partes is AE of CF . Wherefore also the residue EB is of the residue FD (by the 8. of the seuenth) the self same parte

$C \dots F \dots D$
 $A \dots E \dots B$
 or partes

or partes that AB is of CD . Wherefore also (by the 21. definition of this booke) as EB is to FD , so is AB to CD : which was required to be proued.

Demonstration.

¶ The 10. Theoreme.

The 12. Proposition.

If there be a multitude of numbers how many soeuer proportionall: as one of the antecedentes is to one of the consequentes, so are all the antecedentes to all the consequentes.

Suppose that there be a multitude of numbers how many soeuer proportional, namely, A, B, C, D , so that as A is to B , so let C be to D . Then I say that as one of the antecedentes, namely, A is to one of the consequentes, namely, to B , or as C is to D , so are all the antecedentes: namely, A and C to all the consequentes, namely, to B and D . For forasmuch as (by supposition) as A is to B , so is C to D , therefore what parte or partes A is of B , the selfe same part or partes is C of D (by the 21. definition of this booke) wherefore alternately what part or partes A is of C the selfe same parte or partes is B of D (by the ninth and tenth of the seuenth) wherefore both these numbers added together, A and C , are of both these numbers B and D added together, the selfe same part or partes that A is of B (by the 5. and 6. of the seuenth) wherefore (by the 21. definition of the seuenth) as one of the antecedents, namely, A , is to one of the consequentes, namely, to B , so are all the antecedentes A and C to all the consequentes B & D . Which was required to be proued.

This in discrete quantity answereth to the twelfth proposition of the fifth in continual quantity. Demonstration.

¶ The 11. Theoreme.

The 13. Proposition.

If there be foure numbers proportionall: then alternately also they shall be proportionall.

Suppose that there be foure numbers proportional, A, B, C, D , so that as A is to B , so let C be to D . Then I say that alternately also they shalbe proportional, that is. as A is to C , so is B to D . For forasmuch as (by supposition) as A is to B , so is C to D , therefore (by the 21. definition of this booke) what part or partes A is of B the selfe same part or partes is C of D . Therefore alternately what part or partes A is of C the selfe same part or partes is B of D (by the 9. of the seuenth) & also (by the 10. of the same) wherefore as A is to C , so is B to D (by the 21. definition of this booke): which was required to be proued.

This in discrete quantity answereth to the sixteenth proposition of the fifth booke in continual quantities.

Here is to be noted, that although in the foresayd example and demonstration the number A be supposed to be lesse then the number B , and so the number C is lesse then the number D : yet will the same serue also though A be supposed to be greater then B , wherby also C shall be greater then D , as in this example here put. For for that (by supposition) as A is to B , so is C to D , and A is supposed to be greater then B , and C greater then D : therefore (by the 21. definition of this Booke) how multiplex A is to B , so multiplex is C to D , and therefore what part or partes B is of A , the selfe same part or partes is D of C . Wherefore alternately what part or partes B is of D , the selfe same part or partes is A of C , and therefore by the same definition, B is to D , as A is to C . And so must you vnderstand of the former Proposition next going before.

Note.

$A \dots \dots$
 $B \dots$
 $C \dots \dots$
 $D \dots$

Tt. iij. ¶ The

The seventh Booke

The 12. Theoreme.

The 14. Proposition.

If there be a multitude of numbers how many soever, and also other numbers equall vnto them in multitude, which being compared two and two are in one and the same proportion: they shall also of equalitie be in one and the same proportion.

This in discrete quantity answereth to the twety one proposition of the fifth booke in continuall quantitie. Demonstration.



Suppose that there be a multitude of numbers how many soever: namely, A, B, C, and let the other numbers equall vnto them in multitude be D, E, F: which being compared two and two, let be in one and the same proportion: that is, as A to B, so let D be to E: and as B is to C, so let E be to F. Then I say, that of equalitie, as A is to C, so is D to F. For forasmuch as by supposition as A is to B, so is D to E: therefore alternately also (by the 13. of the seventh) as A is to D, so is B to E. Againe, for that as B is to C, so is E to F: therefore alternately also (by the self same) as B is to E, so is C to F. But as B is to E, so is A to D. Wherefore (by the seventh common sentence of the seventh) as A is to D, so is C to F. Wherefore alternately (by the 13. of the seventh) as A is to C, so is D to F: which was required to be demonstrated.

A
B
C
D
E
F

Certaine additions of Campane.

After this Proposition, Campane demonstrateth in numbers these foure kinds of proportionalitie, namely, proportion conuerse, composed, deuided, and euerse: which were in continual quantitie, demonstrated in the 4. 17. 18. and 19. propositions of the fifth booke. And first he demonstrateth conuerse proportion in this maner.

Two cases in this proposition. The first case.

Suppose that the number A be to the number B, as the number C is to the number D. Then I say, that conuersedly B is to A, as D is to C. For if A be lesse then B, C also shall be lesse then D, and what part or partes A is of B, the selfe same part or partes is C of D. Wherefore B is equemultiplex to A, as D is to C. Wherefore (by the 21. definition of this booke) as B is to A, so is D to C.

A
B
C
D

The second case.

But if A be greater then B, C also is greater then D: and what part or partes B is of A, the selfe same part or partes is D of C. Wherefore (by the same definition) as B is to A, so is D to C: which was required to be proued.

Proportionalitie deuided, is thus demonstrated.

Proportionalitie deuided.

Suppose that the number A B be to the number B, as the number C D is to the number D. Then I say, that deuided also, as A is to B, so is C to D. For for that as A B is to B, so is C D to D: therefore alternately (by the 14. of this booke) as A B is to C D, so is B to D. Wherefore (by the 11. of this booke) as A B is to C D, so is A to C. Wherefore as B is to D, so is A to C: and for that as A is to C, so is B to D, therefore alternately as A is to B, so is C to D.

A B ..
C D ...

Proportionalitie composed, is thus demonstrated.

Proportionalitie composed.

If A be vnto B, as C is to D: then shall A B be to B, as C D is to D. For alternately A is to C, as B is to D. Wherefore (by the 13. of this booke) as A B, namely, all the antecedentes are to C D, namely, to all the consequentes, so is B to D, namely, one of the antecedentes to one of the consequentes. Wherefore alternately as A B is to B, so is C D to D.

A B ...
C D ..

Euerse proportionalitie, is thus proued.

Euerse proportionality.

Suppose that A B be to B, as C D is to D: then shall A B be to A, as C D is to C. For alternately A B is

A B is to C D, as E is to D. Wherefore (by the 13. of this booke) A E is to C D, as A is to C. Wherefore alternately A B is to A, as C D is to C : which was required to be proued.

A B . . .
C D . . .

¶ A proportion here added by Campanus.

If the proportion of the first to the second, be as the proportion of the third to the fourth, and if the proportion of the first to the second be as the proportion of the sixth to the fourth: then the proportion of the first and the fifth taken together, shall be to the second, as the proportion of the third and the sixth taken together to the fourth.

As if A be vnto B, as C is to D : and if also E be to B, as F is to D. Then shall A & E taken together, be vnto B, as C and F taken together, are vnto D. For by conuerse proportionalitie, B is to E, as D is to F. Wherefore by proportion of equalitie, as A is to E, so is C to F. Wherefore (by composition) as A and E are to E, so are C and F to F. But (by supposition) as E is to B, so is F to D. Wherefore againe by proportion of equalitie, as A and E are to B, so are C and F to D : which was required to be proued.

And after the same maner may you proue the conuerse of this Proposition. If B be to A, as D is to C : and if also B be vnto E, as D is to F : Then shall B be to A E, as D is to C F. For by conuerse proportionalitie, A is to B, as C is to D. Wherefore of equalitie, A is to E, as C is to F. Wherefore by composition A and E are to E, as C and F are to F. Wherefore conuersedly, E is to A and E, as F is to C and F. But by supposition, B is to E, as D is to F. Wherefore agayne by Proportion of equalitie, B is to A and E, as D is to C and F : which was required to be proued.

A Corollary.

By this also it is manifest that if the proportion of numbers how many soeuer vnto the first, be as the proportion of as many other numbers vnto the second, then shall the proportion of the numbers composed of all the numbers that were antecedentes to the first, be to the first, as the number composed of all the numbers that were antecedentes to the second is to the second. And also conuersedly if the proportion of the first to numbers how many soeuer, be as the proportion of the second to as many other numbers: then shall the proportion of the first to the number composed of all the numbers that were consequentes to it selfe, be as the proportion of the second to the number composed of all the numbers that were consequentes to it selfe.

This proposition in discrete quantity answereib to the 24. proposition of the fifth booke in continual quantity. The conuerse of the same proposition. Demonstration.

A Corollary following those propositions adaded by Campanus.

¶ The 13. Theoreme.

The 15. Proposition.

If vnitie measure any number, and an other number do so many times measure an other number : vnitie also shall alternately so many times measure the third number, as the second doth the fourth.



Suppose that vnitie A do measure the number B C : and let an other number D so many times measure some other number, namely, E F. Then I say, that alternately, vnitie A shall so many times measure the number D, as the number B C doth measure the number E F. For forasmuch as vnitie A doth so many times measure B C, as D doth E F : therefore how many vnities there are in B C, so many numbers are there in E F equall vnto D. Deuide (I say) B C into the vnities which are in it, that is, into B G, G H, and H C. And deuide likewise E F into the numbers equall vnto D, that is, into E K, K L, and L F. Now then the multitude of these B G, G H, and H C, is equall vnto the multitude of these E K, K L, L F. And forasmuch as these vnities B G, G H, and H C, are equall the one to the other, and these numbers E K, K L, & L F, are also equall the one to the other, and the multitude of the vnities B G, G H, and H C, are equall vnto the multitude of the numbers E K, K L, & L F : therefore as vnitie B G is to the number E K, so is vnitie G H to the number K L, and also vnitie H C to the number L F. Wherefore

Construction.

Demonstration.

A .
B . G . H . C .
D . .
E . . K . . L . . F

The seventh Booke

fore (by the 12. of the seventh) as one of the antecedentes is to one of the consequentes, so are all the antecedentes to all the consequentes. Wherefore as unitie B G is to the number E K, so is the number B C to the number E F. But unitie B G is equall vnto unitie A, and the number E K to the number D. VVherefore (by the 7. common sentence) as unitie A is to the number D, so is the number B C to the number E F. VVherefore unitie A measureth the number D, so many times, as B C measureth E F. (by the 21. definition of this booke). which was required to be proued.

The 14. Theoreme.

The 16. Proposition.

If two numbers multiplying them selues the one into the other, produce any numbers: the numbers produced shall be equall the one into the other.



Vppose that there be two numbers A and B: and let A multiplying B produce C, and let B multiplying A produce D. Then I say, that the number C is equall vnto the number D. Take any unitie, namely, E. And forasmuch as A multiplying B produced C, therefore B measureth C by the vnties which are in A. And unitie E measureth the number A by those vnties which are in the number A. VVherefore unitie E so many times measureth A, as B measureth C. VVherefore alternately (by the 15. of the seventh) unitie E measureth the number B, so many times as A measureth C. Againe, for that B multiplying A produced D, therefore A measureth D by the vnties which are in B. And unitie E measureth B by the vnties which are in B. VVherefore unitie E so many times measureth the number B, as A measureth D. But unitie E so many times measureth the number B, as A measureth C. VVherefore A measureth either of these numbers C and D a like. VVherefore (by the 3. common sentence of this booke) C is equall vnto D: which was required to be demonstrated.

The 15. Theoreme.

The 17. Proposition.

If one number multiply two numbers, and produce other numbers, the numbers produced of them, shall be in the selfe same proportion, that the numbers multiplied are.



Vpposethat the number A multiplieng two numbers B and C, do produce the numbers D and E. Then I say that as B is to C, so is D to E. Take unitie, namely, F. And forasmuch as A multiplieng B produced D, therefore B measureth D by those vnties that are in A. And unitie F measureth A by those vnties which are in A. Wherefore unitie F so many times measureth the number A, as B measureth D. VVherefore as unitie F is to the number A, so is the number B to the number D (by the 21. definition of this booke) And by the same reason, as unitie F is to the number A, so is the number C to the number E: wherefore also (by the 7. common sentence of this booke) as B is to D, so is C to E. VVherefore alternately (by the 15. of the seventh) as B is to C, so is D to E. If therfore one number multiply two numbers, and produce other numbers: the numbers produced of them, shall be in the selfe same proportion, that the numbers multiplied are: which was required to be proued.

Here

Here *Flussates* addeth this Corollary.

If two numbers hauing one and the same proportion with two other numbers, do multiply the one the other alternately, and produce any numbers, the numbers produced of them shall be equall the one to the other.

A Corollary added by *Flussates*.

Suppose that there be two numbers *A* and *B*, and also two other numbers *C* and *D*, hauing the same proportion that the numbers *A* and *B* haue: and let the numbers *A* and *B* multiply the numbers *C* & *D* alternately, that is, let *A* multiplieng *D* produce *F*, and let *B* multiplieng *C* produce *E*. Then I say that the numbers produced namely, *E* & *F* are equall. Let *A* and *B* multiply the one the other in such sort, that let *A* multiplieng *B* produce *G*, and let *B* multiplieng *A* produce *H*. Now then the numbers *G* and *H* are equal by the 16. of this booke And forasmuch as *A* multiplieng the two numbers *B* and *D*, produced the numbers *G* and *F*, therefore *G* is to *F*, as *B* is to *D* by this proposition. So likewise *B* multiplieng the two numbers *A* and *C* produced the two numbers *H* and *E*. Wherefore by the same *H* is to *E* as *A* is to *C*. But alternately (by the 13. of this booke) *A* is to *C* as *B* is to *D*, but as *A* is to *C* so is *H* to *E*, and as *B* is to *D*, so is *G* to *F*. Wherefore by the seuenth common sentence, as *H* is to *E*, so is *G* to *F*. Wherefore alternately (by the 13. of this booke) *H* is to *G* as *E* is to *F*. But it is proued that *G* & *H* are equall: wherefore *E* and *F* (which haue the same proportion that *A* and *B* haue) are equall. If therefore there be two numbers, &c. Which was required to be proued.

<i>H</i>		
<i>A</i>	<i>C</i>	<i>F</i>
<i>B</i>	<i>D</i>	<i>E</i>
<i>G</i>		

¶ The 16. Theoreme.

The 18. Proposition.

If two numbers multiply any number, & produce other numbers: the numbers of them produced, shall be in the same proportion that the numbers multiplying are.

 Suppose that two numbers *A* and *B* multiplieng the number *C*, doo produce the numbers *D* and *E*. Then I say that as *A* is to *B*, so is *D* to *E*. For forasmuch as *A* multiplieng *C* produced *D*, therefore *C* multiplieng *A* produceth also *D* (by the 16. of this booke.) And by the same reason *C* multiplieng *B* produceth *E*. Now then one number *C* multiplieng two numbers *A* and *B*, produceth the numbers *D* and *E*. Wherefore by the 17. of the seuenth, as *A* is to *B*, so is *D* to *E*: which was required to be demonstrated.

Demonstration.

<i>A</i>
<i>B</i>
<i>C</i> ...
<i>D</i>
<i>E</i>

This Proposition, and the former touching two numbers, may be extended to numbers how many soeuer. So that if one number multiply numbers how many soeuer, and produce any numbers, the proportion of the numbers produced, and of the numbers multiplied, shall be one and the selfe same. Likewise if numbers how many soeuer multiply one number, and produce any numbers, the proportion of the numbers produced, and of the numbers multiplieng shall be one and the selfe same: which thing by this and the former proposition repeted as often as is needefull, is not hard to proue.

This proposition and the former may be extended to numbers how many soeuer.

¶ The 17. Theoreme.

The 19. Proposition.

If there be foure numbers in proportion: the number produced of the first and the fourth, is equall to that number which is produced of the second and the third. And if the number which is produced of the first and the fourth be equall to that which is produced of the second & the third: those foure numbers shall be in proportion.

Suppose

The ſeuenth Booke

This propoſition in numbers demonſtrateth that which the 16. of the ſixth booke demonſtrateth in lines.
Conſtruction.
Demonſtration.



Suppoſe that there be foure numbers in proportion A, B, C, D, as A is to B, ſo let C be to D. And let A multiplieng D produce E: and let B multiplieng C produce F. Then I ſay that the number E is equall vnto the number F. Let A multiplieng C produce G. Now forasmuch as A multiplieng C, produceth G, & multiplieng D it produceth E: it followeth that the number A multiplieng two numbers C and D, produceth G and E.
Wherefore by the 17. of the ſeuenth, as C is to D, ſo is G to E. But as C is to D, ſo is A to B, wherefore as A is to B, ſo is G to E. Again, forasmuch as A multiplieng C produced G, and B multiplieng C produced F: therefore two numbers A and B, multiplieng one number C, do produce G & F. Wherefore by the 18. of the ſeuenth, as A is to B, ſo is G to F. But as A is to B, ſo is G to E: wherefore as G is to E, ſo is G to F. Wherefore G hath to either of theſe E and F one & the ſame proportion (But if one number haue to two numbers one and the ſame proportion, the ſaide two numbers ſhall be equall). Wherefore E is equall vnto F.

The ſecond part of this propoſition which is the conuerſe of the firſt.
Demonſtration.

But now againe, ſuppoſe that E be equall vnto F. Then I ſay that as A is to B, ſo is C to D. For the ſame order of conſtruction remayning ſtill, forasmuch as A multiplieng C & D produced G and E, therefore by the 17. of the ſeuenth, as C is to D ſo is G to E, but E is equall vnto F (But if two numbers be equall, one number ſhall haue vnto them one and the ſame proportion) wherefore as G is to E, ſo is G to F. But as G is to E, ſo is C to D. Wherefore as C is to D, ſo is G to F, but as G is to F, ſo is A to B by the 18. of the ſeuenth, wherefore as A is to B, ſo is C to D: which was required to be proued.

An aſſumpt added by Campane.

Here Campane addeth, that it is needeles to demonſtrate, that if one number haue to two numbers one and the ſame proportion, the ſaid two numbers ſhall be equall: or that if they be equal, one number hath to them one and the ſame proportion. For (ſaith he) if G haue vnto E and F one and the ſame proportion, the either, what part or partes G is of E, the ſame part or partes is G alſo of F: or how multiplex G is to E, ſo multiplex is G to F (by the 21. definition) And therefore by the 2 and 3 common ſentence, the ſaid numbers ſhall be equall. And ſo conuerſedly, if the two numbers E and F be equal, then ſhall the numbers E and F be either the ſelfe ſame parte or partes of the number G, or they ſhall be equemultiplices vnto it. And therefore by the ſame definition the number G ſhall haue to the numbers E and F one and the ſame proportion.

The 18. Theoreme.

The 20. Propoſition.

If there be three numbers in proportion, the number produced of the extremes, is equall to the ſquare made of the middle number. And if that number which is produced of the extremes, be equall to the ſquare made of the middle number, thoſe three numbers ſhall be in proportion.



This propoſition in numbers demonſtrateth that which the 17. of the ſixth demonſtrateth in lines.
Demonſtration.

Suppoſe there be three numbers in proportion, A, B, C, as A is to B, ſo let B be to C. Then I ſay that the number produced of A and C is equall to the ſquare number which is made of B. Put vnto B an equall number D.
Wherefore as A is to B, ſo is D to C. Wherefore that which is produced of A into C, is equal vnto that which is produced of B into D. But that which is produced of B into D is equal to that which is made of B. (for B is equal vnto D) wherefore that which is produced of A into C is equal to that which is made of B.

But now ſuppoſe that that which is produced of A into C, be equall

to that which is made of B. Then I say that as A is to B, so is B to C. For (the same order of construction remayning) forasmuch as that which is produced of A into C, is equall to that which is made of B, but that which is made of B, is equal to that which is produced of B into D. (For B and D are by supposition equall): therefore that which is produced of A into C, is equall to that which is produced of B into D: wherefore (by the second part of the former proposition) as A is to B, so is D to C: but D is equal to B. Wherefore as A is to B, so is the same B to C: which was required to be proued.

The second part which is the conuerse of the first. Demonstration.

¶ The 19. Theoreme.

The 21. Proposition.

The lest numbers in any proportion, measure any other numbers hauing the same proportion equally, the greater the greater, & the lesse the lesse.



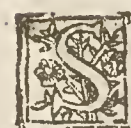
Suppose that C D & E F be the least numbers that haue one & the same proportion with the numbers A and B. Then I say, that the number C D so many times measureth the number A, as the number E F measureth the number B. For forasmuch as by supposition C D is to E F, as A is to B, and C D and E F are also supposed to be lesse then A and B: therefore C D and E F are either part or partes of A and B (by the 4. of this booke, and by the 21. definition of the same). But they are not partes. For if it be possible, let C D be partes of A. Wherefore E F is the selfe same partes of B, that C D is of A. Wherefore how many partes of A there are in C D, so many partes are there of B in E F. Deuide C D into the partes of A, that is, into C G and G D. And likewise deuide E F into the partes of B, that is, into E H and H F. Now then the multitude of these C G and G D, is equall vnto the multitude of these E H & H F. C .. G .. D
And forasmuch as C G and G D are numbers equall the one to the other, E .. H .. F
and these numbers E H and H F are also equall the one to the other, and A
the multitude of these C G and G D, is equall to the multitude of these B
E H and H F: therefore as C G is to E H, so is G D to H F. Wherefore
(by the 12. of the seuenth) as one of the antecedentes is to one of the consequentes, so are all the antecedentes to all the consequentes. Wherefore as C G is to E H, so is C D to E F. Wherefore C G and E H are in the selfe same proportion that C D and E F are, being also lesse then C D and E F: which is impossible. For C D and E F are supposed to be the least that haue one and the same proportion with them. Wherefore C D is not partes of A: wherefore it is a part. Wherefore E F is of B the selfe same part, that C D is of A. Wherefore C D so many times measureth A, as E F doth B: which was required to be demonstrated.

Demonstration leading to an impossibility.

¶ The 20. Theoreme.

The 22. Proposition.

If there be three numbers, and other numbers equall vnto the in multitude, which being compared two and two are in the selfe same proportion, and if also the proportion of them be perturbate, then of equalitie they shall be in one and the same proportion.



Suppose that there be three numbers A, B, and C: and let the other numbers equall vnto them in multitude be D, E, and F. And let two and two compared together be in one and the same proportion: and let the proportion of them be perturbate, so that as A is to B, so let E be to F, and as B is to C, so let D be to E. Then I say, that of equalitie as A is to C, so is D to F. For, for that as A is to B, so is E to F:

A
B
C
D
E
F

Vv. i.

therefore

This proposition in discreet quantitie answereth to the 23. proposition of the fifth booke in continual quantitie.

The ſeuenth Booke

therefore that which is produced of *A* into *F*, is (by the 19. of the ſeuenth) equall to that which is produced of *B* into *E*. Again for that as *B* is to *C*, ſo is *D* to *E*, therefore that which is produced of *D* into *C*, is equall to that which is produced of *B* into *E*. And it is proued, that that which is produced of *A* into *F*, is equal to that which is produced of *B* into *E*. Wherefore that which is produced of *A* into *F*, is equal to that which is produced of *D* into *C*. Wherefore (by the ſecond part of the 19. of the ſeuenth) as *A* is to *C*, ſo is *D* to *F*: which was required to be proued.

A
B
C
D
E
F

The ſame may alſo be proued if in either order be more then three numbers: as it was proued in the 23. of the fiſt touching more magnitudes then three.

¶ The 21. Theoreme.

The 23. Propoſition.

Numbers prime the one to the other: are leaſt of any numbers, that haue one and the ſame proportion with them.

This and the eleuen propoſitions following, declare the paſſions and properties of prime numbers.



*S*uppoſe that *A* and *B* be numbers prime the one to the other. Then I ſay that *A* and *B* are the leaſt of any numbers that haue one and the ſame proportion with them. For if *A* and *B* be not the leaſt of any numbers that haue one and the ſame proportion with them, then are there ſome numbers leſſe then *A* and *B*, being in the ſelfe ſame proportion that *A* and *B* are. Let the ſame be *C* and *D*. Now forasmuch as the leaſt numbers in any proportion meaſure any other numbers hauing the ſame proportion equally, the greater the greater, and the leſſe the leſſe (by the 21. of the ſeuenth) that is, the antecedent the antecedent, and the conſequent the conſequent: therefore *C* ſo many times meaſureth *A*, as *D* meaſureth *B*. How many times *C* meaſureth *A*, ſo many vnities let there be in *E*. Wherefore *D* meaſureth *B* by thoſe vnities which are in *E*. And forasmuch as *C* meaſureth *A* by thoſe vnities which are in *E*, therefore *E* alſo meaſureth *A* by thoſe vnities which are in *C*. And by the ſame reaſon *E* meaſureth *B*, by thoſe vnities which are in *D*. Wherefore *E* meaſureth *A* and *B* being prime numbers the one to the other: which (by the 13. definition of the ſeuenth) is impoſſible. Wherefore there are no other numbers leſſe then *A* and *B*, which are in the ſelfe ſame proportion that *A* and *B* are. Wherefore *A* and *B* are the leaſt numbers that haue one and the ſame proportion with them: which was required to be demonſtrated.

A
B
E
C
D

Demonſtration leading to an impoſſibility.

¶ The 22. Theoreme.

The 24. Propoſition.

The leaſt numbers that haue one and the ſame proportion with them: are prime the one to the other.

This is the conuerſe of the former propoſition.



*S*uppoſe that *A* and *B* be the leaſt numbers that haue one and the ſame proportion with them. Then I ſay that *A* and *B* be prime the one to the other. For if *A* & *B* be not prime the one to the other, then ſhall ſome one number meaſure *A* & *B*. Let the ſame be *C*. And how oftentimes *C* meaſureth *A*, ſo many vnities let there be in *D*: and how oftentimes *C* meaſureth *B*, ſo many vnities let there be in *E*. And forasmuch as *C* meaſureth *A* by

A
B
C
D
E

thoſe

those unities that are in D, therefore C multiplying D produceth A. And by the same reason C multiplying E produceth B. Wherefore the number C multiplying two numbers D and E, produceth A & B. Wherefore (by the 17. of the seventh) as D is to E, so is A to B. But the numbers D and E are lesse then A and B, and are also in the selfe same proportion with the, which is impossible. Wherefore no number measureth these numbers A and B. Wherefore A and B are prime the one to the other: which was required to be demonstrated.

Demonstration leading to an absurditie.

¶ The 23. Theoreme.

The 25. Proposition.

If two numbers be prime the one to the other: any number measuring one of them shall be prime to the other number.

 Suppose that A and B be two prime numbers the one to the other. And let some number, namely, C, measure A. Then I say that C and B are prime numbers the one to the other. For if C and B be not prime the one to the other, let some number measure C and B, and let the same be D. And forasmuch as D measureth C, and C measureth A, therefore D also measureth A (by the fifth common sentence) and D measureth B. Wherefore D measureth A and B being numbers prime the one to the other, which is impossible (by the 13. definition of the seventh): wherefore no number measureth these numbers B and C. Wherefore B and C are numbers prime the one to the other. Which was required to be proved.

A
B
C ...
D ...

Demonstration leading to an absurditie.

¶ The 24. Theoreme.

The 26. Proposition.

If two numbers be prime to any one number, the number also produced of them shall be prime to the selfe same.

 Suppose that the two numbers A and B be prime to any one number, namely, to C, and let A multiplieng B produce D. Then I say that C and D are prime numbers the one to the other. For if C and D be not prime the one to the other then some number shall measure them. Let there be a number that measureth them, and let the same be E. And forasmuch as A and C are prime numbers the one to the other, and the number E measureth C. Therefore E and A are (by the 25. of the seventh) prime the one to the other. And forasmuch as E measureth D. How many times E measureth D, so many unities let there be in F. Wherefore F also measureth D by those unities which are in E. Wherefore E multiplieng F produceth D. But A also multiplieng B produced D. Wherefore that which is produced of E into F is equall to that which is produced of A into B. But if that which is produced of the extremes be equall to that which is produced of the meanes, then are those foure numbers in proportion (by the 19. of the seventh) wherefore as E is to A, so is B to F. But A and E are prime the one to the other. Yea they are prime and the least in the same proportion (by the 23. of the seventh.) But the least numbers in any proportion measure any other numbers hauing the same proportion equally, the greater the greater, and the lesse the lesse, that is, the antecedent the antecedent, and the consequent the consequent by the 21. of the seventh: wherefore E measureth B, and it also measureth C: wherefore E measureth C and B, which are by supposition numbers prime the one to the other, which is impossible by the 13. definition of the seventh.

A .. B ...
C
D
E ...
F ..

Demonstration leading to an absurditie.

The seventh Booke

seuenth. Wherefore no number measureth those numbers C and D. Wherefore C and D are prime the one to the other : which was required to be proued.

¶ The 25. Theoreme.

The 27. Proposition.

If two numbers be prime the one to the other, that which is produced of the one into him selfe, is prime to the other.

Demonstration.



Suppose that there be two numbers prime the one to the other, $A \dots B \dots$
 A and B , and let A multiplieng him selfe produce C . Then I say that B and C are prime the one to the other. For put vnto A an equall number, namely, D . And forasmuch as A & B are prime the one to the other, and A is equall vnto D : therefore D and B also are prime the one to the other. Wherefore either of these numbers D and A is prime to B . Wherefore that which is produced of D into A is prime vnto B by the former proposition. But that number which is produced of D into A is the number C . Wherefore C and B are prime numbers the one to the other : which was required to be proued.

¶ The 26. Theoreme.

The 28. Proposition.

If two numbers be prime to two numbers, eche to either of both: the numbers produced of them shall be prime the one to the other.

Demonstration.



Suppose that there be two numbers A and B prime to two numbers C and D , either of both, to either of both: namely, let either of these A and B be prime to C , and also to D . And let A multiplieng B produce E , and let C multiplieng D produce F . Then I say that E and F are prime numbers the one to the other. For forasmuch as either of these A & B are prime vnto C , therefore that which is produced of A into B is prime vnto C by the 26. of the seuenth. But that which is produced of A into B is the number E ; therefore E and C are prime the one to the other. And by the same reason also E and D are prime the one to the other. Wherefore either of these numbers C and D are prime vnto E : wherefore that also which is produced of C into D is prime vnto E by the same. But that which is produced of C into D is the number F . Wherefore E and F are numbers prime the one to the other : which was required to be demonstrated.

¶ The 27. Theoreme.

The 29. Proposition.

If two numbers be prime the one to the other, and ech multiplying him selfe bring forth certaine numbers: the numbers of them produced shall be prime the one to the other. And if those numbers geuen at the beginning multiplying the sayd numbers produced, produce any numbers : they also shall be prime the one to the other : and so shall it be continuing infinitely.

Suppose



Suppose that there be two numbers A and B prime the one to the other. And let A multiplying him selfe produce C : and multiplying C let it produce E . Likewise let B multiplying him selfe produce D ; and multiplying D let it produce F . The I say, that C and D are numbers prime the one to the other. And likewise that E and F are numbers prime the one to the other. For forasmuch as A and B , are prime the one to the other, and A multiplying him selfe produced C , therefore C and B are prime the one to the other (by the 27. of the seventh). And by the same reason forasmuch as C and B

Demonstration.

$A \dots$

$C \dots \dots \dots$

$E \dots \dots \dots$

$B \dots$

$D \dots \dots \dots$

$F \dots \dots \dots$

are prime the one to the other, and B multiplying him selfe produced D , therefore C and D are prime the one to the other. Again forasmuch as A and B are prime the one to the other, and B multiplying him selfe produced D . Therefore (by the 27. of the seventh) A and D are prime the one to the other. Now then forasmuch as two numbers A and C are prime to two numbers B and D ; either of both to either of both: therefore (by the 28. of the seventh) that which is produced of A into C is prime to that which is produced of B into D . But that which is produced of A into C is the number E , and that which is produced of B into D is the number F . Wherefore E and F are numbers prime the one to the other. And so alwayes if A & B multiplying the numbers E and F do produce any numbers, the numbers produced, may by the former Proposition, be proved to be prime the one to the other: which was required to be proved.

¶ The 28. Theoreme.

The 30. Proposition.

If two numbers be prime the one to the other: then both of them added together, shall be prime to either of them. And if both of them added together be prime to any one of them, then also those numbers geuen at the beginning, are prime the one to the other.



Suppose that these two numbers AB and BC being prime numbers be added together: Then I say, that both these added together, namely, the number ABC , is prime to either of these AB , and BC . For if CA and AB be not prime the one to the other, some number then shall measure them. Let some number measure them, and let the same be D . Now then forasmuch as D measureth the whole CA and the part taken away AB , it measureth also the residue CB (by the 4. common sentence). And it measureth BA . Wherefore D measureth these numbers AB and BC , being prime the one to the other: which is impossible (by the 13. definition of the seventh). Wherefore no number measureth these numbers CA and AB . Wherefore CA and AB are prime the one to the other. And by the same reason also may it be proved, that CA and BC are prime the one to the other. Wherefore the number AC is to either of these numbers AB and BC , prime.

Demonstration of the first part leading to an absurditie.

$A \dots \dots \dots B \dots \dots C$
 $D \dots$

But now suppose that the numbers CA and AB be prime the one to the other. Then I say, that the numbers AB and BC are also prime the one to the other. For if AB & BC be not prime the one to the other: some one number measureth these numbers AB and BC :

The seventh Booke

Demonstration of the second part which is the conuerse of the first, leaauing also to an absurditie.

Let some one number measure them, and let the same be D . And forasmuch as D measureth either of these numbers AB and BC , it shall also measure the whole CA (by the 6. common sentence). And it also measureth AB . Wherefore D measureth these numbers CA and AB being prime the one to the other: which is impossible (by the 13. definition of the seventh). Wherefore no number measureth these numbers AB and BC . Wherefore AB and BC are prime the one to the other, which was required to be proued.

¶ The 29. Theoreme.

The 31. Proposition.

Every prime number is to euery number which it measureth not, prime.

Demonstration leading to an absurditie.



Suppose that there be geuen a prime number, namely, A ; and let B be an other number, which it measureth not. Then I say, that the numbers B & A are prime the one to the other. For if A and B be not prime the one to the other, then some number measureth them. Let there be a number that measureth them, and let the same be C . Now C is no unitie. And forasmuch as C measureth B , but A measureth not B , therefore C is not one and the same number with A . And forasmuch as C measureth A & B , it also measureth A being a prime number, and being not one and the same with it: which is impossible (by the 13. definition of the seventh). Wherefore no number measureth these numbers A and B . And therefore A and B are prime the one to the other: which was required to be proued.

¶ The 30. Theoreme.

The 32. Proposition.

If two numbers multiplying the one the other produce any number, and if also some prime number measure that which is produced of them: then shall it also measure one of those numbers which were put at the beginning.



Suppose that two numbers A and B multiplying the one to the other do produce the number C : and let some prime number, namely, D measure C . Then I say that D measureth one of these numbers either A or B . Suppose that it measure not A : now D is a prime number. Wherefore A and D are prime the one to the other (by the proposition next going before). And how often D measureth C , so many unities let there be in E . And forasmuch as D measureth C by those unities which are in E , therefore D multiplying E produceth C : but A also multiplying B produced C wherefore that which is produced of D into E is equal to that which is produced of A into B . Wherefore (by the 19. of the seventh) as A is to D , so is E to B : but D and A are prime numbers: and therefore the least numbers in that proportion: but the least in any proportion measure the numbers hauing the selfe same proportion with them equally, the greater, the greater, and the lesse, the lesse, that is the antecedent, the antecedent, and the consequent, the consequent (by the 21. of the seventh). wherefore the consequent D measureth the consequent B . In like sort may we proue that if D measure not B it measureth A . Wherefore D measureth one of these numbers A or B : which was required to be proued.

A Corollary.

Hereby it is manifest that if a number measure a number produced of two numbers multiplied the one into the other, or be commensurable to the same, it shall also either

Demonstration.

A Corollary added by Campane.

measure

measure one of the two numbers multiplied, or be commensurable with one of them.

The 31. Theoreme.

The 33. Proposition.

Euery composed number, is measured by some prime number.



Suppose that A be a composed number. Then I say that A is measured by some prime number. For forasmuch as A is a composed number, some number must needs measure it (by the 14. definition of the seuenth). Let there be a number that measureth it, and let the same be B . Now if B be a prime number then is that manifest which we seeke for: but if it be a composed number some number must needs measure it (by the selfe same definition) Let there be a number that measureth it, and let the same be C . And forasmuch as C measureth B , and B measureth A : therefore C also measureth A (by the 5. common sentēce): and if C be a prime number then is that manifest which we sought for. But if it be a composed number some number shall measure it: and the like consideration being had there shall at the length be found some prime number which measureth the number going before, which shall also measure A . For if there be not found any such prime number then shall infinite numbers decreasing measure the sayd number A of which the one is lesse the the other, which is impossible in numbers. Wherefore some prime number shall at the length be found which shall measure the number going before and which shall also measure the number A (by the 5. common sentence). Euery composed number therefore is measured by some prime number: which was required to be proued.

Demonstration leading to an impossibility.

¶ An other way.

Suppose that A be a composed number. Then I say that some prime number measureth it. For forasmuch as A is a composed number, some number shall measure it (by the 14. definition of the seuenth). Let the least number that measureth it be B . Then I say that B is a prime number. For if B be not a prime number some number shall measure it. Let C measure it. Wherefore C is lesse then B . And forasmuch as C measureth B , and B measureth A , therefore C also measureth A being lesse then B , which by supposition is the least number that measureth A , which is absurd. Wherefore B is not a composed number, but a prime number, which was required to be proued.

An other demonstration.

A
 B ...
 C ..

The 32. Theoreme.

The 34. Proposition.

Euery number is either a prime number, or els some prime number measureth it.



Suppose that there be a number A . Then I say that A is either a prime number, or els some prime number measureth it. For if A be a prime number then is that had which is required. But if it be a composed number, some prime number shall measure it (by the 33. of the seuenth). Euery number therefore is either a prime number, or els some prime number measureth it: which was required to be demonstrated.

Demonstration.

A ...
 A

Vv.iiij.

Suppose

The seventh Booke

¶ The 3. Probleme.

The 35. Proposition.

How many numbers soeuer being geuen, to finde out the least numbers that haue one and the same proportion with them.



Suppose that there be a multitude of numbers geuen, namely, A, B, and C. It is required to finde out the least numbers that haue one and the same proportion with these numbers A, B, C. These numbers A, B, C, are either prime the one to the other, or not prime. If A, B, C, be prime the one to the other, then are they the least that haue one and the same proportion with them (by the 23. of the seventh).

A
B
C
D ..
E ...
F
G
H ..
K ...
L
M ...

Two cases in this Proposition.

The first case.

The second case.

Demonstration.

Demonstration leading to an absurditie.

But if they be not prime, take by the 3. of the seventh, vnto A, B, C, the greatest common measure, which let be the number D. And how often D measureth euery one of these A, B, C, so many vnities let there be in euery one of these numbers E F G. Wherefore these numbers E, F, G, do measure these numbers A, B, C, by those vnities which are in D. Wherefore these numbers E, F, G, measure these numbers A, B, C, equally. Wherefore E, F, G, are by the 18. of the seventh, in the selfe same proportion that A, B, C, are. Now then I say that they also are the least. For if E, F, G, be not the least that haue one and the same proportion, with A, B, C, there shall the be some numbers lesse then E, F, G, being in the selfe same proportion that A, B, C, are. Suppose that the same numbers be H, K, L, which shall measure the numbers A, B, C, equally. How many times H measureth A, so many vnities let there be in M. Wherefore either of these K and L measureth either of these B and C by those vnities which are in M (by the 21. of the seventh). And forasmuch as H measureth A by those vnities which are in M, therefore M also measureth A by those vnities which are in H. And by the same reason M measureth either of these B and C by those vnities which are in either of these K and L. Wherefore M measureth these numbers A, B, C. And forasmuch as H measureth A by those vnities which are in M, therefore H multiplieng M, produceth A. And by the same reason E multiplieng D, produceth A. Wherefore that which is produced of E into D is equall to that which is produced of H into M. Wherefore (by the 19. of the seventh) as E is to H, so is M to D. But E is greater then H, wherefore M also is greater then D, and it measureth these numbers A, B, C, which is impossible. For D is supposed to be the greatest common measure vnto A, B, C. Wherefore there shall be no other numbers lesse then E, F, G, and in the selfe same proportion with A, B, C. Wherefore E, F, G, are the lest numbers which haue one and the same proportion with A, B, C : which was required to be done.

✱ A Corollary.

Hereby it is manifest that the greatest common measure to numbers how many soeuer : measureth the sayd numbers by the numbers in the least proportio that the numbers geuen are.

A Corollary added by Campanus.

¶ The 4. Probleme.

The 36. Proposition.

Two numbers being geuen, to finde out the lest number which they measure.

Suppose



Suppose that the two numbers geuen be *A* and *B*. It is required to finde out the lest number which they measure. Now *A* and *B* are either prime the one to the other, or not. Suppose first that *A* and *B* be prime the one to the other: and let *A* multiplying *B* produce *C*: wherefore *B*, multiplying *A* produceth also *C* (by the 16. of the seuenth). Wherefore *A* and *B* measure *C*. Now also I say, that *C* is the lest number which they measure. For if it be not, those numbers *A* and *B* measure some number lesse then *C*: let them measure some number lesse then *C*, and let the same be *D*: and how often *A* measureth *D*, so many unities let there be in *E*: and how often *B* measureth *D*, so many unities let there be in *F*. Wherefore *A* multiplying *E* produceth *D*, and *B* multiplying *F* produceth also *D*. Wherefore that which is produced of *A* into *E*, is equall to that which is produced of *B* into *F*: wherefore (by the 19. of the seuenth) as *A* is to *B*, so is *F* to *E*. But *A* and *B* are prime: yea they are prime and also the lest in that proportion (by the 23. of the seuenth): but the lest numbers in any proportion measure those numbers that haue one and the same proportion with them equally: the greater the greater: and the lesse the lesse (by the 21. of the seuenth). Wherefore *B* measureth *E*, namely, the consequent the consequent. And forasmuch as *A* multiplying *B* and *E* produced *C* and *D*: therefore (by the 17. of the seuenth) as *B* is to *E*, so is *C* to *D*. But *B* measureth *E*. Wherefore *C* also measureth *D*, the greater, the lesse: which is impossible. Wherefore if those numbers *A* and *B* be prime, they shall measure no number lesse then *C*. Wherefore *C* is the lest number which *A* and *B* measure.

Two cases in this proposition.
The first case.

Demonstration leading to an absurditie.

A
B ...
C
D
E
F

But now suppose that *A* and *B* be not prime the one to the other, and take (by the 35. of the seuenth) the lest numbers that haue one and the same proportion with *A* and *B*, and let the same be *F* and *E*. Wherefore that which is produced of *A* into *E*, is equall to that which is produced of *B* into *F* (by the 19. of the seuenth). Let *A* multiplying *E* produce *C*: wherefore *B* multiplying *F* produceth also *C*. Wherefore *A* and *B* measure *C*. Then I say, that *C* is the lest number that they measure. For if it be not, those numbers *A* and *B* shall measure some number lesse then *C*: let them measure a number lesse then *C*, and let the same be *D*. And how often *A* measureth *D*, so many unities let there be in *G*. And how often *B* measureth *D*, so many unities let there be in *H*. Now then *A* multiplying *G* produceth *D*. And *B* multiplying *H* produceth also *D*. Wherefore that which is produced of *A* into *G*, is equall to that which is produced of *B* into *H*. Wherefore (by the 19. of the seuenth) as *A* is to *B*, so is *H* to *G*. But as *A* is to *B*, so is *F* to *E*. Wherefore as *F* is to *E*, so is *H* to *G*: but the lest numbers in any proportion measure the numbers that haue the same proportion with them equally, the greater the greater, & the lesse the lesse (by the 21. of the seuenth). Wherefore *E* measureth *G*. And forasmuch as *A* multiplying *G* and *E* produced *C* and *D*, therefore (by the 17. of the fift) as *E* is to *G*, so is *C* to *D*. But *E* measureth *G*. Wherefore *C* also measureth *D*, the greater the lesse: which is impossible. Wherefore those numbers *A* and *B* do not measure any number lesse then *C*. Wherefore *C* is the lest number that is measured by *A* and *B*: which was required to be done.

The second case.

Demonstration leading to an absurditie.

F ... *A*
E .. *B*
C
D
G ..
H ...

The 33. Theoreme.

The 37. Proposition.

If two numbers measure any number, the least nuber also which they measure, measureth the selfe same number.

Suppose

The ſeuenth Booke

Demonstra-
tion leading to
an impossibi-
litie.

Suppose that there be two numbers geuen A and B: and let them measure the num-
ber CD: and let the least number that they measure be E. The I say that E also
measureth the number CD. For if E do not mea-
sure CD, let E measuring CD, that is subtrahed out of CD $A \dots$
as ofte as you can, as for example, once, leaue a lesse then it selfe, $B \dots$
namely, CF. And let the number subtrahed which E measu- $E \dots \dots$
reth be FD, and forasmuch as A and B measure E, and E $C \dots F \dots \dots D$
measureth DF, therefore A and B also measure DE. And
they measure the whole CD, wherefore by the 4. common sentence of the ſeuenth they mea-
sure also that which remayneth CF being lesse then E: which is impossible. Wherefore E of
necessitie measureth CD, which was required to be proued.

¶ The 5. Probleme.

The 38. Proposition.

Three numbers being geuen, to finde out the least number which they measure.

Two cases in
this propositiō.
The first case.

Suppose that there be three numbers geuen A, B, C. It is required to finde out the
least number which they measure. Take (by the 36. of the ſeuenth) the least number
which A and B measure, and let the same be D. Now then C either measureth D
or els measureth it not. First let it measure it. And the numbers
also A and B measure D: wherefore A, B, C, measure D. Now $A \dots$
then I say that D is the least number which they measure. For $B \dots$
if not, let the numbers A, B, C, measure some number lesse then $C \dots \dots$
D, and let the same be E. And forasmuch as A, B, C, measure $D \dots \dots \dots$
E therefore also A and B measure E, wherefore (by the 37. of $E \dots \dots$
the ſeuenth) the least number which those numbers A and B
measure shall also measure E. But the least number which A and B measure is D. Wherefore
D measureth E, the greater the lesse: which is impossible. Wherefore these numbers A, B, C,
shall not measure any number lesse then D. Wherefore D is the least number that A, B, C,
doo measure.

The second
case.

But now suppose that C measure not D. And take (by the 36. of the ſeuenth) the least
number which those two numbers C and D do measure, and let the same be E. And foras-
much as A and B measure D, and D measureth E, therefore A and B also measure E, and
C also measureth E, wherefore A, B, C, also measure E. I say moreouer that E is the least
number which A, B, C measure. For if it be not, let there be some lesse number then E which
they measure, and let the same be F. And foras-
much as A, B, C, measure F. Therefore A and B $A \dots$
also measure F: wherefore the least number which $B \dots$
these numbers A and B do measure doth also mea- $C \dots \dots$
sure F (by the 37. of the ſeuenth). But the least num- $D \dots \dots \dots$
ber which A and B doo measure is D. Wherefore $E \dots \dots \dots$
D measureth F. And C also measureth F. Wherefore $F \dots \dots \dots$
D and C measure F. Wherefore the least number
which C and D doo measure, shall also (by the selfe same) measure F. But the least number
which C & D measure is E. Wherefore E measureth F, namely, the greater, the lesse, which
is impossible. Wherefore these numbers A, B, C, do not measure any number lesse then E.
Wherefore E is the least number which A, B, C, doo measure: which was required to be
demonstrated.

Demonstra-
tion leading to
an absurditie.

In like maner also how many numbers soeuer being geuen, may be found out the
least

least number which they measure. For if vnto the three numbers A, B, C, be added a forth, then if the sayd forth number measure the number E, then is E the least number which the fower numbers geuen measure. But if it doo not measure E, the by the 37. of this booke must you finde out the least number which E and the forth number measure. Which shall be the number sought for. And so likewise if there be foue, fixe, or how many soeuer geuen.

Corollary.

Hereby it is manifest that the least commo measure to numbers howmanysoeuer, measureth euery number which the sayd numbers howmanysoeuer measure. *A Corollary.*

¶ The 34. Theoreme.

The 39. Proposition.

If a number measure any number : the number measured shall haue a part after the denomination of the number measuring.



Suppose that there be a number B, which let measure the number A. Then I say, that A hath a part taking his denomination of the number B. For how often B measureth A, so many vnities let there be in C. And let D be vnitie. And forasmuch as B measureth A, by those vnities which are in C, and vnitie D measureth C by those vnities which are in C, therefore vnitie D, so many times measureth the number C, as B doth measure A. Wherefore alternately (by the 15. of the seuenth) vnitie D, so many times measureth B, as C doth measure A. Wherefore what part vnitie D is of the number B, the same part is C of A. But vnitie D is a part of B hauing his denomination of B. Wherefore C also is a part of A hauing his denomination of B. Wherefore A hath C as a part taking his denomination of B : which was required to be proued.

A
B
C ...
D .

Demonstration.

The meaning of this Proposition is, that if three measure any number, that number hath a third part, and if foure measure any number the sayd number hath a fourth part. And so forth.

¶ The 35. Theoreme.

The 40. Proposition.

If a number haue any part : the number wherof the part taketh his denomination shall measure it.



Suppose that the number A haue a part, namely, B : and let the part B haue his denomination of the number C. Then I say, that C measureth A. Let D be vnitie. And forasmuch as B is a part of A, hauing his denomination of C : and D being vnitie is also a part of the number C, hauing his denomination of C : therefore what part vnitie D is of the number C, the same part is also B of A : wherefore vnitie D so many times measureth the number C, as B measureth A. Wherefore alternately (by the 15. of the seuenth) vnitie D so many times measureth the number B, as C measureth A. Wherefore C measureth A : which was required to be proued.

A
B
C ..
D .

The conuerse of the former proposition. Demonstration.

This Proposition is the conuerse of the former : and the meaning therof is, that euery number hauing a third part is measured of three, and hauing a fourth part is measured of foure. And so forth.

¶ The

The seventh Booke

The 6. Probleme. Th 41. Proposition.

To finde out the least number, that containeth the partes geuen.

Construction.

Suppose that the partes geuen be A, B, C , namely, let A be an halfe part, B a third part, & C a fourth part. Now it is required to finde out the least nūber which cōtaineth the partes A, B, C . Let the said partes A, B, C , haue their denominations of the numbers D, E, F . And take (by the 38. of the seventh) the least number which the numbers D, E, F , measure, and let the same be G . And forasmuch as the numbers D, E, F measure the number G , therefore the number G hath partes denominatēd of the nūbers D, E, F (by the 39. of the seventh). But the partes A, B, C , haue their denominatiō of the numbers D, E, F . Wherefore G hath those partes A, B, C . I say also that it is the least number which hath these partes. For if G be not the least number which containeth those partes A, B, C , then let there be some number lesse then G which containeth the saide partes A, B, C . And suppose the same to be the number H . And forasmuch as H hath the said partes A, B, C , therefore the numbers that the partes A, B, C , take their denominations of, shall measure H (by the 40. of the seventh). But the numbers wherof the partes A, B, C , take their denominations of, are D, E, F . Wherefore the numbers D, E, F , measure the number H which is lesse then G , which is impossible. For G is supposed to be the least number that the numbers D, E, F , do measure. Wherefore there is no number lesse then G , which containeth these partes A, B, C : which was required to be done.

A an halfe
 B a third
 C a fourth
 D
 E
 F
 G
 H

Demonstratiō
leading to an
absurditie.

Corrolary.

A Corollary
added by
Campane.

Hereby it is manifest that if there be taken the least number, that numbers how many soeuer do measure, the sayd number shall be the least which hath the partes denominatēd of the sayd numbers how many soeuer.

How to finde
out the seconde
least number
and the third,
and so forth
infinitely.

Campane after he hath taught to finde out the first least number that containeth the partes geuen, teacheth also to finde out the second least number, that is, which except the least of all is lesse then all other, and also the third least, and the fourth &c. The second is found out by doubling the number G . For the numbers which measure the number G shall also measure the double thereof (by the 5. common sentence of the seventh). But there cannot be geuen a number greater then the number G , & lesse then the double thereof, whom the partes geuen shall measure. For forasmuch as the partes geuen do measure the whole, namely, which is lesse then the double, and they also measure the part taken away, namely, the number G , they should also measure the residue, namely, a number lesse then G , which is proued to be the least number that they do measure, which is impossible: wherefore the second number which the said partes geuen do measure, must exceeding G , needes reache to the double of G , and the third to the treble, and the fourth to the quadruple, and so infinitely, for those partes can neuer measure any number lesse then the number G .

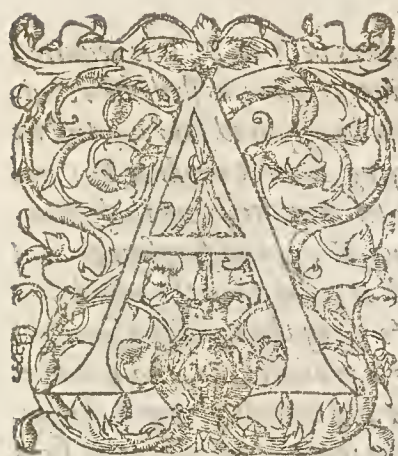
How to find
out the least
number con-
taining the
partes of partes.

By this Proposition also it is easie to find out the least number containing the partes geuen of partes. As if we would finde out the least number which containeth one third part of an halfe part, and one fourth part of a third part, reduce the said diuers fractiōs into simple fraction (by the common rule of reducing of fractions) namely, the third of an halfe into a sixt part of an whole, and the fourth of third into a twelfth part of an whole. And then by this Probleme search out the least number which containeth a sixt part and a twelfth part, and so haue you done.

The end of the seventh booke
of Euclides Elementes.

The

The eighth booke of Euclides Elementes.



After that *Euclide* hath in the seventh booke entreated of the proprieties of numbers in generall, and of certayne kinds thereof more specially, and of prime and composed numbers with others: now in this eight booke he prosecuteth farther, and findeth out and demonstrateth the properties and passions of certayne other kinds of numbers: as of the least numbers in proportion, and how such may be found out infinitely in whatsoeuer proportion: which thing is both delectable, and to great vse. Also here is entreated of playne numbers, and solide: and of theyr sides, and proportion of them. Likewise of the passions of numbers square and cube, and of the natures and conditions of their sides, and of the meane proportionall numbers of playne, solide, square, and cube numbers, with many other thinges very requisite and necessary to be knowne.

The Argument of the eight booke.

The first Theoreme.

The first Proposition.

If there be numbers in continuall proportion howmanysoeuer, and if their extremes be prime the one to the other: they are the least of all numbers that haue one and the same proportion with them.



Suppose that the numbers in continuall proportion be A, B, C, D . And let their extreames namely, A and D be prime the one to the other. Then I say that the numbers A, B, C, D , are the least of all numbers that haue one and the same proportion with the. For if they be not, let E, F, G, H being lesse numbers then A, B, C, D , be in the selfe same proportion that A, B, C, D are. And forasmuch as the numbers A, B, C, D , are in the selfe same proportion that the numbers E, F, G, H , are, & the multitude of these numbers E, F, G, H , is equall to the multitude of these numbers A, B, C, D , therefore of equalitie (by the 14. of the seventh) as A is to D , so is E to H . But A and D are prime the one to the other, yea they are prime and the least that haue the same proportion with them. But the least numbers in any proportion measure the numbers that haue the same proportion with them equally, the antecedent the antecedent, and the consequent the consequent (by the 21. of the seventh) wherefore A measureth E , the greater the lesse: which is impossible. Wherefore the numbers E, F, G, H , being lesse then A, B, C, D , are not in the same proportion that A, B, C, D , are, wherefore A, B, C, D , are the least of all numbers which haue one and the same proportion with them

A
B
C
D
E
F
G
H

Demonstration leading to an absurditie.

The eighth Booke

them: which was required to be demonstrated.

¶ The 1. Probleme.

The 2. Proposition.

To finde out the least numbers in continuall proportion, as many as shall be required, in any proportion geuen.



Construction.

Demonstration.

Suppose that the proportion geuen in the least numbers be A to B . It is required to finde out the least numbers in continuall proportion, as many as shall be required, in the same proportion that A is to B . Let there be required foure. And let A multiplying him selfe produce C : and multiplying B let it produce D : and likewise let B multiplying him selfe produce E . And moreover let A multiplying those numbers C, D, E , produce F, G, H : and let B multiplying E produce K . And forasmuch as A multiplying him selfe produced C , and multiplying B produced D , now then the number A multiplying two numbers A and B produced C & D . Wherefore (by the 17. of the seuenth) as A is to B , so is C to D . Againe, forasmuch as A multiplying B produced D , and B multiplying him selfe produced E , therefore ech of those numbers A and B multiplying B , bringeth forth these numbers D and E . Wherefore (by the 18. of the seuenth) as A is to B , so D to E . But as A is to B , so is C to D . Wherefore as C is to D , so is D to E . And forasmuch as A multiplying C and D produced F and G , therefore (by the 17. of the seuenth) as C is to D , so is F to G . But as C is to D , so is A to B . Wherefore as A is to B , so is F to G . Againe forasmuch as A multiplying D and E produced G and H , therefore (by the 17. of the seuenth) as D is to E , so is G to H . But as D is to E , so is A to B . Wherefore as A is to B , so is G to H .

C

D

E

F

G

H

K

And forasmuch as those numbers A and B multiplying E produced H and K , therefore (by the 18. of the seuenth) as A is to B , so is H to K . And it is proued, that as A is to B , so is F to G , and G to H : wherefore as F is to G , so is G to H , and H to K . Wherefore these numbers C, D, E , and F, G, H, K , are proportionall in the same proportion, that A is to B . Now I say, that they are also the least: For forasmuch as A and B are the least of all numbers that haue the same proportion with them: but the least numbers that haue one & the same proportion with them are prime the one to the other (by the 24. of the seuenth): therefore A and B are prime the one to the other: and ech of these numbers A & B multiplying him selfe produced these numbers C and E , and likewise multiplying ech of these numbers C and E they produced F and K . Wherefore (by the 29. of the seuenth) C, E , and F, K , are prime the one to the other. But if there be numbers in continuall proportion how many soeuer, and if their extremes be prime the one to the other, they are the least of all numbers that haue the same proportion with them (by the first of the eight). Wherefore these numbers C, D, E , and F, G, H, K , are the least of all

of all numbers that haue the same proportion with A and B. And forasmuch as (by the 29. of the seuenth) that alwaies happeneth touching the extremes, namely, that A and B multiplying the numbers produced E and K shall produce other prime numbers, namely, the extremes of five numbers in continuall proportion, therefore (by the first of this booke) all five are the least of that proportion. And so infinitely: which was required to be done.

¶ Corollary.

Hereby it is manifest, that if three numbers being in continuall proportion, be the least of all numbers that haue the same proportion with them, their extremes are squares: and if there be foure their extremes are cubes. For the extremes of three are produced of the multiplying of the nūbers A and B into them selues. And the extremes of foure are produced of the multiplying of the rootes A and B into the squares C and E, whereby are made the cubes F and K.

The 2. Theoreme.

The 3. Proposition.

If there be numbers in continuall proportion how many soeuer, and if they be the least of all numbers that haue one and the same proportion with the: their extremes shall be prime the one to the other.



Suppose that the numbers in continuall proportion being the least of all numbers that haue the same proportion with them be A, B, C, D. Then I say that their extremes A and D are prime the one to the other. Take (by the 2. of the eight, or by the 35. of the seuenth) the two least numbers that are in the same proportion that A, B, C, D, are, and let the same be the numbers E, F. And after that take thre numbers

This proposition is the converse of the first.

A
 B
 C
 D
 E
 F
 G
 H
 K
 L
 M
 N
 O

G, H, K, and so alwayes forward on more (by the former proposition) untill the multitude taken be equall to the multitude of the numbers geuen A, B, C, D. And let those numbers be L, M, N, O. Wherefore (by the 29. of the seuenth) their extremes L, O, are prime the one to the other. For forasmuch as E and F are prime the one to the other, and eche of them mul-

Demonstration.

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tiplieng

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multiplieng himselfe produced G and K , & likewise ech of these G & K multiplieng himselfe produced L & O , therefore (by the 29 of the seuenth) G & K are prime the one to the other, & so likewise are L and O prime the one to the other. And forasmuch as A, B, C, D , are the least of

A
 B
 C
 D
 E
 F
 G
 H
 K
 L
 M
 N
 O

all numbers that haue the same proportion with them, and likewise L, M, N, O , are the least of all numbers that are in the same proportion that A, B, C, D , are, and the multitude of these numbers A, B, C, D , is equall to the multitude of these L, M, N, O : therefore euery one of these A, B, C, D , is equall vnto euery one of these L, M, N, O . Wherefore A is equall vnto L , and D is equall vnto O . And forasmuch as L and O are prime the one to the other, and L is equall vnto A , and O is equall vnto D : therefore A and D are prime the one to the other: which was required to be proued.

The 2. Probleme.

The 4. Proposition.

Proportions in the least numbers how many soeuer beyng geuen, to finde out the least numbers in continuall proportion in the said proportions geue.



Suppose that the proportions in the least numbers geuen, be A to B , C to D , and E to F . It is required to finde out the least numbers in continuall proportion, in the same proportion that A is to B , and that C is to D , and that E is to F . Take the least number whom B and C do measure, and let the same be G . And how often B measureth G , so many times let A measure H . And how oft C measureth G , so many times let D measure K . Now E either measureth K or measureth it not. First let it measure it. And how often E measureth K , so many times let F measure L . And forasmuch as how often A measureth H , so many times doth B measure G : therefore by the 17. of the seuenth, as A is to B , so is H to G . And by the same reason as C is to D , so is G to K , and moreover as E is to F , so is K to L . Wherefore these numbers H, G, K, L , are in continuall proportion, and in the same proportion that A is to B , and that C is to D , and moreover that E is to F . I say also that they are the least in those proportions. For if H, G, K, L , be not the least numbers in continuall proportion, and in the same proportions that A is to B , and C to D , and E to F , then are there some numbers lesse then H, G, K, L , in the same proportions that A is to B and C to D , and E to F , let those numbers be N, X, M, O . And forasmuch as A is to B , so

Two cases in this proposition.
 The first case.

Demonstration leading to an absurditie.

B, so is N to X: and A and B are the least, but the least measure those numbers that haue one and the same proportion with them equally, the greater the greater, and the lesse the lesse, that is, the antecedent, the antecedent, & the consequent the consequent (by the 21. of the seuenth) therefore B measureth X. And by the same reason C also measureth X, wherefore B and C measure X. Wherefore the least number whom B and C measure, shall also by the 37. of the seuenth measure X. But the least number whome B and C measure is G. Wherefore G measureth X, the greater, the lesse, which is impossible. Wherefore there shall not be any lesse numbers then H, G, K, L, in continuall proportion, and in the same proportions that A is to B, and C to D, and E to F.

But now suppose that E measure not K. And by the 36. of the seuenth, take the least number whome E and K measure, and let the same be M. And how often K measureth M, so often let either of these G and H measure either of these N and X. And how often E measureth M, so often let F measure O. And forasmuch as how often G measureth N, so often doth H measure X, therefore as H is to G, so is X to N. But as H is to G, so is A to B. Wherefore as A is to B, so is X to N. And by the same reason as C is to D, so is N to M. Again, forasmuch as how often E measureth M, so often F measureth O, therefore as E is to F, so is M to O. Wherefore X, N, M, O, are in continuall proportion, and in the same proportions that A is to B, and C to D, and E to F. I say also that they are the least in that proportion. For if X, N, M, O, be not the least in continuall proportion, and in the same proportions that A is to B, and C to D, and E to F, then shall there be some numbers lesse then X, N, M, O, in continuall proportion, and in the same proportions that A is to B, and C to D, and E to F. Let the same be the numbers P, R, S, T. And for that as P is to R, so is A to B, and A and B are the least, but the least numbers measure those numbers that haue one & the same proportion with them equally, the greater the greater, and the lesse the lesse, that is, the antecedent the antecedent, and the consequent the consequent by the 21. of the seuenth, therefore B measureth R. And by the same reason also C measureth R. Wherefore B and C measure R. Wherefore the least number whom B and C measure shall also measure R (by the 37. of the seuenth). But the least number whom B and C measure is G, wherefore G measureth R. And as G is to R, so is K to S. Wherefore K measureth S. And E also measureth S. Wherefore E and K measure S. Wherefore the least number whom E and K measure, shall (by the selfe same) measure S. But the least number whom E and K measure is M. Wherefore M measureth S the greater the lesse, which is impossible. Wherefore there are no numbers lesse then X, N, M, O, in continuall proportion, & in the same proportions that A is to B, and C to D, and E to F. Wherefore X, N, M, O, are the least numbers in continuall proportion, and in the same proportions that A is to B, and C to D, and E to F: which was required to be done.

A ...

B ...

C ...

D ...

E ...

F ...

H ...

G ...

K ...

L ...

N ...

X ...

M ...

O ...

The second case.

Demonstration.

A ...

B ...

C ...

D ...

E ...

F ...

H ...

G ...

K ...

X ...

N ...

M ...

O ...

P ...

R ...

S ...

T ...

The eighth Booke

¶ The 3. Theoreme.

The 1. Proposition.

Playne or superficiall numbers are in that proportion the one to the other which is composed of the sides.

This proposition in numbers answereth to the of the sixth touching paralelogrammes.

Construction

Demonstration.

Suppose that A and B be playne or superficiall numbers, and let the sides of A be the numbers C and D, and let the sides of B, be the numbers E and F. Then I say that A is to B in that proportion that is composed of the sides.

Take (by the fourth of the eight) the least numbers in continuall proportion, and in the same proportions that

C is to E, and D to F. And let the same be the numbers

G, H, K: so that as C is to E, so let G be to H, & as D is to F, so let H be to K.

Wherefore those numbers G, H, K, haue the proportions of the sides: but the proportion of G to K is composed of that which G hath to H and of that which H hath to K: wherefore G is unto K in that proportion which is composed of the sides. Now I say that as A is to B, so is G to K. For let D multiplying E produce L. And forasmuch as D multiplying C produced A, and multiplying E produced L: therefore (by the 17. of the seuenth) as C is to E, so is A to L. But as C is to E, so is G to H, wherefore as G is to H, so is A to L. Agayne forasmuch as E multiplying D produced L, & multiplying F produced B: therefore (by the 17. of the seuenth) as D is to F, so is L to B. But as D is to F, so is H to K, wherefore as H is to K, so is L to B. And it is proued that as G is H, so is A to L. Wherefore of equalitie (by the 14. of the seuenth) as G is to K, so is A to B. But G is unto K in that proportion which is composed of the sides, wherefore A is unto B in that proportion which is composed of the sides: which was required to be demonstrated.

A

L

B

C ...

D

E

F

G

H

K

¶ An other demonstration of the same after Campane.

An other demonstration after Campane.

Suppose that A and B be plaine numbers: and let the sides of A be the numbers C and D: and let the numbers E and F be the sides of the number B. And let D multiplying E produce the number G.

Then I say that the proportion of A to B is composed of the proportions of C to E & D to F that is, of the sides of the superficial number A to the sides of the superficial number B. For forasmuch

as D multiplying E produced G, and multiplying C it produced A, therefore by (the 17. of the seuenth) A is to G as C is to E: agayne forasmuch as E multiplying D produced G and multiplying F it produceth B, therefore by the same G is to B as D is to F. Wherefore the proportions of the sides namely, of C to E and of D to F are one and the same with the proportions of A to G and G to B. But (by the fifth definition of the sixth) the proportion of the extremes A to B is composed of the proportions of the meanes, namely, of A to G and G to B, which are proued to be one and the same with the proportions of the sides C to E, and D to F. Wherefore the proportion of the superficial numbers A to B is composed of the proportions of the sides C to E, and D to F. Wherefore playne. &c. which was required to be proued.

A

G

B

C ...

D

E

F

¶ The

¶ The 4. Theoreme.

The 6. Proposition.

If there be numbers in continuall proportion how many soeuer, and if the first measure not the second, neither shall any one of the other measure any one of the other.



Suppose that there be numbers how many soeuer in continuall proportion, namely, five, A, B, C, D, E . And suppose that A measure not B . Then I say, that neither shall any other of the numbers A, B, C, D, E , measure any one of the other. That A, B, C, D, E , do not in continuall order measure one the other, it is manifest: for A measureth not B . Now I say, that neither shall any other of them measure any other of them. I say that A shall not measure C . For how many in multitude A, B, C , are, take so many of the lest numbers that haue one and the same proportion with A, B, C (by the 35. of the seuenth) and let the same be F, G, H . And forasmuch as F, G, H are in the selfe same

Demonstration.

A
 B
 C
 D
 E

 F
 G
 H

proportion that A, B, C , are: and the multitude of these numbers A, B, C , is equall to the multitude of those numbers F, G, H , therefore of equalitie (by the 14. of the seuenth) as A is to C , so is F to H . And for that as A is to B , so is F to G , but A measureth not B , therefore neither doth F measure G . Wherefore F is not vnitie. For if F were vnitie, it should measure any number. But F and H are prime the one to the other (by the 3. of the eight). Wherefore F measureth not H : & as F is to H , so is A to C , wherefore neither doth A measure C . In like sort may we proue that neither shall any other of the numbers A, B, C, D, E , measure any other of the numbers A, B, C, D, E : which was required to be demonstrated.

¶ The 5. Theoreme.

The 7. Proposition.

If there be numbers in continuall proportion how many soeuer, and if the first measure the last, it shall also measure the second.



Suppose that there be a multitude of numbers in continuall proportion, namely, A, B, C, D . And let A the first measure D the last. Then I say, that A the first measureth B the second. For if A do not measure B , neither shall any other measure any other (by the 7. of the eight): which (by supposition) is not true. For A is supposed to measure D . Now then A measuring D , shall also measure B : which was required to be proued.

A ...
 B
 C
 D

Demonstration leading to an impossibility.

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The 6. Theoreme.

The 8. Proposition.

If betwene two numbers there fall numbers in continuall proportion: how many numbers fall betwene them, so many also shall fall in continuall proportion betwene other numbers which haue the selfe same proportion.



Suppose that betwene the two numbers A and B , do fall in continuall proportion on the numbers C and D . And as A is to B , so let E be to F . Then I say, that how many numbers in continuall proportion do fall betwene A and B , so many numbers also in continuall proportion shall there fall betwene E and F . How many. A, B, C, D , are in multitude, take (by the 35. of the seventh) so many of the least numbers that haue one and the same proportion with A, B, C, D , and let the same be G, H, K, L . Wherefore their extremes G and L are prime the one to the other (by the 3. of the eight). And forasmuch as A and C , and D and B , are in the selfe same proportion that G & H , and K and L are, and the multitude of these numbers A, C, D, B , is equall to the multitude of these numbers G, H, K, L : therefore of equalitie (by the 14. of the seventh) as A is to B , so is

Demonstration.

A ...
 C
 D
 B

G .
 H ...
 K
 L

E
 M
 N
 F

G to L . But as A is to B , so is E to F . Wherefore as G is to L , so is E to F . But G and L are prime the one to the other: yea they are prime and the least. But the least numbers measure those numbers that haue the same proportion with them equally, the greater the greater, and the lesse the lesse (by the 21. of the seventh) that is, the antecedent, the antecedent, & the consequent, the consequent. Wherefore how many times G measureth E , so many times L measureth F . How often G measureth E , so often let H measure M and K measure N . Wherefore these numbers G, H, K, L , equally measure these numbers E, M, N, F . Wherefore (by the 18. of the seventh) these numbers G, H, K, L , are in the selfe same proportion that E, M, N, F , are. But G, H, K, L , are in the selfe same proportion that A, C, D, B , are: wherefore those numbers A, C, D, B , are in the selfe same proportion that E, M, N, F , are. But A, C, D, B , are in continuall proportion: wherefore also E, M, N, F , are in continuall proportion. Wherefore how many numbers in continuall proportion fall betwene A and B , so many also in continuall proportion fall there betwene E and F : which was required to be demonstrated.

* A Corollary added by Flußates.

A Corollary added by Flußates.

Betweene two numbers whose proportion is superparticular, or superbipartient, there falleth no meane proportionall. For the least numbers of that proportion differ the one from the other onely by vnitie or by two.

But

But if betwene the greater numbers of that proportion there should fall a meane proportionall, then should there fall also a meane proportionall betwene the least numbers which haue the same proportion by this Proposition. But betwene numbers differing onely by vnitie or by two, there falleth no meane proportionall.

The 7. Theoreme.

The 9. Proposition.

If two numbers be prime the one to the other, and if betwene them shall fall numbers in continuall proportion: how many numbers in continuall proportion fall betwene them; so many also shall fall in continuall proportion betwene either of those numbers and vnitie.



Suppose that there be two numbers prime the one to the other A and B : and let there fall betwene them in continuall proportion these numbers C and D : and let E be vnitie. Then I say, that how many numbers in continuall proportion fall betwene A and B , so many also shall fall in continuall proportion betwene A and vnitie E : and likewise betwene B and vnitie E . Take (by the 35. of the seuenth) the two least numbers that are in the same proportion that A, C, D, B , are: and let the same be F and G : and then take three of the least numbers that are in the same proportion that A, C, D, B , are: and let the same be H, K, L : and so alwaies in order one more, untill the multitude of them be equall to the multitude of these numbers A, C, D, B : and those being so taken let them be M, N, X, O . Now it is manifest, that F multiplying him selfe produced H , and multiplying H produced M . And G multiplying him selfe produced L , and multiplying L produced O . And forasmuch as M, N, X, O , are (by supposition) the least of all numbers that haue the same proportion with G, F : and A, C, D, B , are (by the first of the eight) the least of all numbers that haue the same proportion with G, F : and the multitude of these numbers M, N, X, O , is equall to the multitude of these numbers A, C, D, B : therefore euery one of these numbers M, N, X, O , is equall to euery one of these numbers A, C, D, B . Wherefore M is equall vnto

Construction.

Demonstration.

			M 27	A 27
		H		
	F ...		N 36	C 36
E vnitie		K		
	G ...		X 48	D 48
		L		
			O 64	B 64

A , and O is equall vnto B . And forasmuch as F multiplying him selfe produced H : therefore F measureth H by those vnities which are in F : and vnitie E measureth F by those vnities which are in F : wherefore (by the 15. of the seuenth) vnitie E , so many times measureth the number F , as F measureth H : wherefore as vnitie E is to the number F , so is F to H . Againe, forasmuch as F multiplying H produced M , therefore H measureth M by those vnities which are in F . And vnitie E measureth F by those vnities which are in F : wherefore (by the self same) vnitie E so many times measureth F , as H measureth M . Wherefore as vnitie E is to the number F , so is H to M . But it is proued, that as vnitie E is to the number F , so is F to H : wherefore as vnitie E is to the number F , so is F to H , and H to M . But M is equall vnto A : wherefore as vnitie E is to the number F , so is F to H , & H to A . And by the same reason as vnitie E is to the number G , so is G to L and L to B . Wherefore how many numbers fall in continuall proportion betwene A and B : so many numbers also in continuall proportion fall there betwene vnitie E and the number A , and likewise betwene vnitie E and

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and the number B: which was required to be demonstrated.

The 8. Theoreme.

The 10. Proposition.

If betwene two numbers and vnitie fall numbers in continuall proportion: how many numbers in continuall proportion fall betwene either of them & vnitie so many also shall there fall in continuall proportion betwene them.

This proposition is the converse of the former.

Suppose that betwene the two numbers A, B, and vnitie C, do fall these numbers in continuall proportion D, E, and F, G. Then I say that how many numbers in continuall proportion there are betwene either of these A, B, and vnitie C, so many

			A 27
		E	
	D ...		K 36
C vnitie		H	
			L 48
	F	G	
			B 64

Construction.

Demonstration.

numbers also in continuall proportion shall there fall betwene A and B. Let D multiplying F produce H, and let D multiplying H produce K, and like wise let F multiplying H produce L. And for that by supposition as vnitie C is to the number D, so is D to E, therefore how many times vnitie C measureth the number D, so many times doth D measure E. But vnitie C measureth D by those vnities which are in D: wherefore D measureth E by those vnities which are in D. Wherefore D multiplying himselfe produceth E. Againe for that as vnitie C is to the number D, so is E to A, therefore how many times vnitie C measureth the number D, so many times E measureth A. But vnitie C measureth D, by those vnities which are in D, therefore E measureth A by those vnities which are in D. Wherefore D multiplying E produced A. And by the same reason F multiplying himselfe produced G, and multiplying G produced B. And forasmuch as D multiplying himselfe produced E, and multiplying F produced H, therefore (by the 17. of the seuenth) as D is to F, so is E to H. And by the same reason as D is to F, so is H to G. Wherefore as E is to H, so is H to G. Agayne forasmuch as D multiplying E produced A, and multiplying H produced K, therefore (by the 17. of the seuenth) as E is to H, so is A to K. But as E is to H, so is D to F, therefore as D is to F, so is A to K. Againe forasmuch as D multiplying H produced K, and F multiplying H produced L, therefore (by the 17. of the seuenth) as D is to F, so is K to L. But as D is to F, so is A to K, wherefore as A is to K, so is K to L. Againe forasmuch as F multiplying H produced L and multiplying G produced B, therefore (by the 17. of the seuenth) as H is to G, so is L to B. But as H is to G so is D to F, wherefore as D is to F so is L to B. And it is proued that as D is to F, so is A to K, and K to L, and L to B. Wherefore the numbers A, K, L, B, are continuall proportion. Wherefore how many numbers in continuall proportion fall betwene either of these numbers A, B, & vnitie C, so many also in continuall proportion fall there betwene the numbers A and B: which was required to be proued.

The 9. Theoreme.

The 11. Proposition.

Betwene two square numbers there is one meane proportional number. And a square

a square number to a square, is in double proportion of that which the side of the one is to the side of the other.



*S*uppose that there be two square numbers *A* and *B*, and let the side of *A* be *C*, & let the side of *B* be *D*. Then I say that betwene these square numbers *A* and *B*, there is one meane proportionall number, and also that *A* is unto *B* in double proportion of that which *C* is to *D*. Let *C* multiplieng *D* produce *E*. And foras-

much as *A* is a square nūber, & the side thereof is *C*, therefore *C* multiplieng himselfe produced *A*. And by the same reason *D* multiplieng himselfe produced *B*.

Now forasmuch as *C* multiplieng *C* produced *A*, and multiplieng *D* produced *E*, therefore (by the 17. of the

seuenth) as *C* is to *D*, so is *A* to *E*. Againe forasmuch as *C* multiplieng *D* produced *E*, and *D* multiplieng himselfe produced *B*, therefore these

two numbers *C* and *D* multiplieng one number, namely, *D*, produce *E* and *B*. Wherefore (by the 18. of the seuenth) as *C* is to *D*, so is *E* to *B*. But as *C* is to *D*, so is *A* to *E*. Wherefore as *A* is to *E*, so is *E* to *B*. Wherefore betwene these square numbers *A* and *B*, there is one meane

proportionall number, namely, *E*. Now also I say that *A* is unto *B* in double proportion of that which *C* is to *D*. For forasmuch as there are three numbers in continuall proportion, *A*, *E*, *B*, therefore (by the 10. definition of the fift) *A* is unto *B* in double proportio of that which *A* is to *E*. But as *A* is to *E*, so is *C* to *D*. Wherefore *A* is unto *B* in double proportion of that which the side *C* is unto the side *D*: which was required to be proued.

The first part of this proposition demonstrated.

The second part demonstrated.

¶ The 10. Theoreme.

The 12. Proposition.

Betwene two cube numbers there are two meane proportionall numbers. And the one cube is to the other cube in treble proportion of that which the side of the one is to the side of the other.

*S*uppose that there be two cube numbers *A* and *B*, and let the side of *A* be *C*, and let the side of *B* be *D*. Then I say that betwene those cube numbers *A* and *B*, there are two meane proportionall numbers, and that *A* is unto *B* in treble proportion of that which *C* is to *D*. Let *C* multiplieng himselfe produce *E*, and multiplieng *D* let it produce *F*, and let *D* multiplieng himselfe produce *G*. And let *C* multiplieng *F* produce *H*, and let *D*

Construction.

A

H

K

B

C

D

E

F

G

multiplieng *F* produce *K*. And forasmuch as *A* is a cube number, and the side thereof is *C*, & *C* multiplieng himselfe produceth *E*, therefore *C* multiplieng *E* produceth *A*. And by the same reason

The first part of this proposition demonstrated.

reason for that D multiplieng himselfe produced G, therefore D multiplieng G produceth B. And forasmuch as C multiplieng C and D produced E and F : therefore by the 17. of the fift, as C is to D, so is E to F. And by the same reason also, as C is to D, so is F to G. Againe forasmuch as C multiplieng E and F produced A and H, therefore as E is to F, so is A to H. But as E is to F, so is C to D. Wherefore as C is to D : so is A to H. Againe forasmuch as eche of these

A
H
K
B

C ...
D

E
F
G

numbers C and D multiplieng F produced H and K, therefore (by the 18. of the seuenth) as C is to D, so is H to K. Againe forasmuch as D multiplieng F and G produced K & B : therefore (by the 17. of the seuenth) as F is to G, so is K to B. But as F is to G, so is C to D, wherefore as C is to D, so is K to B. And it is proued that as C is to D, so is A to H, and H to K, and K to B : wherefore betwene these cube numbers A and B, there are two meane proportionall numbers, that is, H and K.

The second
part demon-
strated.

Now also I say, that A is vnto B in treble proportion of that which C is to D. For forasmuch as there are foure numbers proportionall A, H, K, B, therefore (by the 10. definition of the fift) A is vnto B in treble proportion of that which A is vnto H. But as A is vnto H, so is C to D, wherefore A is vnto B in treble proportion of that which C is to D : which was required to be proued.

¶ The 11. Theoreme. The 13. Proposition.

If there be numbers in continuall proportion how many so euer, and ech multiplying himselfe produce certayne numbers, the numbers of them produced shall be proportionall. And if those numbers geuen at the beginning multiplying the numbers produced, produce other numbers, they also shalbe proportionall: and so shall it be continuing infinitely.

Construction.

Demonstra-
tion.

Suppose that there be a multitude of nũbers in cōtinuall proportiō, namely, A, B, C, as A is to B, so let B be to C. And let A, B, C, multiplying ech himself bring forth the nũbers D, E, F, & multiplying the nũbers D, E, F, let the bring forth the nũbers G, H, K. The I say that D, E, F, are in cōtinuall proportiō, and also that G, H, K, are in cōtinuall proportiō. For it is manifest that the nũbers D, E, F, are square numbers, & that the nũbers G, H, K, are cube nũbers. Let A multiplying B produce L. And let A & B multiplying L produce M and N. And againe let B multiplying C produce X: and let B and C multiplying X produce O and P. Now by the discourse of the proposition going before we may

may proue that D, L, E, and also G, M, N, H, are in continuall proportion and in the same proportion that A is to B: and likewise that E, X, F, and also H, O, P, K, are in continuall proportion and in the same proportion that B is to C. But as A is to B, so is B to C. Wherefore D, L, E, are in one and the same proportion with E, X, F: and moreover G, M, N, H, are in one and the same proportion with H, O, P, K: and the multitude of these numbers D, L, E, is equall to the multitude of these numbers E, X, F, and likewise the multitude of these numbers G, M, N, H, is equall to the multitude of these numbers H, O, P, K, wherefore of equality (by the 14. of the seuenth) as D is to E, so is E to F. And as G is to H, so is H to K: which was required to be proued.

A 3	B 4	C 8
D 4	L 8	
E 16	X 32	
F 64		
G 8	M 16	
	N 32	
H 64	O 128	
	P 256	
K 512		

¶ The 12. Theoreme.

The 14. Proposition.

If a square number measure a square number, the side also of the one shall measure the side of the other. And if the side of the one measure the side of the other, the square number also shall measure the square number.



Suppose that there be two square numbers A and B, and let the sides of them be C and D: and let A measure B. Wherefore C also shall measure D. Let C multiplying D produce E. Wherefore (by the 17. and 18. of the seuenth, and 13. of the eight) those numbers A, E, B, are in continuall proportion, and are in the same proportion that C is to D. And forasmuch as A, E, B, are in continuall proportion, and A measureth B, therefore (by the 7. of the eight) A measureth E. But as A is to E, so is C to D: wherefore C measureth D.

The first part of this proposition.

A
 E
 B
 C
 D

But now suppose that the side C do measure the side D. Then I say, that the square number A also measureth the square number B. For the same order of construction remayning, we may in like sort proue, that the numbers A, E, B, are in continuall proportion, & in the same proportion, that C is to D. And for that as C is to D, so is A to E, but C measureth D: therefore A measureth E: and A, E, B, are in continuall proportion: wherefore A measureth B. If therefore a square number measure a square number, the side also of the one shall measure the side of the other. And if the side of the one measure the side of the other, the square number also shall measure the square number: which was required to be demonstrated.

The second part is the conuerse of the first.

¶ The 13. Theoreme.

The 15. Proposition.

If a cube number measure a cube number, the side also of the one shall measure
 Ty. j. sure

sure the side of the other. And if the side of the one measure the side of the other, the cube number also shall measure the cube number.



Suppose that the cube number A do measure the cube number B , and let the side of A be C , and the side of B be D . Then I say, that C measureth D . Let C multiplying him selfe produce E , & multiplying D let it produce F . And let D multiplying him selfe produce G . And moreover let C and D multiplying F produce H and K . Now it is manifest (by the 17. and 18. of the seventh, and 12. of the eighth) that those numbers E, F, G , and also A, H, K, B , are in continuall proportion, & in the same proportion that C is to D . And forasmuch as A, H, K, B , are in continuall proportion, and A measureth B , therefore (by the 7. of the eighth) A also measureth H . But as A is to H , so is C to D . Wherefore C also measureth D .

The first part of this proposition.

A
 H
 K
 B

 C
 D

 E
 F
 G

The second part is the converse of the first.

But now suppose that the side C do measure the side D . Then I say, that the cube number A also measureth the cube number B . For the same order of construction being kept, in like sort may we proue, that A, H, K, B , are in continuall proportion, and in the same proportion that C is to D . And forasmuch as C measureth D , but as C is to D , so is A to H , therefore A measureth H : wherefore A also measureth B . If therefore a cube number measure a cube number, the side also of the one shall measure the side of the other. And if the side of the one measure the side of the other, the cube number also shall measure the cube number: which was required to be proued.

¶ The 14. Theoreme.

The 16. Proposition.

If a square number measure not a square number, neither shall the side of the one measure the side of the other. And if the side of the one measure not the side of the other, neither shall the square number measure the square number.

A negative proportion. The first part of this proposition.



Suppose that A and B be two square numbers, and let the side of A be C : and let the side of B be D . And be it that A measureth not B . Then I say, that neither shall C measure D . For if C do measure D , then (by the 14. of the eighth) A also measureth B . But A by supposition measureth not B : wherefore neither doth C measure D .

A
 B

 C ...
 D

The second part is the converse of the first.

But now againe suppose that the side C measure not the side D . Then I say, that neither shall the square number A measure the square number B . For if A do measure B , then shall
 C (by

C (by the 14. of the eight) measure D. But (by supposition) C measureth not D. Wherefore neither doth A measure B: which was required to be proued.

¶ The 15. Theoreme.

The 17. Proposition.

If a cube number measure not a cube number, neither shall the side of the one measure the side of the other. And if the side of the one measure not the side of the other, neither shall the cube number measure the cube number.

Suppose that the cube number A do not measure the cube number B: and let the side of A be C: and the side of B be D. Then I say, that C shall not measure D. For if C do measure D, then (by the 15. of the eight) A also shall measure B. But (by supposition) A measureth not B: wherefore neither shall C measure D.

But now suppose that the side C measure not the side D. Then I say, that neither shall the cube number A measure the cube number B. For if A do measure B, then also (by the 15. of the eight) shall C measure D. But (by supposition) C measureth not D. Wherefore neither shall A measure B: which was required to be proued.

A negative proposition.

The first part of this proposition.

The second part is the converse of the first.

A
B
C
D

¶ The 16. Theoreme.

The 18. Proposition.

Betweene two like plaine or superficiall numbers there is one meane proportionall number. And the one like plaine number is to the other like plaine number in double proportion of that which the side of like proportion, is to the side of like proportion.

Suppose that there be two like plaine or superficiall numbers A & B. And let the sides of A be the numbers C, D: and the sides of B be the numbers E, F. And forasmuch as like plaine numbers are those which haue their sides proportionall (by the 22. definition of the seuenth) therefore as C is to D, so is E to F. Then I say that betweene A and B there is one meane proportionall number, and that A is unto B in double proportion of that which C is unto E, or of that which D is unto F, that is, of that which side of like proportion is to side of like proportion. For for that as C is to D, so is E to F, therefore alternately (by the 13. of the seuenth) as C is to E, so is D to F. And forasmuch as A is a plaine or superficiall number, and the sides thereof are C and D: therefore D multiplying C produced A.

And by the same reason also E multiplying F produced B. Let D multiplying E produce G. And forasmuch as D multiplying C produced A, and multiplying E produced G, therefore (by the 17. of the seuenth) as C is to E, so is A to G. But as C is to E, so is D to F, wherefore as D is to F, so is A to G. Again forasmuch as E multiplying D produced G, and multiplying F produced B, therefore (by the 17. of the seuenth) as D is to F, so is G to B. But it is proued

Demonstration of the first part of this proposition.

A
G
B
C
D
E
F

The eighth Booke

that as D is to F, so is A to G: wherefore as A is to G, so is G to B. Wherefore these numbers A, G, B, are in continuall proportion. Wherefore betwene A and B there is one meane proportionall number.

Demonstration of the second part.

Now also I say that A is unto B in double proportion of that which side of like proportion is to side of like proportion, that is, of that which C is unto E, or of that which D is unto F. For forasmuch as A, G, B, are in continuall proportion, therefore (by the 10. definition of the fifth) A is unto B in double proportion of that which A is unto G. But as A is to G, so is C to E, and D to F: wherefore A is unto B in double proportion of that which C is to E, or D to F: which was required to be demonstrated.

¶ The 17. Theoreme. Th 19. Proposition.

Betwene two like solide numbers, there are two meane proportionall numbers. And the one like solide number, is to the other like solide number in treble proportion of that which side of like proportion is to side of lyke proportion.



Suppose that there be two like solide numbers A and B. And let the sides of the number A, be the numbers C, D, E. And let the sides of the number B, be the numbers F, G, H. And forasmuch as (by the 22. definition of the seventh) lyke solide numbers haue their sides proportionall, therefore as C is to D, so is F to G, and as D is to E, so is G to H. Then I say that betwene A and B, there are two meane proportionall numbers. And that A is unto B in treble proportion of that which C is to F, or of that which D is to G, or moreouer of that which E is unto H.

A
N
X
B
C ..
D ..
E ..
F ...
G ...
H ...
K
M
L

Demonstration of the first part of this proposition.

For let C multiplieng D produce K. And let F multiplieng G, produce L. And forasmuch as C, D, are in the self same proportion that F, G, are, & of C & D is produced K, and of F and G is produced L, therefore K and L are like plaine numbers. And therefore betwene those numbers K and L, there is one meane proportionall number (by the 18. of the seventh) Let the same be M. Wherefore M is produced of D and F, as it is manifest by the proposition goyng before. Wherefore as K is to M, so is M to L. And forasmuch as D multiplieng C produced K, and multiplieng F produced M: therefore (by the 17. of the seventh) as C is to F, so is K to M, but as K is to M, so is M to L. Wherefore these numbers K, M, L, are in continuall proportion and in the same proportion that C is to D. And for that as C is to D, so is F to G, therefore alternately (by the 13. of the seventh) as C is to F, so is D to G. Again, for that as D is to E, so is G to H, therefore alternately also as D is to G, so is E to H. Wherefore these numbers K, M, L, are in continuall proportion, and in the same proportion that C is to F, and that D is to G, and moreouer that E is to H. Now let E and H multiplieng M produce N and X. And forasmuch as A is a solide number, and the sides thereof are C, D, E, therefore E multiplieng that which is produced of C and D, produceth A. But that which is produced of C and D is K. Wherefore E multiplieng K produceth A. And (by the same reason) H multiplieng that which is produced of F and G, that is multiplieng L produceth

duceth B. And forasmuch as E multiplieng K produced A, and multiplieng M produced N, therefore (by the 17. of the seuenth), as K is to M, so is A to N. But as K is to M, so is C to F, & D to G, and moreover E to H, therefore as C is to F, and D to G, and E to H, so is A to N. Agayne, forasmuch as E multiplieng M produced N, and H multiplieng M, produced X, therefore (by the 18. of the seuenth) as E is to H, so is N to X. But as E is to H, so is C to F, and D to G. Wherefore as C is to F, and D to G, and E to H, so is A to N, and N to X. Agayne forasmuch as H multiplieng M, produced X, and multiplieng L produced B, therefore (by the 17. of the seuenth) as M is to L, so is X to B. But as M is to L, so is C to F, and D to G, and E to H, therefore as C is to F, and D to G, and E to H, so is not onely X to B, but also A to N, and N to X. Wherefore these numbers A, N, X, B, are in continuall proportion, and that in the proportions of the sides. I say moreover that A is unto B in treble proportion of that which side of like proportion, is to side of like proportion, that is, of that which the number C hath to the number F, or of that which D hath to G, or moreover of that which E hath to H. For forasmuch as there are foure numbers in continual proportion, that is, A, N, X, B, therefore (by the 10. definition of the fift) A is unto B in treble proportion of that which A is unto N. But as A is to N, so is it proued that C is to F, and D to G, and moreover E to H. Wherefore A is unto B in treble proportion of that which side of like proportion is unto side of like proportion, that is of that which the number C is to the number F, and of that which D is to G, and moreover of that which E is to H : Which was required to be proued.

The second part.

¶ The 18. Theoreme.

The 20. Proposition.

If betwene two numbers there be one meane proportionall number : those numbers are like plaine numbers.



Vppose that betwene the two numbers A and B there be one meane proportionall number, and let the same be C. Then I say, that those numbers A and B are like plaine numbers. Take (by the 35. of the seuenth) two of the least numbers that haue one & the same proportion with A, C, B : and let the same be the numbers D, E. Wherefore as D is to E, so is A to C, but as A is to C, so is C to B, wherefore as D is to E, so is C to B. Wherefore how many times D measureth A, so many times doth E measure C. How many times D measureth A, so many unities let there be in F. Wherefore F multiplying D, produceth A, and multiplying E it produceth C : wherefore A is a plaine number. and the sides therof are D and F (by the 17. definition of the seuenth). Agayne forasmuch as D and E are the lest numbers that haue one & the same proportion with C, B, therefore (by the 21. of the seuenth) how many times D measureth C, so many times doth E measure B. How often E measureth B, so many unities let there be in G. Wherefore E measureth B by those unities which are in G : wherefore G multiplying E produceth B : wherefore B is a plaine number. (by the 17. definition of the seuenth). And the sides thereof are E and G. Wherefore those two numbers A and B are two plaine numbers. I say moreover that they are like. For forasmuch as F multiplying E produced C, and G multiplying E produced B : therefore (by the 17. of the seuenth) as F is to G, so is C to B, but as C is to B, so is D to E, wherefore as D is to E, so is F to G. Wherefore A and B are like plaine numbers, for their sides are proportionall : which was required to be proued.

This proposition is the conuerse of the 18. proposition. Construction.

Demonstration.

The eighth Booke

The 19. Theoreme.

The 21. Proposition.

If betwene two numbers, there be two meane proportionall numbers, those numbers are like solide numbers.

This proposition is the converse of the 19. proposition.

Construction.



Suppose that betwene two numbers A and B , there be two meane proportionall numbers C, D . Then I say that A and B are like solide numbers. Take (by the 35 of the seuenth, or 2. of the eight) three of the least numbers that haue one and the same proportion with A, C, D, B , and let the same be E, F, G . Wherefore (by the 3. of the eight) their extremes E, G are prime the one to the other. And forasmuch as betwene the numbers E and G there is one meane proportionall number: therefore (by the 20 of the eight) they are like plaine numbers. Suppose that the sides of E , be H and K . And let the sides of G , be L and M . Now it is manifest that these numbers E, F, G , are in continuall

A
 C
 D
 B

 E
 F
 G
 H ...
 K ...
 N ...

 L
 M
 X

Demonstration.

proportion, and in the same proportion that H is to L , and that K is to M . And forasmuch as E, F, G are the least numbers that haue one and the same proportion with A, C, D , therefore of equalitie (by the 14. of the seuenth) as E is to G , so is A to D . But E, G , are (by the 3. of the eight) prime numbers, yea they are prime and the least, but the least numbers (by the 21. of the seuenth) measure those numbers that haue one & the same proportion with them equally, the greater the greater, and the lesse the lesse, that is, the antecedent the antecedent, & the consequent the consequent: therefore how many times E measureth A , so many times G measureth D . How many times E measureth A , so many vnities let there be in N . Wherefore N multiplieng E , produceth A . But E is produced of the numbers H, K . Wherefore N multiplieng that which is produced of H, K , produceth A . Wherefore A is a solide number, and the sides thereof are H, K, N . Agayne, forasmuch as E, F, G , are the least numbers that haue one and the same proportion with C, D, B , therefore how many times E measureth C , so many times G measureth B . How oftentimes G measureth B , so many vnities let there be in X . Wherefore G measureth B by those vnities which are in X . Wherefore X multiplieng G produceth B . But G is produced of the numbers L, M . Wherefore X multiplieng that number which is produced of L and M , produceth B . Wherefore B is a solide number, and the sides thereof are L, M, X . Wherefore A, B are solide numbers. I say moreover that they are like solide numbers. For forasmuch as N and X multiplieng E produced A and C : therefore by the 18. of the seuenth,

as N is to X , so is A to C , that is E to F . But as E is to F , so is H to L , and K to M : therefore as H is to L , so is K to M , and N to X . And H, K, N , are the sides of A , and likewise L, M, X , are the sides of B : wherfore A, B , are like solide numbers: which was required to be proued.

¶ The 20. Theoreme.

The 22. Proposition.

If three numbers be in continuall proportion, and if the first be a square number, the third also shall be a square number.

Suppose that there be three numbers in continuall proportion A, B, C , and let the first be a square number. Then I say that the third is also a square number. For forasmuch as betwene A and C there is one meane proportionall number, namely B , therefore (by the 20. of the eight) A and C are like playne numbers. But A is a square number. Wherfore C also is a square number: which was required to be proued.

A
 B
 C

Demonstration.

¶ The 21. Theoreme.

The 23. Proposition.

If foure numbers be in continuall proportion, and if the first be a cube number, the fourth also shall be a cube number.

Suppose that there be foure numbers in continuall proportion A, B, C, D . And let A be a cube number. Then I say that D also is a cube number. For forasmuch as betwene A and D there are two meane proportionall numbers B, C . Therefore A, D are like solide numbers (by the 21. of this booke) But A is a cube number, wherfore D also is a cube number: which was required to be demonstrated.

A
 B
 C
 D

Demonstration.

¶ The 22. Theoreme.

The 24. Proposition.

If two numbers be in the same proportiō that a square number is to a square number, and if the first be a square number, the second also shall be a square number.

Suppose that two numbers A and B be in the same proportion, that the square number C is vnto the square number D . And let A be a square number. Then I say that B also is a square number. For forasmuch as C and D are square numbers. Therefore C and D are like plaine numbers. Wherfore (by the 18. of the eight) betwene C and D there is one meane proportionall number. But as C is to D , so is A to B . Wherfore betwene A and B there is one meane proportionall number (by the 8. of the eight) But A is a square number. Wherfore (by the 22. of the eight) B also is a square number: which was required to be proued.

A
 B
 C
 D

Demonstration.

The eighth Booke

¶ The 23. Theoreme.

The 25. Proposition.

If two numbers be in the same proportion the one to the other, that a cube number is to a cube number, and if the first be a cube number, the second also shall be a cube number.

Demonstration.



Suppose that two numbers *A* and *B* be in the same proportion the one to the other, that the cube number *C* is unto the cube number *D*. And let *A* be a cube number. Then I say that *B* also is a cube number. For forasmuch as *C, D*, are cube numbers, therefore *C, D* are like solide numbers, wherefore (by the 19. of the eight) betwene

A

E

F

B

C

—96

—144

D, 216

C and *D* there are two proportionall numbers. But how many numbers fall in continual proportion betwene *C* and *D*, so many (by the 8. of the eight) fall there betwene the numbers that haue the same proportion with them. Wherefore betwene *A* and *B* there are two meane proportionall numbers which let be *E* and *F*. And forasmuch as there are foure numbers in continual proportion, namely, *A, E, F, B*, and *A* is a cube number, therefore (by the 23. of the eight) *B* also is a cube number: which was required to be demonstrated.

A Corollary added by Flussates.

✱ A Corollary added by Flussates.
Betwene a square number and a number that is not a square number, falleth not the proportion of one square number to an other. For if the first be a square number, the second also should be a square number which is contrary to the supposition. Likewise betwene a cube number, and a number that is no cube number falleth not the proportion of one cube number to an other. For if the first be a cube number, the second also should be a cube number, which is contrary to the supposition, & therefore impossible.

¶ The 24. Theoreme.

The 26. Proposition.

Like playne numbers, are in the same proportion the one to the other, that a square number is to a square number.

Construction.



Suppose that *A* and *B* be like plaine numbers. Then I say that *A* is unto *B* in the same proportion that a square number is to a square number. For forasmuch as *A, B*, are like plaine numbers, there-

fore betwene *A* and *B* there falleth one meane proportional number (by the 18. of the eight). Let there fall such a number, and let the same be *C*. And (by the 35. of the seventh) take the three least numbers that haue one and the same proportion with *A, C, B*, and let the same be

A

C

B

D

E

F

D, E, F,

D, E, F: wherefore (by the corollary of the 2. of the eight) their extremes, that is D, F, are square numbers. And for that as D is to F, so is A to B, (by the 14. of the seventh): and D, F, are square numbers. Therefore A is vnto B in that proportion, that a square number is vnto a square number: which was required to be proued.

The 25. Theoreme.

The 27. Proposition.

Like solide numbers are in the same proportion the one to the other, that a cube number is to a cube number.

S*uppose that a A and B be like solide numbers. Then I say that A is vnto B, in the same proportion, that a cube number is to a cube number. For forasmuch as A, B, are like solide numbers. Therefore (by the 19. of the eight) betwene A and B there*

Construction.

A
C
D
B
E
F
G
H

fall two meane proportionall numbers. Let there fall two such numbers, and let the same be C and D. And take (by the 35. of the seventh) the least numbers that haue one and the same proportion with A, C, D, B, and equall also with the in multitude, and let the same be E, F, G, H. Wherefore (by the corollary of the 2. of the eight) their extreames, that is, EH, are cube numbers. But as E is to H, so is A to B. Wherefore A is vnto B in the same proportion, that a cube number is to a cube number: which was required to be demonstrated.

Demonstration.

¶ A Corollary added by Flußates.

If two numbers be in the same proportion the one to the other that a square number is to a square number: those two numbers shall be like superfiціаль numbers. And if they be in the same proportion the one to the other that a cube number is to a cube number, they shall be like solide numbers.

A Corollary added by Flußates.

First let the number A haue vnto the number B the same proportion, that the square number C hath to the square number D. Then I say, that A and B are like superfiціаль numbers. For forasmuch as betwene the square numbers C and D there falleth a meane proportionall (by the 11. of this booke) there shall

B D
A C

also betwene A and B (which haue one and the same proportion with C and D) fall a meane proportionall (by the 8. of this booke). Wherefore A and B are like superfiціаль numbers (by the 20. of this booke).

But if A be vnto B, as the cube number C, is to the cube number D. Then are A & B like solide numbers. For forasmuch as C and D are cube numbers, there falleth betwene them two meane proportion-

A C
B D

all numbers (by the 12. of this booke). And therefore (by the 8. of the same) betwene A and B (which are in the same proportion that C is to D) there falleth also two meane proportionall numbers. Wherefore (by the 21. of this booke) A and B are like solide numbers.

* An

The eighth Booke

* An other Corollary added also by Flusates.

Another Corollary added by Flusates.

If a number multiplying a square number, produce not a square number: the sayd number multiplying shall be no square number. For if it should be a square number, then should it and the number multiplied being like superficial numbers (by reason they are square numbers) haue a meane proportionall (by the 18. of thys booke). And the number produced of the sayd meane should be equall to the number

contayned vnder the extremes, which are square numbers (by the 20. of the seuenth). Wherefore the number produced of the extremes being equall to the square number produced of the meane, should be a square number. But the sayd number by supposition, is no square number. Wherefore neither is the number multiplying the square number, a square number.

The first part of the first Corollary is the conuerse of the 26. Proposition of this booke, and hath some vse in the tenth booke. The second part of the same also is the conuerse of the 27. Proposition of the same.

The end of the eighth booke
of Euclides Elementes.



¶ The ninth booke of Eu- clides Elementes.



IN THIS NINTH BOOKE *Euclide* continueth his purpose touching numbers: partly prosecuting thynges more fully, which were before somewhat spoken of, as of square and cube numbers: and partly setting out the natures and proprieties of such kindes of number, as haue not yet bene entreated of: which yet are most necessary to be knowen. As are numbers euen, and odde: whose passions and conditions are in this booke largely taught, with their compositions, and subductions of the one from the other: with many other generall and speciall thinges to be noted, worthy the knowledge.

*The Argumēts
of the ninth
booke.*

¶ The 1. Theoreme.

The 1. Proposition.

If two like plaine numbers multiplying the one the other produce any number: the number of them produced shall be a square number.



Suppose that A and B be two like plaine numbers. And let A multiplying B produce the number C. Then I say, that C is a square number. For let A multiplying him selfe produce D. Wherefore D is a square number. And forasmuch as A multiplying him selfe produced D, and multiplying B produced C, therefore (by the 17. of the seuenth) as A is to B, so is D to C. And forasmuch as A, B, are like plaine numbers, therefore (by the 18. of the eight) betwene A and B there falleth a meane proportionall number. But if betwene two numbers fall num-

*Demonstra-
tion.*

bers in continuall proportion, how many numbers fall betwene them, so many also (by the 8. of the eight) shall there fall betwene the numbers that haue the same proportion with them. Wherefore betwene C and D also there falleth a meane proportionall number. But D is a square number. Wherefore (by the 22. of the eight) C also is a square number: which was required to be proued.

A

B

D

C

¶ The 2. Theoreme.

The 2. Proposition.

If two numbers multiplying the one the other produce a square number: those numbers are like plaine numbers.

Suppose

The eighth

This proposition is the converse of the former.



Suppose that two numbers A
 A and B multiplying the
 one the other do produce B
 C a square number. Then

I say, that A and B are like plaine num-
bers. For let A multiplying him selfe
produce D . Wherefore D is a square
number. And forasmuch as A multi-
plying him selfe produced D , and multiplying B produced C , therefore (by the 17. of the se-
venth) as A is to B , so is D to C . And forasmuch as D is a square number, and so likewise is
 C , therefore D and C are like plaine numbers. Wherefore betwene D and C there is (by the
18. of the eight) one meane proportionall number. But as D is to C , so is A to B . Wherefore
(by the 8. of the eight) betwene A and B there is one meane proportionall number. But if be-
twene two numbers there be one meane proportionall number, those numbers are (by the 20.
of the eight) like plaine numbers. Wherefore A and B are like plaine numbers: which was
required to be proued.

❖ A Corollary added by *Campane*.

*A Corollary
aided by
Campane.*

Hereby it is manifest, that two square numbers multiplyed the one into the other do alwayes produce a square number. For they are like superficial numbers, and therefore the number produced of them, is (by the first of this booke) a square number. But a square number multiplyed into a number not square, produceth a number not square. For if they should produce a square number, they should be like superficial numbers (by this Proposition). But they are not. Wherefore they produce a number not square. But if a square number multiplyed into an other number produce a square number, that other number shall be a square number. For by this Proposition that other number is like vnto the square number which multiplyeth it, and therefore is a square number. But if a square number multiplyed into an other number produce a number not square, neither shall that other number also be a square number. For if it should be a square number, then being multiplyed into the square number it should produce a square number, by the first part of this Corollary.

The 3. Theoreme. The 3. Proposition.

If a cube number multiplying himselfe produce a number, the number produced shall be a cube number.



Suppose that A being a cube number multiplieng himselfe, do produce the number B . Then I say that B is a cube number. Take the side of A , and let the same be the number C , and let C multiplieng himselfe produce the number D . Now it is manifest that C multiplieng D produceth A (by the 20. definition of the seuēth)

Demonstration.

B

A The 2. Proposition.
D
C ..

And forasmuch as C multiplying himselfe produced D, therefore C measureth D by those v-

unities which are in C. But unitie also measureth C by those unities which are in C. Wherefore as unitie is to C, so is C to D. Again forasmuch as C multiplieng D produceth A: therefore D measureth A by those unities which are in C. But unitie measureth C by those unities which are in C: wherefore as unitie is to C, so is D to A. But as unitie is to C, so is C to D, wherefore as unitie is to C, so is C to D & D to A. Wherefore betwene unitie & A there are two meane proportionall numbers, namely, C, D. Again forasmuch as A multiplieng him selfe produced B, therefore A measureth B by those unities which are in A. But unitie also measureth A by those unities which are in A. Wherefore as unitie is to A, so is A to B. But betwene A and unitie, there are two meane proportionall numbers. Wherefore betwene A and B also there are two meane proportionall numbers by the 8. of the eight. But if betwene two numbers, there be two meane proportionall numbers, and if the first be a cube number, the fourth also shall be a cube number by the 21. of the eight. But A is a cube number, wherefore B also is a cube number which was required to be proued.

¶ The 4. Theoreme.

The 4. Proposition.

If a cube number multiplieng a cube number, produce any number, the number produced shall be a cube number.

Suppose that the cube number A multiplieng the cube number B, do produce the number C. Then I say that C is a cube number. For let A multiplieng him selfe produce D. Wherefore D is a cube number (by the proposition going before). And forasmuch as A multiplieng him selfe produced D, and multiplieng B, it produced C: therefore (by the 17. of the seventh) as A is to B, so is D to C. And forasmuch as A and B are cube numbers, therefore A and B are like solide numbers. Wherefore betwene A and B (by the 19. of the eight) there are two meane proportionall numbers. Wherefore also (by the 8. of the same) betwene D and C there are two meane proportionall numbers. But D is a cube number. Wherefore C also is a cube number (by the 23. of the eight) which was required to be demonstrated.

Demonstration.

A	D 64
.....	96
.....	144
B	C 216

¶ The 5. Theoreme.

The 5. Proposition.

If a cube number multiplying any number produce a cube number: the number multiplied is a cube number.

Suppose that the cube number A, multiplying the number B, do produce a cube number, namely, C. Then I say, that B is a cube number. For let A multiplying him selfe produce D. Wherefore (by the 3. of the ninth) D is a cube number. And forasmuch as A multiplying him selfe produced D, and multiplying B, it produced C: therefore (by the 17. of the seventh) as A is to B, so is D to C. And forasmuch as D

Demonstration.

A	
B	
D 729	
C 1728	

and C are cube numbers, they are also like solide numbers. Wherefore (by the 19. of the eight) betwene D and C there are two meane proportionall numbers. But as D is to C, so is A to B.

AA.i.

Wher-

The ninth Booke

Wherefore (by the 8. of the eight) betwene A and B there are two meane proportionall numbers. But A is a cube number. Wherefore B also is a cube number (by the 23. of the eight): which was required to be proued.

¶ A Corollary added by Campane.

A Corollary
added by
Campane.

Hereby it is manifest, that if a cube number multiply a number not cube, it shall produce a number not cube. For if it should produce a cube number, then the number multiplied should also be a cube number (by this Proposition) which is contrary to the supposition. For it is supposed to be no cube number. And if a cube number multiplying a number produce a number not cube, the number multiplied shall be no cube number. For if the number multiplied should be a cube number, the number produced should also be a cube number (by the 4. of this booke): which is contrary to the supposition, and impossible.

¶ The 6. Theoreme.

The 6. Proposition.

If a number multiplieng himselfe produce a cube number: then is that number also a cube number.

Demonstration.

Suppose that the number A multiplieng himselfe, do produce B a cube number. Then I say that A also is a cube number. For let A multiplieng B produce C. And forasmuch as A multiplieng himselfe produced B, & multiplieng B it produced C: therefore C is a cube number. And for that A multiplieng himselfe produced B, and multiplieng B it produced C, therefore (by the 17. of the seuenth) as A is to B, so is B to C. And for that B and C are cube numbers, they are also like solide numbers. Wherefore (by the 19. of the eight) betwene C and B there are two meane proportional numbers. But as B is to C, so is A to B: wherefore (by the 8. of the eight) betwene A and B there are two meane proportional numbers. But B is a cube number. Wherefore A also is a cube number by the 23. of the eight: which was required to be demonstrated.

¶ The 7. Theoreme.

The 7. Proposition.

If a composed number multiplieng any number, produce a number: the number produced shall be a solide number.

Demonstration.

Suppose that the composed number A multiplieng the number B, do produce the number C. Then I say that C is a solide number. For forasmuch as A is a composed number, therefore some number measureth it (by the 14. definition). Let D measure it.

D ...	A	C
E ..	B	

And how often D measureth A, so many vnities let there be in E. Wherefore E multiplieng D produceth A. And forasmuch as two numbers D and E, multiplieng themselues, produce A, which A againe multiplieng B produceth C: therefore C produced of three numbers multiplieng the one the other, namely, D, E, and B is (by the 18. definition of the seuenth) a solide number. And the sides thereof are the numbers D, E, B. If therefore a composed number &c, which was required to be proued.

¶ The 8. Theoreme.

The 8. Proposition.

If from vnitie there be numbers in continuall proportion how many soeuer: the third number from vnitie is a square number, and so are all forward leauing one betwene. And the fourth number is a cube number, and so are all forward leauing two betwene. And the seuenth is both a cube number

and

and also a square number, and so are all forward leauing five betwene.



Vppose that from vnitie there be these numbers in continuall proportion A, B, C, D, E, F . Then I say that the third number from vnitie, namely, B is a square number, and so are all forward leauing one betwene, namely, D and F . And that C the fourth number is a cube number, and so are all forward leauing two betwene. And that F the seuenth number is both a cube number and also a square number, and so are all forward leauing five betwene. For for that as vnitie is to A , so is A to B . Therefore how many times vnitie measureth A , so many times A measureth B . But vnitie measureth A by those vnities which are in A , wherefore A measureth B by those vnities which are in A . And forasmuch as A measureth B by those vnities which are in A . Therefore A multiplieng himselfe produceth B . Wherefore B is a square number. And forasmuch as these numbers B, C, D , are in continuall proportion, and B is a square number, therefore by the 22. of the eight, D also is a square number. And by the same reason also F is a square number. And in like sort may we proue that leauing alwayes one betwene, all the rest forward are square numbers.

Demonstration of the first part.

F 729

E 243

D

C

B

A ...

Vnitie

Now also I say that the fourth number from vnitie, that is, C , is a cube number, and so are all forward leauing two betwene. For for that as vnitie is to the number A , so is B to C , therefore how many times vnitie measureth the number A , so many times B measureth C . But vnitie measureth A by those vnities which are in A , wherefore B measureth C by those vnities which are in A . Wherefore A multiplieng B produceth C . And forasmuch as A multiplieng himselfe produced B , and multiplieng B it produced C , therefore C is a cube number. And forasmuch as C, D, E, F , are in continuall proportion. But C is a cube number, therefore (by the 23. of the eight) F also is a cube number.

The second part demonstrated.

And it is proued, that F being the seuenth number from vnitie is also a square number. Wherefore F is both a cube number, and also a square number. In like sort may we proue, that leauing alwayes five betwene, all the rest forward, are numbers both cube and also square: which was required to be proued.

Demonstration of the third part.

¶ The 9. Theoreme.

The 9. Proposition.

If from vnitie be numbers in continuall proportion how many soeuer: and if that number which followeth next after vnitie be a square number, then all the rest following also be square numbers. And if that number which followeth next after vnitie be a cube number, then all the rest following shall be cube numbers.



Vppose that from vnitie there be these numbers in continuall proportion A, B, C, D, E, F . And let A which followeth next vnto vnitie be a square number. Then I say, that all the rest following also are square numbers. That the third number, namely, B , is a square number, & so all forward leauing one betwene, it is plaine by the Proposition next going before. I say also that all the rest are square numbers. For, forasmuch as A, B, C , are in continuall proportion, and A is a square number, therefore (by the 22. of the eight) C also is a square number. Againe forasmuch as B, C, D , are in continuall

Demonstration of the first part of this proposition.

¶ A. ij.

continuall

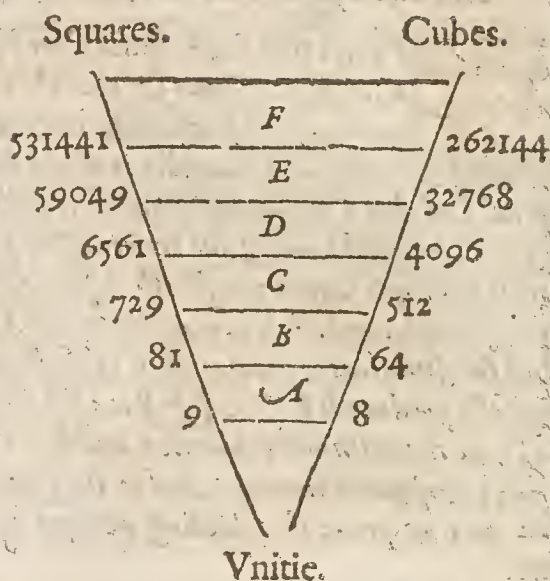
The ninth Booke

in uall proportion, and B is a square number, therefore D also (by the 22. of the eight) is a square number. In like sort may we proue, that all the rest are square numbers.

The second part demonstrated.

But now suppose that A be a cube number. Then I say, that all the rest following are cube numbers. That the fourth from vnitie, that is, C is a cube number, and so all forward leauing two betwene, it is plaine (by the Proposition going before). Now I say, that all the rest also are cube numbers. For, for that as vnitie is to A, so is A to B: therefore how many times vnitie measureth A, so many times A measureth B. But vnitie measureth A by those vnities which are in A. Wherefore A also measureth B by those vnities which are in A. Wherefore A

multiplying him selfe produceth B. But A is a cube number. But if a cube number multiplying him selfe produce any number, the number produced, is (by the 3. of the ninth) a cube number. Wherefore B is a cube number. And forasmuch as there are foure numbers in continuall proportion A, B, C, D, and A is a cube number, therefore D also (by the 23. of the eight) is a cube number. And by the same reason E also is a cube number, and in like sort are all the rest following: which was required to be proued.



The 10. Theoreme.

The 10. Proposition.

If from vnitie be numbers in continuall proportion how many soeuer, and if that number which followeth next after vnitie be not a square number, then is none of the rest following a square number, excepting the third from vnitie, and so all forward leauing one betwene. And if that number which followeth next after vnitie be not a cube number, neither is any of the rest following a cube number, excepting the fourth from vnitie, and so all forward leauing two betwene.

Demonstration of the first part leauing to an absurditie.

Suppose that from vnitie be these numbers in continuall proportion A, B, C, D, E, F. And let A which followeth next after vnitie be no square number. Then I say, that neither is any of the rest a square number, excepting the third from vnitie, & so all forward leauing one betwene, namely, B, D, F, which are square numbers (by the 8. of this booke). For if it be possible, let C be a square number. But B also is a square number. Wherefore B is unto C in that proportion that a square number is to a square number. But as B is

Vnitie .

A ...

B

C

D

E 243

F 729

to C, so is A to B. Wherefore A is vnto B in that proportion that a square number is to a square number. But B is a square number. Wherefore A also is a square number (by the 24. of the eight): which is contrary to the supposition. Wherefore C is not a square number. And by the same reason none of all the other is a square number, excepting the third from vnitie, and so all forward leauing one betwene.

But now suppose that A be not a cube number. Then I say, that none of all the rest is a cube number, excepting the fourth from vnitie, & so all forward leauing two betwene, namely, C, and F, which (by the 8. of this booke) are cube numbers. For if it be possible, let D be a cube number. But C also is a cube number (by the 8. of the ninth). For it is the fourth from vnitie. But as C is to D, so is B to C. Wherefore B is vnto C, in that proportion that a cube number is to a cube number. But C is a cube number. Wherefore B also is a cube number (by the 25. of the eight). And as vnitie is to A, so is A to B. But vnitie measureth A by those vnities which are in A. Wherefore A measureth B by those vnities which are in A. Wherefore A multiplying him selfe produceth B a cube number. But if a number multiplying him selfe produce a cube number, then is that number also a cube number (by the 6. of the ninth). Wherefore A is a cube number: which is contrary to the supposition. Wherefore D is not a cube number. In like sort may we proue, that neither is any of the rest a cube number, excepting the fourth from vnitie, and so all forward leauing two betwene: which was required to be proued.

Demonstration of the second part leauing also an absurditie.

¶ The 11. Theoreme.

The 11. Proposition.

If from vnitie be numbers in continuall proportion how many soeuer, the lesse measureth the greater by some one of them which are before in the said proportionall numbers.



Suppose that from vnitie A be these numbers in continuall proportion B, C, D, E. Then I say that of these numbers B, C, D, E: B being the lesse, measureth E the greater by one of these numbers C or D. For for that as vnitie A is vnto the number B, so is D to E, therefore how many times vnitie A measureth the number B, so many times D measureth E: wherefore alternately (by the 15. of the seuenth) how many times vnitie A measureth the number D, so many times B measureth E. But vnitie A measureth D by those vnities which are in D. Wherefore B also measureth E by those vnities which are in D. Wherefore B the lesse, measureth E the greater by some one of the numbers which went before E in the proportionall numbers. And so likewise may we proue that B measureth D by some one of the numbers B, C, D, namely, by C. And so of the rest. If therefore from vnitie &c. Which was required to be proued.

Demonstration.

¶ The 12. Theoreme.

The 12. Proposition.

If from vnitie be numbers in continuall proportion how many soeuer, how many prime numbers measure the least, so many also shal measure the number which followeth next after vnitie.

AA.iiij.

Suppose

The eighth Booke

Demonstration leading to an absurditie.

Suppose that from unitie be these numbers in continuall proportion A, B, C, D . The I say that how many prime numbers measure D , so many also do measure A . Suppose that some prime number namely, E , do measure D . The I say that E also measureth A , which is next unto unitie. For if E do not measure A , and E is a prime number, but euery number is to euery number which it measureth not a prime number (by the 31. of the seventh). Wherefore A and E are prime numbers the one to the other. And forasmuch as E measureth D , let it measure D by the number F . Wherefore E multiplieng F produceth D . Againie forasmuch as A measureth D by those unities which are in C , therefore A multiplieng C produceth D . But E also multiplieng F produced D , wherfore that which is produced of the numbers A, C is equall to that which is produced of the numbers E, F . Wherefore as A is to E , so is F to C . But A, E , are prime numbers, yea they are prime and the least. But the least numbers measure the numbers that haue one and the same proportion with them equally by the 21. of the seventh, namely, the antecedent the antecedent, and the consequent the consequent. Wherefore E measureth C . Let it measure it by G . Wherefore E multiplieng G produceth C . But A also multiplieng B produceth C . Wherefore that which is produced of the num-

Unitie.

A

B

C

D 256

E

H

G

F 128

bers A, B , is equall to that which is produced of the numbers E, G . Wherefore as A is to E , so is G to B . But A, E are prime numbers, yea they are prime and the least. But the least numbers (by the 21. of the seventh) measure the numbers that haue one and the same proportion with the equally, namely, the antecedent the antecedent, & the consequent the consequent. Wherefore E measureth B . Let it measure it by H . Wherefore E multiplieng H produceth B . But A also multiplieng himselfe produceth B , wherfore that which is produced of the numbers E, H , is equall to that which is produced of the number A . Wherefore as E is to A , so is A to H . But A, E are prime numbers, yea they are prime & the least, but the least numbers (by the 21. of the seventh) measure the numbers that haue one and the same proportion with the equally, namely, the antecedent the antecedent, and the consequent the consequent. Wherefore E measureth A and it also doth not measure it by supposition, which is impossible. Wherefore A and E are not prime the one to the other, wherfore they are composed. But all composed numbers are measured of some prime number, wherfore A and E are measured by some prime number. And forasmuch as E is supposed to be a prime number. But a prime number is not (by the definition) measured by any other number but of himselfe. Wherefore E measureth A and E , wherfore E measureth A , and it also measureth D . Wherefore E measureth these numbers A and D . And in like sort may we proue that how many prime numbers measure D , so many also shall measure A : which was required to be proued.

* An other more briefe demonstration after *Flusates*.

An other demonstration after *Flusates*.

Suppose that from unitie be numbers in continuall proportion how many so euer, namely, A, B, C, D . And let some prime number, namely, E measure the last number which is D . The I say that the same E measureth A which is the next number unto unitie. For if E doo not measure A , then are they prime, the one to the other by the 31. of the seventh. And forasmuch as A, B, C, D , are proportionall from unitie, ther-

therefore A multiplying himselfe produceth B. Wherefore B and E are prime numbers (by the 27. of the seuenth). And forasmuch as A multiplying B produceth C, therefore C is to E also a prime number by the 26. of the seuenth. And likewise infinitely A multiplying C produceth D: wherefore D and E are prime numbers the one to the other (by the same 26. of the seuenth). Wherefore E measureth not D as it was supposed, which is absurd, wherefore the prime number E measureth A, which is nexte vnto vntie: which was required to be proued.

Vntie		
A	4	8
B	16	9
C	64	27
D	256	81
E	2	3

The 13. Theoreme.

The 13. Proposition.

If from vntie be numbers in continuall proportion how many soeuer, and if that which followeth next after vntie be a prime number: then shall no other number measure the greatest number, but those onely which are before in the sayd proportionall numbers.



Suppose that from vntie be these numbers in continuall proportion A, B, C, D, and let that which followeth next after vntie, that is, A, be a prime number. Then I say, that no other number besides these numbers A, B, C, measureth the greatest number of them which is D. For if it be possible, let E measure D. And let E be none of these numbers A, B, C, D. Now it is manifest that E is not a prime number. For if E be a prime number, & do also measure D, it shall likewise measure A being a prime number and not being one and the same with A, by the former Proposition: which is impossible. Wherefore E is not a prime number. Wherefore it is a composed number. But euery composed number (by the 33. of the seuenth) is measured by some prime number.

Demonstration leading to an absurditie.

D
C
B
A ...
Vntie .

E
H
C
F

Now I say, that no other prime number besides A shall measure E. For if any other prime number do measure E, & E measureth D, therefore that number also shall measure D (by the 5. common sentence of the seuenth). Wherefore it shall also measure A (by the proposition next going before) being a prime number and not being one and the same with A: which is impossible. Wherefore onely the prime number A measureth E which measureth the greatest number D.

And forasmuch as E measureth D, let it measure it by F. Now I say, that F is none of these numbers A, B, C. For if F be one and the same with any of these numbers A, B, C, and it measureth D by E, therefore one of these numbers A, B, C, measureth D by E. But one of these numbers A, B, C, measureth D by some one of these numbers A, B, C, therefore E is one and the same with one of these numbers A, B, C: which is contrary to the supposition. Wherefore F is not one and the same with any of these numbers A, B, C.

In like sort may we proue, that onely the prime number A measureth F, prouing first that

AA. iij.

F is

The ninth Booke

F is not a prime number. For if *F* be a prime number, and it measureth *D*, therefore it also measureth *A* being a prime number, and not being one and the same with *A*, by the former Proposition: which is impossible. Wherefore *F* is not a prime number: wherefore it is a composed number, and therefore some prime number shall measure it. Now I say, that no other prime number besides *A* shall measure it. For if any other prime number do measure *F*, and *F* measureth *D*, therefore that number shall measure *D* (by the 5. common sentence of the seventh). Wherefore it shall also measure *A* (by the former Proposition) being a prime number and not being one and the same with *A*: which is impossible. Wherefore onely the prime number *A* measureth *F*. And forasmuch as *E* measureth *D* by *F*, therefore *E* multiplying *F* produceth *D*. But *A* also multiplying *C* produceth *D*, therefore that which is produced

D
E
B
A

Unitie
E
H
G
F

of *A* into *C*, is equall to that which is produced of *E* into *F*. Wherefore proportionally as *A* is to *E*, so is *F* to *C*. But *A* measureth *E*. Wherefore *F* measureth *C*. Let *F* measure *C* by *G*. And in like sort may we proue, that *G* is not one and the same with any of these numbers *A*, *B*, *C*, and that *G* is a composed number, and also that onely the prime number *A* measureth it. And forasmuch as *F* measureth *C* by *G*, therefore *G* multiplying *F* produced *C*. But *A* also multiplying *B* produced *C*. Wherefore that which is produced of *A* into *B*, is equall to that which is produced of *F* into *G*. Wherefore proportionally as *A* is to *F*, so is *G* to *B*. But *A* measureth *F*. Wherefore *G* also measureth *B*. Let *G* measure *B* by *H*. Now in like sort as before may we proue, that *H* is not one and the same with *A*, and that *H* is a composed number, and measured onely of the prime number *A*. And forasmuch as *G* measureth *B* by those unities which are in *H*, therefore *G* multiplying *H* produced *B*. But *A* multiplying him selfe produced *B*. Wherefore that which is produced of *H* into *G*, is equall to the square number which is produced of *A*. Wherefore as *H* is to *A*, so is *A* to *G*. But *A* measureth *G*. Wherefore *H* measureth *A* being a prime number and not being one and the same with it: which is absurde. Wherefore no other number besides these numbers *A*, *B*, *C*, measureth the greatest number *D*: which was required to be demonstrated.

An other demonstration of the same after Campane.

An other demonstration after Campane.

Suppose that *E* not being one and the same with the numbers *A*, *B*, *C*, *D* doo measure the number *D*. And let it measure it by the number *F*. And forasmuch as *A* being a prime number measureth the number *D*, which is produced of *E* into *F*: therefore by the 32. of the seventh, *A* measureth either *B* or *F*. Let it measure *E*. Now forasmuch as *D* is produced of *A* into *C*, and also of *E* into *F*: therefore by the second part of the 19. of the seventh, *A* is to *E*, as *F* is to *C*. But *A* measureth *E*: wherefore *F* measureth *C*. Let it measure it by *G*. Wherefore by the 32. of the seventh *A* shall measure either *F* or *G*. Let it measure *F*. Wherefore as before by the second part of the 19. of the seventh *G* shall measure *B*. Let it measure it by *H*. Now then as before it followeth by the 32. of the seventh that *A* shall measure either *G* or *H*. Suppose that it measure *G*. Wherefore by the second part of the 20. of the seventh *H* shall measure *A* (for of *A* into himselfe is produced *B*, and of *H* into *G* also is produced *B*) If therefore *H* be not equall

equall vnto A , A shall be no prime number. Which is contrary to the supposition. But if it be equall vnto A , then euery one of these numbers G, F, E , shall be some one of the numbers A, B, C, D , by the 12.

Vnitie .

A . . .

B

C

D

E _____

F _____

G _____

H _____

proposition of the ninth repeted as often as neede requireth . Wherefore E is not a number diuerse from them, but is one and the same with some one of them: which is contrary to the supposition, wherefore that is manifest which was required to be proued.

¶ The 14. Theoreme.

The 14. Proposition.

If there be geuen the least number, whom certayne prime numbers geuen, do measure: no other prime number shall measure that nūber, besides those prime numbers geuen.

Suppose that the least number whom these prime numbers B, C, D , do measure, be A . Then I say that no other prime number besides B, C, D , measureth A . For if it be possible, let E being a prime number measure A , and let E be none of these numbers

B, C, D . And forasmuch as E measureth A , let it measure it by F . Wherefore E multiplieng F produceth A . And these prime numbers B, C, D , measure A . But if two numbers multiplieng the one the other produce any number. And if some prime number measure that which is produced, it shall also measure one of those numbers which were put at

the beginning (by the 32. of the seuenth) Wherefore those numbers B, C, D , measure one of these numbers E or F . But they measure not E , for E is a prime number, and is not one and the same with any one of these numbers B, C, D . Wherefore they measure F being lesse then A which is impossible. For A is supposed to be the least whom B, C, D , measure. Wherefore no prime number besides B, C, D , measureth A : which was required to be demonstrated.

A

B ..

C ...

D

E _____

F _____

Demonstration leading to an absurditie

A proposition added by Campane.

If there be numbers how many soeuer in continuall proportion being the least in that proportion: a number measuring one of them, shall be a number not prime to one of the two least numbers in that proportion.

Suppose that there be numbers in continuall proportion how many soeuer namely A, B, C, D, E which let be the least that haue the same proportion with them: and let the two least numbers in that proportion be F and G . And let some number, as H measure some one of the numbers A, B, C, D, E , namely, C . Then I say that H is a number not prime either to F or G . Take (by the 2. of the eight) the three

A proposition added by Campane.

The ninth Booke

three least numbers in the proportion of A to B : which let be P, Q, R . And afterward fower (by the same) which let be K, L, M, N : & so forward till you come to the multitude of the numbers geuen A, B, C, D, E . Now it is manifest (by the demonstration

of the second of the eight) that F multiplyed by P, Q , and R produceth K, L, M : and that F multiplyed by K, L, M, N produceth A, B, C, D . And forasmuch as H measureth C : therefore H is either to F or to M not prime (by the corollary of the 32. of the seuenth added by Capane) If it be not prime vnto F : the is that manifest which was required to be proued. But if H be not prime vnto M . The shal it not be prime either to F or to R (by the same corollary). If agayne it be not prime vnto F , then is that proued which was required. But if it be not prime vnto R , then (by the same corollary) shall it be a number not prime vnto G (which produceth R by the 2. of the eight) but G is one of the two least numbers F or G which are in the proportion of the numbers geuen at the beginning A, B, C, D, E . If therefore there be numbers how many soeuer. &c. which was required to be proued.

¶ The 15. Theoreme.

The 15. Proposition.

If three numbers in continuall proportion be the least of all numbers that haue one and the same proportion with them: euery two of them added together shall be prime to the third.

Construction.

Suppose that there be three numbers in continuall proportion A, B, C , being the least of all numbers that haue one and the same proportion with them. Then I say, that euery two of these numbers A, B, C , added together, are prime to the third: namely, that A, B , is prime to C , and B, C , to A , and A, C , to B . Take (by the 35. of the seuenth) two of the least numbers that haue one and the same proportion with A, B, C , & let the same be the numbers $D E$, and $E F$.

Demonstration.

Now it is manifest (by the sayd 35. Proposition) that $D E$ multiplying him selfe produced A , and multiplying $E F$ produced B , and moreover $E F$ multiplying him selfe produced C .

And forasmuch as $D E$ and $E F$ are the least in that proportion, they are also prime the one to the other (by the 24. of the seuenth). But

if two numbers be prime the one to the other, then both of the added together, shall be prime to either of them (by the 30. of the seuenth).

Wherefore the whole number $D F$ is prime to

either of these numbers $D E$ & $E F$. But $D E$ also is prime vnto $E F$. Wherefore $D F$, & $D E$ are prime vnto $E F$. Wherefore that which is produced of $D F$ into $D E$, is (by the 26. of the seuenth) prime vnto $E F$. But if two numbers be prime the one to the other, that which is produced of the one of the into him selfe, is prime to the other (by the 27. of the seuenth). Wherefore that which is produced of $D F$ into $D E$, is prime to that which is produced of $E F$ into him selfe. But that which is produced of $F D$ into $D E$, is the square number which is produced of $D E$ into him selfe together with that which is produced of $D E$ into $E F$ (by the 3. of the second). Wherefore the square number which is produced of $D E$ together with that which is produced of $D E$ into $E F$, is prime to that which is produced of $E F$ into him selfe. But that which is produced of $D E$ into him selfe, is the number A , & that which is produced of $D E$ into $E F$, is the number B : and that which is produced of $E F$ into him selfe, is the number C . Wherefore the numbers A, B , added together are prime vnto C .

By the

By the like demonstration also may we proue, that the numbers B, C, are prime vnto the number A.

Now also I say, that the numbers A, C, are prime vnto the number B.

For forasmuch as D F is prime to either of these D E and E F: therefore that which is produced of D F into him selfe, is prime to that which is produced of D E into E F. But that which is produced of D F into him selfe, is equall to the square numbers which are produced of D E and E F together with that number which is produced of D E into E F, twise (by the 4. of the second). Wherefore the square numbers which are produced of D E and E F together with that which is produced of D E into E F twise are prime to that which is produced of D E into E F. And by diuision also (by the 30. of the seuenth) the square numbers produced of D E and E F, together with that which is produced of D E into E F once are prime to that which is produced of D E into E F. Againe (by the same 30. of the seuenth) the square numbers produced of D E and E F, are prime to that which is produced of D E into E F. But that which is produced of D E into him selfe is A, and that which is produced of E F into him selfe is C, and that which is produced of D E into E F, is B. Wherefore the numbers A, C, added together are prime vnto the number B: which was required to be demonstrated.

This latter part of the demonstration, which proueth that the numbers A & C are prime vnto B, is somewhat obscurely put of Theon. And therefore I will here make it playner.

Forasmuch as either of the numbers D E, and E F is prime to the whole D F: (as hath before bene proued) therefore that which is produced of D E into E F (which is the number B) is prime vnto D F, by the 26. of the seuenth. Wherefore by the 27. of the same that which is produced of D F into him selfe (which is the number composed of A and C and of the double of B by the 4. of the second) shall be prime vnto B. Wherefore it followeth that the number composed of A and C is prime vnto B. For if a number composed of two numbers, be prime to one of the said two numbers, as here the number composed of A and C taken as one number and of the double of B, is prime vnto the double of B: then the two numbers whereof the number is composed, namely, the number composed of A and C, and the double of B shall be prime the one to the other (by the 30. of the seuenth). And therefore the number composed of A and C shall be prime to B taken once. For if any number should measure the two numbers, namely the number composed of A and C, and the number B, it should also measure the number composed of A and C, and the double of B (by the 5. common sentence of the seuenth): which is not possible, for that they are proued to be prime numbers.

Here haue I added an other demonstration of the former Proposition after Campanes, which proueth that in numbers how many soeuer, which is there proued onely touching three numbers: and the demonstration seemeth somewhat more perspicuous then Theons demonstration. And thus he putteth the proposition.

If numbers how many soeuer being in continuall proportion be the least that haue one & the same proportion with them: every one of them shalbe to the number composed of the rest prime.

Suppose that there be numbers in continuall proportion how many soeuer, and the least in their proportion: namely, A, B, C, D. Then I say that every one of them, as for example first D, is prime to the number composed of the rest, namely, of A, B, C. For if it be not, let some number, namely E measure D, and the number composed of A, B, C. Take the two least numbers in the same proportion that A, B, C, D are (by the 35. of the seuenth) which let be F, G. and forasmuch as E measureth one of these number A, B, C, D, the same E shalbe a number not prime either to F or to G (by the proposition before added by Campane after the 14. proposition) wherefore some number shall measure E and one of these numbers F or G: which let be H. and forasmuch as H measureth E, it shall also measure D, which number D the number E also measureth (by the 5. common sentence of the seuenth). Moreouer forasmuch as H (by supposition) measureth one of these num-

Demonstration to proue that the numbers A and C are prime to B.

Demonstration leading to an absurditie.

		A 8
	L	
F ..		B 12
	I	
G ...		C 18
	K	
		D 24
E ———	ABC	38
H ———	ABD	47

bers.

The ninth Booke

bers F or G, the same H shall measure all the meanes betwene A and D by the same cōmon sentēce. For either of these numbers F or G produceth all the meanes by the next numbers in continuall proportion and in the same proportion with them (as by L, I, K) by the second of the eight. Agayne forasmuch as H measureth E, which (by supposition) measureth the whole A, B, C: the same H shall also measure the whole A, B, C (by the forelayd common sentence) and it measureth the part taken away, namely, the meanes B, C (as it hath bene proued) wherefore it also measureth the residue A (by the 4. common sentence of the seuenth) wherefore H measureth the extreames D and A, which are prime the one to the other (by the 3. of the eight) which were absurd. Wherefore D is a number prime to the number composed of the rest, namely, of A, B, C.

Secondly I say that this is so in euery one of them: namely that C is a prime number to the number composed of A, B, D. For if not, then as before let E measure C, and the number composed of A, B, D: which E shalbe a number not prime either to F or to G (by the former proposition added by Campane) wherefore let H measure them. And forasmuch as H measureth E, it shall also measure the whole A, B, C, D whom E measureth. And forasmuch as H measureth one of these numbers F or G, it shall measure one of the extreames A or D: which are produced of F or G (by the second of the eight) if they be multipl'ed into the meanes L or K. And moreouer the same H shall measure the meanes, B, C (by the 5. common sentence of the seuenth) when as by supposition it measureth either F or G, which measure B, C (by the second of the eight). But the same H measureth the whole A, B, C, D as we haue proued, for that it measureth E. Wherefore it shall also measure the residue, namely, the number composed of the extreames A and D (by the 4. common sentence of the seuenth). And it measureth one of these A or D (for it measureth one of these F or G which produce A and D) wherefore the same H shall measure one of these A or D and also the other of them (by the former common sentence) which numbers A and D are by the 3. of the eight prime the one to the other. Which were absurd. This may also be proued in euery one of these numbers A, B, C, D. Wherefore no number shall measure one of these numbers A, B, C, D and the number composed of the rest. Wherefore they are prime the one to the other: If therefore numbers how many soeuer, &c. which was required to be proued.

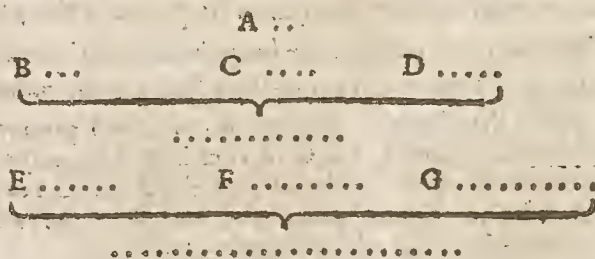
Here as I promised, I haue added *Campanes* demonstrations of those Propositions in numbers, which *Euclide* in the second booke demonstrated in lines. And that in thys place so much the rather, for that *Theon* as we see in the demonstration of the 15. Proposition seemeth to alledge the 3. & 4. Proposition of the second booke: which although they concerne lines onely, yet as we there declared and proued, are they true also in numbers.

¶ The first Proposition added by Campane.

That number which is produced of the multiplication of one number into numbers how many soeuer: is equall to that number which is produced of the multiplication of the same number into the number composed of them.

Demonstration.

This proueth that in numbers which the first of the second proued touching lines. Suppose that the number A being multiplyed into the number B, and into the number C, and into the number D, deo produce the numbers E, F and G. Then I say that the number produced of A multiplyed into the number composed of B, C, and D is equall to the number composed of E, F, and G. For by the converse of the definition of a number multiplyed, what part vntie is of A, the selfe same part is B of E, and C of F, and also D of G. Wherefore by the 5. of the seuenth what part vntie is of A, the selfe same part is the number composed of B, C, and D, of the number composed of E, F, and G. Wherefore by the definition that which is produced of A into the number composed of B, C, D, is equall to the number composed of E, F, G: which was required to be proued.



✱ The second Proposition.

That number which is produced of the multiplication of numbers how many soeuer into one number: is equall to that number which is produced of the multiplication of the number composed of them into the same number.

Thys

This is the conuerse of the former, As if the numbers B and C and D multiplied into the number A doo produce the numbers E and F and G. Then the number composed of B, C, D, multiplied into the number A shall produce the number composed of the numbers E, F, G. Which thing is easily proued by the 16. of the seventh and by the former proposition.

This proposition is the conuerse of the former.

¶ The third Proposition.

That number which is produced of the multiplication of numbers how many soeuer into other numbers how many soeuer, is equall to that number which is produced of the multiplication of the number composed of those first numbers, into the number composed of these latter numbers.

As if the numbers A, B, C doo multiply the numbers D, E, F, each one eche other, and if the numbers produced be added together. Then I say that the number composed of the numbers produced is equall to the number produced of the number composed of the numbers A, B, C into the number composed of the numbers D, E, F. For by the former proposition that which is produced of the number composed of A, B, C into D is equall to that which is produced of euery one of the sayd numbers into D: and by the same reason that which is produced of the number composed of A, B, C into E, is equal to that which is produced of euery one of the sayd numbers into E: and so likewise that which is produced of the number composed of A, B, C into F is equall to that which is produced of euery one of the sayd numbers into F. But by the first of these propositions that which is produced of the number composed of these numbers A, B, C into euery one of these numbers D, E, F is equall to that which is produced of the number composed into the number composed: wherefore that is manifest which was required to be proued.

Demonstration.

¶ The fourth Proposition.

If a number be deuided into partes how many soeuer: that number which is produced of the whole into him selfe, is equall to that number which is produced of the same number into all his partes.

This proueth in numbers that which the second of the second proued in lines. As if the number A be deuided into the numbers B and C, and D. Then I say, that that which is produced of A into him selfe, is equall to that which is produced of A into all the sayd numbers B, C, and D. For putting the number E equall to the number A, it is manifest by the first of these propositions that that which is produced of E into A, is equall to that which is produced of E into all the partes of A. But by the common sentence that which is produced of E into A is equal to that which is produced of A into him selfe: and that which is produced of E into the partes of A is equall to that which is produced of A into the selfe same partes. Wherefore that is manifest which was required to be proued.

This answereth to the 2. of the second.

Demonstration.

¶ The fifth Proposition.

If a number be deuided into two partes: that which is produced of the whole into one of the partes, is equall to that which is produced of the selfe same part into him selfe, and into the other part.

This proueth in numbers that which in the 3. of the second was proued in lines. For let the number A be deuided into the numbers B and C. Then I say that that which is produced of A into C, is equall to that which is produced of C into him selfe and into B. For by the 16. of the seventh, that which is produced of A into C is equall to that which is produced of C into A. Now then put the number D equall to the number C. Wherefore that which is produced of A into C is equall to that which is produced of D into A. But by the first of these propositions that which is produced of D into A is equall to that which is produced of D into B and of D into C. Wherefore forasmuch as that which is produced of D into A and into B and into C is equall to that which is produced of C into A, and into B, and into him selfe, by reason of the equalitie of C and D: that is manifest which was required to be proued.

This answereth to the 3. of the thirde.

Demonstration.

The ninth Booke

¶ The sixt Proposition.

If a number be deuided into two partes: that which is produced of the multiplication of the whole into him selfe, is equall to that which is produced of the multiplication of either of the partes into him selfe, and of the one into the other twise.

This answereth to the 4. of the second.

Demonstration.

This proueth in numbers that which the fourth of the second proued touching lines. As if the number A be deuided into the numbers B and C. Then I say that that which is produced of A into himselfe is equal to that which is produced of B into himselfe, and of C into himselfe, and of B into C twise. For by the 4. of these propositions, that which is produced of A into himselfe, is equall to that which is produced of A into B, and into C. But that which is produced of A into B, is equall to that which is produced of B into himselfe and into C (by the former proposition). And by the same that which is produced of A into C is equall to that which is produced of C into himselfe and into B. And forasmuch as that which is produced of C into B is equall to that which is produced of B into C by the 16. of the seuenth, it is manifest that that is true which was required to be proued.

¶ The seuenth Proposition.

If a number be deuided into two equall partes, & into two vnequall partes: that which is produced of the greater of the vnequall partes into the lesse, together with the square nūber of the number set betwene, is equall to the square number produced of the halfe of the whole.

This answereth to the 5. of the second.

Demonstration.

This proueth in numbers that which the 5. of the second proued in lines. As if the number A B be deuided into two equall numbers, which let be A C, and C B: and also in two vnequal numbers namely, A D and D B, of which let A D be the greater, and D B the lesse. Then I say that that which is produced of the whole A D into D B together with the square number of C D, is equall to the square number of C B. For by the former proposition the square of C B is equall to the square of C D and to the square of D B, and to that which is produced of B D into C D twise. But that which is produced of B D into himselfe and into C D is equall to that which is produced of B D into A C by the first of these propositions, and therefore vnto that which is produced of B D into A C. Wherefore that which is produced of B D into himselfe and into C D twise is equall to that which is produced of B D into A D. Wherefore by the same the square of C B exceedeth that which is produced of B D into A D by the square of C D: wherefore that is manifest which was required to be proued.

¶ The 8. Proposition.

If a number be deuided into two equall partes, and if vnto it be added an other number: that which is produced of the multiplication of the whole being composed into the number added, together with the square of the halfe, is equall to the square of the number composed of the halfe and the number added.

This answereth to the 6. of the second.

Demonstration.

This proueth in numbers that which the 6. of the second proued touching lines. For suppose that the number A B be deuided into equall numbers, which let be A C and C B: and vnto it adde the number B D. Then I say, that that which is produced of the whole A D into D A together with the square of B C, is equall to the square of C D. For by the 6. of these propositions the square of C D is equal to the square of D B, & to the square of B C, and to that which is produced of D B into B C twise. But by the 1. of these propositions, that which is produced of B D into himselfe and into B C twise is equall to that which is produced of B D into D A (for A C and C B are equall) wherefore the square of C D exceedeth that which is produced of B D into D A by the square of C B. Wherefore that is manifest which was required to be proued.

¶ The 9. Proposition.

If a number be deuided into two partes: that which is produced of the whole number into him selfe together with that which is produced of one of the partes into him selfe, is equall to that which is produced of the whole into the sayd part twise together with that which is produced of the other part into him selfe.

This

This proueth in numbers that which the 7. of the second proued in lines. For let the number A be deuided into the numbers B and D . Then I say that the square of A together with the square of D is equall to that which is produced of A into D twise together with the square of B . For it is manifest by the 6. of these propositions that the square of A is equall to the squares of B and D together with that which is produced of B into D twise. Wherefore the square of A together with the square of D , is equall to two squares of D , and to that which is produced of D into B twise together with the square of B . But by the first of these propositions two squares of D , and that which is produced of D into B twise is equall to that which is produced of D into A twise. Wherefore that which is produced of D into A twise together with the square of B , is equall to the square of A together with the square of D : wherefore that is manifest which was required to be proued.

This answereth to the 7. of the second.

Demonstration.

¶ The 10. proposition.

If a number be deuided into two partes, and vnto it be added a number equall to one of the parts: the square of the whole number composed, is equall to the quadruple of that which is produced of the first number into the number added, together with the square of the other part.

This proueth in numbers, that which the 8. of the second proued in lines. Suppose that the number AB be deuided into the numbers AC and CB , vnto which adde the number BD , which let be equall to the number CB . Then I say that the square of the whole number composed, namely, AD , is equall to that which is produced of AB into BD fower times together with the square of AC . For by the 6. of these propositions the square of AD , is equall to the square AB and to the square of BD together with that which is produced of AB into BD twise. And forasmuch as the square of BD is equal to the square CB : therefore the square of AD shall be equall to the square of AB and to the square of CB together with that which is produced of AB into BD twise. But by the former proposition the square of AB together with the square of CB , is equall to the square of AC together with that which is produced of AB into BC twise wherefore the square of AD is equal to that which is produced of AB into BD twise, and to that which is produced of AB into BC twise together with the square of AC . And for that that which is produced of AB into BC is equall to that which is produced of AB into BD , therefore is that manifest which was required to be proued.

This answereth to the 8. of the second.

Demonstration.

✱ The 11. proposition.

If a number be deuided into two equall partes, and into two vnequall partes: the squares of the two vnequall partes taken together, are double to the square of the halfe, and to the square of the excese of the greater part aboue the lesse, the sayd two squares being added together.

This proueth in numbers that which the 9. of the second proued in lines. For suppose that the number AB be deuided into two equall partes: which let be AC and CB , and into two vnequall partes, which let be AD and DB . Then I say that the squares of the two numbers AD & DB , taken together, are double to the two squares of the two numbers AC and CD , taken together. For by the 6. of these propositions the square of AD is equall to the squares of AC and CD , and to that which is produced of AC into CD twise. And forasmuch as the square of AC is equal to the square of CB , the square of AD shall be equall to the square of BC & to the square of CD together with that which is produced of BC into CD twise. Wherefore the square of AD together with the square of DB , is equall to the square of BC , and to the square of CD and to that which is produced of BC into CD twise together with the square of BD . But that which is produced of BC into CD twise together with the square of BD , is equall to the square of BC and to the square of CD by the 9. of these propositions. Wherefore the squares of the two numbers AD and DB are equall to the squares of the two numbers BC and CD , taken twise. And therefore the squares of the two numbers AD and DB are double to the squares of the two numbers BC and CD ; that is AC and CD (for the numbers AC and BC are by supposition equall) wherefore that is manifest which was required to be proued.

This answereth to the 9. of the second.

Demonstration.

✱ The 12. proposition.

If a number be deuided into two equall partes, and vnto it be added an other number: the square of the whole number composed together with the square of the number added, is double to the square of the halfe, together with the square of the number composed of the halfe and the number added.

The ninth Booke

This answereth to the 10. of the second.

Demonstration.

This proueth in numbers that which the 10. of the second proued in lines. Suppose that the number AB be deuided into two equall numbers AC and CB : and vnto it adde the number BD . Then I say that the square of AD together with the square of BD , is double to the square of AC together with the square of CD . For forasmuch as the number CD is deuided into two partes, and vnto it is added the number AC which is equall to one of the partes (namely, to CB) therefore by the 10. of these propositions, the square of AD is equall to that which is produced of CD into CA foure times together with the square of BD . And forasmuch as AC is equall to CB , therefore the square of AD , is equall to that which is produced of DC into CB foure times together with the square of BD . Wherefore the square of AD together with the square of BD , is equall to that which is produced of DC into CB foure times together with two squares of BD . But by the 9. of these propositions, that which is produced of DC into CB foure times together with two squares of BD is double to the square of CD together with the square of CB (for the square of CD together with the square of CB is equal to that which is produced of DC into CB twice together with one square of CB). Wherefore forasmuch as the square of CB is equall to the square of AC , that is manifest: which was required to be proued.

✱ The 13. proposition.

It is impossible to deuide a number in such sorte: that that which is contayned vnder the whole and one of the partes, shall be equall to the square of the other part.

A negative proposition.

Demonstration leading to an impossibility.

That which the 11. of the second taught to be doone in lines is here proued to be impossible to be doone in numbers. For suppose that there be a number whatsoever namely AB . Then I say, that it is impossible to deuide it in such sort as is required in the proposition. For so should it be deuided according to a proportion hauing a meane and two extremes. But if it be possible, let the number AB be so deuided in C . And as AB is to BC , so let BC be to CA . Wherefore AC shall be lesse then CB . Now then take away from CB a number equall to AC which let be CD . And forasmuch as the proportion of the whole AB to the whole BC , is as the proportion of the part taken away from AB , namely, BC to the part taken away from BC , namely, CD : therefore the proportion of the residue of AB , namely, AC , to the residue of BC , namely, to BD , is as the proportion of the whole AB to the whole BC (by the 11. of the seuenth). Wherefore BC is to CD , as CD is to DB . Wherefore CD is greater then DB . Wherefore subtrahing DE out of CD , so that let DE be equall to DB : the proportion of BC to CD is as the proportion of CD to DE . Wherefore the residue of CB , namely, DB , shall be to the residue of CD , namely, to CE , as the proportion of the whole BC to the whole CD . Wherefore CE may be subtrahed out of ED : wherefore there shall be no end of this subtraction: which is impossible.

¶ The 16. Theoreme. The 16. Proposition.

If two numbers be prime the one to the other, the second shall not be to any other number, as the first is to the second.

Demonstration leading to an absurditie.



Suppose that these two numbers A and B be prime the one to the other. Then I say that B is not to any other number as A is to B . For if it be possible, as A is to B , so let B be to C . Now A and B are prime numbers, yea they are prime and the least by the 23. of the seuenth. But (by the 21. of the seuenth) the least measure the numbers that haue one and the same proportion with them equally, the antecedent the antecedent, and the consequent the consequent. Wherefore the antecedent A , measureth the antecedent B , and it measureth also it selfe. Wherefore A measureth these numbers A & B being prime the one to the other, which is impossible. Wherefore as A is to B , so is not B to C : which was required to be proued.

¶ The 17. Theoreme. The 17. Proposition.

If there be numbers in continuall proportion how many soeuer, and if theyr extremes

extremes be prime the one to the other, the lesse shall not be to any other number, as the first is to the second.



Suppose that there be these numbers in continuall proportion A, B, C, D , and let their extremes A and D be prime the one to the other. Then I say that D is not to any other number as A is to B . For if it be possible, as A is to B , so let D be to E . Wherefore alternately by the 13. of the seventh, as A is to D , so is B to E . But

A and D are prime, yea they are prime and the least. But the least numbers (by the 21. of the seventh) measure the numbers that haue one and the same proportiō with them equally, the antecedent the antecedent, and the consequent. Wherefore the antecedent A measureth the antecedent B : but as A is to B , so is B

A
 B
 C
 D
 E ———

Demonstration leading to an absurditie.

to C . Wherefore B also measureth C . Wherefore A also measureth C (by the 5. common sentence of the seventh) and forasmuch as B is to C , so is C to D , but B measureth C . Wherefore C measureth D . But A measureth C . Wherefore A also measureth D by the same common sentence, and it also measureth it selfe. Wherefore A measureth these numbers A and D being prime the one to the other, which is impossible. Wherefore D is not to any other number as A is to B : which was required to be proued.

¶ The 18. Theoreme.

The 18. Proposition.

Two numbers being geuen, to searche out if it be possible a third number in proportion with them.



Suppose that the two numbers geuen be A and B . It is required to searche out if it be possible a third number proportionall with them. Now A, B are either prime the one to the other or not prime. If they be prime, then (by the 16. of the ninth) it is manifest that it is impossible to finde out a third number proportional with them. But now suppose that A, B be not prime the one to the other. And let B multiplieng himselfe produce C . Now A either measureth C , or measureth it not. First, let it measure it and that by D . Wherefore A multiplieng D produceth

C . But B also multiplieng himselfe produced C . Wherefore that which is produced of A into D , is equall to that which is produced of B into himselfe. Wherefore (by the second part of the 19. of the seventh) as A is to B , so is B to D . Wherefore vnto these numbers A, B , is found out a third number in proportion, namely, D .

A
 B
 ———
 A
 B
 D
 C
 A
 B
 D
 C

Three cases in this proposition. The first case.

The second case.

But now suppose that A do not measure C . Then I say that it is impossible to finde out a third number in proportion with these numbers A, B . For if it be possible, let there be found out such a number, and let the same be D .

A
 B
 D
 C

The third case.

The ninth Booke

Wherefore that which is produced of A into D , is equal to that which is produced of B into himselfe, but that which is produced of B into himselfe is C . Wherefore that which is produced of A into D is equal unto C . Wherefore A multiplieng D produced C . Wherefore A measureth C by D . But it is supposed also not to measure it, which is impossible. Wherefore it is not possible to finde out a third number in proportion with A & B , whensoever A measureth not C : which was required to be proued.

¶ The 19. Theoreme.

The 19. Proposition.

Three numbers beyng geuen, to search out if it be possible the fourth number proportionall with them.

Diners cases
in this proposi-
tion.



Suppose that the three numbers geuen be A, B, C . It is required to search out if it be possible a fourth number proportionall with them. Now A, B, C , are either in continuall proportion, and their extremes A, C are prime the one to the other: or they are not in continuall proportion, and their extremes are yet prime the one to the other: or they are in continuall proportion, and their extremes are not prime the one to the other: or they are neither in continuall proportion, nor their extremes are prime the one to the other.

The first case.

If A, B, C , be in continuall proportion, and their extremes be prime the one to the other, it is manifest (by the 17. of the ninth) that it is impossible to finde out a fourth number proportionall with them.

The second
case.

But now suppose that A, B, C , be not in continuall proportion, and yet let their extremes be prime the one to the other. Then I say that so also it is impossible to finde out a fourth number proportional with the. For if it be possible, let there be found such a number, and let the same be D . So that as A is to B , so let C be to D , and as B is to C , so let D be to E . And for that as A is to B , so is C to D , and as B is to C , so is D to E , therefore of equallitie (by the 14. of the seventh) as A is to C , so is C to E . But A and C are prime the one to the other, yea they are prime, and the least: but the lest measure the numbers that haue one & the same proportion with them equally, the antecedent, the antecedent, and the consequent the consequent (by the 21. of the seventh). Wherefore A measureth C , namely, the antecedent the antecedent, and it also measureth it selfe. Wherefore A measureth these numbers A and C being prime the one to the other, which is impossible. Wherefore it is not possible to finde out a fourth number proportionall with these numbers A, B, C .

The third
case.

But now againe suppose that A, B, C , be in continuall proportion, and let A and C not be prime the one to the other. Then I say that it is possible to finde out a fourth number proportionall with them. For let B multiplieng C produce D . Now A either measureth D , or measureth it not. First let it measure it, and that by E . Wherefore A multiplieng E produced D . But B also multiplieng C produced D . Wherefore that which is produced of AE is equal to that which is produced of BC : wherefore in what proportio A is to B , in γ same is C to E . Wherefore there is found out a fourth number, namely, E , proportionall with these numbers A, B, C .

But

But now suppose that A do not measure D . Then I say that it is not possible to finde out a fourth number proportionall with these numbers A, B, C . For if it be possible, let there be found such a number, and let the same be E . Wherefore that which is produced of A into E is equall to that which is produced of B into C . But that which is produced of B into C is D . Wherefore that which is produced of A into E is equall unto D . Wherefore A multiplieng E produced D , wherefore A measureth D , but it also measureth it not, which is impossible. Wherefore it is impossible to finde out a fourth number proportionall, with these numbers A, B, C , whensoever A measureth not D .

But now suppose that A, B, C be neither in continuall proportiō, neither also their extremes be prime the one to the other. And let B mul

A
 B
 C
 E
 D 1350

The fourth case.

tiplieng C produce D . And in like sorte may we proue that if A do measure D , it is possible to finde out a fourth number proportionall with them. But if it do not measure D , the is it impossible: which was required to be proued.

¶ The 20. Theoreme.

The 20. Proposition.

Prime numbers being geuen how many soeuer, there may be geuen more prime numbers.



Suppose that the prime numbers geuen be A, B, C . Then I say, that there are yet more prime numbers besides A, B, C . Take (by the 38. of the seuenth) the left number whom these numbers A, B, C do measure, and let the same be $D E$. And unto $D E$ adde unitie $D F$. Now $E F$ is either a prime number or not.

Two cases in this Proposition.

First let it be a prime number, then are there found these prime numbers A, B, C , and $E F$ more in multitude then the prime numbers first geuen A, B, C .

The first case.

But now suppose that $E F$ be not prime. Wherefore some prime number measureth it (by the 24. of the seuenth): Let a prime number measure it, namely, G .

The second case.

Then I say, that G is none of these numbers A, B, C . For if G be one and the same with any of these A, B, C . But A, B, C measure the nūber $D E$: wherefore G also measureth $D E$: and it also measureth the whole $E F$. Wherefore G being a number shall measure the residue $D F$ being unitie: which is impossible. Wherefore G is not one and the same with any of these prime numbers A, B, C : and it is also supposed to be a prime number. Wherefore there are found these prime numbers A, B, C, G , being more in multitude then the prime numbers geuen A, B, C : which was required to be demonstrated.

¶ A Corollary.

By thys Proposition it is manifest, that the multitude of prime numbers is infinite.

¶ The 21. Theoreme.

The 21. Proposition.

If euen nūbers how many soeuer be added together: the whole shall be eue.

BB.iiij.

Suppose

The ninth Booke

Demonstration.



*S*uppose that these even numbers AB, BC, CD , and DE , be added together. Then I say, that the whole number, namely, AE , is an even number. For forasmuch as every one of these numbers AB, BC, CD , and DE , is an even number, therefore every one of them hath an halfe. Wherefore the whole AE also hath an halfe. But an even number (by the definition) is that which may be devided into two equall partes. Wherefore AE is an even number: which was required to be proued.

$A \dots B \dots C \dots D \dots E$

¶ The 22. Theoreme. The 22. Proposition.

If odde numbers how many soeuer be added together, & if their multitude be even, the whole also shall be even.

Demonstration.



*S*uppose that these odde numbers AB, BC, CD , and DE , being even in multitude, be added together. Then I say, that the whole AE is an even number. For forasmuch as every one of these numbers AB, BC, CD , and DE , is an odde number, if ye take away vnitie from euery one of them, that which remaineth of every one of the is an even number. Wherefore they all added together, are (by the 21. of the ninth) an even number: and the multitude of the vnities taken away is even. Wherefore the whole AE is an even number: which was required to be proued.

$A \dots B \dots C \dots D \dots E$

¶ The 23. Theoreme. The 23. Proposition.

If odde numbers how many soeuer be added together, and if the multitude of them be odde, the whole also shall be odde.

Demonstration.



*S*uppose that these odde numbers, AB, BC , and CD being odde in multitude be added together. Then I say that the whole AD is an odde number. Take away from C vnitie DE , wherefore that which remaineth CE is an even number. But AC also (by the 22. of the ninth) is an even number. Wherefore the whole AE is an even number. But DE which is vnitie being added to the even number AE , maketh the whole AD an odde number: which was required to be proued.

$A \dots B \dots C \dots E \dots D$

¶ The 24. Theoreme. The 24. Proposition.

If from an even number be take away an even number, that which remaineth shall be an even number.

Demonstration.



*S*uppose that AB be an even number, and from it take away an even number CB . Then I say that that which remaineth, namely, AC is an even number. For forasmuch as AB is an even

$A \dots C \dots B$

even

even number, it hath an halfe, and by the same reason also B C hath an halfe. Wherefore the residue C A hath an halfe. Wherefore A C is an even number : which was required to be demonstrated.

¶ The 25. Theoreme.

The 25. Proposition.

If from an even number be taken away an odde number, that which remaineth shall be an odde number.

Suppose that A B be an even number, and take away from it B C an odde number.

Then I say that the residue C A is an odde number. Take away from B C unitie C D. Wherefore D B is an even number. And A B also is an even

number, wherefore the residue A D is an even number (by the former proposition) But C D which is unitie, being taken away from the even number A D maketh the residue A C an odde number : which was required to be proved.

A C . D B

Demonstration.

¶ The 26. Theoreme.

The 26. Proposition.

If from an odde number be taken away an odde number, that which remaineth shall be an even number.

Suppose that A B be an odde number, and from it take away an odde number B C. Then I say that the residue C A is an even number. For forasmuch as

A B is an odde number, take away from it unitie B D. Wherefore the residue A D is even. And by the same reason C D is an even number : wherefore the residue C A is an even number (by the 24. of this booke) : which was required to be proved.

A C D . B

Demonstration.

¶ The 27. Theoreme.

The 27. Proposition.

If from an odde number be taken away an even number, the residue shall be an odde number.

Suppose that A B be an odde number, and from it take away an even number B C. Then I say that the residue C A is an odde number. Take away from A B unitie A D. Wherefore the residue D B is an even number, &

B C is (by supposition) even. Wherefore the residue C D is an even number. Wherefore D A which is unitie, being added unto C D which is an even number maketh the whole A C an odde number : which was required to be proved.

A . D C B

Demonstration.

¶ The 28. Theoreme.

The 28. Proposition.

If an odde number multiplieng an even number produce any number, the number produced shall be an even number.

Suppose

The ninth Booke

Demonstration.



Suppose that A being an odde number multiplieng B being an euen number, do produce the number C. Then I say that C is an euen number. For forasmuch as A multiplieng B produced C, therefore C is composed of so

many numbers equall vnto B as there be in vnities in A. But B is an euen nuber: wherfore C is composed of so many euen numbers, as there are vnities in A. But if eue numbers how many soeuer be added together, the whole (by the 21. of the ninth) is an euen number: wherfore C is an euen number: which was required to be demonstrated.

¶ The 29. Theoreme.

The 29. Proposition.

If an odde number multiplying an odde number produce any number, the number produced shalbe an odde number

Demonstration.



Suppose that A being an odde number multiplying B being also an odde number, doo produce the number C. Then I say that C is an odde number. For forasmuch as A multiplying B produced C, therefore C is composed of so many numbers equall vnto B as there be vnities in A. But either of these numbers A and B is an odde number. Wherfore C is composed of odde numbers, whose multitude also is odde. Wherfore (by the 23. of the ninth) C is an odde nuber: which was required to be demonstrated.

C
B ...
A

✱ A proposition added by Campane.

A proposition added by Campane.

If an odde number measure an euen number, it shall measure it by an euen number.

For if it should measure it by an odde number, then of an odde number multiplyed into an odde number should be produced an odde number, which by the former proposition is impossible.

An other proposition added by him.

An other added by him.

If an odde number measure an odde number, it shall measure it by an odde number.

For if it should measure it by an euen number, then of an odde number multiplyed into an euen number should be produced an odde number which by the 28. of this booke is impossible.

¶ The 30. Theoreme.

The 30. Proposition.

If an odde number measure an euen number, it shall also measure the halfe thereof.

Demonstration leading to an absurdisse.



Suppose that A being an odde number doo measure B being an euen number. Then I say that it shall measure the halfe thereof. For forasmuch as A measureth B let it measure it by C. The I say that C is an euen number. For if not then, if it be possible let it be odde: And forasmuch as A measureth B by C: therefore A multiplying C produceth B. Wherfore B is composed of odde numbers whose multitude also is odde. Wherfore B is an odde number (by the 29. of this booke) which is absurd, for it is supposed to be euen: wherfore C is an euen number. Wherfore A measureth B by an euen number: and C measureth B by A. But either

A ...
C
B

of

of these numbers C and B hath an halfe part: wherefore as C is to B, so is the halfe to the halfe. But C measureth B by A. Wherefore the halfe of C measureth the halfe of B by A: wherefore A multiplying the halfe of C produceth the halfe of B. Wherefore A measureth the halfe of B: and it measureth it by the halfe of C. Wherefore A measureth the halfe of the number B: which was required to be demonstrated.

¶ The 31. Theoreme.

The 31. Proposition.

If an odde number be prime to any number, it shal also be prime to the double thereof.

Suppose that A being an odde number be prime unto the number B: and let the double of B be C. Then I say, that A is prime unto C. For if A and C be not prime the one to the other, some one number measureth them both. Let there be such a number which measureth them both, and let the same be D.

Demonstration.

But A is an odde number. Wherefore D also is an odde number. (For if D which measureth A should be an even number, then should A also be an even number (by the 21. of this booke): which is contrary to the supposition. For A is supposed to be an odde number: & therefore D also is an odde number.) And forasmuch as D being an odde number measureth C, but C is an even number (for that it hath an halfe, namely, B). Wherefore (by the Proposition next going before) D measureth the halfe of C. But the halfe of C is B. Wherefore D measureth B: and it also measureth A. Wherefore D measureth A and B being prime the one to the other: which is absurde. Wherefore no number measureth the numbers A & C. Wherefore A is a prime number unto C. Wherefore these numbers A and C are prime the one to the other: which was required to be proved.

¶ The 32. Theoreme.

The 32. Proposition.

Every number produced by the doubling of two upward, is evenly even onely.

Suppose that A be the number two: and from A upward double numbers how many soever, as B, C, D. Then I say, that B, C, D, are numbers evenly even onely. That every one of them is evenly even, it is manifest: for every one of them is produced by the doubling of two. I say also, that every one of them is evenly even onely. Take unitie E. And forasmuch as from unitie are certaine numbers in continuall proportion, & A which followeth next after unitie is a prime number, therefore (by the 13. of the third) no number measureth D being the greatest number of these numbers A, B, C, D, besides the selfe same numbers in proportion. But every one of these numbers A, B, C, is evenly even. Wherefore D is evenly even onely. In like sort may we proue, that every one of these numbers A, B, C, is evenly even onely: which was required to be proved.

Demonstration.

¶ The 33. Theoreme.

The 33. Proposition.

A number whose halfe part is odde, is evenly odde onely.

Suppose

The ninth Booke

Demonstration leading to an absurditie.

Suppose that *A* be a number whose halfe part is odde. Then I say that *A* is euenly odde onely. That it is euenly odde it is manifest: for his halfe being odde measureth him by an eue number, namely, by 2. (by the definition) I say also that it is euenly odde onely. For if *A* be euenly euen, his halfe also is euen. For (by the definition) an euen number measureth him by an euen number. Wherefore that euen number which measureth him by an euen number shall also measure the halfe thereof being an odde number by the 4. common sentence of the seuenth which is absurd. Wherefore *A* is a number euenly odde onely: which was required to be proued.

An other demonstration to proue the same.

An other demonstration.

Suppose that the number *A* haue to his halfe an od nuber, namely, *B*. The I say that *A* is euenly od onely. That it is euenly odde needeth no profe: forasmuch as the number 2. an euen number measureth it by the halfe thereof which is an odde number. Let *C* be the number 2. by which *B* measureth *A*

$$\begin{array}{ccccccc} & & B & \dots\dots\dots & C & & \\ A & \dots\dots\dots & & & & & \\ & & D & \dots\dots\dots & F & & \end{array}$$

(for that *A* is supposed to be double vnto *B*). And let an euen number, namely, *D* measure *A* (which is possible for that *A* is an euen number by the definition) by *F*. And forasmuch as that which is produced of *C* into *B* is equall to that which is produced of *D* into *F*, therefore by the 19. of the seuenth, as *C* is to *D*, so is *B* to *F*. But *C* the number two measureth *D* being an euen number: wherefore *F* also measureth *B* which is the halfe of *A*. Wherefore *F* is an odde number. For if *F* were an euen number then should it make *B* whome it measureth an odde number also by the 21. of this booke, which is contrary to the supposition. And in like maner may we proue that all the eue nbers which measure the number *A*, do measure it by odde numbers. Wherefore *A* is a number euenly odde onely: which was required to be proued.

¶ The 34. Theoreme.

The 34. Proposition.

If a number be neither doubled from two, nor hath to his halfe part an odde number, it shall be a number both euenly euen, and euenly odde.

Demonstration.

Suppose that the nuber *A* be a nuber neither doubled fro the nuber two, neither also let it haue to his halfe part an odde nuber. Then I say that *A* is a nuber both euenly euen, and euenly odde. That *A* is euenly euen it is manifest, for the halfe thereof is not odde, and is measured by the number 2. which is an euen number. Now I say that it is euenly odde also. For if we deuide *A* into two equall partes, and so continuing still, we shall at the length light vpon a certaine odde number which shall measure *A* by an euen number. For if we should not light vpon such an odde nuber, which measureth *A* by an euen number, we should at the length come vnto the number two, and so should *A* be one of those numbers which are doubled from two upward, which is contrary to the supposition. Wherefore *A* is euenly odde. And it is proued that it is euenly eue: wherefore *A* is a number both euenly euen and euenly odde: whiche was required to be demonstrated.

This proposition and the two former manifestly declare that which we noted vpon the tenth definition of the seuenth booke namely, that *Campane* and *Flussates* and diuers other interpreters of *Euclide* (onely *Theon* except) did not rightly vnderstand the 8. and 9. definitions of the same booke concerning a number euenly euen, and a number euenly odde. For in the one definition they adde vnto *Euclides* wordes extant in the

the Greeke this word *onely* (as we there noted) and in the other this word *all*. So that after their definitions a number can not be evenly euen vnlesse it be measured onely by euen numbers: likewise a number can not be evenly odde vnlesse all the euen numbers which doo measure it, doo measure it by an odde number. The contrary whereof in this proposition we manifestly see. For here *Euclide* proueth that one number may be both evenly euen and evenly odde. And in the two former propositions he proued that some numbers are evenly euen onely, and some evenly odde onely: which word *onely* had bene in vaine of him added, if no number evenly euen could be measured by an odde number, or if all the numbers that measure a number evenly odde must needs measure it by an odde number. Although *Campane* and *Flussates* to auoyde this absurdity haue wrested the 32. proposition of this booke frō the true sence of the Greeke and as it is interpreted of *Theon*. So also hath *Flussates* wrested the 33. proposition. For whereas *Euclide* sayth, *Every nūber produced by the doubling of two vpwārd, is euēly euē onely*: they say, *onely the numbers produced by the doubling of two, are euēly euē*. Likewise whereas *Euclide* saith, *A number whose halfe part is odde, is euēly odde onely*, *Flussates* sayth, *onely a number whose halfe part is od, is euēly od*. Which their interpretatiō is not true, neither can be applyed to the propositions as they are extāt in the Greeke. In dede the sayd 32. and 33. propositions as they put the are true touching those numbers which are evenly euen onely, or euēly od onely. For no number is evenly euen onely, but those onely which are doubled from two vpwārd. Likewise no numbers are evenly odde onely, but those onely whose halfe is an odde number. But this letteth not, but that a number may be evenly euen although it be not doubled from two vpwārd & also that a number may be euēly odde although it haue not to his halfe an odde number. As in this 34. proposition *Euclide* hath plainly proued. Which thing could by no meanes be true, if the foresayd 32. & 33. propositions of this booke should haue that sence and meaning wherein they take it.

¶ The 35. Theoreme.

The 35. Proposition.

If there be numbers in continuall proportion how many soeuer, and if from the second and last be taken away numbers equall vnto the first, as the excessse of the second is to the first, so is the excessse of the last to all the nūbers going before the last.



Suppose that these numbers *A, B, C, D, and E, F,* be in continuall proportion beginning at *A* the least. And from *B, C,* which is the second, take away *C, G,* equall vnto the first, namely, to *A,* and likewise from *E, F* the last take away *F, H,* equall also vnto the first, namely, to *A.* Then I say, that as the excessse *B, G* is to *A* the first, so is *H, E* the excessse, to all the numbers *D, B, C,* and *A,* which go before the last number, namely, *E, F.* Forasmuch as *E, F* is the greater (for the second is supposed greater then the first) put the number *F, L* equall to the number *D,* and likewise the number *F, K* equall to the number *B, C.* And forasmuch as *F, K* is equall vnto *C, B,* of which *F, H* is equall vnto *G, C,* therefore the residue *H, K* is equall vnto the residue *G, B.* And for that as the whole *F, E,* is to the whole *F, L,* so is the part taken away *F, L,* to the part taken away *F, K,* therefore the residue *L, E* is to the residue *K, L,* as the whole *F, E* is to the whole *F, L* (by the 11. of the seventh). So likewise for that *F, L* is to *F, K,* as *F, K* is to *F, H,* *K, L* shall be to *H, K,* as the whole *F, L* is to the whole *F, K* (by the same Proposition). But as *F, E* is to *F, L,* and as *F, L* is to *F, K,* and *F, K* to *F, H,* so were *F, E* to *D,* and *D* to *B, C,* and *B, C* to *A.* Wherefore as *L, E* is to *K, L,* and as *K, L* is to *H, K,* so is *D* to *B, C.* Wherefore alterately (by the 13. of the seventh) as *L, E* is to *D,* so is *K, L* to be *B, C,* and as *K, L* is to *B, C,* so is *H, K* to *A.* Wherefore also as one of the antecedentes is to one of the consequentes, so are

Demonstration.

The ninth Booke

all the antecedentes to all the consequentes . Wherefore as KH is to A , so are HK , KL , and LE , to D , BC , and A (by the 12. of the seuenth) . But it is proued, that KH is equall vnto BG . Wherefore as BG , which is the excesse of the second, is to A , so is EH the excesse of the last vnto the numbers going before D , BC , and A . Wherefore as the excesse of the second is vnto the first, so is the excesse of the last to all the numbers going before the last : which was required to be proued.

¶ The 36. Theoreme.

The 36. Proposition.

If from vnitie be taken numbers how many soeuer in double proportion continually, vntill the whole added together be a prime number, and if the whole multiplying the last produce any number, that which is produced is a perfecte number.

This proposition teacheth how to finde out a perfect number.



Suppose that from vnitie be taken these numbers A, B, C, D , in double proportion continually, so that all those numbers A, B, C, D , & vnitie added together, make a prime number : and let E be the number composed of all those numbers A, B, C, D , & vnitie added together : and let E multiplying D , which is the last number, produce the number FG . Then I say, that FG is a perfect number.

Vnitie .

A ..

B

C

D

O _____

P _____

E _____ 31

H _____ 31 N _____ 31 K 02

L _____ 124

M _____ 148

F _____ 31 X _____ 465 G 490

Construction.

How many in multitude A, B, C, D , are, so many in continuall double proportion take beginning at E , which let be the numbers E, HK, L , and M . VVherefore of equalitie (by the 13. of the seuenth) as A is to D , so is E to M . VVherefore that which is produced of E into D , is equall to that which is produced of A into M . But that which is produced of E into D , is the number FG . VVherefore that which is produced of A into M , is equall vnto FG . VVherefore A multiplying M produceth FG . VVherefore M measureth FG by those vnties which are in A . But A is the number two . VVherefore FG is double to M . And the numbers M, L, HK , and E , are also in continuall double proportion . VVherefore all the numbers E, HK, L, M , and FG , are continually proportionall in double proportion . Take from the second number KH , and from the last FG a number equall vnto the first, namely, to E : and let those numbers taken be HN , & FX . VVherefore (by the Proposition going before) as the excesse of the second number is to the first number, so is the excesse of the last to all the

Demonstration.

numbers

numbers going before it. Wherefore as NK is to E , so is XG to these numbers M, L, K, H , and E . But NK is equall unto E (for it is the halfe of HK , which is supposed to be double to E). Wherefore XG is equall unto these numbers M, L, H, K , and E . But XF is equall unto E , and E is equall unto these numbers A, B, C, D , and unto unitie. Wherefore the whole number FG is equall unto these numbers E, H, K, L, M , and also unto these numbers A, B, C, D , and unto unitie. Moreover I say, that unitie and all the numbers A, B, C, D, E, H, K, L , and M , do measure the number FG . That unitie measureth it, it needeth no proufe. And forasmuch as FG is produced of D into E , therefore D and E do measure it. And forasmuch as the double from unitie, namely, the numbers A, B, C , do measure the number D (by the 13. of this booke) therefore they shall also measure the number FG (whom D measureth) by the 5. common sentence. By the same reason forasmuch as the numbers E, H, K ,

Demonstration leading to an absurditie.

Unitie .

A ..

B

C

D

O _____

P _____

E _____ 31

H _____ 31 N 31 K 62

L _____ 124

M _____ 248

F _____ 31 X 465 G 496

L , and M , are unto FG , as unitie and the numbers A, B, C , are unto D (namely, in subdule proportion) and unitie and the numbers A, B, C , do measure D , therefore also the numbers E, H, K, L , and M , shall measure the number FG . Now I say also, that no other number measureth FG besides these numbers $A, B, C, D, E, H, K, L, M$, and unitie. For if it be possible, let O measure FG . And let O not be any of these numbers A, B, C, D, E, H, K, L , and M . And how often O measureth FG , so many unities let there be in P . Wherefore O multiplying P produceth FG . But E also multiplying D produced FG . Wherefore (by the 19. of the seventh) as E is to O , so is P to D . Wherefore alternately (by the 9. of the seventh) as E is to P , so is O to D . And forasmuch as from unitie are these numbers in continuall proportion A, B, C, D , and the number A which is next after unitie is a prime number, therefore (by the 13. of the ninth) no other number measureth D besides the numbers A, B, C . And it is supposed that O is not one and the same with any of these numbers A, B, C . Wherefore O measureth not D . But as O is to D , so is E to P . Wherefore neither doth E measure P . And E is a prime number. But (by the 31. of the seventh) euery prime number, is to euery number that it measureth not, a prime number. Wherefore E and P are prime the one to the other: yea they are prime and the least. But (by the 21. of the seventh) the least measure the numbers that haue one and the same proportion with them equally, the antecedent the antecedent, and the consequent the consequent. And as E is to P , so is O to D . Wherefore how many times E measureth O , so many times P measureth D . But no other number measureth D besides the numbers A, B, C (by the 13. of this booke). Wherefore P is one and the same with one of these numbers A, B, C . Suppose that P be one and the same with B , & how many B, C, D , are

The ninth Booke

in multitude, so many take from E upward, namely, E, H K, and L. But E, H K, and L, are in the same proportion that B, C, D, are. VVherefore of equalitie, as B is to D, so is E to L. VVherefore that which is produced of B into L, is equall to that which is produced of D into E. But that which is produced of D into E, is equall to that which is produced of P into O. VVherefore that which is produced of P into O, is equall to that which is produced of B into L. VVherefore as P is to B, so is L to O: and P is one & the same with B: wherefore L also is one and the same with O: which is impossible. For O is supposed not to be one and the same with any of the numbers geuen. VVherefore no number measureth F G besides these numbers A, B, C, D, E, H K, L, M, and vnitie. And it is proued, that F G is equall vnto these numbers A, B, C, D, E, H K, L, M, and vnitie, which are the partes thereof (by the 39. of the seuenth). But a perfect nūber (by the definition) is that which is equall vnto all his partes. VVherfore F G is a perfect number: which was required to be proued.

☞ The end of the ninth booke
of Euclides Elementes.



The tenth booke of Eu- clides Elementes.



IN THIS TENTH BOOKE doth Euclide entreat of lines and other magnitudes rationall & irrationall, but chiefly of irrationall magnitudes, commensurable and incommensurable: of which hitherto, in al his former 9. bookes he hath made no mention at all. And herein differeth number from magnitude, or Arithmetike from Geometry: for that although in Arithmetike, certayne numbers be called prime numbers in consideration of themselves, or in respect of an other, and so are called incommensurable, for that no one number measureth them, but onely vnitie. Yet in dede and to speake absolutely and truely, there are no two numbers incommensurable, but haue one common measure which measureth the both, if none other, yet haue they vnitie, which is a common part and measure to all numbers, and all numbers are made of vnities, as of their partes. As hath before bene shewed in the declaration of the definitiōs of the seuenth booke. But in magnitude it is farre otherwise, for although many lines, plaine figures, and bodies, are commensurable, and may haue one measure to measure them, yet all haue not so, nor can haue. For that a line is not made of pointes, as number is made of vnities, and therefore cannot a point be a common part of all lines, and measure them, as vnitie is a common part of all numbers and measureth them. Vnitie taken certayne tymes maketh any number. For there are not in any number infinite vnities: but a point taken certayne tymes, yea as often as ye list, neuer maketh any line, for that in euery line there are infinite pointes. Wherefore lines, figures, and bodies in Geometry, are oftentimes incommensurable and irrationall. Now which are rationall, and which irrationall, which commensurable, and which incommensurable, how many and how sundry sortes and kindes there are of them, what are their natures, passions, and properties, doth Euclide most manifestly shew in this booke, and demonstrate them most exactly.

This tenth booke hath euer hitherto of all men, and is yet thought & accounted, to be the hardest booke to vnderstand of all the bookes of Euclide. Which common receiued opinion, hath caused many to shrinke, and hath (as it were) deterred them from the handeling and treatie thereof. There haue bene in dede in times past, and are presently in these our dayes, many which haue delt, and haue taken great and good diligence in commenting, amending, and restoryng of the sixe first bookes of Euclide, and there haue stayed themselves and gone no farther, beyng deterred and made afraide (as it seemeth, by the opinion of the hardnes of this booke) to passe forth to the bookes following. Truth it is that this booke hath in it somewhat an other & straūger maner of matter entreated of, the the other bookes before had, and the demonstrations also thereof, & the order seeme likewise at the first somewhat straunge and vnaccustomed, which thinges may seeme also to cause the obscuritie therof, and to feare away many from the reading and diligent study of the same, so much that many of the well learned haue much complayned of the darkenes and difficultie thereof, and haue thought it a very hard thing, and in ma-

*The Argūmes
of the tenth
booke.*

*Difference
betwene num
ber and mag
nitude.*

*A line is not
made of pointes
as number is
made of vni
ties.*

*This booke the
hardest to vn
derstand of all
the bookes of
Euclide.*

*In this booke
is entreated of
a straunger
maner of mat
ter then in the
former.*

The tenth Booke

Manyeuen of
the well lear-
ned haue
thought that
this booke can
not well be
vnderstanded
without Al-
gebra.

The nine for-
mer bookes &
the principles
of this booke
well vnder-
stande, this
booke will not
be hard to
vnderstand.

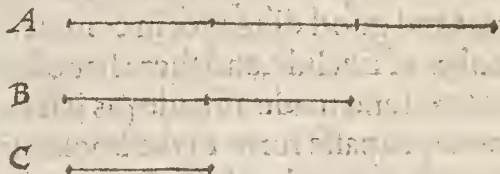
The first
definition.

ner impossible to attayne to the right and full vnderstanding of this booke, without the ayde and helpe of some other knowledge and learning, and chiefly without the knowledge of that more secret and subtile part of Arithmetike, commonly called Algebra, which vndoubtedly first well had and knowne, would geue great light therunto: yet certainly may this booke very well be entred into, and fully vnderstand without any straunge helpe or succour, onely by diligent obseruation of the order, and course of Euclides writings. So that he which diligently hath perused and fully vnderstandeth the 9. bookes goyng before, and marketh also earnestly the principles and definitions of this tenth booke, he shal well perceiue that Euclide is of himselfe a sufficient teacher and instructor, and needeth not the helpe of any other, and shall soone see that this tenth booke is not of such hardnes and obscuritie, as it hath bene hether to thought. Yea, I doubt not, but that by the trauell and industry taken in this translation, and by addicions and emendations gotten of others, there shall appeare in it no hardnes at all, but shall be as easie as the rest of his bookes are.

Definitions.

1 Magnitudes commensurable are such, which one and the selfe same measure doth measure.

First he sheweth what magnitudes are commensurable one to an other. To the better and more cleare vnderstanding of this definition, note that that measure whereby any magnitude is measured, is lesse then the magnitude which it measureth, or at least equall vnto it. For the greater can by no meanes measure the lesse. Farther it behoueth, that that measure if it be equall to that which is measured, taken once make the magnitude which is measured: if it be lesse, then oftentimes taken and repeted, it must precisely render and make the magnitude which it measureth. Which thing in numbers is easely sene, for that (as was before said) all numbers are commensurable one to an other. And although Euclide in this definition comprehendeth purposedly, onely magnitudes which are continuall quantities, as are lines, superficieces, and bodies, yet vndoubtedly the explication of this and such like places, is aptly to be sought of numbers as well rationally as irrationall. For that all quantities commensurable haue that proportion the one to the other, which number hath to numbers. In numbers therefore, 9 and 12 are commensurable, because there is one common measure which measureth them both, namely, the number 3. First it measureth 12, for it is lesse then 12, and being taken certaine times, namely, 4 times, it maketh exactly 12: 3 times 4 is 12, it also measureth 9, for it is lesse then 9. and also taken certaine times, namely, 3 times, it maketh precisely 9: 3 times 3 is 9. Likewise is it in magnitudes, if one magnitude measure two other magnitudes, those two magnitudes so measured, are said to be commensurable. As for example, if the line C. being doubled, do make the line B, and the same lyne C tripled, do make the line A, then are the two lines A and B, lines or magnitudes commensurable. For that one measure, namely, the line C measureth the both. First, the line C is lesse the the line A, and also lesse the the line B, also the line C taken or repeted certaine times, namely, 3 times, maketh precisely the line A, and the same line C taken also certain times, namely, two times, maketh precisely the line B. So that the line C is a common measure to them both, and doth measure them both. And therefore are the two lines A and B lines commensurable. And so imagine ye of magnitudes of other kyndes, as of superficiall figures, and also of bodies.



2 Incommensurable magnitudes are such, which no one common measure doth measure.

The second definition.

This diffinition needeth no explanation at all, it is easely vnderstanded by the diffinition going before of lines commensurable. For contraries are made manifest by comparing of the one to the other: as if the line C, or any other line oftentimes iterated, doo not render precisely the line A, nor the line B, the are the lines A and B incommensurable: Also if the line C, or any other line certayne times repeted, doo exactly render the line A, and doo not measure the line B: or if it measure the line B, and measureth not also the line A, the lines A and B, are yet lines incómensurable: & so of other magnitudes, as of superficiées, and bodyes.

Contraries made manifest by the comparing of the one to the other.

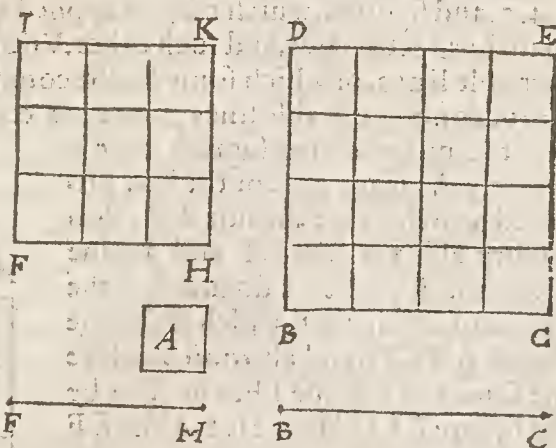
3 Right lines commensurable in power are such, whose squares one and the selfe same superficies, area, or plat doth measure.

The third definition.

To the declaration of this diffinition we must first call to minde what is vnderstanded & ment by the power of a line: which as we haue before in the former bookes noted is nothing ells but the square thereof, or any other plaine figure equall to the square thereof. And so great power & habilitie is a line said to haue, as is the quantitie of the square, which it is able to describe, or a figure superficial equal to the square thereof. This is also to be noted that of lines, some are commensurable in length, the one to the other, and some are commensurable the one to the other in power. Of lines commensurable in length the one to the other, was geuen an example in the declaration of the first diffinitio, namely, the lines A and B, which were commensurable in length, one and the selfe measure, namely, the line C measured the length of either of them. Of the other kinde is geuen this diffinition here set: for the opening of which take this example.

What the power of a line is.

Let there be a certayne line, namely, the line B C, and let the square of that line be the square B C D E. Suppose also an other line, namely, the line F H, & let the square thereof be the square F H I K, and let a certayne superficies, namely, the superficies A, measure the square B C D E, taken 16. times: which is the number of the litle areas, squares, plats, or superficiées cõtained and described within the sayd squares each of which is equall to the superficie: A. Agayne let the same superficies A measure the square F H I K 9. times taken, according to the number of the fieldes or superficieses containd and described in the same. Ye see the that one and the selfe same superficies, namely, the superficies A, is a common measure to both these squares, and by certayne repetitions thereof, measureth them both. Wherefore the two lines B C and F H, which are the sides or lines producing these squares, and whose powers these squares are, are by this diffinition lines commensurable in power.

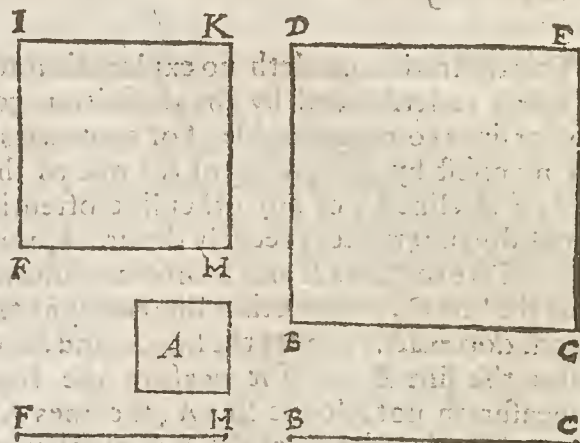


4 Lines incommensurable are such, whose squares no one plat or superficies doth measure.

The fourth definition.

The tenth Booke

This diffinition is easy to be vnderstanded by that which was sayd in the diffinition last set before this, and needeth no farther declaration. And thereof take this example. If neither the superficies A, nor any other superficies doo measure the two squares B C D E, and F H I K: or if it measure the one, namely B C D E, and not the other, F H I K, or if it measure the square F H I K, and not the square B C D E: the two lines B C and F H, are in power incommensurable, and therefore also incommensurable in length. For whatsoever lines are incommensurable in power, the same are also incommensurable in length as shall afterward in the 9. proposition of this booke be proued. And therefore such lines are here defined to be absolutely incommensurable. These things thus standing it may easily appeare, that if a line be assigned and layd before vs, there may be innumerable other lines commensurable vnto it, and other incommensurable vnto it: of commensurable lines some are commensurable in length and power, and some in power onely.



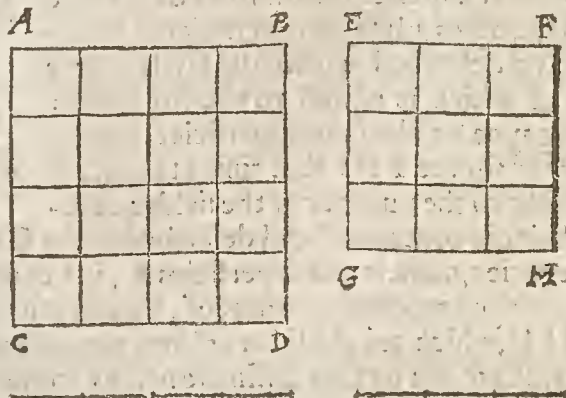
And that right line so set forth is called a rationall line.

The fifth definition.

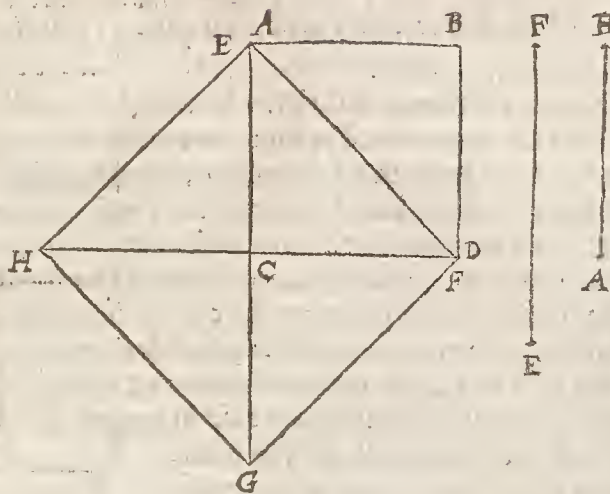
The principles of this booke ought well to be perused, for that they are more straunge then the principles of the former booke.

These principles, diffinitions and groundes of this tenth booke ought well to be payed, they are somewhat more strange and more hard, then are the diffinitions and principles of the other booke of *Euclide* going before, and therefore at the first sight or reading are not straight way conceiued, but ought often to be repeted, and by vse to be confirmed. For the propositions following, bring vnto them much light, and facilitie of vnderstanding. First there is a line supposed, and layd before vs, which may be any line whatsoever, of what length, or shortnes ye will: this line thus first supposed is imagined to haue such diuisions and so many partes as ye list to conceiue in minde, as 3. 4. 5. and so forth, which may be applied to any kinde of measure, as it shall happen, as to inches, feete, pases, and such other. Vnto this line saith *Euclide* may be copared innumerable lines, of which some shalbe commensurable, and other some incommensurable: of commensurable lines, some are commensurable both in length and in power, other some are comensurable in power onely. As if any part of the line proposed which let be the line A B, doo measure also the line E F and againe if any one superficies do measure the square of the line A B, which let be the square A B C D: and also doo measure the square of the line E F, which let be the square E F G H: the line E F to the supposed line & first set, namely, to the line A B, a line commensurable both in length and power, as ye may see in the first example here set.

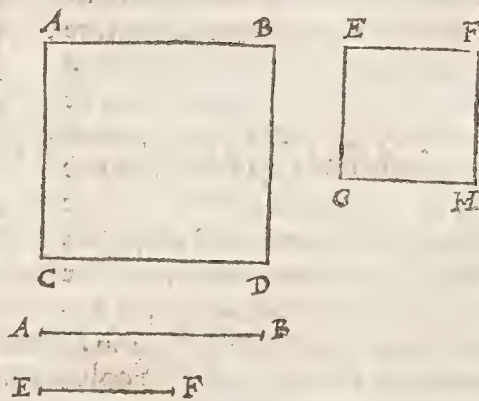
And if it so be that one & the selfe same superficies do measure both the squares of these two lines A B and E F, namely the squares A B C D and E F G H: and no one line do measure the lines A B and E F: then is the line



EF (compared to the supposed and first line AB) commensurable in power onely. As in this example ye may easely perceiue. For the triangle or superficies ACD, twise taken, measureth the square ABCD, namely, the square of the line AB: and the selfe same triangle ACD taken foure times measureth the other square, namely, the square of the line EF. But no one measure or line can be assigned to measure both the lines AB and EF, because the sides of a square and the diameter are incommensurable in length the one to the other, as afterward shalbe shewed. Wherefore they are in length incommensurable, & commensurable in power onely, that is by reason of their squares, which are commensurable the one to the other.



Agayne if it so be that no one line may be found to be a comon measure, to measure both the first line, namely, AB, and also the line EF: nor yet any one superficies to measure the square or powers of these two lines, then is the line EF, to the first line set and supposed, incommensurable both in length and in power. As is supposed to be in this example.



Thus may ye see, how to the supposed line first set may be compared infinite lines, some commensurable both in length & power, and some commensurable in power onely, and incommensurable in length, and some incommensurable both in power & in length. And this first line so set, whereunto, and to whose squares the other lines and their squares are compared, is called a rationall line, commonly of the most part of writers. But some there are, which mislike that it should be called a rationall line, & that not without iust cause. In the Greeke copy it is called *ῥησιμον*, *rete*, which signifieth a thing that may be spokē, & expressed by word, a thing certayne, graunted and appoynted. Wherefore *Flusates*, a man which bestowed great trauell and diligence in restoring of these elementes of Euclide, leauing this word rationall, calleth this line supposed and first set, a line certayne, because the partes thereof into which it is deuided are certayne, and known, and may be expressed by voyce, and also be coumpted by number: other lines, being to this line incommensurable, whose parts are not distinctly known, but are vncertayne, nor can be expressed by name nor assigned by number, which are of other men called irrational, he calleth vncertayne and surd lines. *Petrus Montaneus* although he doth not very wel like of the name, yet he altereth it not, but vseth it in al his booke. Likewise wil we doo here, for that the word hath bene and is so vniuersally receiued. And therefore will we vse the same name, and call it a rationall line. For it is not so great a matter what names we geue to thinges, so that we fully vnderstand the thinges which the names signifie.

This rationall line thus here defined, is the ground and foundation of all the propositions almost of this whole tenth booke. And chiefly from the tenth proposition forwardes. So that vnlesse ye first place this rationall line, and haue a speciall and continuall regard vnto it before ye begin any demonstration, ye shall not easely vnderstand it. For it is as it were the touch and triall of all other lines, by which it is known whether any of them be rationall or not. And this may be called the first rationall line, the line rationall of purpose or a rationall line set in the first place, and so made distinct and secured from other rationall lines, of which shall be spoken afterwarde. And this must ye well commit to memory.

Vnto the supposed line first set may be compared infinite lines.

Why some mislike that the line first set should be called a rationall line.

Flusates calleth this line, a line certayne.

This rationall line the ground in a manner of all the propositions in this tenth booke.

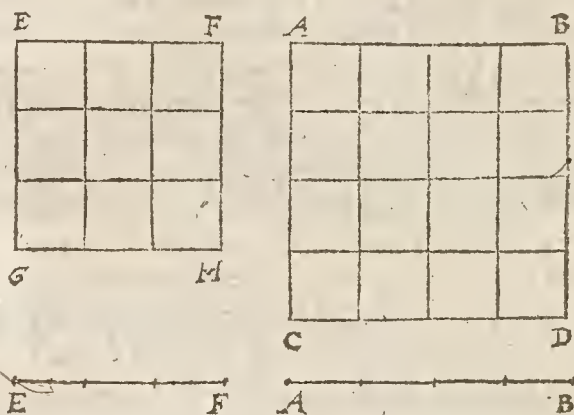
Note. The line Rationall of purpose.

The tenth Booke

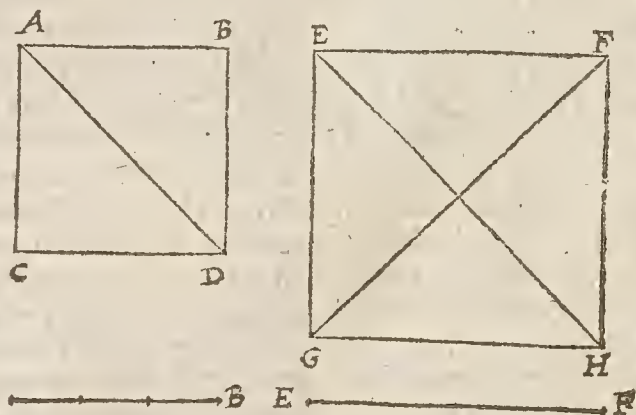
The sixth
definition.

6 Lines which are commensurable to this line, whether in length and power, or in power onely, are also called rationall.

This definition needeth no declaration at all, but is easily perceiued, if the first definition be remembred, which sheweth what magnitudes are commensurable, and the third, which sheweth what lines are commensurable in power. Here note, how aptly & naturally, Euclide in this place vseth these wordes commensurable either in length and power, or in power onely. Because that all lines which are commensurable in length, are also commensurable in power: when he speaketh of lines commensurable in length, he ever addeth and in power, but when he speaketh of lines commensurable in power, he addeth this worde Onely, and addeth not this worde in length, as he in the other added this worde in power. For not all lines which are commensurable in power, are straight way commensurable also in length. Of this definition, take this example. Let the first line rationall of purpose, which is supposed and laide forth, whose partes are certaine & known, and may be expressed, named, and nūbred be AB , the quadrate whereof let be $ABCD$: then suppose againe an other lyne, namely, the line EF , which let be commensurable both in length and in power to the first rationall line, that is, (as before was taught) let one line measure the length of eche line, and also let one superficies measure the two squares of the said two lines, as here in the example is supposed and also appeareth to the eie, then is the line EF also a rationall line.



Moreouer if the lyne EF be commensurable in power onely to the rationall line AB first set and supposed, so that no one line do measure the two lines AB and EF : As in example ye see to be (for that the line EF , is made equall to the line AD , which is the diameter of the square $ABCD$, of which square the line AB is a side, it is certayne that the side of a square is incōmensurable in length to the diameter of the same square: if there be yet founde any one superficies, which measureth the two squares $ABCD$, and $EFGH$ (as here doth the triangle ABD , or the triangle ACD noted in the square $ABCD$, or any of the foure triangles noted in the square $EFGH$, as appeareth somewhat more manifestly in the second example,



in the declaration of the last definition going before) the line EF is also a rationall line. Note that these lines which here are called rationall lines, are not rational lines of purpose, or by supposition, as was the first rationall line, but are rational onely by reason of relation and comparison which they haue vnto it, because they are commensurable vnto it either in length and power, or in power onely. Farther here is to be noted, that these wordes length, and power, and power onely, are ioyned onely with these wordes commensurable or incommensurable, and are neuer ioyned with these woordes rationall or irrational. So that no lines can be called rational in length, or in power, nor likewise can they be called irrational in length, or in power. Wherin vndoubtedly Campanus was deceiued, who vsing those wordes & speeches indifferently, caused & brought in great obscuritie to the propositions and demonstrations of this booke, which he shall easily

Campanus
hath caused
much obscuri-
tie in this
tenth booke.

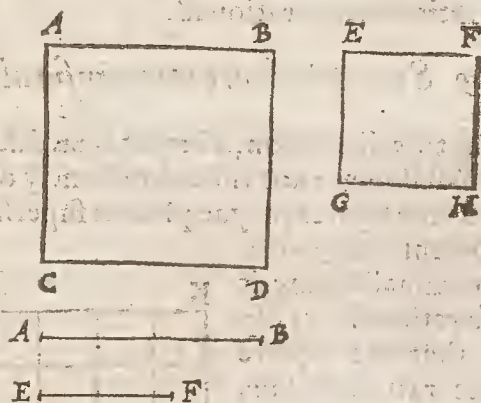
easily see which marketh with diligence the demonstrations of *Campanus* in this booke.

7 Lines which are incommensurable to the rationall line, are called irrational.

The seventh definition.

By lines incommensurable to the rationall line supposed in this place, he vnderstandeth such as be incommensurable vnto it both in length and in power: For there are no lines incommensurable in power onely: for it cannot be that any lines should so be incommensurable in power onely, that they be not also incommensurable in length. What so euer lines be incommensurable in power, the same be also incommensurable in length. Neither can *Euclide* here in this place, meane lines incommensurable in length onely, for in the diffinition before, he called them rationall lines, neither may they be placed amongst irrational lines. Wherefore it remaineth that in this diffinition he speaketh onely of those lines which are incommensurable to the rationall line first geuen and supposed, both in length, and in power. Which by all meanes are incommensurable to the rationall line, & therefore most aptly are they called irrational lines. This diffinition is easy to be vnderstanded by that which hath bene sayd before. Yet for the more plainenes see this example. Let the first rationall line supposed, be the line *AB*, whose square or quadrate, let be *ABCD*. And let there be geuen an other line *EF* which let be to the rationall line incommensurable in length and power, so that let no one line measure the length of the two lines, *AB* and *EF*; and let the square of the line *EF* be *EFGH*. Now if also there be no one superficies which measureth the two squares *ABCD*, and *EFGH*, as is supposed to be in this example, the line *EF* is an irrational line, which word irrational (As before did this word rational) misliketh many learned in this knowledge of Geometry. *Flusates*, as he left the word rational, and in steade thereof vsed this word certaine, so here he leaueth the word irrational, and vseth in place thereof this word *uncertaine*, and euer nameth these lines *uncertaine* lines. *Petrus Montaneus* also misliking the word irrational, would rather haue them to be called surd lines, yet because this word irrational hath euer by custome and long vse, so generally bene receiued, he vseth continually the same. In Greeke such lines are called *αλογoi*, *alogoi*, which signifieth nameles, vspeakeable, *uncertaine*, indeterminate, and with out proportion: not that these irrational lines haue no proportion at all, either to the first rationall line, or betwene them selues: but are so named, for that theyr proportions to the rationall line cannot be expressed in number. That is vndoubtedly very vntrue, which many write, that their proportions are vnknowne both to vs and to nature. Is it not thinke you a thing very absurd to say that there is any thing in nature, and produced by nature, to be hidde from nature, and not to be knowne of nature? it can not be sayd that their proportions are vtterly hidde and vnknowne to vs (much lesse vnto nature) although we cannot geue them their names, and distinctly expresse them by numbers: otherwise should *Euclide* haue taken all this trauell and wonderfull diligence bestowed in this booke, in vaine and to no vse: in which he doth nothing els but teach the proprietie and passions of these irrational lines, and sheweth the proportions which they haue the one to the other.

Here is also to be noted, which thing also *Tartalea* hath before diligently noted, that *Campanus* and many other writers of Geometry, ouer much erred and were deceived in that they wrote and taught, that all these lines whose squares were not signified and might be expressed by a square number (although they might by any other number: as by 11. 12. 14. and such others not square numbers) are irrational lines. Which is manifestly repugnant to the groundes and principles of *Euclide*, who wil, that all lines which are commensurable to the rationall line, whether it be in length and power



Flusates in steede of this word irrational vseth this word *uncertaine*.

Why they are called irrational lines.

The tenth Booke

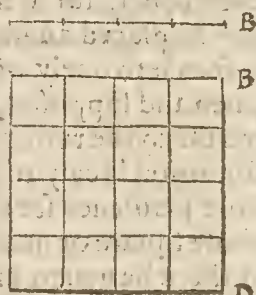
The cause of
the obscurity
and confus-
ednes in this
booke.

The eighth
definition.

power, or in power onely, should be rationall. Vndoubtedly this hath bene one of the chiefest and greatest causes of the wonderfull confusion and darkenes of this booke, which so hath tossed, and tormoyled the wittes of all both writers and readers, masters and scholars, and so ouerwhelmed them, that they could not without infinite trauell and sweate, attayne to the truth and perfect vnderstanding thereof.

8 The square which is described of the rationall right line, supposed, is rationall.

Vntill this diffinition hath Euclide set forth the nature and proprietie of the first kinde of magnitude, namely, of lines how they are rationall or irrationall; now he beginneth to shew how the second kinde of magnitudes, namely superficies are one to the other rationall or irrationall. This diffinition is very playne. Suppose the line A B, to be the rationall line, hauing his parts and diuisions certainly knowen: the square of which line let be the square A B C D. Now because it is the square of the rationall line A B, it is also called rationall: and as the line A B, is the first rationall line, vnto which other lines compared are counted rationall, or irrationall, so is the quadrat or square thereof, the first rationall superficies vnto which all other squares or figures compared, are counted and named rationall or irrationall.

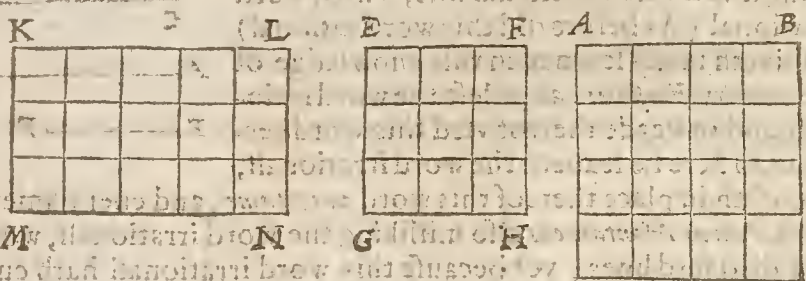


9 Such which are commensurable vnto it, are rationall.

The ninth
definition.

In this diffinition, where it is sayd, such as are commensurable to the square of the rationall line, are not vnderstand onely other squares or quadrates, but all other kindes of rectiline figures playne plats & superficies. What so euer so that if any such figure be comensurable vnto that rationall square, it

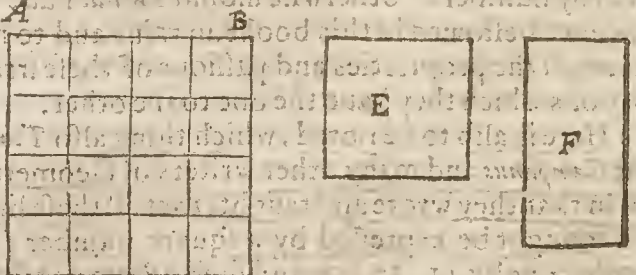
is also rationall. As suppose that the square of the rationall line, which is also rationall, be A B C D: suppose also some other square as the square E F G H, to be commensurable to the same: then is the square E F G H also rational. So also if the rectiline figure K L M N, which is a figure on the one side longer, be commensurable vnto the sayd square as is supposed in this example, it is also a rational superficies and so of all other superficieses.



Such which are incommensurable vnto it, are irrationall.

The tenth
definition.

Where it is sayd in this diffinition such which are incommensurable, it is generally to be taken: as was this word, comensurable in the diffinitio before. For al superficieses, whether they be squares or figures on the one side longer, or otherwise: what maner of right lined figure so euer it be, if they be incommensurable vnto the rationall square supposed, then are they

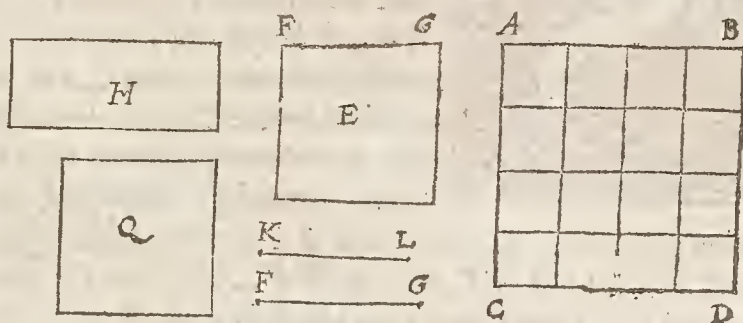


irrationall, As let the square A B C D be the square of the supposed rationall line which square therefore is also rationall: suppose also also an other square, namely the square E, suppose also any other figure, as for example take a figure of one side longer, which let be F. Now if the square E and the figure F, be both incommensurable to the rationall square A B C D, then is ech of these figures E & F irrationall. And so of other.

11 And these lines whose poweres they are, are irrationall. If they be squares, then are their sides irrationall. If they be not squares, but some other rectiline figures, then shall the lines, whose squares are equall to these rectiline figures, be irrationall.

The eleventh definition.

Suppose that the rationall square be A B C D. Suppose also an other square, namely the square E, which let be incommensurable to the rationall square, & therefore is it irrationall: and let the side or line which produceth this square be the line F G: then shall the line F G by this diffinition be an irrationall line: because it is



the side of an irrationall square. Let also the figure H being a figure on the one side longer (which may be any other rectiline figure rectangled or not rectangled, triangle, pentagone, trapezite, or what so ever ells) be incommensurable to the rationall square A B C D, then because the figure H is not a square, it hath no side or roote to produce it yet may there be a square made equall vnto it: for that all such figures may be reduced into triangles, and so into squares, by the 14. of the second. Suppose that the square Q be equall to the irrationall figure H. The side of which figure Q let be the line K L: then shall the line K L be also an irrationall line, because the power or square thereof, is equal to the irrationall figure H: and thus conceiue of others the like.

These irrationall lines and figures are the chiefeft matter and subiect, which is entreated of in all this tenth booke: the knowledge, of which is deepe, and secret, and pertaineth to the highest and most worthy part of Geometrie, wherein standeth the pith and mary of the hole science: the knowlede hereof bringeth light to all the bookes following, with out which they are hard and cannot be at all vnderstoode. And for the more plainenes, ye shall note, that of irrationall lines there be diuers sortes and kindes. But they, whose names are set in a table here following, and are in number 13. are the chiefe, and in this tenth boke sufficiently for Euclides principall purpose, discoursed on.

- 1. A medially line.
 - 2. A binomial line.
 - 3. A first binomial line.
 - 4. A second binomial line.
 - 5. A greater line.
 - 6. A line containing in power a rationall superficies and a medially superficies.
 - 7. A line containing in power two medially superficieses.
 - 8. A residuall line.
 - 9. A first medially residuall line.
 - 10. A second medially residuall line.
 - 11. A lesse line.
 - 12. A line making with a rationall superficies the whole superficies medially.
 - 13. A line making with a medially superficies the whole superficies medially.
- Of all which kindes the diffinitions together with there declarations shalbe set here after in their due places.

- 1.
- 2.
- 3.
- 4.
- 5.
- 6.
- 7.
- 8.
- 9.
- 10.
- 11.
- 12.
- 13.

The tenth Booke

¶ The 1. Theoreme.

The 1. Proposition.

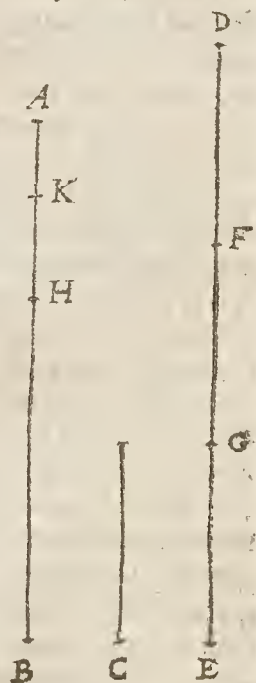
Two vnequall magnitudes being geuen, if from the greater be taken away more then the halfe, and from the residue be againe taken away more then the halfe, and so be done still continually, there shall at length be left a certaine magnitude lesse then the lesse of the magnitudes first geuen.

Construction.



Demonstration.

Suppose that there be two vnequall magnitudes AB , and C , of which let AB be the greater. Then I say, that if from AB , be taken away more then the halfe, and from the residue be taken againe more then the halfe, and so still continually, there shall at the length be left a certaine magnitude, lesse then the lesse magnitude geuen, namely, then C . For forasmuch as C is the lesse magnitude, therefore C may be so multiplied, that at the length it will be greater then the magnitude AB (by the 5. definition of the first booke). Let it be so multiplied, and let the multiplex of C greater then AB , be DE . And deuide DE into the partes equall vnto C , which let be DF , FG , and GE . And from the magnitudes AB take away more then the halfe, which let be BH : and againe from AH , take away more then the halfe, which let be HK . And so do continually untill the diuisions which are in the magnitude AB , be equall in multitude vnto the diuisions which are in the magnitude DE . So that let the diuisions AK , KH , and HB , be equall in multitude vnto the diuisions DF , FG , and GE . And forasmuch as the magnitude DE is greater then the magnitude AB , and from DE is taken away lesse then the halfe, that is, EG (which detraction or taking away is vnderstand to be done by the former diuision of the magnitude DE into the partes equall vnto C : for as a magnitude is by multiplication increased, so is it by diuision diminished) and from AB is taken away more then the halfe, that is, BH : therefore the residue GD is greater then the residue HA (which thing is most true and most easie to conceaue, if we remeber this principle, that the residue of a greater magnitude, after the taking away of the halfe or lesse then the halfe, is euer greater then the residue of a lesse magnitude, after the taking away of more then the halfe). And forasmuch as the magnitude GD is greater then the magnitude HA , and from GD is taken away the halfe, that is, GF : and from AH is taken away more then the halfe, that is, HK : therefore the residue DF is greater then the residue AK (by the foresayd principle). But the magnitude DF is equall vnto the magnitude C . (by supposition). Wherefore also the magnitude C is greater then the magnitude AK . Wherefore the magnitude AK is lesse then the magnitude C . Wherefore of the magnitude AB is left a magnitude AK lesse then the lesse magnitude geuen, namely, then C . which was required to be proued. In like sort also may it be proued if the halfes be taken away.



✱ A Corollary.

A Corollary.

Of this Proposition it followeth, that any magnitude being geuen how litle soeuer it be, there may be geuen a magnitude lesse then it: so that it is impossible that any magnitude

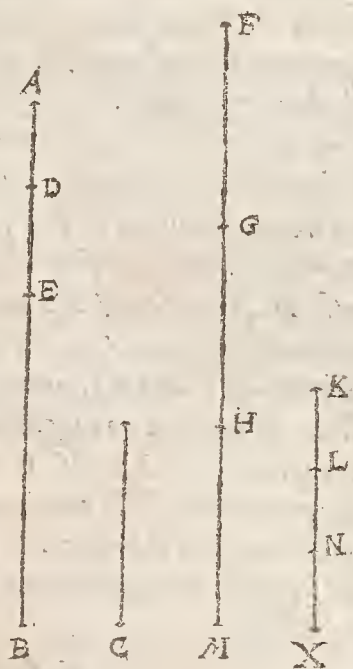
nitide should be geuen then which can be geuen no lesse.

¶ An other demonstration of the same.

Suppose that the two vnequall magnitudes geuen be AB and C . And let C be the lesse. And forasmuch as C is the lesse, therefore C may so be multiplied, that it shall at the length be greater then AB . Let it be so multiplied, and let the multiplex of C exceeding AB be the magnitude FM . And deuide FM into his partes equall vnto C , that is, into the magnitudes MH , HG , and GF . And from AB take away more then the halfe, which let be the magnitude BE : and likewise from EA take away againe more then the halfe, namely, the magnitude ED . And thus do continually untill the diuisions which are in the magnitude FM , be equall in multitude to the diuisions which are in the magnitude AB : and let those diuisions be the magnitudes BE , ED , and DA . And how multiplex the magnitude FM is to the magnitude C , so multiplex let the magnitude KX be to the magnitude DA . And deuide the magnitude KX into the magnitudes equall to the magnitude DA : which let be KL , LN , and NX . Now then the diuisions which are in the magnitude KX , are equall vnto the diuisions which are in the magnitude FM . And forasmuch as BE is greater then the halfe of AB , therefore BE is greater then the residue EA . Wherefore BE is much more greater then DA . But DA is equall vnto XN . Wherefore BE is greater then XN . Again forasmuch as DE is greater then the halfe of EA , therefore DE is greater then the residue DA : but DA is equall vnto LN : wherefore DE is greater then LN . Wherefore the whole magnitude DB is greater then the whole magnitude XL . But DA is equall vnto LK . Wherefore the whole magnitude AB is greater then the whole magnitude KX . And the magnitude MF is greater then the magnitude BA : wherefore MF is much greater then KX . And forasmuch as those magnitudes XN , NL , & LK , are equall the one to the other, & likewise these magnitudes MH , HG , and GF , are equal the one to the other, & the multitude of those magnitudes which are in MF , is equall to the multitude of those magnitudes which are in KX : therefore as KL is to FG , so is LN to GH , and NX to HM . Wherefore (by the 12. of the fifth) as one of the antecedentes, namely, KL , is to one of the consequentes, namely, to FG , so are all the antecedentes, namely, the whole KX to all the consequentes, namely, to the whole FM . But FM is greater then KX . Wherefore FG is greater then LK . But FG is equall vnto C : and KL vnto DA (by supposition). Wherefore the magnitude C is greater then the magnitude AD : which was required to be proued.

Construction.

Demonstration.



¶ The 2. Theoreme.

The 2. Proposition.

Two vnequall magnitudes being geuen, if the lesse be continually taken from the greater, & that which remayneth measureth at no time the magnitude going before: then are the magnitudes geuen incommensurable.

DD ij.

Suppose

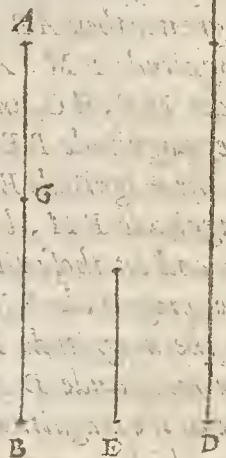
The tenth Booke

This proposition teacheth that incontinually quantitie which the first of the seventh taught in discrete quantity.

Construction.



*S*uppose that there be two unequall magnitudes AB and CD , and let AB be the lesse: and taking away continually by a certaine alternate detraction the lesse from the greater, let not the residue measure the magnitude going before. Then I say, that those two magnitudes AB and CD , are incommensurable. For if they be commensurable, then (by the first definition of the tenth) some one magnitude shall measure them both: Let there be such a magnitude, if it be possible, and let the same be E . And let AB measuring DE , leave a lesse then it selfe, namely, CF (that is, from the greater magnitude CD , take away a certayne part as DE , which let be equall to AB , or if it be not equall vnto it, yet let it be such, that that lesse magnitude AB being more then once repeated may make the magnitude DE : For this is the meaning of this let AB measuring DE . &c. And this detraction made, of the lesse I say out of the greater, let there be left of the greater a certaine portion CF , lesse then the magnitude AB . And this is the meaning of that which in the Theoreme was said, And that which remaineth measureth at no time the magnitude going before. Likewise let CF measuring BG leave a lesse then it selfe, namely, AG : and do this continually as often as neede requireth, untill there be found such a magnitude that is lesse then E : which must needs at the length happen (by the Proposition going before). Let there be found such a magnitude lesse then E , which let be AG . And forasmuch as the magnitude E measureth the magnitude AB , but AB measureth DE : therefore E measureth the magnitude DE (by this common sentence, If a magnitude measure an other magnitude it shall also measure euery magnitude whom it measureth). And it measureth also the whole CD (for it is supposed to be a common measure to the magnitudes AB and CD). Wherefore also it measureth the residue CF (by this common sentence, If a magnitude measure an whole and a part taken away, it shall also measure the residue). And forasmuch as E measureth CF , but CF measureth BG , therefore E also measureth BG (by the first foresaid common sentence): and it measureth the whole AB . Wherefore it shall also measure the residue AG (by the other foresaid common sentence) namely, the greater magnitude shall measure the lesse: which is impossible. Wherefore no magnitude measureth these magnitudes AB and CD . Wherefore the magnitudes AB and CD are incommensurable. If therefore two unequall magnitudes being geuen, the lesse be continually taken from the greater, and that which remaineth measureth at no time the magnitude going before, then are the magnitudes geuen incommensurable: which was required to be demonstrated.



Demonstration leading to an absurditie.

¶ A Corollary added by Montaureus.

By this proposition it is manifest, that if the two unequall magnitudes geuen be not incommensurable, but commensurable, then the lesse being continually subtrahed out of the greater, the residue shall of necessity measure that which went before.

¶ The 1. Probleme.

The 3. Proposition.

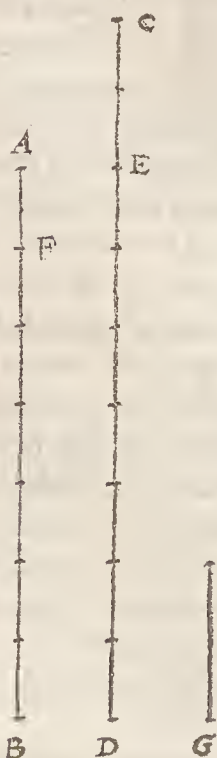
Two magnitudes commensurable being geuen, to finde out their greatest common measure.



Two cases in this proposition. The first case.

*S*uppose that the two commensurable magnitudes geuen be AB & CD , of which let AB be the lesse. It is required to finde out the greatest common measure of the magnitudes AB and CD . Now AB either measureth CD , or not. If it measure it, and seing it also measureth it selfe, wherfore AB is a common measure vnto the

the magnitudes AB and CD . And it is manifest that it is the greatest common measure to them. For no magnitude greater then AB can measure AB . But now suppose that AB do not measure CD . And taking continually the lesse from the greater that which remaineth shall at length measure that which goeth before (by the corollary before added), for that AB and CD are commensurable. Now then let AB measuring ED which is a part of the magnitude CD , leaue a magnitude lesse then it selfe, namely, EC . And let EC measuring the magnitude FB , which is a part of the magnitude AB , leaue a lesse then it selfe, namely, FA , and let FA precisely measure the magnitude CE (And this is the meaning of this, That which remaineth shall at the length measure that which goeth before, when there is nothing left after the measuring made) And forasmuch as AF measureth the magnitude CE , but CE measureth FB : wherfore AF also measureth FB , & it measureth it selfe: wherfore AF measureth also the whole AB . But AB measureth DE , wherfore AF also measureth DE . And AF also measureth CE , wherfore it measureth the whole magnitude CD . Wherfore the magnitude AF measureth both the magnitudes AB and CD . Wherfore AF is a common measure vnto AB and CD . I say also that it is the greatest common measure vnto them. For if



This proposition teacheth, that in continual quantity, which the 2. of the seventh taught in numbers. The second case.

Demonstration leading to an absurditie.

¶ Corollary.

Hereby it is manifest that if a magnitude measure two magnitudes, it shall also measure their greatest common measure. For if it measure the wholes and the partes taken away, it shal also measure the residues, of which one is the greatest common measure, as we may see by the latter end of the former demonstration.

A Corollary.

Montanus reduceth this Probleme into a Theoreme after this maner.

Two Inequall and commensurable magnitudes being geuen, if the lesse do measure the greater, then is it the greatest common measure to them both. But if not, then the lesse being continually by a mutuall detraction (as before hath bene taught) taken out of the greater, whensoever the residue precisely measureth that which went before, leauing nothing, the said residue shall be the greatest common measure to both the magnitudes geuen.

This Probleme reduced to a Theoreme.

¶ The 2. Probleme.

The 4. Proposition.

Three magnitudes commensurable beyng geuen, to finde out their greatest common measure.

DD. iij.

Suppose

The tenth Booke

This proposition teacheth, that in continual quantity, which the 3. of the second taught in numbers.

Construction.

Two cases in this Proposition.
The first case.

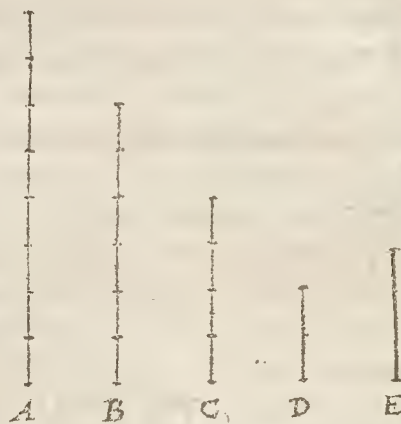
Demonstration leading to an absurditie.

The second case.

A Lemma necessary to be proved before he fall to the demonstration.
Construction.

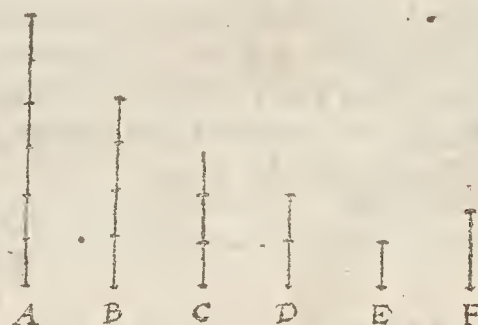
Demonstration leading to an absurditie.

Suppose that the three commensurable magnitudes given be A, B, C . It is required of these three magnitudes to finde out the greatest common measure. Take (by the former proposition) the greatest common measure of the two magnitudes A, B and let the same be D . Now then this magnitude D either measureth the third magnitude, which is C , or not. First let it measure C . And forasmuch as D measureth C , and it measureth also the magnitudes A, B therefore D measureth the three magnitudes A, B, C . Wherefore D is a common measure unto the magnitudes A, B, C . And it is manifest, that it is the greatest common measure. For no magnitude greater then D can measure the magnitudes A, B, C . For if it be possible let the magnitude E being greater then the magnitude D , measure the magnitude A, B, C . And forasmuch as E measureth the magnitudes A, B, C , it measureth the two first magnitudes A, B . Wherefore it shall also (by the Corollary of the former proposition) measure the greatest common measure of the magnitudes A, B which is D , namely, the greater shall measure the lesse, which is impossible.



But now let D not measure the magnitude C . First I say that the magnitudes C, D , are commensurable. For forasmuch as the magnitudes A, B, C are commensurable, some magnitude shall measure them, which shall also measure the magnitudes A, B taken a part. Wherefore it shall also (by the corollary going before) measure the greatest common measure of A, B , that is, D :

and (by supposition) it measureth the magnitude C . Wherefore that said magnitude shall measure the magnitudes C and D . Wherefore the magnitudes C, D are commensurable. Take by the third of this tenth, their greatest common measure, & let the same be the magnitude E . And forasmuch as E measureth D , but D measureth the magnitudes A, B . Wherefore E also measureth the magnitudes A, B , and it also measureth the magnitude C . Wherefore E is a common measure to the three magnitudes A, B, C . I say also that it is the greatest common measure. For if it be possible, let there be a magnitude, namely, F , greater then the magnitude E . And let F measure the three magnitudes A, B, C . And forasmuch as F measureth the magnitudes A, B, C , it also measureth the two first magnitudes A, B . Wherefore (by the corollary going before) it shall also measure the greatest common measure of the magnitudes A, B . But the greatest common measure of the magnitudes A, B , is D . Wherefore F measureth D , and it also measureth C . Wherefore F measureth the magnitudes C, D . Wherefore F shall also measure the greatest common measure of the magnitudes C, D . But the greatest common measure of the magnitudes C, D , is E . Wherefore F measureth E , namely, the greater, the lesse, which is impossible. Wherefore no magnitude greater then E measureth the magnitudes A, B, C . Wherefore E is the greatest common measure of the magnitudes A, B, C . If D do not measure the magnitude C . But if D do measure C , then is D the greatest common measure. Wherefore three magnitudes commensurable being given, there is found their greatest common measure: which was required to be done.



Corollary:

¶ Corollary.

Hereby it is manifest, that if a magnitude measure three magnitudes, it shall also measure their greatest common measure. In like sort also in magnitudes commensurable how many soever being geue, may be found out their greatest common measure, and the corollary will euer be true.

A Corollary.

This Probleme also Montaneus reduceth into a Theoreme after this maner:

Three magnitudes being commensurable, if the greatest common measure to two of them, do measure the third it shall be the greatest common measure to all the three magnitudes geuen. But if it do not measure it, the greatest common measure of the third and of the greatest common measure of the two first, is the greatest common measure of all the three magnitudes.

This Probleme reduced to a Theoreme.

¶ The 3. Theoreme.

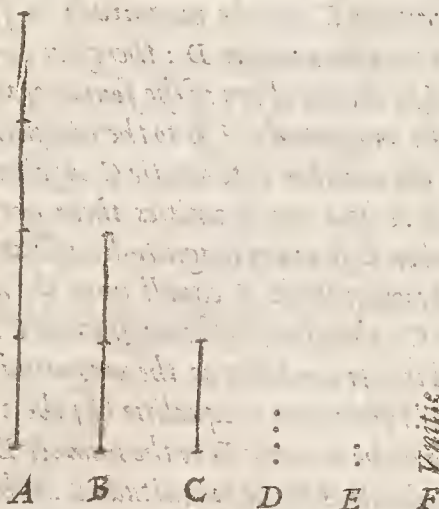
The 5. Proposition.

Magnitudes commensurable, haue such proportion the one to the other, as number hath to number.

Suppose that *A* and *B* be magnitudes commensurable. Then I say that *A* hath unto *B* such proportion as number hath to number. For forasmuch as *A* and *B* are commensurable, therefore some magnitude measureth them, let there be a magnitude that measureth them, and let the same be *C*. And how often *C* measureth *A*, so many vnities let there be in the number *D*. And how often *C* measureth *B*, so many vnities let there be in the number *E*, and let *F* be unitie. And forasmuch as the magnitude *C* measureth the magnitude *A* by those vnities which are in the number *D*, and unitie *F* measureth the number *D* by those vnities which are in the number *D*: therefore how many times unitie measureth the number *D*, so many times doth the magnitude *C* measure the magnitude *A*: wherefore as the magnitude *C* is to the magnitude *A*, so is unitie *F* to the number *D*. Wherefore contrary wise (by the corollary of the fourth proposition of the fifth booke) as the magnitude *A* is to the magnitude *C*, so is the number *D* to unitie *F*. Againe forasmuch as the magnitude *C* measureth the magnitude *B* by those vnities which are in the number *E*, and unitie *F* measureth the number *E* by those vnities which are in the number *E*: therefore how many times unitie *F* measureth the number *E*, so many times doth the magnitude *C* measure the magnitude *B*. Wherefore as the magnitude *C* is to the magnitude *B*, so is unitie *F* to the number *E*, and it is proued that as the magnitude *A* is to the magnitude *C*, so is the number *D* to unitie *F*. Wherefore of equalitie (by the 22. of the fifth) as the magnitude *A* is to the magnitude *B*, so is the number *D* to the number *E*. Wherefore the commensurable magnitudes *A* and *B* haue that proportion the one to the other that the number *D* hath to the number *E*. Magnitudes therefore commensurable, haue such proportion the one to the other, as number hath to number: which was required to be proued.

Construction.

Demonstration.



The tenth Booke

How magnitudes are sayd to be in proportion the one to the other, as number is to number.

Magnitudes are sayd to haue such proportion the one to the other, as nūber hath to number, when as what soeuer proportion is betwene those magnitudes, the same is found betwene some certaine numbers : as if a magnitude be vnto a magnitude either equall, as the number 2. is to the number 2, or double, as the number 4. to the number 2, or triple, as 6. to 2, or in any other multiplex proportion. And so also touching the other kindes of proportion, either superparticular, or superpartient.

¶ The 4. Theoreme.

The 6. Proposition.

If two magnitudes haue such proportion the one to the other, as number hath to number : those magnitudes are commensurable.

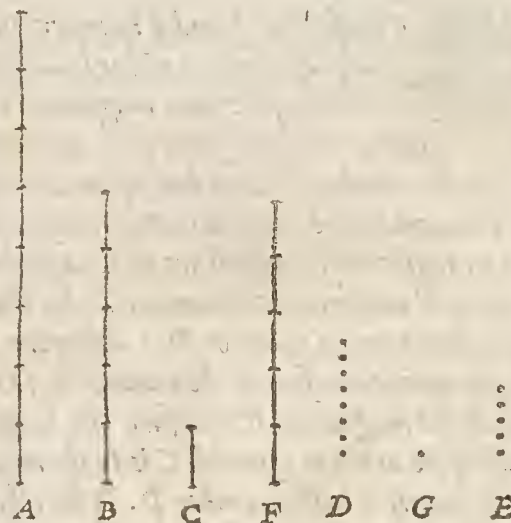
This proposition is the conuerse of former.

Construction.

Demonstration.



Suppose that these two magnitudes *A* and *B* haue that proportion the one to the other, that the number *D* hath to the number *E*. Then I say, that the magnitudes *A*, *B*, are commensurable. How many vnities there are in the number *D*, into so many equall partes deuide the magnitude *A* (by the 9. of the sixth) and let the magnitude *C* be equall to one of the partes therof. And how many vnities there are in the number *E*, of so many magnitudes equall vnto the magnitude *C* let the magnitude *F* be composed. And let *G* be vnitie. Now forasmuch as how many vnities there are in the nūber *D*, so many magnitudes also are there in the magnitude *A* equall vnto the magnitude *C*: therefore what part vnitie *G* is of the nūber *D*, the same part is the magnitude *C* of the magnitude *A*. Wherefore as the magnitude *C* is to the magnitude *A*, so is vnitie *G* to the number *D*. But vnitie *G* measureth the number *D*. Wherefore the magnitude *C* also measureth the magnitude *A*. And for that as the magnitude *C* is to the magnitude *A*, so is vnitie *G* to the number *D*: therefore contrarywise (by the Corollary of the fourth of the fifth) as the magnitude *A* is to the magnitude *C*, so is the number *D* to vnitie *G*. Againe forasmuch as how many vnities there are in the number *E*, so many magnitudes also are there in the magnitude *F* equall vnto the magnitude *C*: therefore as the magnitude *C* is to the magnitude *F*, so is vnitie *G* to the number *E*. And it is proued that as the magnitude *A* is to the magnitude *C*, so is the number *D* to vnitie *G*. Wherefore of equalitie (by the 22. of the fifth) as the magnitude *A* is to the magnitude *F*, so is the number *D* to the number *E*. But as the number *D* is to the number *E*, so is the magnitude *A* to the magnitude *B*. Wherefore (by the 11. of the fifth) as the magnitude *A* is to the magnitude *B*, so is the same magnitude *A* to the magnitude *F*: wherefore *A* hath vnto either of these magnitudes *B* and *F* one and the same proportion. Wherefore (by the 9. of the fifth) the magnitude *B* is equall vnto the magnitude *F*. But the magnitude *C* measureth the magnitude *F*: wherefore it also measureth the magnitude *B*: and it likewise measureth the magnitude *A*. Wherefore the magnitude *C* measureth the magnitudes *A* and *B*. Wherefore the magnitudes *A* & *B* are commensurable. If therefore two magnitudes haue such proportion the one to the other, as number hath to number, those magnitudes are comensurable: which was required to be proued.

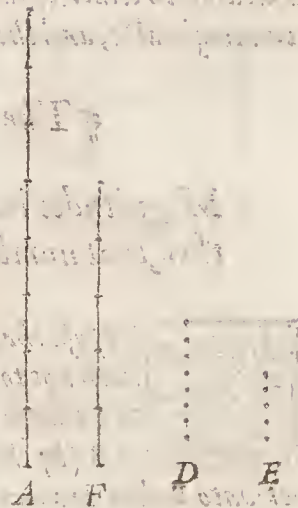


✻ Corollary.

✱ Corollary.

Hereby it is manifest, that if there be two numbers, as D and E , and a right line, as A : it is possible to geue an other line, vnto which the line A shall haue the same proportion, that the number D hath to the number E . For deuide the line A into so many equall partes as there are vnities in the number D (by the 9. of the sixth). And take an other line, as F , which let be composed of so many partes equall to the partes of the line A , as there be vnities in the number E . Wherefore the line A shall be to the line F , as the number D is to the number E . And by this meanes you may vnto any line geue geue an other line commensurable in length. For if two lines be in proportion the one to the other, as number is to number, they shall also be commensurable in length, by this 6. Theoreme.

A Corollary.

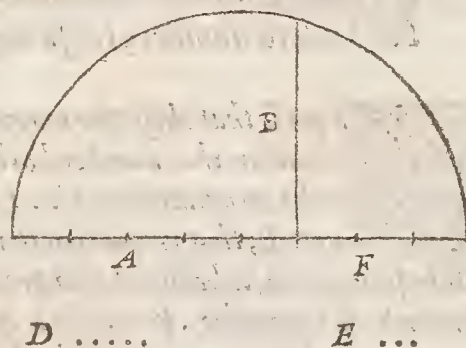


¶ An Assumpt.

Two numbers being geuen, and also a right line.: as the one number is to the other, so to make the square of y line geuen to be to the square of an other line.

Suppose that the numbers geuen be D and E : and let the right line geuen be A . It is required, as the number D is to the number E , so to make the square of the line A to be to the square of an other line. As the number D is to the number E , so let the line A be to the line F (by the former Corollary). And take betwene those two lines A and F the meane proportionall (by the 13. of the sixth) which let be the line B . Now for that as the number D is to the number E , so is the line A to the line F : and as the line A is to the line F , so is the square of the line A to the square of the line F (by the second Corollary of the 20. of the sixth). Wherefore as the number D is to the number E , so is the square of the line A to the square of the line B : which was required to be done.

Construction.



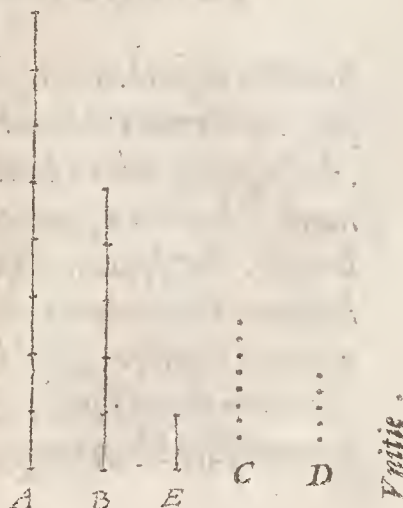
Demonstration.

¶ An other demonstration of the 6. Proposition.

Suppose that these two magnitudes geuen A and B , haue that proportion the one to the other, that the number C hath to the number D . The I say, that those magnitudes are commensurable. How many vnities there are in the number C , into so many equall partes let the magnitude A be deuided, & let the magnitude E be equall vnto one of those partes. Wherefore as vntie is to the number C , so is the magnitude E to the magnitude A . And as the number C is to the number D , so is the magnitude A to the magnitude B . Wherefore of equalitie (by the 22. of the fifth) as vntie is to the number D , so is the magnitude E to the magnitude B . But vntie measureth the number

Construction.

Demonstration.



The tenth Booke

number D. Wherefore the magnitude E measureth the magnitude B. And it also measureth the magnitude A (for that unitie measureth the number C). Wherefore the magnitude E measureth either of these magnitudes A and B. Wherefore the magnitudes A and B are commensurable, and the magnitude E is their common measure.

¶ The 5. Theoreme.

The 7. Proposition.

Magnitudes incommensurable, haue not that proportion the one to the other, that number hath to number.

Demonstration leading to an absurditie.



Suppose that the magnitudes A and B be incommensurable. Then I say, that A hath not to B, that proportion that number hath to number. For if A haue vnto B that proportion that number hath to number, then is A commensurable vnto B (by the 6. of this tenth). But (by supposition) it is not. Wherefore A hath not vnto B that proportion that number hath to number. Magnitudes incommensurable therefore haue not that proportion the one to the other that number hath to number: which was required to be demonstrated.

A —————

B —————

¶ The 6. Theoreme.

The 8. Proposition.

If two magnitudes haue not that proportion the one to the other that number hath to number, those magnitudes are incommensurable.

This is the conuerse of the former, and is proued by an indirect demonstration.



Suppose that these two magnitudes A and B, haue not that proportion the one to the other that number hath to number. Then I say, that A and B are magnitudes incommensurable. For if A and B be commensurable, then shall A haue vnto B, that proportion that number hath to number (by the 5. of this tenth). But (by supposition) it hath not that proportion that number hath to number. Wherefore A and B are incommensurable magnitudes. If therefore two magnitudes haue not that proportion the one to the other that number hath to number, those magnitudes are incommensurable: which was required to be proued.

A —————

B —————

¶ The 7. Theoreme.

The 9. Proposition.

1.

Squares described of right lines commensurable in length, haue that proportion the one to the other, that a square number hath to a square number.

2.

And squares which haue that proportion the one to the other that a square number hath to a square number, shall also haue their sides commensurable in length.

3.

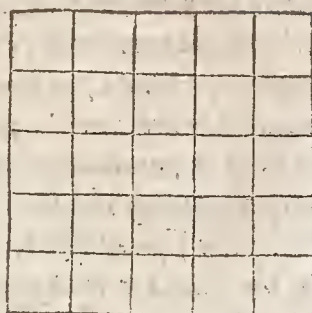
But squares described of right lines incommensurable in length, haue not that proportion the one to the other, that a square number hath to a square number.

4.

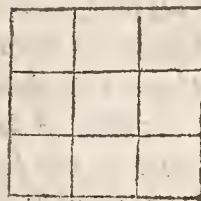
And squares which haue not that proportion the one to the other that a square number hath to a square number, haue not their sides commensurable in length.

Suppose

Suppose that A and B be lines commensurable in length. Then I say that the square of the line A hath unto the square of the line B , that proportion that a square number hath to a square number. For forasmuch as the lines A and B are commensurable in length: therefore the line A hath unto the line B that proportion that number hath to number (by the 5. of this tenth). Let the line A have unto the line B that proportion, that the number C hath to the number D . Now for that as the line A is to the line B , so is the number C to the number D : but the square of the line A is unto the square of the line B in double proportion of that which the line A is unto the line B (for like rectiline figures (by the first corollary of the 20. of the sixth.) are in double proportion of that which the sides of like proportion are) and likewise the square number produced of C is to the square number produced of D , in double proportion of that which the number C is to the number D (for by the 11. of the eighth betwene two square numbers there is one meane proportionall number & a square number is to a square number in double proportion of that which side is unto side). Wherefore as the square of the line A is to the square of the line B , so is the square number produced of the number C , to the square number produced of the number D .



A



B

C D ...

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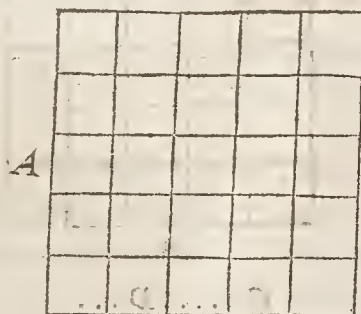
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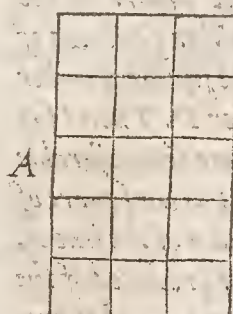
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An other demonstration to proue the same.

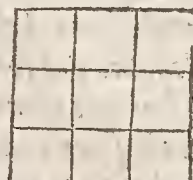
Forasmuch as the lines A and B are commensurable, therefore (by the 5 of this tenth) A hath unto B the same proportion that number hath to number. Let them have that proportion that the number C hath to the number D . And let the number C multiplying himselfe produce the number E , and multiplying the number D , let it produce the number F : and let the number D multiplying himselfe produce the number G . And forasmuch as the number C multiplying himselfe produced the number E , and multiplying the number D it produced the number F : therefore (by the 17. of the seventh) as the number C is to the number D , that is, the line A to the line B so is the number E to the number F . But as the line A is to the line B , so is the square of the line A to the parallelograme contained under the lines A and B (by the first of the sixth) Wherefore as the square of the line A is to that which is contained under the lines A



A



B



B

C

D ...

E

F

G

An other demonstration of the first part.

and

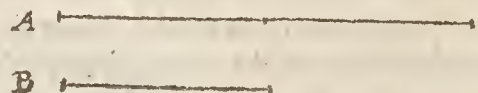
The tenth Booke

and B, so is the number E to the number F. Agayne for as muche as the number C multiplying the number D produced the number F, & the number D multiplying himselfe produced the number G: therefore (by the 17. of the seuenth) as the number C is to the number D, that is, as the line A is to the line B, so is the number F to the number G. But as the line A is to the line B, so is parallelograme contained vnder the lines A and B to the square of the line B (by the first of the sixt). Wherefore as that which is contained vnder the lines A and B is to the square of the line B, so is the number F to the number G. But as the square of the line A is to that which is containd vnder the lines A and B, so is the number E to the number F. Wherefore of equalitie (by the 22. of the fifte) as the square of the line A is to the square of the line B, so is the number E to the number G. But either of these numbers E and G is a square number. For E is produced of the number C multiplied into himselfe, and G is produced of the number D multiplied into himselfe. Wherefore the square of the line A hath vnto the square of the line B that proportion that a square number hath to a square number: which was required to be demonstrated.

An other demonstration of the same first part after Montaureus.

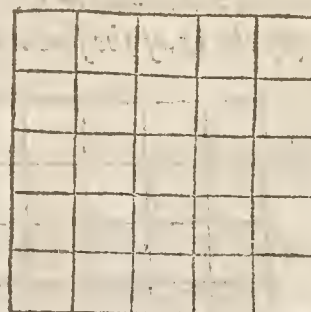
An other demonstration of the same first part after Montaureus.

Suppose that there be two lines commensurable in length A and B. Then I say that the squares described of those lines shalbe in proportion the one to the other as a square number is to a square number. For forasmuch as the lines A and B are commensurable in length, they shalbe in proportion the one to the other as number is to number (by the 5. of this booke). Let A be to B in duple proportion, which is in such proportion as number is to number, namely, as 4. is to 2. and 5. to 3. and so of many other. And (by the second of the eight) take the three least numbers in continuall proportion, and in duple proportion, & let the same be the numbers 4. 2. 1: wherefore by the corollary of the second of the eight, the numbers 4. and 1. shalbe square numbers. (For as 4. is a square number produced of 2. multiplied into himselfe, so is 1. also a square number, for it is produced of vnitie multiplied into himselfe.) I say moreouer that those are the square numbers, whose proportion the squares of the lines A and B haue the one to the other. For as the number 4. is to the number 2. so is the line A to the line B (For either proportion is double by supposition): but as the line A is to the line B, so is the square of the line A to the parallelograme contained vnder the lines A and B (by the first of the sixt). Wherefore as the number 4. is to the number 2.: so shall the square of the line A be to the parallelograme contained vnder the lines A and B. Likewise as the number 2. is to the number 1. so is the line A to the line B (For either proportion is duple by supposition): but as the line A is to the line B, so is the parallelograme contained vnder the lines A and B to the square of the line B (by the selfe same first of the sixt). Wherefore as the number 2. is to 1. so is the parallelograme contained vnder the lines A and B to the square of the line B.: wherefore of equalitie (by the 22. of the fifth) as the square of the line A is to the square of the line B, so is the number 4. to 1. which are proued to be square numbers.



Demonstration of the seconde part which is the conuerse of the former.

But now suppose that the square of the line A, be vnto the square of the line B, as the square number produced of the number C is to the square number produced of the number D. Then I say that the lines A & B are commensurable in length. For for that as the square of the line A is to the square of the line B, so is the square number produced of the number C to the square number produced of the number D: but the proportion of the square of the line A, is vnto the square of the line B double to that proportion which the line A hath vnto the line B (by the corollary of the 20. of



C D ...

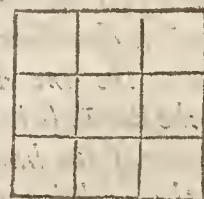
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the

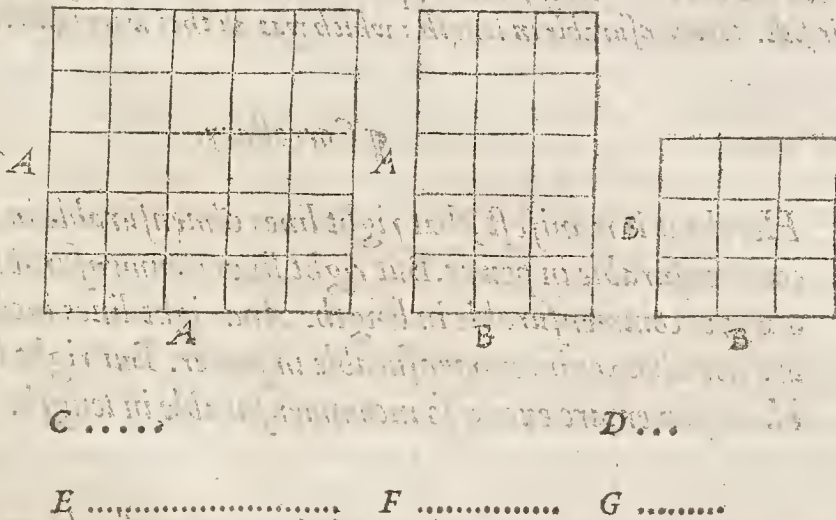
the sixth). And the proportion of the square number which is produced of the number C to the square number produced of the number D is (by the 11. of the eighth) double to that proportion which the number C hath unto the number D. Wherefore as the line A is to the line B, so is the number C to the number D. Wherefore the line A hath unto the line B the same proportion that the number C hath to the number D. Wherefore (by the 6. of this booke) the lines A and B are commensurable in length: which was required to be proved.

An other demonstration to proue the same.

Suppose againe that the square of the line A haue unto the square of the line B, the same proportion, that the square number E hath to the square number G. Then I say that the lines A and B are commensurable in length. For suppose that the side of

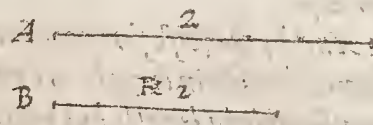
An other demonstration of the second part.

the square number E be the number C, & let the side of the square number G be the number D: and let the number C multiplying the number D produce the number F. Wherefore these numbers E, F, G, are in continuall proportion, and in the same proportion that the number C is to the



number D (by the 17. and 18. of the seventh). And forasmuch as the meane proportionall betwene the squares of the lines A and B is that which is contained vnder the lines A and B. (Which though it might briefly be proued; yet we take it as now). And likewise the meane proportionall betwene the numbers E and G is the number F (by the 20. of the seventh): therefore as the square of the line A is to that which is contained vnder the lines A and B, so is the number E to the number F, and as that which is contained vnder the lines A and B is to the square of the line B, so is the number F to the number G. But as the square of the line A is to that which is contained vnder the lines A and B, so is the line A to the line B (by the first of the sixth). Wherefore the lines A and B are in the same proportion that the number E is to the number F, that is, that the number C is to the number D. Wherefore the lines A and B are commensurable in length (by the 6. of this tenth): which was required to be proued.

But now suppose that the lines A and B be incommensurable in length. Then I say that the square of the line A hath not unto the square of the line B that proportion that a square number hath to a square number. For if the square of the line A haue unto the square of the line B, the same proportion that a square number hath to a square number, then the lines A and B be commensurable in length (by the second part of this proposition). But by supposition they are not. Wherefore the square of the line A, hath not unto the square of the line B that proportion that a square number hath to a square number: which was required to be proued.



This Assump^t followeth as a Corollary of the 25. but (so as it might also be here in Methode, placed) you shall finde it after the 53. of this booke, absolutely demonstrated: for there it serueth to the 54. his demonstration.

Demonstration of the third part.

Again suppose that the square of the line A haue not unto the square of the line B, the

EE. i.

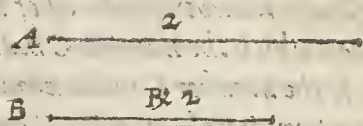
same

Demonstration of the fourth part which is the converse of the 3.

The tenth Booke

Conclusion of
the whole pro-
position.

same proportion that a square number hath to a square number. Then I say that the lines *A* and *B* are incommensurable in length. For if the lines *A* and *B* be commensurable in length, then the square of the line *A* should haue vnto the square of the line *B*, the same proportion that a square number hath to a square number, by the first part of this proposition, but by supposition it hath not, wherfore the lines *A* and *B* are not commensurable in length. Wherfore they are incommensurable in length. Wherfore squares made of right lines commensurable in length, haue that proportion the one to the other, that a square number hath to a square number. And squares which haue that proportion the one to the other, that a square number hath to a square number, shall also haue the sides commensurable in length. But squares described of right lines incommensurable in length, haue not that proportion the one to the other that a square number hath to a square number. And squares which haue not that proportion the one to the other, that a square number hath to a square number, haue not also their sides commensurable in length: which was all that was required to be proued.



¶ Corollary.

A Corollary.

Hereby it is manifest, that right lines commensurable in length, are also euer commensurable in power. But right lines commensurable in power, are not alwayes commensurable in length. And right lines incommensurable in length are not alwayes incommensurable in power. But right lines incommensurable in power, are euer also incommensurable in length.

Profe of the
first part of
the Corollary.

For forasmuch as squares made of right lines commensurable in length, haue that proportion the one to the other, that a square number hath to a square number (by the first part of this proposition): but magnitudes which haue that proportion the one to the other, that number simply hath to number, are (by the sixt of the tenth) commensurable. Wherfore right lines commensurable in length, are commensurable not onely in length, but also in power.

Profe of the
second part.

Againe forasmuch as there are certaine squares which haue not that proportion the one to the other that a square number hath to a square number, but yet haue that proportion the one to the other which number simply hath to number: their sides in dede are in power commensurable, for that they describe squares which haue that proportion which number simply hath to number, which squares are therefore commensurable (by the 6. of this booke): but the said sides are incommensurable in length by the latter part of this proposition. Wherfore it is true that lines commensurable in power, are not straight way commensurable in length also.

Profe of the
third part.

And by the selfe same reason is proued also that third part of the corollary, that lines incommensurable in length, are not alwayes incommensurable in power. For they may be incommensurable in length, but yet commensurable in power. As in those squares which are in proportion the one to the other, as number is to number, but not as a square number is to a square number.

Profe of the
fourth part.

But right lines incommensurable in power, are alwayes also incommensurable in length. For if they be commensurable in length, they shal also be commensurable in power by the first part of this Corollary. But they are supposed to be incommensurable in length, which is absurd. Wherfore right lines incommensurable in power, are euer incommensurable in length.

For

For the better vnderstanding of this proposition and the other following, I haue here added certayne annotacions taken out of *Montaureus*. And first as touching the signification of wordes and termes herein vsed, which are such, that vnlesse they be well marked and peysed, the matter will be obscure and hard, and in a maner inexplicable.

First, this ye must note, that lines to be commensurable in length, and lines to be in proportion the one to the other, as number is to number is all one. So that whatsoeuer lines are commensurable in length, are also in proportion the one to the other, as number is to number. And conuersedly what so euer lynes are in proportion the one to the other, as number is to number, are also commensurable in length, as it is manifest by the 5 and 6 of this booke. Likewise lines to be incommensurable in length, and not to be in proportion the one to the other, as number is to number is all one, as it is manifest by the 7. and 8. of this booke. Wherefore that which is sayd in this Theorème, ought to be vnderstand of lines commensurable in length, and incommensurable in length.

This moreouer is to be noted, that it is not all one, numbers to be square numbers, and to be in proportiō the one to the other, as a square number is to a square number. For although square numbers be in proportion the one to the other, as a square number is to a square number, yet are not all those numbers which are in proportion the one to the other, as a square number is to a square number, square numbers. For they may be like superficiall numbers, and yet not square numbers, which yet are in proportiō the one to the other, as a square number is to a square number. Although all square numbers are like superficiall numbers. For betwene two square numbers there falleth one meane proportionall number (by the 11. of the eight). But if betwene two numbers, there fall one meane proportionall number, those two numbers are like superficiall numbers (by the 20. of the eight). So also if two numbers be in proportion the one to the other, as a square number is to a square number, they shall be like superficiall numbers by the first corollary added after the last proposition of the eight booke.

And now to know whether two superficiall numbers geuen, be like superficiall numbers or no, it is thus found out. First if betwene the two numbers geuen, there fall no meane proportionall, then are not these two numbers like superficiall numbers (by the 18. of the eight). But if there do fall betwene them a meane proportionall, then are they like superficiall numbers (by the 20. of the eight). Moreouer two like superficiall numbers multiplied the one into the other, do produce a square number (by the first of the ninth). Wherefore if they do not produce a square number, then are they not like superficiall numbers. And if the one being multiplied into the other, they produce a square number, then are they like superficiall (by the 2. of the ninth). Moreouer if the said two superficiall numbers be in superparticular, or superbi-partient proportion, then are they not like superficiall numbers. For if they should be like, then should there be a meane proportionall betwene them (by the 20. of the eight). But that is contrary to the Corollary of the 20. of the eight.

And the easilier to conceiue the demonstrations following, take this example of that which we haue sayd.

Suppose that there be a line, namely, C, which imagine to be foure foote long: and let there be another line D, which let be three foote long. And (by the 13. of the sixt) take the meane proportionall betwene the lines A, D, which let be the line, B. Wherefore the square of the line B shall be equall to the rectangle parallelogramme containd vnder the line C and D (by the 17. of the sixt). Which square shall containe 12. foote, & so much also shall the parallelogramme described of the lines C & D containe. Take also two other lines E and F, of which let E be 3. foote long, and let F be a foote long. And let the meane proportionall betwene the lines E and F, be the line A. Now then the square of the line A shall containe 3. foote, as also doth the parallelogramme described of the lines E, F. The I say, that the square of the line B, which containeth 12. foote, is to the square of the line A, which containeth 3. foote, in that proportion that a square number is to a square number. For as the number 12. is to the number 3, so is the square of the line B, which containeth 12. foote, to the square of the line A, which containeth 3. foote. But the numbers 12. and 3. are like superficiall numbers, for the sides of 12. which are 2. and 6, are proportionall with the sides of 3. which are 1. and 3. Wherefore the square of the line B, which containeth 12. foote, shall be vnto the square of the line A, which containeth 3. foote, in that proportion that a like superficiall number is to a like superficiall number. But like superficiall numbers are in proportion the one to the other, as a square number is

Certayne annotations out of Montaureus.

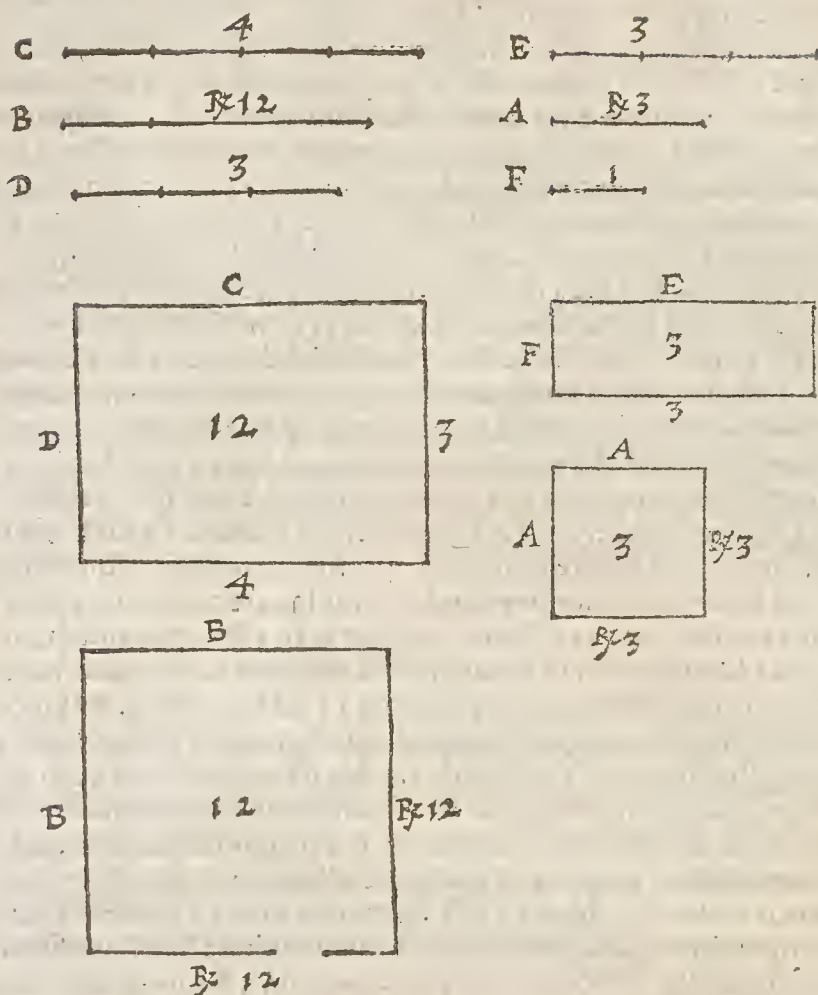
Rules to know whether two superficiall numbers be like or no.

The tenth Booke

to a square nūber, which square numbers are 4. and 1. (by the 26. of the eight). Wherefore the square of the line B, which containeth 12. foote, is to the square of the line A, which containeth 3. foote, in that proportion that a square number is to a square number, namely, that the number 4. is to the number 1: which proportion is quadruple. For the greater square which is 12, containeth the lesse square which is 3, foure times. Wherefore the side of the square 12, which is the line B, is double to the side of the square 3, which is the line A. Wherefore the line B is to the line A, in that proportion that number is to number. Wherefore (by

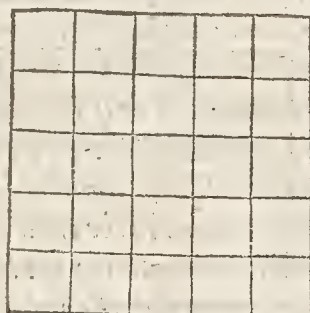
the 5. of this booke) the lines B & A are commensurable in length. Which is a supposition necessary to conclude the first part of this Theoreme, namely, that the squares of such lines are in proportion the one to the other, that a square number is to a square number.

So also the nūber which denominateth the greater terme of the proportion of the line B to the line A, which is 2, if it be multiplyed into it selfe, it maketh a square number, namely, 4. Likewise the number which denominateth the lesse terme, namely, 1. if it be multiplyed into it selfe, it maketh no more but 1. Which vnitie is also in power a square nūber. Wherefore the square of the line B, is to the square of the line A, in that proportion that a square number is to a square number, namely, that 4. is to 1. By this you see (which thing was before noted) that it is not all one, numbers to be square numbers, and to be in proportion the one to the other, as a square number is to a square number. For it is manifest, that the numbers 12. and 3. are not square numbers, when yet the squares expressed by those numbers are in that proportion. But the side of the square 12. although it can not of it selfe be expressed by number distinctly, to say that the side thereof is so many foote long, which feete square taken, make the whole square 12: yet being referred or compared to an other thyng, namely, to the side of the square 3, which side also of it selfe can not be expressed by number, it is vnto the sayde side of the square 3, in double proportion. For the one square being quadruple to the other square (as is the square of the line B, which containeth 12. foote, to the square of the line A, which containeth 3. foote) hath his side double to the side of the other square, by this generall Corollary of the 20. of the sixt, *like rectiline figures are in double proportion the one to the other that their sides of like proportion are*. Now if a man will say, that the side of the square 12. may be measured, for that hys proportion which it hath to the side of the square 3, is measured by 2 (forasmuch as it is dupla proportion: this is to be considered, that in so saying, you say not, that that magnitude can of it selfe be measured, but the proportion thereof. For, that magnitude, namely, the side of the square 12, should by it selfe be measured, when without any respect of the proportion of it to an other thing, we may say that the side of the square, which containeth 12. foote, is so many foote long, the number of which foote multiplyed into it selfe should make that number 12. But this is not possible, for that 12. is not a square number. Wherefore thus you may say: In asmuch as that square 12. is considered by it self, without hauing any respect of the proportion of it to any other thing, but onely as it is 12. foote, it hath no side which of it selfe can be expressed by number. But if it be compared to any other thing, namely, to the square of 3. foote, then may you say that the side of the square 12. is 2, and the side of the square 3. is 1. But this is the denomination of that proportion which is called duple, which proportion can not be or considered

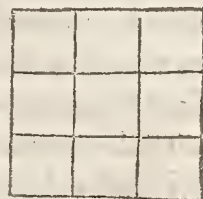


in fewer termes then two, when as it is a relation of one thing to an other thing : wherefore 2. is not the number of such feete, of which there are 12. in the square. Agayne, if the number 2. shoulde be the side of the square 12, so that that side should be 2, then of the multiplication of 2. into it selfe, shoulde not be made that square 12, but an other square which should be 4. foote : as of the number 2. multiplied into him selfe is produced the square number 4. Neither, also if any other number, measure the side of the square 12, and the sayd number be multiplied into him selfe shall it euer make the number 12. When yet all numbers denominating the side of any square number, if they be multiplied into them selues, they make the number which denominateth the square, whose sides they denominate. As 2. multiplied into him selfe maketh 4 : 3. maketh 9 : 4. maketh 16 : and so likewise of all others. Wherefore it is not all one, magnitudes to be in proportion the one to the other, as number is to number, and euery one of them to be measured by him selfe without any respect had of the proportion. As here the side of the square 12. can of it selfe by no meanes be measured, but being compared to any other magnitude, namely, to the side of the square 3, the proportion thereof is expressed by number. So also the side of the square 3, and of all other square figures, whose areas yet can not be expressed by square numbers. And that which we here say, is manifest euen by the wordes of *Euclide* in the 5. 6. 7. and 8. Theoremes of this booke. Where he sayth not, that magnitudes commensurable and incommensurable are of the selues or of their owne nature expressed by numbers, but that either they haue or haue not that proportion which number hath to number. Which thing not being well considered, it should seme hath caused many to erre as hereafter shall be made manifest. And in deede they which haue demonstrated this Theoreme, may seme to some rather to haue demonstrated it particularly & not vniuersally. And doubtles I iudge there are some which vnderstand their sayings otherwise then they ment : when as they thinke, that they suppose certayne lines not onely commensurable in length, as they are supposed to be in the Proposition, but also such, that ech of them apart may be expressed by some certayne number. Wherefore for want of right vnderstanding, this mought they say of their demonstrations: that wheras they thought that they had concluded that generally, which is in this theoreme of *Euclide*, *Squares described of lines comensurable in length, are in proportion the one to the other that a square number is to a square number*: they conclude particularly, thys onely : *Squares described of lines which may by them selues be expressed by some certayne number, are in proportion.* &c. which yet is otherwise, and their demonstrations are right & agreable with the Theoreme. Onely the picture of the figures which the Greeke boke hath, may seme to bring some doubt. For the squares are so, described with certayn litle areas, that the number of them may be denominated by a square number : whereby it mought seme that the lines A & B which describe the squares, ought to be such that they may be expressed by some certayne nuber. As the line A to be 5. foote, and the line B 3. foote. As the two former figures here set declare. Which thyng yet *Euclide* supposeth not, but only requireth that they be commensurable in length, as in the former example of the two squares, the whole area of one of which is 12, and the whole area of the other is 3. For although their sides ca not by them selues be expressed by some certayne number, yet are they commensurable in length. Moreouer thys describing of the squares of the lines A and B, deuided by certayne litle areas, may cause this error, that a man should thinke that it is all one two numbers to be square numbers, and to be in proportion the one to the other as a square number is to a square number. For the number of the areas in the square of the line A is a square number, namely, 25. produced of the roote 5, which is the length of the line A. Likewise the number of the areas of the square of the line B, is a square nuber, namely, 9, which is produced of the roote 3, which is the length of the line B. But we haue before declared that it is not all one, numbers to be called square numbers, and to be in proportion the one to the other, as a square number is to a square number. Wherefore as touching those areas containd in the greater square, which is of the line A, and which are in number 25, they do expresse that square number 25, which is produced of the number 5. multiplied into him selfe, which number 5. is the greater extreme of the proportion betwene 5. and 3, which is the proportion of the lines A and B. And this proportion, namely, of 5. to 3. causeth that the lines A and B are commensurable in length (by the 6. of this booke). The same may be sayd also of the areas of the lesse square. Neither is it of necessitie that you vnderstand those areas to be squares, as either feete square or pases square which make the whole square, although in deede they

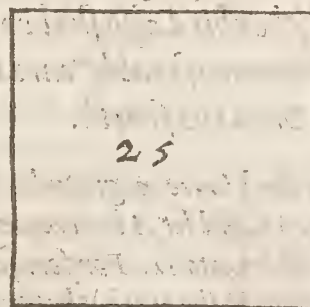
Note.



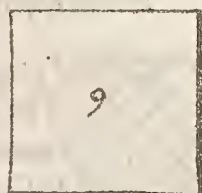
A



B



A

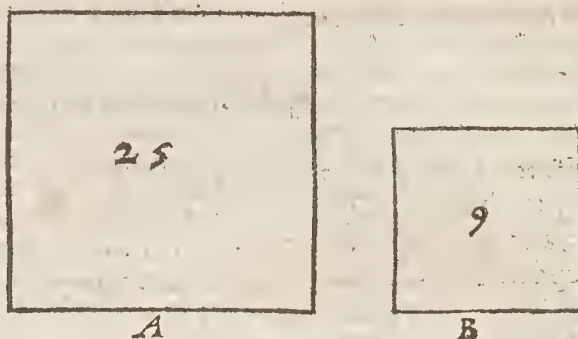


B

EE. iij. may

The tenth Booke

may be such, so that the sides of those squares be so many foote long, as 5. foote or 3. foote. Howbeit this is of necessitie that the numbers which expresse the number of the feete square or pases square, contained in the squares, be either both of the square numbers, as in these square figures of the lines A, B, or that both of them be like superficiall numbers, as in the former squares which were 12. and 3: of which numbers it is manifest by that which hath before bene said, that they are like superficiall numbers, and therefore haue that proportion the one to the other, that a square number hath to a square number. And therefore you may describe the squares of the lines A, B, without any distinction of such little areas, so that the squares may be voyde and emptie, and containd onely of foure right lines, as in the figure here put.



¶ An Assumpt.

Forasmuch as in the eight booke in the 26. proposition it was proued, that like playne numbers haue that proportion the one to the other, that a square number hath to a square number: and likewise in the 24. of the same booke it was proued, that if two numbers haue that proportion the one to the other, that a square number hath to a square number, those numbers are like plaine numbers. Hereby it is manifest, that unlike plaine numbers, that is, whose sides are not proportionall, haue not that proportion the one to the other, that a square number hath to a square number. For if they haue, then should they be like plaine numbers, which is contrary to the supposition. Wherefore unlike plaine numbers haue not that proportion the one to the other, that a square number hath to a square number. And therefore squares which haue that proportion the one to the other, that unlike plaine numbers haue, shall haue their sides incommensurable in length (by the last part of the former proposition) for that those squares haue not that proportion the one to the other that a square number hath to a square number.

This assumpt
as the conuerse
of the 26. of
the eight.

¶ The 8. Theoreme.

The 10. Proposition.

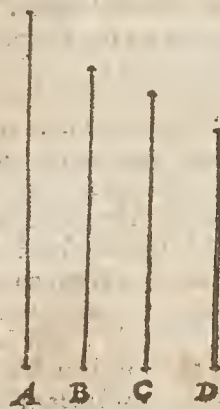
If foure magnitudes be proportionall, and if the first be commensurable vnto the second, the third also shal be commensurable vnto the fourth. And if the first be incommensurable vnto the second, the third shall also be incommensurable vnto the fourth.



Demonstratio
of the first
part.

Suppose that these foure magnitudes A, B, C, D, be proportionall. As A is to B, so let C be to D, and let A be commensurable vnto B. Then I say that C is also commensurable vnto D. For forasmuch as A is commensurable vnto B, it hath (by the fifth of the tenth) that proportion that number hath to number. But as A is to B, so is C to D. Wherefore C also hath vnto D that proportion that number hath to number. Wherefore C is commensurable vnto D (by the 6. of the tenth). But now suppose that the magnitude A be incommensurable vnto the magnitude B. Then I say that the magnitude C also is incommensurable vnto the magnitude D. For forasmuch as A is incommensurable vnto B, therefore (by the 7. of this booke) A hath not vnto B such proportion as number hath to number. But as A is to B, so is C to D. Wherefore C hath not vnto D such proportion as number hath to number. Wherefore (by the 8. of the tenth) C is incommensurable vnto D. If therefore there be foure magnitudes proportionall, and if the first be commensurable vnto the

Demonstra-
tion of the
second part.



second,

second, the third also shall be commensurable vnto the fourth. And if the first be incommensurable vnto the second, the third shall also be incommensurable vnto the fourth: which was required to be proued.

¶ A Corollary added by Montaneus.

If there be foure lines proportionall, and if the two first, or the two last be commensurable in power onely, the other two also shall be commensurable in power onely. This is proued by the 22. of the sixt, and by this tenth proposition. And this Corollary Euclide vseth in the 27. and 28. propositions of this booke, and in other propositions also.

A Corollary.

¶ The 3. Probleme.

The 11. Proposition.

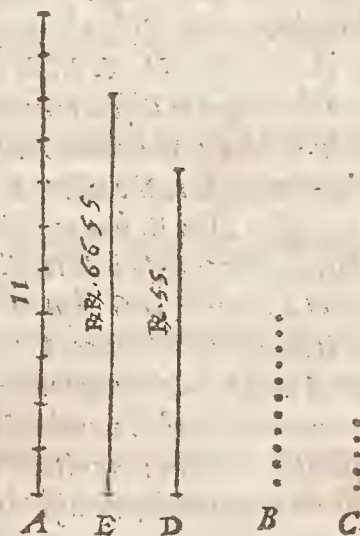
Vnto a right line first set and geuen (which is called a rationall line) to finde out two right lines incommensurable, the one in length onely, and the other in length and also in power.



Vppose that the right line first set and geuen, which is called a rationall line of purpose be *A*. It is required vnto the said line *A*, to finde out two right lines incommensurable, the one in length onely, the other both in length and in power.

Take (by that which was added after the 9. proposition of this booke) two numbers *B* and *C*, not hauing that proportion the one to the other, that a square number hath to a square number, that is, let them not be like plaine numbers (for like plaine numbers by the 26. of the eight haue that proportion the one to the other that a square number hath to a square number). And as the number *B* is to the number *C*, so let the square of the line *A* be vnto the square of an other line, namely, of *D* (how to do this was taught in the assumpt put before the 6. proposition of this booke.) Wherefore the square of the line *A*, is vnto the square of the line *D* commensurable (by the sixt of the tenth.)

And forasmuch as the number *B* hath not vnto the number *C*, that proportion that a square number hath to a square number, therefore the square of the line *A* hath not vnto the square of the line *D*, that proportion that a square number hath to a number. Wherefore by the 9. of the tenth, the line *A* is vnto the line *D* incommensurable in length onely. And so is found out the first line, namely, *D* incommensurable in length onely to the line geuen *A*. Agayne take (by the 13. of the sixt) the meane proportionall betwene the lines *A* and *D*, and let the same be *E*. Wherefore as the line *A* is to the line *D*, so is the square of the line *A* to the square of the line *E* (by the Corollary of the 20. of the sixt). But the line *A* is vnto the line *D* incommensurable in length. Wherefore also the square of the line *A* is vnto the square of the line *E* incommensurable by the second part of the former proposition. Now forasmuch as the square of the line *A* is incommensurable to the square of the line *E*, it followeth (by the definition of incommensurable lynes) that the line *A* is incommensurable in power to the line *E*. Wherefore vnto the right line geuen, and first set, *A*, which is a rationall line, and which is supposed to haue such diuisions and so many partes as ye list to conceyue in minde, as in this example 11, whereunto, as was declared in the 5. definition of this booke, may be compared infinite other lines, either commen-



To finde out the first line incommensurable in length onely to the line geuen.

To finde out the second line incommensurable both in length and in power to the line geuen.

The tenth Booke

surable or incommensurable, is found out the line D incommensurable in length onely. Wherefore the line D is rationally (by the sixth definitiō of this booke) for that it is incommensurable in length onely to the line A, which is the first line set, and is by suppositiō rational. There is also found out the line E, which is vnto the same line A incommensurable, not onely in length but also in power, which line E compared to the rational line A, is by the definition irrational. For Euclide alwayes calleth those lines irrational, which are incommensurable both in length and in power to the line first set, and by supposition rational.

¶ The 9. Theoreme.

The 12. Proposition.

Magnitudes commensurable to one and the selfe same magnitude : are also commensurable the one to the other.

Construction.

Suppose that either of these magnitudes A and B, be commensurable vnto the magnitude C. Then I say that the magnitude A is commensurable vnto the magnitude B. For forasmuch as the magnitude A is commensurable vnto the magnitude C,

therefore (by the 5. of the tenth) A hath vnto C such proportion as number hath to number. Let A haue vnto C that proportion that the number D hath to the number E. Againe forasmuch as B is commensurable vnto C, therefore (by the selfe same) C hath vnto B that proportion that number hath to number. Let C haue vnto B that proportion that the number F hath vnto the number G. Now then take the least numbers in continuall proportion and in these proportions geuen, namely, that the number D hath to the number E, and that the number F hath to the number G (by the 4. of the eighth): which

Demonstration.

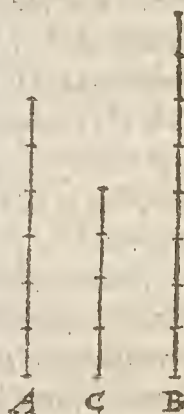
let be the numbers H, K, L. So that as the number D is to the number E, so let the number H be to the number K, and as the number F is to the number G, so let the number K be to the number L. Now for that as A is to C, so is D to E, but as D is to E so is H to K, therefore as A is to C, so is H to K. Againe for that as C is to B, so is F to G, but as F is to G so is K to L: therefore as C is to B, so is K to L. But it is now proued that as A is to C, so is H to K. Wherefore of equalitie (by the 22. of the fifth) as A is to B, so is the number H to the number L. Wherefore A hath vnto B such proportion as number hath to number. Wherefore (by the sixth of the tenth) the magnitude A is commensurable vnto the magnitude B. Magnitudes therefore commensurable to one and the selfe same magnitude, are also commensurable the one to the other: which was required to be proued.

This is with Lambert an Assumpt, but utterly improperly: Flustates maketh it a Corollary, but the Greeke and Montaneus make it a proposition: but euery way an infallible truth is it.

¶ An Assumpt.

If there be two magnitudes compared to one and the selfe same magnitude, and if the one of them be commensurable vnto it, and the other incommensurable: those magnitudes are incommensurable the one to the other.

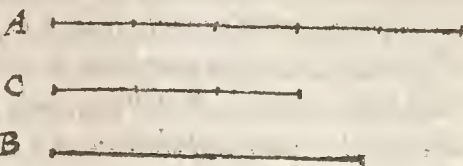
Suppose



D	F
E	G
	H ...
	K ..
	L

Suppose that there be two magnitudes, namely, *A* and *B* and let *C* be a certayne other magnitude. And let *A* be commensurable vnto *C*, and let *B* be commensurable vnto the selfe same *C*. Then I say that the magnitude *A* is incommensurable vnto *B*. For if *A* be commensurable vnto *B*, forasmuch as *A* is also commensurable vnto *C* therefore (by the 12. of the tenth) *B* is also commensurable vnto *C*: which is contrary to the supposition.

Demonstration leading to an absurditie.



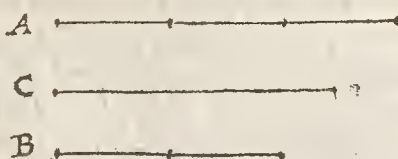
¶ The 10. Theoreme.

The 13. Proposition.

If there be two magnitudes commensurable, and if the one of them be incommensurable to any other magnitude: the other also shall be incommensurable vnto the same.

Suppose that these two magnitudes *A*, *B* be commensurable the one to the other, and let the one of them, namely *A*, be incommensurable vnto an other magnitude, namely, vnto *C*. Then I say that the other magnitude also, namely *B*, is incommensurable vnto *C*. For if *B* be commensurable vnto *C*, then forasmuch as *A* is commensurable vnto *B*, therefore (by the 12. of the tenth) the magnitude *A* also is commensurable vnto the magnitude *C*. But it is supposed to be incommensurable vnto it, which is impossible. Wherefore the magnitudes *B* and *C* are not commensurable. Wherefore they are incommensurable. If therefore there be two magnitudes commensurable, and if the one of them be incommensurable to any other magnitude, the other also shall be incommensurable vnto the same: which was required to be proued.

Demonstration leading to an absurditie.

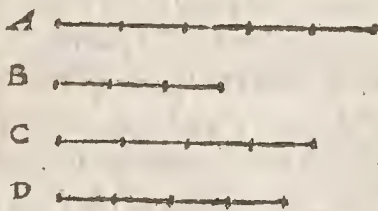


¶ A Corollary added by Montareus.

Magnitudes commensurable to magnitudes incommensurable, are also incommensurable the one to the other.

A Corollary.

Suppose that the magnitudes *A* and *B* be incommensurable the one to the other, and let the magnitude *C* be commensurable to *A*, and let the magnitude *D* be commensurable vnto *B*. Then I say that the magnitudes *C* and *D* are incommensurable the one to the other. For *A* and *C* are commensurable, of which the magnitude *A* is incommensurable vnto *B*, wherefore by this 13. proposition the magnitudes *C* and *B* are also incommensurable: but the magnitudes *B* and *D* are commensurable wherefore by the same, or by the former assumpt, the magnitudes *C* and *D* are incommensurable the one to the other. This corollary, Theon useth often times as in the 22. 26. and 36 propositions of this booke, and in other propositions also.



¶ An Assumpt.

Two vnequall right lines being geuen, to finde out how much the greater is in power more then the lesse.

Suppose that the two vnequall right lines geuen, be *AB* and *C*, of which let *AB* be the greater.

The tenth Booke

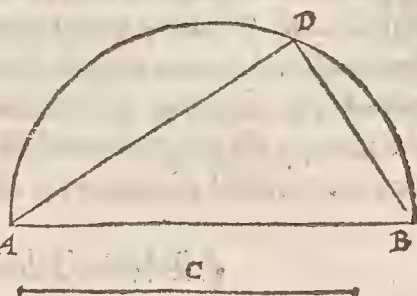
greater. It is required to finde out how much more in power the line AB is then the line C .

Construction. Describe upon the line AB a semicircle ADB . And unto it from the point A apply (by the first of the fourth) a right line AD , equall unto the line C , and draw a right line from D to B . Now it is manifest that the angle ADB is a right angle (by the 31. of the third): and that the line AB is in power more then the line AD , that is, then the line C , by the line DB , by the 47. of the first.

Demonstration.

A Corollary.

And like in sorte, two right lines being geuen, by this meanes may be founde out a right lyne which containeth them both in power. Suppose that the two right lines geuen be AD and DB . It is required to finde out a right lyne that containeth them both in power. Let the lines AB and DB be so put, that they comprehend a right angle ADB , and draw a right line from A to B . Now agayne it is manifest (by the 47. of the first) that the line AB containeth in power the lines AD and DB .



The 11. Theoreme.

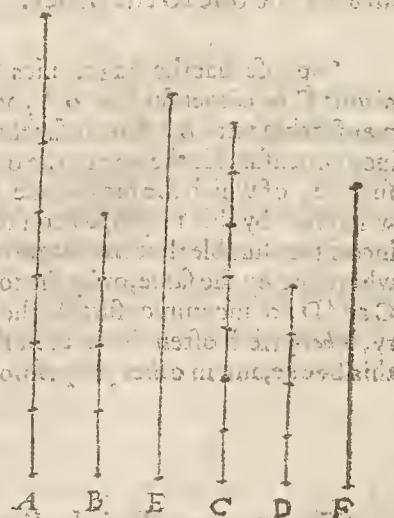
The 14. Proposition.

If there be foure right lines proportionall, and if the first be in power more then the second by the square of a right line commensurable in length vnto the first, the third also shalbe in power more then the fourth, by the square of a right line commensurable vnto the third. And if the first be in power more then the second by the square of a right line incommensurable in length vnto the first, the third also shall be in power more then the fourth by the square of a right line incommensurable in length to the third.



Demonstration.

Suppose that these foure right lines A, B, C, D , be proportionall. As A is to B , so let C be to D . And let A be in power more then B , by the square of the line E . And likewise let C be in power more then D , by the square of the line F . Then I say that if A be commensurable in length vnto the line E , C also shall be commensurable in length vnto the line F . And if A be incommensurable in length to the line E , C also shall be incommensurable in length to the line F . For for that as A is to B , so is C to D , therefore as the square of the line A is to the square of the line B , so is the square of the line C to the square of the line D (by the 22. of the sixth). But by supposition vnto the square of the line A are equall the squares of the lines E and B , and vnto the square of the line C are equall the squares of the of the lines D and F : Wherefore as the squares of the lines E and B (which are equall to the square of the line A) are to the square of the line B , so are the squares of the lines D and F (which are equall to the square of the line C) to the square of the line D (by the seuenth of the fifth). Wherefore (by the 17. of the fifth) as the square of the line E is to the square of the line B , so is the square of the line F to the square of the line D . Wherefore also as the line E is to the line B , so is the line F to the line D (by the second parte of the 22. of the sixth) wherefore contrariwise (by the Corollary of the fourth of the fifth) as B is to E , so is D to F . But (by supposition)



as A is to B , so is C to D , Wherefore of equallitie (by the 22. of the first) as A is to E , so is C to F . If therefore A be commensurable in length vnto E , C also shall be commensurable in length vnto F : and if it be incommensurable in length vnto E , C also shall be incommensurable in length vnto F , by the 10. of this booke. If therefore there be foure right lines proportionall, and if the first be in power more then the second by the square of a right line commensurable in length vnto the first, the third also shall be in power more then the fourth, by the square of a right line commensurable in length vnto the third: and if the first be in power more then the second, by the square of a right line incommensurable in length vnto the first, the third also shall be in power more then the fourth, by the square of a right line incommensurable in length to the third: which was required to be proued.

Note that the line A may be proued to be in proportion to the line E , as the line C is to the line F , by another way, namely, by conuersion of proportion (of some, as we haue before noted, called inuerse proportion) by the 19. of the first. For, forasmuch as the foure lines A, B, C, D , are proportionall: therefore (by the 22. of the first) their squares also are proportionall. And forasmuch as the antecedent, namely, the square of the line A exceedeth the consequent, namely, the square of the line B , by the square of the line E : and the other antecedent, namely, the square of the line C , exceedeth the other consequent, namely, the square of the line D , by the square of the line F , therefore as the square of the line A is to the excessse, namely, to the square of the line E , so is the square of the line C to the excessse, namely, to the square of the line F . Wherefore (by the second part of the 22. of the first) as the line A is to the line E , so is the line C to the line F .

Another way to proue that the lines A, E, C, F , are proportionall.

¶ The 12. Theoreme.

The 15. Proposition.

If two magnitudes commensurable be composed, the whole magnitude composed also shall be commensurable to either of the two partes. And if the whole magnitude composed be commensurable to any one of the two partes, those two partes shall also be commensurable.

Et these two commensurable magnitudes AB and BC , be composed or added together. Then I say, that the whole magnitude AC is commensurable to either of these partes AB and BC . For forasmuch as AB and BC are commensurable, therefore (by the first definition of the tenth) some one magnitude measureth them both.

Let there be a magnitude that measureth them, and let the same be D . Now forasmuch as D measureth

AB and BC , it shall also measure the whole magnitude composed AC , by this common sentence, what

soeuer magnitude measureth two other magnitudes,

shall also measure the magnitude composed of them. But the same D measureth AB and BC (by supposition). Wherefore D measureth AB, BC , and AC . Wherefore AC is commensurable to either of these magnitudes AB and BC .

But now suppose that the whole composed magnitude AC be commensurable to any one of these two magnitudes AB or BC , let it be commensurable I say vnto AB . Then I say, that

the two magnitudes AB and BC are commensurable. For forasmuch as AB and AC are commensurable, some one magnitude measureth them (by the first definition of the tenth).

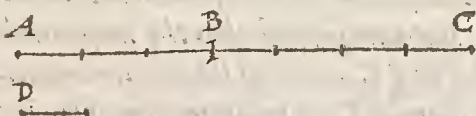
Let some magnitude measure them, and let the same be D . Now forasmuch as D measureth AB and AC , it also measureth the residue BC , by this common sentence, what soeuer mea-

sureth the whole and the part taken away, shall also measure the residue. But the same D measureth the magnitude AB (by supposition). Wherefore D measureth either of these magnitudes AB and BC . Wherefore the magnitudes AB and BC are commensurable. If ther-

fore two magnitudes commensurable be composed, the whole magnitude composed also shall be commensurable to either of the two partes. And if the whole magnitude composed be com-

mensurable

Demonstration of the first part.



Demonstration of the second part which is the conuerse of the first.

The tenth Booke

mensurable to any one of the two partes, those two partes shall also be commensurable: which was required to be demonstrated.

¶ A Corollary added by Montaureus.

A Corollary.

If an whole magnitude be commensurable to one of the two magnitudes which make the whole magnitude, it shall also be commensurable to the other of the two magnitudes. For if the whole magnitude AC be commensurable vnto the magnitude BC , then by the 2. part of this 15 Proposition: the magnitudes AB and BC are commensurable. Wherefore (by the first part of the same) the magnitude AC shall be commensurable to either of these magnitudes AB and BC . This Corollary Theon useth in the demonstration of the 17. Proposition and also of other Propositions. Howbeit Euclide left it out, for that it seemed easie as in a maner do all other Corollaries.

¶ The 13. Theoreme.

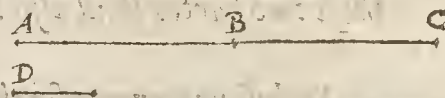
The 16. Proposition.

If two magnitudes incommensurable be composed, the whole magnitude also shall be incommensurable vnto either of the two partes componentes. And if the whole be incommensurable to one of the partes componentes, those first magnitudes also shall be incommensurable.

Demonstration of the first part by an argument leading to an absurditie.



Et these two incommensurable magnitudes AB & BC , be composed, or added together. Then I say, that the whole magnitude AC , is incommensurable to either of these magnitudes AB and BC . For if AC and AB be not incommensurable, then some one magnitude measureth them (by the first definition of the tenth). Let there be such a magnitude, if it be possible, and let the same be D . Now forasmuch as D measureth CA and AB , it also measureth the residue BC , & it likewise measureth AB . Wherefore D measureth AB and BC . Wherefore (by the first definition of the tenth) the magnitudes AB and BC are commensurable. But it is supposed that they are incommensurable: which is impossible. Wherefore no magnitude doth measure the magnitudes AB and AC . Wherefore the magnitudes CA and AB are incommensurable. In like sort also may we proue, that the magnitudes AC and CB are incommensurable.



Demonstration of the second part leading also to an impossibilitie. And this second part is the conuerse of the first.

But now suppose that the magnitude AC be incommensurable to one of these magnitudes AB or BC , and first let it be incommensurable vnto AB . Then I say, that the magnitudes AB and BC are incommensurable. For if they be commensurable some one magnitude measureth them. Let some one magnitude measure them, & let the same be D . Now forasmuch as D measureth AB and BC , it also measureth the whole magnitude AC . And it measureth AB : Wherefore D measureth these magnitudes CA and AB . Wherefore CA & AB are commensurable. And they are supposed to be incommensurable: which is impossible. Wherefore no magnitude measureth AB and BC . Wherefore the magnitudes AB and BC are incommensurable. And in like sort may they be proued to be incommensurable, if the magnitude AC be supposed to be incommensurable vnto BC . If therefore there be two magnitudes incommensurable composed, the whole also shall be incommensurable vnto either of the two partes component, and if the whole be incommensurable to one of the partes component, those first magnitudes shall be incommensurable: which was required to be proued.

¶ A Corollary added by Montaureus.

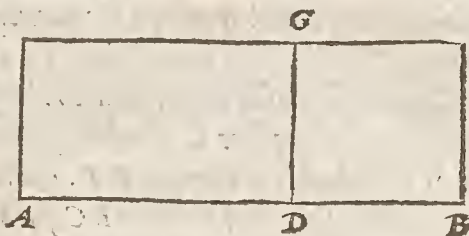
If an whole magnitude be incommensurable to one of the two magnitudes which make the whole magnitude, it shall also be incommensurable to the other of the two magnitudes. For if the whole

whole magnitude AC be incōmensurable vnto the magnitude BC , then by the 2. part of this 16. Theoreme, the magnitudes AB and BC shall be incommensurable. Wherefore by the first part of the same Theoreme, the magnitude AC shall be incommensurable to either of these magnitudes AB and BC . This Corollary Theoreme vseth in the demonstration of the 73. Theoreme, & also of other Propositions.

¶ An Assumpt.

If vpon a right line be applied a parallelogramme wanting in figure by a square: the parallelogramme so applied, is equall to that parallelogramme which is containd vnder the segmentes of the right line, which segmentes are made by reason of that application.

Suppose that vpon a right line AB be applied a parallelogramme AG , wanting in forme by the square GB . Then I say, that AG is equall vnto that which is containd vnder AD and DB , which thing is of it self manifest. For forasmuch as GB is a square, therefore the line DG is equall vnto the line DB : and the parallelogramme AG is that which is containd vnder the lines AD and DG , that is, vnder the lines AD and DB . If therefore vpon a right line be applied a parallelogramme wanting in figure by a square: the parallelogramme applied is equall to the parallelogramme which is containd vnder the segmentes of the right line, which are made by reason of that application. which was required to be demonstrated.



This Assumpt I before added as a Corollary out of *Flussates* after the 28. Proposition of the first booke.

¶ The 14. Theoreme.

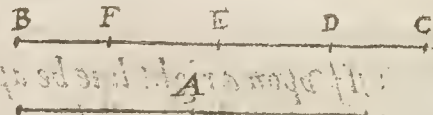
The 17. Proposition.

If there be two right lines vnequall, and if vpon the greater be applied a parallelogramme equall vnto the fourth part of the square of the lesse line, and wanting in figure by a square, if also the parallelogramme thus applied deuide the line where vpon it is applied into partes commensurable in length: then shall the greater line be in power more then the lesse, by the square of a line commensurable in length vnto the greater. And if the greater be in power more then the lesse by the square of a right line commensurable in length vnto the greater, and if also vpon the greater be applied a parallelogramme equall vnto the fourth part of the square of the lesse line, and wanting in figure by a square: then shall it deuide the greater line into partes commensurable.



Suppose that these two right lines A and BC , be vnequall: of which let BC be the greater. And vpon the line BC let there be applied (by the 28. of the sixth) a parallelogramme equall vnto the fourth part of the square of the line A being the lesse (that is, equall vnto the square described vpon halfe of

the line A) and wanting in figure by a square. And let the same parallelogramme be that which is contained under the lines $B D$ and $D C$. And (by supposition) let the lines $B D$ and $D C$ be commensurable in length. Then I say, that the line $B C$ is in power more then the line A , by the square of a line commensurable in length unto the sayd line $B C$. Deuide (by the 10. of the first) the line $B C$ into two equall partes in the point E . And (by the third of the first) unto the line $D E$ put an equall line $E F$. Wherefore the residue $D C$ is equall unto the residue $B F$. And forasmuch as the right line



Construction.

Demonstration
of the first
part.

$B C$ is deuided into two equall partes in the point E , and into two vnequall partes in the point D , therefore (by the 5. of the second) the rectangle figure comprehended under the lines $B D$ and $D C$ together with the square of the line $E D$, is equall to the square of the line $E C$. And in the same proportion are they eche being taken fower times by the 15. of the fifth. Wherefore that which is contained under the lines $B D$ and $D C$ taken fower times together with the square of the line $E D$ taken also fower times, is equall to the square of the line $E C$ taken fower times. But unto that which is contained under the lines $B D$ & $D C$ fower times is equal the square of the line A by supposition: for the parallelogramme contained under the lines $B D$ and $D C$ once is supposed to be equall to the fourth part of the square of the line A . And unto the square of the line $D E$ taken fower times is equall the square of the line $D F$, for the line $D F$ is double to the line $D E$. And unto the square of the line $E C$ fower times taken, is equall the square of the line $B C$, for the line $B C$ is also double to the line $C E$. Wherefore the squares of the lines A and $D F$ are equall unto the square of the line $B C$. Wherefore the square of the line $B C$, is greater then the square of the line A , by the square of the line $D F$. Wherefore the greater line $B C$ is in power more then the lesse line A , by the square of the line $D F$. Now resteth to proue that the line $B C$ is commensurable in length unto the line $D F$. Forasmuch as by supposition the line $B D$ is commensurable in length unto $D C$, therefore (by the 15. of the tenth) the whole line $B C$ is commensurable in length unto the line $D C$: but the line $D C$ equall to the line $B F$. Wherefore the whole line $B C$ is commensurable in length unto the lines $B F$ & $C D$. Let the two lines $B F$ and $C D$ be imagined to be so composed that they make one line. Now forasmuch as the whole line $B C$ is commensurable in length to the two lines $B F$ and $C D$ taken as one line: therefore the lines $B F$ and $C D$ taken as one line are commensurable in length to the line $F D$ (by the 2. part of the 15. of the tenth). Wherefore also the whole line $B C$ is commensurable in length to the line $F D$ by the first part of the same: this may also be proued by the corollary put after the 16. proposition of this booke. Wherefore the line $B C$ is in power more then the line A by the square of a line commensurable in length unto the line $B C$.

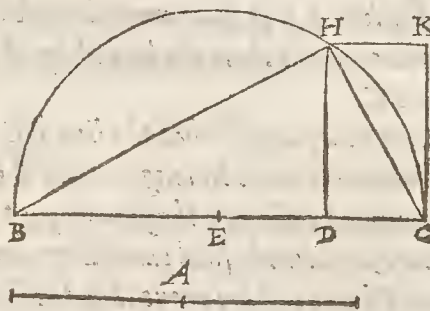
Demonstration
of the
second part
which is the
conuerse of
the first.

But now suppose that the line $B C$ be in power more then the line A , by the square of a line commensurable in length unto the line $B C$. And upon the line $B C$ let there be applied a rectangle parallelogramme equall unto the fourth part of the square of the line A , and wanting in figure by a square, and let the sayd parallelogramme be that which is contained under the lines $B D$ and $D C$. Then must we proue that the line $B D$ is unto the line $D C$ commensurable in length. The same constructions and suppositions, that were before, remayning, we may in like sort proue that the line $B C$ is in power more then the line A , by the square of the line $F D$. But by supposition the line $B C$ is in power more then the line A by the square of a line commensurable unto it in length. Wherefore the line $B C$ is unto the line $F D$ commensurable in length. Wherefore the line composed of the two lines $B F$ and $D C$ is commensurable in length unto the line $F D$ (by the second part of the 15. of the tenth). Wherefore (by the 12. of the tenth or by the first part of the 15. of the tenth) the line $B C$ is commensurable in length to the line composed of $B F$ and $D C$. But the whole line composed of $B F$ and $D C$ is commensurable in length unto $D C$. For $B F$ (as before hath bene proued) is equall to $D C$. Wherefore the line

line BC is commensurable in length vnto the line DC (by the 12. of the tenth). Wherefore also the line BD is commensurable in length vnto the line DC (by the second part of the 15. of the tenth). If therefore there be two right lines vnequall, and if vpon the greater be applied a parallelograme equall vnto the fourth part of the square of the lesse and wanting in figure by a square, if also the parallelograme thus applied deuide the line whereupon it is applied into partes commensurable in length: then shall the greater line be in power more then the lesse by the square of a line commensurable in length vnto the greater. And if the greater be in power more then the lesse by the square of a line commensurable in length vnto the greater, and if also vpon the greater be applied a parallelograme equall vnto the fourth part of the square made of the lesse and wanting in figure by a square: then shall it deuide the greater line into partes commensurable in length: which was required to be proued.

Campane after this proposition teacheth how we may redily apply vpon the line BC a parallelograme equall to the fourth part of the square of halfe of the line A , and wanting in figure by a square, after this maner.

Deuide the line BC into two lines in such sort that halfe of the line A shalbe the meane proportionall betwene those two lines, which is possible, when as the line BC is supposed to be greater then the line A , and may thus be done. Deuide the line BC into two equal partes in the point E and describe vpon the line BC a semicircle BHC . And vnto the line BC , and from the point C erect a perpendicular line CK and put the line CK equall to halfe of the line A . And by the point K draw vnto the line EC a parallel line KH cutting the semicircle in the point H ; (which it must needes cut, forasmuch as the line BC is greater then the line A). And fro the point H draw vnto the line BC a perpendicular line HD : which line HD , forasmuch as by the 34. of the first it is equall vnto the line CK , shall also be equall to halfe of the line A : draw the lines BH and HC . Now then by the 31. of the third the angle BHC is a right angle. Wherefore by the corollary of the eight of the sixth booke the line HD is the meane proportionall betwene the lines BD and DC . Wherefore the halfe of the line A which is equall vnto the line HD is the meane proportionall betwene the lines BD and DC . Wherefore that which is contained vnder the lines BD and DC is equall to the fourth part of the square of the line A . And so if vpon the line BD be described a rectangle parallelograme hauing his other side equall to the line DC , there shalbe applied vpon the line BC a rectangle parallelograme equall vnto the square of halfe of the line A , and wanting in figure by a square: which was required to be done.



How to de-
uide the line
 BC redely in
such sort as is
required in
the proposition.

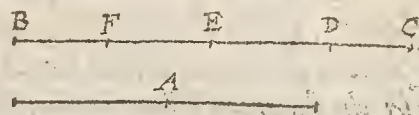
The 15. Theoreme.

The 18. Proposition.

If there be two right lines vnequall, and if vpon the greater be applied a parallelograme equall vnto the fourth part of the square of lesse, and wanting in figure by a square, if also the parallelograme thus applied deuide the line whereupon it is applied into partes incommensurable in length: the greater line shalbe in power more then the lesse line by the square of a line incommensurable in length vnto the greater line. And if the greater line be in power more then the lesse line, by the square of a line incommensurable in length vnto the greater, and if also vpon the greater be applied a parallelograme equall vnto the fourth part of the square of the lesse and wanting in figure by a square: then shall it deuide the greater line into partes incommensurable in length.

The tenth Booke

Suppose that these two right lines A and BC be unequall the one to the other, of which let BC be the greater. And upon the same BC apply a parallelogramme equall unto the fourth part of the square of the line A , and wanting in figure by a square: which how to doo was before taught in the end of the former proposition. And let the sayd parallelogramme be that which is contained under the lines BD and DC . And let BD be incommensurable in length unto DC . Then I say that the line BC is in power more then the line A , by the square of a right line incommensurable in length unto the line BC . First let the same order of construction, and demonstration be obserued in this which was in the former proposition. And we may in like sort proue that the line BC is in power more then the line A by the square of the line DF . Now then must we proue that the lines BC and DF are incommensurable in length: Forasmuch as by supposition the line BD is incommensurable in length unto the line DC : therefore (by the 16. of the tenth) the line BC is incommensurable in length unto the line CD . But DC is commensurable to these two lines BF and DC added together. For BF is equall unto DC . Wherefore (by the 13. of the tenth) BC is incommensurable unto these two lines BF and DC composed. Wherefore by the second part of the 16. of the tenth the line composed of the lines BF and DC taken as one line is incommensurable in length unto the line FD . Wherefore by the first part of the same 16. proposition the line BC is incommensurable in length unto the line FD . Wherefore the line BC is in power more then the line A by the square of a line incommensurable in length unto the line BC .



Demonstration of the first part.

Demonstration of the second part which is the conuerse of the former.

But now suppose that the line BC be in power more then the line A by the square of a line incommensurable in length unto BC . And upon the line BC let there be applied a parallelogramme equall unto the fourth part of the square of the line A , and wanting in figure by a square, and let the said parallelogramme be that which is contained under the lines BD and DC . Then must we proue that the line BD is unto the line DC incommensurable in length. The same order of construction and demonstration being kept, we may in like sort proue that the line BC is in power more then the line A by the square of the line FD . But now (by supposition) the line BC is in power more then the line A by the square of a line incommensurable in length unto BC . Wherefore the line BC is unto the line FD incommensurable in length. Wherefore the line composed of BF and DC taken as one line, shall be incommensurable in length to the line FD (by the second part of the 16. of the tenth): wherefore also by the first part of the same, the line BC shall be incommensurable in length to the line composed of the lines BF and DC . But the line composed of the lines BF and DC is commensurable in length to the line DC (for that BF (as before hath bene proued) is equall to DC). Wherefore the line BC is incommensurable in length to the line DC (by the 13. of the tenth). Wherefore by the second part of the 16. of the tenth, the line BD is incommensurable in length unto the line DC . If therefore there be two right lines unequall, and if upon the greater be applied a parallelogramme equall unto the fourth part of the square of the lesse line, & wanting in figure by a square, if also the parallelogramme thus applied deuide the line wherupon it is applied into partes incommensurable in length: the greater line shall be in power more then the lesse line by the square of a line incommensurable in length unto the greater. And if the greater line be in power more then the lesse, by the square of a line incommensurable in length unto the greater, and if also upon the greater be applied a parallelogramme equall unto the fourth part of the square of the lesse line, and wanting in figure by a square, then shall it deuide the greater line into partes incommensurable in length: which was required to be demonstrated.

This Proposition may also be demonstrated by the former proposition, namely, the first part of this by

by the second part of the former, and the second part of this by the first part of the former, by an argument leading to an absurditie. For as touching the first part of this proposition, the line B C containyng in power more then the line A by the square of the line F D, if the line B C be not incommensurable vnto the line F D, then is it commensurable vnto it. Wherefore (by the second part of the 17 proposition) the lines B D and D C also are commensurable, which is impossible, for they are supposed to be incommensurable. So likewise as touching the second parte of the same, the line B C containyng in power more then the line A by the square of the line F D, if the line D B be not incommensurable to the line D C, then is it commensurable vnto it: wherefore (by the first part of the 17. proposition) the lines B C and F D are also commensurable, which were absurde. For the lines B C and F D are supposed to be incommensurable: which was required to be proued.

An other demonstration by an argument leading to an absurditie.

¶ An assumpt.

Forasmuch as it hath bene proued that lines commensurable in length, are alwayes also commensurable in power, but lines commensurable in power are not alwayes commensurable in length, but may be in length both commensurable and also incommensurable: it is manifest, that if vnto the line propounded, which is called rationall of purpose, a certayne line be commensurable in length, it ought to be called rationall and commensurable vnto it, not only in length, but also in power: for lines commensurable in length are also alwayes commensurable in power. But if vnto the line propounded which is called rationall of purpose, a certayne line be commensurable in power, then if it be also commensurable vnto it in length, it is called rationall and commensurable vnto it both in length and in power. But againe if vnto the said line geuen which is called rationall, a certayne line be commensurable in power, and incommensurable in length, that also is called rationall, commensurable in power onely.

An Assumpt.

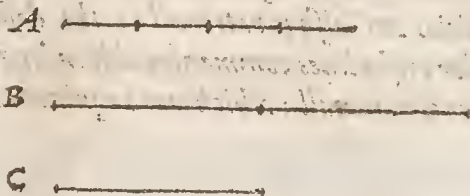
An annotacion of Proclus.

He calleth those lines rationall, which are vnto the rationall line first set commensurable in length & in power, or in power onely. And there are also other right lines, which are vnto the rationall line first set, incommensurable in length, and are vnto it commensurable in power onely, and therefore they are called rationall, & commensurable the one to the other: for which cause they are rationall. But euen these lines may be commensurable the one to the other, either in length, and therefore in power, or els in power onely. Now if they be commensurable in length, then are those lines called rationall, commensurable in length, but yet so that they be vnderstand to be in power commensurable: but if they be commensurable the one to the other in power onely, they also are called rationall commensurable in power onely.

¶ A Corollary.

And that two lines or more being rationall and commensurable in length to the rational line first set, are also commensurable the one to the other in length, hereby it is manifest: for forasmuch as they are rationall and commensurable in length to the rational line first set, but those magnitudes which are commensurable to one and the selfe same magnitude, are also commensurable the one to the other (by the 12. of the tenth) wherefore the rationall lines, commensurable in length to the rational line first set, are also commensurable in length the one to the other. And as touching those which are rationall commensurable in power onely to the rational line first set, they also must needs be at the least commensurable in power the one to the other. For forasmuch as their squares are rationall they shall bee commensurable to the square of the rational line first set. Wherefore by the 12. of this booke, they are also commensurable the one to the other. Wherefore their lines are at the least commensurable in power the one to the other. And it is possible also that they may be commensurable in length the one to the other. For suppose that A be a rational line first set, and let the line B be vnto the same rational line A commensurable in power onely, that is, incommensurable in length vnto it. Let there be also an other line C commensurable in power onely to the same rational line A.

A Corollary added by Montaurens.



The tenth Booke

furable in length to the lyne B (which is possible by the principles of this booke.) Now by the 13. of the tenth, it is manifest that the line C is incommensurable in length vnto the line A. But the square of the line A is commensurable to the square of the line B by supposition, and the square of the line C is also commensurable to the square of the line B by supposition. Wherefore by the 12. of this booke, the square of the line C is commensurable to the square of the line A. Wherefore by the definition, the line C shall be rationally commensurable in power onely to the line A, as also is the line B. Wherefore there are geuen two rationally commensurable in power onely to the rationally line first set, and commensurable in length the one to the other.

Cause.

Cause of increasing the difficulty of this booke.
Note.

Here is to be noted, which thing also we before noted in the definitions, that Campane and others which followed him, brought in these phrases of speeches, to call some lynes rationally in power onely, and other some rationally in length and in power, which we cannot finde that Euclide euer vsed. For these wordes in length and in power are neuer referred to rationalitie or irrationalitie, but alwayes to the commensurabilitie or incommensurabilitie of lines. Which perverting of wordes (as was there declared) hath much increased the difficulty and obscures of this booke. And now I thinke it good agayne to put you in minde, that in these propositions which follow, we must euer haue before our eyes the rationally line first set, vnto which other lines compared are either rationally or irrational, according to their commensurability or incommensurability.

¶ The 16. Theoreme.

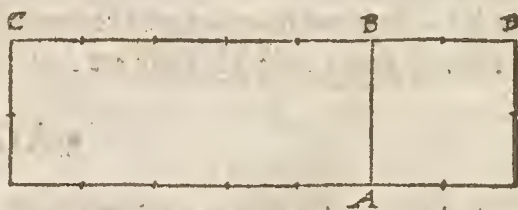
The 19. Proposition.

A rectangle figure comprehended vnder right lines commensurable in lengthe, being rationally according to one of the foresaide wayes: is rationally.

Construction.

Demonstration.

Suppose that this rectangle figure AC be comprehended vnder these right lines AB and BC being commensurable in length, and rationally according to one of the foresaid wayes. Then I say that the superficies AC is rational, describe (by the 46. of the first) vpon the line AB a square AD. Wherefore that square AD is rational by the definition. And forasmuch as the line AB is commensurable in length vnto the line BC, and the line AB is equall vnto the lyne BD, therefore the lyne BD is commensurable in length vnto the line BC. And as the line BD is to the line BC, so is the square DA to the superficies AC (by the first of the sixth): but it is proued that the line BD is commensurable vnto the line BC, wherefore (by the 10. of the tenth) the square DA is commensurable vnto the rectangle superficies AC. But the square DA is rational, wherefore the rectangle superficies AC also is rational by the definition. A rectangle figure therefore comprehended vnder right lines commensurable in length, being rationally according to one of the foresaid wayes is rational: which was required to be proued.

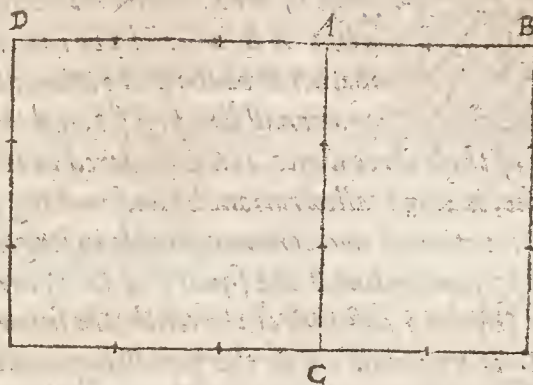


Diuers cases in this proposition.
The second case.

Where as in the former demonstration the square was described vpon the lesse line, we may also demonstrate the Proposition, if we describe the square vpon the greater line, and that after thys maner: Suppose that the rectangle superficies BC be containd of these vnequall lines AB and AC, which let be rationally commensurable the one to the other in length. And let the line AC be the greater. And

vpon

Upon the line AC describe the square DC . Then I say, that the parallelogramme BC is rationally. For the line AC is commensurable in length vnto the line AB by supposition, and the line DA is equall to the line AC . Wherefore the line DA is commensurable in length to the line AB . But what proportion the line DA hath to the line AB , the same hath the square DC to the parallelogramme CB (by the first of the sixth). Wherefore (by the 10. of this booke) the square DC is commensurable to the parallelogramme CB . But it is manifest, that the square DC is rationally, for that it is the square of a rationally line, namely, AC . Wherefore (by the definition) the parallelogramme also CB is rationally.

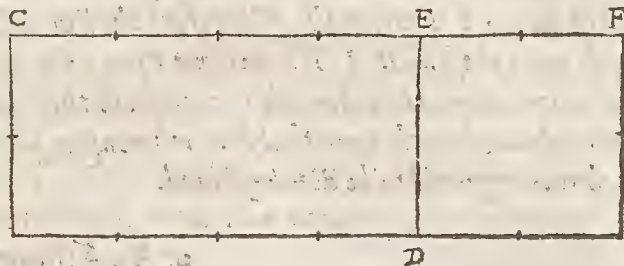


The first kind of rationally lines commensurable in length.

Moreouer, forasmuch as those two former demonstrations seeme to speake of that parallelogramme which is made of two lines, of which any one may be the line first set, which is called the first rationally line, from which (we sayd) ought to be taken the measures of the other lines compared vnto it, and the other is commensurable in length to the same first rationally line, which is the first kinde of rationally lines commensurable in length: I thinke it good here to set an other case of the other kinde of rationally lines; of lines I say rationally commensurable in length compared to an other rationally line first set, to declare the generall truth of this Theoreme, and that we might see that this particle according to any of the foresayd wayes was not here in vaine put. Now then suppose first a rationally line AB . Let there be also a parallelogramme CD contayned vnder the lines

This particle in the proposition (according to any of the foresayd wayes) was not in vaine put.

CE and ED , which lines let be rationally, that is commensurable in length to the first rationally line propounded AB . Howbeit, let those two lines CE and ED be diuers and vnequall lines vnto the first rationally line AB . Then I say, that the parallelogramme CD is rationally. Describe the square of the line DE , which let be DF . First it is manifest (by the 12. of this booke) that the lines CE & ED , are commensurable in length the one to the other.



The second kinde of rationally lines commensurable in length.

For either of them is supposed to be commensurable in length vnto the line AB . But the line ED is equall to the line EF . Wherefore the line CE is commensurable in length to the line EF . But as the line CE is to the line EF , so is the parallelogramme CD to the square DF (by the first of the sixth). Wherefore (by the 10. of this booke) the parallelogramme CD shall be commensurable to the square DF . But the square DF is commensurable to the square of the line AB which is the first rationally line propounded. Wherefore (by the 12. of this booke) the parallelogramme CD is commensurable to the square of the line AB . But the square of the line AB is rationally (by the definition). Wherefore by the definition also of rationally figures, the parallelogramme CD shall be rationally.

The third case.

Now resteth an other case of the thirde kinde of rationally lines commensurable in length the one to the other, which are to the rationally line AB first set commensurable in power onely, and yet are therefore rationally lines. And let the lines CE and ED be commensurable in length the one to the other. Now then let the selfe same construction remaine that was in the former: so that let the lines CE and ED be rationally commensurable in power onely vnto the line AB : But let them be commensurable in length the one to the other. Then I say, that in this case also the parallelogramme CD is rationally. First it may be proued as before, that the parallelogramme CD is commensurable to the square DF . Wherefore (by the 12. of this booke) the parallelogramme CD shall be commensurable to the square of the line AB . But the square of the line AB is rationally. Wherefore (by the definition) the parallelogramme CD shall be also rationally. This case is well to be noted. For it serueth to the demonstration and vnderstanding of the 25. Proposition of this booke.

The third kinde of rationally lines commensurable in length. The fourth case.

The 17. Theoreme.

The 20. Proposition.

If vpon a rationally line be applied a rationally rectangle parallelogramme: the other side that maketh the breadth thereof shall be a rationally line and commensurable in length vnto that line wherupon the rationally parallelogramme is applied.

The tenth Booke

This proposition is the conuerse of the former proposition.

Construction.

Demonstration.



Suppose that this rationall rectangle parallelogramme AC , be applied vpon the line AB , which let be rationall according to any one of the foresaid wayes (whether it be the first rationall line set, or any other line commensurable to the rationall line first set, and that in length and in power, or in power onely: for

one of these three wayes, as was declared in the Assumpt put before the 19. Proposition of this booke, is a line called rationall) and making in breadth the line BC . Then I say, that the line

BC is rationall and commensurable in length vnto the line BA . Describe (by the 46. of the first) vpon the line BA a square AD . Wherefore (by the 9. definitio of the tenth) the square

AD is rationall. But the parallelogramme

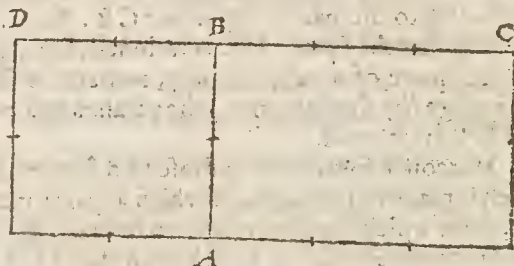
AC also is rationall (by supposition). Wherefore (by the conuerse of the definition of rationall figures, or by the 12. of this booke) the

square DA is commensurable vnto the parallelogramme AC . But as the square DA is

to the parallelogramme AC , so is the line DB to the line BC (by the first of the sixth). Wherefore (by the 10. of the tenth) the line DB is commensurable vnto the line BC . But the line

DB is equall vnto the line BA . Wherefore the line AB is commensurable vnto the line BC . But the line AB is rationall. Wherefore the line BC also is rationall and commensurable in

length vnto the line BA . If therefore vpon a rationall line be applied a rationall rectangle parallelogramme, the other side that maketh the breadth thereof shall be a rationall line commensurable in length vnto that line whereupon the rationall parallelogramme is applied: which was required to be demonstrated.



¶ An Assumpt.

A line containyng in power an irrationall superficies, is irrationall.

An Assumpt.

Suppose that the line AB containe in power an irrationall superficies, that is, let the square described vpon the line AB , be equall vnto an irrationall superficies. Then I say, that the line AB is irrationall. For if the line AB be rationall, the shall the square of the line AB be also rationall. For so was it put in the definitions. But (by supposition) it is not. Wherefore the line AB is irrationall. A line therefore containyng in power an irrationall superficies, is irrationall.

¶ The 18. Theoreme. The 21. Proposition.

A rectangle figure comprehended vnder two rationall right lines commensurable in power onely, is irrationall. And the line which in power containeth that rectangle figure is irrationall, & is called a medially line.



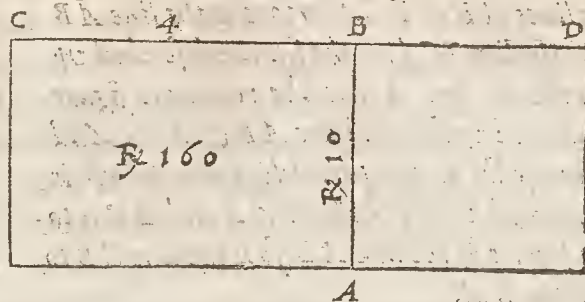
Construction.
Demonstration.

Suppose that this rectangle figure AC be comprehended vnder these rationall right lines AB and BC commensurable in power onely. Then I say, that the superficies AC is irrationall: and the line which containeth it in power is irrationall, and is called a medially line. Describe (by the 46. of the first) vpon the line

AB a square AD . Wherefore the square AD is rationall. And forasmuch as the line AB is vnto the line BC incommensurable in length, for they are supposed to be commensurable in power onely, and the line AB is equall vnto the line BD , therefore also the line BD is

vnto

unto the line BC incommensurable in length. And as the line DB is to the line BC , so is the square AD to the parallelogramme AC (by the first of the sixth). Wherefore (by the 10. of the tenth) the square DA is unto the parallelogramme AC incommensurable. But the square DA is rationall. Wherefore the parallelogramme AC is irrational. Wherefore also the line that containeth the superficies AC in power, that is whose square is equall unto the parallelogramme AC , is (by the Assumpt going before) irrational. And it is called a medially line, for that the square which is made of it, is equall to that which is containd under the lines AB and BC , and therefore it is (by the second part of the 17. of the sixth) a meane proportionall line betwene the lines AB and BC . A rectangle figure therefore comprehended under rationall right lines which are commensurable in power onely, is irrational. And the line which in power containeth that rectangle figure is irrational, and is called a medially line.



At this Proposition doth Euclide first entreate of the generation and production of irrational lines. And here he searcheth out the first kinde of them, which he calleth a medially line. And the definition thereof is fully gathered and taken out of this 21. Proposition, which is this. A medially line is an irrational line whose square is equall to a rectangled figure containd of two rationall lines commensurable in power onely. It is called a medially line, as Theon rightly sayth, for two causes, first for that the power or square which it produceth is equall to a medially superficies or parallelogramme. For as that line which produceth a rationall square, is called a rationall line, and that line which produceth an irrational square, or a square equall to an irrational figure generally is called an irrational line: so is that line which produceth a medially square, or a square equall to a medially superficies, called by speciall name a medially line. Secondly it is called a medially line, because it is a meane proportionall betwene the two lines commensurable in power onely which comprehend the medially superficies.

Definition of
a medially line.

¶ A Corollary added by Flussates.

A rectangle parallelogramme containd under a rationall line and an irrational line, is irrational. For if the line AB be rationall, and if the line CB be irrational, they shall be incommensurable. But as the line BD (which is equall to the line BA) is to the line BC , so is the square AD to the parallelogramme AC . Wherefore the parallelogramme AC shall be incommensurable to the square AD which is rationall (for that the line AB wherupon it is described is supposed to be rationall). Wherefore the parallelogramme AC which is containd vnder the rationall line AB , and the irrational line BC , is irrational.

A Corollary.

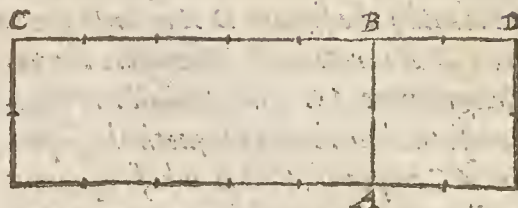
¶ An Assumpt.

If there be two right lines, as the first is to the second, so is the square which is described vpon the first, to the parallelogramme which is contained vnder the two right lines.

Suppose that there be two right lines AB and BC . Then I say that as the line AB is to the line BC , so is the square of the line AB , to that which is contained vnder the lines AB and BC . Describe (by the 46. of the first) vpon the line AB a square AD . And make perfect the parallelogramme AC . Now for that as the line AB is to the line BC (for the line AB is equall to the line BD), so is the square AD to the parallelogramme CA by the first of the sixth and

This assumpt
is nothing els
but a part of
the first pro-
position of the
sixt booke.

and AD is the square which is made of the line AB , and AC is that which is contained under the lines BD and BC , that is, under the lines AB & BC : therefore as the line AB is to the line BC , so is the square described upon the line AB to the rectangle figure contained under the lines AB & BC . And conuersedly as the parallelograme which is contained under the lines AB and BC is to the square of the line AB , so is the line CB to the line BA .



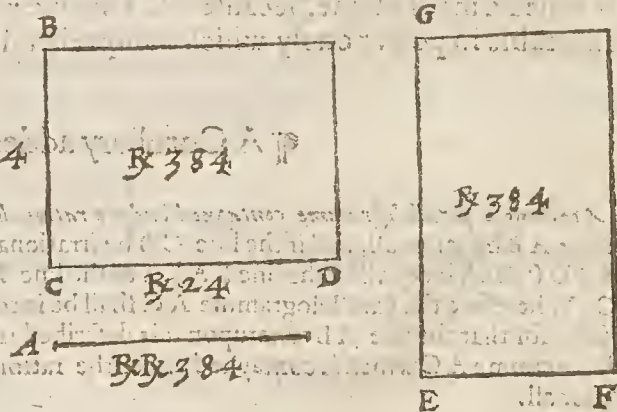
The 19. Theoreme.

The 22. Proposition.

If upon a rationall line be applied the square of a mediall line: the other side that maketh the breadth thereof shalbe rationall, and incommensurable in length to the line wherupon the parallelograme is applied.

Demonstration.

Suppose that A be a mediall line, and let BC be a line rationall, and upon the line BC describe a rectangle parallelograme equall vnto the square of the line A , and let the same be BD making in breadth the line CD . Then I say that the line CD is rationall and incommensurable in length vnto the line CB . For forasmuch as A is a mediall line, it containeth in power (by the 21. of the tenth) a rectangle parallelograme comprehended under rationall right lines commensurable in power onely. Suppose that it containe in power the parallelograme GF : and by supposition it also containeth in power the parallelograme BD . Wherefore the parallelograme BD is equall vnto the parallelograme GF : and it is also equiangle vnto it, for that they are ech rectagle. But in parallelogrames equall and equiangle the sides which containe the equall angles, are reciprocal (by the 14. of the sixth). Wherefore what proportio the line BC hath to the line EG , the same hath the line EF to the line CD . Therefore (by the 22. of the sixth) as the square of the line BC is to the square of the line EG , so is the square of the line EF to the square of the line CD . But the square of the line BC is commensurable vnto the square of the line EG (by supposition). For either of them is rationall. Wherefore (by the 10. of the tenth) the square of the line EF is commensurable vnto the square of the line CD . But the square of the line EF is rationall. Wherefore the square of the line CD is likewise rationall. Wherefore the line CD is rational. And forasmuch as the line EF is incommensurable in length vnto the line EG (for they are supposed to be commensurable in power onely). But as the line EF is to the line EG , so (by the assumpt going before) is the square of the line EF to the parallelograme which is contained under the lines EF and EG . Wherefore (by the 10. of the tenth) the square of the line EF is incommensurable vnto the parallelograme which is contained under the lines FE and EG . But vnto the square of the line EF , the square of the line CD is commensurable, for it is proved that either of them is a rationall



a rationall line. And that which is contained vnder the lines DC and CB is commensurable vnto that which is contained vnder the lines FE and EG . For they are both equall to the square of the line A . Wherefore (by the 13. of the tenth) the square of the line CD is incommensurable to that which is contained vnder the lines DC and CB . But as the square of the line CD is to that which is contained vnder the lines DC and CB , so (by the assumpt going before) is the line DC to the line CB . Wherefore the line DC is incommensurable in length vnto the line CB . Wherefore the line CD is rationall and incommensurable in length vnto the line CB . If therefore vpon a rationall line be applied the square of a medially line, the other side that maketh the breadth thereof shalbe rationall, and incommensurable in length to the line whereupon the parallelogramme is applied: which was required to be proued.

A square is sayd to be applied vpon a line, when it, or a parallelograme equall vnto it, is applied vpon the sayd line. If vpon a rationall line geuen we will apply a rectangle parallelograme equall to the square of a medially line geuen, and so of any line geuen, we must, by the 11. of the sixth, finde out the third line proportionall with the rationall line and the medially line geuen: so yet that the rationall line be the first, and the medially line geuen, (which containeth in power the square to be applied) be the second. For then the superficies contained vnder the first and the third, shalbe equall to the square of the middle line, by the 17. of the sixth.

How a square is sayde to be applied vpon a line.

The 20. Theoreme.

The 23. Proposition.

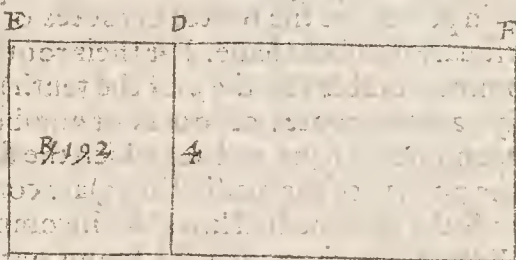
A right line commensurable to a medially line, is also a medially line.



Suppose that A be a medially line. And vnto the line A let the line B be commensurable, either in length, & in power, or in power only. Then I say that B also is a medially line. Let there be put a rationall line CD . And vpon the line CD , apply a rectangle parallelograme CE , equall vnto the square of the line A , and making in breadth the line ED . Wherefore (by the proposition going before) the line ED is rationall and incommensurable in length vnto the line CD . And againe vpon the line CD apply a rectangle parallelograme CF equall vnto the square of the line B , and making in breadth the line DF . And forasmuch as the line A is commensurable vnto the line B , therefore the square of the line A is commensurable to the square of the line B . But the parallelograme EC is equall to the square of the line A , and the parallelograme CF is equall to the square of the line B : wherefore the parallelograme EC is commensurable vnto the parallelograme CF . But as the parallelograme EC , is to the parallelograme CF , so is the line ED to the line DF (by the first of the sixth). Wherefore (by the 10. of the tenth) the line ED is commensurable in length vnto the line DF . But the line ED is rationall and incommensurable in length vnto the line DC , wherefore the line DF is rationall and incommensurable in length vnto the line DC (by the 13. of the tenth). Wherefore the lines CD and DF are rationall commensurable in power onely. But a rectangle figure comprehended vnder rationall right lines commensurable in power onely, is (by the 21. of the tenth) irrational, and the line that containeth it in power is irrational, and is called a medially line. Wherefore the line that containeth in power that which is comprehended vnder the lines CD and DF is a medially line. But the line B containeth in power the parallelograme which is comprehended vnder

Construction.

Demonstration.



The tenth Booke

der the lines C D and D F Wherefore the line B is a medially line: A right line therefore commensurable to a medially line, is also a medially line: which was required to be proved.

¶ Corollary.

Hereby it is manifest that a superficies commensurable unto a medially superficies, is also a medially superficies.

For the lines which containe in power those superficieses are commensurable in power, of which the one is a medially line (by the definitio of a medially line in the 21. of this tenth): Wherefore the other also is a medially line by this 23. propositio. And as it was sayd of rationally lines so also is it to be sayd of medially lines, namely, that a line commensurable to a medially line, is also a medially line, a line I say which is commensurable unto a medially line, whether it be commensurable in length, and also in power, or els in power onely. For vniuersally it is true, that lines commensurable in length, are also commensurable in power. Now if unto a medially line there be a line commensurable in power, if it be commensurable in length, the are those lines called medially lines commensurable in length & in power. But if they be commensurable in power onely, they are called medially lines commensurable in power onely.

There are also other right lines incommensurable in length to the medially line, and commensurable in power onely to the same: and these lines are also called medially, for that they are commensurable in power to the medially line. And in as much as they are medially lines, they are commensurable in power the one to the other. But being compared the one to the other, they may be commensurable either in length, and therefore in power, or els in power onely. And then if they be commensurable in length, they are called also medially lines commensurable in length, and so consequently they are vnderstanded to be commensurable in power. But if they be commensurable in power onely, yet notwithstanding they also are called medially lines commensurable in power onely.

Flussates after this proposition teacheth how to come to the vnderstanding of medially superficieses and lines, by surd numbers, after this maner. Namely to expresse the medially superficieses by the rootes of numbers which are not square numbers: and the lines containing in power such medially superficieses, by the rootes of rootes of numbers not square. Medially lines also commensurable, are expresse by the rootes of rootes of like superficial numbers, but yet not square, but such as haue that proportion that the squares of square numbers haue. For the rootes of those numbers and the rootes of rootes are in proportion as numbers are, namely, if the squares be proportionall the sides also shalbe proportionall (by the 22. of the sixt). But medially lines incommensurable in power, are the rootes of rootes of numbers, which haue not that proportion, that square numbers haue. For their rootes are the powers of medially lines, which are incommensurable (by the 9. of the tenth). But medially lines commensurable in power onely, are the rootes of rootes of numbers, which haue that proportion that simple square numbers haue, and not which the squares of squares haue. For the rootes (which are the powers of the medially lines) are commensurable, but the rootes of rootes (which expresse the sayd medially lines) are incommensurable.

Wherefore there may be found out infinite medially lines incommensurable in power, by comparing infinite vnlike playne numbers the one to the other. For vnlike playne numbers, which haue not the proportion of square numbers, doo make the rootes which expresse the superficieses of medially lines incommensurable (by the 9. of the tenth). And therefore the medially lines containing in power those superficieses are incommensurable in length. For lines incommensurable in power, are alwayes incommensurable in length (by the corollary of the 9. of the tenth).

¶ The 21. Theoreme.

The 24. Proposition.

A rectangle parallelogramme comprehended vnder medially lines commensurable in length, is a medially rectangle parallelogramme.

Suppose

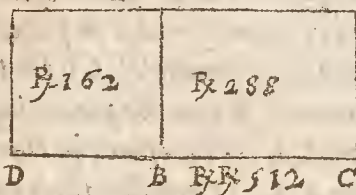


Suppose that the rectangle parallelogramme AC , be comprehended under these medially right lines AB and BC , which let be commensurable in length. Then I say, that AC is a medially rectangle parallelogramme. Describe (by the 46. of the first) upon the line AB a square AD . Wherefore the square AD is a medially superficies. And forasmuch as the line AB is commensurable in length unto the line BC , and the line AB is equall unto the line BD , therefore the line BD is commensurable in length unto the line BC . But as the line DB is to the line BC , so is the square DA to the parallelogramme AC (by the first of the sixth). Wherefore (by the 10. of the tenth) the square DA is commensurable unto the parallelogramme AC . But the square DA is medially, for that it is described upon a medially line. Wherefore AC also is a medially parallelogramme (by the former Corollary). A rectangle. &c: which was required to be proved.

Construction.

Demonstration.

Ex 162. A



The 22. Theoreme.

The 25. Proposition.

A rectangle parallelogramme comprehended under medially right lines commensurable in power onely, is either rationally, or medially.

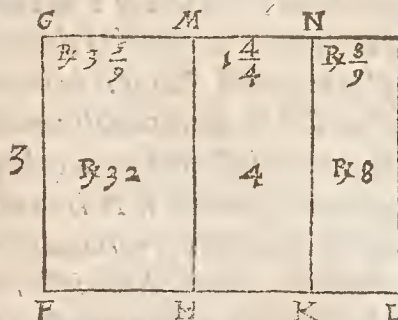
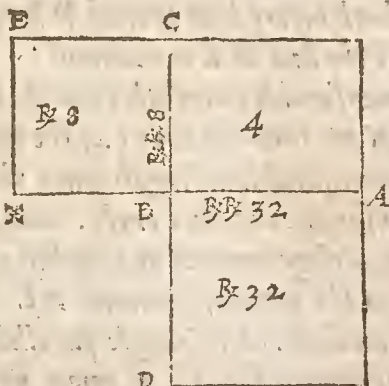


Medially line being given, there may be found an other line commensurable unto it, in power onely (by the 11. of this booke) as was taught there touching rationally lines. Now then suppose that the rectangle parallelogramme AC be comprehended under these medially right lines AB & BC . Then I say, that the parallelogramme AC is either rationally, or medially. Describe (by the 46. of the first) upon the lines AB and BC , their squares AD and BE . Wherefore either of these squares AD and BE is medially (by the 21. of the tenth). Let there be put a rationally line FG . And upon the line FG , let there be applied a rectangle parallelogramme GH equall to the square AD , and making the breadth the line FH . (How to do this was taught in the 22. of this booke). And upon the line HM , apply a rectangle parallelogramme MK , equall to the parallelogramme AC , and making in breadth the line HK : (to do this ye must take a fourth line proportionall with the lines HM , AB & BC (by the 12. of the sixth) which fourth line let be HK : wherefore (by the 26. of the sixth) that which is contained under the extremes HM and HK , is equall to the parallelogramme contained under the meanes AB and BC). And more-

Construction.

Demonstration.

over upon the line KN apply a rectangle parallelogramme NL equall to the square BE , and making in breadth the line KL . Wherefore the lines FH , HK & KL , are in one and the selfe same right line. (For those parallelogrammes so applied vpon the lines



FG , HM , and KN , are rectangle, and the angles FHM and KHM are equall to two right angles, for they are right angles: wherefore the lines FH and HK are in one right line, by the 14. of the first. So also may be sayd of the angles HKM and LKN). And

GG. i.

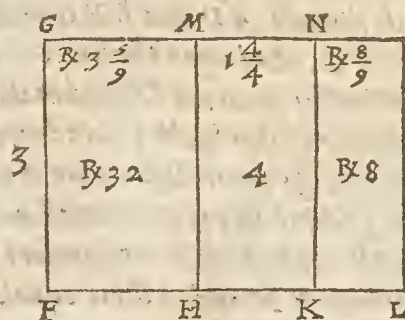
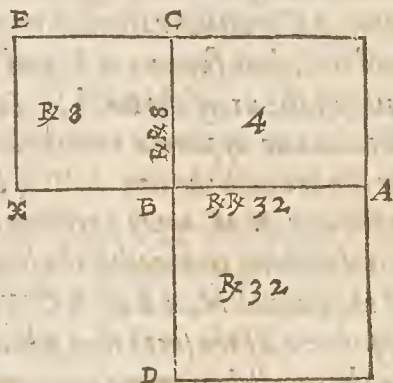
forasmuch

The tenth Booke

forasmuch as either of these squares AD and BE is medially, and the square AD is equall to the parallelogramme GH , and the square BE to the parallelogramme NL : wherefore either of these parallelogrammes GH and NL is medially. And they are applyed vpon a rationall line, namely, FG . Wherefore (by the 22. of the tenth) either of these right lines FH and KL is a rationall line, and incōmensurable in length vnto the line FG . And forasmuch as the square AD is commensurable to the square BE (for the lines AB and BC are supposed to be commensurable in power) therefore the parallelogramme GH is commensurable to the parallelogramme NL (for they are equall vnto the sayd squares). But as the parallelogramme GH is to the parallelogramme NL , so (by the first of the sixth) is the line FH to the line KL . Wherefore (by the 10. of the tenth) the line FH is commensurable in length to the line KL . Wherefore these right lines FH and KL , are rationall commensurable in length the one to the other (commensurable in length, I say, the one to the other, for vnto the line FG , by reason of which they are rationall, they are incōmensurable in length as it hath bene proued). Wherefore the parallelogramme containd vnder the lines FH and KL , is rationall (by the 19. of the tenth).

In these
fewe words
consisteth
the full de-
monstratiō
of the corol-
lary follow-
ing.

And forasmuch as the line DB is equall to the line BA , and the line XB to the line BC : therefore as the line DB is to the line BC , so is the line AB to the line BX . But as the line DB is to the line BC , so (by the first of the sixth) is the square DA to the parallelogramme AC . And as the line AB is to the line BX , so is the parallelogramme AC to the square CX . Wherefore as the square DA is to the parallelogramme AC , so is the parallelogramme AC to the square CX . But the square AD is equall to the parallelogramme GH , and to the parallelogramme AC is the parallelogramme MK also equall, and to the parallelogramme NL is equall the square BE : Wherefore as the parallelogramme GH is to the parallelogramme MK , so is the parallelogramme MK to the parallelogramme NL . Wherefore (by the first of the sixth) as the line FH is to the line HK , so is the line HK to the line KL . Wherefore (by the 17. of the sixth) the parallelogramme containd vnder the lines FH and KL , is equall to the square of the line HK . But the parallelogramme containd vnder the



lines FH and KL , is rationall, as hath before bene proued. Wherefore the square of the line HK is also rationall. Wherefore also the line HK is rationall.

Note.

And now if the line HK be commensurable in length vnto the line HM , that is, vnto the line FG , which is equall to the line HM , then (by the 19. of the tenth) the parallelogramme NH is rationall. But if it be incommensurable in length vnto the line FG , then the lines HK and HM are rationall commensurable in power onely. And so shall the parallelogramme HN be medially. Wherefore the parallelogramme HN is either rationall, or medially. But the parallelogramme HN is equall to the parallelogramme AC . Wherefore the parallelogramme AC is either rationall, or medially. A rectangle parallelogramme therefore comprehended vnder medially right lines commensurable in power onely, is either rationall, or medially: which was required to be demonstrated.

How to finde medially lines commensurable in power onely containyng a rationall parallelogramme, and also other medially lines commensurable in power containyng a medially

mediall parallelogramme, shall afterward be taught in the 27. and 28. Propositions of this booke.

¶ A Corollary.

Hereby it is manifest, that a rectangle parallelogramme containd vnder two right lines, is the meane proportionall betwene the squares of the sayd lines. As it was manifest (by the first of the sixth) that that which is containd vnder the lines AB and BC , is the meane proportionall betwene the squares AD and CX . This Corollary is put after the 23. Proposition of this booke as an Assumpt, and there demonstrated, which there in his place you shall finde. But because it followeth of this Proposition so euidently and briefly without farther demonstration, I thought it not amisse here by the way to note it.

A Corollary.

¶ The 23. Theoreme.

The 26. Proposition.

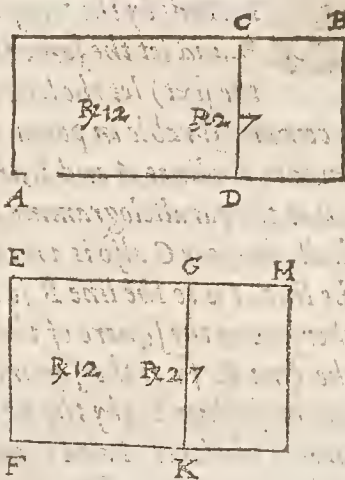
A mediall superficies exceedeth not a mediall superficies; by a rationall superficies.



Or if it be possible, let AB being a mediall superficies, exceede AC being also a mediall superficies, by DB being a rationall superficies: And let there be put a rationall right line EF . And vpon the line EF apply a rectangle parallelogramme FH , equall vnto the mediall superficies AB , whose other side let be EH : and from the parallelogramme FH take away the parallelogramme FG , equall vnto the mediall superficies AC . Wherefore (by the third common sentence) the residue BD is equall to the residue KH . But (by supposition) the superficies DB is rationall. Wherefore the superficies KH is also rationall. And forasmuch as either of these superficies AB and AC is mediall, and AB is equall vnto FH , & AC vnto FG : therefore either of these superficies FH and FG is mediall: and they are applied vpon the rationall line EF . Wherefore (by the 22. of the tenth) either of these lines HE and EG is rationall & incommensurable in length vnto the line EF . And forasmuch as the superficies DB is rationall, and the superficies KH is equall vnto it: therefore KH is also rationall: and it is applied vpon the rationall line EF (for it is applied vpon the line GK , which is equall to the line EF). Wherefore (by the 20. of the tenth) the line GH is rationall and commensurable in length vnto the line GK . But the line GK is equall to the line EF . Wherefore the line GH is rationall and commensurable in length vnto the line EF . But the line EG is rationall and incommensurable in length to the line EF . Wherefore (by the 13. of the tenth) the line EG is incommensurable in length vnto the line GH . And as the line EG is to the line GH , so is the square of the line EG to the parallelogramme containd vnder the lines EG and GH (by the Assumpt put before the 21. of the tenth). Wherefore (by the 10. of the tenth) the square of the line EG is incommensurable vnto the parallelogramme containd vnder the lines EG and GH . But vnto the square of the line EG are commensurable the squares of the lines EG and GH , for either of them is rationall, as hath before bene proued. Wherefore the squares of the lines EG and GH are incommensurable vnto the parallelogramme containd vnder the lines EG and GH . But vnto the parallelogramme containd vnder the lines EG and GH , is commensurable that which is containd vnder the lines EG and GH twise (for they are in proportion the one to the other as number is to number,

Construction.

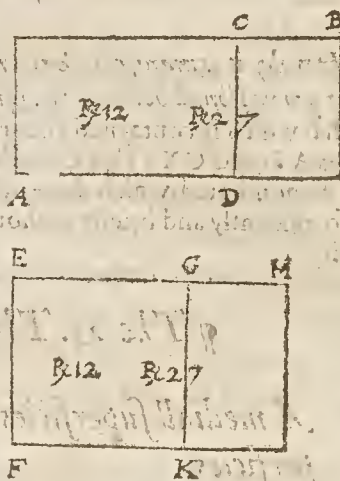
Demonstration leading to an absurditie.



GG.ij. namely,

The tenth Booke

namely, as unitie is to the number 2, or as 2 is to 4: and therefore (by the 6. of this booke) they are commensurable). Wherefore (by the 13. of the tenth) the squares of the lines EG and GH are incommensurable vnto that which is containd vnder the lines EG and GH twice. (This is more briefly concluded by the corollary of the 13. of the tenth). But the squares of the lines EG and GH together with that which is containd vnder the lines EG and GH twice are equall to the square of the line EH (by the 4. of the second). Wherefore the square of the line EH is incommensurable to the squares of the lines EG and GH (by the 16. of the tenth). But the squares of the lines EG & GH are rationall. Wherefore the square of the line EH is irrational. Wherefore the line also EH is irrational. But it hath before bene proued to be rationall: which is impossible. Wherefore a medially superficies exceedeth not a medially superficies by a rationall superficies: which was required to be proued.



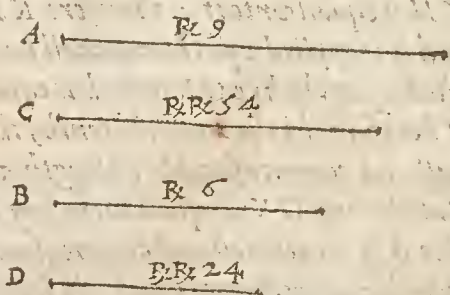
¶ The 4. Probleme. The 27. Proposition.

To finde out medially lines commensurable in power onely, containyng a rationall parallelogramme.

Construction.

Demonstration.

If there be put two rationall lines commensurable in power onely, namely, A and B. And (by the 13. of the sixth) take the meane proportionall betwene the lines A and B, and let the same line be C. And as the line A is to the line B, so (by the 12. of the sixth) let the line C be to the line D. And forasmuch as A and B are rationall lines commensurable in power onely, therefore (by the 21. of the tenth) that which is containd vnder the lines A and B, that is, the square of the line C. For the square of the line C is equall to the parallelogramme containd vnder the lines A and B (by the 17. of the sixth) is medially, therefore C also is a medially line. And for that as the line A is to the line B, so is the line C to the line D, therefore as the square of the line A is to the square of the line B, so is the square of the line C to the square of the line D (by the 22. of the sixth). But the squares of the lines A and B are commensurable, for the lines A and B are supposed to be rationall commensurable in power onely. Wherefore also the squares of the lines C and D are commensurable (by the 10. of the tenth) wherefore the lines C and D are commensurable in power onely. And C is a medially line. Wherefore (by the 23. of the tenth) D also is a medially line. Wherefore C and D are medially lines commensurable in power onely. Now also I say that they containe a rationall parallelogramme. For for that as the line A is to the line B, so is the line C to the line D: therefore alternately also (by the 16. of the fifth) as the line A is to the line C, so is the line B to the line D. But as the line A is to the line C, so is the line C to the line B: wherefore as the line C is to the line B, so is the line B to the line D. Wherefore the parallelogramme containd vnder the lines C and D is equal to the square of the line B. But the square of the line B is rationall. Wherefore the parallelogramme which is containd vnder the lines C and D is also rationall. Wherefore there are found out medially lines commensurable



ble

ble in power onely, contayning a rationall parallelogramme : which was required to be done.

The 5. Probleme. The 28. Proposition.

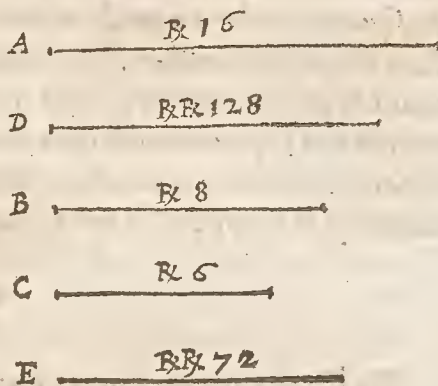
To finde out mediall right lynes commensurable in power onely, contayning a mediall parallelogramme.

Et there be put three rationall right lines commensurable in power only, namely, A, B, and C, and (by the 13. of the sixth) take the meane proportional betwene the lines A and B, & let the same be D. And as the line B is to the line C, so (by the 12. of the sixth) let the line D be to the line E. And forasmuch as the lines A and B are rationall commensurable in power onely, therefore (by the 21. of

Construction.

Demonstration.

the tenth) that which is contained under the lines A and B, that is the square of the line D, is mediall. Wherefore D is a mediall line. And forasmuch as the lines B and C are commensurable in power onely, and as the line B is to the line C, so is the line D to the line E : wherefore the lines D and E are commensurable in power onely (by the corollary of the tenth of this booke) but D is a mediall line. Wherefore E also is a mediall line (by the 23. of this booke.) Wherefore D & E are mediall lines commensurable in power onely. I say also that they containe a mediall parallelograme.



For for that as the line B is to the line C, so is the line D to the line E. therefore alternately (by the 16. of the fifth) as the line B is to the line D, so is the line C to the line E. But as the line B is to the line D, so is the line D to the line A, by converse proportion (which is proved by the corollary of the fourth of the fifth) Wherefore as the line D is to the line A, so is the line C to the line E. Wherefore that which is contained under the lines A & C, is (by the 16. of the sixth) equall to that which is contained under the lines D & E. But that which is contained under the lines A and C is mediall (by the 21. of the tenth.) Wherefore that which is contained under the lines D and E is mediall. Wherefore there are found out mediall lines commensurable in power onely, containing a mediall superficies : which was required to be done.

An Assumpt.

To finde out two square numbers, which added together make a square number.

Let there be put two like superficiall numbers AB and BC (which how to finde out, hath bene taught after the 9. proposition of this booke) And let them both be either even numbers or odde. And let the greater number be AB. And forasmuch as if from any even number be taken away an even number, or from an odde number be taken away an odde number, the residue shall be even (by the 24. and 26

A D C B

of the ninth). If therefore from AB being an even number be taken away BC an even number, or from AB being an odde number be taken away BC being also odde : the residue AC shall be even. Deuide the number AC into two equall partes in D : wherefore the number which is produced of AB into BC together with the square number of CD, is (by the sixth of the second, as Barlaam demonstrateth it in numbers) equall to the square number of BD.

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The tenth Booke

But that which is produced of AB into BC is a square nūber. For it was proued (by the first of the ninth) that if two like plaine numbers multiplieng the one the other, produce any nūber, the number produced shal be a square number. Wherefore there are found out two square numbers, the one being the square number which is produced of AB into BC , and the other the square number produced of CD , which added together make a square number, namely, the square number produced of BD multiplied into himselfe, forasmuch as they were demonstrated equall to it.

A Corollary.

To finde out two square numbers exceeding the one the other by a square number.

* A Corollary.

And hereby it is manifest, that there are found out two square numbers, namely, the one the square number of BD , and the other the square number of CD , so that that number wherein thone exceedeth the other, the number (I say) which is produced of AB into BC , is also a square number: namely, when AB & BC are like playne numbers. But when they are not like playne numbers, then are there found out two square numbers, the square number of BD , and the square number of DC , whose excesse, that is, the number wherby the greater exceedeth the lesse, namely, that which is produced of AB into BC , is not a square number.

$A \dots D \dots C \dots B$

¶ An Assumpt.

To finde out two square numbers which added together make not a square number.

An Assumpt.

Let AB and BC be like playne numbers, so that (by the first of the ninth) that which is produced of AB into BC is a square number, and let AC be an euen number. And deuide C into two equall partes in D . Now by that which hath before bene sayd in the former assumpt, it is manifest that the square number produced of AB into BC , together with the square number of CD , is equall to the square number of BD . Take away from CD unitie DE . Wherefore that which is produced of AB into BC together with the square of CE is lesse then the square number of BD . Now then I say that the square number produced of AB into BC added to the square number of CE , make not a square number. For if they do make a square number, then that square number which they make, is either greater the the square number of BE , or equall vnto it, or lesse then it. First, greater it cannot be, for it is already proued that the square number produced of AB into BC , together with the square number of CE , is lesse then the square number of BD . But betwene the square number of BD , and the square number of BE , there is no meane square number. For the number BD exceedeth the number BE onely by unitie: which unitie can by no meanes be deuided into numbers. Or if the number produced of AB into BC together with the square of the nūber CE , should be greater then the square of the number BE , then should the selfe same number produced of AB into BC together with the square of the number CE , be equall to the square of the number BD , the contrary wherof is already proued. Wherefore if it be possible, let that which is produced of AB into BC together with the square number of the number CE be equall to the square number of BE . And let GA be double to unitie DE , that is, let it be the number two. Now forasmuch as the whole number AC is by supposition double to the whole number CD , of which the number AG is double to unitie DE , therefore (by the 7. of the ninth

ninth

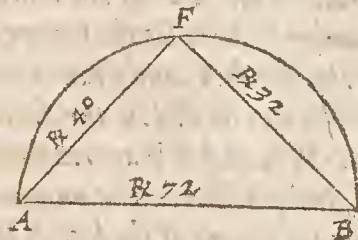
uenth) the residue, namely, the number GC is double to the residue, namely, to the number EC . Wherefore the number GC is devided into two equall partes in E . Wherefore that which is produced of GB into BC together with the square number of CE is equall to the square number of BE . But that which is produced of AB into BC , together with the square number of CE , is supposed to be equal to the square number of BE : wherefore that which is produced of GB into BC together with the square number of CE is equal to that which is produced of AB into BC , together with the square number of CE . Wherefore taking away the square number of CE , which is common to them both, the number AB shall be equall to the number GB , namely, the greater to the lesse, which is impossible. Wherefore that which is produced of AB into BC together with the square number of CE , is not equall to the square number of BE . I say also that that which is produced of AB into BC together with the square number of CE is not lesse then the square number of BE . For if it be possible, the shall it be equal to some square number lesse then the square number of BE . Wherefore let the number produced of AB into BC together with the square of the number CE be equal to the square number of BF . And let the number HA be double to the number DF . The also it followeth that the number HC is double to the number CF , so that HC also is devided into two equall partes in F , and therefore also the number which is produced of HB into BC , together with the square number of FC , is equall to the square number of the number BF . But by supposition, the number which is produced of AB into BC together with the square number of CE is equall to the square number of BF . Wherefore it followeth that the number produced of AB into BC together with the square number of CE , is equall to that which is produced of HB into BC together with the square number CF , which is impossible. For if it should be equall, then forasmuch as the square of CF is lesse then the square of CE , the number produced of HB into BC should be greater then the number produced of AB into BC . And so also should the number HB be greater then the number AB , when yet it is lesse then it. Wherefore the number produced of AB into BC together with the square number of CE , is not lesse then the square number of BE . And it is also proued that it cannot be equall to the square number of BE , neither greater then it. Wherefore that which is produced of AB into BC added to the square number of CE , maketh not a square number. And although it be possible to demonstrate this many other wayes, yet this semeth to vs sufficient, least the matter beyng ouer long, should seeme to much tedious.

The 6. Probleme.

The 29. Proposition.

To finde out two such rationall right lynes commensurable in power only, that the greater shall be in power more then the lesse, by the square of a right line commensurable in length vnto the greater.

Let there be put a rational line AB , and take also two such square numbers CD and DE , that their excesse CE be not a square number (by the corollary of the first assumpt of the 28. of the tenth) And vpon the line AB describe a semicircle AFB . And by the corollary of the 6. of the tenth, as the number DC is to the number CE , so let the square of the lyne BA be to the square of the line AF . And draw a line from F to B . Now for that as the square of the line BA is to the square of the line AF , so is the number CD to the num-



Construction.

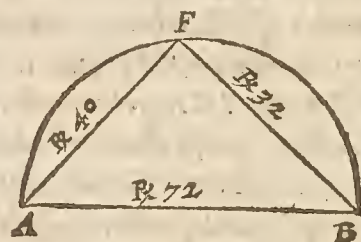
Demonstration.

GG. iij.

ber

The tenth Booke

ber CE , therefore the square of the line BA hath to the square of the line AF , that proportion that the number CD hath to the number CE . Wherefore the square of the line BA is commensurable to the square of the line AF (by the 6. of the tenth). But the square of the line AB is rational. Wherefore also the square of the line AF is rational. Wherefore also the line AF is rational. And forasmuch as the number CD hath not unto the number CE that proportion that a square number hath to a square number, therefore neither also hath the square of the line AB to the square of the line AF that proportion that a square number hath to a square number. Wherefore (by the 9. of the tenth) the line AB is unto the line AF incommensurable in length. Wherefore the lines AF and AB are rational commensurable in power onely. And for that as the number DC is to the number CE , so is the square of the line AB to the square of the line AF : therefore by conuersion or euerse proportion which is demonstrated (by the corollary of the 19. of the fifth) as the number CD is to the number DE , so is the square of the line AB to the square of the line BF , which is the excessse of the square of the line AB about the square of the line AF (by the assumpt put before the 14. of this booke). But the number CD hath to the number DE that proportion that a square number hath to a square number: wherefore the square of the line AB hath to the square of the line BF , that proportion that a square number hath to a square number. Wherefore (by the 9. of the tenth) the line AB is commensurable in length unto the line BF . And (by the 47. of the first) the square of the line AB is equal to the squares of the lines AF and FB . Wherefore the line AB is in power more then the line AF by the square of the line BF which is commensurable in length unto the line AB . Wherefore there are found out two such rational lines commensurable in power onely, namely, AB and AF , so that the greater line AB is in power more then the lesse line AF , by the square of the line FB , which is commensurable in length unto the line AB : which was required to be done.



$C \dots \dots E \dots \dots D$

The 7. Theoreme.

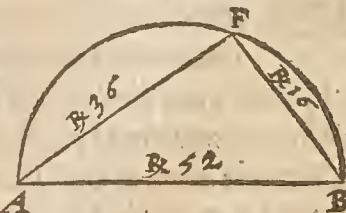
The 30. Proposition.

To finde out two such rational lines commensurable in power onely, that the greater shalbe in power more then the lesse by the square of a right line incommensurable in length to the greater.

Montaureus maketh this an Assumpt: as the Greeke text seemeth to do likewise but without a cause.
Construction.

Demonstration.

Let there be put a rational line AB , and take also (by the 2. assumpt of the 28. of the tenth) two square numbers CE and ED , which being added together make not a square number, and let the numbers CE and ED added together make the number CD . And upon the line AB describe a semicircle AFB . And (by the corollary of the 6. of the tenth) as the number DC is to the number CE , so let the square of the line AB be to the square of the line AF , and draw a line from F to B . And we may in like sort, as we did in the former proposition, proue that the lines BA and AF are rational commensurable in power onely. And for that as the number DC is to the number CE , so is the square of the line AB to the square of the line AF : therefore by conuersion (by the corollary of the 19. of the fifth) as the number CD is to the number DE , so is the square of the line AB to the square of the line FB . But the number C



$C \dots \dots \dots E \dots \dots D$

D hath

D hath not to the number DE that proportion that a square number hath to a square number. Wherefore neither also the square of the line AB hath to the square of the line BF that proportion that a square number hath to a square number. Wherefore the line AB is (by the 9. of the tenth) incommensurable in length to the line BF. And the line AB is in power more then the line AF by the square of the right line BF, which is incommensurable in length unto the line AB. Wherefore the lines AB and AF are rationally commensurable in power onely. And the line AB is in power more then the line AF by the square of the line FB which is commensurable in length unto the line AB: which was required to be done.

¶ An Assumpt.

If there be two right lines hauing betwene them selues any proportion: as the one right line is to the other, so is the parallelograme contained vnder both the right lines to the square of the lesse of those two lines.

Suppose that these two right AB and BC be E
in some certaine proportion. Then I say that as the
line AB is to the line BC, so is the parallelograme
contained vnder AB and BC to the square of BC.
Describe the square of the line BC and let the
same be CD, and make perfect the parallelograme
AD now it is manifest that as the line AB is to C
the line BC, so is the parallelograme AD to the
parallelograme or square BE (by the first of the sixth). But the parallelograme AD is that
which is contained vnder the lines AB and BC, for the line BC is equall to the line BD
and the parallelograme BE is the square of the line BC. Wherefore as the line AB is to the
line BC so is the parallelograme contained vnder the lines AB and BC to the square of
the line BC, which was required to be proued.

This Assump^{ti}on
settieth forth
nothing els
but that
which the first
of the sixth set-
teth forth, and
therefore in
some exam-
plars it is not
founde.

¶ The 8. Probleme.

The 31. Proposition.

To finde out two mediall lines commensurable in power onely, comprehending a rationally superficies, so that the greater shall be in power more then the lesse by the square of a line commensurable in length vnto the greater.

Et there be taken (by the 29. of the tenth) two rationally lines commensurable in power onely A and B, so that let the line A being the greater be in power more then the line B, being the lesse by the square of a line commensurable in length vnto the line A. And let the square of the line C be equall to the parallelograme contained vnder the lines A and B which is done by finding out the meane proportionall line, namely the line C betwene the lines A and B (by the 13. of the sixth). Now the parallelograme contained vnder the lines A and B is mediall (by the 21. of this booke). Wherefore (by the corollary of the 23. of the tenth) the square also of the line C is mediall. Wherefore the line C also is mediall: Vnto the square of the line B let the parallelograme contained vnder the lines C and D be equall (by finding out a third line proportionall) namely the line D to the two lines C and B (by the 11. of the sixth). But the square of the line B is rationally. Wherefore the parallelograme contained vnder the line C and D is rationally. And for that as the line A is to the line B,

Construction.

Demonstration.

so

The tenth Booke

so is the parallelograme contained vnder the lines A and B to the square of the line B (by the assumpt going before). But vnto the parallelograme contained vnder the lines A and B is equall the square of the line C, and vnto the square of the line B is equal the parallelograme contained vnder the lines C and D, as it hath now bene proved: therefore as the line A is to the line B, so is the square of the line C to the parallelograme contained vnder the lines C, D. But as the square of the line C is to that which is containd vnder the lines C and D, so is the line C to the line D. Wherefore as the line A is to the line B, so is the line C to the line D. But (by supposition) the line A is commensurable vnto the line B in power onely. Wherefore (by the 11. of the tenth) the line C also is vnto the line D commensurable in power onely. But the line C is medially. Wherefore by the 23. of the tenth) the line D also is medially. And for that as the line A is to the line B, so is the line C to the line D: but the line A is in power more then the line B, by the square of a line commensurable in length vnto the line A (by supposition). Wherefore the line C also is in power more then the line D by the square of a line commensurable in length vnto the line C. Wherefore there are found out two medially lines C and D commensurable in power onely comprehending a rationall superficies, and the line C is in power more then the line D, by the square of a line commensurable in length vnto the line C. And in like sort may be found out two medially lines commensurable in power onely containning a rationall superficies, so that the greater shalbe in power more then the lesse by the square of a line incommensurable in length to the greater, namely, when the line A is in power more then the line B by the square of a line incommensurable in length vnto the line A, which to do is taught by the 30. of this booke.

The selfe same construction remaining, that part of this proposition fro these wordes. And for that as the line A is to the line B, to these wordes. But (by supposition) the line A is commensurable vnto the line B, may more easely be demonstrated after this maner. The lines C, B, D, are in continuall proportion by the second part of the 17. of the sixt. But the lines A, C, D are also in continuall proportion by the same. Wherefore by the 11. of the fifth, as the line A is to the line C, so is the line B to the line D. Wherefore alternately as the line A is to the line B so is the line C to the line D. &c. which was required to be doone.

¶ An assumpt.

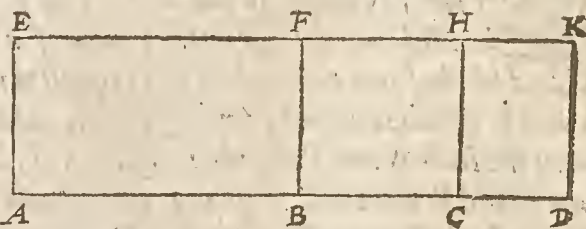
If there be three right lines hauing betwene them selues any proportion: as the first is to the third, so is the parallelograme contained vnder the first and the second, to the parallelograme contained vnder the second and the third.

Suppose that these three lines A B, B C, and C D be in some certayne proportion. Then I say that as the line A B is to the line C D, so is the parallelograme containd vnder the lines A B and B C to the parallelograme containd vnder the lines B C and C D. From the point A raise up vnto the line A B a perpendicular line A E,

Construction.

and let A E be equall to the line B C: and by the poynt E draw vnto the line A D a parallel line E K: and by euery one of the poyntes B, C, and D draw vnto the line A E parallel lines B F, C H, and D K. And for that as the line A B is to the line B C so is the parallelograme A F to the parallelograme B H (by the first of the sixt): and as the line B C is to the line

Demonstration.



line CD , so is the parallelograme BH to the parallelograme CK . Wherefore of equalitie as the line AB is to the line CD , so is the parallelograme AF to the parallelograme CK . But the parallelograme AF is that which is containd vnder the lines AB and BC , for the line AE is put equall to the line BC . And the parallelograme CK is that which is contained vnder the lines BC and CD , for the line BC is equal to the line CH , for that the line CH is equall to the line AE (by the 34. of the first). If therefore there be three right lines ha- uing betwene them selues any proportion : as the first is to the third, so is the parallelograme contained vnder the first and the second, to the parallelogramme cotained vnder the second and the third: which was required to be demonstrated.

¶ The 9. Probleme.

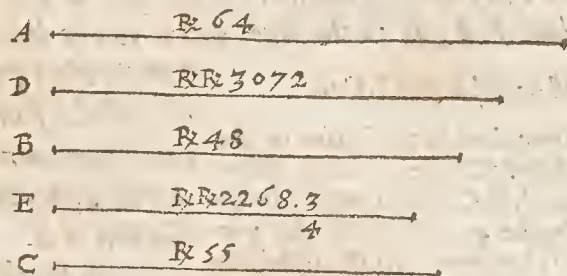
The 32. Proposition.

To finde out two mediall lines commensurable in power onely, comprehending a mediall superficies, so that the greater shall be in power more then the lesse, by the square of a line commensurable in length vnto the greater.



Et there be taken three rationall lines cōmensurable in power onely, A, B, C , so, that (by the 29. of the tenth) let the line A be in power more then the line C , by the square of a line commensurable in length vnto the line A . And vnto the parallelogramme containd vnder the lines A & B , let the square of the line D be equall. But that which is containd vnder the lines A and B is mediall. Wherefore (by the Corollary of the 23. of the tenth) the square of the line D also is mediall. Wherefore the line D also is mediall. And vnto that which is containd vnder the lines B and C , let be equall that which is containd vnder the lines D and E (which is done by finding out a fourth line proportionall vnto the lines D, B, C , which let be the line E). And for that (by the Assumpt going before) as that which is containd vnder the lines A and B is to that which is containd vnder the lines B and C , so is the line A to the line C . But vnto that which is containd vnder the lines A & B , is equall the square of the line D , and vnto that which is containd vnder the lines B & C , is equall that which is containd vnder the lines D and E . Wherefore as the line A is to the line C , so is the square of the line D , to that which is containd vnder the lines D and E . But as the square of the line D is to that which is containd vnder the lines D and E , so is the line D to the line E (by the Assumpt put before the 22. of the tenth). Wherefore as the line A is to the line C , so is the line D to the line E . But the line A is vnto the line C commensurable in power onely. Wherefore the line D is vnto the line E commensurable in power onely. But D is a mediall line. Wherefore (by the 23. of the tenth) E also is a mediall line. And for that as the line A is to the line C , so is the line D to the line E , and the line A is in power more then the line C , by the square of a line commensurable in length vnto the line A . Wherefore (by the 14. of the tenth) D is in power more then E , by the square of a line commensurable in length vnto the line D . I say moreover that that which is containd vnder the lines D and E is mediall: For forasmuch as that which is containd vnder the lines B & C , is equall to that which is containd vnder the lines D and E : but that which is containd vnder the lines B and C is mediall. Wherefore that which is containd vnder the lines D and E is also mediall. Wherefore there are found out two mediall lines D and E , cōmensurable in power onely, comprehending a mediall superficies, so that the greater is in power more then the lesse, by the square of a line commensurable in length to the greater: which was required to be done

Construction.



Demonstration.

Corollary.

done. And thus is it evident, how in like sorte may be founde out two medially lines commensurable in power only, containing a medially superficies, so that the greater shall be in power more then the lesse, by the square of a line incommensurable in length unto the greater. When the line A is in power more then the line C , by the square of a line incommensurable in length unto the line A : as the thirteth teacheth us.

¶ An Assumpt.

1.

2.

3.

4.

Suppose that there be a rectangle triangle ABC , having the angle BAC a right angle. And (by the 12. of the first) from the poynt A to the right line BC , a perpendicular line being drawn AD : then I say first, that the parallelogramme containd under the lines CB and BD , is equall to the square of the line BA . Secondly I say, that the parallelogramme containd under the lines BC and CD , is equall to the square of the line CA . Thirdly I say, that the parallelogramme containd under the lines BD and DC , is equall to the square of the line AD . And fourthly I say, that the parallelogramme containd under the lines BC & AD , is equall to the parallelogramme containd under the lines BA & AC .

As touching the first, that the parallelogramme containd under the lines CB and BD , is equall to the square of the line AB , is thus proued.

For forasmuch as in the rectangle triangle BAC , from the right angle unto the base is drawn a perpendicular line AD , therefore (by the 8. of the sixth) the triangles ABD and ADC , are like to the whole triangle ABC , and are also like the one to the other. And for that the triangle ABC is like to the triangle ADB , therefore both the triangles are equiangle by the definition of like figures. Wherefore (by the 4. of the sixth) as the line CB is to the line BA , so is the line AB to the line BD .

* A Corollary added by I. Dee.

* Therefore if you deuide the square of the side AB by the side BC : the quotient will be BD . Which maketh DC also known: by either of which (by the 47. of the first) the perpendicular AD is known: which consideration in practise, is very profitable, where the three sides AB , AC , and BC , are known or geuen.

I. Dee * The second Corollary.

* Therefore if you deuide the square of the side AC , by the side BC , the portion DC , will be the product. &c. as in the former Corollary.

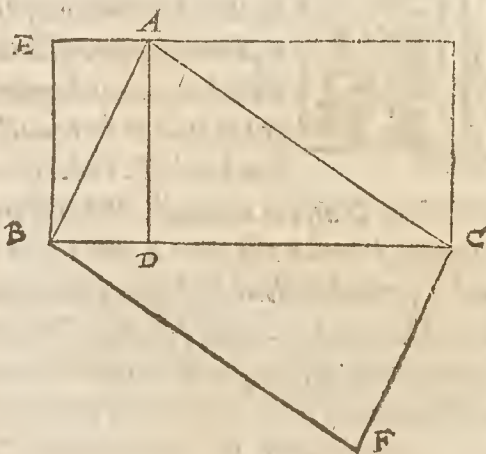
* Wherefore (by the 17. of the sixth) the

parallelogramme contained under the lines BC & BD , is equall to the, square of the line AB .

As touching the second, that the parallelogramme contained under the lines BC and CD , is equall to the square of the line AC is by the selfe same reason proued. For the triangle ABC is like to the triangle ADC . Wherefore as the line BC is to the line AC , so is the line, AC to the line DC .

* Wherefore the parallelogramme contained under the lines BC and CD , is equall to the square of the line AC . As touching the third, that the parallelogramme contained under the lines BD and DC , is equall to the square of the line DA , is thus proued. For, forasmuch as if in a rectangle triangle be

drawn frō the right angle to the base a perpendicular line, the perpendicular so drawn is the meane proportional betwene the segmēts of the base (by the corollary of the 8. of the sixth): therefore as the line BD is to the line DA , so is the line AD to the line DC . Wherefore (by the 17. of the sixth) the parallelogramme containd under the lines BD and DC , is equall to the square of the line DA . As touching the fourth, that the parallelogramme contained under the lines BC and AD , is equall to the parallelogramme contained under the lines BA and AC , is thus proued. For forasmuch as (as we haue already declared) the triangle ABC is like, and therefore equiangle, to the triangle ABD , therefore as the line BC is to the



the line AC , so is the line BA to the line AD (by the 4. of the sixth). * But if there be foure right lines proportionall, that which is contained vnder the first and the last, is equall to that which is contained vnder the two meanes (by the 16. of the sixth). Wherefore that which is contained vnder the lines BC and AD , is equall to that which is contained vnder the lines BA and AC .

I say moreover, that if there be made a parallelogramme complete, contained vnder the lines BC and AD , which let be EC : and if likewise be made complete the parallelogramme contained vnder the lines BA and AC , which let be AF , it may by an other way be proved that the parallelogramme EC is equall to the parallelogramme AF . For, forasmuch as either of them is double to the triangle ACB (by the 41. of the first): and things which are double to one and the selfe same thing, are equall the one to the other. Wherefore that which is contained vnder the lines BC and AD , is equall to that which is contained vnder the lines BA and AC .

I. Dee *The thirde Corollary.

* Therefore if the parallelogramme of BA , and AC , be deuied by BC , the product will geue the perpendicular DA . These three Corollaries in practise Logistickall and Geometrickall are profitable.

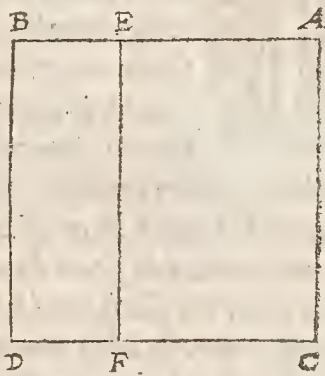
An other demonstration of this fourth part of the determinatio.

2. ¶ An Assumpt.

If a right line be deuied into two vnequall partes: as the greater part is to the lesse, so is the parallelogramme contained vnder the whole line and the greater part, to the parallelogramme contained vnder the whole line and the lesse part.

An Assumpt.

Deuide the right line AB into two vnequall partes in the point E : And let AE be the greater part. Then I say, that as the line AE is to the line EB , so is the parallelogramme contained vnder the lines BA and AE to the parallelogramme contained vnder the lines BA & BE . Describe the square of the line AB , and let the same be $ACDB$. And from the point E draw vnto either of these lines AC and DB a parallell line EF . Now it is manifest, that as the line AE is to the line EB , so is the parallelogramme AF to the parallelogramme BF (by the first of the sixth). But the parallelogramme AF is contained vnder the lines BA and AE (for the line AC is equall to the line AB) and the parallelogramme BF is contained vnder the lines AB and BE (for the line DB is equall to the line AB). Wherefore as the line AE is to the line EB , so is the parallelogramme contained vnder the lines BA and AE , to the parallelogramme contained vnder the lines AB and BE : which was required to be demonstrated.



This Assumpt differeth litle from the first Proposition of the sixth booke.

3. ¶ An Assumpt.

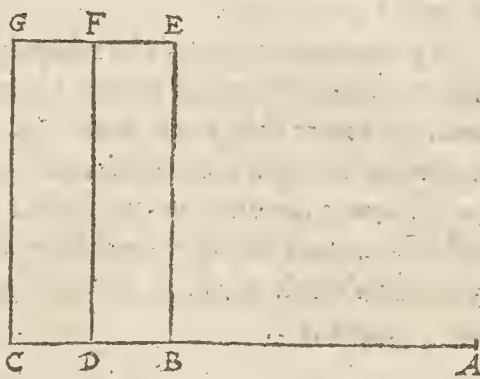
If there be two vnequall right lines, and if the lesse be deuied into two equall partes: the parallelogramme contained vnder the two vnequall lines, is double to the parallelogramme contained vnder the greater line & halfe of the lesse line.

HH. 3.

Suppose

The tenth Booke

Suppose that there be two unequal right lines AB and BC , of which let AB be the greater, and deuide the line BC into two equal partes in the point D . Then I say, that the parallelogramme contained vnder the lines AB & BC , is double to the parallelogramme contained vnder the lines AB and BD . From the point B raise vp vpon the right line BC , a perpendicular line BE , and let BE be equall to the line BA . And drawing from the point C and D , the lines CG and DF parallels and equall to BE : and then drawing the right line GFE , the figure is complete. Now for that as the line DB is to the line DC , so is the parallelogramme BF to the parallelogramme DG (by the 1. of the sixth): therefore by composition of proportion, as the whole line BC is to the line DC , so is the parallelogramme BG to the parallelogramme DG (by the 18. of the fifth). But the line BC is double to the line DC . Wherefore the parallelogramme BG is double to the parallelogramme DG . But the parallelogramme BG is contained vnder the lines AB and BC for the line AB is equall to the line BE (and the parallelogramme DG is containd vnder the lines AB and BD) for the line BD is equall to the line DC , and the line AB to the line DF : which was required to be demonstrated.



¶ The 10. Probleme.

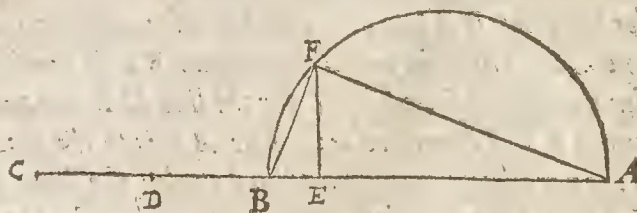
The 33. Proposition.

To finde out two right lines incommensurable in power, whose squares added together make a rationall superficies, and the parallelogramme contained vnder them make a mediall superficies.

Construction.



Take by the 30. of the tenth, two rationall right lines commensurable in power onely, namely, AB and BC , so that let the line AB , being the greater, be in power more then the line BC being the lesse, by the square of a line incommensurable in length vnto the line AB . And by the 10. of the first, deuide the line BC into two equal partes in the point D . And vpon the line AB apply a parallelogramme equall to the square either of the line BD or of the line DC , and wanting in figure by a square, by the 28. of the sixth, and let that parallelogramme be that which is contained vnder the lines AE and EB . And vpon the line AB describe a semicircle AFB . And by the 11. of the first, from the point E , raise vp vnto the line AB , a perpendicular line EF , cutting the circumference in the point F . And draw lines from A to F , and from E to B . And forasmuch as there are two unequal right lines AB and BC , and the line AB is in power more then the line BC , by the square of a line incommensurable in length vnto AB , and vpon the line AB is applied a parallelogramme equall to the fourth part of the square of the line BC , that is, to the square of the halfe of the line BC , and wanting in figure by a square, and the said parallelogramme is that which is contained vnder the lines AE and EB . wherefore by the 2. part of the 18. of the tenth, the line AE is incommensurable in length vnto the line EB . But as the line AE is to the line EB , so is the parallelogramme contained vnder the lines BA and AE , to the parallelogramme containd vnder the lines AB and BE by the second assumpt before put. And that which is contained vnder the



Demonstration.

the line $B A$ and $A E$ is equall to the square of the line $A F$, by the second part of the first assumpt before put. And that which is contained under the lines $A B$ and $B E$ is by the first part of the same assumpt, equall to the square of the line $B F$. Wherefore the square of the line $A F$ is incommensurable to the square of the line $B F$. Wherefore the lines $A F$ and $B F$ are incommensurable in power. And forasmuch as $A B$ is a rationall line. (by supposition) therefore (by the 7. definition of the tenth) the square of the line $A B$ is rationall. Wherefore also the squares of the lines $A F$ and $F B$ added together make a rationall superficies. For (by the 47. of the first) they are equal to the square of the line $A B$. Again forasmuch as (by the third part of the first assumpt going before) that which is contained under the lines $A E$ and $E B$, is equall to the square of the line $E F$. But by supposition that which is contained under the lines $A E$ and $E B$ is equall to the square of the line $B D$. Wherefore the line $F E$ is equall to the line $B D$. Wherefore the line $B C$ is double to the line $F E$. Wherefore (by the third assumpt. going before) that which is contained under the lines $A B$ and $B C$, is double to that which is contained under the lines $A B$ and $E F$. But that which is contained under the lines $A B$ and $B C$, is by supposition medially. Wherefore (by the corollary of the 23. of the tenth) that which is contained under the lines $A B$ and $E F$ is also medially, but that which is contained under the lines $A B$ and $E F$, is (by the last parte of the first assumpt going before) equall to that which is contained under the lines $A F$ and $F B$. Wherefore that which is contained under the lines $A F$ & $F B$ is a medially superficies. And it is proued, that that which is composed of the squares of the lines $A F$ and $F B$ added together is rationall. Wherefore there are found out two right lines $A F$ and $F B$ incommensurable in power, whose squares added together, make a rationall superficies, and the parallelogramme contained under them, is a medially superficies: which was required to be done.

The first parte of the determination concluded.

The second part concluded.

The totall conclusion.

¶ The 11. Probleme.

The 34. Proposition.

To finde out two right lines incommensurable in power, whose squares added together make a medially superficies, and the parallelogramme contained under them, make a rationall superficies.



Take (by the 31. of the tenth) two medially lines $A B$ and $B C$, commensurable in power onely, comprehending a rationall superficies, so that let the line $A B$ be in power more then the line $B C$ by the square of a line incommensurable in length vnto the line $A B$. And describe vpon the line $A B$ a semicircle $A D B$. And by

Construction.

the 10. of the first, deuide the line $B C$ into two equall partes in the point E .

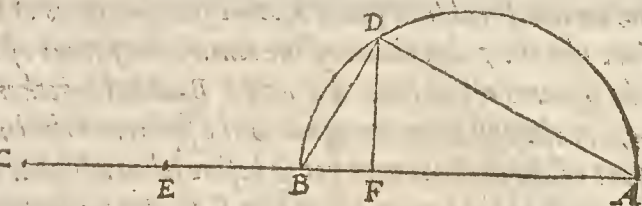
And (by the 28. of the sixth) vpon the line $A B$ apply a parallelogramme equall to the square of the line $B E$, and wantyng in figure by a square, and let that parallelogramme be that which is contained under the lines $A F$ and $F B$. Wherefore

the line $A F$ is incommensurable in length vnto the line $F B$ (by the 2. part of the 18. of the tenth). And from the point F vnto the right line $A B$, raise vp (by the 11. of the first) a perpendicular line $F D$, and draw lines from A to D , and from D to B . And forasmuch as the line $A F$ is incommensurable vnto the line $F B$: but (by the second assumpt going before the 33. of the tenth) as the line $A F$ is to the line $F B$, so is the parallelogramme contained under the lines $B A$ and $A F$, to the parallelogramme contained under the lines $B A$ and $B F$, wherefore (by the tenth of the tenth) that which is contained under the lines $B A$ and $A F$ is incommensurable to that which is contained under the lines $A B$ and $B F$: but that which is

Demonstration.

The tenth Booke

contained under the lines $B A$ and $A F$ is equall to the square of the line $A D$, and that which is contained under the lines $A B$ and $B F$ is also equall to the square of the line $D B$ (by the second part of the first assumpt going before the 33. of the tenth) wherefore the square of the line $A D$ is incommensurable to the square of the line $D B$. Wherefore the lines $A D$ and $D B$ are incommensurable in power. And forasmuch as the square of the line $A B$ is medially, therefore also the superficies made of the squares of the lines $A D$ and $D B$ added together is medially. For the squares



The first part
of the deter-
mination con-
cluded.

of the lines $A D$ and $D B$ are (by the 47. of the first) equall to the square of the line $A B$. And forasmuch as the line $B C$ is double to the line $F D$ (as it was proved in the proposition going before) therefore the parallelogramme contained under the lines $A B$ and $B C$ is double to the parallelogramme contained under the lines $A B$ and $F D$ (by the third assumpt going before the 33. proposition) wherefore it is also commensurable unto it (by the sixt of the tenth) But that which is contained under the lines $A B$ and $B C$ is supposed to be rationally. Wherefore that which is contained under the lines $A B$ and $F D$ is also rationally. But that which is contained under the lines $A B$ and $F D$, is equall to that which is contained under the lines $A D$ and $D B$ (by the last part of the first assumpt going before the 33. of the tenth) Wherefore that which is contained under the lines $A D$ and $D B$ is also rationally. Wherefore there are found out two right lines $A D$ and $D B$ incommensurable in power, whose squares added together, make a medially superficies, and the parallelogramme contained under them, make a rationally superficies: which was required to be done.

The second
part concluded.
The totall
conclusion.

¶ The 12. Probleme.

The 35. Proposition.

To finde out two right lines incommensurable in power, whose squares added together, make a medially superficies, and the parallelogramme contained under them, make also a medially superficies, which parallelogramme moreouer, shall be incommensurable to the superficies made of the squares of those lines added together.

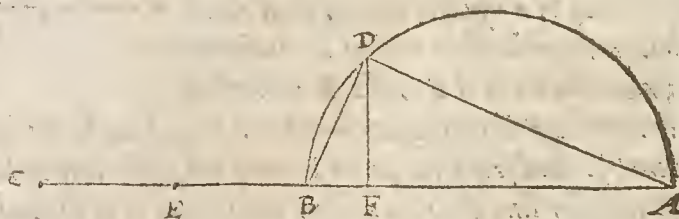
Construction.

Take (by the 32. of the tenth) two medially lines $A B$ and $B C$ commensurable in power only, comprehending a medially superficies, so that let the line $A B$ be in power more

than the line $B C$ by the square of a line incommensurable in length unto the line $A B$. And upon the line $A B$ describe a semicircle $A D B$, and let the rest of the construction be as it was in the two former propositions. And forasmuch as (by the 2 part of the 18. of the tenth) the line $A F$ is incommensurable in length unto the line $F B$, therefore the line $A D$ is

Demonstration.

incommensurable in power unto the line $D B$ (by that which was demonstrated in the proposition going before). And forasmuch as the square of the line $A B$ is medially, therefore that also which is composed of the squares of the lines $A D$ and $D B$ (which squares are equall to the square of the line $A B$ by the 47. of the first) is medially. And forasmuch as that which is



The first part
concluded.

con-

contained under the lines AF and FB , is equall to either of the squares of the lines EB and FD , for by supposition the parallelogramme contained under the lines AF and FB is equall to the square of the line EB , and the same parallelogramme is equall to the square of the line FD (by the third part of the first assumpt going before the 33. of the tenth). Wherefore the line BE is equall to the line FD . Wherefore the line BC is double to the line FD . Wherefore that which is contained under the lines AB and BC is double to that which is contained under the lines AB and FD . Wherefore they are commensurable by the sixth of this booke: but that which is contained under the lines AB and BC is medially by supposition. Wherefore also that which is contained under the lines AB and FD is medially (by the corollary of the 23. of the tenth) but that which is contained under the lines AB and FD , is (by the fourth part of the first assumpt going before the 33. of the tenth) equall to that which is contained under the lines AD and DB : wherefore that which is contained under the lines AD and DB is also medially. And forasmuch as the line AB is incommensurable in length unto the line BC . But the line BC is commensurable in length unto the line BE . Wherefore (by the 13. of the tenth) the line AB is incommensurable in length unto the line BE . Wherefore the square of the line AB is incommensurable to that which is contained under the lines AB and BE (by the first of the sixth and 10. of this booke). But unto the square of the line AB are equall the squares of the lines AD and DB added together (by the 47. of the first): and unto that which is contained under the lines AB and BE , is equall that which is contained under the lines AB and FD , that is, which is contained under AD and DB . For the parallelogramme contained under the lines AB and FD is equall to the parallelogramme contained under the lines AD and DB (by the last part of the first assumpt going before the 33. of this tenth booke). Wherefore that which is composed of the squares of the lines AD and DB is incommensurable to that which is contained under the lines AD and DB . Wherefore there are found out two right lines AB and DB incommensurable in power, whose squares added together, make a medially superficies, and the parallelogramme contained under them, make also a medially superficies, which parallelogramme moreover is incommensurable to the superficies composed of the squares of those lines added together, which was required to be done.

The second part concluded.

The third part concluded.

The totall conclusion.

The beginning of the Senaries by Composition.

The 25. Theoreme.

The 36. Proposition.

If two rationall lines commensurable in power onely be added together: the whole line is irrational, and is called a binomium, or a binomial line.

The first Senary by composition.



Suppose that these two rationall right lines AB and BC being commensurable in power onely be added together. (the 11. of the tenth teacheth to finde out two such lines). Then I say that the whole line AC is irrational. For forasmuch as the line AB is incommensurable in length unto the line BC . (for they are supposed to be commensurable in power onely). But as the line AB is to the line BC , so (by the assumpt put before the 22. of the tenth) is the parallelogramme contained under the lines AB and BC to the square of the line BC . Wherefore (by the 10. of this booke) the parallelogramme contained under the lines AB and BC is incommensurable to the square of the line BC . But the parallelogramme contained under the lines AB and BC is equall to the square of the line AC . (by the 47. of the first). Therefore the square of the line AC is incommensurable to the square of the line BC . But the line AC is commensurable in power with the line BC . Therefore the line AC is irrational. HH. iij. B and

Demonstration.

The tenth Booke

B and *BC* is incommensurable to the square of the line *BC*. But vnto the parallelograme contained *A* under the lines *AB* and *BC* is commensurable the parallelograme contained vnder *AB* and *BC* twice.

(by the 6. of the tenth) : wherefore that which is contained vnder *AB* and *BC* twice is incommensurable to the square of the line *BC* (by the 13. of the tenth). But vnto the square of the line *BC* is commensurable that which is composed of the squares of the lines *AB* and *BC* (by the 15. of the tenth), for by supposition the lines *AB* and *BC* are commensurable in power onely. Wherefore (by the 13. of the tenth) that which is composed of the squares of the lines *AB* and *BC* added together is incommensurable to that which is contained vnder the lines *AB* and *BC* twice. Wherefore (by the 16. of the tenth) that which is contained vnder *AB* and *BC* twice together with the squares of the lines *AB* and *BC*, which (by the 4. of the second) is equall to the square of the whole line *AC*, is incommensurable to that which is composed of the squares of *AB* and *BC* added together. But that which is composed of the squares of *AB* and *BC* added together is rationally, for it is commensurable to either of the squares of the lines *AB* and *BC* of which either of them is rationally by supposition: wherefore the square of the line *AC* is (by the 10. definition of the tenth) irrational. Wherefore the line *AC* also is irrational, and is called a binomial line.

Definition of
a binomial
line.

This proposition sheweth the generation and production of the second kinde of irrational lines which is called a binomium, or a binomial line. The definition whereof is fully gathered out of this proposition, and that thus.

A binomium or a binomial line, is an irrational line composed of two rational lines commensurable the one to the other in power onely. And it is called a binomium, that is, hauing two names, because it is made of two such lines as of his partes which are onely commensurable in power and not in length: and therefore ech part or line, or at the least the one of them, as touching length, is vncertaine and vnkowne. Wherefore being ioyned together their quantitie cannot be expressed by any one number or name, but ech part remaineth to be seuerally named in such sort as it may. And of these binomial lines there are sixe seuerall kindes, the first binomial, the second, the third, the fourth, the fifth, and the sixth, of what nature and condition ech of these is shalbe knowne by their definitious which are afterward set in their due place.

Sixe kindes of
binomial
lines.

¶ The 25. Theoreme.

The 37. Proposition.

If two medially lines commensurable in power onely containing a rational superficies, be added together: the whole line is irrational, and is called a first binomial line.

Et these two medially lines *AB* and *BC* being commensurable in power onely, and containyng a rational superficies (the 27. of the tenth teacheth to finde out two such lines) be composed. Then I say that the whole line *AC* is irrational. For as it was sayd in the proposition next going before that which is composed of the squares of the lines *AB* and *BC* is incommensurable to that which is contained vnder the lines *AB* and *BC* twice: wherefore (by the 16. of the tenth) that which is composed of the squares of the lines *AB* and *BC* together with that which is contained vnder the lines *AB* and *BC* twice, that is, the square of the line *AC* is incommensurable to that which is contained vnder the lines *AB* and *BC* twice. But that which is contained vnder the lines *AB* and *BC* twice is commensurable to that which is contained vnder the lines *AB* and *BC* once (by the 6. of the tenth) wherefore

Demonstration.

wherefore the square of the whole line AC , is (by the 13. of the tenth) incommensurable to that which is contained under the lines AB and BC once. But by supposition the lines AB and BC comprehend a rationall superficies. Wherefore the square of the whole line AC is irrationall: wherefore also the line AC is irrationall. And it is called a first bimediall line.

The third irrational line which is called a first bimediall line, is shewed by this proposition, and the definition thereof is by it made manifest, which is this. A first bimediall line, is an irrationall line, which is composed of two mediall lines commensurable in power onely containing a rationall parallelograme. It is called a first bimediall line, because the two mediall lines or partes whereof it is composed containe a rationall superficies, which is preferred before an irrationall.

Definition of a first bimediall line.

¶ The 26. Theoreme.

The 38. Proposition.

If two mediall lines commensurable in power onely containing a mediall superficies, be added together: the whole line is irrationall, and is called a second bimediall line.



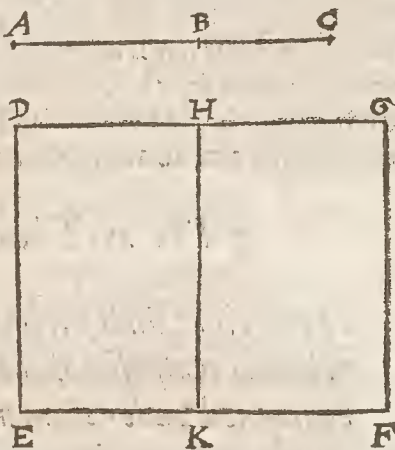
Et these two medial lines AB and BC being commensurable in power onely, and containing a mediall superficies (the 28. of the tenth teacheth to finde out two such lines) be added together. Then I say that the whole line AC is irrationall. Take a rationall line DE . And (by the 44. of the first) upon the line DE apply the parallelograme DF , equal to the square of the line AC , whose other side let be the line DG . And forasmuch as the square of the

Construction.

line AC is (by the 4. of the second) equall to that which is composed of the squares of the lines AB and BC , together with that which is contained under the lines AB and BC twice: but the square of the line AC is equall to the parallelograme DF . Wherefore the parallelograme DF is equall to that which is composed of the squares of the lines AB and BC together with that which is contained under the lines AB and BC twice. Now then againe (by the 44. of the first) upon the line DE apply the parallelograme EH equall to the squares of the lines AB and BC . Wherefore the parallelograme remayning, namely, HF , is equall to that which is contained under the lines AB and BC twice. And forasmuch as either

of these lines AB and BC is mediall, therefore the squares of the lines AB and BC are also mediall. And that which is contained under the lines AB and BC twice is (by the corollary of the 24. of the tenth) mediall. For by the 6. of this booke it is commensurable to that which is contained under the lines AB and BC once, which is by supposition medial. But unto the squares of the lines AB and BC is equall the parallelograme EH , and unto that which is contained under the lines AB and BC twice is equall the parallelograme HF : wherefore either of these parallelogrames HE and AF is mediall: and they are applied upon the rationall line ED . Wherefore (by the 22. of the tenth) either of these lines DH and HG is a rationall line, and incommensurable in length unto the line DE . And forasmuch as (by supposition) the line AB is incommensurable in length unto the line BC . But as the line AB is to the line BC , so is the square of the line AB to the parallelograme which is contained under the lines AB and BC (by the first of the sixth). Wherefore (by the 10. of this booke) the square of the line AB is incommensurable to the parallelograme contained under the lines

Demonstration.

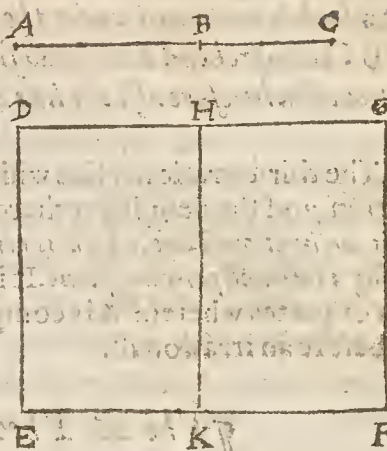


HH. iij.

AB

The tenth Booke

AB and BC. But to the square of the line AB is commensurable that which is composed of the squares of the lines AB and BC (by the 15. of the tenth). For the squares of the lines AB and BC are commensurable (when as the lines AB and BC are put to be commensurable in power onely). And to that which is contained under the lines AB and BC is commensurable that which is contained under the lines AB and BC twice (by the 6. of the tenth) wherefore that which is composed of the squares of the lines AB and BC is incommensurable to that which is contained under the lines AB and BC twice. But to the squares of the lines AB and BC is equall the parallelogramme EH. And to that which is contained under the lines AB and BC twice is equall the parallelogramme FH. Wherefore the parallelogramme EH is incommensurable to the parallelogramme FH. Wherefore the line DH is incommensurable in length to the line HG (by the 1. of the sixth and 10. of this booke). And it is proved that they are rationall lines. Wherefore the lines DH & HG are rationall commensurable in power onely. Wherefore (by the 36. of the tenth) the whole line DG is irrational. And the line DE is rationall. But a recteangle superficies comprehended under a rationall line and an irrational line is (by the corollary added after the 21. of the tenth) irrational. Wherefore the superficies DE is irrational. And the line also which containeth it in power is irrational. But the line AC containeth in power the superficies DF. Wherefore the line AC is irrational. And it is called a second bimediall line.



Definition of
a second bimediall line.

This Proposition sheweth the generation of the fourth irrational line, called a second bimediall line. The definition wherof is euident by this Proposition, which is thus. A second bimediall line is an irrational line, which is made of two mediall lines commensurable in power onely ioyned together, which comprehend a mediall superficies. And it is called a second bimediall, because the two mediall lines of which it is composed, containe a mediall superficies, and not a rationall. Now a mediall is by nature & in knowledge after a rationall.

¶ The 27. Theoreme.

The 39. Proposition.

If two right lines incommensurable in power be added together, hauing that which is composed of the squares of them rationall, and the parallelogramme contained under them mediall: the whole right line is irrational, and is called a greater line.

Demonstration.



Et these two right lines AB and BC being incommensurable in power onely, and making that which is required in the Proposition (The 33. of the tenth teacheth to finde out two such lines) be added together. Then I say, that the whole line AC is irrational. For forasmuch as (by supposition) the parallelogramme contained under the lines AB and BC is mediall, therefore the parallelogramme contained twice under the lines AB and BC is mediall. (For that which is contained under AB and BC twice, is commensurable to that which is contained under AB and BC once (by the 6. of the tenth). Wherefore (by the Corollary of the 23. of the tenth) that which is contained under AB & BC twice, is mediall). But by supposition that which is composed of the squares of the lines AB and BC, is rationall. Wherefore that which is contained under the lines AB and BC twice is incommensurable to that which is composed of the

of the squares of the lines AB and BC . Wherefore (by the 16. of the tenth) that which is composed of the squares of the lines AB and BC together with that which is contained under the lines AB & BC twice, which is (by the 4. of the second) equall to the square of the line AC , is incommensurable to that which is composed of the squares of the lines AB and BC . But that which is composed of the squares of the lines AB and BC , is rationall. Wherefore the square of the whole line AC , is irrational. Wherefore the line AC also is irrational. And is called a greater line. And it is called a greater line for that that which is composed of the squares of the lines AB & BC which are rationall, is greater then that which is contained under the lines AB and BC twice, which are medially. Now it is meete that the name should be given according to the propertie of the rationall.



An Assumpt.

And that the superficies composed of the squares of the lines AB and BC , is greater then that which is contained under the lines AB and BC twice, may thus be proued. First it is manifest, that the lines AB and BC are unequal: for if they were equall, then the squares of the lines AB and BC should be equall to that which is contained under the lines AB and BC twice, so that that which is contained under AB and BC , should also be rationall: which is contrary to the supposition. Wherefore the lines AB and BC are unequal. Suppose then that the line AB be the greater, & let the line BD be equall to the line BC . Wherefore (by the 7. of the second) the squares of the lines AB and BD , are equall to that which is contained under the lines AB and BC twice and to the square of the line AD . But the line DB is equall to the line BC . Wherefore the squares of the lines AB and BC , are equall to that which is contained under the lines AB and BC twice and to the square of the line AD . Wherefore the squares of the lines AB and BC are greater then that which is contained under the lines AB and BC twice, by the square of the line AD : which was required to be proued.

An Assumpt.



This Proposition teacheth the production of the fifth irrational line, which is called a greater line: which is by the sense of this Proposition thus defined.

A greater line is an irrational line, which is composed of two right lines which are incommensurable in power, the squares of which added together, make a rational superficies, and the parallelogramme which they containe, is medially. It is therefore called a greater line, as Theon sayth, because the squares of the two lines of which it is composed, added together being rationall, are greater then the medially superficies contained vnder them twice. And it is conuenient that the denomination be taken of the proprietie of the rationall part, rather then of the medially part.

Definition of a greater line.

¶ The 28. Theoreme.

The 40. Proposition.

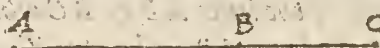
If two right lines incommensurable in power be added together, hauing that which is made of the squares of them added together medially, and the parallelogramme contained vnder them rationall: the whole right line is irrational, and is called a line containing in power a rational and a medially superficies.

Let

The tenth Booke

Demonstration.

Et these two right lines AB and BC being incommensurable in power, & making that which is required in the Proposition be added together. (The 34. of the tenth teacheth to finde out two such lines). Then I say, that the whole line AC is irrational. For forasmuch as that which is composed of the squares of the lines AB and BC is medially, but that which is contained under the lines AB and BC twice is rational, therefore that which is composed of the squares of the lines AB and BC , is incommensurable to that which is contained under the lines AB and BC twice. Wherefore (by the 16. of the tenth) that which is composed of the squares of the lines AB and BC together with that which is contained under the lines AB and BC twice, which (by the 4. of the second) is the square of the whole line AC , is incommensurable to that which is contained under the lines AB and BC twice. But that which is contained under the lines AB and BC twice, is rational, for that which is contained under the lines AB and BC once is put to be rational. Wherefore the square of the whole line AC is irrational. Wherefore the line AC also is irrational. And it is therefore called a line containing in power a rational and a medially superficies, because the power thereof containeth two superficieses, whereof the one is rational, namely, that which is contained under the two partes, and the other medially, namely, that which is made of the squares of the partes added together. And by reason that the rational is in order of nature & of knowledge before the medially, therefore the first part of the denomination is taken of it, and the second part is taken of the medially: which was required to be proved.



In this Proposition is taught the generation of the sixth irrational line, which is called a line whose power is rational and medially. The definition of which is gathered of this Proposition after this manner.

Definition of a line whose power is rational and medially.

A line whose power is rational and medially, is an irrational line which is made of two right lines incommensurable in power added together, whose squares added together make a medially superficies, but that superficies which they containe is rational. The reason of the name is before set forth in the Proposition.

¶ The 29. Theoreme.

The 41. Proposition.

If two right lines incommensurable in power be added together, hauing that which is composed of the squares of them added together medially, and the parallelogramme contained under them medially, and also incommensurable to that which is composed of the squares of them added together: the whole right line is irrational, and is called a line containing in power two medials.

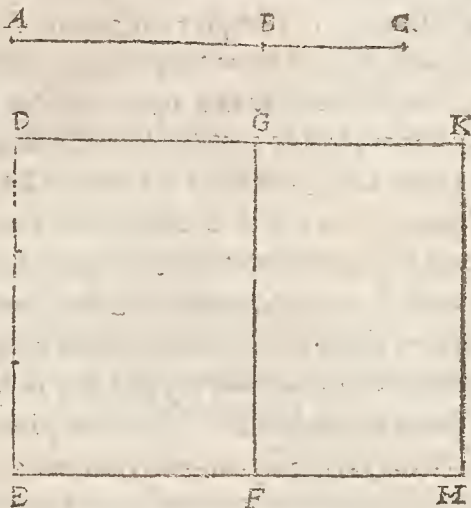
Construction.

Et these two right lines AB and BC being incommensurable in power, and hauing that which is composed of the squares of the lines AB and BC medially, and the parallelogramme which is contained under the lines AB and BC medially, and also incommensurable to that which is composed of the squares of the lines AB and BC added together, be added together (the 35. of the tenth teacheth to finde out two such lines) Then I say that the whole line AC is irrational. Take a rational line DE , and (by the 44. of the first) upon the line DE apply the parallelogramme DF equall to the squares of the lines AB and BC . And upon the line GF , which is equall to the line DE , apply the parallelogramme GH equall to that which is contained under the lines AB and BC twice. Wherefore the whole

whole parallelogramme DH is equall to the square of the line AC (by the 4. of the second.) And forasmuch as that which is composed of the squares of the lines AB and BC is medially and is equall to the parallelogramme DF : therefore DF also is medially (by that which was said in the 38. proposition of this booke). And it is

Demonstration.

applied upon the rationally line DE . Wherefore the line DG is rationally and incommensurable in length unto DE (by the 22. of the tenth). And by the same reason the line GK is rational & incommensurable in length unto the line GF , that is, unto the line DE . And forasmuch as that which is composed of the squares of the lines AB & BC added together, is by supposition incommensurable to that which is contained under the lines AB and BC twice, therefore also the parallelogramme DF is incommensurable unto the parallelogramme GH . Wherefore also the line DG is incommensurable unto the line GK (by the first of the sixth) and by the tenth of the tenth. But it is now proved that they are rationally. Wherefore the lines DG and GK are rationally commensurable in power onely. Wherefore (by the 36. of the tenth) the whole



line DK is rationally, and is called a binomial line, but the line DE is irrational. Wherefore the parallelogramme DH is irrational (by the corollary added after the 22. proposition of the tenth). Wherefore also the line which containeth it in power is irrational: but the line AC containeth it in power. Wherefore the line AC is irrational, and is called a line containing in power two medials. It is called a line containing in power two medials, for that it containeth in power two medially superficieses, one of which is composed of the squares of the lines AB and BC added together, and the other is that which is contained under the lines AB & BC twice: which was required to be demonstrated.

In this proposition is taught the nature of the 7. kinde of irrational lines which is called a line whose power is two medials. The definition whereof is taken of this proposition after this manner.

A line whose power is two medials, is an irrational line which is composed of two right lines incommensurable in power, the squares of which added together, make a medially superficies, and that which is contained under them is also medially, and moreover it is incommensurable to that which is composed of the two squares added together.

Definition of a line containing in power two medials.

The reason why this line is called a line whose power is two medials, was before in the end of the demonstration declared.

And that the said irrational lines are divided one way onely, that is, in one point onely, into the right lines of which they are composed, and which make every one of the kinds of those irrational lines, shall straight way be demonstrated: but first will we demonstrate two assumptes here following.

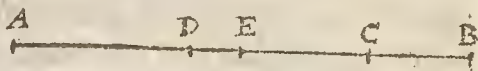
¶ An Assumpt.

Take a right line and let the same be AB , and divide it into two unequal partes in the point C , and againe divide the same line AB into two other unequal partes, in an other point namely, in D , and let the line AC (by supposition) be greater then the line DB . Then I say that

An Assumpt.

The tenth Booke

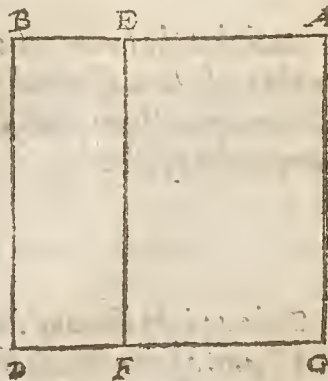
that the squares of the lines AC and BC added together, are greater then the squares of the lines AD and DB added together. Deuide the line AB (by the 10. of the first) into two equall partes in the point E . And forasmuch as the line AC is greater then the line DB , take away the line DC which is common to them both: wherfore the residue AD is greater then the residue CB , but the line AE is equall to the line EB . Wherfore the line DE is lesse then the line EC . Wherfore the pointes C and D are not equally distant from the point E , which is the point of the section into two equall partes. And forasmuch as (by the 5. of the second) that which is contained vnder the lines AC and CB together with the square of the line EC is equall to the square of the line EB . And by the same reason that which is contained vnder the lynes AD and DB together with the square of the line DE , is also equall to the self same square of the line EB : wherfore that which is contained vnder the lines AC and CB together with the square of the line EC is equall to that which is contained vnder the lines AD and DB together with the square of the line DE : of which the square of the line DE is lesse then the square of the line EC (for it was proued that the line DE is lesse then the line EC). Wherfore the parallelogramme remayning, contained vnder the lines AC and CB is lesse then the parallelogramme remayning contained vnder the lines AD and DB . Wherfore also that which is contained vnder the lines AC and CB twise is lesse then that which is contained vnder the lines AD and DB twise. But (by the fourth of the second) the square of the whole line AB is equall to that which is composed of the squares of the lines AC and CB together with that which is contained vnder the lynes AC and CB twise, and by the same reason the square of the whole line AB is equall to that which is composed of the squares of the lines AD and DB together with that which is contained vnder the lynes AD and DB twise: wherfore that which is composed of the squares of the lynes AC and CB together with that which is contained vnder the lynes AC and CB twise, is equall to that which is composed of the squares of the lynes AD and DB , together with that which is contained vnder the lynes AD and DB twise. But it is already proued that that which is contained vnder the lynes AC and CB twise, is lesse then that which is contained vnder the lines AD & DB twise. Wherfore the residue, namely, that which is composed of the squares of the lines AC and CB is greater then the residue, namely, then that which is composed of the squares of the lines AD and DB : which was required to be demonstrated.



¶ An Assumpt.

A ratiōall superficies exceedeth a ratiōall superficies, by a ratiōall superficies.

Let AD be a ratiōall superficies, and let it exceede AF being also a ratiōall superficies by the superficies ED . Then I say that the superficies ED is also ratiōall. For the parallelogramme AD is commensurable to the parallelogramme AF , for that either of them is ratiōall. Wherfore (by the second part of the 15. of the tenth) the parallelogramme AF is commensurable to the parallelogramme ED . But the the parallelogramme AF is ratiōall. Wherfore also the parallelogramme ED is ratiōall.



¶ The 30. Theoreme.

The 42. Proposition.

A binomial line is in one point onely deuided into his names.

The second Se-
nary by com-
position.



Suppose that AB be a binomial line, and in the point C let it be deuided into his names, that is, into the lines wherof the whole line AB is composed. Wherefore these lines AC and CB are rational commensurable in power onely. Now I say that the line AB cannot in any other point besides C be deuided into two ra-

tional lines commensurable in power onely. For

if it be possible, let it be deuided in the point D ,

so that let the lines AD and DB be rational

commensurable in power onely. First it is manifest, that neither of these pointes C and D deuideth the right line AB into two equall partes. Otherwise the lines AC and CB should be rational commensurable in length, and so likewise should the lines AD and DB be. For every line measureth it selfe, and any other line equall to it selfe. Moreover the line DB is either one and the same with the line AC , that is, is equall to the line AC , or els it is greater then the line AC , either els it is lesse then it. If DB be equall to the line AC , then putting the line DB vpon the line AC eche endes of the one, shall agree with eche endes of the other. Wherefore putting the point B vpon the point A , the point D also shall fall vpon the point C , and the line AD which is the rest of the line AC , shall also be equall to the line CB , which is the rest of the line DB . Wherefore the line AB is deuided into his names in the point C . And so also shal the line AB being deuided in the point D be deuided in the self same point that the self same line AB was before deuided in the point C , which is contrary to the suppositiō. For by suppositiō it was deuied in sundry pointes, namely, in C & D . But if the line DB be greater the the line AC , let the line AB be deuided into two equal partes in the point E . Wherefore the points C & D shal not equally be distant frō the point E . Now (by the first assumpt going before this propositiō) that which is cōposed of the squares of y lines AD & DB is greater the that which is composed of the squares of the lines AC & CB . But that which is composed of the squares of the lines AD & DB together with that which is cōtained vnder the lines AD & DB twise, is equall to that which is composed of the squares of the lines AC & CB together with that which is contained vnder the lines AC and CB twise, for either of them is equall to the square of the whole line AB (by the 4. of the second): wherefore how much that which is cōposed of the squares of the lines AD and DB added together is greater then that which is composed of the squares of the lines AC and CB added together, so much is that which is contained vnder the lines AC and CB twise greater then that which is contained vnder the lines AD and DB twise. But that which is composed of the squares of the lines AD and DB exceedeth that which is composed of the squares of the lines AC and CB by a rational superficies (by the 2. assumpt going before this proposition.) For that which is composed of the squares of the lines AD and DB is rational, and so also is that which is composed of the squares of the lines AC and CB : for the lines AD and DB are put to be rational commensurable in power onely, and so likewise are the lines AC and CB . Wherefore also that which is contained vnder the lines AC and CB twise, exceedeth that which is contained vnder the lines AD & DB twise by a rational superficies, whē yet notwithstanding they are both medial superficieses (by the 21. of the tenth): which (by the 26. of the same) is impossible. And if the line DB be lesse then the line AC , we may by the like demonstration proue the selfe same impossibilitie. Wherefore a binomial line is in one point onely deuided into his names: Which was required to be demonstrated.



Demonstrati-
on leading to
an impossibi-
liti.

The tenth Booke

Corollary added by *Flussates*.

A Corollary.

Two rationall lines commensurable in power onely being added together cannot be equall to two other rationall lines commensurable in power onely added together. For either of them should make a binomial line, and so should a binomial line be deuided into his names in moe poyntes then one: which by this proposition is proued to be impossible. The like shall follow in the five next irrational lines as touching their two names.

¶ The 31. Probleme.

The 43. Proposition.

A first bimediall line is in one poynt onely deuided into his names.

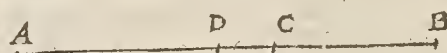


Suppose that AB be a first bimediall line, and let it be deuided into his partes in the poynt C , so that let the lines AC and CB be mediall cōmensurable in power onely, and containing a rationall superficies. Then I say that the line AB can not be deuided into his names in any other poynt then in C . For if it be possible let it

Demonstration leading to an impossibility.

be deuided into his names in the poynt D , so that let

AD & DB be mediall lines commensurable in power onely, comprehending a rationall superficies. Now for-



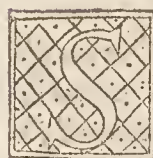
asmuch as how much that which is containd vnder

the lines AD and DB twise differreth from that which is containd vnder the lines AC and CB twise, so much differreth that which is composed of the squares of the lines AD and DB from that which is composed of the squares of the lines AC and CB : but that which is containd vnder the lines AD and DB twise differreth from that which is containd vnder the lines AC and CB twise, by a rationall superficies (by the second assumpt going before the 41. of the tenth). For either of those superficieses is rationall. Wherefore that which is composed of the squares of the lines AC and CB differreth from that which is composed of the squares of the lines AD and DB by a rationall superficies, when yet they are both mediall superficieses: which is impossible. Wherefore a first bimediall line is in one poynt onely deuided into his names: which was required to be proued.

¶ The 32. Theoreme.

The 44. Proposition.

A second bimediall line is in one poynt onely deuided into his names.



Suppose that the line AB being a second bimediall line, be deuided into his names in the poynt C : so that let the lines AC and CB be mediall lines commensurable in power onely, comprehending a mediall superficies. It is manifest that the poynt C deuideth not the whole line AB into two equall partes. For

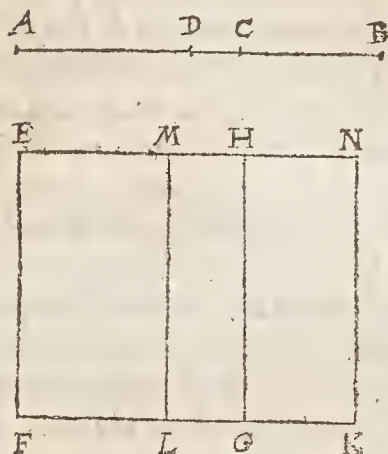
Demonstration leading to an impossibility.

the lines AC and CB are not commensurable in length the one to the other. Now I say that the line AB cannot be deuided into his names in any other poynt but onely in C . For if it be possible, let it be deuided into his names in the poynt D , so that let not the line AC be one and the same, that is, let it not be equall, with the line DB . But let it be greater then it. Now it is manifest (by the first assumpt going before the 42. proposition of this booke) that the squares of the lines AC and CB are greater then the squares of the lines AD and DB . And also that the lines AD and DB are mediall lines commensurable in power onely, comprehending a mediall superficies. Take a rationall line EF . And (by the 44. of the first) vpon the line EF apply a rectangle parallelograme EK equall to the square of the line AB . From which parallelograme take away the parallelograme EG equall to the squares of the lines AC and CB . Wherefore the residue, namely, the parallelograme HK is equall to that which is contained

wed under the lines AC and CB twise. Agayne from the the parallelograme EK take away the parallelograme EL equall to the squares of the lines AD and DB which are lesse then the squares of the lines AC and CB . Wherefore the residue, namely, the parallelograme MK is equall to that which is contayned under the lines AD and DB twise.

And forasmuch as the squares of the lines AC and CB are medially, therefore the parallelograme EG also is medially. And it is applyed upon the rationall line EF : wherefore the line EH is rationall and incommensurable in length to the line EF . And by the same reason, the parallelograme HK is medially (for that which is equall unto it, namely, that which is contayned under the lines AC and CB twise is medially) therefore the line HN is also rationall and incommensurable in length unto the line EF . And forasmuch as the lines AC and CB are medially lines commensurable in power onely, therefore

the line AC is incommensurable in length unto the line CB . But as the line AC is to the line CB , so is the square of the line AC to that which is contayned under the lines AC and CB (by the 1. of the sixt). Wherefore the square of the line AC is incommensurable to that which is contayned under the lines AC and CB . But (by the 16. of the tenth) unto the square of the line AC are commensurable the squares of the lines AC and CB added together, for the lines AC and CB are commensurable in power onely. And unto that which is contayned under the lines AC and CB is commensurable that which is contayned under the lines AC and CB twise. Wherefore that which is composed of the squares of the lines AC and CB , is incommensurable to that which is contained under the lines AC and CB twise. But to the squares of the lines AC and CB is equall the parallelograme EG , and to that which is contained under the lines AC and CB twise is equall the parallelograme HK . Wherefore the parallelograme EG is incommensurable to the parallelograme HK . Wherefore also the line EH is incommensurable in length to the line HN . And the lines EH and HN are rationall. Wherefore they are rationall commensurable in power onely: but if two rationall lines commensurable in power onely be added together, the whole line is irrational, and is called a binomial line (by the 36. of the tenth). Wherefore the binomial line EN is in the poynt H devided into his names. And by the same reason also may it be proued that the lines EM and MN are rationall lines commensurable in power onely. Wherefore EN being a binomial line is devided into his names in sundry poyntes, namely, in H and M , neither is the line EH one and the same, that is, equal with MN . For the squares of the lines AC and CB are greater then the squares of the lines BD and AD (by the 1. assumpt put after the 41. of the tenth). But the squares of the lines AD and DB are greater then that which is contayned under the lines AD and DB twise (by the assumpt put after the 39. of the tenth). Wherefore the squares of the lines AC and CB , that is, the parallelograme EG is much greater then that which is contained under the lines AD and DB twise, that is then the parallelograme MK . Wherefore (by the first of the sixt) the line EH is greater then the line MN . Wherefore EH is not one and the same with MN . Wherefore a binomial line is in two sundry poyntes devided into his names. Which is impossible. The selfe same absurditie also will follow if the line AC be supposed to be lesse then the line DB . A second binomial line therefore is not devided into his names in sundry poyntes. Wherefore it is devided in one onely: which was required to be demonstrated.



Demonstration leading to an impossible line.

¶ The 33. Theoreme.

The 45. Proposition.

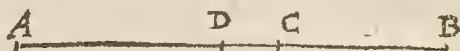
A greater line is in one poynt onely devided into his names.

The tenth Booke

Demonstrati-
on leading to
an impossibi-
litie.



Et AB being a greater line be deuided into his names in the poynt C , so that let the lines AC and CB be rationally incommensurable in power, hauing that which is composed of the squares of the lines AC and CB rationally, and that which is contained under the lines AC and CB medially. Then I say that the line AB can not in any other poynt then in C be deuided into his names. For if it be possible, let it be deuided into his names in the poynt D , so that let AD and DB be lines incommensurable in power, hauing that which is composed of the squares of the lines AD and DB rationally, and that which is contained under the lines AD and DB medially. Now forasmuch as how much the squares of the lines AC and CB differ from the squares of the lines AD and DB , so much differeth that which is contained under the lines AD and DB twice from that which is contained under the lines AC and CB twice, by those things which haue bene sayd in the demonstration of the 42. proposition. But the squares of the lines AC and CB exceede the squares of the lines AD and DB by a rationally superficies (for they are either of them rational). Wherefore that which is contained under the lines AD and DB twice exceedeth that which is contained under the lines AC and CB twice by a rationally superficies: when as either of them is a medially superficies. Which is impossible (by the 26. of the tenth). Wherefore a greater line is in one poynt onely deuided into his names: which was required to be proued.



¶ The 34. Theoreme.

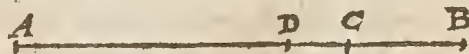
The 46. Proposition.

A line containing in power a rational and a medially, is in one point onely deuided into his names.

Demonstrati-
on leading to
an impossibi-
litie.



Et AB being a line containing in power a rational and a medially, be deuided into his names in the point C , so that let the lines AC & CB be incommensurable in power, hauing that which is composed of the squares of the lines AC and CB medially, and that which is contained under the lines AC and CB rationally. Then I say, that the line AB can not in any other point be deuided into his names but onely in the point C . For if it be possible, let it be deuided into his names in the point D , so that let the lines AD and DB be incommensurable in power, hauing that which is composed of the squares of the lines AD and DB medially, and that which is contained under the lines AD and DB rationally. Now forasmuch as how much that which is contained under the lines AD & DB twice differeth from that which is contained under the lines AC and CB twice, so much differ the squares of the lines AC & CB added together from the squares of the lines AD and DB added together. But that which is contained under the lines AC and CB twice exceedeth that which is contained under the lines AD and DB twice by a rationally superficies (for either of them is rational). Wherefore also the squares of the lines AC and CB added together, exceede the squares of the lines AD and DB added together by a rationally superficies, when yet ech of them is a medially superficies: which is impossible. Wherefore a line containing in power a rational and a medially, is in one point onely deuided into his names: which was required to be demonstrated.



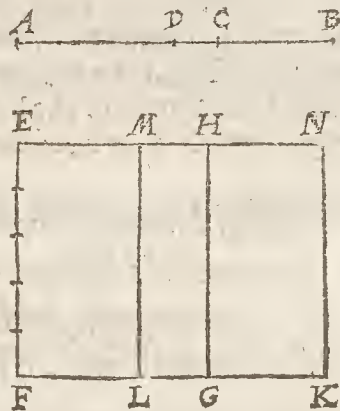
¶ The

A line containning in power two medials, is in one point onely divided into his names.



Suppose that AB being a line containing in power two medials, be divided into his names in the point C , so that let the lines AC and CB be incommensurable in power, having that which is composed of the squares of the lines AC & CB mediall, and that also which is contained under the lines AC and CB mediall; and moreouer incommensurable to that which is composed of the squares of the lines AC and CB . Then I say, that the line AB can in no other point be divided into his names but onely in the point C . For if it be possible, let it be divided into his names in the point D , so that let not the line AC be one and the same; that is, equall with the line DB : but by supposition let the line AC be the greater. And take a rationall line EF . And (by the 43. of the first) upon the line EF apply a rectangle parallelograme EG equall to that which is composed of the squares of the lines AC and CB : and likewise upon the line HG , which is equall to the line EF , apply the parallelogramme HK equall to that which is contained under the lines AC and CB twice. Wherefore the whole parallelogramme EK is equall to the square of the line AB . Again upon the same line EF describe the parallelogramme EL equall to the squares of the lines AD and DB . Wherefore the residue, namely, that which is contained under the lines AD and DB twice, is equall to the parallelogramme remaining, namely, to MK . And forasmuch as that which is composed of the squares of the lines AC and CB , is (by supposition) mediall, therefore the parallelogramme EG which is equall unto it, is also mediall: and it is applied upon the rationall line EF . Wherefore (by the 22. of the tenth) the line HE is rationall and incommensurable in length unto the line EF . And by the same reason also the line HN is rationall and incommensurable in length to the same line EF . And forasmuch as that which is composed of the squares of the lines AC and CB is incommensurable to that which is contained under the lines AC and CB twice (for it is supposed to be incommensurable to that which is contained under the lines AC and CB once): therefore the parallelogramme EG is incommensurable to the parallelogramme HK . Wherefore the line EH also is incommensurable in length to the line HN , and they are rationall lines: wherefore the lines EH and HN are rationall commensurable in power onely. Wherefore the whole line EN is a binomiall line, and is divided into his names in the point H . And in like sort may we proue, that the same binomiall line EN is divided into his names in the point M , and that the line EH is not one and the same that is equall with the line MN , as it was proued in the end of the demonstration of the 44. of this booke. Wherefore a binomiall line is divided into his names in two sundry pointes: which is impossible (by the 42. of the tenth). Wherefore a line containing in power two medials, is not in sundry pointes divided into his names. Wherefore it is divided in one point onely: which was required to be demonstrated.

Construction.



Demonstration leading to an absurditie.

The tenth Booke

¶ Second Definitions.

Sixe kindes of
binomiall
lines.



It was shewed before that of binomiall lines there were sixe kindes, the definitions of all which are here now set, and are called second definitiōs. All binomiall lines, as all other kindes of irrationall lines, are cōceaued, cōsidered, and perfectly vnderstanded onely in respecte of a rationall line (whose partes as before is taught, are certayne, and knowen, and may be distinctly expressed by number) vnto which line they are compared. Thys rationall line must ye euer haue before your eyes, in all these definitions, so shall they all be easie inough.

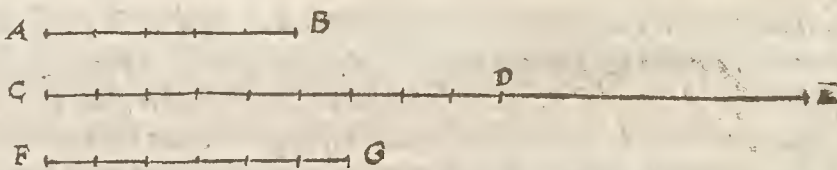
A binomiall
line consisteth
of two partes.

A binomiall line (ye know) is made of two partes or names, wherof the one is greater then the other. Wherefore the power or square also of the one is greater then the power or square of the other. The three first kindes of binomiall lines, namely, the first, the second, & the third, are produced, when the square of the greater name or part of a binomiall exceedeth the square of the lesse name or part, by the square of a line which is commensurable in length to it, namely, to the greater. The three last kindes, namely, the fourth, the fift, and the sixt, are produced, when the square of the greater name or part exceedeth the square of the lesse name or part, by the square of a line incommensurable in length vnto it, that is, to the greater part.

First defini-
tion.

A first binomiall line is, whose square of the greater part exceedeth the square of the lesse part by the square of a line commensurable in length to the greater part, and the greater part is also commensurable in length to the rationall line first set.

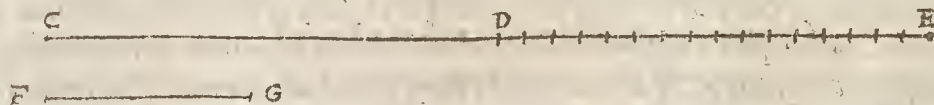
As let the rationall line first set be AB : whose partes are distinctly knowen: suppose also that the line CE be a binomiall line, whose names or partes let be CD and DE . And let the square of the line CD the greater part exceede the square of the line DE the lesse part by the square of the line FG : which line FG , let bee commensurable in length to the line CD , which is the greater part of the binomiall line. And moreouer let the line CD the greater part be commensurable in length to the rationall line first set, namely, to AB . So by this definition the binomiall line CE is a first binomiall line.



Second defini-
tion.

A second binomiall line is, when the square of the greater part exceedeth the square of the lesse part by the square of a line commensurable in length vnto it, and the lesse part is commensurable in length to the rationall line first set.

As (supposing euer the rationall line) let CE be a binomiall line deuided in the poynt D . The square of whose greater part CD let exceede the square of the lesse part DE by the square of the line FG , which line FG let be cōmensurable in length vnto the line CD the greater part of the binomiall line. And let also the line DE the lesse part of the binomiall line be commensurable in length to the rationall line first set AB . So by this definition the binomiall line CE is a second binomiall line.

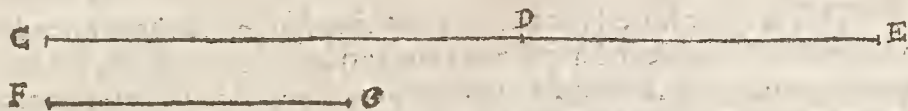


Third defini-
tion.

A third binomiall line is, when the square of the greater part exceedeth the square

square of the lesse part, by the square of a line cōmensurable in length vnto it. And neither part is commensurable in length to the rationall line geuē.

As suppose the line C E to be a binomial line: whose partes are ioyned together in the poynt D: and let the square of the line C D the greater part exceede the square of the lesse part D E by the square of the line F G, and let the line F G be commensurable in length to the line C D the greater part of the binomial.

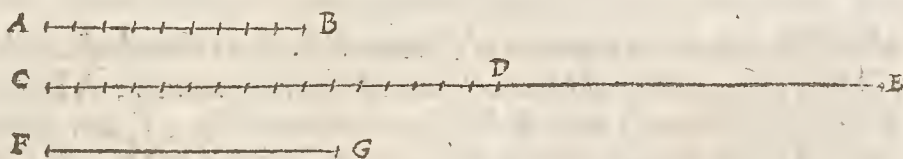


Moreouer, let neither the greater part C D, nor the lesse part D E, be commensurable in length to the rationall line A B, then is the line C E by this definition a third binomial line.

A fourth binomial line is, when the square of the greater part exceedeth the square of the lesse by the square of a line incommensurable in length vnto the greater part. And the greater is also commensurable in length to the rationall line.

Fourth definition.

As let the line C E be a binomial line, whose partes let be C D & D E, & let the square of the line C D the greater part exceede the square of the line D E the lesse, by the square of the line F G. And let the line F G be incommensurable in length to the line C D the greater.

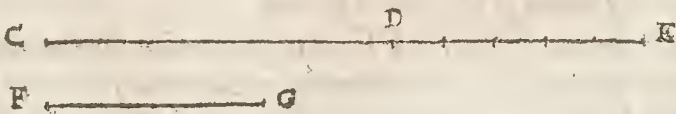


Let also the line C D the greater part be commensurable in length vnto the rationall line A B. Then by this definition the line C E is a fourth binomial line.

A fift binomial line is, when the square of the greater part exceedeth the square of the lesse part, by the square of a line incommensurable vnto it in length. And the lesse part also is commensurable in length to the rationall line geuen.

Fifth definition.

As suppose that C E be a binomial line, whose greater part let be C D, and let the lesse part be D E. And let the square of the line C D exceede the square of the line D E by the square of the line F G, which let be incōmensurable in length vnto the line C D the greater part of the binomial line. And let the line D E the second part of the binomial line be commensurable in length vnto the rationall line A B. So is the line C E by this definition a fift binomial line.



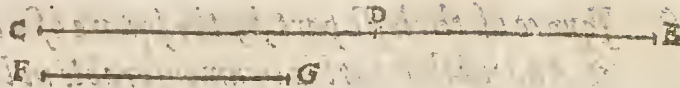
A sixt binomial line is, when the square of the greater part exceedeth the square of the lesse, by the square of a line incommensurable in length vnto it. And neither part is commensurable in length to the rationall line geuen.

Sixth definition.

As let the line C E be a binomial line, deuided into his names in the point D. The square of whose greater part C D let exceede the square of the lesse part D E by the square of the line F G, and let the line F G be incommensurable in length to the line C D the greater part of the binomial line. Let also

The tenth Booke

neither CD the greater part, nor DE the lesse part be commensurable in length to the rationall line AB . And so by this definition the line CE is a first binomial line. So ye see that by these definitions, & their examples, and declarations, all the kindes of binomial lines are made very playne.



This is to be noted that here is nothing spoken of those lines, both whose portions are commensurable in length vnto the rationall line first set, for that such lines cannot be binomial lines. For binomial lines are composed of two rationall lines commensurable in Power onely (by the 36. of this booke). But lines both whose portions are commensurable in length to the rationall line first set are not binomial lines. For that the partes of such lines should by the 12. of this booke be commensurable in length the one to the other. And so should they not be such lines as are required to the composition of a binomial line. Moreouer such lines should not be irrational but rationall, for that they are commensurable to ech of the parts whereof they are composed (by the 15. of this booke). And therefore they should be rationall for that the lines which compose them are rationall.

¶ The 13. Probleme.

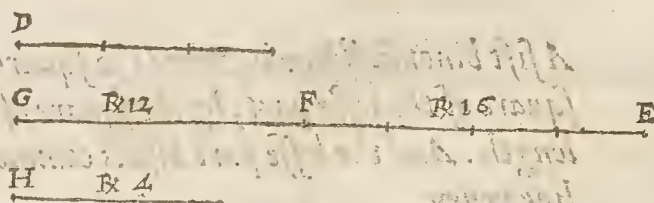
The 48. proposition.

To finde out a first binomial line.

The third Secondary by composition.

Construction.

Take two numbers AC and CB , & let them be such, that the number which is made of them both added together, namely, AB , haue vnto one of them, namely, vnto BC that proportion that a square number hath to a square number. But vnto the other, namely, vnto CA let it not haue that proportion that a square number hath to a square number (such as is euery square number which may be deuided into a square number and into a number not square). Take also a certayne rationall line, and let the same be D . And vnto the line D let the line EF be commensurable in length. Wherefore the line EF is rationall. And as the number AB is to the number AC , so let the square of the line EF be to the square of another line, namely, of FG (by the corollary of the sixt of 5th teth). Wherefore the square of the line EF hath



$A \dots\dots C \dots\dots B$

Demonstration.

to the square of the line FG that proportion that number hath to number. Wherefore the square of the line EF is commensurable to the square of the line FG (by the 6. of this booke). And the line EF is rationall. Wherefore the line FG also is rationall. And forasmuch as the number AB hath not to the number AC , that proportion that a square number hath to a square number, neither shal the square of the line EF haue to the square of the line FG that proportion that a square number hath to a square number. Wherefore the line EF is incommensurable in length to the line FG (by the 9. of this booke). Wherefore the lines EF and FG are rationall commensurable in power onely. Wherefore the whole line EG is a binomial line (by the 36. of the tenth). I say also that it is a first binomial line. For for that as the number BA is to the number AC , so is the square of the line EF to the square of the line FG : but the number BA is greater then the number AC : wherefore the square of the line EF is also greater then the square of the line FG . Vnto the square of the line EF let the squares of the lines FG and H be equall (which how to finde out is taught in the assumpt put after

after the 13. of the tenth). And forthat as the number BA is to the number AC , so is the square of the line EF to the square of the line FG : therefore (by conuersion or euerfion of proportion (by the corollary of the 19. of the fift) as the number AB is to the number BC , so is the square of the line EF to the square of the line H . But the number AB hath to the number BC that proportion that a square number hath to a square number. Wherefore also the square of the line EF hath to the square of the line H that proportion that a square number hath to a square number. Wherefore the line EF is commensurable in length to the line H (by the 9. of this booke. Wherefore the line EF is in power more then the line FG by the square of a line commensurable in length to the line EF . And the lines EF and FG are rationall commensurable in power onely. And the line EF is commensurable in length to the rationall line D . Wherefore the line EG is a first binomial line: which was required to be doone.

The 14. Probleme.

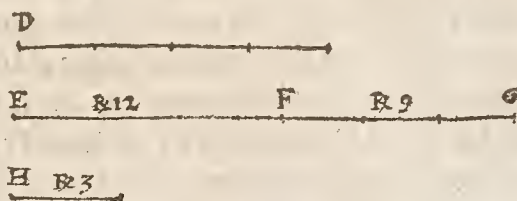
The 49. Proposition.

To finde out a second binomial line.

CTake two numbers AC and CB , and let them be such that the number made of them both added together, namely, AB haue vnto BC that proportiō that a square number hath to a square number, and vnto the number CA let it not haue that proportion that a square number hath to a square number, as it was declared in the former proposition. Take also a rationall line, and let the same be D , and vnto the line D let the line FG be commensurable in length. Wherefore FG is a rationall line. And as the number CA is to the number AB so let the square of the line GF be to the square of the line FE (by the 6. of the tenth), Wherefore the square of the line GF is commensurable to the square of the line FE . Wherefore also FE is a rationall line. And forasmuch as the nūber CA hath not vnto the number AB that proportiō that a square number hath to a square nūber, therefore neither also the square of the line GF hath to the square of the line FE that proportion that a square number hath to a square number. Wherefore the line GF is incommensurable in length vnto the line FE (by the 9. of the tenth): wherefore the lines FG and FE are rationall commensurable in power onely. Wherefore the whole line EG is a binomial line. I say moreouer that the line EG is a second binomial line. For for that by contrary proportion as the number BA is to the number AC , so is the square of the line EF to the square of the line FG . But the number BA is greater then the number AC , wherefore also the square of the line EF is greater then the square of the line FG . Vnto the square of the line EF , let the squares of the lines GF and H be equall. Now by conuersion (by the corollary of the 19. of of the fift) as the number AB is to the number BC , so is the square of the line EF to the square of the line H . But the number AB hath to the number BC that proportion that a square number hath to a square number: Wherefore the square of the line EF hath to the square of the line H , that proportion that a square number hath to a square number. Wherefore (by the 9. of the tenth) the line EF is commensurable in length vnto the line H . Wherefore the line EF is in power more then the line FG by the square of a line commensurable in length vnto the line EF : and the lines

Construction.

Demonstration.



$A \dots \dots \dots C \dots \dots B$

The tenth Booke

EE and FG are rationally commensurable in power only, and FG being the lesse name is commensurable in length unto the rational line geuen, namely, to D. Wherefore EG is a second binomial line: which was required to be done.

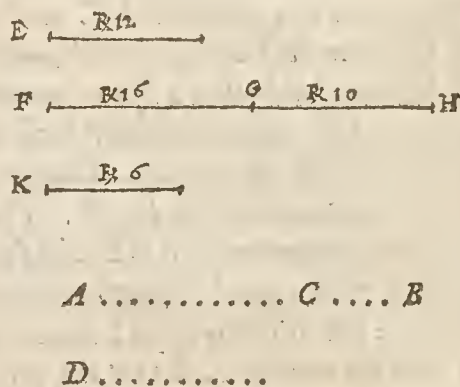
¶ The 15. Probleme.

The 50. Proposition.

To finde out a third binomial line.

Construction.

Take two numbers AC and CB, and let them be such that the number made of them both added together, namely, AB, haue to the number BC that proportion that a square number hath to a square number. But to the number AC let it not haue that proportion that a square number hath to a square number, as it was declared in the two former. And take also some other number that is not a square number, and let the same be D, and let not the number D haue either to the number BA, or to the number AC that proportion that a square number hath to a square number. And take a rational line and let the same be E. And as the number D is to the number AB, so let the square of the line E be to the square of the line FG. Wherefore the square of the line E is commensurable to the square of the line FG, but the line E is rational, wherefore the line FG also is rational. And for that the number D hath not to the number AB that proportion that a square number hath to a square number, neither also shall the square of the line E haue to the square of the line FG that proportion that a square number hath to a square number. Wherefore the line E is incommensurable in length to the line FG (by the 9. of the tenth.) Now againe as the number AB is to the number AC, so let the square of the line FG be to the square of the line GH. Wherefore the square of the line FG is commensurable to the square of the line GH. And the line FG is rational. Wherefore also the line GH is rational. And for that the number BA hath not to the number AC, that proportion that a square number hath to a square number, therefore neither also hath the square of the line FG to the square of the line GH that proportion that a square number hath to a square number. Wherefore the line FG is incommensurable in length to the line GH. Wherefore the lines FG & GH are rational commensurable in power only. Wherefore the whole line FH is a binomial line. I say moreouer that it is a third binomial line. For for that as the number D is to the number AB, so is the square of the line E to the square of the line FG: but as the number AB is to the number AC, so is the square of the line FG to the square of the line GH: therefore of equalitie (by the 22. of the fifth) as the number D is to the number AC, so is the square of the line E to the square of the line GH. But the number D hath not to the number AC that proportion that a square number hath to a square number. Wherefore neither also hath the square of the line E to the square of the line GH that proportion that a square number hath to a square number. Wherefore the line E is incommensurable in length to the line GH. And for that as the number AB is to the number AC, so is the square of the line FG to the square of the line GH, therefore the square of the line FG is greater then the square of the line GH. Vnto the square of the line FG, let the squares of the lines GH and K be equal. Wherefore (by euerse proportion by the corollary of the 19. of the fifth) as the number AB is to the number BC, so is the square of the line FG to the square of the line K. But the number AB hath to the number



Demonstration.

BC

B.C. that proportio that a square number hath to a square number. Wherefore also the square of the line F G hath to the square of the line K that proportion that a square number hath to a square number. Wherefore the line F G is commensurable in length to the line K. Wherefore the line F G is in power more then the line G H by the square of a line commensurable in length vnto it. And the lines F G and G H are rationall commensurable in power onely, and neither of them is commensurable in length vnto the rationall line E: wherefore the lyne F-H is a third binomiall line: which was required to be done.

¶ The 16. Probleme.

The 51. Proposition.

To finde out a fourth binomiall line.

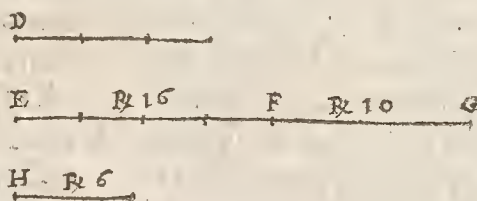


Ake two numbers A C and C B, & let the be such that the nūber made of the both added together, namely, A B, haue to neither of the nūbers A C and C B that proportio that a square number hath to a square nūber (such as is enery square nūber to two nūbers not square, which are lesse the it & make the said square nūber). And take a rationall line, and let the same be D. And vnto the

Construction.

line D let the line E F be cōmensurable in length.

Wherefore E F is a rationall line, and as the number B A is to the number A C, so let the square of the line E F be to the square of y. line F G. Wherefore the square of the line E F is commensurable to the square of the line F G, and the line E F is a rationall line. Wherefore also the line F G is a rationall line. And for that the number B A hath not to the number A C that proportion that a square number hath to a square number, neither



Demonstration.

also shall the square of the line E F haue to the square of the line F G that proportion that a square number hath to a square number. Wherefore the line E F is incommensurable in length to the line F G. Wherefore the lines E F and F G are rationall commensurable in power onely. Wherefore the whole line E G is a binomiall line. I say moreouer that it is a fourth binomiall lyne. For for that as the number B A is to the number A C, so is the square of the line E F to the square of the line F G. But the number B A is greater then the number A C. Wherefore also the square of the line E F is greater then the square of the line F G. Vnto the square of the line E F let the squares of the lines F G and H be equall. Wherefore by conuerſion (by the corollary of the 19. of the fift) as the number A B is to the number B C, so is the square of the line E F to the square of the line H. But the number A B hath not to the number B C that proportion that a square number hath to a square number: therefore neither also hath the square of the line E F to the square of the line H that proportio that a square nūber hath to a square nūber. Wherefore (by the 9. of the tēth) the line E F is incōmensurable in length vnto the line H. Wherefore the line E F is in power more the the line F G by the square of a line incommensurable in length vnto it. And the lynes E F and F G are rationall commensurable in power onely, and the line E F is commensurable in length to the rationall line D. Wherefore the line E G is a fourth binomiall line: which was required to be found out.

A C B

¶ The 17. Probleme.

The 52. Proposition.

To finde out a fift binomiall lyne.

Take

The tenth Booke

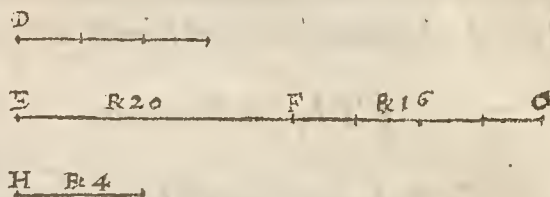
Construction.



Take two numbers AC and CB , and let them be such, that the number AB haue to neither of the numbers AC or CB that proportion that a square number hath to a square number, as in the former proposition. And take a rationall line and let the same be D . And vnto the line D let the line FG be commensurable in length. Wherefore the line FG is rationall. And as the number

Demonstration.

CA is to the number AB , so let the square of the line GF be to the square of the line EF . Wherefore the square of the line GF is commensurable to the square of the line FE . Wherefore also the line FE is rationall. And for that the number CA hath not to the number AB that proportion that a square number hath to a square number, therefore neither also hath the square



$A \dots\dots\dots C \dots\dots B$

of the line GF to the square of the line FE that proportion that a square number hath to a square number. Wherefore (by the 9. of the tenth) the line GF is incommensurable in length to the line FE . Wherefore the lines EF and FG are rationall commensurable in power only. Wherefore the whole line EG is a binomiall line. I say moreouer that it is a fift binomiall line. For for that as the number CA is to the number AB , so is the square of the line GF to the square of the line FE , therefore contrariwise, as the number BA is to the number AC , so is the square of the line EF to the square of the line FG . but the number BA is greater then the number AC . Wherefore also the square of the line EF is greater then the square of the line FG . Vnto the square of the line EF , let the squares of the lines FG and H be equall. Wherefore by conuersiō (by the corollary of the 19. of the fift) as the nūber AB is to the number BC , so is the square of the line EF to the square of the line H . But the nūber AB hath not to the number BC that proportion that a square number hath to a square number. Wherefore neither also hath the square of the line EF to the square of the line H that proportion that a square number hath to a square number. Wherefore (by the 9. of the tenth) the line EF is incommensurable in length to the line H . Wherefore the line EF is in power more then the line FG by the square of a line incommensurable in length vnto it. And the lines EF and FG are rationall commensurable in power onely. And the line FG being the lesse name, is commensurable in length to the rationall line geuen, namely, to D . Wherefore the whole line EG is a fift binomiall line: which was required to be found out.

¶ The 18. Probleme.

The 13. Proposition.

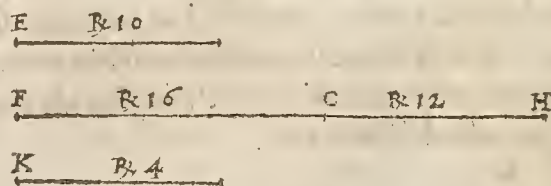
To finde out a sixt binomiall line.

Construction.



Take two numbers AC & CB , and let the be such that the number which is made of them both added together, namely, AB , haue to neither of the numbers AC or CB that proportion that a

square nūber hath to a square number. Take also any other number which is not a square number, and let the same be D . And let not the number D haue to any one of these numbers AB and AC that proportion that a square number hath to a square nūber. Let there be put moreouer a rationall line,



$A \dots\dots\dots C \dots\dots B$
 $D \dots\dots\dots$

and

and let the same be E . And as the number D is to the number AB , so let the square of the line E be to the square of FG . Wherefore (by the 6. of the tenth) the line E is commensurable in power to the line FG , & the line E is rationall. Wherefore also the line FG is rationall. And for that the number D hath not to the number AB that proportion that a square nūber hath to a square number, therefore neither also shall the square of the line E haue to the square of the line FG that proportion that a square number hath to a square number. Wherefore the line FG is incommensurable in length to the line E . Againe, as the number BA is to the number AC , so let the square of the line FG be to the square of the line GH . Wherefore (by the 6. of the tenth) the square of the line FG is commensurable to the square of the line GH . And the square of the line FG is rationall. Wherefore the square of the line GH is also rationall. Wherefore also the line GH is rationall. And for that the number AB hath not to the number AC , that proportion that a square number hath to a square number: therefore neither also hath the square of the line FG to the square of the line GH , that proportion that a square number hath to a square number. Wherefore the line FG is incommensurable in length to the line GH . Wherefore the lines FG and GH are rationall commensurable in power onely. Wherefore the whole line FH is a binomial line. I say moreouer, that it is a sixt binomial line. For for that as the number D is to the number AB , so is the square of the line E to the square of the line FG . And as the number BA is to the number AC , so is the square of the line FG to the square of the line GH . Wherefore of equalitie (by the 22. of the fift) as the number D is to the number AC , so is the square of the line E to the square of the line GH . But the number D hath not to the nūber AC that proportion that a square number hath to a square number. Wherefore neither also hath the square of the line E to the square of the line GH that proportion that a square number hath to a square number. Wherefore the line E is incommensurable in length to the line GH . And it is already proued, that the line FG is also incommensurable in length to the line E . Wherefore either of these lines FG and GH is incommensurable in length to the line E . And for that as the number BA is to the number AC , so is the square of the line FG to the square of the line GH : therefore the square of the line FG is greater then the square of the line GH . Vnto the square of the line FG , let the squares of the lines GH and K be equall. Wherefore by euerſion of proportion, as the number AB is to the number BC , so is the square of the line FG to the square of the line K . But the number AB hath not to the number BC that proportion that a square number hath to a square number. Wherefore neither also hath the square of the line FG to the square of the line K that proportion that a square number hath to a square number. Wherefore the line FG is incommensurable in length vnto the line K . Wherefore the line FG is in power more then the line GH , by the square of a line incommensurable in length to it. And the lines FG and GH are rationall commensurable in power onely. And neither of the lines FG & GH is commensurable in length to the rationall line geuen, namely, to E . Wherefore the line FH is a sixt binomial line: which was required to be found out.

Demonſtration.

¶ A Corollary added out of *Fluſſates*.

By the 6. former Propositions it is manifeſt, how to deuide any right line geuen into the names of euery one of the ſixe foreſayd binomial lines. For if it be required to deuide a right line geuen into a firſt binomial line, then by the 48. of this booke finde out a firſt binomial line. And this right line being ſo found out deuided into his names, you may by the 10. of the ſixt, deuide the right line geuen in like ſort. And ſo in the other fiue following.

A Corollary added by *Fluſſates*.

Although I here note vnto you this Corollary out of *Fluſſas*, yet, in very conſcience and of gratefull minde, I am enforced to certifie you, that, many yeares, before the trauailes of *Fluſſas* (vpō Euclides Geometricall Elementes) were publiſhed, the order how to deuide, not onely the 6. Binomial lines into their names, but alſo to adde to the 6. Reſiduals their due partes: and farthermore to deuide all the o-

The tenth Booke

*M. Dee his
booke called
Tirocinium
Mathemati-
cum.*

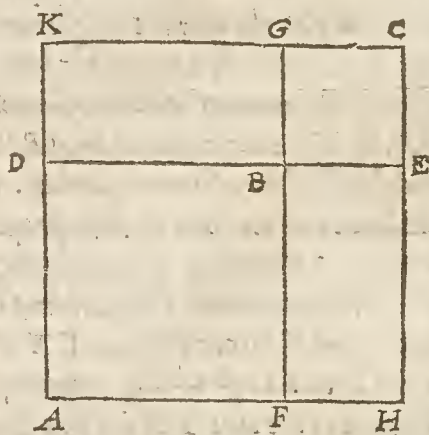
ther irrational lines (of this tenth booke) into the partes distinct, of which they are composed: with many other straunge conclusions Mathematicall, to the better vnderstanding of this tenth booke and other Mathematicall bookes, most necessary, were by *M. Iohn Dee* inuented and demonstrated: as in his booke, whose title is *Tirocinium Mathematicum* (dedicated to *Petrus Nonnius*, An. 1559.) may at large appeare. Where also is one new arte, with sundry particular pointes, whereby the Mathematicall Sciences, greatly may be enriched. Which his booke, I hope, God will one day allowe him opportunitie to publishe: with diuers other his Mathematicall and Metaphysicall labours and inuentions.

¶ An Assumpt.

If a right line be deuided into two partes how soeuer: the rectangle parallelogramme contayned vnder both the partes, is the meane proportionall betwene the squares of the same parts. And the rectangle parallelogramme contained vnder the whole line and one of the partes, is the meane proportionall betwene the square of the whole line and the square of the sayd part.

*This Assumpt
as was before
noted, follow-
eth most brief-
ly without
farther demon-
stration of the
25. of this
booke.
Demonstra-
tion.*

Suppose that there be two squares *AB* and *BC*, and let the lines *DB* and *BE* so be put that they both make one right line. Wherefore (by the 14. of the first) the lines *FB* and *BG* make also both one right line. And make perfect the parallelogramme *AC*. Then I say, that the rectangle parallelogramme *DG* is the meane proportionall betwene the squares *AB* and *BC*: and moreover, that the parallelogramme *DC* is the meane proportionall betwene the squares *AC* and *CB*. First the parallelogramme *AC* is a square. For forasmuch as the line *DB* is equall to the line *BF*, and the line *BE* vnto the line *BG*, therefore the whole line *DE* is equall to the whole line *FG*. But the line *DE* is equall to either of these lines *AH* & *KC*, and the line *FG* is equall to either of these lines *AK* and *HC* (by the 34. of the first). Wherefore the parallelogramme *AC* is equilateral, it is also rectangle (by the 29. of the first). Wherefore (by the 46. of the first) the parallelogramme *AC* is a square. Now for that as the line *FB* is to the line *BG*, so is the line *DB* to the line *BE*. But as $\frac{FB}{BG}$ line *FB* is to the line *BG*, so (by the 1. of the sixth) is the parallelogramme *AB*, which is the square of the line *DB*, to the parallelogramme *DG*, and as the line *DB* is to the line *BE*, so is the same parallelogramme *DG* to the parallelogramme *BC*, which is the square of the line *BE*. Wherefore as the square *AB* is to the parallelogramme *DG*, so is the same parallelogramme *DG* to the square *BC*. Wherefore the parallelogramme *DG* is the meane proportionall betwene the squares *AB* and *BC*. I say moreover, that the parallelogramme *DC* is the meane proportionall betwene the squares *AC* and *CB*. For for that as the line *AD* is to the line *DK*, so is the line *KG* to the line *GC* (for they are ech equall to eche). Wherefore by composition (by the 18. of the fifth) as the line *AK* is to the line *KD*, so is the line *KC* to the line *CG*. But as the line *AK* is to the line *KD*, so is the square of the line *AK*, which is the square *AC*, to the parallelogramme cotayned vnder the lines *AK* and *KD*, which is the parallelogramme *CD*: and as the line *KC* is to the line *CG*, so also is the parallelogramme *DC* to the square of the line *GC*, which is the square *BC*. Wherefore as the square *AC* is to the parallelogramme *DC*, so is the parallelogramme *DC* to the square *BC*. Wherefore the parallelogramme *DC* is the meane proportionall betwene the squares *AC* and *BC*: which was required to be demonstrated.

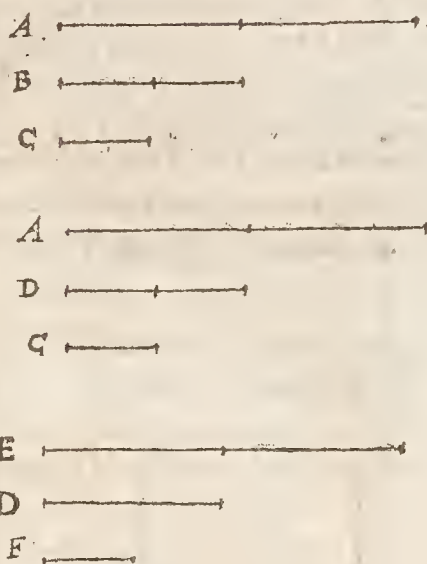


¶ An Assumpt.

Magnitudes that are meane proportionalls betwene the selfe same or equall magnitudes, are also equall the one to the other.

Suppose that there be three magnitudes A, B, C. And as A is to B, so let B be to C. And likewise as the same magnitude A is to D, so let D be to the same magnitude C. Then I say that B and D are equall the one the other. For the proportion of A unto C is double to that proportion which A hath to B (by the 10. definition of the fift) and likewise the selfe same proportion of A to C is (by the same definition) double to that proportion which A hath to D. But magnitudes whose equemultiplices are either equall or the selfe same, are also equall. Wherefore as A is to B, so is A to D. Wherefore (by the 9. of the fift) B and D are equall the one to the other. So shall it also be if there be other magnitudes equal to A and C, namely, E and F, betwene which let the magnitude D be the meane proportionall.

An Assumpt.



¶ The 36. Theoreme.

The 14. Proposition.

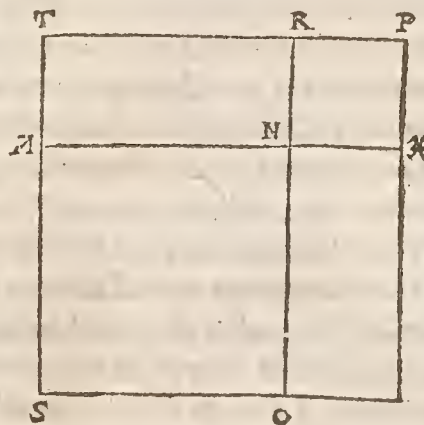
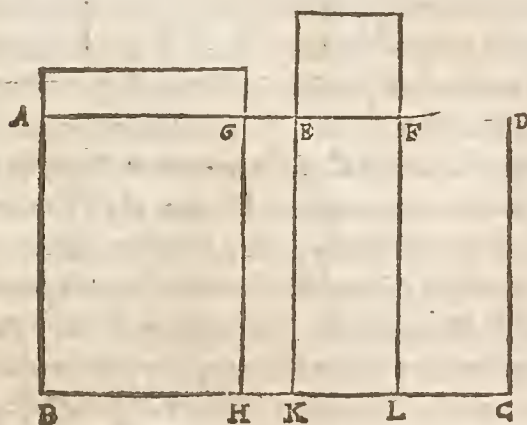
If a superficies be contained vnder a rationall line & a first binomial line: the line which containeth in power that superficies is an irrational line, & a binomial line.

The fourth Senary by composition.



Suppose that the superficies A B C D, be contained vnder the rational line A B, and vnder a first binomial line A D. Then I say that the line which containeth in power the superficies A C, is an irrational line, and a binomial line. For forasmuch as the line A D is a first binomial line, it is in one only point devided into

Construction.



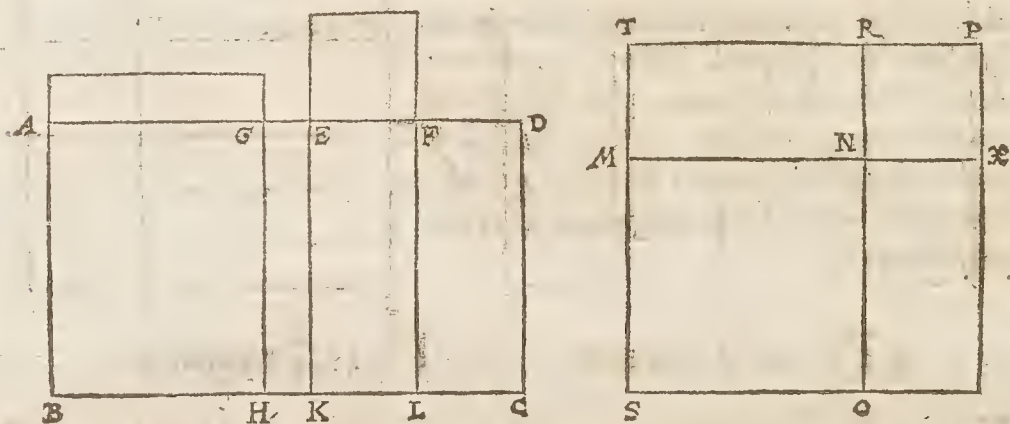
his names (by the 42. of this tenth): let it be devided into his names in the point E. And let A E be the greater name. Now it is manifest that the lines A E and E D are rationall commensurable.

KK. ij.

men.

The tenth Booke

incommensurable in power onely, and that the line AE is in power more then the line ED , by the square of a line commensurable in length to the line AE , and moreover that the line AE is commensurable in length to the rationall line given AB by the definition of a first Binomial line set before the 48. proposition of this tenth. Deuide (by the 10. of the first) the line ED into two equall partes in the point F . And forasmuch as the line AE is in power more then the line ED by the square of a line commensurable in length unto the line AE , therefore if upon the greater line, namely, upon the line AE be applied a parallelogramme equall to the fourth part of the square of the lesse line, that is, to the square of the line EF , & wanting in forme by a square, it shall deuide the greater line, namely, AE into two partes commensurable in length the one to the other (by the second part of the 17. of the tenth.) Apply therefore upon the line AE a parallelogramme equall to the square of the line EF , and wanting in forme by a square by the 28. of the sixth, and let the same be that which is contained under the lines AG and GE . Wherefore the line AG is commensurable in length to the line

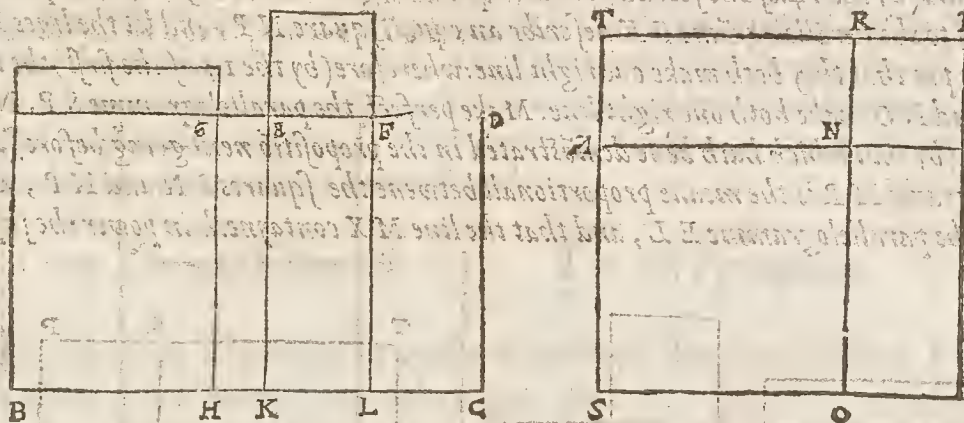


GE . Draw by the pointes G, E , and F , to either of these lines AB and DC these parallel lines GH, EK , and FL (by the 31. of the first) And (by the 14. of the second) unto the parallelogramme AH describe an equall square SN . And unto the parallelogramme GK describe (by the same) an equal square NP . And let these lines MN & NX be so put, that they both make one right line. Wherefore (by the 14. of the first) the lines RN and NO make also both one right line. Make perfect the parallelogramme SP . Wherefore the parallelogramme SP is a square by those things which were demonstrated after the determination in the first assumpt going before. And forasmuch as that which is contained under the lines AG and GE is equal to the square of the line EF (by construction) : therefore as the line AG is to the EF , so is the line EF to the line EG (by the 14. or 17. of the sixth) Wherefore also (by the 1. of the sixth) as the parallelogramme AH is to the parallelogramme EL , so is the parallelogramme EL to the parallelogramme GK . Wherefore the parallelogramme EL is the meane proportionall betwene the parallelogrammes AH and GK . But the parallelogramme AH is equal to the square SN , and the parallelogramme GK is equal to the square NP by construction. Wherefore the parallelogramme EL is the meane proportionall betwene the squares SN and NP (by the 7. of the fifth) But (by the first assumpt going before) the parallelogramme MR is the meane proportionall betwene the squares SN and NP . Wherefore the parallelogramme MR is equal to the parallelogramme EL (by the last assumpt going before) But the parallelogramme MR is equal to the parallelogramme OX (by the 43. of the first) and the parallelogramme EL is equal to the parallelogramme FC by construction, and by the first of the sixth. Wherefore the whole parallelogramme EC is equal to the two parallelogrammes MR & OX . And the parallelogrammes AH and GK are equal to the squares SN and NP by construction. Wherefore the whole parallelogramme AC is equal to the whole square SP , that

Demonstration.

is, to the square of the line MX . Wherefore the line MX containeth in power the parallelogramme AC . I say moreover that the line MX is a binomiall line. For forasmuch as (by the 17. of the tenth) the line AG is commensurable in length to the line EG . Therefore (by the 15. of the tenth) the whole line AE is commensurable in length to either of these lines AG and GE . But by supposition the line AE is commensurable in length to the line AB . Wherefore (by the 12. of the tenth) either of the lines AG & GE are commensurable in length to the line AB . But the line AB is rationally. Wherefore either of these lines AG and GE is rationally. Wherefore (by the 19. of the tenth) either of these parallelogrammes AH and GK is rationally. Wherefore (by the first of the sixth, and 10. of the tenth) the parallelogramme AH is commensurable to the parallelogramme GK . But the parallelogramme AH is equall to the

The first part of this demonstration concluded.



square SN , and the parallelogramme GK is equall to the square NP . Wherefore the squares SN and NP , which are the squares of the lines MN and NX are rationally and commensurable. And forasmuch as (by supposition) the line AE is incommensurable in length to the line ED . But the line AE is commensurable in length to the line AG . And the line DE is commensurable in length to the line EF , for it is double to it by construction. Wherefore (by the 13. of the tenth) the line AG is incommensurable in length to the line EF . Wherefore the parallelogramme AH is incommensurable to the parallelogramme EL . But the parallelogramme AH is equal to the square SN , and the parallelogramme EL is equall to the parallelogramme MR . Wherefore the square SN is incommensurable to the parallelogramme MR . But as the square SN is to the parallelogramme MR , so is the line ON to the line NR by the 1. of the sixth. Wherefore the line ON is incommensurable to the line NR . But the line ON is equall to the line MN , and the line NR to the line NX . Wherefore the line MN is incommensurable to the line NX . And it is already proved that the squares of the lines MN and NX are rationally and commensurable. Wherefore the lines MN and NX are rationally commensurable in power onely. Wherefore the whole line MX is a binomiall line, and it containeth in power the parallelogramme AC : which was required to be proved.

The second part of the demonstration concluded.

The third part concluded.

The totall conclusion.

The 37. Theoreme.

The 55. Proposition.

If a superficies be comprehended vnder a rationall line and a second binomiall line: the line that containeth in power that superficies is irrational, and is a first bimediall line.

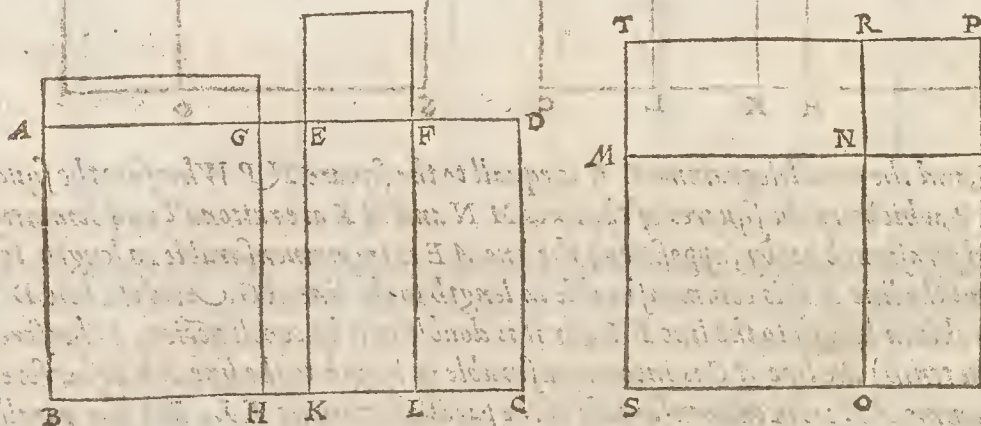


Suppose that the superficies $ABCD$ be contained vnder a rationall line AB , and vnder a second binomiall line AD . Then I say that the line that containeth in power the superficies AC is a first bimediall line. For forasmuch as AD is a second binomiall line, it can in one onely point be devided into his names, by the

The tenth Booke

43. of this tenth: thlet it therefore by supposition be deuided into his names in the poynt E, so that let A E be the greater name. Wherefore the lines A E and E D are rationall commensurable in power onely, and the line A E is in power more then the line E D by the square of a line commensurable in length to A E, and the lesse name, namely, E D is commensurable in length to the line A B by the definition of a second binomiall line, set before the 48. proposition of this tenth. Deuide the line E D (by the tenth of the first) into two equall partes in the poynt F. And (by the 28. of the sixth) upon the line A E apply a parallelogramme equal to the square of the line E F, and wanting in figure by a square. And let the same parallelogramme be that which is containd vnder the lines A G and G E. Wherefore (by the second part of the 17. of this tenth) the line A G is commensurable in length to the line G E. And (by the 31. of the first) by the poyntes G, E, F, draw vnto the lines A B and C D these parallel lines, G H, E K, & F L. And (by the 14. of the second) vnto the parallelograme A H describe an equall square S N. And to the parallelograme G K describe an equall square N P, and let the lines M N & N X be so put that they both make one right line: wherefore (by the 14. of the first) the lines also R N, and N O make both one right line. Make perfect the parallelogramme S P. Now it is manifest (by that which hath bene demonstrated in the proposition next going before) that the parallelograme M R is the meane proportionall betwene the squares S N and N P, and is equall to the parallelogramme E L, and that the line M X containeth in power the superficies

Demonstration.



AC. Now resteth to proue that the line M X is a first bimediall line. Forasmuch as the line A E is incommensurable in length to the line E D, and the line E D is commensurable in length to the line A B, therefore (by the 13. of the tenth) the line A E is incommensurable in length to the line A B. And forasmuch as the line A G is commensurable in length to the line G E therefore the whole line A E is (by the 15. of the tenth) commensurable in length to either of these lines A G and G E. But the line A E is rationall: wherefore either of these lines A G and G E is rationall. And forasmuch as the line A E is incommensurable in length to the line A B, but the line A E is commensurable in length to either of these lines A G and G E: wherefore either of the lines A G and G E are incommensurable in length to the line A B (by the 13. of the tenth). Wherefore the lines A B, A G, and G E are rationall commensurable in power onely. Wherefore (by the 21. of the tenth) either of these parallelogrames A H and G K is a medially superficies. Wherefore also either of these squares S N and N P is a medially superficies by the corollary of the 23. of the tenth. Wherefore the lines M N & N X are medially lines by the 21. of this tenth. And forasmuch as the line A G is commensurable in length to the line G E, therefore (by the 1. of the sixth and 11. of the tenth) the parallelograme A H is commensurable to the parallelogramme G K that is, the square S N to the square N P, that is, the square of the line M N to the square of the line N X. Wherefore the lines M N and N X are medially commensurable in power. And forasmuch as the line A E is incommensurable

The first part
of this demon-
stration con-
cluded.
The third
part concluded.

in length to the line ED , but the line AE is commensurable in length to the line AG , and the line ED is commensurable in length to the line EF , therefore (by the 13. of the tenth) the line AG is incommensurable in length to the line EF . Wherefore (by the 1. of the sixth and 11. of the tenth) the parallelogramme AH is incommensurable to the parallelogramme EL , that is, the square SN to the parallelogramme MR , that is, the line ON is incommensurable to the line NR , that is the line MN to the line NX . And it is proued that the lines MN and NX are medially lines commensurable in power. Wherefore the lines MN and NX are medially lines commensurable in power onely. Now I say moreover that they comprehend a rational superficies. For forasmuch as by supposition the line DE is commensurable in length to either of these lines AB and EF , therefore the line FE is commensurable in length to the line EK which is equall to the line AB (by the 12. of the tenth). And either of these lines EF and EK is a rationally line. Wherefore the parallelogramme EL , that is the parallelogramme MR , is a rationally superficies (by the 19. of the tenth). But the parallelogramme MR is that which is containd vnder the lines MN and NX . But if two medially lines commensurable in power onely and comprehending a rationally superficies be added together, the whole line is irrational and is called a first bimedially (by the 37. of the tenth). Wherefore the line MX is a first bimedially line: which was required to be demonstrated.

The fourth part concluded.

The fifth part concluded.

The totall conclusion.

¶ The 38. Theoreme.

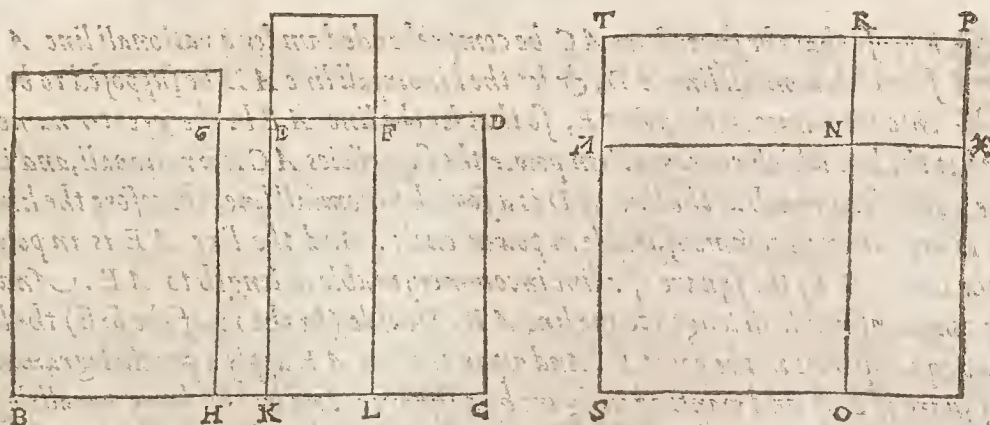
The 16. Proposition.

If a superficies be containd vnder a rationally line and a third binomially line: the line that containeth in power that superficies is irrational, and is a second bimedially line.



Suppose that the superficies $ABCD$ be comprehended vnder the rationally line AB and a third binomially line AD , and let the line AD be supposed to be diuided into his names in the point E , of which let AE be the greater name. Then I say, that the line that containeth in power the superficies AC is irrational, and is a second bimedially line. Let the same construction of the figures be in this, that was in the two Propositions next going before. And now forasmuch as the line AD is a third binomially line, therefore these lines AE and ED are rationally commensurable in power onely. And

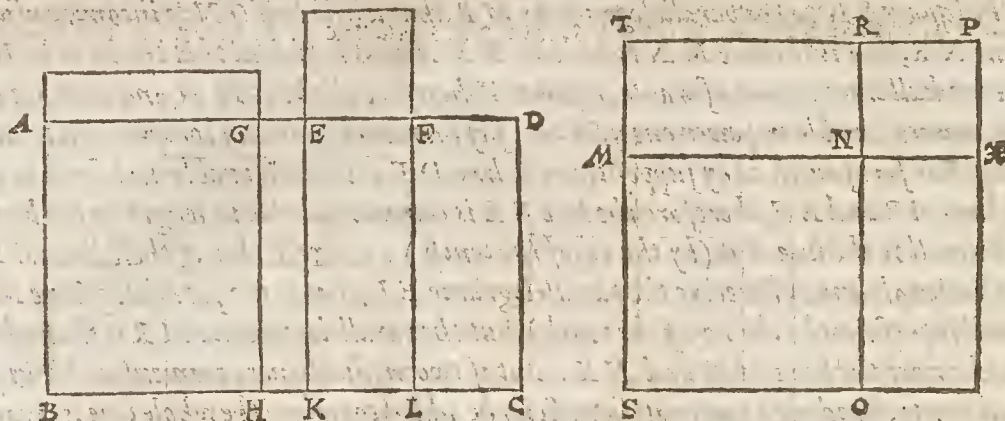
Demonstration.



the line AE is in power more the line ED by the square of a line commensurable in length to the line AE , and neither of the lines AE nor ED is commensurable in length to the line AB by the definition of a third binomially line set before the 48. Proposition. As in the former Propositions it was demonstrated, so also may it in this Proposition be proued, that the

The tenth Booke

line MX containeth in power the superficies AC , and that the lines MN and NX are medially lines commensurable in power onely. Wherefore the line MX is a bimediall line. Now resteth to proue that it is a second bimediall line. Forasmuch as the line DE is (by supposition)



incommensurable in length to the line AB , that is, to the line EK . But the line ED is commensurable in length to the line EF . Wherefore (by the 13. of the tenth) the line EF is incommensurable in length to the line EK . And the lines FE and EK are rationally. For by supposition the line ED is rationally, unto which the line FE is commensurable. Wherefore the lines FE and EK are rational lines commensurable in power onely. Wherefore (by the 21. of the tenth) the parallelogramme EL , that is, the parallelogramme MR which is contained under the lines MN and NX is a mediall superficies. Wherefore that which is contained under the lines MN and NX is a mediall superficies. Wherefore the line MX is a second bimediall line (by the 38. Proposition and definition annexed thereto): which was required to be proued.

¶ The 39. Theoreme.

The 57. Proposition.

If a superficies be contained under a rationall line, and a fourth binomiall line: the line which containeth in power that superficies is irrationall, and is a greater line.

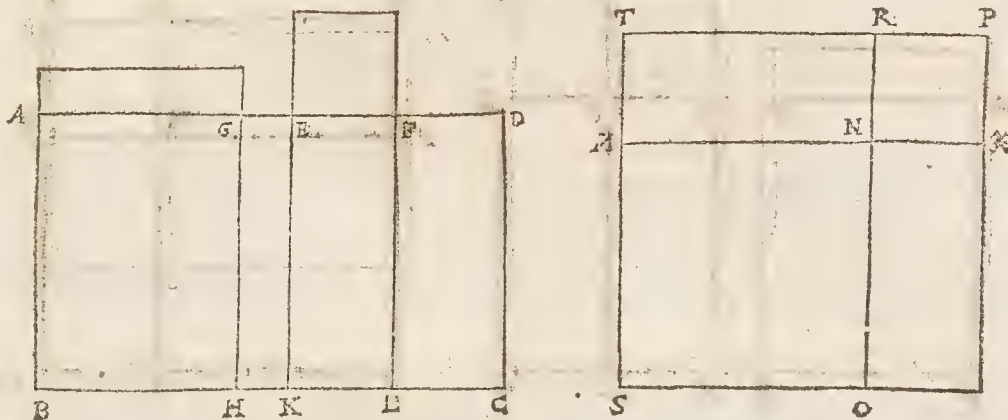
Construction.

Suppose that the superficies AC be comprehended under a rationall line AB and a fourth binomiall line AD , & let the binomiall line AD be supposed to be deuided into his names in the point E , so that let the line AE be the greater name. Then I say, that the line which containeth in power the superficies AC is irrationall, and is a greater line. For, forasmuch as the line AD is a fourth binomiall line, therefore the lines AE and ED are rationall commensurable in power onely. And the line AE is in power more then the line ED by the square of a line incommensurable in length to AE . And the line AE is commensurable in length to the line AB . Deuide (by the 10. of the first) the line DE into two equall partes in the point F . And vpon the line AE apply a parallelogramme equall to the square of EF and wanting in figure by a square: and let the same parallelogramme be that which is contained under the lines AG & GE . Wherefore (by the second part of the 18. of the tenth) the line AG is incommensurable in length to the line EG . Draw vnto the line AB , by the pointes G, E, F , parallell lines GH, EK , and FL , and let the rest of the construction be as it was in the three former Propositions. Now it is manifest, that the line MX containeth in power the superficies AC : Now resteth to proue that the line MX is an irrationall

Demonstration.

irrationall line, and a greater line. Forasmuch as the line AG is incommensurable in length to the line EG , therefore (by the 1. of the sixth, and 11. of the tenth) the parallelogramme AH is incommensurable to the parallelogramme GK , that is, the square SN to the square NP . Wherefore the lines MN and NX are incommensurable in power. And forasmuch as the line AE is commensurable in length to the rationall line AB , therefore the parallelo-

2.



gramme AK is rationall. And it is equall to the squares of the lines MN and NX . Wherefore that which is composed of the squares of the lines MN and NX added together is rationall. And forasmuch as the line ED is incommensurable in length to the line AB , that is, to the line EK , but the line ED is commensurable in length to the line EF , therefore the line EF is incommensurable in length to the line EK . Wherefore the lines EK and EF are rationall commensurable in power onely. Wherefore (by the 21. of the tenth) the parallelogramme LE , that is, the parallelogramme MR is medially. And the parallelogramme MR is that which is contained under the lines MN , and NX . Wherefore that which is contained under the lines MN and NX is medially. And that which is composed of the squares of the lines MN & NX is proved to be rationall, & the line MN is demonstrated to be incommensurable in power to the line NX . But if two lines incommensurable in power be added together, having that which is made of the squares of them added together rationall, & that which is under them medially, the whole line is irrationall, and is called a greater line (by the 39. of the tenth). Wherefore the line MX is irrationall, and is a greater line, and it containeth in power the superficies AC : which was required to be demonstrated.

3.

4.

¶ The 40. Theoreme.

The 58. Proposition.

If a superficies be contained under a rationall line and a fifth binomiall line: the line which containeth in power that superficies is irrationall, and is a line containyng in power a rationall and a medially superficies.

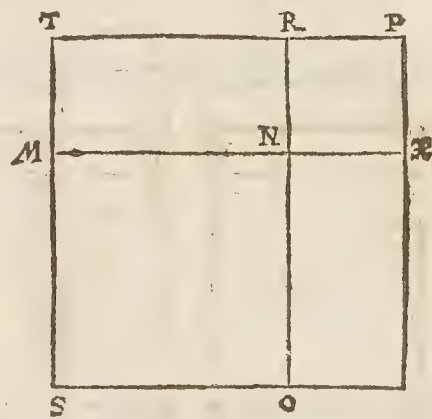
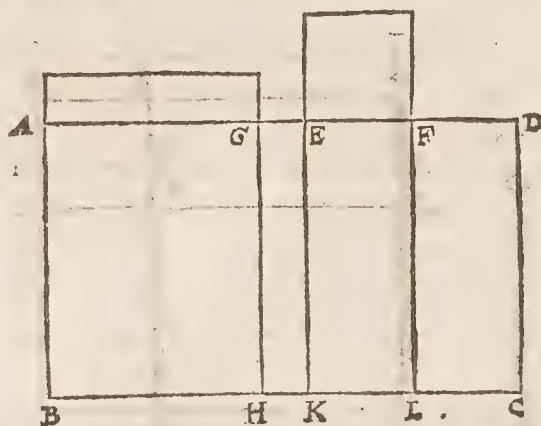


Suppose that the superficies AC be contained under the rationall line AB , and under a fifth binomiall line AD : and let the same line AD be supposed to be divided into his names in the poynt E , so that let the line AE be the greater name. Then I say, that the line which containeth in power the superficies AC is irrationall, and is a line containyng in power a rationall and a medially superficies. Let the selfe same constructions be in this, that were in the foure Proposition next going before. And it is manifest, that the line MX containeth in power the superficies AC . Now resteth to proue that the line MX is a line containyng in power a rationall & a medially superficies. Forasmuch

Demonstration.

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asmuch as the line AG is incommensurable in length to the line GE , therefore (by the 1. of the sixth, and 10. of the tenth) the parallelogramme AH is incommensurable to the parallelogramme HE , that is, the square of the line MN to the square of the line NX . Wherefore the lines MN and NX are incommensurable in power. And forasmuch as the line AD is a fifth binomial line, and his lesse name or part is the line ED , therefore the line ED is com-



mensurable in length to the line AB . But the line AE is incommensurable in length to the line ED . Wherefore (by the 13. of the tenth) the line AB is incommensurable in length to the line AE . Wherefore the lines AB and AE are rationally commensurable in power only. Wherefore (by the 21. of the tenth) the parallelogramme AK is medial, that is, that which is composed of the squares of the lines MN & NX added together. And forasmuch as the line DE is commensurable in length to the line AB , that is, to the line EK , but the line DE is commensurable in length to the line EF , wherefore (by the 12. of the tenth) the line EF is also commensurable in length to the line EK . And the line EK is rational. Wherefore (by the 19. of the tenth) the parallelogramme EL , that is, the parallelogramme MR , which is contained under the lines MN and NX is rational. Wherefore the lines MN and NX are incommensurable in power, having that which is composed of the squares of them added together, Medial, and that which is contained under them, Rational. Wherefore (by the 40. of the tenth) the whole line MX is a line containing in power a rational and a medial superficies, and it containeth in power the superficies AC : which was required to be proved.

¶ The 41. Theoreme.

The 59. Proposition.

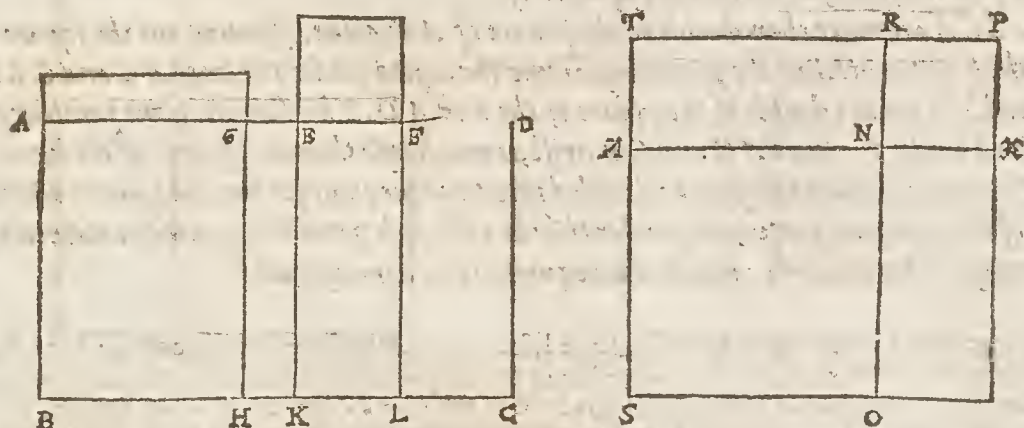
If a superficies be contained under a rational line, and a sixth binomial line, the line which containeth in power that superficies, is irrational, & is called a line containing in power two medials.

Demonstration.

- S**uppose that the superficies $ABCD$ be contained under the rational line AB , and under a sixth binomial line AD , and let the line AD be supposed to be divided into his names in the point E , so that let the line AE be the greater name. Then I say that the line that containeth in power the superficies AC is irrational, and is a line containing in power two medials. Let the selfe same constructions be in this, that were in the former propositions. Now it is manifest that the line MX containeth in power the superficies AC , and that the line MN is incommensurable in power to the line NX . And forasmuch as the line AE is incommensurable in length to the line AB , therefore the lines AE and AB are
1. irrational
 2. rational

rationall commensurable in power only. Wherefore (by the 21. of the tenth) the parallelogramme AK that is, that which is composed of the squares of the lines MN and NX added together is medially. Again forasmuch as the line ED is incommensurable in length to the line AB , therefore also the line EF is incommensurable in length to the line EK . Wherefore the lines EF and EK are rationall commensurable in power onely. Wherefore the parallelogramme EL , that is, the parallelogramme MR which is contained under the lines MN and NX is me-

3.



dially. And forasmuch as the line AE is incommensurable in length to the line EF , therefore the parallelogramme AK is also incommensurable to the parallelogramme EL (by the first of the sixth, and 10. of the tenth.) But the parallelogramme AK is equal to that which is composed of the squares of the lines MN and NX added together. And the parallelogramme EL is equal to that which is contained under the lines MN and NX . Wherefore that which is composed of the squares of the lines MN and NX added together, is incommensurable to that which is contained under the lines MN and NX : and either of them, namely, that which is composed of the squares of the lines MN and NX added together, and that which is contained under the lines MN and NX , is proved medially, and the lines MN and NX are proved incommensurable in power. Wherefore (by the 41. of the tenth) the whole line MX is a line containing in power two medials, and it containeth in power the superficies AC : which was required to be demonstrated.

4.

5.

*An Assumpt.

If a right line be divided into two unequal partes, the squares which are made of the unequal partes, are greater then the rectangle parallelogramme contained under the unequal partes, twice.

Suppose that AB be a right line, and let it be divided into two unequal partes in the point C . And let the line AC be the greater part. Then I say, that the squares of the lines AC and CB , are greater then that which is contained under the lines AC and CB , twice. Divide (by the 10. of the first) the line AB into two equal partes, in the point D . Now forasmuch as the right line AB is divided into two equal partes in the point D , and into two unequal partes in the point C , therefore (by the 5. of the second) that which is contained under the lines AC and CB , together with the square of the line CD , is equal to the square of the line AD . Wherefore that which is contained under the



the

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the lines AC and CB , (omitting the square of the line CD) is lesse then the square of the AD (by the 9. common sentence, and the seventh of the fifth.) Wherefore that which is contained under the lines AC and CB , twice, is lesse then the double of the square of the line AD (that is, the twice the square of the line AD)

by * alternate proportion, and the 14. of the fifth.

But the squares of the lines AC and CB are double to the squares of the lines AD and DC .

(by the 9. of the second.) Therefore the squares of

AC and CB are more then double to the square of AD alone, (leaving out the square of DC) by the 8. of the fifth. But the parallelogramme contained under the lines AC and CB twice, is proued lesse then the double of the square of the line AD . Therefore the same parallelogramme contained under the lines AC and CB twice, is much lesse then the squares of the lines AC and CB . If a right line therefore be deuided into two vnequall partes, the squares which are made of the vnequall partes, are greater then the rectangle parallelogramme contained under the vnequall partes, twice: which was required to be demonstrated.

* In numbers I neede not haue so alleaged, for the 17. of the seventh had confirmed the doubles to be one to the other, as their singles were, but in our magnitudes, it likewise is true and euident by alternate proportion, thus. As the parallelogramme of the lines AC and CB is to his double, so is the square of the line AD to his double (eche being halfe). Wherefore, alternately, as the parallelogramme is to the square, so is the parallelogramme his double to the double of the square. But the parallelogramme was proued lesse then the square: wherefore his double is lesse then the square his double, by the 14. of the fifth.

This Assumpt is in some bookes not read, for that in maner it semeth to be all one with that which was put after the 39. of this booke: but for the diuers maner of demonstrating, it is necessary. For the feate of inuentiō is therby furthered. And though Zambert did in the demonstration hereof, omittē that which P. Montaurēus could not supply, but plainly doubted of the sufficiencie of this prooffe, yet M. Dee, by onely allegation of the due places of credite, whose pithe & force, Theon his wordes do containe, hath restored to the demonstration sufficiently, both light and authoritie, as you may perceiue, and chiefly such may iudge, who can compare this demonstration here (thus furnished) with the Greeke of Theon, or latine translation of Zambert.

¶ The 42. Theoreme.

The 60. Proposition.

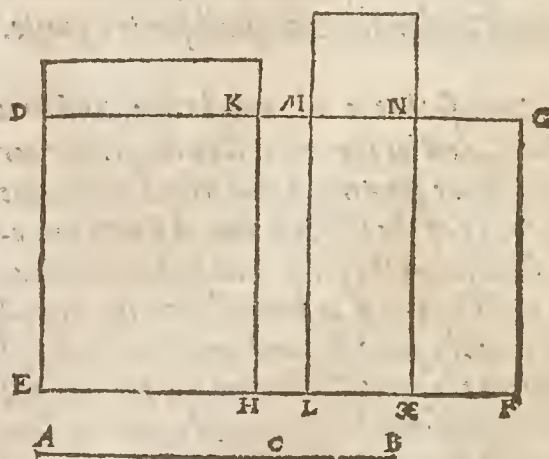
The square of a binomial line applyed vnto a rational line, maketh the breadth or other side a first binomial line.



Suppose that the line AB be a binomial line, and let it be supposed to be deuided into his names in the poynt C , so that

Construction.

let AC be the greater name. And take a rational line DE . And (by the 44. of the first) vnto the line DE apply a rectangle parallelogramme $DEFG$ equall to the square of the line AB and making in breadth the line DG . Then I say that the line DG is a first binomial line. Vnto the line DE apply the parallelogramme DH equall to the square of the line AC , and vnto the line KH which is equall to the



line

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the Assumpt
concluded at
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ing of this
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The vse of
this Assumpt
is in the next
proposition, &
other follow-
ing.

The fifth Se-
nary by com-
position.

line DE apply the parallelograme K L equall to the square of the line BC. Wherefore the residue namely, that which is containd under the lines AC & CB twise is equall to the residue, namely, to the parallelograme MF, by the 4. of the second. Deuide (by the 10. of the first) the line MG into two equall partes in the poynt N. And (by the 31. of the first) draw the line NX parallel to either of these lines ML and GF. Wherefore either of these parallelogrammes MX and NF is equall to that which is containd under the lines AC and CB once, by the 15. of the fifth. And forasmuch as the line AB is a binomialline, and is deuided into his names in the poynt C therefore the lines AC and CB are rationall commensurable in power onely. Wherefore the squares of the lines AC and CB are rationall, and therefore commensurable the one to the other. Wherefore (by the 15. of the tenth) that which is made of the squares of the lines AC and CB added together is commensurable to either of the squares of the lines AC or CB; wherefore that which is made of the squares of the lines AC and CB added together is rationall. And it is equall to the parallelograme DL by construction. Wherefore the parallelograme DL is rationall. And it is applied vnto the rational line DE: wherefore (by the 20. of the tenth) the line DM is rationall, and commensurable in length to the line DE. Agayne forasmuch as the lines AC and CB are rationall commensurable in power onely, therefore that which is containd under the lines AC and CB twise, that is, the parallelograme MF is medially by the 21. of the tenth, and it is applied vnto the rationall line ML. Wherefore the line MG is rationall and incommensurable in length to the line ML (by the 22. of this tenth) that is, to the line DE. But the line MD is proued rationall and commensurable in length to the line DE. Wherefore (by the 13. of the tenth) the line DM is incommensurable in length to the line MG. Wherefore the lines DM & MG are rationall commensurable in power onely. Wherefore (by the 36. of the tenth) the whole line DG is a binomialline. Now resteth to proue that it is a first binomialline. Forasmuch as (by the the assumpt going before the 54. of the tenth) that which is containd under the lines AC and CB is the meane proportionall betwene the squares of the lines AC and CB, therefore the parallelograme MX is the meane proportionall betwene the parallelogrammes DH and KL. Wherefore as the parallelogame DH is to the parallelograme MX, so is the parallelograme MX to the parallelograme KL, that is, as the line DK is to the line MN, so is the line MN to the line MK. Wherefore that which is containd under the lines DK and KM is equall to the square of the line MN. And forasmuch as the square of the line AC is commensurable to the square of the line CB, the parallelograme DH is commensurable to the parallelograme KL. Wherefore (by the 1. of the sixth and 10. of the tenth) the line DK is commensurable in length to the line KM. And forasmuch as the squares of the lines AC and CB are greater then that which is containd under the lines AC and CB twise by the assumpt going before this proposition, or by the assumpt after this 39. of the tenth, therefore the parallelograme DL is greater then the parallelograme MF. Wherefore (by the first of the sixth) the line DM is greater then the line MG. And that which is containd under the lines DK and KM is equall to the square of the line MN, that is to the fourth part of the square of the line MG. But (by the 17. of the tenth), if there be two vnequall right lines, and if vpon the greater be applied a parallelograme equall to the fourth part of the square made of the lesse line and wanting in figure by a square, if also the parallelograme thus applied deuide the line whereupon it is applied into parts commensurable in length, then shall the greater line be in power more then the lesse by the square of a line commensurable in length to the greater. Wherefore the line DM is in power more then the line MG by the square of a line commensurable in length vnto the line DM. And the lines DM and MG are proued rationall commensurable in power onely. And the line DM is proued the greater name and commensurable in length to the rationall line geuen DE. Wherefore by the definition of a first binomialline set before the 48. proposition of this booke, the line DG is a first binomialline

Demonstration.

2.

Concluded that DG is a binomialline.

3.

4.

5.

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line: which was required to be proved.

This proposition and the five following are the conuerses of the sixe former propositions.

¶ The 43. Theoreme.

The 61. Proposition.

The square of a first bimediall line applied to a rationall line, maketh the breadth or other side a second binomiall line.



Suppose that the line AB be a first bimediall line, and let it be supposed to be divided into his partes in the point C , of which let AC be the greater part. Take also a ratiōnall line DE , and (by the 44. of the first) apply to the line DE the parallelogramme DF equall to the square of the line AB , & making in breadth

Construction. the line DG . Then I say, that the line DG is a second binomial line. Let the same constructions be in this, that were in the Proposition going before. And forasmuch as the line

Demonstration. AB is a first binomial line, and is divided into his partes in the point C , therefore (by the 37. of the tenth) the lines AC and CB are

37. of the tenth) the lines AC and CB are
mediall commensurable in power onely,

comprehending a rationall superficies. Wherefore also the squares of the lines AC and

CB are medially. Wherefore the parallelogramme DL is medially (by the Corollary

of the 23. of the tenth) and it is applied up-
pon the rationall line D E. Wherefore (by

the 22. of the tenth) the line MD is rati-
onall and incommensurable in length to

the line DE . Again forasmuch as that which is contained under the lines AC and

which is bisected under the lines AC and CB twice is rationally, therefore also the parallelogramme ME is rationally, and it is

applied unto the rationall line ML . Where
ble in length to the line ML that is to t

the line DM is incommensurable in length. Wherefore the lines DM and MC are ratio-

Wherefore the lines DM and MG are ratio
the whole line DG is a binomialall line. Now
Therefore, as the squares of the lines AG

Forasmuch as the squares of the lines AC
under the lines AC and CB twice (by the

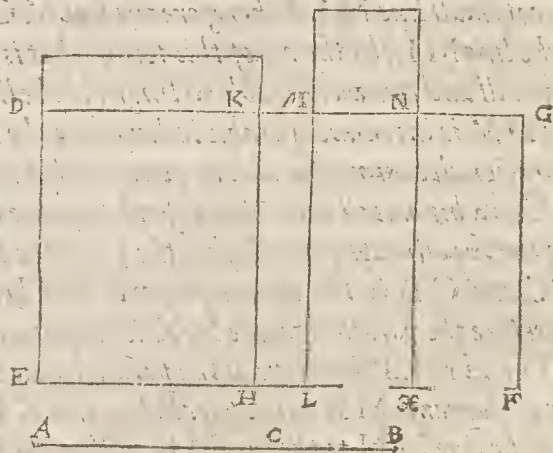
the parallelogramme D L is greater then the
first of the sixt) the line D M is greater the

the line AC is commensurable to the square
 BDH is commensurable to the parallelogram

measurable in length to the line $K M$. And $K M$ is equal to the square of the line $M N$

line MG . Wherefore (by the 17. of the ten
 MG , by the square of a line commensura

MG is commensurable in length to the rational
line *DG* is a second binomial line : which



The

¶ The 44. Theoreme.

The 62. Proposition.

The square of a second bimediall line, applied vnto a rationall line: maketh the breadth or other side therof, a third binomiall lyne.

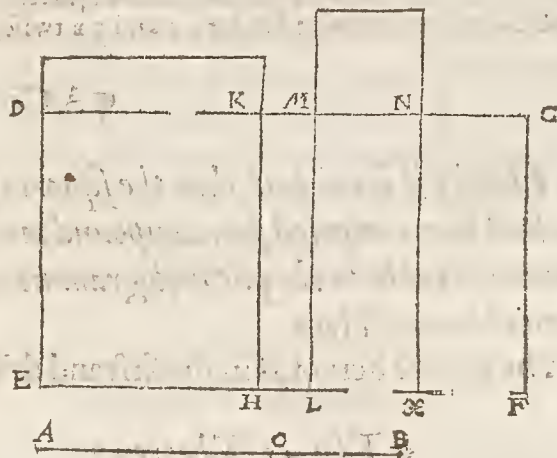


Suppose that AB be a second bimediall line, and let AB be supposed to be divided into his partes in the point C , so that let AC be the greater part. And take a rationall line DE . And (by the 44. of the first) vnto the line DE apply the parallelogramme DF equall to the square of the line AB , and making in breadth the line DG . Then I say that the line DG is a third binomiall line. Let the selfe same constructions be in this that were in the propositions next going before. And forasmuch as the line AB is a second bimediall line, and is divided

Construction.

into his partes in the point C , therefore (by the 38. of the tenth) the lines AC and CB are medials commensurable in power only, comprehending a medially superficies. Wherefore† that which is made of the squares of the lines AC and CB added together, is medially, and it is equall to the parallelogramme DL by construction. Wherefore the parallelogramme DL is medially, and is applied vnto the rationall line DE , wherefore (by the 22. of the tenth) the line DM is rationall and incommensurable in length to the line DE . And by the lyke reason also* the line MG is rationall and incommensurable in length to the line ML , that is, to the line DE . Wherefore either of these

Demonstration.



†

lines DM and MG is rational, and incommensurable in length to the line DE . And forasmuch as the line AC is incommensurable in length to the line CB , but as the line AC is to the line CB so (by the assumpt going before the 22. of the tenth) is the square of the line AC to that which is contained under the lines AC and CB . Wherefore the square of the line AC is incommensurable to that which is contained under the lines AC and CB . Wherefore that † that which is made of the squares of the lines AC and CB added together, is incommensurable to that which is contained under the lines AC and CB twise, that is, the parallelogramme DL to the parallelogramme MF . Wherefore (by the first of the sixth, and 10. of the tenth) the line DM is incommensurable in length to the line MG . And they are proved both rationall, wherefore the whole line DG is a binomiall line by the definition in the 36. of the tenth. Now resteth to proue that it is a third binomiall line. As in the former propositions, so also in this may we conclude that the line DM is greater then the line MG , and that the line DK is commensurable in length to the line KM . And that that which is contained under the lines DK and KM is equall to the square of the line MN . Wherefore the line DM is in power more then the line MG by the square of a line commensurable in length vnto the line DM , and neither of the lines DM nor MG is commensurable in length to the rational line DE . Wherefore (by the definition of a third binomiall line) the line DG is a third binomiall line: which was required to be proued.

††

DG , concluded a binomiall line.

¶ Here follow certaine annotations by M. Dee, made vpon three places in the demonstration, which were not very euident to yong beginners.

† (The squares of the lines a and c are medials (as is taught after the 21. of this tenth) and therefore forasmuch as they are (by supposition) commensurable th'one to the other: (by the 15. of the tenth)

LL. ij.

the

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the compound of them both is commensurable to each part. But the partes are medials, therefore (by the corollary of the 23. of the tenth) the compound shall be mediall.

* For that mx is equall (by construction) to that which is containd vnder the lines ac and cb , which is proued mediall: therefore (by the corollary of the 23. of this tenth) mx is mediall, and therefore (by the same corollary) his double MF is mediall. And it is applied to a rationall line, ML (beyng equall to DE) therefore by the 22. of the tenth, the line MG is rationall and incommensurable in length to ML , that is, to DE .

†† Because the compound of the two squares (of the lines ac and cb) beyng commensurable one to the other, is also to eyther square (by the 15.) commensurable, therefore to the square of ac : But the square of ac is proued incommensurable to that which is contained vnder ac & cb once. Wherefore (by the 13. of the tenth) the compound of the two squares (of the lines ac and cb) is incommensurable, to that which is contained vnder the lines ac and cb once. But to that which is twice contained vnder the same lines ac and cb , the parallelogramme once containd, is commensurable (for it is as 1. is to 2.) therefore that which is made of the squares of the lines ac and cb is incommensurable to the parallelogramme contained vnder ac and cb twice, by the sayd 13. of this tenth.

¶ A Corollary.

A Corollary
added by M.
Dec.

Hereby it is euident, that the squares made of the two partes of a second bimediall line, composed, is a compound mediall, and that the same compound is incommensurable to the parallelogramme containd vnder the two partes of the second bimediall lyne.

The prooffe hereof, is in the first and third annotations here before annexed.

¶ The 45. Theoreme.

The 63. Proposition.

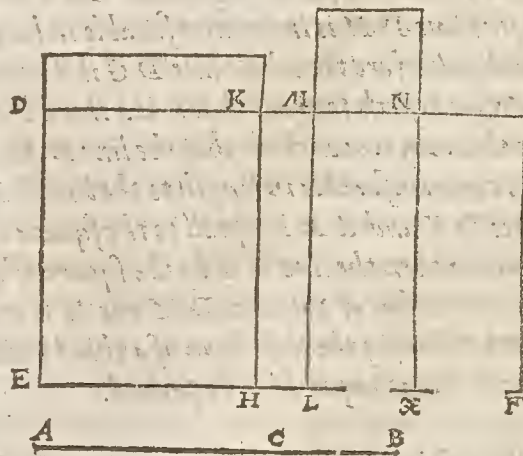
The square of a greater line applied vnto a rationall line, maketh the breadth or other side a fourth binomiall line.

Suppose that the line AB be a greater line, and let it be supposed to be deuided into his partes in the point C , so that let AC be the greater part. And take a rationall line DE . And (by the 44. of the first) vnto the line DE , apply the parallelogramme DE equall to the square of the line AB , and making in breadth the line DG . Then I say, that the line DG is a fourth binomiall line. Let the selfe same construction be in this, that

Construction.

Demonstration.

was in the former Propositions. And forasmuch as the line AB is a greater line, & is deuided into his partes in the point C : therefore the lines AC and CB are incommensurable in power, hauing that which is made of the squares of them added together rationall, and the parallelogramme which is containd vnder them, mediall. Now forasmuch as that which is made of the squares of the lines AC and CB added together is rationall, therefore the parallelogramme DL is rationall. Wherefore also the line MD is rationall and commensurable in length to the line DE (by the 20. of this tenth). Again forasmuch as that which is contained vnder the lines AC and CB twice is mediall, that is, the parallelogramme



MF ,

M F, and it is applied vnto the rationall line M L, therefore (by the 22. of the tenth) the line M G is rationall and incommensurable in length to the line D E. Therefore (by the 13. of the tenth) the line D M is incommensurable in length to the line M G. Wherefore the lines D M and M G are rationall commensurable in power onely. Wherefore the whole line D G is a binomial line. Now resteth to proue, that it is also a fourth binomial line. Euen as in the former Propositions, so also in this may we conclude, that the line D M is greater then the line M G. And that that which is containd vnder the lines D K and K M is equall to the square of the line M N. Now forasmuch as the square of the line A C is incommensurable to the square of the line C B, therefore the parallelogramme D H is incommensurable to the parallelogramme K L. Wherefore (by the 1. of the sixth, and 10. of the tenth) the line D K is incommensurable in length to the line K M. But if there be two vnequall right lines, and if vpon the greater be applied a parallelogramme equall to the fourth part of the square made of the lesse, and wanting in figure by a square, and if also the parallelogramme thus applied deuide the line wherupon it is applied into partes incommensurable in length, the greater line shall be in power more then the lesse, by the square of a line incommensurable in length to the greater (by the 18. of the tenth). Wherefore the line D M is in power more then the line M G, by the square of a line incommensurable in length to D M. And the lines D M and M G are proued to be rationall commensurable in power onely. And the line D M is commensurable in length to the rationall line geuen D E. Wherefore the line D G is a fourth binomial line: which was required to be proued.

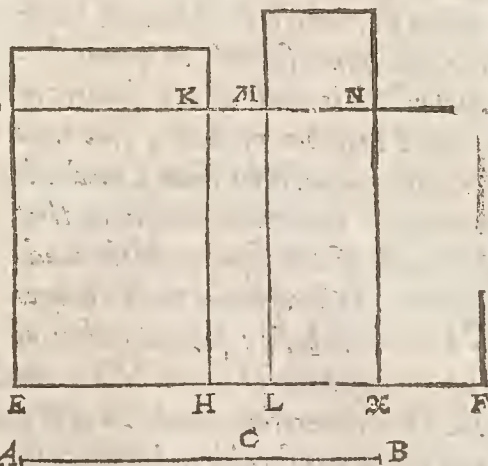
¶ The 46. Theoreme.

The 64. Proposition.

The square of a line containning in power a rationall and a medially superficies applied to a rationall line, maketh the breadth or other side a fift binomial line.



*V*ppose that the line A B be a line containning in power a rationall and a medially superficies, and let it be supposed to be deuided into his partes in the point C, so that let A C be the greater part, and take a rationall line D E. And (by the 44. of the first) vnto the line D E apply the parallelogramme D F equall to the square of the line A B, and making in breadth the line D G. Then I say, that the line D G is a fift binomial line. Let the selfe same construction be in this, that was in the former. And forasmuch as A B is a line containning in power a rationall and a medially superficies, and is deuided into his partes in the poynt C, therefore the lines A C & C B are incommensurable in power, hauing that which is made of the squares of the added together medially, and that which is containd vnder them rationall. Now forasmuch as that which is made of the squares of the lines A C and C B added together is medially, therefore also the parallelogramme D L is medially. Wherefore (by the 22. of the tenth) the line D M is rationall and incommensurable in length to the line D E. Againe forasmuch as that which is containd vnder the lines A C and C B twise, that is, the parallelogramme M F, is rationall, therefore by the 20. the line M G is rationall & comensurable in length to the



Construction.

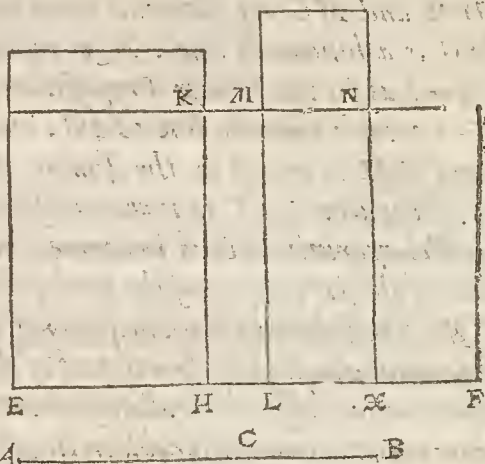
Demonstration.

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2.

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to the line DE . Wherefore (by the 13. of the tenth) the line DM is incommensurable in length to the line MG . Wherefore the lines DM and MG are rationally commensurable in power onely. Wherefore the whole line DG is a binomial line. I say moreouer, that it is a sixth binomial. For, as in the former, so also in this may it be proued, that that which is contained under the lines DK and KM is equall to the square of MN the halfe of the lesse: and that the line DK is incommensurable in length to the line KM . Wherefore (by the 18. of the tenth) the line DM is in power more then the line MG by the square of a line incommensurable in length to the line DM . And the lines DM and MG are rationally commensurable in power onely, and the lesse line, namely, MG is commensurable in length to the rational line DE . Wherefore the line DG is a sixth binomial line: which was required to be demonstrated.



The 47. Theoreme.

The 65. Proposition.

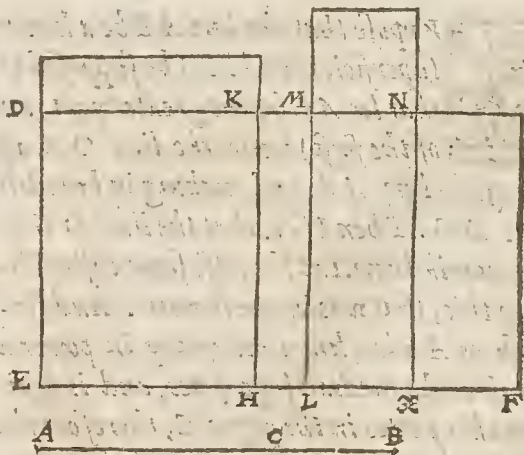
The square of a line containyng in power two medials applyed vnto a rational line, maketh the breadth or other side a sixth binomial line.

Suppose that the line AB be a line containyng in power two medials, and let it be supposed to be devided into his partes in the poynt C . And take a rational line DE . And (by the 44. of the first) vnto the rational line DE apply the parallelograme $DEFG$ equall to the square of the line AB and making in breadth the line DG . Then I say that the line DG is a sixth binomial line.

Construction.

Demonstration.

Let the selfe same construction be in this that was in the former. And forasmuch as the line AB is a line containyng in power two medials, and is devided into his partes in the poynt C , therefore the lines AC & CB are incommensurable in power having that which is made of the squares of them added together mediall, and that which is contained under them, mediall, and moreouer incommensurable to that which is made of the squares of them added together. Wherefore by those things which haue bene before proued, either of



these parallelogrames DL and MF is mediall, and either of them is applyed vpon the rational line DE . Wherefore (by the 22. of the tenth) either of these lines DM and MG is rational and incommensurable in length to the line DE . And forasmuch as that which is made of the squares of the lines AC and CB , added together is incommensurable to that which is contained under the lines AC and CB twise, therefore the parallelograme DL is incommensurable to the parallelograme MF . Wherefore (by the 1. of the sixth and 10. of the tenth) the line DM is incommensurable in length to the line MG . Wherefore the lines DM and MG are rationally commensurable in power onely. Wherefore the whole line DG

is a binomial line. I say also that is a sixth binomial line. For euen as in the other propositions it hath bene proued so also in this may it be proued, that that which is containd vnder the lines D K and K M is equall to the square of the line M N, and that the line D K is incommensurable in length to the line K M, and therfore (by the 18. of the tenth) the line D M is in power more then the line M G by the square of a line incommensurable in length to the line D M. And neither of these lines D M nor M G is commensurable in length to the rational line geuen D E. Wherefore the line D G is a sixth binomial line: which was required to be demonstrated.

¶ The 48. Theoreme.

The 66. Proposition.

A line commensurable in length to a binomial line, is also a binomial line of the selfe same order.

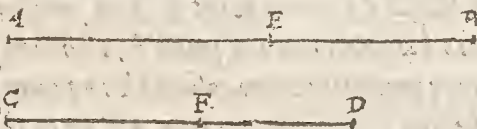


Suppose that the line A B be a binomial line, and vnto the line A B let the line C D be commensurable in length. Then I say that the line C D is a binomial line and of the selfe same order that the line A B is. For forasmuch as A B is a binomial line let it be deuided into his names in the poynt E, and let A E be the

The sixth Senary.

Construction.

greater name. Wherefore the lines A E and E B are rational commensurable in power onely. And as the line A B is to the line C D, (so by the 12. of the sixth) let the line A E be to the line C F. Wherefore (by the 19. of the fifth) the residue, namely, the line E B is to the residue, namely, to the line F D, as the line A B is to the line C D.



Demonstration.

But (by supposition) the line A B is commensurable in length to the line C D. Wherefore (by the 10. of the tenth) the line A E is commensurable in length to the line C F, and the line E B to the line F D. And the lines A E and E B are rational. Wherefore the lines C F and F D are also rational. And for that as the line A E is to the line C F, so is the line E B to the line F D, therefore alternately (by the 16. of the fifth) as the line A E is to the line E B, so is the line C F to the line F D. But the lines A E and E B are commensurable in power onely, wherefore the lines C F and F D are also commensurable in power onely, and they are rational. Wherefore the whole line C D is a binomial line. I say also that it is of the selfe same order of binomial lines that the line A B is. For the line A E is in power more then the line E B either by the square of a line commensurable in length to the line A E, or by the square of a line incommensurable in length to the line A E. If the line A E be in power more then the line E B by the square of a line commensurable in length to the line A E, the line also C F (by the 14. of the tenth) shalbe in power more then the line F D by the square of a line commensurable in length to C F. And if the line A E be commensurable in length to a rational line geuen, the line C F also shalbe commensurable in length to the same (by the 12. of the tenth). And so either of these lines A B and C D is a first binomial line, that is, they are both of one and the selfe same order. But if the line E B be commensurable in length to the rational line put, the line F D also shalbe commensurable in length to the same. And by that meanes agayne the lines A B and C D are both of one and the selfe same order, for either of them is a second binomial line. But if neither of the lines A E nor E B be commensurable in length to the rational line put, neither also of these lines C F nor F D shalbe commensurable in length to the same. And so either of the lines A B and C D is a third binomial line. But if the line A E be in power more then the line E B by the square of a line incommensurable in length to the line A E, the line also C F shalbe in power more then the line F D by the square of a line incommensurable in length to the line C F, (by the 14. of

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3.

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- the tenth). And then if the line AE be commensurable in length to the rationall line put the line CF also shalbe commensurable in length to the same, and so either of the lines AB , and CD shalbe a fourth binomial line. And if the line EB be commensurable in length to the rationall line geuen, the line FD also shalbe commensurable in length to the same. And so either of the lines AB and CD shalbe a fift binomial line. But if neither of the lines AE nor EB be commensurable in length to the rationall line geuen, neither also of the lines CF nor FD shalbe commensurable in length to the same, and so either of the lines AB and CD shalbe a sixt binomial line. A line, therefore commensurable in length to a binomial line, is also a binomial line of the selfe same order: which was required to be proued.

¶ The 49. Theoreme. The 67. Proposition.

A line commensurable in length to a bimediall line, is also a bimediall lyne and of the selfe same order.

Construction.



Demonstration.

Suppose that the line AB be a bimediall line, And vnto the line AB , let the lyne CD be commensurable in length. Then I say that the line CD is a bimediall line, and of the self order that the line AB is. Deuide the line AB into his partes in the point E . And forasmuch as the line AB is a bimediall line, and is deuided into his partes in the point E , therefore (by the 37. and 38. of the tenth) the lines AE and EB are medials commensurable in power onely. And (by the 12. of the sixt) as the line AB is to the line CD , so let the line AE be to the line CF .

Wherfore (by the 19. of the fift) the residue, namely, the line EB is to the residue, namely, to the line FD , as the line AB is to the line CD . But the line AB is commensurable in length to the lyne CD . Wherfore the line AE is commensurable in length to the line CF , and the line EB to the line FD . Now the lines AE and EB are medials, wherfore (by the 23. of the tenth) the lines CF and FD are also medials. And for that as the line AE is to the line EB , so is the line CF to the line FD . But the lines AE and EB are commensurable in power onely, wherfore the lines CF and FD are also commensurable in power onely. And it is proued that they are medials. Wherfore the lyne CD is a bimediall line. I say also that it is of the selfe same order that the line AB is. For, for that as the line AE is to the line EB , so is the line CF to the line FD , but as the line CF is to FD , so is the square of the lyne CF to the parallelogramme contained vnder the lynes CF and FD , by the first of the sixt. Therefore as the line AE is to the line EB , so (by the 11. of the fift) is the square of the line CF to the parallelogramme contained vnder the lines CF and FD : but as AE is to EB , so by the 1. of the sixt, is the square of the line AE , to the parallelogramme contained vnder the lines AE and EB , therefore (by the 11. of the fift) as the square of the line AE is to that which is contained vnder the lines AE and EB , so is the square of the line CF to that which is contained vnder the lines CF and FD . Wherfore alternately (by the 16. of the fift) as the square of the line AE is to the square of the line CF , so is that which is contained vnder the lines AE and EB to that which is contained vnder the lines CF & FD . But the square of the line AE is commensurable to the square of the line CF , because AE and CF are commensurable in length. Wherfore that which is contained vnder the lines AE and EB is commensurable to that which is contained vnder the lines CF and FD . If therefore that which is contained vnder the lines AE and EB be rationall, that is, if the line

AB

AB be a first binomial line, that also which is contained under the lines *CF* and *FD* is rational. Wherefore also the line *CD* is a first binomial line. But if that which is contained under the lines *AE* and *EB* be binomial, that is, if the line *AB* be a second binomial line, that also which is contained under the lines *CF* and *FD* is binomial: wherefore also the line *CD* is a second binomial line. Wherefore the lines *AB* and *CD* are both of one and the selfe same order: which was required to be proued.

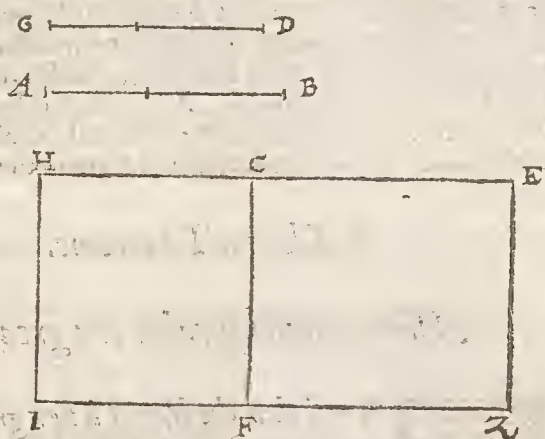
3.

4.

¶ A Corollary added by *Flussates*: but first noted by *P. Montaurens*.

A line commensurable in power onely to a binomial line, is also a binomial line, and of the selfe same order.

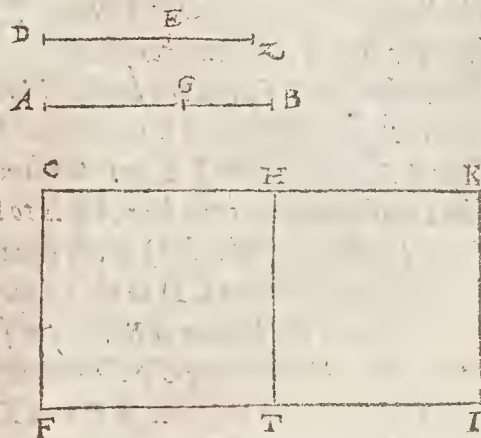
Suppose that *AB* be a binomial line, either a first or a second, whereunto let the line *GD* be commensurable in power onely. Take also a rational line *EZ*, vpon which (by the 45. of the first) apply a rectangle parallelogramme equall to the square of the line *AB*, which let be *EZFC*, and let the rectangle parallelogramme *CFIH* be equall to the square of the line *GD*. And forasmuch as vpon the rational line *EZ* is applied a rectangle parallelogramme *EF* equall to the square of a first binomial line, therefore the other side therof, namely, *EC*, is a second binomial line, by the 61. of this booke. And forasmuch as by supposition the squares of the lines *AB* & *GD* are commensurable, therefore the parallelogrammes *EF* and *CI* (which are equall vnto them) are also commensurable. And therefore by the 1. of the sixth, the lines *EC* and *CH* are commensurable in length. But the line *EC* is a second binomial line. Wherefore the line *CH* is also a second binomial line, by the 66. of this booke. And forasmuch as the superficies *CI* is contained vnder a rational line *EZ* or *CF*, and a second binomial line *CH*, therefore the line which containeth it in power, namely, the line *GD* is a first binomial line, by the 55. of this booke. And so is the line *GD* in the selfe same order of binomial lines that the line *AB* is.



A Corollary added by Flussates.

The like demonstration also will serue if the line *AB* be supposed to be a second binomial line. For so shall it make the breadth *EC* a third binomial line whereunto the line *CH* shall be commensurable in length, and therefore *CH* also shall be a third binomial line, by means whereof the line which containeth in power the superficies *CI*, namely, the line *GD* shall also be a second binomial line. Wherefore a line commensurable either in length, or in power onely to a binomial line, is also a binomial line of the selfe same order.

But so is it not of necessitie in binomial lines, for if their powers onely be commensurable, it followeth not of necessitie that they are binomials of one and the selfe same order, but they are eche binomials eyther of the three first kindes, or of the three last. As for example. Suppose that *AB* be a first binomial line, whose greater name let be *AG*, and vnto *AB* let the line *DZ* be commensurable in power onely. Then I say, that the line *DZ* is not of the selfe same order that the line *AB* is. For if it be possible, let the line *DZ* be of the selfe same order that the line *AB* is. Wherefore the line *DZ* may in like sort be deuided as the line *AB* is, by that which hath bene demonstrated in the 66. Proposition of this booke: let it be so deuided in the poynt *E*. Wherefore it can not be so deuided in any other poynt, by the 42. of this booke. And for that the line *AB* is to the line *DZ*, as the line *AG* is to the line *DE*, but the lines *AG* & *DE*, namely, the greater names, are commensurable in length the one to the other (by the 10. of this booke) for that they are commensurable in length to one and the selfe same rational line, by the first definition of binomial lines. Wherefore the lines *AB* and *DZ* are commensurable in length, by the 13. of this booke. But by supposition they are commensurable in power onely: which is impossible.

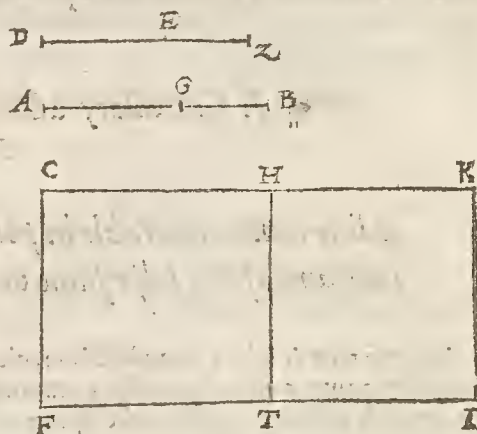


Note.

The

The tenth Booke

The selfe same demonstration also will serue, if we suppose the line AB to be a second binomial line: for the lesse names GB and EZ being commensurable in length to one and the selfe same rational line, shall also be commensurable in length the one to the other. And therefore the lines AB and DZ which are in the selfe same proportion with them, shall also be commensurable in length the one to the other: which is contrary to the supposition. Farther, if the squares of the lines AB and DZ be applied vnto the rational line CF , namely, the parallelogrammes CT and HL , they shall make the breadthes CH and HK first binomial lines, of what order soeuer the lines AB & DZ (whose squares were applied vnto the rational line) are, (by the 60. of this booke). Wherefore it is manifest, that vnder a rational line and a first binomial line, are confusedly containned all the powers of binomial lines (by the 54. of this booke). Wherefore the onely commensuration of the powers doth not of necessitie bring forth one and the selfe same order of binomial lines. The selfe same thyng also may be proued, if the lines AB and DZ be supposed to be a fourth or fifth binomial line, whose powers onely are commensurable, namely, that they shall as the first bring forth binomial lines of diuers orders. Now forasmuch as the powers of the lines AG and GB , and DE and EZ are commensurable & proportionall, it is manifest, that if the line AG be in power more then the line GB by the square of a line commensurable in length vnto AG , the line DE also shall be in power more then the line EZ by the square of a line commensurable in length vnto the line DE (by the 16. of this booke). And so shall the two lines AB and DZ be eche, of the three first binomial lines. But if the line AG be in power more then the line GB by the square of a line incommensurable in length vnto the line AG , the line DE shall also be in power more then the line EZ by the square of a line incommensurable in length vnto the line DE , by the selfe same Proposition. And so shall eche of the lines AB and DZ be of the three last binomial lines. But why it is not so in the third and sixth binomial lines, the reason is: For that in them neither of the names is commensurable in length to the rational line put FC .



¶ The 50. Theoreme.

The 58. Proposition.

A line commensurable to a greater line, is also a greater line.

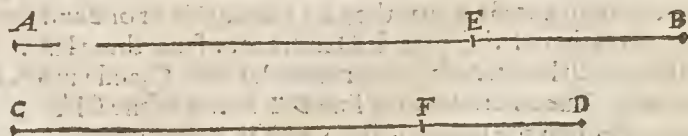
Construction.



Suppose that the line AB be a greater line. And vnto the line AB let the line CD be commensurable. Then I say that the line CD also is a greater line. Deuide the line AB into his partes in the point E . Wherefore (by the 39. of the tenth) the lines AE and EB are incommensurable in power, hauing that which is made of the squares of them added together rational, and that which is contained vnder the mediall. And let the rest of the construction be in this, as it was in the former. And for that as

Demonstration.

the line AB is to the line CD , so is the line AE to the line CF , & the line EB to the line FD , but the line AB is commensurable to the line CD by suppositiō. Wherefore the line AE is commensura-

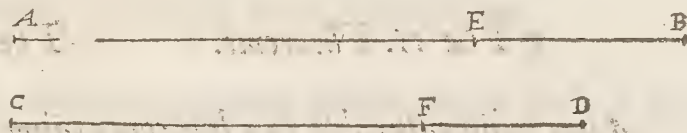


ble to the line CF , and the line EB to the line FD . And for that as the line AE is to the line CF , so is the line EB to the line FD . Therefore alternately (by the 16. of the fifth) as the line AE is to the line EB , so is the line CF to the line FD . Wherefore by composition also (by the 18. of the fifth) as the line AB is to the line EB , so is the line CD to the line FD . Wherefore (by the 22. of the sixth) as the square of the line AB is to the square of the line EB , so is the square of the line CD to the square of the line FD . And in like sort may we proue that as the square of the line AB is to the square of the line AE , so is the square of the line CD , to the square of the line CF . Wherefore (by the 11. of the fifth) as the square of the line AB is to the squares of the lines AE and EB , so is the square of the line CD to the squares of the lines CF and FD . Wherefore alternately (by the 16. of the fifth) as the square of the line AB

is to the square of the line $C D$, so are the squares of the lines $A E$ and $E B$ to the squares of the lines $C F$ and $F D$. But the square of the line $A B$ is commensurable to the square of the line $C D$ (for the line $A B$ is commensurable to the line $C D$ by suppositio). Wherefore also the squares of the lines $A E$ and $E B$ are commensurable to the squares of the lines $C F$ and $F D$. But the squares of the lines $A E$ and $E B$ are incommensurable, and being added together are rationall. Wherefore the squares of the lines $C F$ and $F D$ are incommensurable, & being added together, are also rationall. And in like sort may we proue that that which is contained vnder the lines $A E$ and $E B$ twise, is commensurable to that which is contained vnder the lines $C F$ and $F D$ twise. But that which is contained vnder the lines $A E$ and $E B$ twise, is mediall, wherefore also that which is contained vnder the lines $C F$ and $F D$ twise is mediall. Wherefore the lines $C F$ and $F D$ are incommensurable in power, hauing that which is made of the squares of them added together rationall, and that which is contained vnder the medially. Wherefore (by the 39. of the tenth) the whole line $C D$ is irrational, & is called a greater line. A line therefore commensurable to a greater line, is also a greater line.

An other demonstration of Peter Montaurens to proue the same.

Suppose that the line $A B$ be a greater line, and vnto it let the line $C D$ be commensurable any way, that is, either both in length and in power, or els in power onely. Then I say that the line $C D$ also is a greater line. Deuide the line $A B$ into his partes in the point E . and let the rest of the construction be in this as it was in the former. And for that as the line $A B$ is to the line $C D$, so is the line $A E$ to the lyne $C F$, and the line $E B$ to the line $F D$,



therefore as the line $A E$ is to the lyne $C F$, so is the line $E B$ to the line $F D$, but the line $A B$ is commensurable to the line $C D$. Wherefore also the lyne $A E$ is commensurable to the lyne $C F$, and likewise the line $E B$ to the line $F D$. And for that as the line $A E$ is to the line $C F$, so is the line $E B$ to the line $F D$, therefore alternately as the lyne $A E$ is to the line $E B$, so is the line $C F$ to the lyne $F D$. Wherefore (by the 22. of the sixth) as the square of the lyne $A E$ is to the square of the line $E B$, so is the square of the line $C F$ to the square of the lyne $F D$. Wherefore by composition (by the 18. of the fifth) as that which is made of the squares of the lynes $A E$ and $E B$ added together is to the square of the lyne $E B$, so is that which is made of the squares of the lynes $C F$ and $F D$ added together to the square of the lyne $F D$. Wherefore by contrary proportion as the square of the line $E B$ is to that which is made of the squares of the lines $A E$ and $E B$ added together, so is the square of the lyne $F D$ to that which is made of the squares of the lynes $C F$ and $F D$ added together. Wherefore alternately as the square of the line $E B$ is to the square of the lyne $F D$, so is that which is made of the squares of the lynes $A E$ and $E B$ added together to that which is made of the squares of the lynes $C F$ and $F D$ added together. But the square of the lyne $E B$ is commensurable to the square of the lyne $F D$, for it hath already bene proued that the lines $E B$ and $F D$ are commensurable. Wherefore that which is made of the squares of the lines $A E$ & $E B$ added together is commensurable to that which is made of the squares of $C F$ & $F D$ added together. But that which is made of the squares of the lines $A E$ and $E B$ added together is rationall by suppositio. Wherefore that which is made of the squares of the lynes $C F$ and $F D$ added together is also rationall. And as the lyne $A E$ is to the lyne $E B$, so is the line $C F$ to the lyne $F D$. But as the lyne $A E$ is to the lyne $E B$, so is the square of the line $A E$ to the parallelogramme containd vnder the lynes $A E$ and $E B$: therefore as the lyne $C F$ is to the lyne $F D$, so is the square of the lyne $A E$ to the parallelogramme containd vnder the lines $A E$ and $E B$: & as the lyne $C F$ is to the lyne $F D$, so is the square of the lyne $C F$ to the parallelogramme containd vnder the lynes $C F$ & $F D$. Wherefore as the square of the lyne $A E$ is to the parallelogramme containd vnder the lines $A E$ and $E B$, so is the square of the lyne $C F$ to the parallelogramme cotayned vnder the lynes $C F$ and $F D$. Wherefore alternately as the square of the line $A E$ is to the square of the lyne $C F$, so is the parallelogramme containd vnder the lynes $A E$ and $E B$ to the parallelogramme containd vnder the lines $C F$ and $F D$. But the square of the lyne $A E$ is commensurable to the square of the lyne $C F$, for it is already proued that the lynes $A E$ and $C F$ are commensurable. Wherefore the parallelogramme containd vnder the lynes $A E$ and $E B$ is commensurable to the parallelogramme containd vnder the lynes $C F$ and $F D$. But the parallelogramme containd vnder the lines $A E$ and $E B$ is medially by suppositio. Wherefore the parallelogramme containd vnder the lynes $C F$ and $F D$ also is medially. And (as it hath already bene proued) as the line $A E$ is to the lyne $E B$, so is the lyne $C F$ to the lyne $F D$. But the lyne $A E$ was by suppositio incommensurable in power to the line $E B$. Wherefore (by the 10. of the tenth) the lyne $C F$ is incommensurable in power to the lyne $F D$. Wherefore the lynes

An other demonstration after P. Montaurens.

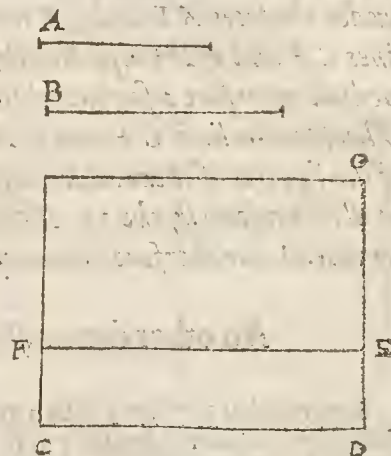
The tenth Booke

lynes C F and F D are incommensurable in power, hauing that which is made of the squares of them added together rationall, and that which is containd vnder them medially. Wherefore the whole lyne C D is (by the 39. of the tenth) a greater lyne. Wherefore a lyne commensurable to a greater lyne is also a greater lyne: which was required to be demonstrated.

An other more briefe demonstration of the same after Campane.

An other demonstration after Campane.

Suppose that A be a greater line, vnto which let the line B be commensurable, either in length and power, or in power onely. And take a rational line C D. And vpon it apply the superficies C E equall to the square of the line A: and also vpon the line B E (which is equall to the rational line C D) apply the parallelogramme F G equall to the square of the line B. And forasmuch as the squares of the two lines A and B are commensurable by supposition, the superficieses C E, shalbe commensurable vnto the superficieses F G: and therefore by the first of the sixth and tenth of this booke, the line D E is commensurable in length to the line G E. And forasmuch as (by the 63. of this booke) the line D E is a fourth binomial line, therefore by the 66. of this booke the line G E is also a fourth binomial line: wherefore by the 57. of this booke the line B which containeth in power the superficieses F G is a greater line.



¶ The 51. Theoreme.

The 69. Proposition.

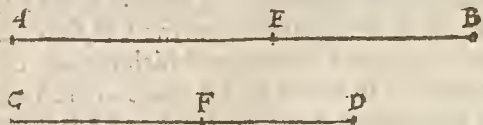
A line commensurable to a line containyng in power a rationall and a medially is also a line containyng in power a rationall and a medially.



Construction.

Vppose that AB be a line containyng in power a rationall and a medially. And vnto the line AB let the line CD be commensurable, whether in length and power, or in power onely. The I say that the line CD is a line cōtaynyng in power a rationall & a medially. Diuide the line AB into his parts in the poynt E. Where-

fore (by the 40. of the tenth) the lines AE and EB are incommensurable in power, hauing that which is made of the squares of them added together medially, and that which is containd vnder the rationall.



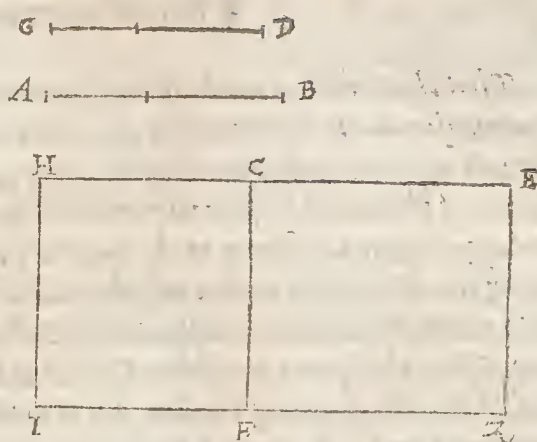
Demonstration.

Let the same construction be in this that was in the former. And in like sort we may proue that the lines C F and F D are incommensurable in power, and that that which is made of the squares of the lines AE and EB is commensurable to that which is made of the squares of the lines C F and F D, and that that also which is containd vnder the lines AE and EB is commensurable to that which is containd vnder the lines C F and F D. Wherefore that which is made of the squares of the lines C F and F D is medially, and that which is containd vnder the lines C F and F D is rationall. Wherefore the whole line C D is a line containyng in power a rationall and a medially: which was required to be demonstrated.

An other demonstration of the same after Campane.

Suppose

Suppose that AB be a line containyng in power a rationall and a medially: whereunto let the line GD be commensurable either in length and power, or in power onely. Then I say that the line GD is a line containyng in power a rationall and a medially. Take a rational line EZ , vpo which by the 45. of the first apply a rectāge parallelograme $EZFC$ equall to the square of the line AB : and vpon the line CF (which is equall to the line EZ) apply the parallelograme $FCHI$ equall to the square of the line GD : and let the breadths of the sayd parallelogrammes be the lines EC and CH . And forasmuch as the line AB is commensurable to the line GD at the least in power onely, therefore the parallelogrammes EF and FH (which are equall to their squares) shalbe commensurable. Wherefore by the 1. of the sixt the right lines EC and CH are comēsurable in lēgth. And forasmuch as the parallelograme EF (which is equall to the square of the line AB which containeth in power a rationall and a medially) is applied vpon the rational EZ , making in breadth the line EC , therefore the line EC is a fifth binomiall line (by the 64. of this booke) vnto which line EC the line CH is comēsurable in length, wherefore by the 66. of this booke the line CH is also a fifth binomiall line. And forasmuch as the superficies CI is containd vnder the rational line EZ (that is CF) and a fifth binomiall line CH , therefore the line which containeth in power the superficies CI , which by supposition is the line GD is a line containyng in power a rationall and a medially by the 58. of this booke. A line therefore commensurable to a line containyng in power a rationall and a medially. &c.



An other demonstration after Campanus.

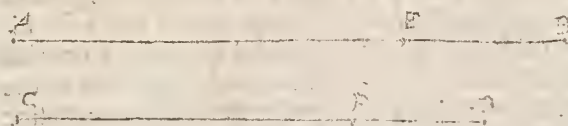
¶ The 52. Theoreme.

The 70. Proposition.

A line commensurable to a line containyng in power two mediallys, is also a line containyng in power two mediallys.



Vppose that AB be a line containyng in power two mediallys. And vnto the line AB let the line CD be commensurable, whether in length & power, or in power onely. Then I say, that the line CD is a line containyng in power two mediallys. Forasmuch as the line AB is a line containyng in power two mediallys, let it be deuided into his partes in the point E . Wherefore (by the 41. of the tenth) the lines AE and EB are incommensurable in power, hauing that which is made of the squares of them added together medially, and that also which is contained vnder them medially, and that which is made of the squares of the lines AE & EB is incommensurable to that which is contained vnder the lines AE and EB . Let the selfe same construction be in this, that was in the former. And in like sort may we proue, that the lines CF & FD are incommensurable in power, and that that which is made of the squares of the lines AE and EB added together, is commensurable to that which is made of the squares of the lines CF and FD added together, and that that also which is contained vnder the lines AE and EB is commensurable to that which is contained vnder the lines CF and FD . Wherefore that which is made of the squares of the lines CF and FD is medially (by the Corollary of the 23. of the tenth): and that which is containd vnder the lines CF and FD is medially (by the same Corollary): and moreover, that which is made of the squares of the lines CF & FD is incommensurable to that which is contained vnder the lines CF and FD . Wherefore the line CD is a line containyng in power two mediallys: which was required to be proued.



Construction.

Demonstration.

The tenth Booke

¶ An Assumpt added by Montaireus.

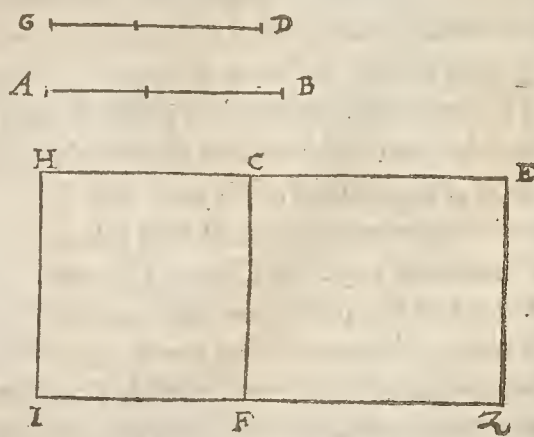
An Assumpt.

That that which is made of the squares of the lines CF and FD added together, is incommensurable to that which is contained under the lines CF and FD is thus proued. For, because as that which is made of the squares of the lines AE and EB added together is to the square of the line AE , so is that which is made of the squares of the lines CF and FD added together, to the square of the line CF , as it was proued in the Propositions going before: therefore alternately, as that which is made of the squares of AE and EB added together is to that which is made of the squares of CF and FD added together, so is the square of the line AE to the square of the line CF . But before, namely, in the 68. Proposition, it was proued, that as the square of the line AE is to the square of the line CF , so is the parallelogramme contained under the lines AE and EB to the parallelogramme contained under the lines CF and FD . Wherefore as that which is made of the squares of the lines AE and EB is to that which is made of the squares of the lines CF and FD , so is the parallelogramme contained under the lines AE and EB to the parallelogramme contained under the lines CF and FD . Wherefore alternately, as that which is made of the squares of the lines AE and EB is to the parallelogramme contained under the lines AE and EB , so is that which is made of the squares of the lines CF and FD to the parallelogramme contained under the lines CF and FD . But by supposition that which is made of the squares of the lines AE and EB , is incommensurable to the parallelogramme contained under the lines AE & EB . Wherefore that which is made of the squares of the lines CF and FD added together, is incommensurable to the parallelogramme contained under the lines CF and FD : which was required to be proued.

✱ An other demonstration after Campane.

An other demonstration after Campane.

Suppose that AB be a line containyng in power two medials: wherunto let the line GD be commensurable either in length, and in power, or in power onely. Then I say, that the line GD is a line containyng in power two medials. Let the same construction be in this, that was in the former. And forasmuch as the parallelogramme EF is equall to the square of the line AB , and is applyed vpon a rationall line EZ , it maketh the breadth EC a sixt binomial line, by the 65. of this booke. And forasmuch as the parallelogrammes EF & CI (which are equall vnto the squares of the lines AB and GD , which are supposed to be commensurable) are commensurable, therefore the lines EC and CH are commensurable in length, by the first of the sixt. But EC is a sixt binomial line: Wherefore CH also is a sixt binomial line, by the 66. of this booke. And forasmuch as the superficies CI is containd vnder the rationall line CF and a sixt binomial line CH , therefore the line which cōtayneth in power the superficies CI , namely, the line GD is a line containyng in power two medials, by the 59. of this booke. Wherefore a line commensurable to a line containyng in power two medials. &c.



✱ An Annotation.

Hetherto hath bene spoken of sixe Senarys, of which the first Senary containeth the production of irrationall lines by composition: the second, the diuision of them, namely, that those lines are in one point onely deuided: the third, the finding out of binomial lines, of the first, I say, the second, the third, the fourth, the fift, and the sixt: after that beginneth the fourth Senary, containing the difference of irrationall lines betwene them selues. For by the nature

nature of euery one of the binomiall lines are demonstrated the differences of irrational lines. The sixth entreateth of the applications of the squares of euery irrational line, namely, what irrational lines are the breadthes of euery superficies so applied. In the sixth Senary is proued, that any line commensurable to any irrational line, is also an irrational line of the same nature. And now shall be spoken of the seventh Senary, wherein againe are plainly set forth the rest of the differences of the said lines betwene them selues.

And there is euen in those irrational lines an arithmetically proportionalitie. And that line which is the arithmetically mean proportional betwene the partes of any irrational line, is also an irrational line of the selfe same kinde. First it is certaine, that there is an arithmetically proportion betwene those partes. For suppose that the line AB be any of the foresaid irrational lines, as for example, let it be a binomial line, & let it be deuided into his names in the point C . And let AC be the greater name, from which take away the line AD equall to the lesse name, namely, to CB . And deuide the line CD into two equall partes in the point E . It is manifest that the line AE is equall to the line EB . Let the line FG be equall to either of AE or EB . It is plaine that how much the line AC differeth from the line FG , so much the same line FG differeth from the line CB : for in each is the difference of the line DE or EC , which is the propertie of arithmetically proportionalitie. And it is manifest, that the line FG is commensurable in length to the line AB , for it is the halfe thereof. Wherefore (by the 66. of the tenth) the line FG is a binomial line. And after the selfe same manner may it be proued touching the rest of the irrational lines.

Note.

¶ The 53. Theoreme.

The 71. Proposition.

If two superficieses, namely, a rational and a medially superficies be composed together, the line which containeth in power the whole superficies is one of these foure irrational lines, either a binomial line, or a first bimediall lyne, or a greater lyne, or a lyne containing in power a rational and a medially superficieses.



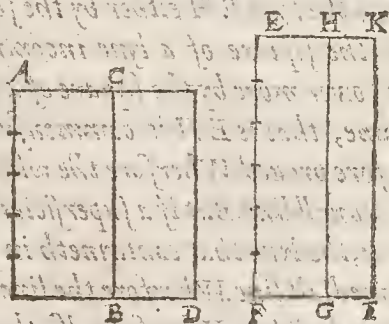
Suppose that there be two superficieses AB and CD , of which let the superficies AB be rational, & the superficies CD medially. The I say that the lyne containing in power the whole superficies AD , is either a binomial line, or a first bimediall line, or a greater line, or a lyne containing in power a rational & a medially superficies. For the superficies AB is either greater or lesse the the superficies CD (for they can by no means be equal, whe as the one is rational, and the other medial). First let it be greater, and take a rational line EF . And (by the 44. of the first) vnto the line EF apply the parallelogramme EG equal to the superficies AB , & making in breadth the line EH , and to the same line EF , that is, to the line HG apply the parallelogramme HI equal to the superficies CD , and making in breadth the line HK . And forasmuch as the superficies AB is rational, and is equal to the parallelogramme EG , therefore the parallelogramme EG is also rational, and it is applied vnto the rational line EF , making in breadth the line EH . Wherefore the line EH is rational and commensurable in length to the line EF (by the 20. of the tenth). Againe forasmuch as the superficies CD is medially, and is e-

The seventh Senary.

Construction.

Two cases in this proposition, of which each also hath his two cases. First case.

Demonstration.

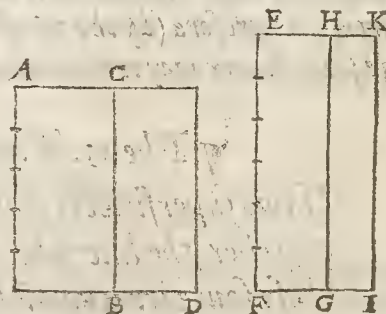


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The tenth Booke

equal to the parallelogramme HI ; therefore the parallelogramme HI is also medially, and is applied unto the rationall line EF , that is, unto the lyne HG making in breadth the line HK . Wherefore the lyne HK is rationall and incommensurable in length to the line EF (by the 22 of the tenth.) And forasmuch as the superficies CD is medially, and the superficies AB is rationall, therefore the superficies AB is incommensurable to the superficies CD . Wherefore also the parallelogramme EG is incommensurable to the parallelogramme HI . But as the parallelogramme GE is to the parallelogramme HI , so (by the 1. of the sixth) is the line EH to the lyne HK . Wherefore (by the 10. of the tenth) the line EH is incommensurable in length to the line HK , and they are both rationall. Wherefore the lines EH and HK are rationall commensurable in power onely. Wherefore the whole line EK is a binomial line, and is divided into his names in the poynt H . And forasmuch as the superficies AB is greater then the superficies CD , but the superficies AB is equall to the parallelogramme EG , and the superficies CD to the parallelogramme HI . Wherefore the parallelogramme EG is greater then the parallelogramme HI . Wherefore the line EH is greater then the line HK . Wherefore the line EI is in power more then the line HK either by the square of a line commensurable in length to the lyne EH , or by the square of a lyne incommensurable in length to the lyne EH . First let it be in power more by the square of a lyne commensurable in length unto the line EH . Now the greater name, namely, EH is commensurable in length to the rationall line geuen EF , as it hath already bene proued. Wherefore the whole line EK is a first binomial line. And the lyne EF is a rationall lyne. But if a superficies be containned under a rationall line, and a first binomial lyne, the lyne that containeth in power the same superficies, is (by the 54. of the tenth) a binomial line. Wherefore the lyne containing in power the parallelogramme EI is a binomial line. Wherefore also the line containing in power the superficies AD is a binomial line.



First part of the first case.

Second part of the first case.

But now let the lyne EH be in power more then the line HK by the square of a line incommensurable in length to the line EH : now the greater name that is, EH is commensurable in length to the rationall line geuen EF . Wherefore the line EK is a fourth binomial line. And the line EF is rationall. But if a superficies be contained under a rationall line and a fourth binomial line, the line that containeth in power the same superficies is (by the 57. of the tenth) irrational, and is a greater line. Wherefore the line which containeth in power the parallelogramme EI is a greater line. Wherefore also the line containing in power the superficies AD is a greater lyne.

The second case.

First part of the second case.

But now suppose that the superficies AB which is rationall, be lesse then the superficies CD which is medially. Wherefore also the parallelogramme EG is lesse then the parallelogramme HI . Wherefore also the line EH is lesse then the line HK . Now the line HK is in power more then the lyne EH either by the square of a line commensurable in length to the line HK , or by the square of a lyne incommensurable in length unto the lyne HK . First let it be in power more by the square of a line commensurable in length unto HK : now the lesse name, that is EH is commensurable in length to the rationall line geuen EF , as it was before proued. Wherefore the whole line EK is a second binomial line. And the line EF is a rationall line. But if a superficies be contained under a rationall line and a second binomial lyne, the lyne that containeth in power the same superficies, is (by the 55. of the tenth) a first bimediall line. Wherefore the line which containeth in power the parallelogramme EI is a first bimediall line. Wherefore also the line that containeth in power the superficies AD is a first bimediall lyne.

Second part of the second case.

But now let the line HK be in power more then the line EH , by the square of a line incommen-

comensurable in length to the lyne $H K$, now the lesse name, that is, $E H$ is comensurable in length to the rationall lyne geuen $E F$. Wherefore the whole line $E K$ is a fift binomiall lyne. And the lyne $E F$ is rationall. But if a superficies be containd under a rationall lyne, and a fift binomiall lyne, the line that containeth in power the same superficies, is (by the 58. of the tenth) a line containing in power a rationall and a mediall. Wherefore the lyne that containeth in power the parallelogramme $E I$ is a line containing in power a rationall and a mediall. Wherefore also the lyne that containeth in power the superficies $A D$ is a lyne containing in power a rationall and a mediall. If therfore a rationall and a mediall superficies be added together, the lyne which containeth in power the whole superficies, is one of these foure irrational lines, namely, either a binomiall line, or a first bimediall line, or a greater lyne, or a lyne containing in power a rationall and a mediall: which was required to be demonstrated.

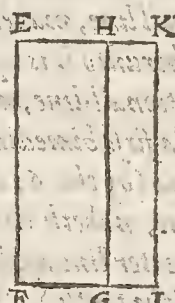
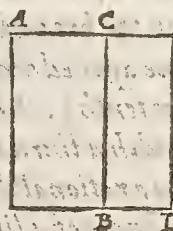
¶ The 54. Theoreme.

The 72. Proposition.

If two mediall superficieces incommensurable the one to the other be composed together: the line containyng in power the whole superficies is one of the two irrational lines remaynyng, namely, either a second bimediall line, or a line containyng in power two medials.



Et these two mediall superficieces $A B$ and $C D$ being incommensurable the one to the other be added together. Then I say, that the line which containeth in power the superficies $A D$ is either a second bimediall line, or a line containyng in power two medials. For the superficies $A B$ is either greater or lesse then the superficies $C D$ (for they can by no meanes be equall, when as they are incommensurable). First let the superficies $A B$ be greater then the superficies $C D$. And take a rationall line $E F$. And (by the 44. of the first) unto the line $E F$ apply the parallelogramme $E G$ equall to the superficies $A B$, and making in breadth the line $E H$; and unto the same line $E F$, that is, to the line $H G$, apply the parallelogramme $H I$ equall to the superficies $C D$, & making in breadth the line $H K$. And forasmuch as either of these superficieces $A B$ & $C D$ is mediall, therefore also either of these parallelogrammes $E G$ and $H I$ is mediall. And they are eche applied to the rationall line $E F$, making in breadth the lines $E H$ and $H K$. Wherefore (by the 22. of the tenth) either of these lines $E H$ and $H K$ is rationall and incommensurable in length to the line $E F$. And forasmuch as the superficies $A B$ is incommensurable to the superficies $C D$, and the superficies $A B$ is equall to the parallelogramme $E G$, and the superficies $C D$ to the parallelogramme $H I$: therefore the parallelogramme $E G$ is incommensurable to the parallelogramme $H I$. But (by the 1. of the sixth) as the parallelogramme $E G$ is to the parallelogramme $H I$, so is the line $E H$ to the line $H K$. Wherefore (by the 10. of the tenth) the line $E H$ is incommensurable in length to the line $H K$. Wherefore the lines $E H$ and $H K$ are rationall commensurable in power onely. Wherefore the whole line $E K$ is a binomiall line. And as in the former Proposition so also in this may it be proued, that the line $E H$ is greater then the line $H K$. Wherefore the line $E H$ is in power more then the line $H K$, either by the square of a line commensurable in length to the line $E H$, or by the square of a line incommensurable in length to the line $E H$. First let it be greater by the square of a line commensurable in length unto the line $E H$. Now neither of these lines $E H$ and $H K$ is commensurable in length to the rationall line geuen $E F$. Wherefore the whole line $E K$ is a third binomiall



Construction.

Two cases in this Proposition.
The first case.

The first part of the first case.

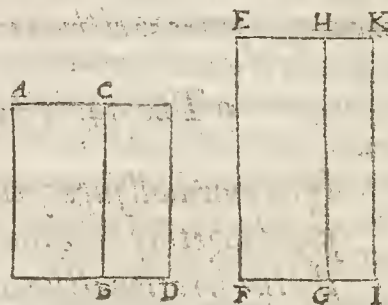
The tenth Booke

nomiall line. And the line EF is a rationall line. But if a superficies be containd under a rationall line & a third binomiall line, the line that containeth in power the same superficies, is (by the 56. of the tenth) a second bimediall line. Wherefore the line that containeth in power the superficies EI , that is, the superficies AD , is a second bimediall line.

The second
part of the
first case.

The second
case.

But now suppose that the line EH be in power more then the line HK by the square of a line incommensurable in length to the line EH . And forasmuch as either of these lines EH and HK is incommensurable in length to the rationall line given EF , therefore the line EK is a sixth binomiall line. But if a superficies be contained under a rationall line and a sixth binomiall line, the line that containeth in power the same superficies, is (by the 59. of the tenth) a line containing in power two medials. Wherefore the line that containeth in power the superficies AD , is a line containyng in power two medials. And after the selfe same maner, if the superficies AB be lesse then the superficies CD , may we proue, that the line that containeth in power the superficies AD , is either a second bimediall line, or a line containing in power two medials. If therefore two mediall superficieses incommensurable the one to the other be added together, the line containyng in power the whole superficies is one of the two irrational lines remainyng, namely, either a second bimediall line, or a line containyng in power two medials: which was required to be proued.



¶ A Corollary following of the former Propositions.

A Corollary.

A binomiall line and the other irrational lines following it, are neither mediall lines, nor one and the same betwene them selues. For the square of a mediall line applied to a rationall line, maketh the breadth rationall and incommensurable in length to the rationall line, wherunto it is applied (by the 22. of the tenth). The square of a binomiall line applied to a rationall line, maketh the breadth a first binomiall line (by the 60. of the tenth). The square of a first bimediall line applied vnto a rationall line, maketh the breadth a second binomiall line (by the 61. of the tenth). The square of a second bimediall line applied vnto a rationall line, maketh the breadth a third binomiall line (by the 62. of the tenth). The square of a greater line applied to a rationall line, maketh the breadth a fourth binomiall line (by the 63. of the tenth). The square of a line containing in power a rationall & a mediall superficies, maketh the breadth a fifth binomiall line (by the 64. of the tenth). And the square of a line containing in power two medials, applied vnto a rationall line, maketh the breadth a sixth binomiall line (by the 65. of the tenth). Seing therefore that these foresaid breadthes differ both from the first breadth, for that it is rationall, and differ also the one from the other, for that they are binomials of diuers orders: it is manifest that those irrational lines differ also the one from the other.

¶ Here beginneth the Senaries by subtraction.

¶ The 55. Theoreme.

The 73. Proposition.

If from a rationall line be taken away a rationall line commensurable in power onely to the whole line: the residue is an irrational line, and is called a residuall line.

Suppose



Suppose that AB be a rationall line, and from AB take away a rationall line BC commensurable in power onely to the whole line AB . Then I say that the line remayning, namely AC is irrationall and is called a residuall line. For forasmuch as the line AB is incommensurable in length vnto the line BC , and (by the as-

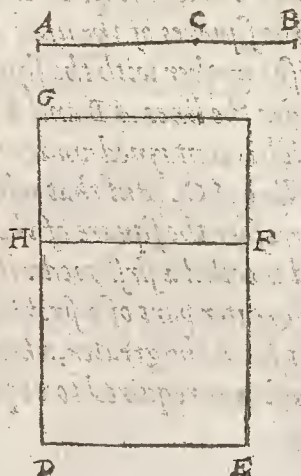
The first Secondary by subtraction.

Demonstration.

sumpt going before the 22. of the tenth) as the line AB is to the line BC , so is the square of the line AB to that which is containd under the lines AB and BC : wherefore (by the 10. of the tenth) the square of the line AB is incommensurable to that which is containd under the lines AB and BC . But vnto the square of the line AB are commensurable the squares of the lines AB and BC (by the 15. of the tenth). Wherefore the squares of the lines AB and BC are incommensurable to that which is containd under the lines AB and BC . But vnto that which is containd under the lines AB and BC is commensurable that which is containd under the lines AB and BC twise. Wherefore the squares of the lines AB and BC are incommensurable to that which is containd under the lines AB and BC twise. But the squares of the lines AB and BC are equall to that which is containd under the lines AB and BC twise, and to the square of the line AC (by the 7. of the second). Wherefore that which is containd under the lines AB and BC twise together with the square of the line AC is incommensurable to that which is containd under the lines AB and BC twise. Wherefore (by the 2 part of the 16. of the tenth) that which is containd under the lines AB and BC twise, is incommensurable to the square of the line AC . Wherefore (by the first part of the same) that which is containd under the lines AB and BC twise together with the square of the line AC , that is, the squares of the lines AB and BC are incommensurable to the square of the line AC . But the squares of the lines AB and BC are rationall, for the lines AB and BC are put to be rationall: wherefore the line AC is irrationall and is called a residuall line: which was required to be proued.

* An other demonstration after Campane.

Campane demonstrateth this Proposition by a figure more briefly after this maner. Let the superficies EG be equall to the squares of the lines AB and BC added together: which shall be rationall (for that the lines AB and BC are supposed to be rationall commensurable in power onely). From which superficies take away the superficies DF equall to that which is containd under the lines AB & BC twise, which shall be medially (by the 21. of this booke). Now by the 7. of the second, the superficies FG is equall to the square of the line AC . And forasmuch as the superficies EG is incommensurable to the superficies DF (for that the one is rationall and the other medially): therefore (by the 16. of this booke) the same superficies EG is incommensurable to the superficies FG . Wherefore the superficies FG is irrationall. And therefore the line AC which containeth it in power is irrationall: which was required to be proued.



An other demonstration after Campane.

An annotation of P. Montaireus.

This Theoreme teacheth nothing els but that that portion of the greater name of a binomial line which remayneth after the taking away of the lesse name from the greater name is irrationall, which is called a residuall line, that is to say, if from the greater name of a binomial line, which greater name is a rationall line commensurable in power onely to the lesse name, be taken away the lesse name, which selfe lesse name is also commensurable in power onely to the greater name (which greater name this Theoreme calleth the whole line) the rest of the line which remaineth is irrational, which he calleth

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a residuall line. Wherefore all the lines which are intreated in this Theoreme, and in the five other which follow are the portions remayning of the greater partes of the whole lines which were intreated of in the 36. 37. 38. 39. 40. 41. propositions, after the taking away the lesse part from the greater.

In this proposition is set forth the nature of the eight kinde of irrational lines which is called a residuall line, the definition whereof by this proposition is thus.

Definition of the eight irrational line.

A residuall line is an irrational line which remayneth, when from a rational line given, is taken away a rational line commensurable to the whole line in power onely.

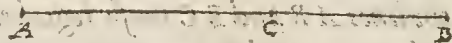
The 56. Theoreme.

The 74. Proposition.

If from a medially line be taken away a medially line commensurable in power onely to the whole line, and comprehending together with the whole line a rational superficies: the residue is an irrational line, and is called a first medially residuall line.

Suppose that AB be a medially line. And from the line AB take away a medially line BC commensurable in power onely to the whole line AB and comprehending together with the line AB a rational superficies, that is, let that which is comprehended under the lines AB and BC be rational. Then I say that the line remayning, namely, AC is irrational and is called a first medially residuall line. For forasmuch as the lines AB and BC are medially, therefore also the squares of the lines AB and BC are medially. But that which is contained under the lines AB and BC twice is rational. Wherefore that which is composed of the squares of the lines AB and BC , that is, that which is contained under AB and BC twice together with the square of the line AC is incommensurable to that which is contained under the lines AB and BC twice. Wherefore (by the second part the 16. of the tenth) that which is contained under the lines AB and BC twice is incommensurable to the square of the line AC . But that which is contained under the lines AB and BC twice is rational, wherefore the square of the line AC is irrational. Wherefore also the line AC is irrational: and is called a first medially residuall line. This first medially residuall line is also that part of the greater part of a first bimedially line, which remayneth after the taking away of the lesse part from the greater, wherof it hath also his name, and is called a first medially residuall line: which was required to be proved.

Demonstration.



Out of this proposition is taken the definition of the ninth kinde of irrational lines, which is called a first residuall medially line the definition whereof is thus.

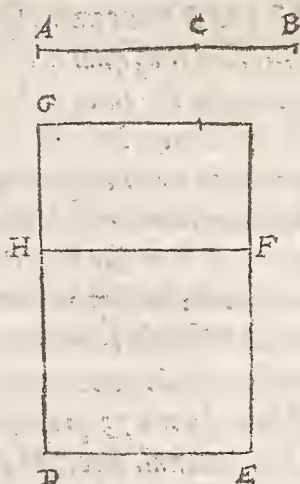
Definition of the ninth irrational line.

A first residuall medially line is an irrational line which remayneth, when from a medially line is taken away a medially line commensurable to the whole in power onely, and the part taken away and the whole line containe a medially superficies.

Another demonstration after Campane.

Let the line DE be rational, upon which apply the superficies DE equal to that which is contained

ned vnder the lynes A B and B C twise, and let the superficies G E be equal to that which is composed of the squares of the lynes A B and B C: wherfore by the 7. of the second, the superficies F G is equal to the square of the lyne A C. And forasmuch as (by supposition) the superficies E G is medially, therefore (by the 22. of the tenth) the lyne D G is rationally commensurable in power onely to the rational lyne D E. And forasmuch as by supposition the superficies E H is rational, therefore by the 20. of the tenth, the lyne D H is rational commensurable in length vnto the rational lyne D E. Wherfore the lynes D G and D H are rationally commensurable in power only (by the assumpt put before the 13. of this booke). Wherfore by the 73 of this booke, the lyne G H is a residuall lyne, and is therefore irrational. Wherfore (by the corollary of the 21. of this booke) the superficies F G is irrational. And therefore the lyne A C which cōtayneth it in power is irrational, and is called a first medially residuall lyne.



An other demonstration after Campanus.

The 75. Theoreme.

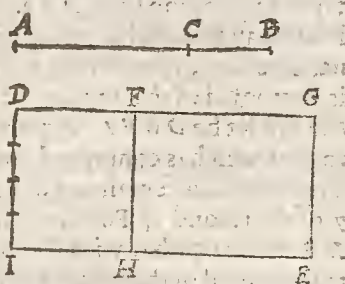
The 75. Proposition.

If from a medially lyne be taken away a medially lyne commensurable in power only to the whole lyne, and comprehending together with the whole lyne a medially superficies, the residue is an irrational lyne, and is called a second medially residuall lyne.



Suppose that A B be a medially lyne, and from A B take away a medially lyne C B commensurable in power onely to the whole lyne A B, and comprehending together with the whole lyne A B a medially superficies, namely, the parallelogramme contained vnder the lines A B and B C. Then I

say that the residue, namely, the lyne A C is irrational, and is called a second medially residuall lyne. Take a rational lyne D I, and (by the 44. of the first) vnto the lyne D I apply the parallelogramme D E equal to the squares of the lines A B & B C, and making in breadth the lyne D G. And vnto the same lyne D I apply the parallelogramme D H equal to that which is contained vnder the lines A B & B C twise, and making in breadth the lyne D F. Now the parallelogramme D H is lesse then the parallelogramme D E, for that also the square of the lines A B and B C are greater then that which is contained vnder the lines A B and B C twise, by the square of the lyne A C by the 7. of the second. Wherfore the parallelogramme remayning, namely, F E, is equal to the square of the lyne A C. And forasmuch as the squares of the lines A B and B C are medially, therefore also the parallelogramme D E is medially, and is applied to the rational lyne D I, making in breadth the lyne D G. Wherfore (by the 22. of the tenth) the lyne D G is rational and incommensurable in length to the lyne D I. Againe forasmuch as that which is contained vnder the lines A B and B C is medially, therefore also that which is contained vnder the lines A B and B C twise is medially, but that which is contained vnder the lines A B and B C twise is equal to the parallelogramme D H. Wherfore the parallelogramme D H is medially and is applied to the rational lyne D I making in breadth the lyne D F. Wherfore the lyne D F is rational and incommensurable in length to the lyne D I. And forasmuch as the lines A B and B C are commensurable in power onely, therefore the lyne A B is incommensurable in length to the lyne B C. Wherfore (by the assumpt going before the 22. of the tenth, and by the 10. of the tenth) the square of the lyne A B is incommensurable to that which is contained vnder the lines A B and B C. But vnto the square of the lyne A B are commensurable the squares of A B and B C (by the 15. of the tenth) And vnto that which is contained vnder the lines



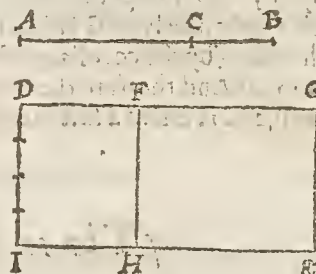
Construction.

Demonstration.

A B

The tenth Booke

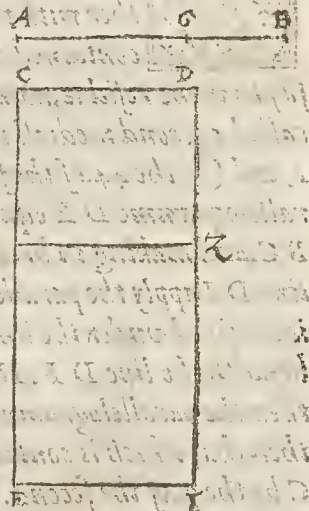
AB and BC is commensurable to that which is contained under the lines AB and BC twice. Wherefore the squares of the lines AB and BC are incommensurable to that which is contained under the lines AB and BC twice. But unto the squares of the lines AB and BC is equal the parallelogramme DE, and to that which is contained under the lines AB and BC twice, is equal the parallelogramme DH. Wherefore the parallelogramme DE is incommensurable to the parallelogramme DH. But as the parallelogramme DE is to the parallelogramme DH, so is the line GD to the line DF. Wherefore the line GD is incommensurable in length to the line DF. And either of the is rational. Wherefore the lines GD and DF are rational commensurable in power only. Wherefore the line FG is a residual line (by the 73. proposition of the tenth) And the line DE is a rational line. But a superficies comprehended under a rational line, and an irrational line is irrational (by the 21 of the tenth) and the line which containeth in power the same superficies is irrational (by the assumpt going before the same) Wherefore the parallelogramme FE is irrational. But the line AC containeth in power the parallelogramme FE. Wherefore the line AC is an irrational line and is called a second medially residual line. And this second medially residual line is that part of the greater part of a binomial line which remaineth after the taking away of the lesse part from the greater: which was required to be proued.



And another demonstration more briefe after Campanus.

An other demonstration after Campanus.

Suppose that *AB* be a medially line, from which take away the medially line *GB* commensurable unto the whole line *AB* in power only and containing with it a medially superficies, namely, that which is contained under the lines *AB* and *BG*. Then I say that the residue *AG* is an irrational line, and is called a second medially residual line. Take a rational line *DC*, upon which apply a parallelogramme equal to that which is composed of the squares of the lines *AB* and *BG*, which by the 45. of the first let be *DCFI*. Agayne let the parallelogramme *ZEFI* be equal to that which is contained under the lines *AB* and *BG* twice. Wherefore the superficies remaining *DF* is equal to the square of the line *AG* by the 7. of the second. (For that which is contained under the lines *AB* and *BG* twice together with the square of the line *AG* is equal to that which is composed of the squares of the lines *AB* and *BG*). And forasmuch as the squares of the lines *AB* and *BG* are medially, for that they are described of medially lines: the parallelogramme *DE* which is equal unto the, shall also be medially. And forasmuch as that which is contained under the lines *AB* and *BG* is by supposition medially, therefore the superficies *ZE* which is double unto it is also medially. But the medially superficies *DE* exceedeth not the medially superficies *ZE* by a rational superficies (by the 26. of this booke). Wherefore the excess, namely, the superficies *DF* is irrational, unto which the square of the line *AG* is equal: wherefore the square of the line *AG* is irrational, and therefore the line *AG* which containeth it in power is irrational by the assumpt put before the 21. of this booke, and is called a second medially residual line.



Definition of the tenth irrational line.

This proposition setteth forth the nature of the tenth kinde of irrational lines, which is called a second residual medially line, which is thus defined.

A second residual medially line is an irrational line which remaineth when from a medially line is taken away a medially line commensurable to the whole in power only, and the part taken away & the whole line containe a medially superficies.

The 58. Theoreme.

The 76. Proposition.

If from a right line be taken away a right line incommensurable in power to the whole, and if that which is made of the squares of the whole line and of the line taken away added together be rational, and the parallelogramme

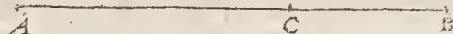
contained vnder the same lines medially: the line remayning is irrationall, and is called a lesse line.



Suppose that AB be a right line, and from the right line AB take away a right line BC incommensurable in power to the whole line, namely, to AB , and let that which is composed of the squares of the lines AB and BC be rationall, and let the parallelogramme containd vnder the same lines AB and BC be medially.

Then I say, that the line remayning, namely, the line AC is irrationall, & is called a lesse line. For forasmuch as that which is composed of the squares of the lines AB and BC is rationall, and that

which is containd vnder the lines AB and BC twice is medially, therefore that which is composed of the squares of the lines AB and BC is incommensurable to that which is contained vnder the lines AB and BC twice. Wherefore the squares of the lines AB and BC , are incommensurable to the square of AC , as it was said in the 73. Proposition. But that which is made of the squares of the lines AB and BC is rationall. Wherefore the square of the line AC is irrationall: wherefore also the line AC is irrationall: and is called a lesse line. And is therefore so called, for that it is that portio of the greater part of a greater line, which remaineth after the taking away of the lesse part from the greater: which was required to be proued.



Demonstration.

In this Proposition is containd the definition of the eleventh kinde of irrational lines, which is called a lesse line, whose definition is thus.

A lesse line is an irrational line which remaineth, when from a right line is taken away a right line incommensurable in power to the whole, and the square of the whole line, & the square of the part taken away added together, make a rational superficies, and the parallelogramme containd of them is medially.

Definition of the eleventh irrational line.

This Proposition may after Campanus way be demonstrated, if you remember well the order & positions which he in the three former Propositions vsed.

¶ The 59. Theoreme.

The 77. Proposition.

If from a right line be taken away a right line incommensurable in power to the whole line, and if that which is made of the squares of the whole line and of the line taken away added together be medially, and the parallelogramme contained vnder the same lines rationall: the line remaining is irrationall, and is called a line making with a rational superficies the whole superficies medially.

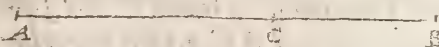


Suppose that AB be a right line, and from the right line AB take away a right line BC incommensurable in power to the whole line AB , and let that which is made of the squares of the lines AB and BC added together, be medially, and the parallelogramme contained vnder the same lines rationall. Then I say, that the

line remayning, namely, the line AC , is irrationall, and is called a line making with a rational

superficies the whole superficies medially. For forasmuch as that which is made of the squares of

the lines AB and BC added together is medially, and that which is contained vnder the lines

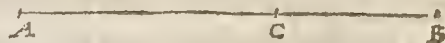


Demonstration.

AB and

The tenth Booke

AB and BC twise is rationall, therefore that which is made of the squares of the lines AB and BC added together, is incommensurable to that which is contained under the lines AB and BC twise. Wherefore (by the 16. of the tenth) the residue, namely, the square of the line AC is incommensurable to that which is contained under the lines AB and BC twise. But that which is contained under the lines AB and BC twise is rationall. Wherefore the square of the line AC is irrational. Wherefore also the line AC is irrational: and is called a line making with a rationall superficies the whole superficies medially: and is therefore so called for that that which is made of the squares of the lines AB and BC added together is medially, & is a certaine whole superficies, part whereof is that which is contained under the lines AB & BC, which is a rationall superficies. For the squares of the lines AB and BC, are equall to that which is contained under the lines AB and BC twise, and to the square of the line AC (by the 7. of the second). Or it is therefore so called for that the square thereof added to a rationall superficies, maketh the whole superficies medially, as shall be proued by the 109. Proposition of this booke: which was required to be proued.



In this Proposition is declared the nature of the twelue kind of irrational lines, which is called a line making with a rationall superficies the whole superficies medially, whose definition is thus.

Definition of the twelue kind of irrational line.

A line making with a rationall superficies the whole superficies medially, is an irrational line which remaineth, when from a right line is taken away a right line incommensurable in power to the whole line, and the square of the whole line & the square of the part taken away added together make a medially superficies, and the parallelogramme contained of them is rationall.

This Proposition also may after Campanes way be demonstrated, observing the former caution.

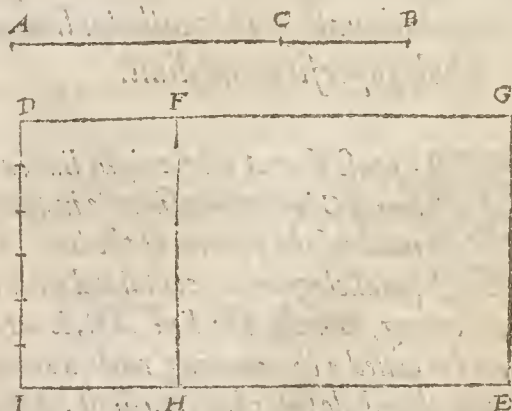
¶ The 60. Theoreme.

The 78. Proposition.

If from a right line be taken away a right line incommensurable in power to the whole line, and if that, which is made of the squares of the whole line and of the line taken away added together be medially, and the parallelogramme contained under the same lines be also medially, and incommensurable to that which is made of the squares of the sayd lines added together: the line remayning is irrational, and is called a line making with a medially superficies the whole superficies medially.



Suppose that AB be a right line, and from AB take away a right line BC incommensurable in power to the whole line AB. And let that which is made of the squares of the lines AB and BC added together be medially, and let the parallelogramme contained under the lines AB and BC be also medially, and let that which is made of the squares of the lines AB and BC added together be incommensurable to that which is contained under the lines AB and BC. The I say that the line remayning, namely, the line AC is



irrational

irrationall and is called a line making with a mediall superficies the whole superficies mediall. Take a rationall line DI . And (by the 44. of the first) vnto the line DI apply the parallelogramme DE equall to that which is made of the squares of the lines AB and BC added together, and making in breadth the line DG . And vnto the same line DI apply the parallelogramme DH equall to that which is containned vnder the lines AB and BC twice and making in breadth the line DF . Wherefore the parallelogramme remainning, namely the parallelogramme FE , is equall to the square of the line AC . Wherefore the line AC containeth in power the parallelogramme FE . And forasmuch as that which is made of the squares of the lines AB and BC added together is mediall, and is equall to the parallelogramme DE , therefore also the parallelogramme DE is mediall. And the parallelogramme DE is applied to the rationall line DI making in breadth the line DG . Wherefore (by the 22. of the tenth) the line DG is rationall and incommensurable in length to the line DI . Agayne forasmuch as that which is containned vnder the lines AB and BC twice is mediall and is equall to the parallelogramme DH , therefore the parallelogramme DH is mediall. And the parallelogramme DH is applied vnto the rationall line DI making in breadth the line DF , wherefore the line DF is rationall and incommensurable in length to the line DI . And forasmuch as that which is made of the squares of the lines AB and BC added together is incommensurable to that which containned vnder the lines AB & BC twice, therefore the parallelogramme DE is incommensurable to the parallelogramme DH . But as the parallelogramme DE is to the parallelogramme DH , so (by the first of the sixth) is the line DG to the line DF : wherefore the line DG is incommensurable in length to the line DF . And they are both rationall lines. Wherefore the lines DG and DF are rationall commensurable in power onely, wherefore the line FG is a residuall line by the 73. of this booke. But the line FH is rationall for that it is equall vnto the line DI . But a rectangle parallelogramme containned vnder a rationall line and an irrationall line is irrationall, and the line also that containeth in power the same parallelogramme is irrationall (by the 21. of the tenth). But the line CA containeth in power the parallelogramme FE . Wherefore the line AC is irrationall and is called a line making with a mediall superficies the whole superficies mediall. And is therefore so called, for that that which is made of the squares of the lines AB and BC added together is mediall, & is a certayne whole superficies, part whereof is that which is containned vnder the lines AB and BC , which is also mediall: you shall also by the 110. proposition of this booke vnderstand an other cause why it is so called.

Construction.

Demonstration.

This proposition may thus more briefly be demonstrated: forasmuch as that which is composed of the squares of the lines AB and BC is mediall, and that also which is containned vnder them is mediall, therefore the parallelogrammes DE and DH which are equall vnto them are mediall: but a mediall superficies exceedeth not a mediall superficies by a rationall superficies. Wherefore the superficies FE which is the excessse of the mediall superficies DE about the mediall superficies DH is irrationall. And therefore the line AC which containeth it in power is irrationall. &c.

In this proposition is shewed the condition and nature of the thirteenth and last kinde of irrationall lines, which is called a line making with a mediall superficies the whole superficies mediall, whose definition is thus.

A line making with a mediall superficies the whole superficies mediall is an irrationall line which remainneth, when from a right line is taken away a right line incommensurable in power to the whole line, and the squares of the whole line and of the line taken away added together make a mediall superficies, and the parallelogramme containned of the is also a mediall superficies, moreover the squares of them are incommensurable to the parallelogramme containned of them.

Definition of the thirteenth and last irrationall line.

An assumpt of Campane.

NN.i.

If

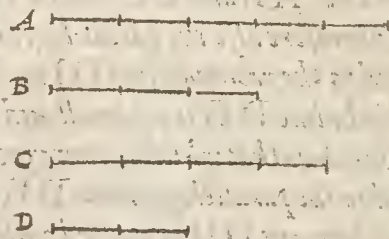
The tenth Booke

If there be fower quantities, & if the difference of the first to the second, be as the difference of the third to the fourth, then alternately, as the difference of the first is to the third, so is the difference of the second to the fourth.

*An Assumpt
of Campane.*

*I. Dee.
Though Cam-
panes lemma
be true, yet the
maner of de-
monstrating
it, (narrowly
considered) is
not artificiall.*

This is to be vnderstand of quantities in like sort referred the one to the other, that is if the first be greater then the second, the third ought to be greater then the fourth and if the first be lesse then the second, the third ought to be lesse then the fourth: and is also to be vnderstand in arithmetical proportionality. As for example let the difference of A be vnto B as the difference of C is to D. Then I say that as the difference of A is to C, so is the difference of B to D. For (by this common sentence, the difference of the extreames is composed of the differences of the extremes to the meanes), the difference of A to C is composed of the difference of A to B and of the difference of B to C. And (by the same common sentence) the difference of B to D is composed of the difference of B to C, and of the difference of C to D. And forasmuch as (by supposition) the difference of A to B is as the difference of C to D, and the difference of B to C is common to them both. Wherefore it followeth, that as the difference of A is to C, so is the difference of B to D: which was required to be proued.



¶ The 61. Theoreme.

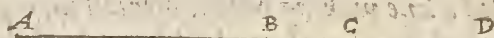
The 79. Proposition.

Vnto a residuall line can be ioyned one onely right lyne rational, and commensurable in power onely to the whole lyne.

*Second Sena-
ry.*

*Demonstrati-
on leading to
an impossibi-
litie.*

If Et AB be a residuall line, and vnto it let the line BC be supposed to be ioyned, so that let the lines AC and BC be rationall commensurable in power onely. Then I say that vnto the line AB cannot be ioyned any other rationall line commensurable in power onely to the whole line. For if it be possible, let BD be such a line added vnto it. Wherefore the lines AD and DB are rationall commensurable in power onely. And forasmuch as how much the squares of the lines AD and DB do exceede that which is contained vnder the lines AD and DB twise, so much also do the squares of the lines AC and CB exceede that which is contained vnder the lines AC and CB twise, for the excesse of eche is one and the same, namely, the square of the line AB (by the 7. of the second.) Wherefore alternately (by the former assumpt of Campanus) how much the squares of the lines AD and DB do exceede the squares of the lines AC & CB, so much also exceedeth that which is contained vnder the lines AD and DB twise, that which is contained vnder the lines AC and CB twise. But that which is made of the squares of the lines AD and DB added together, exceedeth that which is made of the squares of the lines AC and CB added together by a rationall superficies: (for they are either of them rational). Wherefore that which is contained vnder the lines AD and DB twise, exceedeth that which is contained vnder the lines AC and CB twise by a rationall superficies. But that which is contained vnder the lines AD and DB twise, is medially, for it is commensurable to that which is contained vnder the lines AD and DB once, which superficies is medially (by the 21. of the tenth) and by the same reason also that which is contained vnder the lines AC and CB twise is medially. Wherefore a medially superficies differeth from a medially superficies by a rationall superficies, which (by the 26. of the tenth) is impossible. Wherefore vnto the lyne AB cannot be ioyned any other rationall line besides BC commensurable in power onely, to the whole line. Wherefore vnto a residuall line can be ioyned one onely right line rational and commensurable in power onely to the whole lyne: which was required to be demonstrated.



¶ The

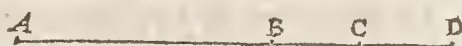
¶ The 62. Theoreme.

The 80. Proposition.

Vnto a first medially residuall line can be ioyned one onely medially right lyne, commensurable in power onely to the whole lyne, and comprehending wyth the whole lyne a rationally superficies.

Suppose that AB be a first medially residuall line, & vnto AB ioyned the lyne BC , so that let the lynes AC and BC be medially commensurable in power onely, & let that which is contained vnder the lynes AC and BC be rationally. Then I say that vnto the lyne AB cannot be ioyned any other medially line commensurable in power onely to the whole lyne, and comprehending together with the whole lyne a rationally superficies. For if it be possible let the lyne BD be such a lyne. Wherefore the lynes AD and DB are medially commensurable in power onely, and that which is contained vnder the lynes AD and DB is rational. And forasmuch as how much the squares of the lynes AD and DB exceede that which is contained vnder the lynes AD and DB twise, so much also exceede the squares of the lynes AC & BC , that which is contained vnder the lynes AC and CB twise (for the excesse of eche is one and the same, namely, the square of the lyne AB). Wherefore alternately (as it was sayd in the former proposition) how much the squares of the lynes AD and DB exceede the squares of the lynes AC and CB , so much also that which is contained vnder the lynes AD and DB twise, exceedeth that which is contained vnder the lynes AC and CB twise. But that which is contained vnder the lynes AD and DB twise, exceedeth that which is contained vnder the lynes AC and CB twise by a rationally superficies, for they are either of them a rationally superficies. Wherefore that which is made of the squares of the lynes AD & DB exceedeth that which is made of the squares of the lynes AC & CB by a rationally superficies, which (by the 26. of the tenth) is impossible. For they are either of them medially (for those foure lynes were put to be medially.) Wherefore vnto a first medially residuall line can be ioyned onely one right medially line commensurable in power onely to the whole line, and comprehending with the whole line a rationally superficies: which was required to be proued.

Demonstration leading to an absurditie.

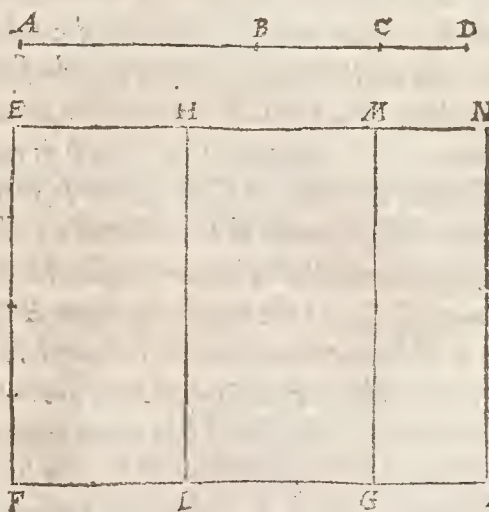


¶ The 63. Theoreme.

The 81. Proposition.

Vnto a second medially residuall line can be ioyned onely one medially right lyne, commensurable in power onely to the whole line, and comprehending with the whole line a medially superficies.

Suppose that AB be a second medially residuall line, & vnto the lyne AB ioyned the lyne BC , so that let the lynes AC and CB be medially commensurable in power onely, and let that which is comprehended vnder the lynes AC and CB be medially. Then I say, that vnto the lyne AB can not be ioyned any other medially right lyne commensurable in power onely to the whole line, and comprehending together with the whole line a medially superficies. For if it be possible, let the lyne BD be such a lyne. Wherefore the lynes AD & DB are medially commensurable in power onely, and that



NN. ij.

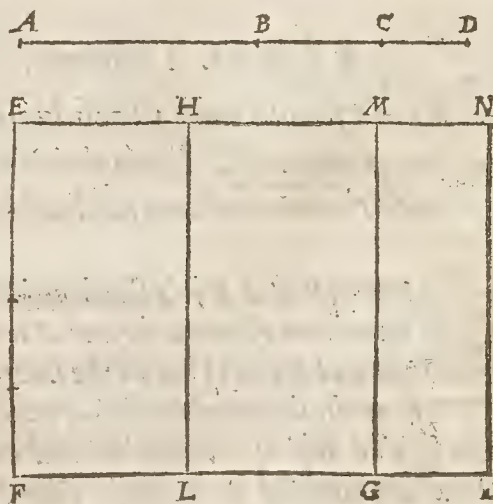
which

The tenth Booke

Construction.

Demonstration leading to an absurditie.

which is contained under the lines AD and DB is also medially. Take a rational line EF . And (by the 44. of the first) unto the line EF apply the parallelogramme EG equall to the squares of the lines AC and CB , and making in breadth the line EM : and from that parallelogramme EG take away the parallelogramme HG equall to that which is contained under AC and CB twice, and making in breadth the line HM . Wherefore the parallelogramme remayning, namely, EL , is (by the 7. of the second) equall to the square of the line AB . Wherefore the line AB containeth in power the parallelogramme EL . Againe, unto the line EF apply (by the 44. of the first) the parallelogramme EI equall to the squares of the lines AD and DB , and making in breadth the line EN . But the squares of the lines AD and DB are equall to that which is contained under the lines AD and DB twice, and to the square of the line AB . Wherefore the parallelogramme EI is equall to that which is contained under the lines AD & DB twice, and to the square of the line AB . But the parallelogramme EL is equall to the square of the line AB . Wherefore the parallelogramme remayning, namely, HI , is equall to that which is contained under the lines AD and DB twice. And forasmuch as the lines AC and CB are medially, therefore the squares also of the lines AC and CB are medially: and they are equall to the parallelogramme EG : wherefore the parallelogramme EG is (by that which was spoken in the 75. Proposition) medially: and it is applied unto the rational line EF , making in breadth the line EM . Wherefore (by the 22. of the tenth) the line EM is rational, and incommensurable in length to the line EF . Againe, forasmuch as that which is contained under the lines AC and CB is medially, therefore (by the Corollary of the 23. of the tenth) that which is contained under the lines AC and CB twice is also medially: and it is equall to the parallelogramme HG : wherefore also the parallelogramme HG is medially, and is applied to the rational line EF , making in breadth the line HM . Wherefore (by the 22. of the tenth) the line HM is rational, and incommensurable in length to the line EF . And forasmuch as the lines AC and CB are commensurable in power onely, therefore the line AC is incommensurable in length to the line CB . But as the line AC is to the line CB , so (by the Assumpt going before the 22. of the tenth) is the square of the line AC to that which is contained under the lines AC & CB . Wherefore (by the 10. of the tenth) the square of the line AC is incommensurable to that which is contained under the lines AC and CB . But unto the square of the line AC are commensurable the squares of AC & CB , and unto that which is contained under the lines AC and CB , is commensurable that which is contained under the lines AC and CB twice. Wherefore the squares of the lines AC & CB are incommensurable to that which is contained under the lines AC and CB twice. But unto the squares of the lines AC and CB is equall the parallelogramme EG , and unto that which is contained under the lines AC & CB twice, is equall the parallelogramme HG . Wherefore the parallelogramme EG is incommensurable to the parallelogramme HG . But as the parallelogramme EG is to the parallelogramme HG so is the line EM to the line HM . Wherefore the line EM is incommensurable in length to the line HM . And they are both rational lines. Wherefore the lines EM and HM are rational commensurable in power onely. Wherefore the line EH is a residuall line, and unto it is ioyned a rational line HM commensurable in power onely to the whole line EM . In like sort also may it be proued, that unto the line EH is ioyned the line HN , being also rational, and commensurable in power onely to the whole line



line EN . Wherefore vnto a residuall line is ioyned more then one onely line commensurable in power onely to the whole line: which (by the 79. of the tenth) is impossible. Wherefore vnto a second medially residuall line can be ioyned onely one medially right line commensurable in power onely to the whole line, and comprehending with the whole line a medially superficies: which was required to be demonstrated.

¶ The 64. Theoreme.

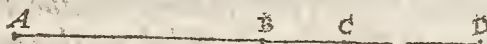
The 82. Proposition.

Vnto a lesse line can be ioyned onely one right line incommensurable in power to the whole lyne, and making together with the whole lyne that which is made of their squares added together rationally, and that which is contained vnder them medially.



Suppose that AB be a lesse line, and to AB ioyne the line BC , so that let BC be such a line as is required in the Theoreme. Wherefore the lines AC and CB are incommensurable in power, hauing that which is made of the squares of them added together rationally, and that which is contained vnder them medially. Then

I say that vnto AB cannot be ioyned any other such right line. For if it be possible, let the lyne BD be such a line. Wherefore the lines AD & DB are incommensurable in power, hauing that which is made of the squares of them added together, rationally, and that which is contained vnder them medially. And for that how much the squares of the lines AD and DB exceede the squares of the lines AC and CB , so much that which is contained vnder the lines AD and DB twise, exceedeth that which is contained vnder the lines AC and CB twise (by those things which were spoken in the 79. proposition) But that which is made of the squares of the lines AD and DB added together exceedeth that which is made of the squares of the lines AC and CB added together by a rationally superficies, for they are either of them rationally by supposition. Wherefore that which is contained vnder the lines AD and DB twise, exceedeth that which is contained vnder the lines AC and CB twise by a rationally superficies: which (by the 26. of the tenth) is impossible, for either of them is medially by supposition. Wherefore vnto a lesse line can be ioyned onely one right line incommensurable in power to the whole line, and making together with the whole line that which is made of their squares added together rationally, and that which is contained vnder them medially: which was required to be demonstrated.



Demonstration leading to an absurditie.

¶ The 65. Theoreme.

The 83. Proposition.

Vnto a line making with a rationally superficies the whole superficies medially, can be ioyned onely one right lyne incommensurable in power to the whole lyne, and making together with the whole line that which is made of their squares added together medially, and that which is contained vnder them rationally.



Suppose that AB be a line making with a rationally superficies the whole superficies medially, and vnto it let the line BC be ioyned, so that let BC be such a line as is required in the Theoreme. Wherefore the lines AC and CB are incommensurable in

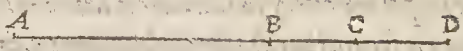
NN. iij.

power

The tenth Booke

Demonstratio
leading to an
impossibilitie.

power, hauing that which is made of the squares of the lines AC and CB added together medially, and that which is contained vnder the lynes AC and CB rationally. Then I say that vnto the lyne AB cannot be ioyned any other such line. For if it be possible, let the line BD be such a line. Wherefore the lines AD and DB are incommensurable in power, hauing that which is made of the squares of the lines AD and DB added together medially, and that which is contained vnder the lines AD and DB rationally. Now for that how much the squares of the lines AD and DB exceede the squares of the lines AC and CB , so much that which is contained vnder the lines AD and DB twise exceedeth that which is contained vnder the lynes AC and CB twise, by that which was spoken in the 79. proposition. But that which is contained vnder the lines AD and DB twise, exceedeth that which is contained vnder the lines AC and CB twise by a rationally superficies, for they are either of them rationally by suppositio. Wherefore that which is made of the squares of the lines AD and DB added together, exceedeth that which is made of the squares of the lines AC and CB added together by a rationally superficies, which by the 26. of the tenth, is impossible, for they are either of them medially by supposition. Wherefore vnto the line AB cannot be ioyned any other lyne besides BC , making that which is required in the proposition. Wherefore vnto a line making with a rationally superficies the whole superficies medially can be ioyned onely one right line incommensurable in power to the whole line, and making together with the whole line that which is made of their squares added together medially, and that which is contained vnder them rationally: which was required to be proued.



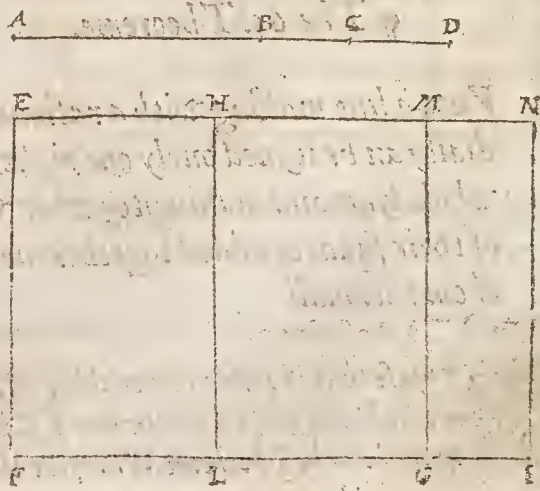
The 66. Theoreme.

The 84. Proposition.

Vnto a line making with a medial superficies the whole superficies medially, can be ioyned onely one right line incommensurable in power to the whole line, and making together with the whole line that which is made of their squares added together medially, and that which is contained vnder them medially, and moreouer making that which is made of the squares of them added together incommensurable to that which is contained vnder them.



Suppose that AB be a line making with a medial superficies the whole superficies medially, and vnto it let the line BC be ioyned, so that let BC be such a line as is required in the Theoreme. Wherefore the lines AC and CB are incommensurable in power, hauing that which is made of the squares of the lines AC and CB added together medially, & that which is contained vnder the lines AC and CB medially, and moreouer that which is made of the squares of the lines AC and CB is incommensurable to that which is contained vnder the lines AC and CB . Then I say, that vnto the line AB can be ioyned no other such line. For if it be possible, let BD be such a line. Wherefore the lines AD and DB are incommensurable in power, hauing that which is made of the squares of the lines AD and DB



DB added together medially, and that which is contained under the lines AD and DB medially, and moreover that which is made of the squares of the lines AD and DB added together, is incommensurable to that which is contained under the lines AD and DB . Take a rationally line EF . And (by the 44. of the first) unto the line EF apply the parallelogramme EG equall to the squares of the lines AC and CB , and making in breadth the line EM : and from the parallelogramme EG take away the parallelogramme HG equall to that which is contained under the lines AC & CB twice, and making in breadth the line HM . Wherefore the residue, namely, the square of the line AB is equall to the parallelogramme EL (by the 7. of the second). Wherefore the line AB containeth in power the parallelogramme EL . Againe (by the 44. of the first) unto the line EF apply the parallelogramme EI equall to the squares of the lines AD and DB , and making in breadth the line EN . But the square of the line AB is equall to the parallelogramme EL . Wherefore the residue, namely, the parallelogramme HI is equall to that which is contained under the lines AD and DB twice. And forasmuch as that which is made of the squares of the lines AC and CB is medially, and is equall to the parallelogramme EG , therefore also the parallelogramme EG is medially. And it is applied unto the rationally line EF , making in breadth the line EM . Wherefore (by the 22. of the tenth) the line EM is rationally and incommensurable in length to the line EF . Againe, forasmuch as that which is contained under the lines AC and CB twice is medially, and is equall to the parallelogramme HG . Wherefore the parallelogramme HG is medially, which parallelogramme HG is applied to the rationally line EF , making in breadth the line HM . Wherefore the line HM is rationally and incommensurable in length to the line EF . And forasmuch as the squares of the lines AC and CB are incommensurable to that which is contained under the lines AC and CB twice, therefore the parallelogramme EG is incommensurable to the parallelogramme HG . Wherefore the line EM is incommensurable in length to the line MH , and they are both rationally. Wherefore the lines EM and MH are rationally commensurable in power onely. Wherefore EH is a residuall line. And the line ioyned unto it is HM . And in like sort may we proue, that the line EH is a residuall line, and that the line HN is ioyned unto it. Wherefore unto a residuall line is ioyned two sundry lines, being eche commensurable in power onely to the whole line: which (by the 79. of the tenth) is impossible. Wherefore unto the line AB can not be ioyned any other right line besides the line BC , which shall be incommensurable in power to the whole line, & haue together with the whole line that which is made of their squares added together medially, and that which is contained under them medially, and moreover incommensurable to that which is made of their squares added together. Wherefore unto a line making with a medially superficies the whole superficies medially, can be ioyned onely one right line incommensurable in power to the whole line, and making together with the whole line that which is made of their squares added together medially, and that which is contained under them medially, and moreover making that which is made of the squares of them added together incommensurable to that which is contained under the: which was required to be proued.

Construction.

Demonstration leading to an absurditie.

¶ Third Definitions.



S of binomiall lynes, there are 6. diuers kindes, so also of residuall lynes which are correspondent vnto them and depend of them (for a residuall line is nothing els (as was before said) but that which remaineth whē the lesse part of a binomiall line is taken from the greater part or name thereof,) there are likewise sixe seuerall kindes. All which are knowne and con-

The tenth Booke

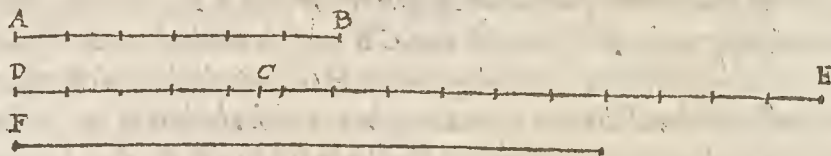
Six kindes of residuall lines.

sidered in comparison to a rationall line set forth & appointed, and these residual line haue the selfe same order of production that the binomials had. For as the three first kindes of binomiall lines, namely, the first, second, and third, were produced when the square of the greater part of the binomiall excedeth the square of the lesse part thereof by the square of a line commensurable vnto it in length: so in likewise, the first three kindes of residuall lines, namely, the first, second, and third, are produced, when the square of the whole, namely, of that which is made of the residuall line, and the line ioyned vnto it added together, excedeth the square of the line ioyned to the residuall, by the square of a lyne which is commensurable vnto it in length. And as the three last kindes of binomials, namely, the fourth, fifth, and sixth were produced when the square of the greater part excedeth the square of the lesse, by the square of a line incommensurable in length vnto it, euen so the three last kyndes of residuall lynes are produced when the square of the whole excedeth the square of the lyne adioined, by the square of a line incommensurable vnto it in length. As ye may perceiue by their definitions following.

First diffini- tion.

A first residuall line is, when the square of the whole excedeth the square of the lyne adioyned, by the square of a lyne commensurable vnto it in length, and also the whole is commensurable in length to the rationall line first set.

As let A B be a rationall line, whose partes are certaine, distinct, & to be expressed by number. And let the residual lyne be C D, and let the

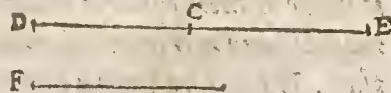


line ioyned vnto it be E C, and let the whole being composed of the residuall C D, and the line adioyned E C, be the line E D: let moreouer the square of the whole line E D, excede the square of the line adioyned E C by the square of the line F, which line F let be commensurable in length to the whole lyne E D, and let the whole line E D be also commensurable in length to the rationall line A B: then is the residuall lyne C D by this definition a first residuall line.

Second diffi- nition.

A second residuall line is, when the square of the whole excedeth the square of the line adioyned, by the square of a line commensurable vnto it in length, and also the line adioyned is commensurable in length to the rationall lyne.

As suppose the line C D to be a residuall, and let the line adioyned vnto it be E C, and the whole made of them both, let be the line E D: & let the square of E D the whole line excede the square of the lyne adioyned E C, by the square of the lyne F, and let the lyne F be commensurable in length to the whole lyne E D, moreouer let the line adioyned E C be commensurable in length to the rationall line A B: then by this definition, the residuall line C D is a second residuall line.



Third diffini- tion.

A third residuall line is, when the square of the whole excedeth the square of the lyne adioyned, by the square of a line commensurable vnto it in length and neither the whole line, nor the line adioyned is comensurable in length to the rationall lyne.

As (the former supposition standing) suppose that the square of the whole lyne E D excede the square of the lyne adioyned E C by the square of the lyne F, and let the lyne F be commensurable in length to the whole lyne E C, and let neither the whole lyne E D, nor the line adioyned E C be commensurable in length to the rationall lyne A B, then by this definition the residuall lyne C D is a third residuall lyne.

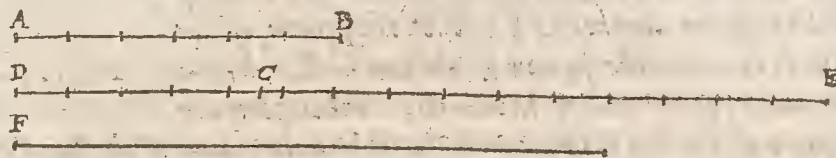


A fourth

A fourth residuall line is, when the square of the whole lyne exceedeth the square of the lyne adioyned, by the square of a lyne incommensurable vnto it in length, and the whole lyne is also commensurable in length to the rationall lyne.

Fourth diffinition.

As the residuall lyne beyng as before C D, & the lyne adioyned E C, and the whole E D, let the square of the whole lyne E D exceede the

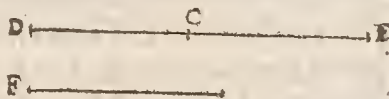


square of the line adioyned E C by the square of the lyne F, and let the lyne F be incommensurable in length to the whole line E D, and let E D the whole line be commensurable in length to the rationall lyne A B, then is the residuall line C D by this declaration a fourth residuall lyne.

A fifth residuall line is, when the square of the whole lyne exceedeth the square of the lyne adioyned, by the square of a lyne incommensurable vnto it in length, and the lyne adioyned is commensurable in length to the rationall lyne.

Fifth diffinition.

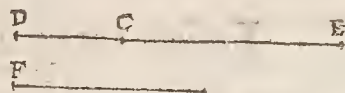
As the residuall line beyng c d, the lyne adioyned e c, and the whole lyne E D, let the square of the whole lyne E D exceede the square of the line adioyned e c by the square of the lyne F, and let the line F be incommensurable in length to the whole lyne E D, and let also e c the lyne adioyned be commensurable in length to the rationall line A B, then shall the residuall c d be by this definition a fifth residuall lyne.



A sixth residuall line is when the square of the whole line, exceedeth the square of the line adioyned, by the square of a line incommensurable vnto it in length, and neither the whole line nor the line adioyned is commensurable in length to the rationall line.

Sixth diffinition.

As suppose the residuall line to be c d, and the lyne adioyned to be e c, and the whole lyne composed of them let be e d, and let the square of the whole lyne e d exceede the square of the line adioyned by the square of the lyne F, which line F let be incommensurable in length to the whole lyne e d: moreouer let neither the whole lyne c d nor the line adioyned e c, be commensurable in length to the rationall line A B, then shall the residuall lyne c d be by this explication a sixth residuall lyne, and the last.



¶ The 19. Probleme.

The 85. Proposition.

To finde out a first residuall line.



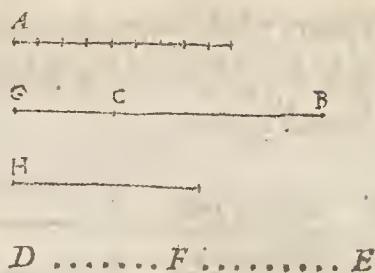
Take a rationall line and let the same be A, and vnto it let the line B G be commensurable in length. Wherefore the line B G also is rationall. And take two square numbers D E and E F which let be such, that the exceſſe of the greater, namely, of D E,

Third Senary. Construction.

The tenth Booke

Demonstration.

DE, above the lesse EF (which excesse let be the number DF) be no square number (by the corollary of the first assumpt of the 28. of the tenth). Wherefore the number ED hath not to the number DF that proportion that a square number hath to a square number (by the 24. of the eighth). And as the number ED is to the number DF, so let the square of the line BG, be to the square of the line GC (by the corollary of the sixt of the tenth). Wherefore the square of the line BG is commensurable to the square of the line GC. But the square of the line BG is rationall, wherefore also the square of the line GC is rationall. Wherefore the line GC is also rationall. And forasmuch as the number ED hath not to the number DF, that proportion that a square number hath to a square number, therefore neither also hath the square of the line BG to the square of the line GC that proportion that a square number hath to a square number. Wherefore (by the 9. of the tenth) the line BG is incommensurable in length to the line GC. And they are both rationall. Wherefore the lines BG and GC are rational commensurable in power onely. Wherefore the line BC is a residuall line. I say moreover that it is a first residuall line. For forasmuch as the square of the line BG is greater then the square of the line GC (that it is greater it is manifest, for by supposition the square of the line BG is to the square of the line GC, as the greater number, namely, ED is to the number DF) unto the square of the line BG let the squares of the lines GC and H be equall. And for that as the number DE is to the number DF, so is the square of the line BG to the square of the line GC, therefore by conuersion of proportion (by the corollary of the 9. of the fifth) as the number DE is to the number EF, so is the square of the line BG to the square of the line H. But the number DE hath to the number EF that proportion that a square number hath to a square number, for either of them is a square number, wherefore also the square of the line BG hath to the square of the line H that proportion that a square number hath to a square number. Wherefore the line GB is commensurable in length to the line H. Wherefore the line GB is in power more then the line GC by the square of a line commensurable in length to the line GB: and the whole line, namely, GB is commensurable in length to the rationall line A. Wherefore the line BC is a first residuall line. Wherefore there is founde out a first residuall line: which was required to be done.



¶ The 20. Probleme.

The 86. Proposition.

To finde out a second residuall line.

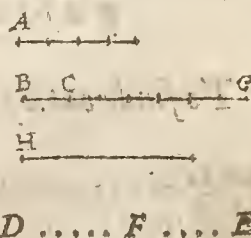
Construction.



Ake a rationall line, and let the same be A, and unto it let the line GC be commensurable in length. And take two square numbers DE and EF, and let them be such that the excesse of the greater, namely, DF, be no square number. And as the number DF is to the number DE, so let the square of the line GC be to the

Demonstration.

square of the line GB. Wherefore both the squares are commensurable. And forasmuch as the square of the line GC is rationall, therefore the square of the line BG is also rationall: Wherefore also the line BG is rationall. And forasmuch as the squares of the lines BG & GC haue not that proportion the one to the other that a square number hath to a square number, therefore the lines BG and GC are incommensurable in length, and they are both rationall. Wherefore the lines BG and GC are rational commensura-



ble

ble in power onely. Wherefore the line BC is a residuall line. I say moreover, that it is a second residuall line. For forasmuch as the square of the line BG is greater then the square of the line GC , unto the square of the line BG let the squares of the lines GC & H be equall. And for that as the number DE is to the number DF , so is the square of the line GB to the square of the line GC , therefore (by conuersion of proportion) as the number DE is to the number EF , so is the square of the line BG to the square of the line H . But either of these numbers DE and EF is a square number. Wherefore the line GB is commensurable in length to the line H . Wherefore the line BG is in power more then the line GC , by the square of a line commensurable in length to the line BG : and the line GC that is ioyned to the residuall line is commensurable in length to the rationall line A . Wherefore the line BC is a second residuall line. Wherefore there is found out a second residuall line: which was required to be done.

¶ The 21. Probleme.

The 87. Proposition.

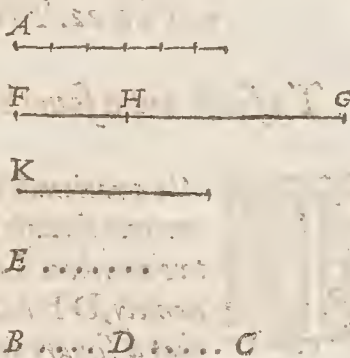
To finde out a third residuall line.



Take a rationall line, & let the same be A . and take three numbers E, BC , and CD , not hauing the one to the other that proportion that a square number hath to a square number: and let the number BC haue to the number BD that proportion that a square number hath to a square number. And let the number BC be greater then the number CD . And as the number E is to the number

Construction.

BC , so let the square of the line A be to the square of the line FG : and as the number BC is to the number CD , so let the square of the line FG be to the square of the line HG . Wherefore the square of the line A is commensurable to the square of the line FG . But the square of the line A is rationall: Wherefore also the square of the line FG is rationall: wherefore the line FG is also rationall. And forasmuch as the number E hath not to the number BC that proportion that a square number hath to a square number, therefore neither also hath the square of the line A to the square of the line FG that proportion that a square number hath to a square number. Wherefore the line A is incommensurable in length to the line FG . Again for that as the number BC is to the number CD , so is the square of the line FG to the square of the line HG , therefore the square of the line FG is commensurable to the square of the line HG . But the square of the line FG is rationall. Wherefore also the square of the line HG is rationall. Wherefore also the line HG is rationall. And for that the number BC hath not to the number CD , that proportion that a square number hath to a square number, therefore neither also hath the square of the line FG to the square of the line HG , that proportion that a square number hath to a square number. Wherefore the line FG is incommensurable in length to the line HG : and they are both rationall. Wherefore the lines FG & HG are rationall commensurable in power onely. Wherefore the line FG is a residuall line. I say moreover, that it is a third residuall line. For for that as the number E is to the number BC , so is the square of the line A to the square of the line FG : and as the number BC is to the number CD , so is the square of the line FG to the square of the line HG : therefore by equalitie of proportion, as the number E is to the number CD , so is the square of the line A to the square of the line HG : but the number E hath not to the number CD that proportion that a square number hath to a square number, therefore

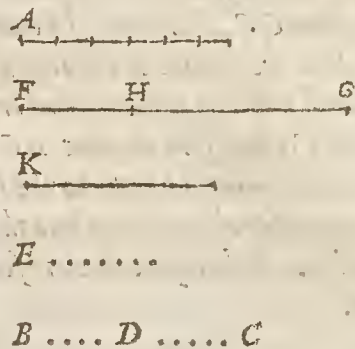


Demonstration.

fore

The tenth Booke

fore neither also hath the square of the line A to the square of the line HG that proportion that a square number hath to a square number, therefore the line A is incommensurable in length to the line HG . Wherefore neither of the lines FG and HG is commensurable in length to the rationall line A . And forasmuch as the square of the line FG is greater then the square of the line HG (that the line FG is greater then the line HG it is manifest, for by supposition the number BC is greater then the number CD), unto the square of the line FG let the squares of the lines HG & K be equall. And for that as the nūber BC is to the number CD , so is the square of the line FG to the square of the line HG , therefore (by conuersion of proportion) as the number BC is to the number BD , so is the square of the line FG to the square of the line K . But the nūber BC hath to the number BD that proportion that a square number hath to a square number. Wherefore the square of the line FG hath to the square of the line K that proportion that a square number hath to a square number. Wherefore the line FG is commensurable in length to the line K . Wherefore the line FG is in power more then the line HG , by the square of a line commensurable in length to the line FG , and neither of the lines FG and GH is commensurable in length to the rationall line A : when yet notwithstanding either of the lines FG and GH is rationall. Wherefore the line FH is a third residuall line. Wherefore there is found out a third residuall line: which was required to be done.



¶ The 22. Probleme.

The 88. Proposition.

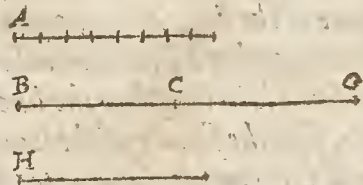
To finde out a fourth residuall line.

Construction.



Take a rationall line and let the same be A : and unto it let the line BG be commensurable in length. Wherefore the line BG is rationall. And take two numbers DF and FE , and let them be such that the whole number, namely, DE haue to neither of the numbers DF and FE that proportion that a square number hath to a square

number. And as the number DE is to the number EF , so let the square of the line BG be to the square of the line GC : wherefore the square of the line BG is commensurable to the square of the line GC , wherefore also the square of the line GC is rationall, and the line GC is also rationall. And for that the number DE hath not



Demonstration.

the number EF that proportion that a square number hath to a square number, therefore the line BG is incommensurable in length to the line GC . And they are both rationall: wherefore the line BC is a residuall line. I say moreover that it is a fourth residuall line. For forasmuch as the square of the line BG is greater then the square of the line GC , unto the square of the line BG let the squares of the lines CG and H be equall. And for that as the number DE is to the number EF , so is the square of the line BG to the square of the line GC , therefore by conuersion of proportion as the number DE is to the number DF , so is the square of the line BG to the square of the line H . But the numbers DE and DF haue not the one to the other that proportion that a square number hath to a square number. Wherefore the line BG is incommensurable in length to the line H . Wherefore the line BG is in power more then

$D \dots F \dots E$

the

the line GC by the square of a line incommensurable in length to the line BG : and the whole line BG is commensurable in length to the rationall line A . Wherefore the line BC is a fourth residuall line. Wherefore there is founde out a fourth residuall line: which was required to be doone.

¶ The 23. Probleme.

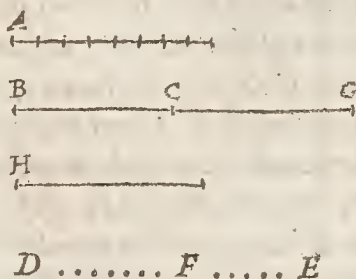
The 89. Proposition.

To finde out a fift residuall lyne.



Take a rational line and let the same be A , and vnto it let the line CG be commensurable in length. Wherefore the line CG is rational. And take two numbers DE and FE , which let be such, that the number DE haue to neither of these numbers DF nor FE that proportion that a square number hath to a square number. And as the number FE is to the number DE , so let the square of the line CG be to the square of the line BG . Wherefore the square of the line CG is commensurable to the square of the line BG . Wherefore the square of the line BG is rational, and the line BG is also rational. But the numbers DE and EF haue not that proportion the one to the other that a square number hath to a square number. Wherefore the lines BG and GC are rational commensurable in power onely. Wherefore the line BC is a residuall line. I say moreover that it is a fift residuall line. For forasmuch as the square of the line BG is greater then the square of the line GC , vnto the square of the line BG let the squares of the lines GC and H be equal. Now therefore for that as the number DE is to the number EF , so is the square of the line BG to the square of the line GC , therefore by conuerſion of proportion, as the number DE is to the number DF , so is the square of the line BG to the square of the line H . But the numbers DE & DF haue not that proportion the one to the other that a square number hath to a square number. Wherefore the line BG is incommensurable in length to the line H . Wherefore the line BG is in power more then the line CG by the square of a line incommensurable in length to the line BG , and the line CG which is ioyned to the residuall line is commensurable in length to the rationall line A . Wherefore the line BC is a fift residuall line. Wherefore there is found out a fift residuall line: which was required to be done.

Construction.



Demonstration.

¶ The 24. Probleme.

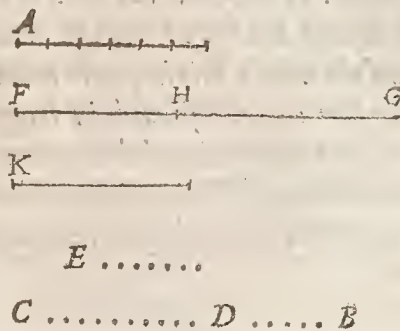
The 90. Proposition.

To finde out a sixth residuall line.



Take a rational line and let the same be A . And take three numbers E , BC , and CD , not haueing the one to the other that proportion that a square number hath to a square number. And let not the number BC haue to the number BD that proportion that a square number hath to a square number. And let the number BC be greater then the number CD , & as the number E is to the number BC , so let the square of the line A be to the square of the lyne FG . And as the number BC is to the number CD , so let the square

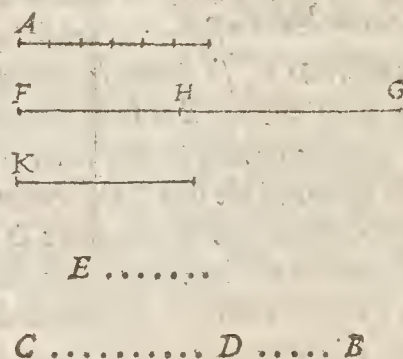
Construction.



The tenth Booke

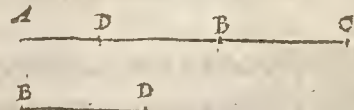
**Demonstra-
cion.**

of the line FG be to the square of the line GH . Now therefore for that as the number E is to the number BC , so is the square of the line A to the square of the line FG , therefore the square of the line A is commensurable to the square of the line FG . Wherefore the square of the line FG is rationall, and the line FG is also rationall. And for that the number E hath not to the number BC that proportion that a square number hath to a square number, therefore the line A is incommensurable in length to the line FG . Again for that as the number BC is to the number CD , so is the square of the line FG to the square of the line GH , therefore the square of the line FG is commensurable to the square of the line GH . But the square of the line FG is rationall, wherefore the square also of the line GH is rationall, wherefore the line GH is also rationall. And for that the number BC hath not to the number CD that proportion that a square number hath to a square number, therefore the line FG is incommensurable in length to the line GH , and they are both rationall. Wherefore the lines FG and GH are rationall commensurable in power onely. Wherefore the line FH is a residuall line. I say moreover that it is a sixt residuall line. For for that as the number E is to the number BC , so is the square of the line A to the square of the line FG , and as the number BC is to the number CD , so is the square of the line FG to the square of the line GH , therefore by equalitie of proportion as the number E is to the number CD , so is the square of the line A to the square of the line GH . But the number E hath not to the number CD that proportion that a square number hath to a square number. Wherefore the line A is incommensurable in length to the line GH , and neither of these lines FG nor GH is commensurable in length to the rationall line A . And forasmuch as the square of the line FG is greater then the square of the line GH , vnto the square of the line FG let the squares of the lines GH and K be equall. Now therefore for that as the number BC is to the number CD , so is the square of the line FG to the square of the line GH , therefore by conuersion of proportion as the number BC is to the number BD , so is the square of the line FG to the square of the line K . But the number BC hath not to the number BD that proportion that a square number hath to a square number, therefore the line FG is incommensurable in length to the line K . Wherefore the line FG is in power more then the line GH by the square of a line incommensurable in length to the line FG , and neither of the lines FG nor GH is commensurable in length to the rationall line A . Wherefore the line FH is a sixt residuall line. Wherefore there is found out a sixt residuall line: which was required to be done.



**An other
more redie
way to finde
out the fixe re-
siduall lines.**

There is also a certayne other redier way to finde out euery one of the forsayd fixe residuall lines which is after this maner. Suppose that it were required to finde out a first residuall line. Take a first binomial line AC , & let the greater name thereof be AB . And vnto the line BC let the line BD be equall. Wherefore the lines AB and BC , that is the lines AB and BD are rationall commensurable in power onely, and the line AB is in power more then the line BC , that is, then the line BD by the square of a line commensurable in length to the line AB . And the line AB is commensurable in length to the rationall line $geuen$. For the line AC is put to be a first binomial line. Wherefore the line AD is a first residuall line. And in like maner may ye finde out a second, a third, a fourth, a fift, and a sixt residuall line, if ye take for eche a binomial line of the same order.



The 67. Theoreme.

The 91. Proposition.

If a superficies be containd vnder a rationall line & a first residuall line: the line which containeth in power that superficies is a residuall line.

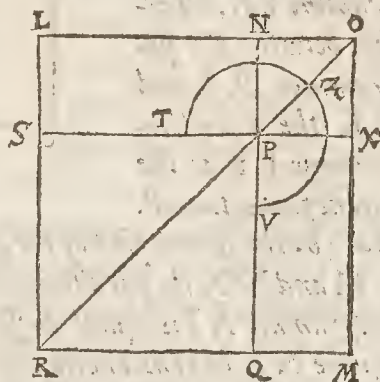
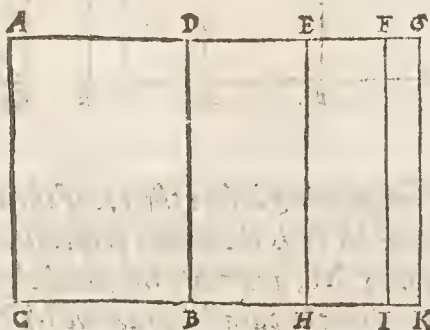


Suppose that there be a rectangle superficies AB containd vnder a rationall line AC and a first residuall line AD. Then I say that the line which containeth in power the superficies AB is a residuall line. For forasmuch as AD is a first residuall line, let the line ioyned vnto it be DG (by the line ioyned vnto it vnder-

Fourth Sense-ry.

The first part of the Construction.

stand such a line as was spoken of in the end of the 79. proposition). Wherefore the lines AG and GD are rationall commensurable in power only, & the whole line AG is commensurable in length to the rationall line AC, and the line AG



is in power more then the line GD by the square of a line commensurable in length vnto AG, by the definition of a first residuall line. Denide the line GD into two equall partes in the poynt E. And vpon the line AG apply a parallelogramme equall to the square of the line EG and wanting in figure by a square, and let the sayd parallelogramme be that which is containd vnder the lines AF and FG. Wherefore the line AF is commensurable in length to the line FG (by the 17. of the tenth). And by the poyntes E, F and G, draw vnto the line AC these parallel lines EH, FI, and GK. And make perfect the parallelograme AK. And forasmuch as the line AF is commensurable in length to the line FG, therefore also the whole line AG is commensurable in length to either of the lines AF and FG (by the 15. of the tenth). But the line AG is commensurable in length to the line AC. Wherefore either of the lines AF and FG is commensurable in length to the line AC. But the line AC is rationall, wherefore either of the lines AF and FG is also rationall. Wherefore (by the 19. of the tenth) either of the parallelogrammes AI and FK is also rationall. And forasmuch as the line DE is commensurable in length to the line EG, therefore also (by the 15. of the tenth) the line DG is commensurable in length to either of the lines DE and EG. But the line DG is rationall, wherefore either of the lines DE and EG is rationall, and the selfe same line DG is incommensurable in length to the line AC (by the definition of a first residuall line, or by the 13. of the tenth). For the line DG is incommensurable in length to the line AG, which line AG is commensurable in length to the line AC: wherefore either of the lines DE and EG is rationall and incommensurable in length to the line AC. Wherefore (by the 21. of the tenth) either of these parallelogrammes DH and EK is medially. Vnto the parallelogramme AI let the square LM be equall, and vnto the parallelogramme FK let the square NX be equall, being taken away from the square LM, and hauing the angle LOM common to them both. (And to doo this, there must be founde out the meane proportionall betwene the lines FI and FG. For the square of the meane proportionall is equall to the parallelogramme containd vnder the lines FI and FG. And from the line LO cut of a line equall to the meane proportionall so founde out, and describe the square thereof). Wherefore both the squares LM and NX are about one and the selfe same diameter (by the 26. of the

The first part of the demonstration.

Note. AI and FK concluded rational parallelogramme.

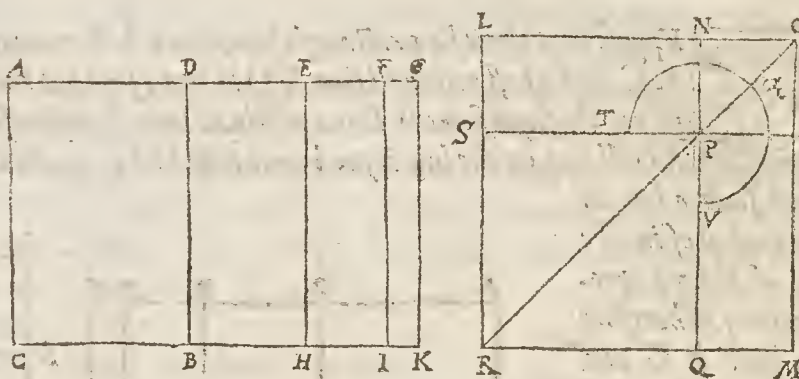
Note. DH and FK parallelogrammes medially.

Second part of the construction.

The tenth Booke

Second part of
the demonstra-
tion.

sixth) let their diameter be OR and describe the figure as it is here set forth. Now then forasmuch as the parallelogramme containd vnder the lines AF & FG is equal to the square of the line EG , therefore (by the 17. of the sixth) as the line AF is to the line EG , so is the line EG to the line FG . But as the line AF is to the line EG , so is the parallelogramme AI to the parallelogramme EK . And as the line EG is to the line FG , so is the parallelogramme EK to the parallelogramme FK . Wherefore betwene the parallelogrammes AI and FK the parallelogramme EK is the meane proportionall.



But (by the second part of the assumpt going before the 54. of the tenth) betwene the squares LM and NX the parallelogramme MN is the meane proportionall. And vnto the parallelogramme AI is equall the square LM , and vnto the parallelogramme FK is equall the square NX by construction. Wherefore the parallelogramme MN is equall to the parallelogramme EK (by the 2. assumpt going before the 54. of the tenth). But the parallelogramme EK is (by the first of the sixth) equall to the parallelogramme DH , and the parallelogramme MN is (by the 43. of the first) equall to the parallelogramme LX . Wherefore the whole parallelogramme DK is equall to the gnomon VTZ (which gnomon consisteth of those parallelogrammes by which ye see in the figure passeth a portion of a circle greater then a semicircle) and moreover to the square NX : and the parallelogramme AK is equall to the squares LM and NX by construction: and it is now proued, that the parallelogramme DK is equall to the gnomon VTZ , and moreover to the square NX . Wherefore the residue namely the parallelogramme AB is equall to the square SQ which is the square of the line LN . Wherefore the square of the line LN is equall to the parallelogramme AB . Wherefore the line LN containeth in power the parallelogramme AB . I say moreover that the line LN is a residuall line. For forasmuch as either of these parallelogrammes AI and FK is rationall, as it is before sayd, therefore the squares LM and NX which are equall vnto them, that is, the squares of the lines LO and ON are rationall. Wherefore the lines LO and ON are also rationall. Agayne forasmuch as the parallelogramme DH that is LX is medially, therefore the parallelogramme LX is incommensurable to the square NX . Wherefore (by the 1. of the sixth, and 10. of the tenth) the line LO is incommensurable in length to the line ON : and they are both rationall. Wherefore they are lines rationall commensurable in power onely. Wherefore LN is a residuall line by the definition, and it containeth in power the parallelogramme AB . If therefore a superficies be containd vnder a rationall line and a first residuall line, the line which containeth in power that superficies, is a residuall line: which was required to be demonstrated.

LN , is the
onely line that
we sought to
consider.

¶ The 68. Theoreme.

The 92. Proposition.

If a superficies be contained vnder a rationall line and a second residuall line: the line which containeth in power that superficies, is a first medially residuall line.

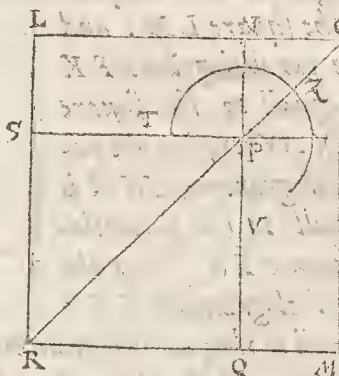
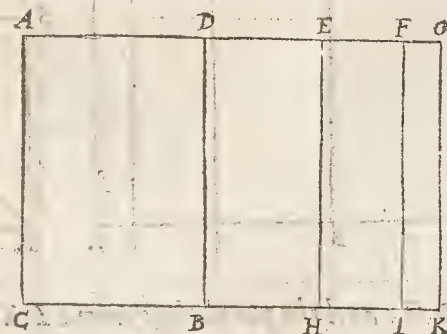
Suppose



Suppose that AB be a superficies contained under a rationall line AC , and a second residuall line AD . Then I say, that the line that containeth in power the superficies AB is a first medially residuall line. For let the line ioyned to the line AD be DG . Wherefore the lines AG and GD are rationall commensurable

First part of the construction.

in power onely, and the line that is ioyned to the residuall line, namely, the line DG is commensurable in length to the rationall line AC : and the line AG is in power more then the line DG , by the square of a line commensurable in length to the line AG . Deuide the line DG into two equal parts in the point E .



And unto the line AG apply a parallelogramme equall to the fourth part of the square of the line DG , that is, equall to the square of the line EG , and wanting in figure by a square, and let that parallelogramme be that which is contained under the lines AF and FG . Wherefore (by the 17. of the tenth) the line AF is commensurable in length to the line FG . And by the pointes E, F , and G , draw unto the line AC these parallell lines EH, FI , and GK : and forasmuch as the line AF is commensurable in length to the line FG , therefore the whole line AG is commensurable in length to either of these lines AF and FG . But the line AG is rationall and incommensurable in length to the line AC . Wherefore either of these lines AF and FG are rationall and incommensurable in length to the line AC . Wherefore either of these parallelogrammes AI and FK is (by the 21. of the tenth) medially. Againe, forasmuch as the line DE is commensurable in length to the line EG , therefore the line DG is commensurable in length to either of these lines DE and EG . But the line DG is commensurable in length to the rationall line AC . Wherefore either of these lines DE and EG is rationall and commensurable in length to the line AC . Wherefore (by the 19. of the tenth) either of these parallelogrammes DH and EK is rationall. Vnto the parallelogramme AI describe an equall square LM , and unto the parallelogramme FK let the square NX be equall, as in the Proposition going before. Wherefore the squares LM and NX are both about one and the same diameter. Let the diameter be OR , and describe the figure as is in the former Proposition expressed. Now therefore forasmuch as the parallelogrammes AI and FK are medially and ^{*}commensurable the one to the other, and the squares of the lines LO & ON which are equall to those parallelogrammes, are medially, therefore the lines LO and ON are also medially commensurable in power. (And it is manifest, that the lines LO and ON are commensurable in power, for their squares are commensurable, and those squares, namely, the squares of the lines LO & ON are commensurable, for they are equall to the parallelogrammes AI and FK , which are commensurable the one to the other: and that those parallelogrammes AI and FK are commensurable the one to the other, hereby it is manifest, for that it was before proued that the lines AF and FG are commensurable in length. Wherefore (by the 1. of the sixth, and 10. of the tenth) the parallelogrammes AI and FK are commensurable the one to the other. Wherefore it is now manifest by the way of resolution, that the lines LO & ON are commensurable in power). And forasmuch as the parallelogramme contained under the lines AF and FG is equall to the square of the line EG , therefore as the line AF is to the line EG , so is the line EG to the line FG . But as the line AF is to the line EG , so is the parallelogramme AI to the parallelogramme $E-K$: and as the line EG is to the line FG , so is the parallelogramme $E-K$ to

The first part of the demonstration.

AI and FK concluded parallelogrammes medially.

DH & EK , rationall.

The second part of the construction.

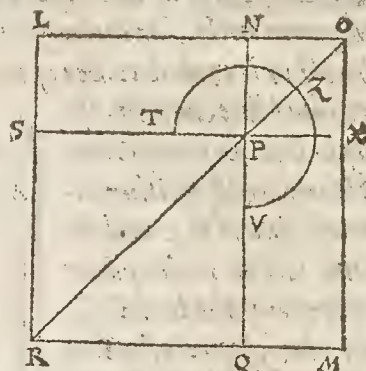
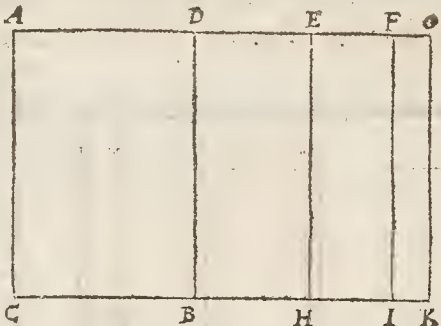
The second part of the demonstration.

^{*}Analytically the proof hereof followeth among other things.

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“

The tenth Booke

the parallelogramme FK . Wherefore the parallelogramme EK is the meane proportionall betwene the parallelogrammes AI and FK : and the parallelogramme MN is also the meane proportionall betwene the squares LM & NX : and the parallelogramme AI is equall to the square LM : and the parallelogramme FK is equall to the square NX . Wherefore the parallelogramme MN is equall to the parallelogramme EK . But the parallelogramme EK is



The line LN found, which is the principall drift of all the former discourse.

equall to the parallelogramme DH : and the parallelogramme LX is equall to the parallelogramme MN . Wherefore the whole parallelogramme DK is equall to the Gnomon VTZ , and to the square NX . Wherefore the residue, namely, the parallelogramme AB is equall to the square SQ , that is, to the square of the line LN . Wherefore the line LN containeth in power the superficies AB . I say moreover, that the line LN is a first medially residuall line. For forasmuch as the parallelogramme EK is rationall, and is equall to the parallelogramme MN , that is, LX , therefore LX , that is, the parallelogramme contained under the lines LO and ON is rationall. But the square NX is medially, for it is already proved that the parallelogramme FK which is equall to the square NX , is medially. Wherefore the parallelogramme LX is incommensurable to the square NX . But as the parallelogramme LX is to the square NX , so is the line LO to the line ON (by the 1. of the sixth). Wherefore (by the 10. of the tenth) the lines LO and ON are incommensurable in length. And it is already proved, that they are medially commensurable in power. Wherefore the lines LO & ON are medially commensurable in power onely containing a rationall superficies. Wherefore the line LN is a first medially residuall line, and containeth in power the superficies AB , which is contained under a rationall line and a second residuall line. If therefore a superficies be contained under a rationall line and a second residuall line: the line which containeth in power that superficies, is a first medially residuall line: which was required to be demonstrated.

¶ The 69. Theoreme.

The 93. Proposition.

If a superficies be contained under a rationall line and a third residuall line: the line that containeth in power that superficies is a second medially residuall line.

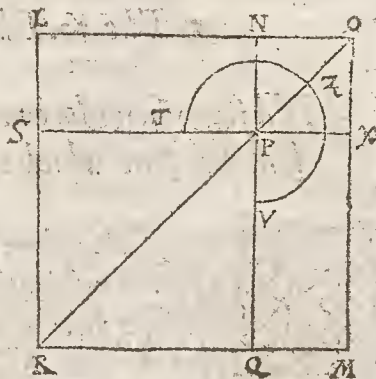
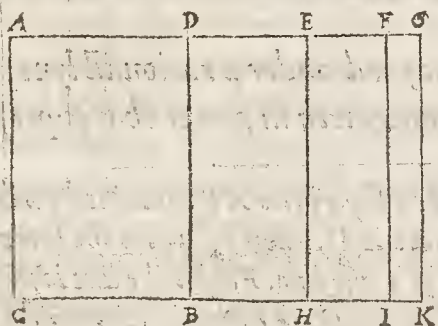


Suppose that AB be a superficies contained under a rationall line AC , & a third residuall line AD . Then I say, that the line which containeth in power the superficies AB is a second medially residuall line. Let the line ioyned vnto AD , be DG . Wherefore the lines AG and GD are rationall commensurable in power onely, and neither of the lines AG nor GD is commensurable in length to the rationall line AC , and the whole line AG is in power more then the line GD , by the square of a line commensurable in length to the line AG . Let the rest of the construction be as it was in the former Propositions. Wherefore the lines AF and FG are commensurable in length: and the parallelogramme AI is commensurable to the parallelogramme FK . And forasmuch as the lines AF and FG are commensurable in length, therefore the whole line AG is commensurable

The first part of the Construction. The first part of the demonstration.

able in length to either of these lines AF and FG . But the line AG is rationally and incommensurable in length to the line AC . Wherefore either of these lines AF and FG is rationally and incommensurable in length to the line AC . Wherefore (by the 21. of the tenth) either of these parallelogrammes AI and FK is medially. Again, forasmuch as the line DE is commensurable in length to the line EG , therefore also the whole line DG is commensurable in length to either of these lines DE and EG . But the line DG is rationally commensurable in power only to the line AC .

Wherefore also either of the lines DE and EG is rationally and commensurable in power only to the line AC . Wherefore either of these parallelogrammes DH and EK is medially. Again forasmuch as



Note.
 AI and FK
medially.

Note
 DH and EK
medially.

the lines AG and DG are commensurable in power only, therefore they are incommensurable in length. But the line AG is commensurable in length to the line AF : and the line DG is commensurable in length to the line GE . Wherefore the line AF is incommensurable in length to the line EG . But as the line AF is to the line EG , so is the parallelogramme AI to the parallelogramme EK . Wherefore the parallelogramme AI is incommensurable to the parallelogramme EK . Unto the parallelogramme AI describe an equall square LM : and unto the parallelogramme FK describe an equall square NX : and describe the figure as you did in the former Proposition. Now forasmuch as the parallelogramme contained under the lines AF and FG is equall to the square of the line EG , therefore as the line AF is to the line EG , so is the line EG to the line FG . But as the line AF is to the line EG , so is the parallelogramme AI to the parallelogramme EK : and as the line EG is to the line FG , so is the parallelogramme EK to the parallelogramme FK . Wherefore as the parallelogramme AI is to the parallelogramme EK , so is the parallelogramme EK to the parallelogramme FK . Wherefore the parallelogramme EK is the meane proportionall betwene the parallelogrammes AI and FK . But the parallelogramme MN is the meane proportionall betwene the squares LM and NX . Wherefore the parallelogramme EK is equall to the parallelogramme MN . Wherefore the whole parallelogramme DK is equall to the Gnomon VTZ , & to the square NX . And the parallelogramme AK is equall to the squares LM and NX . Wherefore the residue, namely, the parallelogramme AB is equall to the square QS , that is, to the square of the line LN . Wherefore the line LN containeth in power the superficies AB . I say moreover, that the line LN is a second medially residuall line. For for that as it is proued, the parallelogrammes AI and FK are medially, therefore the squares that are equall unto them, namely, the squares of the lines LO and ON are also medially. Wherefore either of these lines LO and ON is medially. And forasmuch as the parallelogramme AI is * commensurable to the parallelogramme FK , therefore the squares that are equall to them, namely, the squares of the lines LO and ON are also commensurable. Again, forasmuch as it is proued, that the parallelogramme AI is incommensurable to the parallelogramme EK , therefore the square LM is incommensurable to the parallelogramme MN , that is, the square of the line LO to the parallelogramme contained under the lines LO & ON . Wherefore also the * line LO is incommensurable in length to the line ON . Wherefore the lines LO and ON are medially commensurable in power only. I say moreover, that they containe a medially superficies. For forasmuch as it is proued, that the parallelogramme EK is medially, therefore the

Note.
 AI incommensurable to EK ,
Second part of
the Constrution.

The principall
line, LN founde.

* Because the
lines AF and FG
are proued
commensurable
in length.

* By the first of
the sixth and
tenth of the
tenth.

The tenth Booke

parallelogramme which is equall unto it, namely, the parallelogramme contained under the lines LO and ON is also mediall. Wherefore the line LN is a second mediall residuall line, and containeth in power the superficies AB . Wherefore the line that containeth in power the superficies AB is a second mediall residuall line. If therefore a superficies be contained under a rationall line and a third residuall line, the line that containeth in power that superficies, is a second mediall residuall line: which was required to be demonstrated.

The 70. Theoreme.

The 94. Proposition.

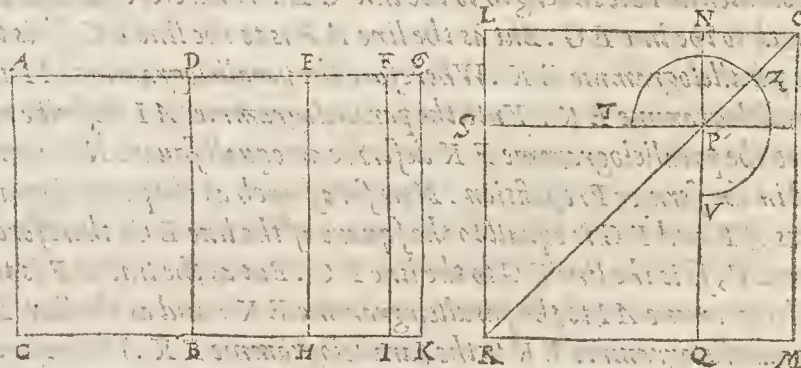
If a superficies be containd vnder a rationall lyne, and a fourth residuall lyne: the lyne which containeth in power that superficies, is a lesse lyne.



Suppose that there be a superficies AB contained vnder a rationall line AC , and a fourth residuall line AD . Then I say that the line which containeth in power the superficies AB is a lesse line. For let the line ioyned vnto it be DG . Wherefore the lines AG and DG are rationall commensurable in power only, and the

line AG is in power more then the line DG , by the square of a line incommensurable in length to the line AG , and the line AG is commensurable in length to the line AC . Denide the line DG into two equall partes in the point E . And vnto the line AG apply a parallelogramme equal to the

square of the line EG , and wanting in figure by a square, and let that parallelogramme be that which is containd vnder the lines AF and FG . Wherefore (by the 18. of the tenth) the line AF is incommensurable in length to



the line FG . Draw by the pointes $E, F, \& G$ vnto the lines AC and DB these parallel lines EH, FI , and GK . Now forasmuch as the line AG is rational, and commensurable in length to the line AC , therefore the whole parallelogramme AK is (by the 19. of the tenth) rational.

Againe forasmuch as the line DG is incommensurable in length to the line AC (for if the line DG were commensurable in length to the line AC , then forasmuch as the line AG is commensurable in length to the same line AC , the lines AG and DG should be commensurable in length the one to the other, when yet they are put to be commensurable in power only) and both these lines AC and DG are rational. Wherefore the parallelograme DK is mediall.

Againe forasmuch as the line AF is incommensurable in length to the line FG , therefore the parallelogramme AI is incommensurable to the parallelogramme FK . Vnto the parallelogramme AI describe an equal square LM , and vnto the parallelogramme FK describe an equal square NX , and let the angle LOM be common to both those squares. Wherefore the squares LM and NX are about one and the selfe same diameter. Let their diameter be

OR , and describe the figure. And forasmuch as the parallelograme contained vnder the lines AF and FG is equal to the square of the line EG , therefore proportionally as the line AF is to the line EG , so is the line EG to the line FG , but as the line AF is to the line EG , so is the parallelogramme AI to the parallelogramme EK (by the 1. of the sixt.) And as the line EG is to the line FG , so is the parallelogramme EK to the parallelogramme FK . Wherefore

the parallelogramme AI is equal to the parallelogramme EK . And as the line AF is to the line EG , so is the parallelogramme AI to the parallelogramme EK . And as the line EG is to the line FG , so is the parallelogramme EK to the parallelogramme FK . Wherefore

The first part of the construction.

The first part of the demonstration.

Note. AK rational.

Note. DK mediall. AI and FK incommensurable.

The second part of the construction.

The second part of the demonstration.

the parallelogramme $E K$ is the meane proportionall betwene the parallelogrammes $A I$ and $F K$: wherefore as it was said in the former propositions, the parallelogramme $M N$ is equal to the parallelogramme $E K$: but the parallelogramme $D H$ is equall to the parallelogramme $E K$, and the parallelogramme $M N$ to the parallelogramme $L X$. Wherefore the whole parallelogramme $D K$ is equall to the gnomon $V T Z$, and to the square $N X$. Wherefore the residue, namely, the parallelogramme $A B$ is equall to the residue, namely, to the square $S Q$, that is to the square of the line $L N$. I say moreover that $L N$ is that irrationall line which is called a lesse line. For forasmuch as the parallelogramme $A K$ is rationall, and is equall to the squares of the lines $L O$ and $O N$, therefore that which is made of the squares of the lines $L O$ and $O N$ added together is rationall. Again forasmuch as the parallelogramme $D K$ is medially, and is equall to that which is contained under the lines $L O$ and $O N$ twice, therefore that which is contained under the lines $L O$ and $O N$ twice, is also medially. And forasmuch as the parallelogramme $A I$ is incommensurable to the parallelogramme $F K$, therefore the squares which are equall unto them, namely, the squares of the lines $L O$ and $O N$ are incommensurable the one to the other. Wherefore the lines $L O$ and $O N$ are incommensurable in power, having that which is made of their squares added together rationall, and that which is contained under them twice medially, which is commensurable to that which is contained under them once. Wherefore that which is contained under them once is also medially. Wherefore $L N$ is that irrationall line which is called a lesse line, and it containeth in power the superficies $A B$. If therefore a superficies be contained under a rationall line and a fourth residuall line, the line which containeth in power that superficies is a lesse line: which was required to be demonstrated.

$L N$ (the chiefe line of this theoreme) founde.

¶ The 71. Theoreme.

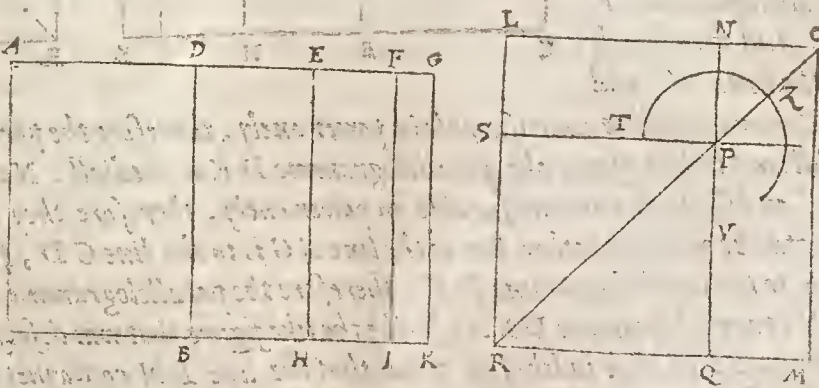
The 95. Proposition.

If a superficies be contained under a rationall line and a fift residuall line: the line that containeth in power the same superficies, is a line making with a rationall superficies, the whole superficies medially.



Suppose that there be a superficies $A B$ contained under a rationall line $A C$ and a fift residuall line $A D$. The I say that the line that containeth in power the superficies $A B$, is a line making with a rationall superficies the whole superficies medially. For unto the line $A D$ let the line $D G$ be ioyned, which shal be commensurable in length to the rational line $A C$. And let the rest of the construction be as in the proposition next going before. And forasmuch as the line $A G$ is incommensurable in length to the line $A C$ and they are both rational, therefore the parallelogramme $A K$ is medial. Again forasmuch as the line $D G$ is rational and commensurable in length to the line $A C$, therefore the parallelogramme $D K$ is rational. Unto the parallelogramme $A I$ describe an equall square $L M$, and unto the parallelogramme $F K$ describe an equall square $N X$, and as in the proposition next go-

Demonstration.



ing

The tenth Booke

The line LN.

ing before, so also in this may we proue, that the line LN containeth in power the superficies AB. I say moreouer that that line LN is a line making with a rationall superficies the whole superficies mediall. For forasmuch as the parallelogramme AK is mediall, therefore that which is equall vnto it, namely, that which is made of the squares of the lines LO and ON added together is also mediall. Againe forasmuch as the parallelogramme DK is rationall, therefore that which is equall vnto it, namely, that which is contained vnder the lines LO and ON twise, is also rationall. And forasmuch as the line AF is incommensurable in length to the line FG, therefore (by the 1. of the sixth, & 10. of the tenth) the parallelogramme AL is incommensurable to the parallelogramme FK, wherfore also the square of the lyne LO is incommensurable to the square of the line ON. Wherfore the lines LO and ON are incommensurable in power hauing that which is made of their squares added together mediall, and that which is containd vnder them twise rationall. Wherfore the line LN is that irrationall line which is called a lyne making with a rationall superficies the whole superficies mediall, and it containeth in power the superficies AB. Wherfore the line containyng in power the superficies AB, is a line making with a rationall superficies the whole superficies mediall. If therfore a superficies be containd vnder a rationall lyne & a sixth residuall line, the line that containeth in power the same superficies, is a line makyng with a rationall superficies the whole superficies mediall: which was required to be proued.

¶ The 72. Theoreme.

The 96. Proposition.

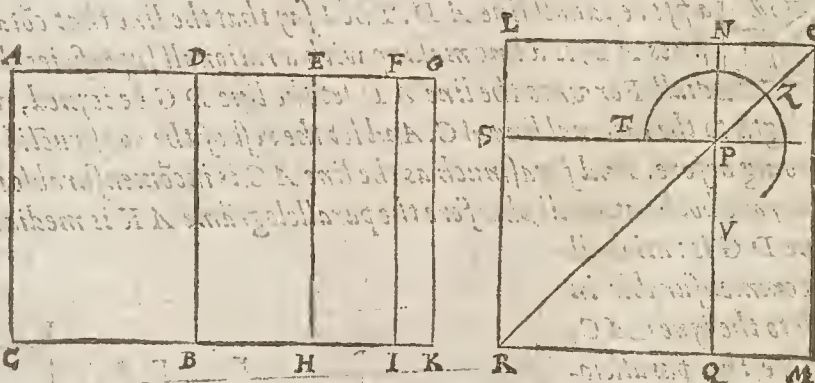
If a superficies be containd vnder a rationall line and a sixth residuall line, the line which containeth in power the same superficies is a line making with a mediall superficies the whole superficies mediall.



*S*uppose that AB be a superficies containd vnder a rationall line AC, & a sixth residuall line AD. Then I say that the line which containeth in power the superficies AB, is a line making with a mediall superficies the whole superficies mediall. For vnto the line AD let the line DG be ioyned. And let the rest be as in

Demonstration.

the propositions going before. And forasmuch as the line AF is incommensurable in length to the line FG, therefore the parallelogramme AL is incommensurable to the parallelogramme FK. And forasmuch as the lines AG and



AC are rationall commensurable in power onely, therefore the parallelogramme AK is mediall, and in like maner the parallelogramme DK is mediall. Now forasmuch as the lines AG and GD are commensurable in power onely, therefore they are incommensurable in length the one to the other. But as the line AG is to the line GD, so is the parallelogramme AK to the parallelogramme DK, therefore the parallelogramme AK is incommensurable to the parallelogramme DK. Describe the like figure that was described in the former propositions, and we may in like sort proue that the line LN containeth in power the superficies AB. I say moreouer that it is a line making with a mediall superficies the whole superficies mediall

mediall. For the parallelogramme AK is mediall, wherefore that which is equal vnto it, namely, that which is made of the squares of the lines LO and ON added together is also mediall. And forasmuch as the parallelogramme DK is mediall, therefore that which is equall vnto it, namely, that which is containd vnder the lines LO and ON twise is also mediall. And forasmuch as the parallelogramme AK is incommensurable to the parallelogramme DK , therefore the squares of the lines LO and ON are incommensurable to that which is containd vnder the lines LO and ON twise. And forasmuch as the parallelogramme AI is incommensurable to the parallelogramme FK , therefore also the square of the line LO is incommensurable to the square of the line ON . Wherefore the lines LO and ON are incommensurable in power, hauing that which is made of the squares of the lines LO and ON mediall, and that which is containd vnder them twise mediall, and moreover that which is made of the squares of them is incommensurable to that which is containd vnder them twise. Wherefore the line LN is that irrational line which is called a line making with a mediall superficies the whole superficies mediall, and it containeth in power the superficies AB . Wherefore the line which containeth in power the superficies AB is a line making with a mediall superficies the whole superficies mediall. If therefore a superficies be containd vnder a rationall line and a sixth residuall line, the line which containeth in power the same superficies is a line making which a mediall superficies the whole superficies mediall. which was required to be demonstrated.

¶ The 73. Theoreme.

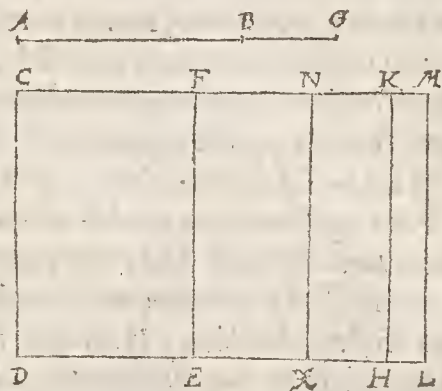
The 97. Proposition.

The square of a residuall line applyed vnto a rationall line, maketh the breadth or other side a first residuall line.

The fineth Senary.

Suppose that AB be a residuall line, and let CD be a rationall line. And vnto the line CD apply the parallelogramme CE equall to the square of the line AB , and making the breadth the line CF . Then I say that the line CF is a first residuall line. For vnto the line AB let the line conueniently ioyned be supposed to be BG (which selfe line is also called a line ioyned, as we declared in the end of the 79. proposition). Wherefore the lines AG and GB are rationall commensurable in power onely. And vnto the line CD apply the parallelogramme CH equall to the square of the line

AG , and vnto the line KH (which is equall to the line CD) apply the parallelogramme KL equall to the square of the line BG . Wherefore the whole parallelogramme CL is equall to the squares of the lines AG and GB . And the parallelogramme CE is equall to the square of the line AB , wherefore the parallelogramme remayning, namely, the parallelogramme FL is equall to that which is containd vnder the lines AG and GB twise. For (by the 7. of the second) the squares of the lines AG and GB are equall to that which is containd vnder the lines AG & GB twise, and to the square of the line AB . Denide the line FM into two equall partes in the point N . And by the poynt N draw vnto the line CD a parallell line NX . Wherefore either of the parallelogrammes FX and NL is equall to that which is containd vnder the lines AG and GB once. And forasmuch as the squares of the lines AG and GB are rationall, vnto which squares the parallelogramme CL is equall, therefore the parallelogramme CL also is rationall: wherefore the line GM is rationall and* commensurable in length



These sixe propositions following are the conuerfes of the sixe former propositions. Construction.

Demonstration.

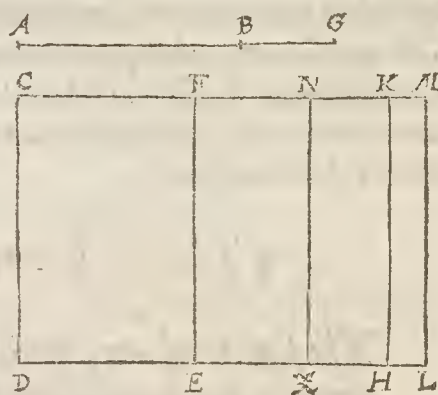
*By the 20. of the tenth.

The tenth Booke

**By the 21.
of the tenth.
*By the 22. of
the tenth.

C F concluded &
residual line.

length to the line CD . Agayne forasmuch as that which is containd under the lines AG and GB twise, is ** medially, therefore the parallelogramme equall vnto it, namely the parallelogramme FL is also medially. Wherefore the line FM is rationally and * incommensurable in length to the line CD . And forasmuch as the squares of the lines AG and GB are rationally, and that which is containd under the lines AG and GB twise, is medially, therefore the squares of the lines AG and GB are incommensurable to that which is containd under the lines AG and GB twise. But vnto the squares of the lines AG and GB is equall the parallelogramme CL , and to that which is containd under the lines AG and GB twise is equall the parallelogramme FL , wherefore the parallelogramme CL is incommensurable to the parallelogramme FL . Wherefore also the line CM is incommensurable in length to the line FM : and they are both rationally. Wherefore the lines CM and FM are rationally commensurable in power onely: and therefore the line CF is a residual line (by the 73. of the tenth). I say moreover that it is a first residual line. For forasmuch as that which is containd under the lines AG and GB is the meane proportionall betwene the squares of the lines AG and GB (by the assumpt going before the 54. of the tenth). And vnto the square of the line AG is equall the parallelogramme CH , and vnto that which is containd under the lines AG and GB is equall the parallelogramme NL , and vnto the square of the line GB is equall the parallelogramme KL . Wherefore the parallelogramme NL is the meane proportionall betwene the parallelogrammes CH and KL . Wherefore as CH is to NL , so is NL to KL . But as CH is to NL , so is the line CK to the line NM & as NL is to KL , so is the line NM to the line KM . Wherefore as the line CK is to the line NM , so is the line NM to the line KM . Wherefore the parallelogramme containd under the lines CK and KM is equall to the square of the line NM , that is to the fourth part of the square of the line FM . And forasmuch as the square of the line AG is commensurable to the square of the line GB , therefore the parallelogramme CH is commensurable to the parallelogramme KL . But as CH is to KL , so is the line CK to the line KM : wherefore the line CK is commensurable in length to the line KM . Wherefore (by the 17. of the tenth) the line CM is in power more then the line FM by the square of a line commensurable in length to the line CM . But the line CM is commensurable in length to the rationally line CD . Wherefore the line CF is a first residual line. Wherefore the square of a residual line applyed vnto a rationally line, maketh the breadth or other side a first residual line: which was required to be demonstrated.



¶ The 74. Theoreme.

The 98. Proposition.

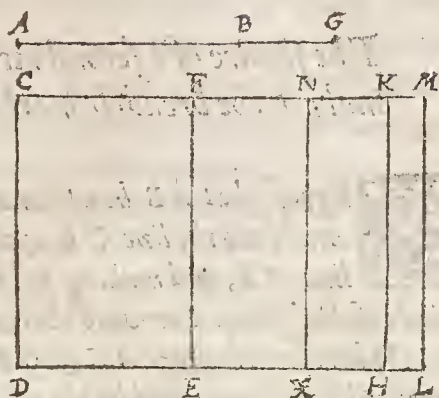
The square of a first medially residual line applyed to a rationally line, maketh the breadth or other side a second residual line.

Construction.



Suppose that AB be a first medially residual line, and let CD be a rationally line. And vnto the line CD apply the parallelogramme CE equall to the square of the line AB , and making in breadth the line CF . Then I say that the line CF is a second residual

residuall line. For vnto the line AB , let the lyne
conueniently ioyned be supposed to be BG . Where-
fore the lines AG and BG are medially commen-
surable in power onely, comprehending a rationall
superficies. And vnto the line CD apply the pa-
rallelogramme CH equal to the square of the line
 AG , and making in bredth the line CK , and vn-
the line KH (which is equall to the line CD) ap-
ply the parallelogramme KL equall to the square
of the line GB and making in breadth the line K
 M . Wherefore the whole parallelogramme CL is e-
quall to both the squares of the lines AG and G



Demonstra-
tion.

B which are medially & commensurable the one to the other. Wherefore the parallelogrammes
 CH and KL are medially and commensurable the one to the other. Wherefore (by the 15. of the
tenth) the whole parallelogramme CL is commensurable to either of these parallelogrammes
 CH and KL . Wherefore (by the corollary of the 23. of the tenth) the whole parallelogramme
 CL is also medially. Wherefore (by the 22. of the tenth) the line CM is rationall and incommen-
surable in length to the line CD . And forasmuch as the parallelogramme CL is equal
to the squares of the lines AG and GB : and the squares of the lines AG and GB are equall
to that which is contained vnder the lines AG and GB twise, together with the square of the
line AB (by the 7. of the second): and vnto the square of the line AB is equall the parallelo-
gramme CE . Wherefore the residue, namely, that which is contained vnder the lines AG and
 GB twise, is equal to the residue, namely, to the parallelogramme FL . But that which is contain-
ed vnder the lines AG & GB twise is rational. Wherefore the parallelogramme FL is also
rational. Wherefore the line FM is rational and commensurable in length to the line CD (by
the 20. of the tenth.) Now forasmuch as the parallelogramme CL is medially, and the paral-
lelogramme FL is rational, therefore they are incommensurable the one to the other. Wherefore
also the lyne CM is incommensurable in length to the lyne FM , and they are both rational.
Wherefore the line CF is a residuall line. I say moreouer that it is a second residuall lyne. For
denide the line FM into two equall partes in the point N , from which point draw vnto the
line CD a parallel line NX . Wherefore either of these parallelogrammes FX and NL is e-
quall to the parallelogramme contained vnder the lines AG and GB . And forasmuch as the
parallelogramme contained vnder the lines AG and GB is the meane proportionall betwene
the squares of the lines AG and GB : Therefore the parallelogramme NL is the meane pro-
portionall betwene the parallelogrammes CH and KL . But as CH is to NL , so is the line C
 K to the line NM , and as NL is to KL , so is the line NM to the line KM . Wherefore as the
line CK is to the lyne NM , so is the lyne NM to the line KM . Wherefore the parallelogramme
contayned vnder the lines CK and KM is equall to the square of the line NM , that is, to
the fourth part of the square of the lyne FM . But the parallelogramme CH is commensura-
ble to the parallelogramme KL . Wherefore also the lyne CK is commensurable in length to
the lyne KM . Wherefore (by the 17. of the tenth) the line CM is in power more then the line
 FM , by the square of a line commensurable in length to the line CM . And the line FM
which is the line conueniently ioyned, is commensurable in length to the rationall lyne CD .
Wherefore the lint CF is a second residuall lyne. Wherefore the square of a first medially res-
iduall line applied to a rationall line, maketh the breadth or other side a second residuall lyne:
which was required to be proued.

CF concluded
a residuall
line.

The tenth Booke

¶ The 75. Theoreme.

The 99. Proposition.

The square of a second mediall residuall line applied vnto a rationall line,
maketh the breadth or other side a third residuall line.

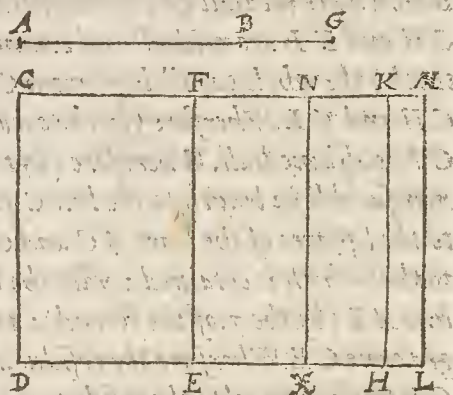
Construction.



Suppose that AB be a second mediall residuall line, and let CD be a rationall line. And vnto the line CD apply the parallelogramme CE equall to the square of the line AB , and making in breadth the line CF . Then I say, that the line CF is a third residuall line. For vnto the line AB let the line conveniently ioyned be supposed to be BG . Wherefore the lines AG & GB are mediall commensurable in power onely containing a mediall superficies. And let the rest of the construction be as in the Proposition next going before.

Demonstration.

Wherefore the line CM is rationall and incommensurable in length to the rationall line CD . And either of the parallelogrammes FX and NL is equall to that which is contained vnder the lines AG and GB . But that which is contained vnder the lines AG & GB is mediall. Wherefore that which is contained vnder the lines AG and GB twise is also mediall. Wherefore the whole parallelogramme FL is also mediall. Wherefore the line FM is rationall and incommensurable in length to the line CD . And forasmuch as the lines AG and GB are incommensurable in length, therefore also the square of the line AG is incommensurable to the parallelogramme contained vnder the lines AG and GB . But vnto the square of the line AG are commensurable the squares of the lines AG and GB : and vnto the parallelogramme contained vnder the lines AG and GB , is commensurable that which is contained vnder the lines AG and GB twise. Wherefore the squares of the lines AG and GB are incommensurable to that which is contained vnder the lines AG and GB twise. Wherefore the parallelogrammes which are equall vnto them, namely, the parallelogrammes CL and FL are incommensurable the one to the other. Wherefore also the line CM is incommensurable in length to the line FM : and they are both rationall. Wherefore the line CF is a residuall line. I say moreover, that it is a third residuall line. For forasmuch as the square of the line AG , that is, the parallelogramme CH is commensurable to the square of the line BG , that is, to the parallelogramme KL , therefore the line CK is commensurable in length to the line KM . And in like sort as in the former Proposition, so also in this may we proue, that the parallelogramme contained vnder the lines CK and KM , is equall to the square of the line NM , that is, to the fourth part of the square of the line FM . Wherefore the line CM is in power more then the line FM , by the square of a line commensurable in length to the line CM : and neither of the lines CM nor FM is commensurable in length to the rationall line CD . Wherefore the line CF is a third residuall line. Wherefore the square of a second mediall residuall line applied vnto a rationall line, maketh the breadth or other side a third residuall line: which was required to be demonstrated.



CF concluded
residual line.

¶ The 76. Theoreme.

The 100. Proposition.

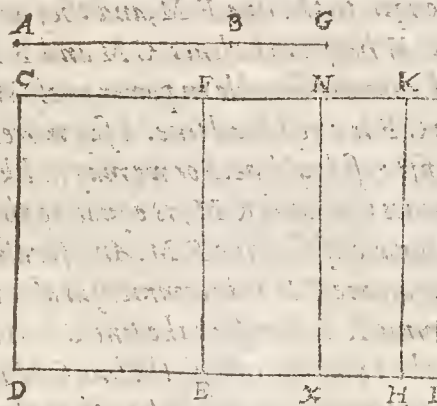
The square of a lesse line applied vnto a rationall line, maketh the breadth
or other side a fourth residuall line.

Suppose

Suppose that AB be a lesse line, and let CD be a rationall line. And vnto the line CD apply the parallelogramme CE equall to the square of the line AB , and making in breadth the line CF . Then I say, that the line GF is a fourth residuall line. For vnto the line AB let the line conueniently ioyned be supposed to be BG . Wherefore the lines AG & GB are incommensurable in power, hauing that which is made of their squares added together rationall, and that which is contained vnder them mediall. And let the rest of the construction be as in the Propositions going before. Wherefore the whole parallelogramme CL is rationall. Wherefore the line CM is also rationall, and commensurable in length to the line CD . And forasmuch as that which is contained vnder the lines AG and GB twise is mediall, therefore the parallelogramme which is equall vnto it, namely, the parallelogramme FL , is also mediall. Wherefore the line FM is rationall and incommensurable in length to the line CD . But the line CM is commensurable in length to the line CD . Wherefore (by the 13. of the tenth) the line CM is incommensurable in length to the line FM : and they are both rationall. Wherefore the lines CM and FM are rationall commensurable in power onely. Wherefore the line CF is a residuall line: I say moreover, that it is a fourth residuall line. For forasmuch as the lines AG and GB are incommensurable in power, therefore the squares of them, that is, the parallelogrammes, which are equall vnto them, namely, the parallelogrammes CH and KL , are incommensurable the one to the other. Wherefore also the line CK is incommensurable in length to the line KM . And in like sort may we proue, that the parallelogramme contained vnder the lines CK and KM , is equall to the square of the line NM , that is, to the fourth part of the square of the line FM . Wherefore (by the 18. of the tenth) the line CM is in power more then the line FM , by the square of a line incommensurable in length to the line CM . And the whole line CM is commensurable in length to the rationall line CD . Wherefore the line CF is a fourth residuall line. Wherefore the square of a lesse line applied vnto a rationall line, maketh the breadth or other side a fourth residuall line: which was required to be proued.

Construction.

Demonstration.



CF proued a residuall line.

¶ The 77. Theoreme.

The 101. Proposition.

The square of a lyne making with a rationall superficies the whole superficies mediall applied vnto a rational line, maketh the breadth or other side a fift residuall lyne.

Suppose that AB be a line making with a rationall superficies the whole superficies mediall, and let CD be a rationall line. And vnto the line CD apply the parallelogramme CE equall to the square of the line AB , and making in breadth the line CF . Then I say that the line CF is a fift residuall line. For vnto the line AB let the line conueniently ioyned be supposed to be BG . Wherefore the lines AG and GB are incommensurable in power, hauing that which is made of their squares added together mediall, and that which is contained vnder them rationall. Let the rest of the construction be in this as it was in the former propositions. Wherefore the whole parallelogramme CL is mediall. Wherefore the line CM is rationall and incommensurable in length to the line CD . And either of the

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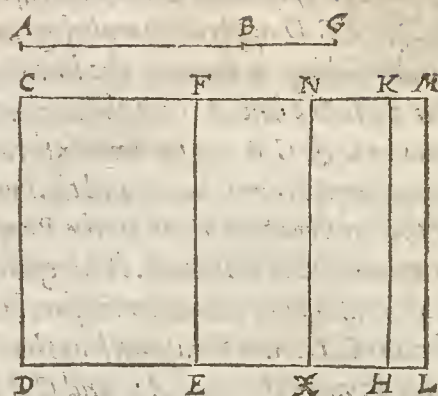
paral.

The tenth Booke

parallelogramme FX & NL is rationall. Wherefore the whole parallelogramme FL is also rationall. Wherefore also the line FM is rationall and commensurable in length to the line CD . And forasmuch as the parallelogramme CL is medially, and the parallelogramme FL is rationall, therefore CL and FL are incommensurable the one to the other, and the line CM is incommensurable in length to the line FM , and they are both rationall. Wherefore the lines CM and FM are rationall commensurable in power onely. Wherefore the lyne CF is a residuall line. I say moreover that it

CF proved a residuall line.

is a fift residuall line. For we may in like sort proue, that the parallelograme contained vnder the lines CK and KM , is equall to the square of the line NM , that is, to the fourth part of the square of the lyne FM . And forasmuch as the square of the line AG , that is, the parallelogramme CH is incommensurable to the square of the line BG , that is to the parallelogramme KL , therefore the line CK is incommensurable in length to the line KM . Wherefore (by the 18. of the tenth) the line CM is in power more then the line FM , by the square of a line incommensurable in length to the line CM . And the line conveniently ioyned, namely, the line FM is commensurable in length to the rationall line CD . Wherefore the line CF is a fift residuall line. Wherefore the line CF is a fift residuall line. Wherefore the square of a line making with a rationall superficies the whole superficies medially, applied vnto a rational line maketh the breadth or other side a fift residuall lyne: which was required to be demonstrated.



¶ The 78. Theoreme.

The 102. Proposition.

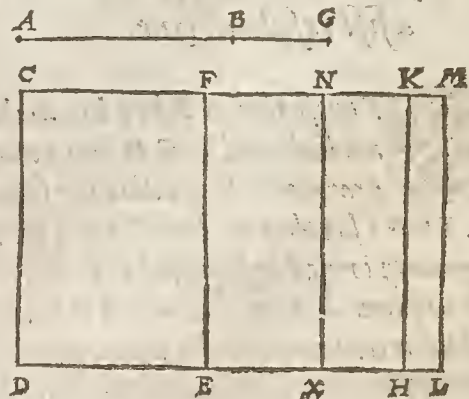
The square of a lyne making with a medially superficies, the whole superficies medially applied to a rationall line, maketh the breadth or other side, a sixt residuall line.

Construction.



Suppose that AB be a line making with a medially superficies, the whole superficies medially, and let CD be a rationall line. And vnto the line CD apply the parallelogramme CE equall to the square of the line AB and making in breadth the line CF . Then I say that the line CF is a sixt residuall line. For vnto the line

AB let the line conveniently ioyned be BG . Wherefore the lines AG and BG are incommensurable in power hauing that which is made of their squares added together medially, & that which is contained vnder them medially, and moreover that which is made of their squares added together is incommensurable to that which is contained vnder them. Let the rest of the construction be in this, as it was in the propositions going before. Wherefore the whole parallelogramme CL is medially, (for it is equall to that which is made of the squares of the lines AG & BG added together, which is supposed to be medially). Wherefore the line CM is rationall and incommensurable in length to the line CD : and in like manner the parallelogramme FL is medially. Wherefore also the line FM is rationall and incommensurable in length to the line CD . And forasmuch as that



which

Demonstration.

which is made of the squares of the lines AG and GB added together, is incommensurable to that which is contained under the lines AG and GB twice, therefore the parallelogrammes equall to them, namely, the parallelogrammes CL and FL are incommensurable the one to the other. Wherefore also the lines CM and FM are incommensurable in length, and they are both rational. Wherefore they are rationall commensurable in power only. Wherefore the line CF is a residuall line. I say moreover that it is a sixt residuall line. Let the rest of the demonstration be as it was in the former propositions. And forasmuch as the lines AG and GB are incommensurable in power, therefore their squares, that is, the parallelogrammes which are equall unto them, namely, the parallelogrammes CH and KL are incommensurable the one to the other. Wherefore also the line CK is incommensurable in length to the line KM . Wherefore (by the 18. of the tenth) the line CM is in power more then the line FM by the square of a line incommensurable in length to the line CM . And neither of the lines CM nor FM is commensurable in length to the rationall line CD . Wherefore the line CF is a sixt residuall line. Wherefore the square of a line making with a medial superficies the whole superficies medially applied to a rationall line, maketh the breadth or other side a sixt residuall line: which was required to be demonstrated.

CF proved a residuall.

¶ The 79. Theoreme.

The 103. Proposition.

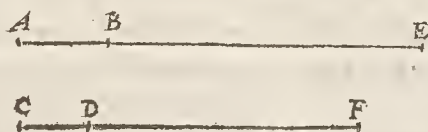
A line commensurable in length to a residuall line: is it selfe also a residuall line of the selfe same order.



Suppose that AB be a residuall line, unto which let the line CD be commensurable in length. Then I say, that the line CD is also a residuall line, and of the selfe same order of residuall lines that the line AB is. For forasmuch as the line AB is a residuall line, let the line conveniently ioyned unto it be supposed to be BE . Wherefore the lines AE and BE are rationall commensurable in power onely. As the line AB is to the line CD , so (by the 12. of the sixth) let the line BE be to the line DF . Wherefore (by the 12. of the fifth) as one of the antecedentes is to one of the consequentes, so are all the antecedentes to all the consequentes. Wherefore as the line AB is to the line CD , so is the whole line AE to the whole line CF , and the line BE to the line DF . Wherefore (by the 10. of the tenth) the line AE is commensurable in length to the line CF , and the line BE to the line DF . But the line AE is rationall. Wherefore the line CF is also rationall. And in like sort the line DF is rationall, for that the line BE , to whom it is commensurable, is also rationall. And for that as the line BE is to the line AE , so is the line DF to the line CF . But the lines BE and AE are commensurable in power onely: Wherefore the lines CD and DF are commensurable in power onely. Wherefore the line CD is a residuall line. I say moreover, that it is a residuall line of the selfe same order that the line AB is. For for that as we haue before said, as the line AE is to the line CF , so is the line BE to the line DF , therefore alternately, as the line AE is to the line BE , so is the line CF to the line DF . But the line AE is in power more then the line EB , either by the square of a line commensurable in length to the line AE , or by the square of a line incommensurable in length to the line AE . If AE be in power more then BE , by the square of a line commensurable in length to AE , then the line CF shall also (by the 14. of the tenth) be in power more then the line DF , by the square of a line commensurable in length to the line CF , and so if the line AE be commensurable in length to the rationall line put, forasmuch as the line

The sixt Secondary.

Construction.



Demonstration.

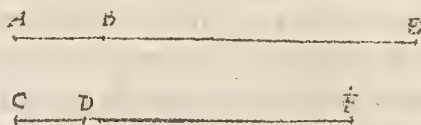
CD concluded a residuall line.

PP. iij.

AE

The tenth Booke

1. *AE* is commensurable in length to the line *CF*, therefore (by the 12. of the tenth) the line *CF* shall also be commensurable in length to the same rationall line. Wherefore either of the lines *AB* and *CD* is a first residuall line. And if the line *BE* be commensurable in length to the rationall line put, forasmuch as the line *BE* is commensurable in length to the line *DF*, therefore the line *DF* shall also be commensurable in length to the rationall line put: and then either of the lines *AB* and *CD* is a second residuall line. And if neither of the lines
2. *AE* nor *BE* be commensurable in length to the rationall line put, then neither of the lines *CF* nor *DF* shall be commensurable in length to the same rationall line (by the 13. of the tenth). And so either of the lines *AB* & *CD* is a third residuall line. But
3. if the line *AE* be in power more then the line *BE*, by the square of a line incommensurable in length to the line *AE*, the line *CF* shall in like sort (by the 14. of the tenth) be in power more then the line *DF*, by the square of a line incommensurable in length to the line *CF*: and then if the line *AE* be commensurable in length to the rationall line, the line *CF* shall also in like sort be commensurable in length to the same rationall line: and so either of the lines *AB* and *CD*
4. is a fourth residuall line. And if the line *BE* be commensurable in length to the rationall line, the line *DF* shall also be commensurable in length to the same line: and so either of the lines *AB* & *CD*
5. is a fifth residuall line. And if neither of the lines *AE* nor *BE* be commensurable in length to the rationall line, in like sort neither of the lines *CF* nor *DF* shall be commensurable in length to the same rationall line. And so either of the lines *AB* & *CD* is a sixth residuall line.
6. Wherefore the line *CD* is a residuall line of the selfe same order that the line *AB* is. A line therefore commensurable in length to a residuall line, is it selfe also a residuall line of the selfe same order: which was required to be proued.



Note.

As before touching binomial lines, so also touching residuall lines, this is to be noted, that a line commensurable in length to a residuall line, is alwayes a residuall line of the selfe same order that the residuall line is, vnto whom it is commensurable, as hath before in this 103. proposition bene proued. But if a line be commensurable in power only to a residuall line, then followeth it not, yea it is impossible, that that line should be a residuall of the selfe same order that the residuall line, is vnto whom it is commensurable in power onely. Howbeit those two lines shall of necessitie be both either of the three first orders of residuall lines, or of the three last orders: which is not hard to proue, if ye marke diligently the former demonstration, and that which was spoken of binomial lines as touching this matter.

¶ The 80. Theoreme.

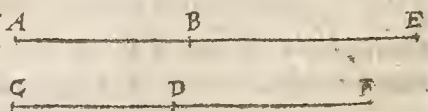
The 104. Proposition.

A line commensurable to a medially residuall line, is it selfe also a medially residuall line, and of the selfe same order.

Construction.



Suppose that *AB* be a medially residuall line, vnto whome let the line *CD* be commensurable in length and in power, or in power onely. Then I say that *CD* is also a medially residuall line, and of the selfe same order. For forasmuch as the line *AB* is a medially residuall line, let the line conveniently ioyned vnto it be *BE*: wherefore the lines *AE* and *BE* are medially commensurable in power onely. As *AB* is to *CD*, so by the 22. of the sixth) let *BE* be to *DF*. And in like sort as in the former so also in this may we proue, that the line *AE* is commensurable in length and in power, or in power onely



Demonstration.

ly vnto

by vnto the line CF , & the line BE to the line DF . Wherefore (by the 23. of the tenth) the line CF is a medially line, and the line DF is also a medially line, for that it is commensurable to the medially line BE . And in like sort the lines CF and DF are commensurable in power onely: for that they haue the selfe same proportiō the one to the other, that the lines AE and EB haue, which are commensurable in power onely. Wherefore the line CD is a medially residuall line. I say moreover that it is of the selfe same order that the line AB is. For for that as the line AE is to the line BE , so is the line CF to the line DF . But as the line AE is to the line BE , so is the square of the line AE to the parallelogramme containd vnder the lines AE and BE (by the first of the sixth): and as the line CF is to the line DF , so is the square of the line CF to the parallelogramme containd vnder the lines CF and DF . Wherefore as the square of the line AE is to the parallelogramme containd vnder the lines AE and BE , so is the square of the line CF to the parallelogramme containd vnder the lines CF and DF . Wherefore alternately as the square of the line AE is to the square of the line CF , so is the parallelogramme containd vnder the lines AE and BE , to the parallelogramme containd vnder the lines CF and DF . But the square of the line AE is commensurable to the square of the line CF (for the line AE is commensurable to the line CF). Wherefore also the parallelogramme containd vnder the lines AE and BE , is commensurable to the parallelogramme containd vnder the lines CF and DF . Wherefore if the parallelogramme containd vnder the lines AE and EB be rationally, the parallelogramme also containd vnder the lines CF and FD shall be rationally. And then either of the lines AB and CD is a first medially residuall line. But if the parallelogramme containd vnder the lines AE and BE be medially, the parallelogramme also containd vnder the lines CF and FD shall be also medially (by the corollary of the 23. of the tenth): and so either of the lines AB and CD is a second medially residuall line. Wherefore the line CD is a medially residuall line of the selfe same order that the line AB is. A line therefore commensurable to a medially residuall line, is it selfe also a medially residuall line of the selfe same order. which was required to be demonstrated.

CD proued a medially.

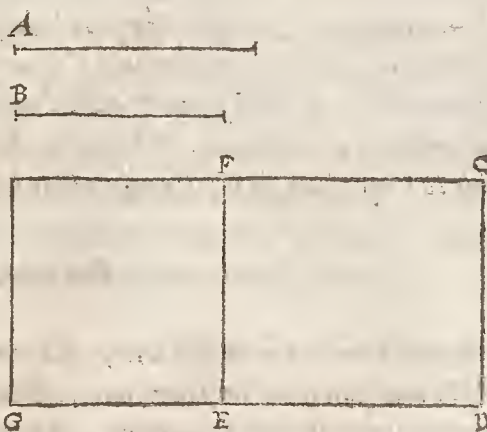
1.

2.

This Theoreme is vnderstanded generally, that whether a line be commensurable in length & in power, or in power onely to a medially residuall line, it is it selfe also a medially residuall line, and of the selfe same order, which thing also is to be vnderstanded of the three Theoremes which follow.

An other demonstration after Campane.

Suppose that A be a medially residuall line, vnto whome let the line B be commensurable in length, or in power onely. And take a rationally line CD , vnto which apply the parallelogramme CE equall to the square of the line A , and vnto the line FE (which is equall to the line CD) apply the parallelogramme FG equall to the square of the line B . Now then the parallelogrammes CE and FG shall be commensurable, for that the lines A, B are commensurable in power: wherefore by the 1. of the sixth and 10. of this booke, the lines DE and FG are commensurable in length. Now then if A be a first medially residuall line, then is the line DE a second residuall line by the 98. of this booke: and if the line A be a second medially residuall line, then is the line DE a third residuall line by the 99. of this booke. But if DE be a second residuall line, GE also shall be a second residuall line (by the 103. of this booke). And if DE be a third re-



Construction.

Demonstration.

The tenth Booke

residuell line, GE also shall (by the same) be also a third residuell line. Wherefore it followeth by the 92. and 93. of this booke, that B is either a first medial residuell line or a second medial residuell line, according as the line A is supposed to be: which was required to be proued.

¶ The 81. Theoreme.

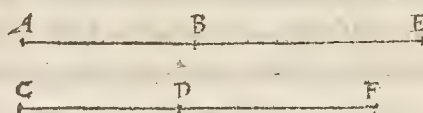
The 105. Proposition.

A line commensurable to a lesse line: is it selfe also a lesse line.

Construction.

Demonstration.

Suppose that AB be a lesse line, unto whom let the line CD be commensurable. Then I say, that the line CD is also a lesse line. For let the same construction be in this, that was in the former Propositions. And forasmuch as the lines AE and BE are incommensurable in power, therefore (by the 22. of the sixt, and 10. of the tenth) the lines CF & FD are incommensurable in power. Again (by the 22. of the sixt) as the square of the line AE is to the square of the line BE , so is the square of the line CF to the square of the line DF . Wherefore by composition as the squares of the lines AE and BE are to the square of the line BE , so are the squares of the lines CF and DF , to the square of the line DF : and alternately, as the squares of the lines AE and BE are to the squares of the lines CF and DF , so is the square of the line BE to the square of the line DF . But the square of the line BE is commensurable to the square of the line DF (for the lines BE and DF are commensurable). Wherefore that which is made of the squares of the lines AE and BE added together, is commensurable to that which is made of the squares of the lines CF and DF added together. But that which is made of the squares of the lines AE and BE added together, is rationally. Wherefore that which is made of the squares of the lines CF and DF added together, is also rational. Again, for that as the square of the line AE is to the parallelogramme contained under the lines AE and BE , so is the square of the line CF to the parallelogramme contained under the lines CF and DF (as we declared in the Proposition next going before): therefore alternately, as the square of the line AE is to the square of the line CF , so is the parallelogramme contained under the lines AE and BE , to the parallelogramme contained under the lines CF and DF . But the square of the line AE is commensurable to the square of the line CF , for the lines AE & CF are commensurable. Wherefore the parallelogramme contained under the lines AE and BE , is commensurable to the parallelogramme contained under the lines CF and DF . But the parallelogramme contained under the lines AE and BE is medial. Wherefore the parallelogramme contained under the lines CF and DF is also medial. Wherefore the lines CF and DF are incommensurable in power, having that which is made of their squares added together rational, and the parallelogramme contained under them medial. Wherefore the line CD is a lesse line. A line therefore commensurable to a lesse line, is it selfe also a lesse line: which was required to be proued.

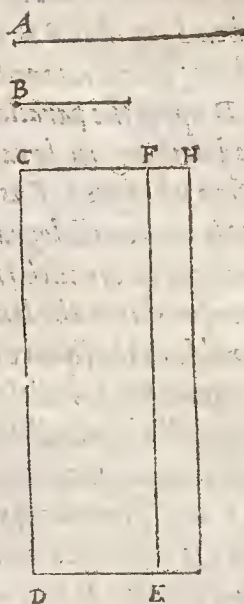


An other demonstration.

Construction.

Suppose that A be a lesse line, and unto A let the line B be commensurable whether in length and power, or in power onely. Then I say that B is a lesse line. Take a rational line C . And unto the line C apply (by the 44. of the first) the parallelogramme CE equal to the square of the line A , and making in breadth the line CF . Wherefore (by the 100. proposition)

tion) the line $C F$ is a fourth residuall line. Vnto the line $F E$ apply (by the same) the parallelogramme $E H$ equall to the square of the line B , and making in breadth the line $F H$. Now forasmuch as the line A is commensurable to the line B , therefore also the square of the line A is commensurable to the square of the line B . But vnto the square of the line A is equall the parallelogramme $C E$, & vnto the square of the line B is equal the parallelogramme $E H$. Wherfore the parallelogramme $C E$ is commensurable to the parallelogramme $E H$. But as the parallelogramme $C E$ is to the parallelogramme $E H$, so is the line $C F$ to the line $F H$. Wherfore the line $C F$ is commensurable in length to the line $F H$. But the line $C F$ is a fourth residuall line. Wherfore the line $F H$ is also a fourth residuall line (by the 103. of the tenth): and the line $F E$ is rationall. But if a superficies be contained vnder a rationall line, and a fourth residuall lyne, the line that containeth in power that superficies is (by the 94. of the tenth) a lesse lyne. But the line B containeth in power the superficies $E H$. Wherfore the line B is a lesse line: which was required to be proved.



Demonstration.

¶ The 82. Theoreme.

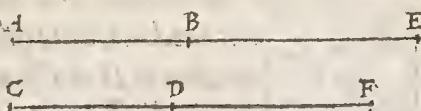
The 106. Proposition.

A line commensurable to a lyne making with a rationall superficies the whole superficies medially, is it selfe also a lyne making with a rationall superficies the whole superficies medially.



Suppose that $A B$ be a line making with a rationall superficies the whole superficies medially, vnto whom let the line $C D$ be commensurable. Then I say that the line $C D$ is a line making with a rationall superficies the whole superficies medially. Vnto the line $A B$ let the line conveniently ioyned be $B E$. Wherfore the lines $A E$ and $E B$ are incommensurable in power, hauing that which is made of their squares added together medially, and the parallelogramme contained vnder them rationall. Let the construction be in this as it was in the former propositions. And in like sort may we proue that as the line $A E$ is to the line $B E$, so is the line $C F$ to the line $D F$, and that that which is made of the squares of the lines $A E$ and $B E$ added together is commensurable to that which is made of the squares of the lines $C F$ and $D F$ added together, and that that which is contained vnder the lines $A E$ and $E B$, is in like sort commensurable to that which is contained vnder the lines $C F$ and $D F$. Wherfore also the lines $C F$ and $D F$ are commensurable in power, hauing that which is made of their squares added together medially, and that which is contained vnder them rationall. Wherfore the line $C D$ is a lyne making with a rationall superficies the whole superficies medially. Wherfore a line commensurable to a line making with a rationall superficies the whole superficies medially, is it selfe also a lyne making with a rationall superficies the whole superficies medially: which was required to be demonstrated.

Construction.



Demonstration.

An other demonstration.

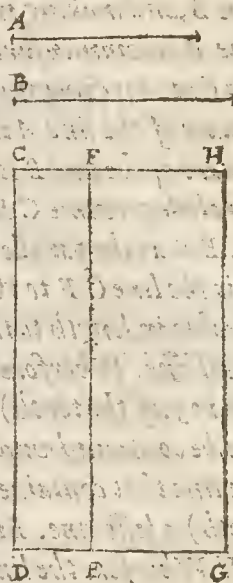
Suppose that A be a line making with a rationall superficies the whole superficies medially, and

The tenth Booke

Construction.

Demonstration.

and vnto it let the lyne B be commensurable either in length and in power, or in power onely. Then I say that B is a lyne making with a rationall superficies the whole superficies medially. Take a rational line C D, and vnto the line C D apply the parallelogramme C E equal to the square of the line A and making in breadth the lyne C F. Wherefore (by the 101. proposition) the lyne C F is a fifth residual lyne. Againe vnto the line F E apply the parallelogramme F G equal to the square of the line B, and making in breadth the lyne F H. Now forasmuch as the line A is commensurable to the lyne B, therefore the square of the lyne A is commensurable to the square of the lyne B. But vnto the square of the lyne A is equal the parallelogramme C E, and vnto the square of the lyne B is equal the parallelogramme F G. Wherefore the parallelogramme C E is commensurable to the parallelogramme F G. Wherefore the line C F is also commensurable in length to the line F H. But the line C F is a fifth residual line. Wherefore also the line F H is a fifth residual line. And the line F E is rational. But if a superficies be contained vnder a rational line and a fifth residual lyne, the lyne that containeth in power that superficies, is (by the 95. of the tenth) a lyne making with a rational superficies the whole superficies medially. But the lyne B containeth in power the parallelogramme F G. Wherefore the lyne B is a lyne making with a rational superficies, the whole superficies medially: which was required to be demonstrated.



¶ The 83. Theoreme.

The 107. Proposition.

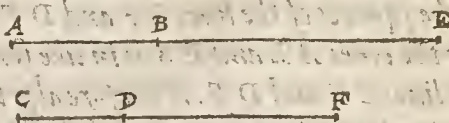
A line commensurable to a line, making with a medially superficies, the whole superficies medially, is it selfe also a line making with a medially superficies the whole superficies medially.

Construction.

Demonstration.



Suppose that A B be a line making with a medially superficies the whole superficies medially, vnto whome let the line C D be commensurable. Then I say that the line C D is also a line making with a medially superficies the whole superficies medially. For vnto the line A B let the line conveniently ioyned be B E. And let the rest of the construction be in this as it was in the former propositions. Wherefore the lines A E and B E are incommensurable in power, hauing that which is made of their squares added together medially, and that which is contained vnder them also medially, and moreover that which is made of their squares added together is incommensurable to that which is contained vnder them. But the lines A E and B E (as we haue before proued) are commensurable to the lines C F & D F, and that which is made of the squares of the lines A E and B E added together, is commensurable to that which is made of the squares of the lines C F and F D added together, and the parallelogramme contained vnder the lines A E and B E is commensurable to the parallelogramme contained vnder the lines C F and D F. Wherefore the lines C F and D F are incommensurable in power hauing that which is made of their squares added together medially, and that which is contained vnder them also medially, and moreover that which is made of their squares added together, is incommensurable to that which is contained vnder them. Wherefore the line C D is a line making with a medially superficies the whole superficies medially. A line therefore commensurable to a lyne making



making with a medially superficies the whole superficies medially, is it selfe also a line making with a medially superficies the whole superficies medially: which was required to be proued.

This proposition may also be another way demonstrated, as the three former propositions were. If vpon a rationally line you apply parallelogrammes equall to the squares of the lines AB and CD , the breadthes of which parallelogrammes shall be eche a sixth residual line, and therefore the lines which containe them in power, namely, the lines AB and CD shall be both such lines as is required in the proposition, which is easy to conclude marking the order of the demonstration in the three former propositions.

¶ The 84. Theoreme.

The 108. Proposition.

If from a rationally superficies be taken away a medially superficies, the line which containeth in power the superficies remainyng, is one of these two irrational lines, namely, either a residuall line, or a lesse line.

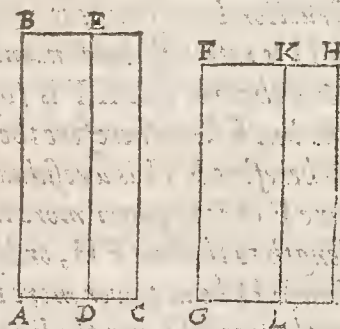


Vppose that BC be a rationally superficies, and from it take away a medially superficies, namely, BD . Then I say, that the line which containeth in power the superficies remainyng, namely, the superficies EC , is one of these two irrational lines, namely, either a residuall line, or a lesse line. Take a rationally line FG . And vpon FG describe (by the 44. of the first) a rectangle parallelogramme GH equall to the superficies BC . And from the parallelogramme GH take away the parallelogramme GK equall to the superficies BD . Wherefore (by the third common sentence) the superficies remainyng, namely, EC , is equall to the parallelogramme remainyng, namely, to LH . And forasmuch as BC is rationally, and BD is medially, and BC is equall to the parallelogramme GH , and BD to the parallelogramme GK : therefore GH is rationally, and GK is medially: and the parallelogramme GH is applied vnto the rationally line FG . Wherefore (by the 20. of the tenth) the line FH is rationally and commensurable in length to the line FG . And the parallelogramme GK is also applied vnto the rationally line FG . Wherefore (by the 22. of the tenth) the line FK is rationally and incommensurable in length to the line FG . Wherefore (by the Assumpt of the 12. of the tenth) the line FH is incommensurable in length to the line FK . And they are both rationally. Wherefore the lines FH and FK are rationally commensurable in power onely. Wherefore the line KH is a residuall line: and the line conveniently ioyned vnto it is KF . Now the line FH is in power more then the line KF , either by the square of a line commensurable in length to the line FH , or by the square of a line incommensurable in length to the line FH . First let it be in power more then the line KF , by the square of a line commensurable in length to the line FH , and the whole line FH is commensurable in length to the rationally line put, namely, to FG . Wherefore the line KH is a first residuall line. But if a superficies be contained vnder a rationally line, and a first residuall line, the line that containeth in power that superficies, is (by the 91. of the tenth) a residuall line. Wherefore the line which containeth in power LH , that is, the superficies EC , is a residuall line. But if the line FH be in power more then the line KF , by the square of a line incommensurable in length to the line FH , and the whole line FH is commensurable in length to the rationally line geuen FG . Wherefore the line KH is a fourth residuall line. But a line containing in power a superficies contained vnder a rationally line and a fourth residuall line, is a lesse line (by the 94. of the tenth). Wherefore the line that containeth in power the superficies LH , that is, the superficies EC , is a lesse line. If therefore from a rationally superficies be taken away a medially superficies, the line which containeth

Seuenth Senary.

Construction.

Demonstration.



The tenth Booke

in power the superficies remainyng, is one of these two irrationall lines, namely, either a residuall line, or a lesse line : which was required to be proued.

¶ The 85. Theoreme.

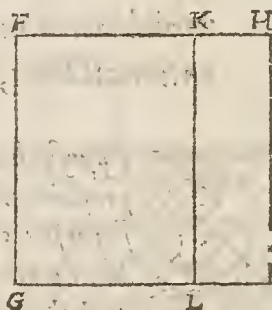
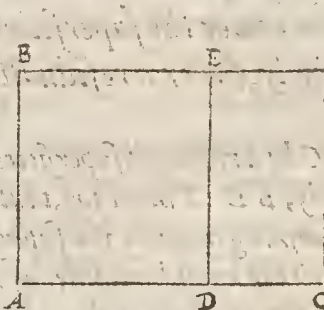
The 109. Proposition.

If from a medially superficies be taken away a rationally superficies, the line which containeth in power the superficies remainyng is one of these two irrationall lines, namely either a first medially residuall line, or a line making with a rationally superficies the whole superficies medially.



Suppose that B C be a medially superficies and from it take away a rationally superficies, namely,

BD. Then I say that the line which containeth in power the superficies remainyng, namely, the superficies EC is one of these two irrationall lines, either a first medially residuall line, or a line making with a rational



Construction.

Demonstration.

1.

2.

Take a rationally line FG, and let the rest of the construction be in this as it was in the former proposition. Wherefore it followeth that the line FH is rationally and incommensurable in length to the line FG (by the 22. of the tenth). And that the line KF is (by the 20. of the tenth) rationally and commensurable in length to the line FG. Wherefore the lines FH and FK are rationally commensurable in power onely. Wherefore KH is a residuall line. And the line conveniently ioyned vnto it is FK. Now the line FH is in power more then the line FK, either by the square of a line commensurable in length to the line FH, or by the square of a line incommensurable in length vnto it. If the line FH be in power more then the line FK by the square of a line commensurable in length to the line FH and the line conveniently ioyned vnto it, namely, FK, is commensurable in length to the rationally line FG. Wherefore the line KH is a second residuall line. And the line FG is a rationally line. But a line containyng in power a superficies comprehended vnder a rationally line and a second residuall line is (by the 92. of the tenth) a first medially residuall line. Wherefore the line that containeth in power the superficies LH, that is, the superficies CE is a first medially residuall line. But if the line HF be in power more then the line FK by the square of a line incommensurable in length to the line FH, and the line conveniently ioyned, namely, the line FK is commensurable in length to the rationally line put, namely, to FG: wherefore the line KH is a first residuall line. Wherefore (by the 95. of the tenth) the line that containeth in power the superficies LH, that is, the superficies EC, is a line making with a rationally superficies, the whole superficies medially: which was required to be proued.

¶ The 86. Theoreme.

The 110. Proposition.

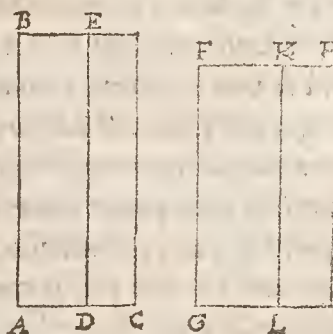
If from a medially superficies be taken away a medially superficies incommensurable to the whole superficies, the line which containeth in power the superficies which remaineth, is one of these two irrationall lines, namely, either a second medially residuall line, or a line making with a medially superficies the whole superficies medially.

As



*S*n the former descriptions, so here also takeaway fro the medially superficies BC a medially superficies BD , and let BD be incommensurable to the whole superficies BC . Then I say, that the line which containeth in power the superficies EC , is one of these two irrational lines, namely, either a second medially residuall line, or a line making with a medially superficies the whole superficies medially.

For forasmuch as either of these superficieses BC and BD is medially, and BC is incommensurable to BD , it followeth (by the 22. of the tenth) that either of these lines FH and FK is rationally and incommensurable in length to the line FG . And forasmuch as the superficies BC is incommensurable to the superficies BD , that is, the superficies GH , to the superficies GK , therefore (by the first of the sixth, & 10. of the tenth) the line FH is incommensurable in length to the line FK . Wherefore the lines HF and FK are rational commensurable in power onely. Wherefore (by the 73. of the tenth) the



Demonstration.

line KH is a residuall line, and the line conveniently ioyned vnto it is FK . Now the line HF is in power more then the line FK , either by the square of a line commensurable in length to the line HF , or by the square of a line incommensurable in length vnto it. If the line HF be in power more then the line FK , by the square of a line commensurable in length to the line FH , and neither of the lines HF nor FK is commensurable to the rational line put FG . Wherefore the line KH is a third residuall. But the line GF , that is, the line KL , is rational. And a rectangle superficies contained vnder a rational line and a third residuall line, is irrational, and the line which containeth in power that superficies, is (by the 93. of the tenth) a second medially residuall line. Wherefore the line that containeth in power the superficies LH , that is, the superficies EC is a second medially residuall line. But if the line HF be in power more then the line FK , by the square of a line incommensurable in length to the line FH , and neither of the lines HF nor FK is commensurable in length to the line FG . Wherefore the line HK is a sixth residuall line. But a line containing in power a superficies contained vnder a rational line and a sixth residuall line, is (by the 96. of the tenth) a line making with a medially superficies the whole superficies medially. Wherefore the line that containeth in power the superficies LH , that is, the superficies EC , is a line making with a medially superficies the whole superficies medially. If therefore from a medially superficies be taken away a medially superficies, incommensurable to the whole superficies, the line that containeth in power the superficies which remaineth, is one of the two irrational lines remaining, namely, either a second medially residuall line, or a line making with a medially superficies the whole superficies medially: which was required to be proued.

¶ The 87. Theoreme.

The 111. Proposition.

A residuall line, is not one and the same with a binomially lyne.

*S*uppose that AB be a residuall line. Then I say that AB is not one and the same with a binomially line. For if it be possible let it be a binomially line. And take a rational line DC . And (by the 44. of the first) vnto the line CD apply a rectangle parallelogramme CE equal to the square of the line AB , and making in bredth the line DE . And forasmuch as AB is a residuall line, therefore (by the 97. of the tenth) the line DE is a first residuall line. Let the line conveniently ioyned vnto it be EF . Wherefore the lines D

Construction.

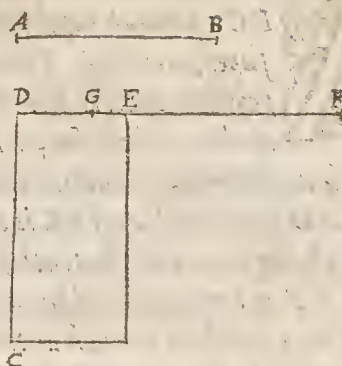
Demonstration leading to an impossibility.

Q. Q. i.

F and

The tenth Booke

F and *F E* are ratiōall commensurable in power onely, and the line *D F* is in power more then the line *F E* by the square of a line commensurable in length to the line *D E* & the line *D F*, is cōmensurable in length to the rational line put *D C*. Again forasmuch as *A B* is by position a binomial line, therefore (by the 60. of the tenth) the line *D E* is a first binomial line. Deuide it into his names in the point *G*. And let *D G* be the greater name. Wherefore the lines *D G* and *G E* are ratiōall commensurable in power onely. And the line *D G* is in power more then the line *G E* by the square of a line commensurable in length to the lyne



D G, and the line *D G* is commensurable in length to the rational line put *D C*. Wherefore the line *D F* is commensurable in length to the line *D G*. Wherefore (by the 15. of the tenth) the whole line *D F* is commensurable in length to the line remaining, namely, to the line *G F*. And forasmuch as the line *D F* is cōmensurable to the line *F G*, but the line *F D* is rational. Wherefore the line *FG* is also rational. And forasmuch as the line *F D* is commensurable in length to the line *F G*, but the line *D F* is incommensurable in length to the line *F E*. Wherefore the line *F G* is incommensurable in length to the line *F E* (by the 13. of the tenth) and they are both rational lines. Wherefore the lines *G F* and *F E* are rational commensurable in power onely. Wherefore (by the 73. of the tenth) the line *E G* is a residual line, but it is also rational (as before hath bene proued): which is impossible, namely, that one & the same line should be both rational and irrational. Wherefore a residual line is not one and the same with a binomial line, that is, is not a binomial line: which was required to be demonstrated.

A Corollary.

¶ A Corollary.

A residual lyne and the other five irrational lynes following it, are neither mediall lines, nor one and the same betwene themselues. that is, one is utterly of a diuers kinde frō an other. For the square of a mediall line applied to a rational line, maketh the breadth rational and incommensurable in length to the rational lyne, whereunto it is applied (by the 22. of the tenth) The square of a residual line applied to a rational line, maketh the breadth a first residual line (by the 97. of the tenth). The square of a first mediall residual line applied to a rational line, maketh the breadth a second residual lyne (by the 98. of the tenth) The square of a second mediall residual line applied vnto a rational line, maketh the breadth a third residual line (by the 99. of the tenth) The square of a lesse line applied to a rational line, maketh the breadth a fourth residual line (by the 100. of the tenth) The square of a line making with a rational superficies the whole superficies mediall applied to a rational line, maketh the breadth a fift residual line (by the 101. of the tenth) And the square of a line making with a mediall superficies the whole superficies mediall applied to a rational line, maketh the breadth a sixt residual line (by the 102. of the tenth) Now forasmuch as these foresaid sides which are the breadthes differ both from the first breadth, for that it is rational, and differ also the one frō the other, for that they are residuals of diuers orders and kindes, it is manifest that those irrational lines differ also the one from the other. And forasmuch as it hath bene proued in the 111. proposition, that a residual line is not one and the same with a binomial line, and it hath also bene proued that the squares of a residual line and of the five irrational lines that follow it being applied to a rational line do make their breadthes one of the residuals of that order of which they were, whose squares were applied to the rational line, likewise also the squares of a binomial line, and of the five irrational lines which follow it, being applied to a rational line, do make the breadthes one

of

of the binomials of that order of which they were, whose squares were applied to the rationall line. Wherefore the irrational lines which follow the binomial line, and the irrational lines which follow the residual line, differ the one from the other, so that all the irrational lines are 13 in number, namely, these.

- 1 A medially line.
- 2 A binomial line.
- 3 A first bimedially line.
- 4 A second bimedially line.
- 5 A greater line.
- 6 A line containing in power a rationall superficies and a medially superficies.
- 7 A line containyng in power two medially superficieses.
- 8 A residual line.
- 9 A first medially residual line.
- 10 A second medially residual line.
- 11 A lesse line.
- 12 A line making with a rationall superficies the whole superficies medially.
- 13 A line making with a medially superficies the whole superficies medially.

¶ The 88. Theoreme.

The 112. Proposition.

The square of a rationall line applyed vnto a binomial line, maketh the breadth or other side a residual line, whose names are commensurable to the names of the binomial line, & in the selfe same proportiō: & moreover that residual line is in the selfe same order of residual lines, that the binomial line is of binomial lines.



Suppose that A be a rationall line, and B C a binomial line whose greater name let be C D. And vnto the square of the line A let the parallelogramme containd vnder the lines B C and E F (so that E F be the breadth) be equall. Then I say that E F is a residual line, whose names are commensurable to the names of the

binomial line B C, which names let be C D and D B, and are in the same proportion with them: and moreover the line E F is in the selfe same order of residual lines, that the line B C is of binomial lines. Vnto the square of the line A let the parallelogramme containd vnder the lines B D and G be equal. Now forasmuch as that which is containd vnder the lines B C & E F is equal to that which is containd vnder the lines B D and G, therefore reciprocally (by the 14. of the sixth)

as the line C B is to the B D, so is the line G to the line E F. But the line B C is greater then the line B D, wherefore the line G is greater then the line E F. Vnto the line G let the line E H be equall. Wherefore (by the 11. of the fifth) as the line C B is to the line B D, so is the line H E to the line F E. Wherefore by deuision (by the 19. of the fifth) as the line C D is to the line B D, so is the line H F to the line F E. As the line H F is to the F E, so let the line F K be to the line K E (how this is to be done we will declare at the end of this demonstration). Wherefore (by the 12. of the fifth) the whole line H K is to the whole line K F as the line F K is to the line K E. For as one of the antecedentes is to one of the consequentes, so are all the antecedentes to all the consequentes. But as the line F K is the line K E, so is the line C D to the line D B (for F K is to E K as H F is to F E, and H F is to F E as C D is D B). Wherefore (by the 11.

The determination hath sundry partes orderly to be proued.

Construction.

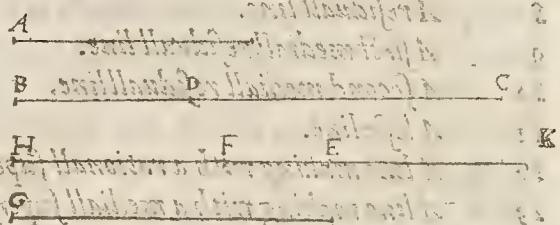
Demonstration.

This is an Assumpt, problematick, artificially used and demonstrated. *Therefore those three lines are in continuall proportion.

The tenth Booke

of the fifth) as the line HK is to the line KF , so is the line CD to the line DB . But the square of the line CD is commensurable to the square of the line DB : wherefore, (by the 10. of the tenth) the square of the line HK is commensurable to the square of the line FK . But these three lines HK , FK , and EK are proportionall in continuall proportion (as it hath already bene proued). Wherefore (by the second corollary of the 20. of the sixth) the square of the line HK is to the square of the line FK as the line HK is to the line EK : wherefore the line HK is commensurable in length to the line EK . Wherefore (by the 15. of the tenth) the line HE is commensurable in length to the line FK . And forasmuch as the square of the line A is equal to that which is containd vnder the lines EH and BD , but the square of the line A is rationally, wherefore that which is containd vnder the lines EH and BD is rationally. And it is applyed vnto the rationally line BD . Wherefore (by the 20. of the tenth) the line EH is rationally and commensurable in length to the line BD . Wherefore also the line EK which is commensurable in length to the line HE is

1. rational and commensurable in length to the line BD . Now for that as the line CD is to the line DB , so is the line FK to the line EK (for it was before proued, that as CD is to DB , so is HE to EE , and as HE is to FE , so is FK to EK) but the lines CD and



2. DB are commensurable in power onely, wherefore (by the 10. of the tenth) the lines FK and KE are also commensurable in power onely. And for that as the line CD is to the line DB , so is the line FK to the line EK , therefore by contrary proportion as DB is to CD , so is EK to FK , and alternately as DB is to EK , so is CD to FK : but the lines BD and EK are commensurable in length (as it hath already bene proued). Wherefore also the lines CD and FK are commensurable in length. But the line CD is rationally: wherefore also the line FK is rationally. Wherefore the lines FK and EK are rationally commensurable in power onely. Wherefore the line FE is a residuall line: whose names FK and KE are commensurable to the names CD and BD of the binomial line BC and in the same proportion as is proued.

FE concluded
a residuall
line, which is
somewhat pre-
posterously, in
respect of the
order propoun-
ded, both in 1.
the propositio,
and also in the
determinatio.

3. I say moreover that it is a residuall line of the selfe same order that the binomial line is. For the line CD is in power more then the line BD either by the square of a line commensurable in length to the line CD , or by the square of a line incommensurable in length. Now if the line CD be in power more then the line BD by the square of a line commensurable in length vnto the line CD , then (by the 13. of the tenth) the line FK is in power more then the line EK by the square of a line commensurable in length to the line FK . And so if the line CD be commensurable in length to the rationally line put, the line FK also shalbe commensurable in length to the same rationally line: wherefore then the line BC is a first binomial line, & the line FE is likewise a first residuall line. And if the line BD be commensurable in length to the rationally line, the line EK is also commensurable in length to the same, and then the line BC is a second binomial line, and the line FE a second residuall line. And if neither of the lines CD nor DB be commensurable in length vnto the rationally line, neither of the lines FK nor EK are commensurable in length vnto the same, and then the line BC is a third binomial line, & the line FE is a third residuall line. And if the line CD be in power more then the line BD , by the square of a line incommensurable in length to the line CD , the line FK is also (by the 14. of the tenth) in power more then the line EK by the square of a line incommensurable in length to the line FK . And so if the line CD be commensurable in length to a rationally line put, the line FK also is commensurable in length to the same, wherefore the line BC is a fourth binomial line, and the line FE is a fourth residuall line. And if the line BD be commensurable in length to the rationally line, the line EK is likewise commensurable in length to the same, and then the line BC is a fifth binomial line and the line

EF

EF a fifth residuall line. And if neither of the lines CD nor DB be commensurable in length to the rationall line, neither also of the lines FK nor EK is commensurable in length to the same, and then the line BC is a sixth binomial line, and the line FE a sixth residuall line. Wherefore the line FE is a residuall line, whose names, namely, FK and EK are commensurable to the names of the binomial line, namely, to the names CD and DB, and are in the selfe same proportion, and the residuall line EF is in the selfe same order of residuall lines, that the binomial line BC is of binomial lines. Wherefore the square of a rationall line applied vnto a binomial line maketh the breadth (or other side) a residuall line, whose names are commensurable to the names of the binomial line, and in the selfe same proportion, and moreover that residuall line is in the selfe same order of residuall lines, that the binomial line is of binomial lines: which was required to be demonstrated.

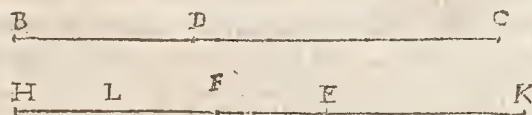
6.

✱ Here is the Assumpt (of the foregoing Proposition) confirmed.

Now let vs declare how as the line HF is to the line FE, so to make the line FK to the line EK. The line CD is greater then the line BD by supposition. Wherefore also the line HF is greater then the line FE (by alternate proportion, and the 14. of the fifth). From the line HF take away the line FL equall to the line FE. Wherefore the line remayning, namely, HL, is lesse then the line HF, for the line HF is equall to the lines HL & LF. As HL is to HF, so (by the 12. of the sixth) let FE be to FK. Wherefore by contrary proportion (by the Corollary of the 4. of the fifth) as HF is to HL, so is FK to FE. Wherefore by conuerſion of proportion (by the Corollary of the 19. of the fifth) as HF is to LF, that is, to the line equall vnto it, namely, to FE, so is the line FK to the line EK.

Construction.

Demonstration.



M. Dee, of this Assumpt, maketh ($\pi\omicron\rho\sigma\mu\alpha\lambda\iota\sigma\tau\epsilon\varsigma$, that is, Acquisitively,) a Probleme vniuersall, thus:

Two vnequall right lines being propounded, to adioyne vnto the lesse, a right line, which takē with the lesse (as one right line) shall haue the same proportion, to the line adioyned, which, the greater of the two propounded, hath to the lesse.

1.

The construction and demonstration hereof, is worde for worde to be taken, as it standeth here before: after these wordes: The line HF is greater then the line FE.

¶ A Corollary also noted by I. Dee.

It is therefore eident, that thus are three right lines (in our handling) in continuall proportion: it is to Weete, the greater, the lesse and the adioyned, make the first, the lesse with the adioyned, make the second: and the adioyned line is the third.

2.

This is proued in the beginning of the demonstration, after the Assumpt vsed.

✱ An other demonstration after Flussas.

Take a rationall line A, and let GB be a binomial line, whose greater line let be GD: and vpon the line GB apply (by the 45. of the first) the parallelograme BZ equall to the square of the line A, and making in breadth the line GZ. Likewise vpon the line DB (by the same) apply the parallelograme BI equall also to the square of the line A, and making in breadth the line DI: and put the line GZT equall to the line DI. Then I say, that GZ is such a residuall line as is required in the Proposition.

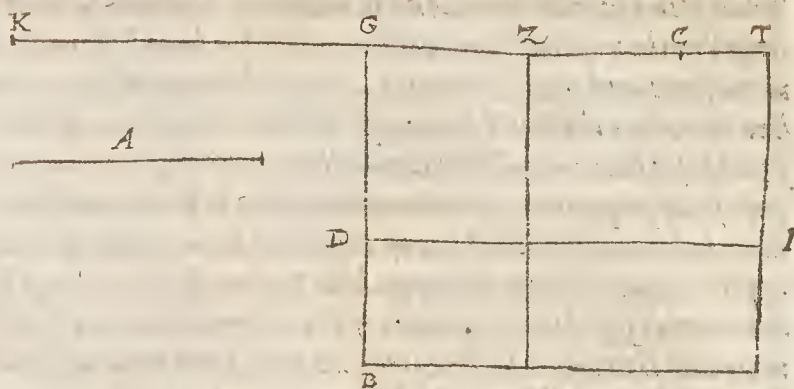
Construction.

QQ. iij.

Forasmuch

The tenth Booke

Forasmuch as the parallelogrammes BZ & BI are equall, therefore (by the 14. of the sixth) reciprocally as the line GB is to the line BD , so is the line DI or the line GT , (which is equall vnto it) vnto the line GZ . Wherefore by diuision, as the line GD is to the line DB , so is the line TZ to the line ZG (by the 17. of the fifth). Wherefore the line TZ is



Demonstration.

greater then the line ZG . (For the line GD is the greater name of the binomial line GB). Vnto the line ZG put the line ZC equall. And as the line TC is to the line TZ , so (by the 11. of the sixth) let the line ZG be to the line ZK . Wherefore contrarywise (by the Corollary of the 4. of the fifth) the line TZ is to the line TC , as the line ZK is to the line ZG . Wherefore by conuersion of proportion (by the 19. of the fifth) as the line TZ is to the line ZC (that is, to ZG , which is equall vnto it) so is the line ZK to the line KG . But the line TZ is to the line ZG , as the line GD is to the line DB . Wherefore (by the 11. of the fifth) the line ZK is to the line KG , as the line GD is to the line DB . But the lines GD and DB are commensurable in power onely. Wherefore also the lines ZK and KG are commensurable in power onely, by the 10. of this booke. Farther, forasmuch as the line TZ is to the line ZG , as the line ZK is to the line KG , therefore by the 12. of the fifth, all the antecedentes, namely, the whole line TK are to all the consequentes, namely, to the line KZ , as one of the antecedentes, namely, the line ZK is to one of the consequentes, namely, to the line KG . Wherefore the line ZK is the meane proportionall betwene the lines TK and KG . And therefore (by the Corollary of the 20. of the sixth) as the first, namely, the line TK , is to the third, namely, to the line KG : so is the square of the line TK to the square of the second, namely, of the line KZ . And forasmuch as the parallelograme BI (which is equall to the square of the rational line A) is applied vpon the rational line DB , it maketh the breadth DI rationall and commensurable in length vnto the line DB , by the 20. of the tenth. And therefore the line GT (which is equall vnto the line DI) is commensurable in length to the same line DB . And for that as the line GD is to the line DB , so is the line KZ to the line KG , but as the line KZ is to the line KG , so is the line TK to the line KZ , therefore (by the 11. of the fifth) as the line GD is to the line DB , so is the line TK to the line KZ . Wherefore (by the 22. of the sixth) as the square of the line GD is to the square of the line DB , so is the square of the line TK to the square of the line KZ . But the square of the line GD is commensurable to the square of the line DB (for the names GD and DB of the binomial line GB are commensurable in power). Wherefore the square of the line TK shall be commensurable to the square of the line KZ , by the 10. of this booke. But as the square of the line TK is to the square of the line KZ , so is it proued, that the right line TK is to the right line KG . Wherefore the right line TK is commensurable in length to the right line KG . Wherefore it is also commensurable in length to the line TG (by the 15. of the tenth). Which line TG is (as it hath bene proued) a rational line, and equall to the line DI . Wherefore the lines TK and KG are rationall commensurable in length. And forasmuch as it hath bene proued, that the line ZK is commensurable in power onely vnto the rational line KG , therefore the lines ZK and KG are rationall commensurable in power onely. Wherefore the line GZ is a residuall line. And forasmuch as the rational line TG is commensurable in length to either of these lines DB and KG . Wherefore the lines DB & KG shall be commensurable in length, by the 12. of the tenth. But the line ZK is to the line KG , as the line GD is to the line DB . Wherefore alternately, by the 16. of the fifth, the line KZ is to the line GD , as the line KG is to the line DB . Wherefore the line ZK is commensurable in length vnto the line GD . Wherefore the lines ZK and KG (the names of the residuall line GZ) are commensurable in length to the lines GD and DB , which are the names of the binomial line GB : and the line ZK is to the line KG in the same proportion, that the line GD is to the line DB . Wherefore if the whole line ZK be in power more then the line conueniently ioyned KG , by the square of a line commensurable in length to the line ZK , then the greater name GD shall be in power more then the lesse name DB , by the square of a line commensurable in length to the line GD , by the 14. of the tenth. And if the line ZK be in power more then the line KG , by the square of a line incommensurable in length to the line ZK , the line also GD shall be in power more then the line DB , by the square of a line incommensurable in length vnto the line GD (by the same Proposition). And if the greater or lesse name of the one be commensurable in length to the rational line put, the greater or lesse name also of the other shall be commensurable in length to the same rational line, (by the 12. of this booke). But if neither name of the one be commensurable in length to the rational line put, neither name of the other also shall be commensurable in length to the same rational line put (by the 13. of the same). Wherefore the residuall line GZ shall be in the selfe same order of residuall lines, that the binomial line GB is of binomial lines (by the definitions of residuall and binomial lines.

1.
Here are the
four partes
of the proposi-
tion more order-
ly handled 2.
then in the
former de- 3.
monstration.

lines. The square therefore of a rationall line applied to a binomial line. &c : which was required to be proued.

¶ The 89. Theoreme.

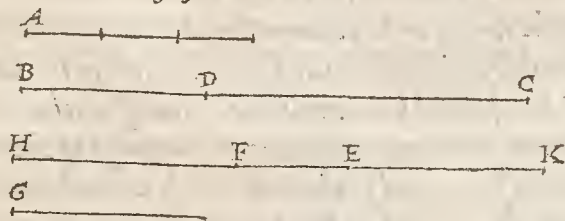
The 113. Proposition.

The square of a rational line applied vnto a residuall, maketh the breadth or other side a binomial line, whose names are commensurable to the names of the residuall line, and in the selfe same proportion: and moreouer that binomial line is in the selfe same order of binomial lynes, that the residuall line is of residuall lynes.



Suppose that A be a rationall line, and $B D$ a residuall line. And vnto the square of the line A let that which is contained vnder the lines $B D$ and $K H$ be equal. Wherfore the square of the rationall line A applied vnto the residuall line $B D$ maketh the breadth or other side $K H$. Then I say that the line $K H$ is a binomi-

all line, whose names are commensurable to the names of the residuall line $B D$, and in the selfe same proportion, and that the line $K H$ is in the selfe same order of binomial lines, that the line $B D$ is of residuall lines. Vnto the line $B D$ let the line conueniently ioyned be $D C$. Wherfore the lines $B C$ and



Construction.

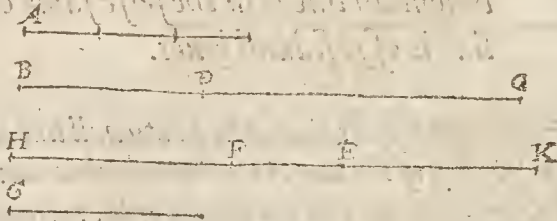
$D C$ are rationall commensurable in power onely. And vnto the square of the line A let the parallelogramme contained vnder the lines $B C$ and G be equall. But the square of the line A is rationall. Wherfore the parallelogramme contained vnder the lines $B C$ and G is also rationall. Wherfore also the line G is rationall and commensurable in length to the line $B C$ (by the 20. of the tenth). Now forasmuch as the parallelogramme contained vnder the lines $B C$ and G is equall to that which is contained vnder the lines $B D$ and $K H$, therefore (by the 16. of the sixth) as the line $B C$ is to the line $B D$, so is the line $K H$ to the line G . But the line $B C$ is greater then the line $B D$. Wherfore also the line $K H$ is greater then the line G . Vnto the line G let the line $K E$ be equall. Wherfore the line $K E$ is rationall and commensurable in length to the line $B C$, as also the line G was (by the 12. of the tenth) And for that as $B C$ is to $B D$, so is $K H$ to $K E$. Wherfore by conuersion of proportiō (by the corollary of § 19. of the fifth) as $B C$ is to $D C$, so is $K H$ to $E H$. As $K H$ is to $E H$, so let the line $F H$ be to the line $E F$ (how this is to be done, we will decare at the ende of this demonstration). Wherfore the residue $K F$ is to the residue $F H$, as the whole $K H$ is to the whole $H E$ (by the 19. of the fifth) that is, as the line $B C$ is to the line $C D$. But the lines $B C$ and $C D$ are commensurable in power onely. Wherfore also the lines $K F$ and $F H$ are commensurable in power only. And for that as $K H$ is to $H E$, so is $K F$ to $F H$, but as $K H$ is to $H E$, so is also $H F$ to $F E$; therefore as $K F$ is to $F H$, so is $F H$ to $F E$. Wherfore (by the corollary of the 19. of the sixth) as the first is to the third, so is the square of the first, to the square of the second. Wherfore as $K F$ is to $F E$, so is the square of the line $K F$ to the square of the line $F H$, but these squares are commensurable, for the lines $K F$ and $F H$ are commensurable in power. Wherfore the lines $K F$ and $F E$ are commensurable in length. Wherfore (by the second part of the 15. of the tenth) the lines $K E$ and $E F$ are commensurable in length. Wherfore (by the same) the lines $K F$ and $F E$ are commensurable in length. But the line $K E$ is rationall and commensurable in length to the line $B C$: wherfore the line $K F$ is also rationall and commensurable in length to the line $B C$. And for that as the line $B C$ is to the line $C D$, so is $K F$ to $F H$, therefore alternately (by the 16. of the fifth) as $B C$ is to $K F$, so is $C D$ to $F H$. But the line $B C$ is commensurable in length

Demonstration.

An Assumpe.

The tenth Booke

1. to the line $K F$. Wherefore the line $C D$ is commensurable in length to the line $F H$. But the
 2. line $C D$ is rationall. Wherefore also the line $F H$ is rationall. And the lines $B C$ and $C D$ are
 3. rationall commensurable in power onely. Wherefore the lines $K F$ and $F H$ are rationall com-
 4. mensurable in power onely. Wherefore the line $K H$ is a binomial line, whose names are com-
 mensurable to the names of the residuall line, and in the same proportion. I say moreover that
 it is a binomial of the selfe same order of binomial lines, that the line $B D$ is of residuall lines.
 For if the line $B C$ be in power more then the line $C D$ by the square of a line commensurable
 in length to the line $B C$, the line $K F$ is also in power more then the line $F H$ by the square
 of a line commensurable in length to the
 line $K F$ (by the 14. of the tenth). And if
 the line $B C$ be commensurable in length
 to the rationall line put, the line $K F$ is also
 (by the 12. of the tenth) commensurable
 in length to the rationall line, and so the
 lyne $B D$ is a first residuall lyne, and the
 line $K H$ is in like sort a first binomial line.



1. If the line $C D$ be commensurable in length to the rational line, the line $F H$ is also commen-
 2. surable in length to the same line, and so the line $B D$ is a second residuall line, and the line
 $K H$ a second binomial line. And if neither of the lines $B C$ nor $C D$ be commensurable in
 3. length to the rational line, neither also of the lines $K F$ nor $F H$ is commensurable in length
 to the same, and so the line $B D$ is a third residuall line, and the line $K H$ a third binomial
 line. But if the line $B C$ be in power more then the line $C D$ by the square of a line incommen-
 surable in length to the line $B C$, the line $K F$ is in power more then the line $F H$ by the square
 of a line incommensurable in length to the line $K F$ (by the 14. of the tenth). And if the line
 4. $B C$ be commensurable in length to the rational line put, the line $K F$ is also commensura-
 5. ble in length to the same line, and so the line $B D$ is a fourth residuall line, and the line $K H$
 a fourth binomial line. And if the line $C D$ be commensurable in length to the rational line, the
 line $F H$ is also commensurable in length to the same, & so the line $B D$ is a fifth residuall line, &
 the line $K H$ a fifth binomial line. And if neither of the lines $B C$ nor $C D$ be commensurable
 6. in length to the rational line, neither also of the lines $K F$ nor $F H$ is commensurable in
 length to the same, and so the line $B D$ is a sixth residuall line, and the line $K H$ is a sixth bino-
 mial line. Wherefore $K H$ is a binomial line, whose names $K F$ and $F H$ are commensurable
 to the names of the residuall line $B D$, namely, to $B C$ and $C D$, and in the selfe same propor-
 tion, and the binomial line $K H$ is in the selfe same order of binomial lines, that the residu-
 all $B D$, is of residuall lines. Wherefore the square of a rational line applied vnto a residuall
 line, maketh the breadth or other side a binomial line, whose names are commensurable to
 the names of the residuall line, and in the selfe same proportion, and moreover the binomial
 line is in the selfe same order of binomial lines, that the residuall line is of residuall lines:
 which was required to be demonstrated.

✻ The Assumpt confirmed.

Now let vs
 declare how, as
 the line $K H$ is to
 the line $E H$, so



to make the line $H F$ to the line $F E$. Adde vnto the line $K H$ directly a line equall to $H E$,
 and let the whole line be $K L$, and (by the tenth of the sixth) let the line $H E$ be divided as the
 whole line $K L$ is divided in the point H : let the line $H E$ be so divided in the point F . Where-
 fore

fore as the line KH is to the line HL , that is, to the line HE , so is the line HF to the line FE .

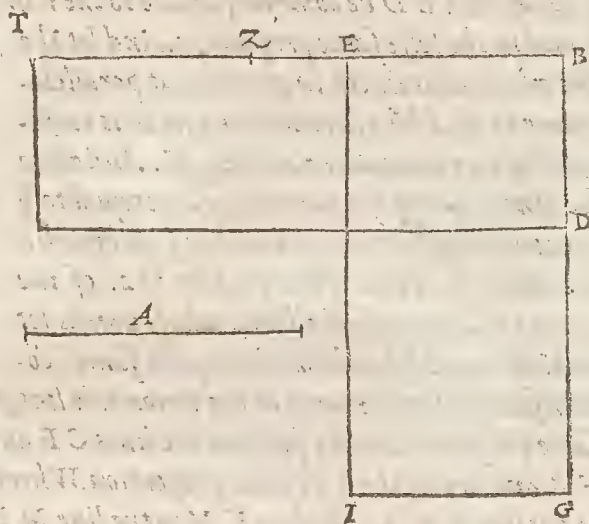
An other demonstration after Fluffus.

Suppose that A be a rationall line, and let BD be a residuall line. And vpon the line BD apply the parallelogramme DT equall to the square of the line A (by the 45. of the first) making in breadth the line BT . Then I say that BT is a binominall line such a one as is required in the proposition. Forasmuch as BD is a residuall line, let the line conveniently ioyned vnto it be GD . Wherefore the lines BG and GD are rationall commensurable in power onely. Vpon the rationall line BG apply the parallelogramme BY equall to the square of the line A and making in breadth the line BE . Wherefore the line BE is rationall and commensurable in length to the line BG (by the 20. of the tenth). Now forasmuch as the parallelogrammes BI and TD are equall (for that they are eche equall to the square of the line A): therefore reciprokally (by the 14. of the sixth) as the line BT is to the line BE , so is the line BG to the line BD . Wherefore by conuersion of proportion (by the corollary of the 19. of the fifth) as the line BT is to the line TE , so is the line BG to the line GD . As the line BG is to the line GD , so let the line TZ be to the line ZE by the corollary of the 10. of the sixth. Wherefore by the 11. of the fifth the line BZ is to the line TE , as the line TZ is to the line ZE . For either of them are as the line BG is to the line GD . Wherefore the residue BZ is to the residue ZT , as the whole BT is to the whole TE by the 19. of the fifth. Wherefore by the 11. of the fifth the line BZ is to the line ZT as the line ZT is to the line ZE . Wherefore the line TZ is the meane proportionall betwene the lines BZ and ZE . Wherefore the square of the first, namely, of the line BZ , is to the square of the second, namely, of the line ZT , as the first, namely, the line BZ , is to the third, namely, to the line ZE (by the corollary of the 20. of the sixth).

And for that as the line BG is to the line GD , so is the line TZ to the line ZE : but as the line TZ is to the line ZE , so is the line BZ to the line ZT . Wherefore as the line BG is to the line GD , so is the line BZ to the line ZT (by the 11. of the fifth). Wherefore the lines BZ and ZT are commensurable in power onely, as also are the lines BG and GD (which are the names of the residuall line BD) by the 10. of this booke. Wherefore the right lines BZ and ZE are comensurable in length, for we haue proued that they are in the same proportion that the squares of the lines BZ and ZT are. And therefore (by the corollary of the 13. of this booke) the residue BE (which is a rationall line) is commensurable in length vnto the same line BZ . Wherefore also the line BG (which is commensurable in length vnto the line BE) shall also be commensurable in length vnto the same line BZ (by the 12. of the tenth). And it is proued that the line BZ is to the line ZT commensurable in power onely. Wherefore the right lines BZ and ZT are rationall commensurable in power onely. Wherefore the whole line BT is a binomiall line (by the 36. of this booke). And for that as the line BG is to the line GD , so is the line BZ to the line ZT : therefore alternately (by the 16. of the fifth) the line BG is to the line BZ , as the line GD is to the line ZT . But the line BG is commensurable in length vnto the line BZ : Wherefore (by the 10. of this booke) the line GD is commensurable in length vnto the line ZT . Wherefore the names BG and GD of the residuall line BD are commensurable in length vnto the names BZ and ZT of the binomiall line BT : and the line BZ is to the line ZT in the same proportion that the line BG is to the line GD as before it was more manifest. And that they are of one and the selfe same order is thus proued. If the greater or lesse name of the residuall line, namely, the right lines BG or GD be comensurable in length to any rationall line put: the greater name also or lesse, namely, BZ or ZT shalbe commensurable in length to the same rationall line put by the 12. of this booke. And if neither of the names of the residuall line be commensurable in length vnto the rationall line put, neither of the names of the binomiall line shalbe commensurable in length vnto the same rationall line put (by the 13. of the tenth). And if the greater name BG be in power more then the lesse name by the square of a line commensurable in length vnto the line BG , the greater name also BZ shalbe in power more then the lesse by the square of a line commensurable in length vnto the line BZ . And if the one be in power more by the square of a line incommensurable in length, the other also shalbe in power more by the square of a line incommensurable in length by the 14. of this booke. The square therefore of a rationall line. &c. which was required to be proued.

An other demonstration after Fluffus. Construction.

Demonstration.



1.

2.

3.

4.

The

The tenth Booke

¶ The 90. Theoreme. The 114. Proposition.

This is in a
maner the con
uerse of both
the former pro
positions, ioynt
ly.

If a parallelograme be cotained vnder a residuall line & a binomiall lyne, whose names are commensurable to the names of the residuall line, and in the selfe same proportion: the lyne which containeth in power that superficies is rationally.



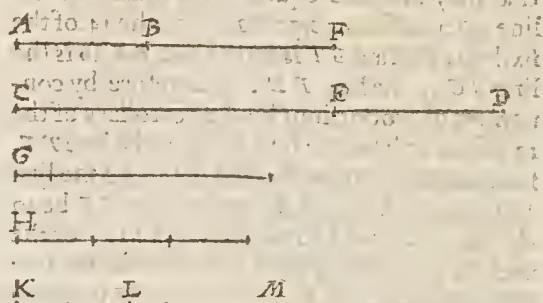
Suppose that a parallelogramme be contained vnder a residuall line AB and a binomiall line CD , and let the greater name of the binomiall line be CE , and the lesse name be ED , and let the names of the binomiall line, namely, CE and ED be commensurable to the names of the residuall line, namely, to AF and FB , and in the selfe same proportion. And let the

line which containeth in power that parallelograme be G . The I say that the line G is rational. Take a rational line, namely, H . And vnto the line CD apply a parallelograme equal to $\frac{1}{2}$ square of the line H , and making in breadth the line KL . Wherefore (by the 112. of the

Construction.

Demonstration.

tenth) KL is a residuall line, whose names let be KM and ML , which are (by the same) commensurable to the names of the binomiall line, that is to CE and ED , and are in the selfe same proportio. But by position the lines CE and ED are commensurable to the lines AF and FB , and are in the selfe same proportion. Wherefore (by the 12. of the tenth) as the line AF is to the line FB , so is the line KM to the line ML . Wherefore alternately (by the 16. of the fifth) as the line AF is to the line KM , so is the line BF to the line LM . Wherefore the residue AB is to the residue KL , as the whole AF is to the whole KM . But the line AF is commensurable to the line KM , for either of the lines AF and KM is commensurable to the line CE . Wherefore also the line AB is commensurable to the line KL . And as the line AB is to the line KL , so (by the first of the sixth) is the parallelogramme contained vnder the lines CD and AB to the parallelogramme contained vnder the lines CD and KL . Wherefore the parallelogramme contained vnder the lines CD and AB is commensurable to the parallelogramme contained vnder the lines CD and KL . But the parallelogramme contained vnder the lines CD and KL is equall to the square of the line H . Wherefore the parallelogramme contained vnder the lines CD & AB is commensurable to the square of the line H . But the parallelogramme contained vnder the lines CD and AB is equall to the square of the line G . Wherefore the square of the line H is commensurable to the square of the line G . But the square of the line H is rational. Wherefore the square of the line G is also rational. Wherefore also the line G is rational, and it containeth in power the parallelogramme contained vnder the lines AB and CD . If therefore a parallelogramme be contained vnder a residuall line and a binomiall line, whose names are commensurable to the names of the residuall line, and in the selfe same proportion, the line which containeth in power that superficies, is rational: which was required to be proued.



¶ Corollary.

Hereby it is manifest, that a rationally parallelogramme may be contained vnder irrational lines.

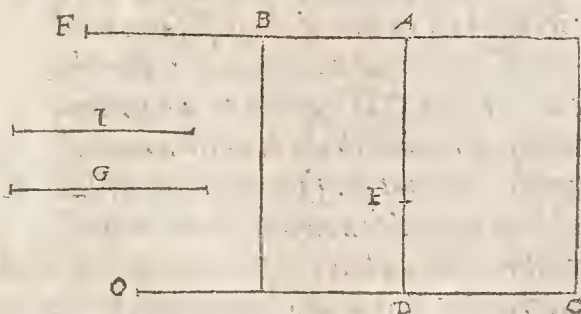
An

¶ An other demonstration after *Flußas*.

Suppose that the superficies DB be containd vnder a residuall line AB , and a binomial line AD : whose names AE and ED let be commensurable in length vnto the names of the residuall line AB , which let be AF and FB . And let the line AE be to the line ED , in the same proportion that the line AF is to the line FB . And let the right line ι containe in power the superficies DB . Then I say, that the line ι is a rationall line. Take a rationall line, which let be G . And vpon the line AD describe (by the 45. of the first) a parallelogramme equall to, the square of the line G , and making in breadth the line DC . Wherefore (by the 112. of this booke) CD is a residuall line, whose names (which let be CO and OD) shall be cōmensurable in lēgth vnto the names AE and ED , and the line CO shall be vnto the line OD , in the same proportion that the line AE is to the line ED . But as the line AE is to the line ED , so by supposition, is the line AF to the line FB . Wherefore as the line CO is to the line OD , so is the line AF to the line FB . Wherefore the lines CO and OD are commensurable with the lines AF and FB (by the 12. of this booke). Wherefore the residue, namely, the line CD is to the residue, namely, to the line AB , as the line CO is to the line AF (by the 19. of the fifth). But it is proued, that the line CO is cōmensurable vnto the line AF . Wherefore the line CD is commensurable vnto the line AB . Wherefore (by the first of the sixth) the parallelogramme CA is commensurable to the parallelogramme DB . But the parallelogramme CA is (by construction) rationall (for it is equall to the square of the rationall line G). Wherefore the parallelogramme DB is also rationall. Wherefore the line ι which by supposition cōtaineth in power the superficies DB , is also rationall. If therfore a parallelograme be containd. &c: which was required to be proued.

Construction.

Demonstration.



¶ The 91. Theoreme.

The 115. Proposition.

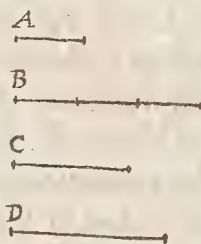
Of a medially line are produced infinite irrationall lines, of which none is of the selfe same kinde with any of those that were before.



Suppose that A be a medially line. Then I say, that of the line A may be produced infinite irrationall lines, of which none shall be of the selfe same kinde with any of those that were before. Take a rationall line B . And vnto that which is contained vnder the lines A and B let the square of the line C be equall (by the 14.

of the second). Wherefore the line C is irrationall. For a superficies contained vnder a rationall line and an irrationall line, is (by the Assumpt following the 38. of the tenth) irrationall: and the line which containeth in power an irrationall superficies, is (by the Assumpt going before the 21. of the tenth) irrationall. And it is not one and the selfe same with any of those thirtene that were before. For none of the lines that were before applied to a rationall line maketh the breadth medially.

Againe vnto that which is contained vnder the lines B and C , let the square of D be equall. Wherefore the square of D is irrationall. Wherefore also the line D is irrationall and not of the selfe same kinde with any of those that were before. For the square of none of the lines which were before, applied to a rationall line, maketh the breadth the line C . In like sort also shall it so followe, if a man proceede infinitely. Wherefore it is manifest, that of a medially line are produced infinite irrationall lines, of which none is of the selfe same kinde with any of those that were before: which was required to be proued.



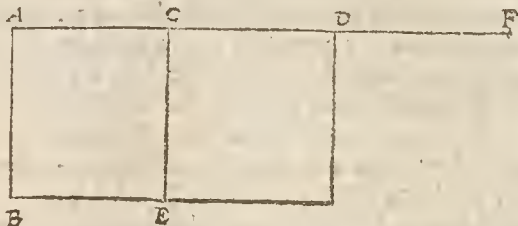
Demonstration.

The tenth Booke

An other demonstration.

Another demonstration.

Suppose that AC be a mediall line: Then I say, that of the line AC may be produced infinite irrationall lines, of which none shall be of the selfe same kinde with any of those irrationall lines before named. Vnto the line AC and from the point A , draw (by the 11. of the first) a perpendicular line AB , and let AB be a rationall line, and make perfecte the parallelogramme BC . Wherefore BC is irrationall, by that which was declared and proued (in maner of an Assumpt) in the end of the demonstration of the 38: and the line that containeth it in power is also irrationall. Let the line CD containe in power the superficies BC . Wherefore CD is irrationall & not of the selfe same kind with any of those that were before: for the square of the line CD applied to a rationall line, namely, AB , maketh the breadth a mediall line, namely, AC . But the square of none of the foresaid lines applied to a rationall line maketh the breadth a mediall line. Againe, make perfecte the parallelogramme ED . Wherefore the parallelogramme ED is also irrationall (by the sayd Assumpt in the end of the 38. his demonstration briefly proued) and the line which containeth it in power is irrationall: let the line which containeth it in power be DF . Wherefore DF is irrationall and not of the selfe same kinde with any of the foresaid irrationall lines. For the square of none of the foresaid irrationall lines applied vnto a rationall line, maketh the breadth the line CD . Wherefore of a mediall line are produced infinite irrationall lines, of which none is of the selfe same kinde with any of those that were before: which was required to be demonstrated.



¶ The 92. Theoreme.

The 116. Proposition.

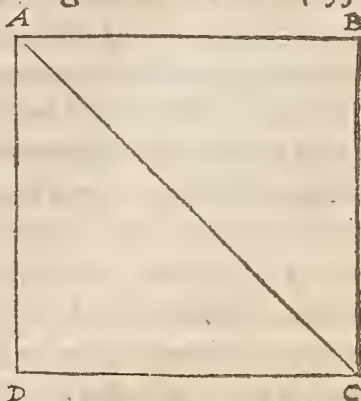
Now let vs proue that in square figures, the diameter is incommensurable in length to the side.

Demonstratio leading to an impossibilitie.



Suppose that $ABCD$ be a square, and let the diameter thereof be AC . Then I say that the diameter AC is incommensurable in length to the side AB . For if it be possible, let it be commensurable in length. I say that the this will follow, that one and the selfe same number shall be both an euen number & an odde number.

It is manifest (by the 47. of the first) that the square of the line AC is double to the square of the line AB . And for that the line AC is commensurable in length to the line AB (by supposition), therefore the lyne AC hath vnto the line AB that proportion that a number hath to a number (by the 5. of the tenth). Let the lyne AC haue vnto the line AB that proportion that the number EF hath to the number G . And let EF and G be the least numbers that haue one and the same proportion with them. Wherefore EF is not unitie. For if EF be unitie, and it hath to the number G that proportion that the line AC hath to the lyne AB , and the line AC is greater then the lyne AB . Wherefore unitie EF is greater then the number G , which is impossible. Wherefore FE is not unitie, wherefore it is a number. And for that as the square of the line AC is to the square of the lyne AB , so is the square number of the number EF , to the square number of the



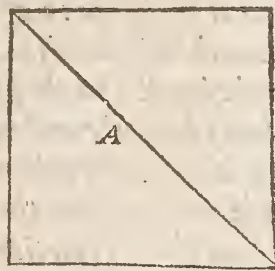
$F \dots H \dots E$
 $G \dots$

of the number G , for in eche is the proportion of their sides doubled (by the corollary of the 20 of the sixt and 11. of the eight): and the proportion of the line AC to the line AB doubled, is equal to the proportiō of the nūber EF to the number G , doubled, for as the line AC is to the line AB , so is the nūber EF to the number G . But the square of the line AC is double to the square of the line AB . Wherefore the square number produced of the number EF is double to the square number produced of the number G . Wherefore the square number produced of EF is an euen number. Wherefore EF is also an euen number. For if EF were an odde number, the square number also produced of it, should (by the 23. and 29. of the ninth) be an odde number. For if odde numbers how many soeuer be added together, and if the multitude of the be odde, the whole also shal be odde. Wherefore EF is an euen number. Deuide the number EF into two equall partes in H . And forasmuch as the numbers EF and G are the lest numbers in that proportion, therefore (by the 24. of the seuenth) they are prime numbers the one to the other. And EF is an euen number. Wherefore G is an odde number. For if G were an euen number, the number two should measure both the number EF and the number G (for e- uery euen nūber hath an halfe part by the definition) but these numbers EF & G are prime the one to the other. Wherefore it is impossible that they should be measured by two or by any other number besides vnitie. Wherefore G is an odde number. And forasmuch as the number EF is double to the number EH , therefore the square number produced of EF is quadruple to the square number produced of EH . And the square number produced of EF is double to the square number produced of G . Wherefore the square number produced of G is double to the square number produced of EH . Wherefore the square number produced of G is an euen number. Wherefore also by those thinges which haue bene before spoken, the number G is an euen number, but it is proued that it is an odde number, which is impossible. Wherefore the line AC is not commensurable in length to the line AB , wherefore it is incommensurable.

An other demonstration.

We may by an other demonstration proue, that the diameter of a square is incommensurable to the side thereof. Suppose that there be a square, whose diameter let be A and let the side thereof be B . Then I say that the line A is incommensurable in length to the line B . For if it be possible let it be commensurable in length. And agayne as the line A is to the line B so let the number EF be to the number G : and let them be the least that haue one and the same proportion with them: wherefore the numbers EF and G , are prime the one to the other. First I say that G is not v- nitie. For if it be possible let it be vnitie. And for that the square of the line A is to the square of the line B , as the square number produced of EF is to the square number produced of G (as it was proued in the former demonstration) but the square of the line A is double to the square of the line B . Wherefore the square nūber produced of EF is double to the square number produced of G . And by your supposition G is vnitie. Wherefore the square number produced of EF is the number two which is impossible. Wherefore G is not vnitie. Wherefore it is a number. And for that as the square of the line A is to the square of the line B , so is the square number produced of EF to the square number produced of G . Wherefore the square number produced of EF is double to the square number produced of G . Wherefore the square number produced of G measureth the square number produced of EF . Wherefore also (by the 14. of the eight) the number G measureth the number EF : and the number G also measureth it selfe. Wherefore the number G measureth these numbers EF and G , when yet they are prime the one to the other

An other demonstration leading to an impossibilitie.



$F \dots \dots E$
 $G \dots$

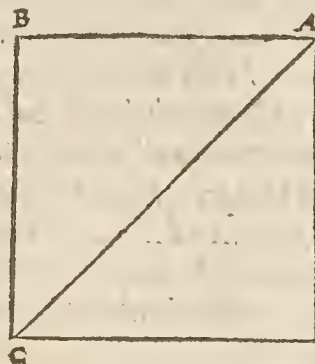
RR.i. other

The tenth Booke

other: which is impossible. Wherefore the diameter A is not commensurable in length to the side B. Wherefore it is incommensurable: which was required to be demonstrated.

An other demonstration after *Flussas*.

Suppose that vpon the line AB be described a square whose diameter let be the line AC. Then I say that the side AB is incommensurable in length vnto the diameter AC. Forasmuch as the lines AB and BC are equall, therefore the square of the line AC is double to the square of the line AB by the 47. of the first. Take by the 2. of the eight nũbers how many soeuer in continuall proportion frõ vnitie, and in the proportion of the squares of the lines AB and AC. Which let be the numbers D, E, F, G. And forasmuch as the first from vnitie namely E is no square number, for that it is a prime number, neither is also any other of the sayd numbers a square number except the third from vnitie and so all the rest leuing one betwene, by the 10. of the ninth. Wherefore D is to E, or E to F, or F to G, in that proportion that a square number is to a number not square. Wherefore by the corollary of the 25. of the eight, they are not in that proportion the one to the other that a square number is to a square number. Wherefore neither also haue the squares of the lines AB and AC (which are in the same proportion) that proportion that a square number hath to a square number. Wherefore by the 9. of this booke their sides, namely, the side AB and the diameter AC are incommensurable in length the one to the other which was required to be proued.

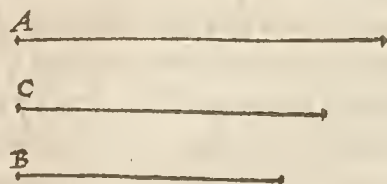


D .
E ..
F
G

This demonstration I thought good to adde, forthat the former demonstrations seme not so full, and they are thought of some to be none of *Theons*, as also the proposition to be none of *Euclides*.

Here followeth an instruction by some studious and skilfull Grecian
(perchance *Theon*) which teacheth vs of farther vse and
fruite of these irrational lines.

Seing that there are founde out right lines incommensurable in length the one to the other, as the lines A and B, there may also be founde out many other magnitudes hauing length and breadth (such as are playne superficieses) which shalbe incommensurable the one to the other. For if (by the 13. of the sixth) betwene the lines A and B there be taken the meane proportionall line, namely C, then (by the second corollary of the 20. of the sixth) as the line A is to the line B, so is the figure described vpon the line A to the figure described vpon the line C, being both like and in like sort described, that is, whether they be squares (which are alwayes like the one to the other), or whether they be any other like rectiline figures, or whether they be circles about the diameters A and C. For circles haue that proportion the one to the other, that the squares of their diameters haue (by the 2. of the twelfth). Wherefore (by the second part of the 10. of the tenth) the figures described vpon the lines A and C being like and in like sort described are incommensurable the one to the other. Wherefore by this meanes there are founde out superficieses incommensurable the one to the other. In like sort there may be founde out figures commensurable the one to the other, if ye put the lines A and B to be commensurable in length the one to the other. And seing that it is so, now let vs also proue that euen in solides also or bodyes there are some commensurable the one to the other, and other some incommensurable the one to the other. For if from eche of the squares of the lines A and B, or from any other rectiline figures equal to these squares



be erected solides of equall altitude, whether those solides be composed of equidistant superficieses; or whether they be pyramids, or prisms, those solides so erected shalbe in that proportion the one to the other that theyr bases are (by the 32. of the eleventh and 5. and 6. of the twelfth). Howbeit there is no such proposition concerning prisms. And so if the bases of the solides be commensurable the one to the other, the solides also shal be commensurable the one to the other, and if the bases be incommensurable the one to the other, the solides also shal be incommensurable the one to the other (by the 10. of the tenth). And if there be two circles A and B. and upon each of the circles be erected Cones or Cilinders of equal altitude, those Cones & Cilinders shal be in that proportion the one to the other that the circles are, which are their bases (by the 11. of the twelfth): and so if the circles be commensurable the one to the other, the Cones and Cilinders also shal be commensurable the one to the other. But if the circles be incommensurable the one to the other, the Cones also and Cilinders shal be incommensurable the one to the other, (by the 10. of the tenth). Wherefore it is manifest that not onely in lines and superficieses, but also in solides or bodies is found commensurabilitie or incommensurability.

An aduertisement by Iohn Dee.

Although this proposition were by Euclide to this booke allotted, (as by the auncient grecian published vnder the name of *Aristoteles* *Περὶ ἀριθμῶν γενικῶν*, it would seme to be, and also the property of the same, agreable to the matter of this booke, and the proposition it selfe, so famous in Philosophy and Logicke, as it was, would in maner craue his elemental place, in this tenth booke) yet the dignitie & perfection of Mathematicall Method can not allow it here: as in due order following: But most aptly after the 9. proposition of this booke, as a Corollary of the last part thereof. And vndoubtedly the proposition hath for this 2000. yeares bene notably regarded among the greke Philosophers: and before *Aristotles* time was concluded with the very same inconuenience to the gaynefayer, that the first demonstration here induceth, namely, *Odde number to be equal to euen*: as may appeare in *Aristotles* worke, named *Analitica prima*, the first booke and 40. chapter. But els in very many places of his workes he maketh mention of the proposition. Eident also it is that *Euclide* was about *Aristotles* time, and in that age the most excellent Geometrician among the Grekes. Wherefore, seing it was so publike in his time, so famous, and so appertayning to the property of this booke: it is most likely, both to be knowne to *Euclide*, and also to haue bene by him in apt order placed. But of the disordring of it, can remayne no doubt, if ye consider in *Zamberts* translation, two other propositions going next before it, so farre misplaced, that where they are, word for word, before duely placed, being the 105. and 106. yet here (after the booke ended), they are repeated with the numbers of 116. and 117. proposition. *Zambert* therein was more faythfull to follow as he found in his greke example, than he was skilfull or carefull to doe what was necessary.

Yea and some greke written auncient copyes haue them not so: Though in deede they be well demonstrated, yet truth disordred, is halfe disgraced: especially where the patterne of good order, by profession is auouched to be. But through ignorance, arrogancy and temeritie of vnskilfull Methode Masters, many thinges remayne yet, in these Geometricall Elementes, vnduely tumbled in:

though true, yet with disgrace: which by helpe of so

many wittes and habilitie of such, as now may

haue good cause to be skilfull herein,

will (I hope) ere long be taken a-

way: and thinges of impor-

tance (wanting) sup-

plied.

RR. ij.

The end of the tenth booke
of Euclides Elementes



The eleuenth booke of Euclides Elementes.

The argument
of the eleuenth
booke.

A point the be-
ginning of all
quantitie con-
tinuall.
The methode
vsed by Euclide
in the ten for-
mer bookes.
First booke.
Second booke.

Third booke.
Fourth booke.

Fifth booke.

Sixth booke.

Seuenth booke
Eight booke.
Ninth booke.

Tenth booke.

What is entrea-
ted of in the
fise bookes fol-
lowing.
Five regular bo-
dies, the finall
ende & scope of
Euclides Geo-
metricall Ele-
mentes.
Comparison of
the first booke
and this eleuenth
booke together.



HITHERTO HATH EVCLIDE IN THESE former bookes with a wonderfull Methode and order entreated of such kindes of figures superficial, which are or may be described in a superficies or plaine. And hath taught and set forth their properties, natures, generations, and productions euen from the first roote, ground, and beginning of them: namely, from a point, which although it be indiuisible, yet is it the beginning of all quantitie, and of it and of the motion and flowing thereof is produced a line, and consequently all quantitie continuall, as all figures playne and solide what so euer. *Euclide* therefore in his first booke began with it, and from thence went he to a line, as to a thing most simple next vnto a point, then to a superficies, and to angles, and so through the whole first booke, he intreated of these most simple and plaine groundes. In the second booke he entreated further, and went vnto more harder matter, and taught of diuisions of lines, and of the multiplication of lines, and of their partes, and of their passions and properties. And for that rightlined figures are far distant in nature and propertie from round and circular figures, in the third booke he instructeth the reader of the nature and condition of circles. In the fourth booke he compareth figures of right lines and circles together, and teacheth how to describe a figure of right lines with in or about a circle: and contrariwise a circle with in or about a rectiline figure. In the fifth booke he searcheth out the nature of proportion (a matter of wonderfull vse and deepe consideration), for that otherwise he could not compare figure with figure, or the sides of figures together. For whatsoeuer is compared to any other thing, is compared vnto it vndoubtedly vnder some kinde of proportion. Wherefore in the sixth booke he compareth figures together, one to an other, likewise their sides. And for that the nature of proportion, can not be fully and clearely sene without the knowledge of number, wherein it is first and chiefly found: in the seuenth, eight, and ninth bookes, he entreateth of number, & of the kindes and properties thereof. And because that the sides of solide bodyes, for the most part are of such sort, that compared together, they haue such proportion the one to the other, which can not be expressed by any number certayne, and therefore are called irrational lines, he in the tenth booke hath writté & taught which lines are comensurable or incomensurable the one to the other, and of the diuersitie of kindes of irrational lines, with all the conditions & proprieties of them. And thus hath *Euclide* in these ten foresayd bookes, fully & most pléteously in a meruelous order taught, whatsoeuer semed necessary, and requisite to the knowledge of all superficial figures, of what sort & forme so euer they be. Now in these bookes following he entreateth of figures of an other kinde, namely, of bodely figures: as of Cubes, Pyramids, Cones, Columnes, Cilinders, Parallelipedons, Spheres and such others: and sheweth the diuersitie of the, the generation, and production of them, and demonstrateth with great and wonderfull art, their proprieties and passions, with all their natures and conditions. He also compareth one of them to an other, whereby to know the reason and proportion of the one to the other, chiefly of the fise bodyes which are called regular bodyes. And these are the thinges of all other entreated of in Geometrie, most worthy and of greatest dignitie, and as it were the end and finall entent of the whole are of Geometrie, and for whose cause hath bene written, and spoken whatsoeuer hath hitherto in the former bookes bene sayd or written. As the first booke was a ground, and a necessary entrie to all the rest following, so is this eleuenth booke a necessary entrie and ground to the rest which follow. And as that contayned the declaration of wordes, and definitions of thinges requisite

requisite to the knowledge of superficial figures, and entreated of lines (and of their diuisions and sections) which are the termes and limites of superficial figures: so in this booke is set forth the declaration of wordes and definitions of thinges pertayning to solide and corporall figures: and also of superficieses which are the termes & limites of solides: morcouer of the diuision and intersection of them, and diuers other thinges, without which the knowledge of bodely and solide formes can not be attayned vnto. And first is set the definitions as followeth.

Definitions.

A solide or body is that which hath length, breadth, and thicknes, and the terme or limite of a solide is a superficies.

First definition

There are three kindes of continuall quantitie, a line, a superficies, and a solide or body: the beginning of all which (as before hath bene sayd) is a poynt, which is indiuisible. Two of these quantities, namely, a line and a superficies, were defined of *Euclide* before in his first booke. But the third kinde, namely, a solide or body he there defined not, as a thing which pertayned not then to his purpose: but here in this place he setteth the definitiō therof, as that which chiefly now pertayneth to his purpose, and without which nothing in these thinges can profitably be taught. *A solide* (sayth he) *is that which hath length, breadth, and thicknes, or depth.* There are (as before hath bene taught) three reasons or meanes of measuring, which are called cōmonly dimensions, namely, length, breadth, and thicknes. These dimensions are ascribed vnto quantities onely. By these are all kindes of quantitie defined, & are counted perfect or imperfect, according as they are pertaker of fewer or more of them. As *Euclide* defined a line, ascribing vnto it onely one of these dimensions, namely, length: Wherefore a line is the imperfectest kinde of quantitie. In defining of a superficies, he ascribed vnto it two dimensions, namely, length, and breadth: whereby a superficies is a quantitie of greater perfection then is a line, but here in the definitiō of a solide or body, *Euclide* attributeth vnto it all the three dimensions, length, breadth, and thicknes. Wherefore a solide is the most perfectest quantitie, which wanteth no dimension at all, passing a lyne by two dimensions, and passing a superficies by one. This definition of a solide is without any designation of forme or figure easily vnderstanded, onely conceiuing in minde, or beholding with the eye a piece of timber or stone, or what matter so euer els, whose dimensions let be equall or vnequall. For example let the length therof be 5. inches, the breadth 4. and the thicknes 2. if the dimensions were, equall, the reason is like, and all one, as it is in a Sphere and in a cube. For in that respect and consideration onely, that it is long, broade, and thicke, it beareth the name of a solide or body, and hath the nature and properties therof. There is added to the ende of the definition of a solide, that the terme and limite of a solide is a superficies. Of thinges infinitie there is no Arte or Science. All quantities therfore in this Arte entreated of, are imagined to be finite, and to haue their endes and borders as hath bene shewed in the first booke, that the limites and endes of a line are pointes, and the limites or borders of a superficies are lines, so now he saith that the endes, limites, or borders of a solide are superficieses. As the side of any square piece of timber, or of a table, or die, or any other like, are the termes and limites of them.

A solide the most perfect quantitie.

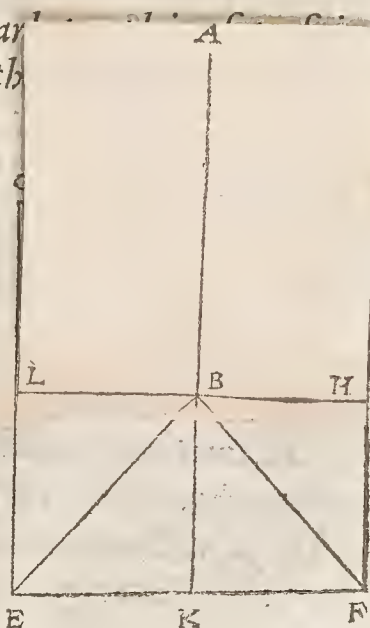
No science of thinges infinite.

2 *A right line is then erected perpendicular, when the right line maketh right angles with all the drawen vpon the ground plaine superficies.*

where
d are

Second definition.

Suppose that vpon the grounde playne superficies *CDEF* from the point *B* be erected a right line, namely, *BA*, so that let the point *A* be a loft in the ayre. Drawe also from the poynte *B* in the playne superficies *CDEF*, as many right lines as ye list, as the lines *BC*, *BD*, *BE*, *BF*, *BG*, *BK*, *BH*, and *BL*. If the erected line *BA* with all these lines drawen in the superficies *CDEF* make a right angle, so that all these angles *ABC*, *ABD*, *ABE*, *ABF*, *ABG*, *ABK*, *ABH*, *ABL*, and so of others, be right angles, then by this definition, the line *AB* is a line erected vpon the superficies *CDEF*: it is also called commonly a perpendicular line or a plumb line, vnto or vpon a superficies.



requisite to the knowledge of superficial figures, and entreated of lines (and of their diuisions and sections) which are the termes and limites of superficial figures: so in this booke is set forth the declaration of wordes and definitions of thinges pertayning to solide and corporall figures: and also of superficieses which are the termes & limites of solides: moreouer of the diuision and interfection of them, and diuers other thinges, without which the knowledge of bodely and solide formes can not be attayned vnto. And first is set the definitions as followeth.

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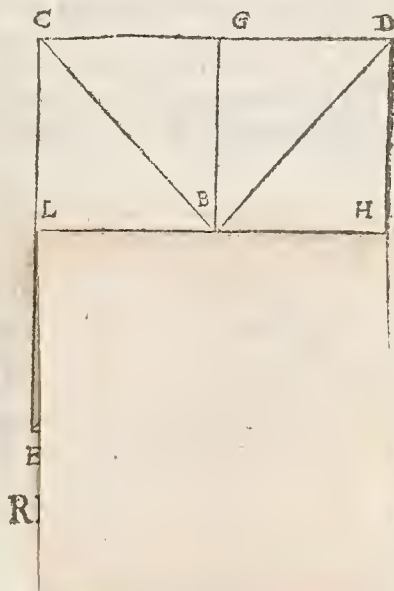
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No science of thinges infinite.

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Suppose that vpon the grounde playne superficies $CDEF$ from the pointe B be erected a right line, namely, BA , so that let the point A be a loft in the ayre. Drawe also from the poynte B in the playne superficies $CDEF$, as many right lines as ye list, as the lines BC , BD , BE , BF , BG , BK , BH , and BL . If the erected line BA with all these lines drawn in the superficies $CDEF$ make a right angle, so that all these angles ABC , ABD , ABE , ABF , ABG , ABK , ABH , ABL , and so of others, be right angles, then by this definition, the line AB is a line erected vpon the superficies $CDEF$: it is also called commonly a perpendicular line or a plumb line, vnto or vpon a superficies.



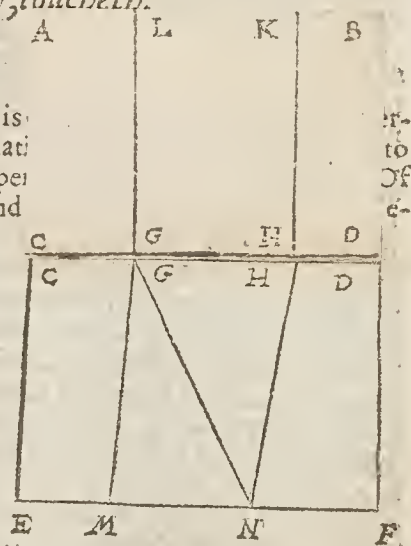
The eleventh Booke

Third defini-
tion.

3 A plaine superficies is then vpright or erected perpendicularly to a plaine superficies, when all the right lines drawn in one of the plaine superficieses vnto the common section of those two plaine superficieses, making therewith right angles, do also make right angles to the other plaine superficies. Inclination or leaning of a right line, to a plaine superficies, is an acute angle, contained vnder a right line falling from a point aboue to the plaine superficies, and vnder an other right line, from the lower end of the sayd line (let downe) drawn in the same plaine superficies, by a certaine point assigned, where a right line from the first point aboue, to the same plaine superficies falling perpendicularly, toucheth.

Two defini-
tions included
in this defi-
nition.
Declaratiō of
the first part.

In this third definition are included two definitions: the first is perpendicularly vpon a plaine superficies. The second is of the inclination a superficies: of the first take this example. Suppose ye haue two superficieses which let the superficies CDEF be a ground plaine superficies, and erected vnto it, and let the line CD be a common terme or intersection to them both, that is, let it be the end or bound of either of them, & be drawn in either of them: in which line note at pleasure certaine pointes, as the point G, H. From which pointes vnto the line CD, draw perpendicular lines in the superficieses ABCD, which let be GL and HK, which falling vpon the superficieses CDEF, if they cause right angles with it, that is, with lines drawn in it from the same pointes G and H, as if the angle LGM or the angle LGN contained vnder the line LG drawn in the superficieses erected, and vnder the GM or GN drawn in the ground superficieses CDEF, lying flat, be a right angle, then by this definition, the superficieses ABCD is vpright or erected vpon the superficieses CDEF. It is also commonly called a superficies perpendicular vpon or vnto a superficies.



Declaration
of the second
part.

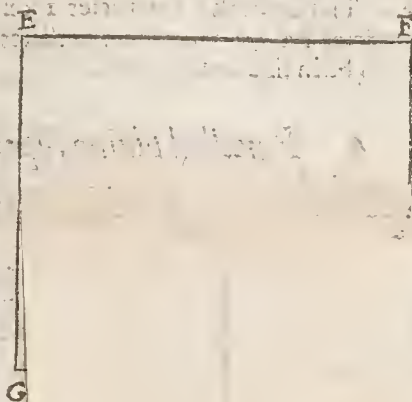
For the second part of this definition, which is of the inclination of a right line vnto a plaine superficies, take this example. Let ABCD be a ground plaine superficies, vpon which from a point being a loft, namely, the point E, suppose a right line to fall, which let be the line EG, touching the plaine superficies ABCD at the poynt G. Again, from the point E, being the toppe or higher limite and end of the inclining line EG, let a perpendicular line fall vnto the plaine superficies ABCD, which let be the line EF, and let F be the point where EF toucheth the plaine superficies ABCD. Then from the point of the fall of the line inclining vpon the superficies vnto the point of the falling of the perpendicular line vpon the same superficies, that is, from the point G to the point F, draw a right line GF. Now by this definition, the acute angle EGF is the inclination of the line EG vnto the superficies ABCD. Because it is contained of the inclining line, and of the right line drawn in the superficies, from the point of the fall of the line inclining to the point of the fall of the perpendicular line: which angle must of necessitie be an acute angle. For the angle EFG is by construction a right angle, and three angles in a triangle. Wherefore the other two angles, namely, the angles EGF, and GEF. Wherefore either of them is lesse then a right angle. Wherefore the angl

Fourth defi-
nition.

4 Inclination of a plaine superficies to a plaine superficies, is an acute angle contained vnder the right lines, which being drawn in either of the plaine superficieses to one & the self same point of the cōmon section, make with the section right angles.

Suppose

Suppose that there be two superficieses $A B C D$ & $E F G H$, and let the superficies $A B C D$ be supposed to be erected not perpendicularly, but somewhat leaning and inclining vnto the plaine superficies $E F G H$, as much or as little as ye will: the comon terme or section of which two superficieses let be the line $C D$. From some one point, as from the point M assigned in the common section of the two superficieses, namely, in the line $C D$, draw a perpendicular line in either superficies. In the ground superficies $E F G H$ draw the line $M K$, and in the superficies $A B C D$ draw the line $M L$. Now if the angle $L M K$ be an acute angle, then is that angle the inclination of the superficies $A B C D$ vnto the superficies $E F G H$, by this definition, because it is contained of perpendicular lines drawn in either of the superficieses to one and the self same point being the common section of them both.

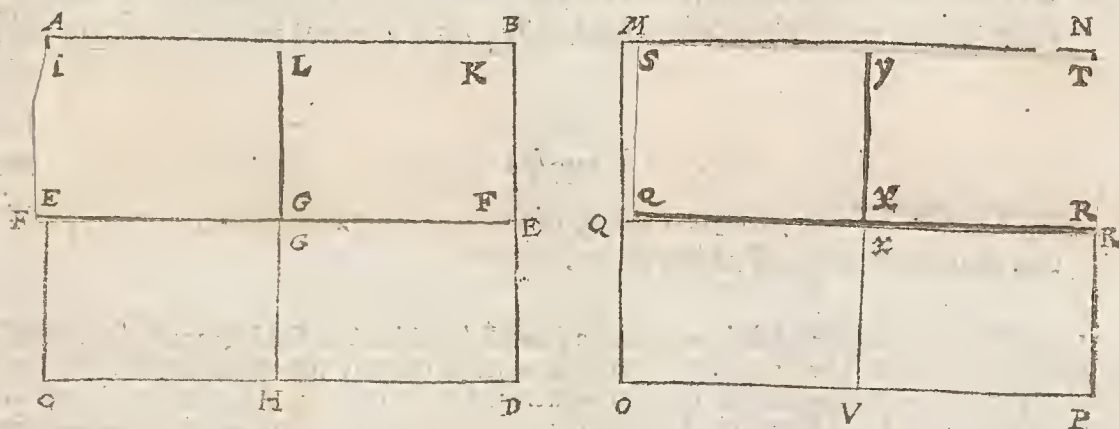


s Plaine superficieses are in like sort inclined
the sayd angles of inclination are equall the one to the other.

n Fifth definition.

This definition needeth no declaration at all, but is most manifest by the definition last going before. For in considering the inclinations of diuers superficieses to others, if the acute angles containd vnder the perpendicular lines drawn in them from one point assigned in ech of their common sections be equall, as if to the angle $L M K$ in the former example be geuen an other angle in the inclination of two other superficieses equall, then is the inclination of these superficieses like, and are by this definition sayd in like sort to incline the one to the other.

Theodosius geueth an other definition of like inclination of plaine superficieses the one to the other, after this maner. One plaine superficies is like inclined to an other, as an other superficies is to an other, when in either of the plaine superficieses right lines being drawn, and making right angles with their common section, containe in the same point equall angles. This definition is in substance the same with that geuen of Euclide, and is an elucidation of it. For example let $A B C D$ be a ground plaine superficies, vnto which let the superficies $E F I K$ incline and leane. And let the common section of these two superficieses be the line $E F$. Then drawe in eche of these superficieses right lines to some one point of the common section $E F$, which let be the point G : with which section let them make right angles. As in the super-



ficies $A B C D$ draw the line $H G$, which in the point G let it make with the common section a right angle $H G F$ or $H G E$. Also in the superficies $E F I K$ draw the line $L G$, which in the point G together with the common section $E F$ let make also a right angle $L G F$, or the right angle $L G E$.

Now also let there be an other ground plaine superficies, namely, the superficies $M N O P$, vnto whom also let leane and incline the superficies $Q R S T$, and let the common section or segment of them be the line $Q R$. And draw in the superficies $M N O P$ to some one point of the comon section as to the point x the line $v x$, making with the common section right angles, namely, the angle $v x R$, or the angle $v x Q$: also in the superficies $S T Q R$ draw the right line $r x$ to the same point x in the common section, making therewith right angles, as the angle $r x R$, or the angle $r x Q$. Now (as sayth the definition) if the angles containd vnder the right lines drawn in these superficieses & making right angles with the common section, be in the pointes, that is, in the pointes of their meting in the common section, equall: then is the inclination of the superficieses equall. As in this example, if the angle $L G H$ containd vnder the line $L G$ being in the inclining superficies $I K E F$ and vnder the line $H G$ being in the ground superficies $A B C D$, be equall, to the angle $r x v$ containd vnder the line $v x$ being in the

The eleventh Booke

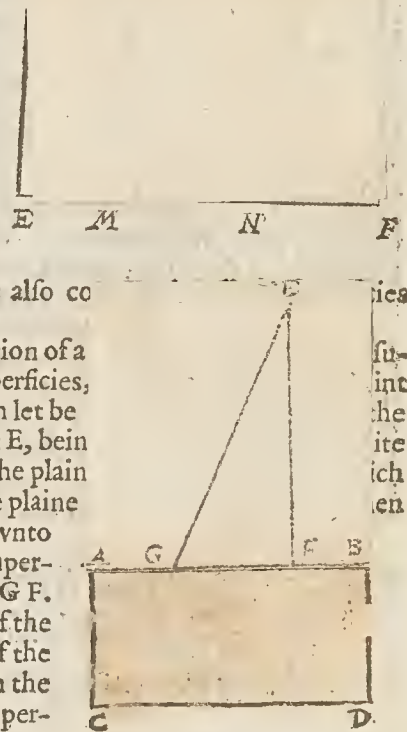
Third defini-
tion.

3. *A plaine superficies is then vpright or erected perpendicularly to a plaine superficies, when all the right lines drawn in one of the plaine superficieses vnto the common section of those two plaine superficieses, making therewith right angles, do also make right angles to the other plaine superficies. Inclination or leaning of a right line, to a plaine superficies, is an acute angle, contained vnder a right line falling from a point aboue to the plaine superficies, and vnder an other right line, from the lower end of the sayd line (let downe) drawn in the same plaine superficies, by a certaine point assigned, where a right line from the first point aboue, to the same plaine superficies falling perpendicularly, toucheth.*

Two defini-
tions included
in this defini-
tion.

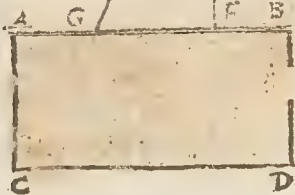
Declaratiō of
the first part.

In this third definition are included two definitions: the first is of a plaine superficies erected perpendicularly vpon a plaine superficies. The second is of the inclination or leaning of a right line vnto a superficies: of the first take this example. Suppose ye haue two superficieses $ABCD$ and $CDEF$. Of which let the superficies $CDEF$ be a ground plaine superficies, and let the superficies $ABCD$ be erected vnto it, and let the line CD be a common terme or intersection to them both, that is, let it be the end or bound of either of them, & be drawn in either of them: in which line note at pleasure certaine pointes, as the point G, H . From which pointes vnto the line CD , draw perpendicular lines in the superficies $ABCD$, which let be GL and HK , which falling vpon the superficies $CDEF$, if they cause right angles with it, that is, with lines drawn in it from the same pointes G and H , as if the angle LGM or the angle LGK contained vnder the line LG drawn in the superficies erected, and vnder the GM or GN drawn in the ground superficies $CDEF$, lying flat, be a right angle, then by this definition, the superficies $ABCD$ is vpright or erected vpon the superficies $CDEF$. It is also co perpendicular vpon or vnto a superficies.



Declaration
of the second
part.

For the second part of this definition, which is of the inclination of a superficies, take this example. Let $ABCD$ be a ground plaine superficies, being a loft, namely, the point E , suppose a right line to fall, which let be plaine superficies $ABCD$ at the point G . Againe, from the point E , being end of the inclining line EG , let a perpendicular line fall vnto the plain let be the line EF , and let F be the point where EF toucheth the plaine from the point of the fall of the line inclining vpon the superficies vnto the point of the falling of the perpendicular line vpon the same superficies, that is, from the point G to the point F , draw a right line GF . Now by this definition, the acute angle EGF is the inclination of the line EG vnto the superficies $ABCD$. Because it is contained of the inclining line, and of the right line drawn in the superficies, from the point of the fall of the line inclining to the point of the fall of the perpendicular line: which angle must of necessitie be an acute angle. For the angle EFG is by construction a right angle, and three angles in a triangle are equall to two right angles. Wherefore the other two angles, namely, the angles EGF , and GFE , are equall to one right angle. Wherefore either of them is lesse then a right angle. Wherefore the angle EGF is an acute angle.



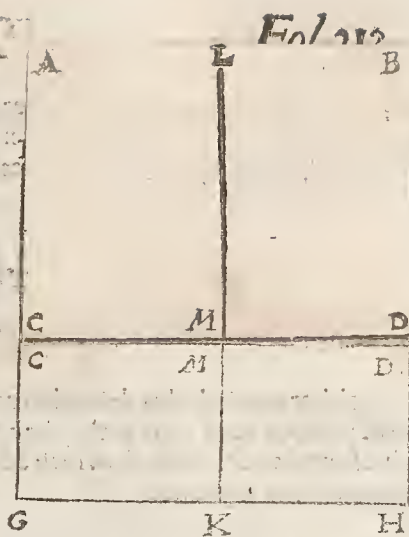
Fourth defini-
tion.

4. *Inclination of a plaine superficies to a plaine superficies, is an acute angle contained vnder the right lines, which being drawn in either of the plaine superficieses to one & the self same point of the cōmon section, make with the section right angles.*

Suppose

of Euclides Elementes.

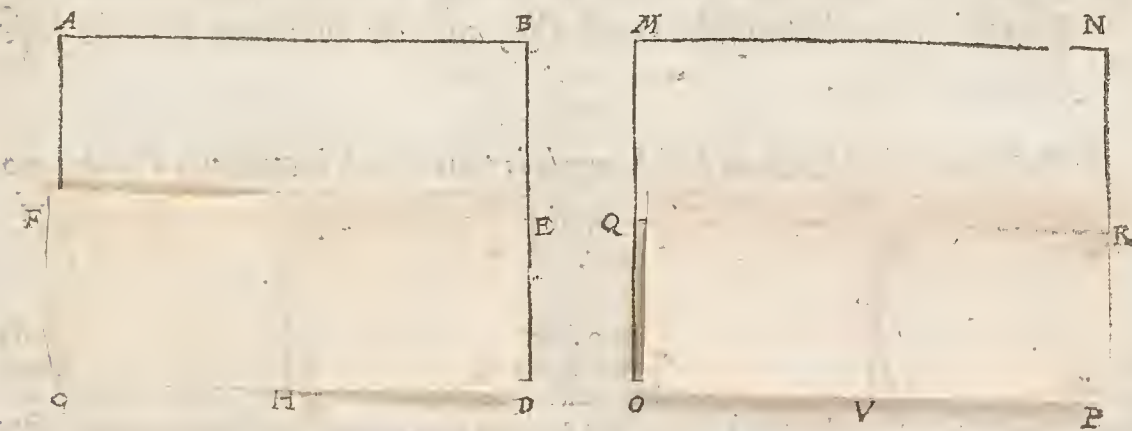
Suppose that there be two superficieses $ABCD$ & $EFGH$, and let the superficies $ABCD$ be supposed to be erected not perpendicularly, but somewhat leaning and inclining vnto the plaine superficies $EFGH$, as much or as little as ye will: the comon terme or section of which two superficieses let be the line CD . From some one point, as from the point M assigned in the common section of the two superficieses, namely, in the line CD , draw a perpendicular line in either superficies. In the ground superficies $EFGH$ draw the line MK , and in the superficies $ABCD$ draw the line ML . Now if the angle LMK be an acute angle, then is that angle the inclination of the superficies $ABCD$ vnto the superficies $EFGH$, by this definition, because it is contained of perpendicular lines drawn in either of the superficieses to one and the self same point being the common section of them both.



s Plaine superficieses are in like sort inclined the one to the other, when the sayd angles of inclination are equall the one to the other. Fifth definition.

This definition needeth no declaration at all, but is most manifest by the definition last going before. For in considering the inclinations of diuers superficieses to others, if the acute angles containd vnder the perpendicular lines drawn in them from one point assigned in ech of their common sections be equall, as if to the angle LMK in the former example be geuen an other angle in the inclination of two other superficieses equall, then is the inclination of these superficieses like, and are by this definition sayd in like sort to incline the one to the other.

Theodosius geueth an other definition of like inclination of plaine superficieses the one to the other, after this maner. One plaine superficies is like inclined to another, as an other superficies is to another, when in either of the plaine superficieses right lines being drawen, and making right angles with their common section, containe in the same pointes equall angles. This definition is in substance the same with that geuen of Euclide, and is an elucidation of it. For example let $ABCD$ be a ground plaine superficies, vnto which let the superficies $EFGH$ incline and leane. And let the common section of these two superficieses be the line EF . Then drawe in eche of these superficieses right lines to some one point of the common section EF , which let be the point G : with which section let them make right angles. As in the super-



ficies $ABCD$ draw the line HG , which in the point G let it make with the common section a right angle HGF or HGE . Also in the superficies $EFGH$ draw the line LG , which in the point G together with the common section EF let make also a right angle LGF , or the right angle LGE .

Now also let there be an other ground plaine superficies, namely, the superficies $MNOP$, vnto whom also let leane and incline the superficies $QRST$, and let the common section or segment of them be the line QR . And draw in the superficies $MNOP$ to some one point of the comon section as to the point x the line vx , making with the common section right angles, namely, the angle vxR , or the angle vxQ : also in the superficies $QRST$ draw the right line yx to the same point x in the common section, making therewith right angles, as the angle yxR , or the angle yxQ . Now (as sayth the definition) if the angles containd vnder the right lines drawn in these superficieses & making right angles with the common section, be in the pointes, that is, in the pointes of their meting in the common section, equall: then is the inclination of the superficieses equall. As in this example, if the angle LGH containd vnder the line LG being in the inclining superficies $EFGH$ and vnder the line HG being in the ground superficies $ABCD$, be equall, to the angle yxv containd vnder the line vx being in the

The eleuenth Booke

ground superficies $MNOP$ and vnder the line TX being in the inclining superficies $STQR$: then is the inclination of the superficies $TXEF$ vnto the superficies $ABCD$, like vnto the inclination of the superficies $STQR$ vnto the superficies $MNOP$. And so by this definition these two superficieses are sayd to be in like sort inclined.

Sixth diffinition.

6 *Parallell plaine superficieses are those, which being produced or extended any way neuer touch or concurre together.*

Neither needeth this definition any declaration, but is very easie to be vnderstanded by the definition of parallell lines: for as they being drawn on any part, neuer touch or come together: so parallell plaine superficieses are such, which admitte no touch, that is, being produced any way infinitely neuer meete or come together.

Seuenth diffinition.

7 *Like solide or bodily figures are such, which are contained vnder like plaine superficieses, and equall in multitude.*

What plaine superficieses are called like, hath in the beginning of the sixth booke, bene sufficiently declared. Now when solide figures or bodies be contained vnder such like plaine superficieses as there are defined, and equall in number, that is, that the one solide haue as many in number as the other, in their sides and limites: they are called like solide figures, or like bodies.

Eighth diffinition.

8 *Equall and like solide (or bodely) figures are those which are contained vnder like superficieses, and equall both in multitude and in magnitude.*

In like solide figures it is sufficient, that the superficieses which containe them be like and equall in number onely, but in like solide figures and equall, it is necessary that the like superficieses contaynyng them, be also equal in magnitude. So that besides the likenes betwene them, they be (eche being compared to his correspondent superficies) of one greatnes, and that their areas or fieldes be equal. When such superficieses contayne bodies or solides, then are such bodies equall and like solides or bodies.

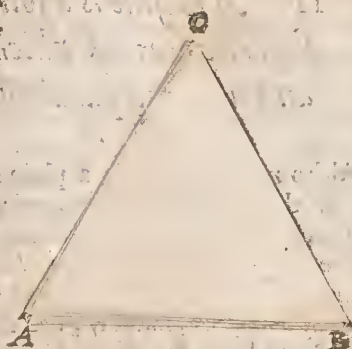
Ninth diffinition.

9 *A solide or bodily angle, is an inclination of moe then two lines to all the lines which touch themselues mutually, and are not in one and the selfe same superficies.*

Or els thus: *A solide or bodily angle is that which is contayned vnder moe then two playne angles, not being in one and the selfe same plaine superficies, but consisting all at one point.*

Of a solide angle doth Euclide here geue two seuerall definitiōs. The first is geuen by the concurse and touch of many lines. The second by the touch & concurse of many superficiall angles. And both these definitions tende to one, and are not much different, for that lynes are the limites and termes of superficieses. But the second geuen by superficiall angles is the more naturall definition, because that superficieses are the next and immediate limites of bodies, and so are not lines. An example of a solide angle cannot wel and at fully be geue or described, in a plaine superficies. But touchyng this first definitiō, lay before you a cube or a die, and cōsider any of the corners or angles therof, so shal ye see that at euery angle there concurre thre lines (for two lines cōcurring cannot make a solide angle) namely, the line or edge of his breadth, of his lēgth, and of his thicknes, which their so inclining & cōcurring together, make a solide angle, and so of others. And now cōcerning the second definitiō, what superficial or plain angles be, hath bene taught before in the first boke, namely, that it is the touch of two right lines. And as a superficial or playne angle is caused & cōtained of right lines, so is a solide angle caused & cōtained of plaine superficial angles. Two right lines touching together, make a plaine angle, but two plaine angles ioyned together can not make a solide angle, but according to the definitiō, they must be moe thē two, as three, foure, siue, or moe: which also must not be in one & the selfe same superficies, but must be in diuers superficieses, meeting at one point. This definition is not hard, but may easily be cōceiued in a cube or a die, where ye see three angles of any three superficieses or sides of the die concurre and meete together in one point, which three playne angles so ioyned together, make a solide angle. Likewise in a Pyramis or a spire of a steple or any other such thing, all the sides therof tēding vpward

ward narrower and narrower, at length ende their angles (at the height or toppe therof) in one point. So all their angles there ioyned together, make a solide angle. And for the better sight thereof, I haue set here a figure wherby ye shall more easily conceiue it, the base of the figure is a triangle, namely, $A B C$, if on euery side of the triangle $A B C$, ye rayse vp a triangle, as vpon the side $A B$, ye raise vp the triangle $A F B$, and vpon the side $A C$ the triangle $A F C$, and vpon the side $B C$, the triangle $B F C$, and so bowing the triangles raised vp, that their toppes, namely, the pointes F meete and ioyne together in one point, ye shal easily and plainly see how these three superficial angles $A F B$, $B F C$, $C F A$, iojne and close together, touching the one the other in the point F , and so make a solide angle.

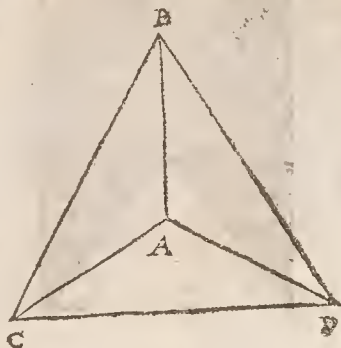


10 A Pyramis is a solide figure contained vnder many playne superficieses set vpon one playne superficies, and gathered together to one point. Tenth definition.

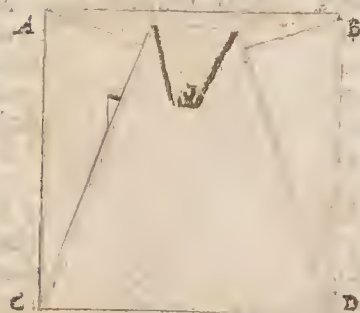
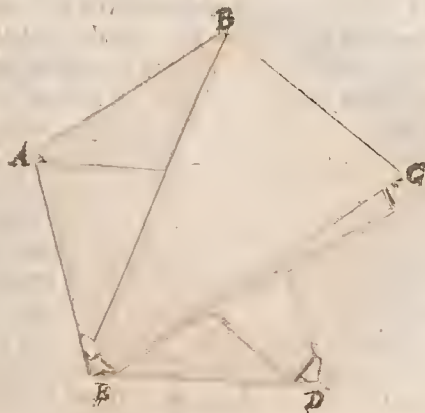
Two superficieses rayfed vpon any ground can not make a Pyramis, for that two superficial angles ioyned together in the toppe, cannot (as before is sayd) make a solide angle. Wherefore whē thre, foure, fve, or moe (how many soeuer) superficieses are raised vp frō one superficies being the ground, or base, and euer ascēding diminish their breadth, till at the lēgth all their angles cōcurre in one point, making there a solide angle: the solide inclosed, bounded, and terminated by these superficieses is called a Pyramis, as ye see in a taper of foure sides, and in a spire of a towre which containeth many sides, either of which is a Pyramis.

And because that all the superficieses of euery Pyramis ascend from one playne superficies as from the base, and tende to one poynt, it must of necessitie come to passe, that all the superficieses of a Pyramis are trianguler, except the base, which may be of any forme or figure except a circle. For if the base be a circle, then it ascendeth not with sides, or diuers superficieses, but with one round superficies, and hath not the name of a Pyramis, but is called (as hereafter shall appeare) a Cone.

Of Pyramids there are diuers kindes. For according to the varietie of the base is brought forth the varietie and diuersitie of kindes of Pyramids. If the base of a Pyramis be a triangle, then is it called a triangled Pyramis. If the base be a figure of foure angles, it is called a quadrangled Pyramis. If the base be a Pentagon, then is it a Pentagonall or fveangled Pyramis. And so forth according to the increase of the angles of the base infinitely. Although the figure of a Pyramis can not be well expressed in a playne superficies, yet may ye sufficiently conceaue of it both by the figure before set in the definition of a solide angle; and by the figure here set, if ye imagine the point A together with the lines $A B$, $A C$, and $A D$, to be eleuated on high. And yet that the reader may more clerely see the forme of a Pyramis, I haue here set two sundry Pyramids which will appeare bodilike, if ye erecte the papers wherin are drawn the triangular sides of eche Pyramis, in such fort that the pointes of the angles F of ech triangle may in euery Pyramis concurre in one point, and make a solide angle: one of which hath to his base a foure sided figure, and the other a fve sided figure. The forme of a triangled Pyramis ye may before beholde in the example of a solide angle. And by these may ye conceaue of all other kindes of Pyramids.



11 A



ward narrower and narrower, at length ende their angles (at the height or toppe therof) in one point. So all their angles there ioyned together, make a solide angle. And for the better sight thereof, here a figure wherby ye shall more easily conceiue it, the figure is a triangle, namely, ABC , if on euery side of the triangle, ye rayse vp a triangle, as vpon the side AB , ye raise vp the triangle AFB , and vpon the side AC the triangle AFC , and vpon the side BC , the triangle BFC , and so bowing the triangles raised vp, that their toppes, namely, the pointes F meete and ioyne together in one point, ye shal easily and plainly see how these three superficial angles AFB , BFC , CFA , ioyne and close together, touching the one the other in the point F , and so make a solide angle.

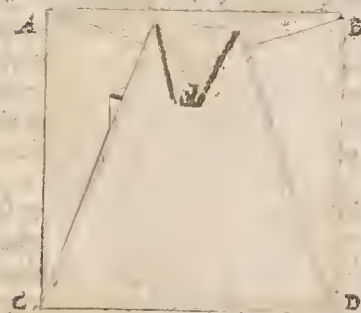
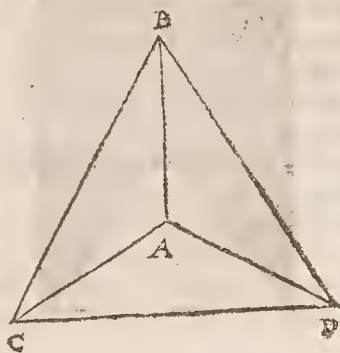
10 A Pyramis is a solide figure contained vnder many superficieses, and gathered together to one point.

iciences Tenth definition.

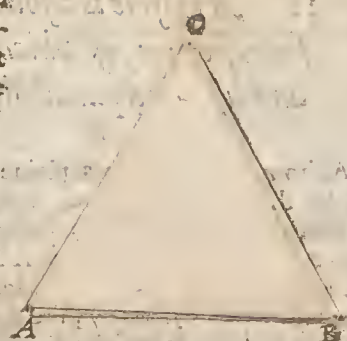
Two superficieses rayfed vpon any ground can not make a Pyramis, for that two superficial angles ioyned together in the toppe, cannot (as before is sayd) make a solide angle. Wherefore, whether three, foure, five, or moe (how many soeuer) superficieses are raised vp from one superficies being the ground, or base, and euer ascending diminish their breadth, till at the length all their angles cocurre in one point, making there a solide angle: the solide inclosed, bounded, and terminated by these superficieses is called a Pyramis, as ye see in a taper of foure sides, and in a spire of a towre which containeth many sides, either of which is a Pyramis.

And because that all the superficieses of euery Pyramis ascend from one playne superficies as from the base, and tende to one point, it must of necessitie come to passe, that all the superficieses of a Pyramis are triangler, except the base, which may be of any forme or figure except a circle. For if the base be a circle, then it ascendeth not with sides, or diuers superficieses, but with one round superficies, and hath not the name of a Pyramis, but is called (as hereafter shall appeare) a Cone.

Of Pyramids there are diuers kindes. For according to the varietie of the base is brought forth the varietie and diuersitie of kindes of Pyramids. If the base of a Pyramis be a triangle, then is it called a triangled Pyramis. If the base be a figure of foure angles, it is called a quadrangled Pyramis. If the base be a Pentagon, then is it a Pentagonall or fiveangled Pyramis. And so forth according to the increase of the angles of the base infinitely. Although the figure of a Pyramis can not be well expressed in a playne superficies, yet may ye sufficiently conceaue of it both by the figure before set in the definition of a solide angle, and by the figure here set, if ye imagine the point A together with the lines AB , AC , and AD , to be eleuated on high. And yet that the reader may more clerely see the forme of a Pyramis, I haue here set two sundry Pyramids which will appeare bodilike, if ye erecte the papers wherein are drawn the triangular sides of eche Pyramis, in such fort that the pointes of the angles F of ech triangle may in euery Pyramis concurre in one point, and make a solide angle: one of which hath to his base a foure sided figure, and the other a five sided figure. The forme of a triangled Pyramis ye may before beholde in the example of a solide angle. And by these may ye conceaue of all other kindes of Pyramids.



ward narrower and narrower, at length ende their angles (at the heighth or toppe therof) in one point. So all their angles there ioyned together, make a solide angle. And for the better sight thereof, I haue set here a figure wherby ye shall more easily conceiue it, the base of the figure is a triangle, namely, $A B C$, if on euery side of the triangle $A B C$, ye raise vp a triangle, as vpon the side $A B$, ye raise vp the triangle $A F B$, and vpon the side $A C$ the triangle $A F C$, and vpon the side $B C$, the triangle $B F C$, and so bowing the triangles raised vp, that their toppes, namely, the pointes F meete and ioyne together in one point, ye shal easily and plainly see how these three superficiall angles $A F B$, $B F C$, $C F A$, iojne and close together, touching the one the other in the point F , and so make a solide angle.

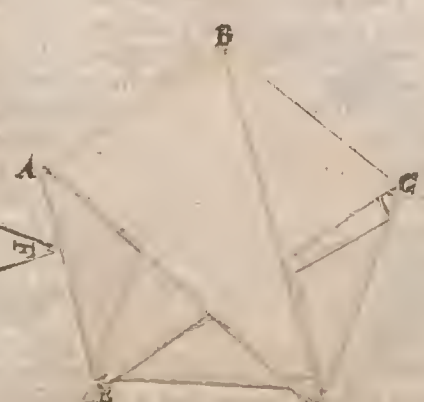
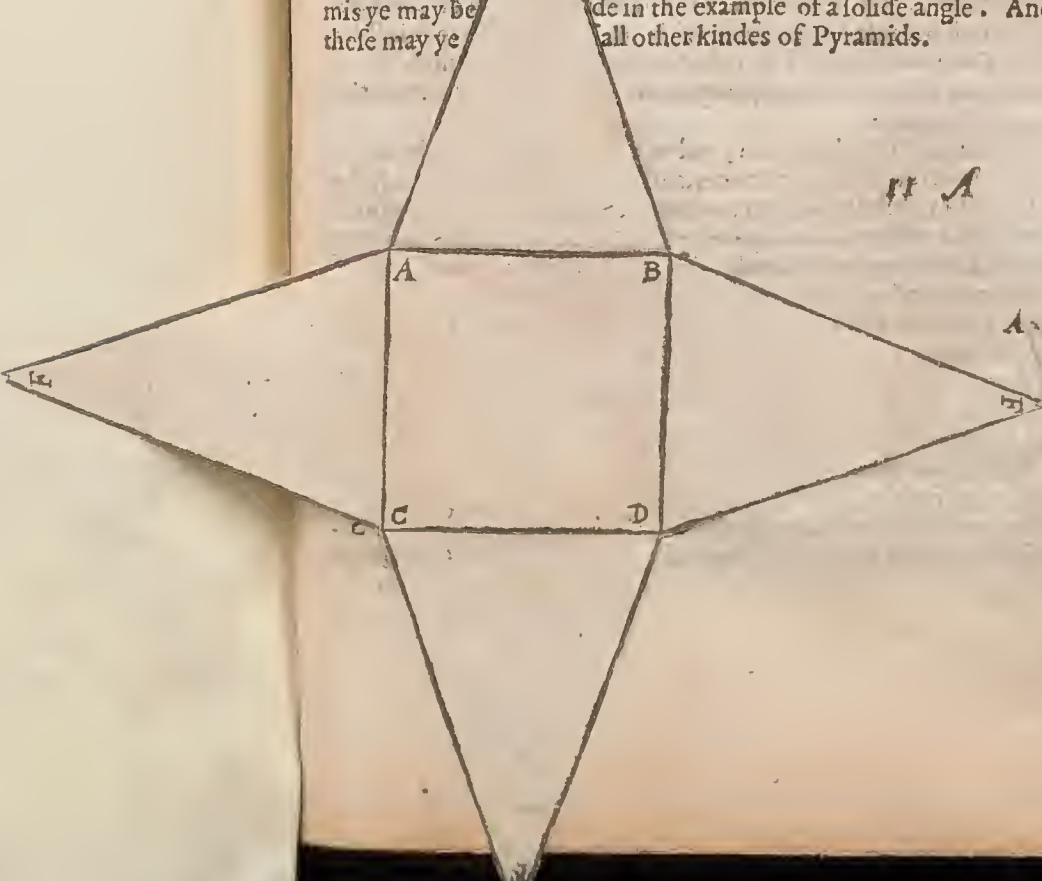
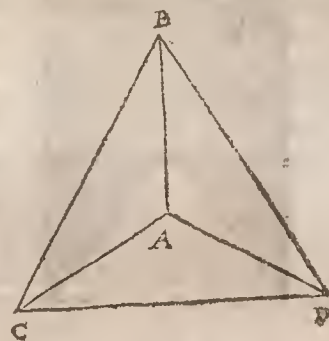


10 *A Pyramis is a solide figure contained vnder many playne superficiees set vpon one playne superficies, and gathered together to one point.* Tenth diffinition.

Two superficiees rayfed vpon any ground can not make a Pyramis, for that two superficiall angles ioyned together in the toppe, cannot (as before is sayd) make a solide angle. Wherefore whe three, foure, fve, or moe (how many soeuer) superficiees are raised vp fro one superficies being the ground, or base, and euer ascēding diminish their breadth, till at the lēgth all their angles cōcurre in one point, making there a solide angle: the solide inclosed, bounded, and terminated by these superficiees is called a Pyramis, as ye see in a taper of foure sides, and in a spire of a towre which containeth many sides, either of which is a Pyramis.

And because that all the superficiees of euery Pyramis ascend from one playne superficies as from the base, and tende to one pōynt, it must of necesitie come to passe, that all the superficiees of a Pyramis are trianguler, except the base, which may be of any forme or figure except a circle. For if the base be a circle, then it ascendeth not with sides, or diuers superficiees, but with one round superficies, and hath not the name of a Pyramis, but is called (as hereafter shall appeare) a Cone.

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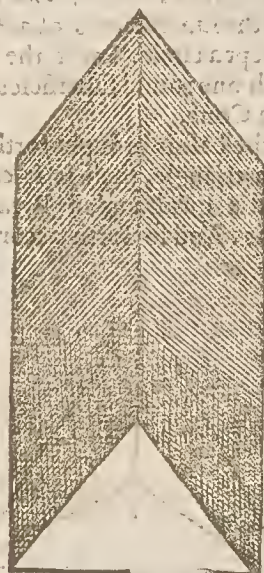
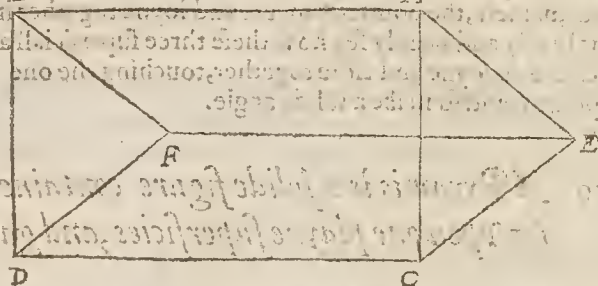
The eleuenth Booke

Eleuenth diffi-
nition.

11 A prisme is a solide or a bodily figure contained vnder many plaine superficieses, of which the two superficieses which are opposite, are equall and like, and parallels; & all the other superficieses are parallelogrames.

This diffinitio-
on agreeth to
the generall
name of a
prisme, which
also may be
applied to Pa-
rallelepedons
(as may ap-
peare in the
12. booke fol-
lowing & 10.
propositio, his
demonstratio)
and to some o-
ther bodie
likewise.

Although you may in a plaine superficies by this figure here set, without any hardnes conceaue what a prisme is, namely, if ye imagine the superficies $ABDC$ to be the ground & base of the solide, and the two superficieses, namely, the superficies $A E F B$, and the superficies $C E F D$ to be erected vp on the sides of the base, the one on the one side, namely, on the line $A B$, and the other on the other side, namely, on the line $D C$, not perpendicularly, but inclining and bending the one to the other, till they meete in the toppe, namely, on the line $E F$. For so ye see that this solide figure is contained vnder many plaine superficieses, of which two, namely, the superficies $B F D$, which are the endes of the solide, and opposite the one to the other, and parallels, and all the other superficieses, namely, the base $A B C D$, & the superficies $A E F B$, and the superficies $C E F D$ are parallelogrames, to make the thing more clere vnto the reader, I haue here set a Prisme



like, if you erecte bending wise the papers wherein are drawn the parallelogrames $A B E F$, & $C D E F$, that they may concur in the line $E F$ in the toppe, and so erect the papers wherein are drawn the triangles $A C E$ and $B D F$, that the side $A E$ of the one triangle may exactly agree with the side $A E$ of the one parallelogramme $A B E F$, and the side $C E$ of the same triangle, with the side $C E$ of the parallelogramme $C D E F$: and moreover, the side $B F$ of the other triangle, with the side $D F$ of the parallelogramme $C D E F$: and finally, the side $B F$ of the same triangle, with the side $B F$ of the parallelogramme $A B E F$. And so shall you most easilie see the forme of a Prisme: that it consisteth of two equall, like, and parallell triangular superficieses, and of three parallelogrammes: wherof the one is the base, and the other two are erected bending wise. Here also behold the forme thereof as it is by arte described in a plaine to appeare bodilike.

Flussas here noteth that Theon and Campane disagree in defining a Prisme, and he preferreth the definition geuen of Campane before the definition geuen of Euclide (which because he may seme with out lesse offence to reiect, he calleth it Theons definition) and following Campane he geueth an other definition, which is this.

A Prisme is a solide figure, which is contayned vnder onely playne superficieses, of which two are triangles, like, equall, and parallels, and the rest are parallelogrammes.

The example before set agreeth likewise with this definition, and manifestly declareth the same. For in it were five superficieses, the base, the two erected superficieses, and the two endes: of which the two endes are triangles like, equall and parallels, and all the other are parallelogrammes as this definition requireth. The cause why he preferreth the definion of Campane before the definition of Theon (as he calleth it, but in very deede it is Euclides definition, as certainly, as are all those which are geuen of him in the former bookes, neither is there any cause at all, why it should be doubted in this one definition more then in any of the other) as he him selfe alledgeth, is, for that it is (as he sayth) to large, and comprehendeth many mo kindes of solide figures besides Prismes, as Columnes hauing sides, and all Parallelipedons, which a definition should not doo: but should be conuertible with the thing defined, and declare the nature of it onely, and stretch no farther.

Me thinketh Flussas ought not to haue made so much a doo in this matter, nor to haue bene so sharpe in sight and so quicke as to see and espy out such faultes, which can of no man that will see rightly without affection be espyed for such great faultes. For it may well be answered that these faultes which he noteth (if yet they be faultes) are not to be found in this definion. It may be sayd that it extendeth it selfe not farther then it should, but declareth onely the thing defined, namely, a Prisme. Neither doth

An other diffi-
nition of a
prisme, which
is a speciall
diffinition of
a prisme: as it
is commonly
called and
vsed.

doth it agree (as *Flussas* cauilleth) with all Parallelipipeds and Columnes hauing sides. All Parallelipipeds what so euer right angled, or not right angled which are described of equidistant sides or superficieses, haue their sides opposit. So that in any of them there is no one side, but it hath a side opposit vnto it. So likewise is it of euē sided Columnes, eche hath his opposite side directly agaynst it, which agreeth not with this definition of *Euclide*. Here it is evidently sayd, that of all the superficieses, the two which are opposite are equall, like, and parallels, meaning vndoubtedly onely two & no moe. Which is manifest by that which followeth. The other (sayth he) are parallelogrammes, signifying most evidently that none of the rest besides the two aforesayd, which are equall, like, and parallels, are opposite: but two of necessitie are rayfed vp, and concur in one common line, and the other is the base. So that it containeth not vnder it the figures aforesayd, that is sided Columnes, & al Parallelipipeds, as *Flussas* hath not so aduisedly noted.

Agayne where *Flussas* setteth in his definition, as an essentiall part thereof, that of the fūe superficieses, of which a Prisme is containd, two of them must be triangles, that vndoubtedly is not of necessitie, they may be of some other figure. Suppose that in the figure before geuen that in the place of the two opposite figures, which there were two triangles, were placed two pentagōs: yet should the figure remayne a Prisme still, and agree with the definition of *Euclide*, and falleth not vnder the definition of *Flussas*. So that his definitiō semeth to be too narrow and stretcheth not so farre as it ought to do, nor declareth the whole nature of the thing defined. Wherefore it is not to be preferrd before *Euclides* definition, as he woulde haue it. This figure of *Euclide* called a Prisme, is called of *Campanus* and certayne others *Figura Serratilis*, for that it representeth in some maner the forme of a Sawe. And of some others it is called *Cuneus*, that is, a Wedge, because it beareth the figure of a wedge.

This bodie
called *Figura
Serratilis*.

Moreover although it were so, that the definitiō of a Prisme should be so large, that it should cōtaine all these figures noted of *Flussas* as sided Columnes, & all Parallelipipeds: yet should not *Flussas* haue so great a cause to finde so notably a fault, so vtterly to reiect it. It is no rare thing in all learnings, chiefly in the Mathematicalls, to haue one thing more generall then an other. Is it not true that euery Iosceles is a triangle, but not euery triangle is an Iosceles? And why may not likewise a Prisme be more generall, then a Parallelepipedon, or a Columne hauing sides (and containe them vnder it as a triangle cōtaineth vnder it an Iosceles and other kinds of triangles). So that euery Parallelepipedon, or euery sided Columne be a Prisme, but not euery Prisme a Parallelepipedon or a sided Columne. This ought not to be so much offensiue. And indeede it semeth manifestly of many, yea & of the learned so to be takē, as clearely appeareth by the wordes of *Psellus* in his Epitome of Geometrie, where he entreateth of the produccion and constitution of these bodyes. His wordes are these. *All rectiline figures being erected vpon their playnes or bases by right angles, make Prismes.* Who perceaueth not but that a Pentagon erected vpon his base of fūe sides maketh by his motion a sided Columne of fūe sides? Likewise an Hexagon erected at right angles produceth a Columne hauing sixe sides: and so of all other rectiline figures. All which solides or bodyes so produced, whether they be sided Columnes or Parallelepipedons, be here in most plaine wordes (of this excellēt and auncient Greke author *Psellus*) called Prismes. Wherefore if the definitiō of a Prisme geuē of *Euclide* should extend it selfe so largely as *Flussas* imagineth, and should enclude such figures or bodyes, as he noted: he ought not yet for all that so much to be offended, and so narrowly to haue sought faultes. For *Euclide* in so defining mought haue that meaning & sense of a Prisme which *Psellus* had. So ye see that *Euclide* may be defended either of these two wayes, either by that that the definition extendeth not to these figures, and so not to be ouer generall nor stretch farther then it ought: or ells by that that if it should stretch so far it is not so haynous. For that as ye se many haue takē it in that sense. In deede cōmonly a Prisme is taken in that significatiō and meaning in which *Campanus*, *Flussas* and others take it. In which sense it semeth also that in diuers propositions in these bookes following it ought of necessitie to be taken.

Psellus.

12 A Sphere is a figure which is made, when the diameter of a semicircle Twelue dis-
abiding fixed, the semicircle is turned round about, vntill it returne vnto finition.
the selfe same place from whence it began to be moued.

To the end we may fully and perfectly vnderstand this definition, how a Sphere is produced of the motion of a semicircle, it shall be expedient to cōsider how quantities Mathematically are by imagination conceaued to be produced, by flowing and motion, as was somewhat touched in the beginning of the first booke. Euer the lesse quantitie by his motion bringeth forth the quātitie next aboue it. As a point mouing, flowing, or gliding, bringeth forth a line, which is the first quantitie, and next to a point. A line mouing produceth a superficies, which is the second quantitie, and next vnto a line. And last of all, a superficies mouing bringeth forth a solide or body, which is the third & last quantitie. These thinges well marked, it shall not be very hard to attaine to the right vnderstanding of this definition. Vpon the line A B being the diameter, describe a semicircle



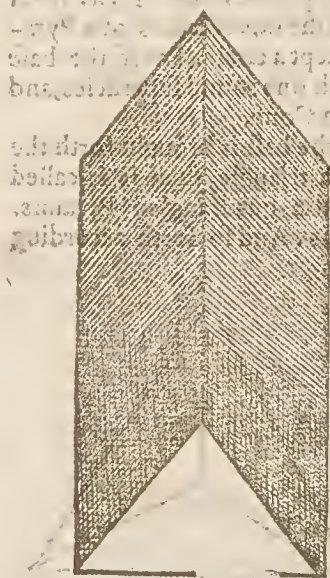
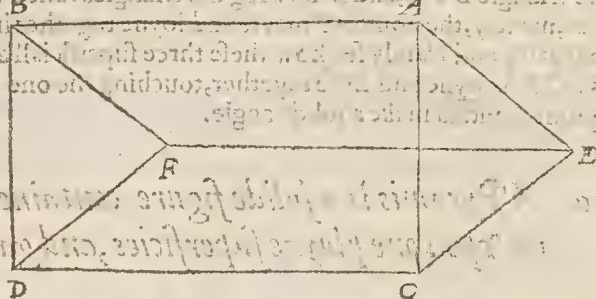
The eleuenth Booke

**Eleuenth diffi-
nition.**

11 A prisme is a solide or a bodily figure contained vnder many plaine superficieses, of which the two superficieses which are opposite, are equall and like, and parallels, & all the other superficieses are parallelogrames.

This diffinitio-
on agreeth to
the generall
name of a
prisme, which
also may be
applied to Pa-
rallelepedons
(as may ap-
peare in the
12. booke fol-
lowing & 10.
propositiō, his
demonstratiō)
and to some o-
ther bodie
likewise.

Although you may in a plaine superficies by this figure here set, without any hardnes conceaue what a prisme is, namely, if ye imagine the superficies $ABDC$ to be the ground & base of the solide, and the two superficieses, namely, the superficies $AEFB$, and the superficies $CEFD$ to be erected vp on the sides of the base, the one on the one side, namely, on the line AB , and the other on the other side, namely, on the line DC , not perpendicularly, but inclining and bending the one to the other, till they meete in the toppe, namely, on the line EF . For so ye see that this solide figure is contained vnder many plaine superficieses, of which two, namely, the superficies AEC , and the superficies BFD , which are the endes of the solide, and opposite the one to the other, are equall like and parallels, and all the other superficieses, namely, the base $ABCD$, & the two erected superficieses, that is, the superficies $AEFB$, and the superficies $CEFD$ are parallelogrammes. Yet notwithstanding, to make the thing more clere vnto the reader, I haue here set a Prisme which will appeare bodi-



like, if you erecte bending wise the papers wherein are drawn the parallelogrammes $AEBF$, & $CEFD$, that they may concur in the line EF in the toppe, and so erect the papers wherein are drawn the triangles ACE and BDF , that the side AE of the one triangle may exactly agree with the side AE of the one parallelogramme $AEBF$: and moreover, the side CE of the same triangle, with the side CE of the same parallelogramme $CEFD$: and the side BF of the same triangle, with the side BF of the same parallelogramme $AEBF$. And so shall you moste easily see that this Prisme: that it consisteth of two equall, like, and parallell triangular superficieses, and of three parallelogrammes: wherof the one is the base, and the other two are erected bending wise. Here also behold the forme thereof as it is by arte described in a plaine to appeare bodilike.

Flussas here noteth that *Theon* and *Campane* disagree in defining a Prisme, and he preferreth the definition geuen of *Campane* before the definition geuen of *Euclide* (which because he may seme with out lesse offence to reiect, he calleth it *Theons* definition) and following *Campane* he geueth an other definition, which is this.

A Prisme is a solide figure, which is containcd vnder fīue playne superficieses, of which two are triangles, like, equall, and parallels, and the rest are parallelogrammes.

The example before set agreeth likewise with this definition, and manifestly declareth the same. For in it were fīue superficieses, the base, the two erected superficieses, and the two endes: of which the two endes are triangles like, equall and parallels, and all the other are parallelogrammes as this definition requireth. The cause why he preferreth the definition of *Campane* before the definition of *Theon* (as he calleth it, but in very deede it is *Euclides* definition, as certainly, as are all those which are geuen of him in the former bookes, neither is there any cause at all, why it should be doubted in this one definition more then in any of the other) as he him selfe alledgeth, is, for that it is (as he sayth) to large, and comprehendeth many mo kindes of solide figures besides Prismes, as Columnes hauing sides, and all Parallelipedons, which a definition should not doo: but should be conuertible with the thing defined, and declare the nature of it onely, and stretch no farther.

Me thinketh *Flussas* ought not to haue made so much a doo in this matter, nor to haue bene so sharpe in sight and so quicke as to see and espy out such faultes, which can of no man that will see rightly without affection be espyed for such great faultes. For it may well be answered that these faultes which he noteth (if yet they be faultes) are not to be found in this definition. It may be sayd that it extendeth it selfe not farther then it should, but declareth onely the thing defined, namely, a Prisme. Neither doth

An other diffi-
nition of a
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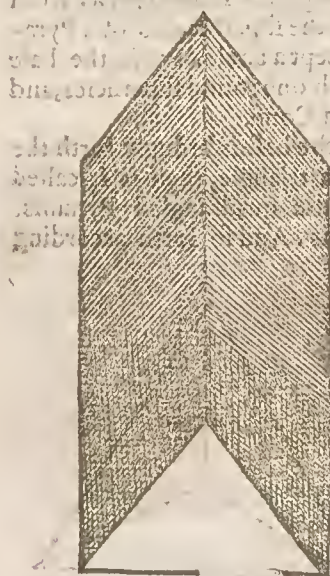
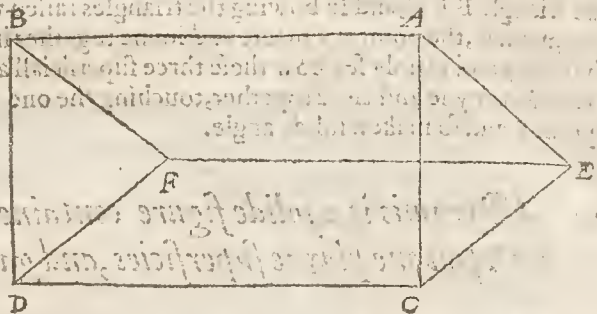
The eleventh Booke

Eleventh diffi-
nition.

11 A prisme is a solide or a bodily figure contained vnder many plaine superficieses, of which the two superficieses which are opposite, are equall and like, and parallels, & all the other superficieses are parallelogrames.

This diffinition agreeth to the generall name of a prisme, which also may be applied to Parallelepipedons (as may appeare in the 12. booke following & 10. proposition, his demonstratiō) and to some other bodie likewise.

Although you may in a plaine superficies by this figure here set, without any hardnes conceaue what a prisme is, namely, if ye imagine the superficies $ABDC$ to be the ground & base of the solide, and the two superficieses, namely, the superficies $AEFB$, and the superficies $CEFD$ to be erected vp on the sides of the base, the one on the one side, namely, on the line AB , and the other on the other side, namely, on the line DC , not perpendicularly, but inclining and bending the one to the other, till they meete in the toppe, namely, on the line EF . For so ye see that this solide figure is contained vnder many plaine superficieses, of which two, namely, the superficieses BEF , which are the endes of the solide, and opposite the one to the other, and parallels, and all the other superficieses, namely, the base $ABCD$, & that is, the superficies $AEFB$, and the superficies $CEFD$ are parallelogrammes, to make the thing more clere vnto the reader, I haue here set a Prisme



like, if you erecte bending wise the papers wherein are drawn the parallelogrammes $ABEF$, & $CDEF$, that they may concur in the line EF in the toppe, and so you shall see the papers wherein are drawn the triangles ACE and BDF , that the side AE of the one triangle may exactly agree with the side AE of the one parallelogramme $CDEF$: and moreover, the side CE of the same triangle, with the side DF of the parallelogramme $ABEF$: and the side BF of the same triangle, with the side AE of the parallelogramme $ABEF$. And so shall you more easily see that it consisteth of two equall, like, and parallell triangular superficieses, and of three parallelogrammes: whereof the one is the base, and the other two are erected bending wise. Here also behold the forme thereof as it is by arte described in a plaine to appeare bodilike.

Flussas here noteth that Theon and Campane disagree in defining a Prisme, and he preferreth the definition geuen of Campane before the definition geuen of Euclide (which because he may seme with out lesse offence to reiect, he calleth it Theons definition) and following Campane he geueth an other definition, which is this.

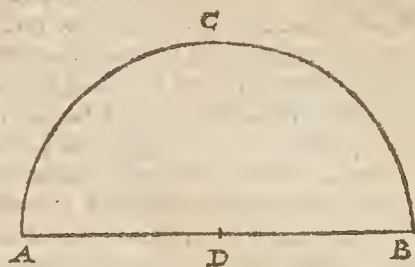
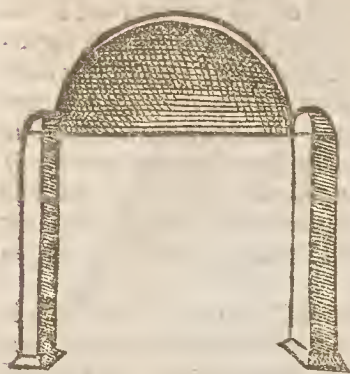
A Prisme is a solide figure, which is contayned vnder five playne superficieses, of which two are triangles, like, equall, and parallels, and the rest are parallelogrammes.

The example before set agreeth likewise with this definition, and manifestly declareth the same. For in it were five superficieses, the base, the two erected superficieses, and the two endes: of which the two endes are triangles like, equall and parallels, and all the other are parallelogrammes as this definition requireth. The cause why he preferreth the definition of Campane before the definition of Theon (as he calleth it, but in very deede it is Euclides definition, as certainly, as are all those which are geuen of him in the former bookes, neither is there any cause at all, why it should be doubted in this one definition more then in any of the other) as he him selfe alledgeth, is, for that it is (as he sayth) to large, and comprehendeth many mo kindes of solide figures besides Prismes, as Columnes hauing sides, and all Parallelipipedons, which a definition should not doo: but should be conuertible with the thing defined, and declare the nature of it onely, and stretch no farther.

Me thinketh Flussas ought not to haue made so much a doo in this matter, nor to haue bene so sharpe in sight and so quicke as to see and espy out such faultes, which can of no man that will see rightly without affection be espyed for such great faultes. For it may well be answered that these faultes which he noteth (if yet they be faultes) are not to be found in this definition. It may be sayd that it extendeth it selfe not farther then it should, but declareth onely the thing defined, namely, a Prisme. Neither doth

An other diffinition of a prisme, which is a speciall diffinition of a prisme: as it is commonly called and vsed.

The eleuenth Booke



semicircle A C B, whose centre let be D : the diameter A B being fixed on his endes or^r pointes, imagine the whole superficies of the semicircle to moue round from some one point assigned, till it returne to the same point againe . So shall it produce a perfect Sphere

or Globe, the forme whereof you see in a ball or bowle . And it is fully round and solide, for that it is described of a semicircle which is perfectly round, as our country man *Iohannes de Sacro Busco* in his booke of the Sphere, of this definition which he taketh out of *Euclide*, doth

What is to be taken heede of in the definition of a sphere geuen by Iohannes de Sacro Busco.

well collecte . But it is to be noted and taken heede of, that none be deceaued by the definition of a Sphere geuen by *Iohannes de Sacro Busco* : A Sphere (sayth he) is the passage or mouing of the circumference of a semicircle, till it returne vnto the place where it beganne, which agreeth not with *Euclide* . *Euclide* plainly sayth, that a Sphere is the passage or motion of a semicircle, and not the passage or motion of the circumference of a semicircle : neither can it be true that the circumference of a semicircle, which is a line, should describe a body . It was before noted that euery quantitie moued, describeth and produceth the quantitie next vnto it . Wherefore a line moued can not bring forth a body, but a superficies onely . As if ye imagine a right line fastened at one of his endes to moue about from some one point till it returne to the same againe, it shall describe a plaine superficies, namely, a circle . So also if ye likewise conceaue of a crooked line, such as is the circumference of a semicircle, that his diameter fastened on both the endes it should moue from a point assigned till it returne to the same againe, it should describe & produce a round superficies onely, which is the superficies and limite of the Sphere, and should not produce the body and soliditie of the Sphere . But the whole semicircle, which is a superficies, by his motion, as is before said, produceth a body, that is, a perfect Sphere . So see you the errour of this definition of the author of the Sphere : which whether it happened by the author him selfe, which I thinke not : or that that particle was thrust in by some one after him, which is more likely, it is not certaine . But it is certaine, that it is vnaptly put in, and maketh an vntrue definition : which thing is not here spoken, any thing to derogate the author of the booke, which assuredly was a man of excellent knowledge : neither to the hindrance or diminishing of the worthines of the booke, which vndoubtedly is a very necessary booke, then which I know none more meete to be taught and red in scholes touching the groundes and principles of Astronomie and Geographie : but onely to admonishe the young and vnskilfull reader of not falling into errour . *Theodosius* in his booke *De Sphericis* (a booke very necessary for all those which will see the groundes and principles of Geometrie and Astronomie, which also I haue translated into our vulgare tounge, ready to the presse) defineth a Sphere after thys maner :

Theodosius definition of a sphere.

A Sphere is a solide or body contained vnder one superficies, in the middle whereof there is a point, from which all lines drawn to the circumference are equall . This definition of *Theodosius* is more essentiall and naturall, then is the other geuen by *Euclide* . The other did not so much declare the inward nature and substance of a Sphere, as it shewed the industry and knowledge of the producing of a Sphere, and therefore is a causall definition geuen by the cause efficient, or rather a description then a definition . But this definition is very essentiall, declaring the nature and substance of a Sphere . As if a circle should be thus defined, as it well may : A circle is the passage or mouing of a line from a point till it returne to the same point againe : it is a causall definition, shewing the efficient cause whereof a circle is produced, namely, of the motion of a line . And it is a very good description fully shewing what a circle is . Such like description is the definition of a Sphere geuen of *Euclide* by the motion of a semicircle . But when a circle is defined to be a plaine superficies, in the midst whereof is a point, from which all lines drawn to the circumference thereof, are equall : this definition is essentiall and formall, and declareth the very nature of a circle . And vnto this definition of a circle, is correspondent the definition of a Sphere geue by *Theodosius*, saying : that it is a solide or body, in the midst whereof there is a point, from which all the lines drawn to the circumference are equall . So see you the affinitie betwene a circle and a Sphere . For what a circle is in a plaine, that is a Sphere in a Solide . The fulnes and content of a circle is described by the motion of a line moued about : but the circumference thereof, which is the limite and border thereof, is described of the end and point of the same line moued about . So the fulnes, content, and body of a Sphere or Globe is described of a semicircle moued about . But the Sphericall superficies, which is the limite and border of a Sphere, is described of the circumference of the same semicircle moued about . And this is the superficies ment in the definition, when it is sayd, that it is contained vnder one superficies, which superficies is called of *Iohannes de Sacro Busco* & others, the circumference of the Sphere .

The circumference of a sphere.

Galens definition of a sphere.

The dignitie of a sphere.

Galene in his booke *de diffinitionibus medicis*, geueth yet an other definitio of a Sphere, by his proper tie or comon accidence of mouing, which is thus . A Sphere is a figure most apt to all motion, as hauing no base whereon to stay . This is a very plaine and witty definition, declaring the dignitie thereof about all figures generally . All other bodies or solides, as Cubes, Pyramids, and others haue sides, bases, and angles, all which are stayes to rest vpon, or impedimentes and lets to motion . But the Sphere hauing no side or base

bale to stay one, nor angle to let the course thereof, but onely in a poynt touching the playne wherein it standeth, moueth freely and fully with out let. And for the dignity and worthines thereof, this circular and Sphericall motion is attributed to the heauens, which are the most worthy bodyes. Wherefore there is ascribed vnto them this chiefe kinde of motion. This solide or bodely figure is also commonly called a Globe.

A sphere called a Globe.

13 The axe of a Sphere is that right line which abideth fixed, about which the semicircle was moued.

Thirteenth diffinition.

As in the example before geuen in the definition of a Sphere, the line A B, about which his endes being fixed, the semicircle was moued (which line also yet remayneth after the motion ended) is the axe of the Sphere described of that semicircle. *Theodosius* defineth the axe of a Sphere after this maner. The axe of a Sphere is a certayne right line drawn by the centre, ending on either side in the superficies of the Sphere, about which being fixed the Sphere is turned. As the line A B in the former example. There nedeth to this definition no other declaration, but onely to consider, that the whole Sphere turneth vpon that line A B, which passeth by the centre D, and is extended one either side to the superficies of the Sphere, wherefore by this definition of *Theodosius* it is the axe of the Sphere.

Theodosius diffinition of the axe of a sphere.

14 The centre of a Sphere is that poynt which is also the centre of the semicircle.

Fourteenth diffinition.

This definition of the centre of a Sphere is geuen as was the other definition of the axe, namely, hauing a relation to the definition of a Sphere here geuen of *Euclide*: where it was sayd that a Sphere is made by the reuolution of a semicircle, whose diameter abideth fixed. The diameter of a circle and of a semicircle is all one. And in the diameter either of a circle or of a semicircle is contained the center of either of them, for that the diameter of eche euer passeth by the centre. Now (sayth *Euclide*) the poynt which is the center of the semicircle, by whose motion the Sphere was described, is also the centre of the Sphere. As in the example there geuen, the poynt D is the centre both of the semicircle & also of the Sphere. *Theodosius* geueth an other definition of the centre of a Sphere which is thus. The centre of a Sphere is a poynt with in the Sphere, from which all lines drawn to the superficies of the Sphere are equall. As in a circle being a playne figure there is a poynt in the midst, from which all lines drawn to the circumference are equall, which is the centre of the circle: so in like maner with in a Sphere which is a solide and bodely figure, there must be conceaued a poynt in the midst thereof, from which all lines drawn to the superficies thereof are equall. And this poynt is the centre of the Sphere by this definition of *Theodosius*. *Flussas* in defining the centre of a Sphere comprehendeth both those definitions in one, after this sort. The centre of a Sphere is a poynt assigned in a Sphere, from which all the lines drawn to the superficies are equall, and it is the same which was also the centre of the semicircle which described the Sphere. This definition is superfluous and containeth more the nedeth. For either part thereof is a full and sufficient diffinition, as before hath bene shewed. Or ells had *Euclide* bene insufficient for leauing out the one part, or *Theodosius* for leauing out the other. Parauenture *Flussas* did it for the more explication of either, that the one part might open the other.

Theodosius diffinition of the center of a sphere.

Flussas diffinition of the center of a sphere.

15 The diameter of a Sphere is a certayne right line drawn by the centre, and one eche side ending at the superficies of the same Sphere.

Fifteenth diffinition

This definitiō also is not hard, but may easely be couceaued by the definitiō of the diameter of a circle. For as the diameter of a circle is a right line drawne frō one side of the circūfrence of a circle to the other, passing by the centre of the circle: so imagine you a right line to be drawn from one side of the superficies of a Sphere to the other, passing by the center of the Sphere, and that line is the diameter of the Sphere. So it is not all one to say, the axe of a Sphere, and the diameter of a Sphere. Any line in a Sphere drawn from side to side by the centre is a diameter. But not euery line so drawn by the centre is the axe of the Sphere, but onely one right line about which the Sphere is imagined to be moued. So that the name of a diameter of a Sphere is more general, then is the name of an axe. For euery axe in a Sphere is a diameter of the same: but not euery diameter of a Sphere is an axe of the same. And therefore *Flussas* setteth a diameter in the definition of an axe as a more generall word in this maner. The axe

Difference betweene the diameter & axe of a sphere.

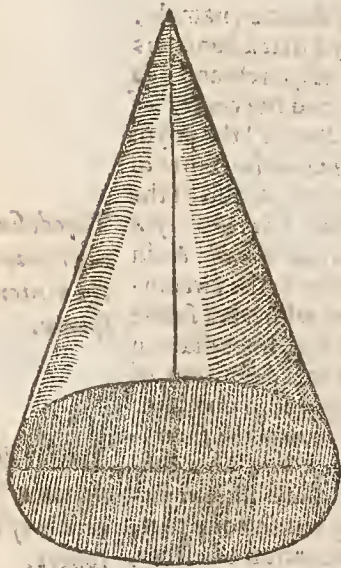
The eleuenth Booke

of a Sphere, is that fixed diameter about which the Sphere is moued. A Sphere (as also a circle) may haue infinite diameters, but it can haue but onely one axe.

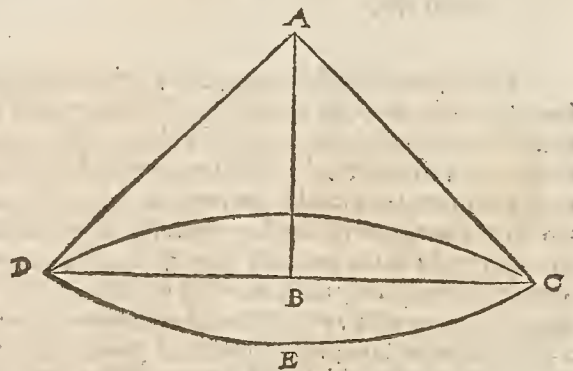
*Seuententh
definition.*

16 A cone is a solide or bodely figure which is made, when one of the sides of a rectangle triangle, namely, one of the sides which contayne the right angle, abiding fixed, the triangle is moued about, vntill it returne vnto the selfe same place from whence it began first to be moued. Now if the right line which abideth fixed be equall to the other side which is moued about and containeth the right angle: then the cone is a rectangle cone. But if it be lesse, then is it an obtuse angle cone. And if it be greater, the is it an acute angle cone,

This definition of a Cone is of the nature and condition that the definition of a Sphere was, for either is geuen by the motion of a superficies. There, as to the production of a Sphere was imagined a semicircle to moue round, from some one point till it returned to the same point againe: so here must ye imagine a rectangle triangle to moue about till it come againe to the place where it beganne. Let ABC be a rectangle triangle, hauing the angle ABC a right angle, which let be contained vnder the lines AB and BC .



Now suppose the side AB , namely, one of the lines which cōtaine the right angle ABC to be fastened, and about it suppose the triangle ABC to be moued from some one poynt assigned till it returne to the same



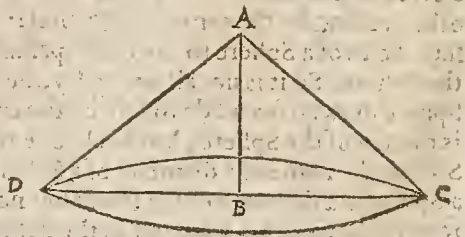
agayne (as vppon the diameter in the definition of a Sphere ye imagined a semicircle to moue about): so shall the solide or body thus described be a perfect Cone. As you may imagine by this figure here set. And the forme of a Cone you may sufficiently conceaue by the figure set in the margent. There are of Cones three kindes, namely, a rectangle Cone, an obtuseangle Cone, and an acute angle Cone, all which were before in the former definitiō defined: Namely, the first kinde after this maner.

If the right line which abideth fixed, be equall to the other side which moueth round about, and containeth the right angle, then the Cone is a rectangle Cone.

*First kinde
of Cones.*

As suppose in the former example, that the line AB which is fixed, and about which the triangle was moued, and after the motion yet remayneth, be equall to the line BC , which is the other line containing the right angle, which also is moued about together with the whole triangle: then is the Cone described, as the Cone ADC in this example, a right angled Cone: so called for that the angle at the toppe of the Cone is a right angle. For forasmuch as the lines AB and BC of the triangle ABC are equall, the angle BAC is equall to the angle BCA (by the 5. of the first). And eche of them is the halfe of the right angle ABC (by the 32. of the first). In like sort may it be shewed in the triangle ABD , that the angle BDA is equall to the angle BAD , and that eche of them is the halfe of a right angle. Wherefore the whole angle CAD , which is composed of the two halfe right angles, namely, DAB and CAB is a right angle. And so haue ye what is a right angled Cone.

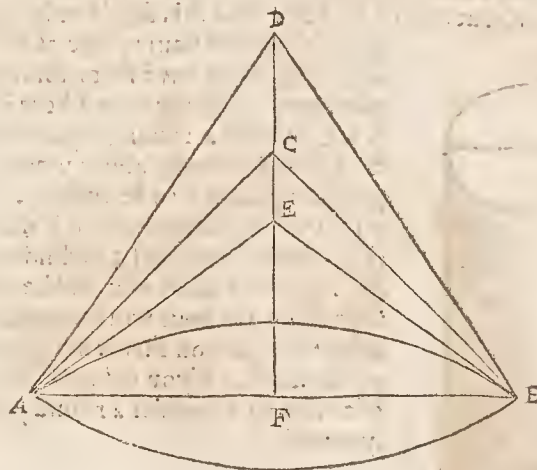
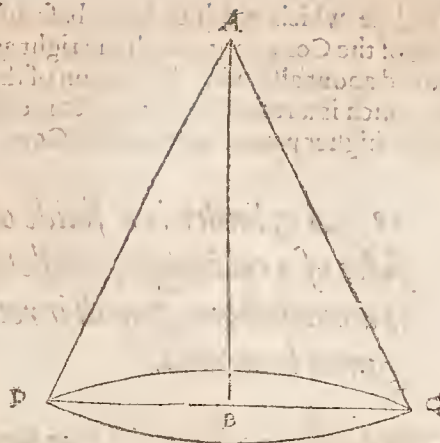
But if it be lesse, then is it an obtuseangle Cone. As in this example, the line AB fixed is lesse then the line BC moued about. Wherefore the Cone described of the circumuolutiō of the triangle ABC about the line AB , is an obtuseangle Cone, for that the angle at the toppe DAC is greater then a right angle. Wherefore it is an obtuseangle. And therefore the Cone is called an obtuse angle Cone.



And

And if it be greater, then is it an acuteangle Cone. As in this figure, the line AB fastened, is greater then the line BC moued about. Wherefore the Cone described by the motion and turning of the triangle ABC about AB is an acuteangle Cone, hauing the angle at the toppe BAC an acute angle. Of whome the Cone is called an acuteangle Cone. For the easier sight & cōsideration of all these kindes of Cones, and also for the plainer demonstration of the varieties of their angles in their toppes, I haue described them all three in one playne figure, of which the Cone ACB is a right angled Cone, hauing his fixed side CF equall to the line FB , and hys angle ACB a right angle: the Cone AEB is an obtuse angle Cone, and ADB an acuteangle Cone. By which figure ye may easily demonstrate (by the 21. of the first) that the angle ADB of the Cone ADB , whose fixed line DF is greater then the side FB , is lesse then the right angle ACB , and so is an acute angle. And also (by the same 21. of the first) ye shall with like facilitie perceauē how the angle AEB of the Cone AEB whose fixed line EF is lesse then the side FB , is greater then the right angle ACB : and therefore is an obtuse angle.

This figure of a Cone is of *Campane*, of *Vitellio*, and of others which haue written in these latter times, called a round Pyramis, which is not so aptly. For a Pyramis, and a Cone, are farre distant, & of sundry natures. A Cone is a regular body produced of one circumuolution of a rectangle triangle, and limited and bordered with one onely round superficies. But a Pyramis is terminated and bordered with diuers superficieces. Therefore can not a Cone by any iust reason beare the name of a Pyramis. This solide of many is called *Turbo*, which to our purpose may be Englished a *Top* or *Ghyg*: and moreouer, peculiarly *Campane* calleth a Cone the Pyramis of a round Columne, namely, of that Columne which is produced of the motion of a parallelogramme (contained of the lines AB and BC) moued about, the line AB being fixed. Of which Columnes shall be shewed hereafter.

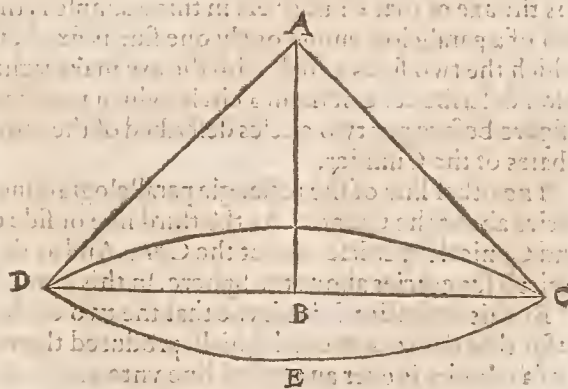


A Cone called of *Campane* a rounde Pyramis.

17 The axe of a Cone is that line, which abideth fixed, about which the triangle is moued. And the base of the Cone is the circle which is described by the right line which is moued about.

Senententh diffinition,

As in the example the line AB is supposed to be the line about which the right angled triangle ABC (to the production of the Cone) was moued: and that line is here of *Euclide* called the axe of the Cone described. The base of the Cone is the circle which is described by the right line which is moued about. As the line AB was fixed and stayed, so was the line BC (together with the whole triangle ABC) moued and turned about. A line moued; as hath bene sayd before, produceth a superficies: and because the line BC is moued about a point, namely, the point B , being the end of the axe of the Cone AB , it produceth by his motion, and reuolution a circle, which circle is the base of the Cone: as in this example, the circle CDE .



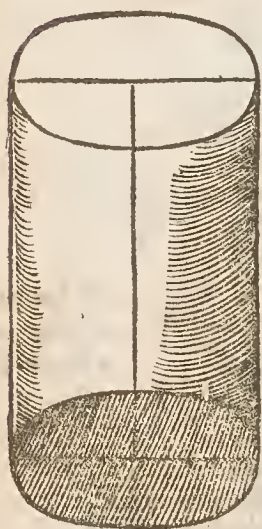
The eleuenth Booke

A conicall superficies.

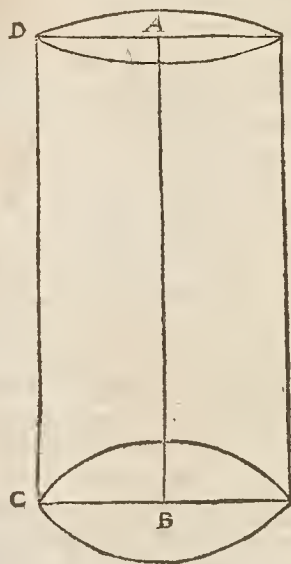
The line which produceth the base of the Cone, is the line of the triangle which together with the axe of the Cone containeth the right angle. The other side also of the triangle, namely, the line AC , is moued about also with the motion of the triangle, which with his reuolution describeth also a superficies, which is a round superficies, & is erected vpon the base of the Cone, & endeth in a point, namely, in the higher part or toppe of the Cone. And it is commonly called a Conicall superficies.

Eighteenth definition.

18 *A cylinder is a solide or bodely figure which is made, when one of the sides of a rectangle parallelogramme, abiding fixed, the parallelogramme is moued about, vntill it returne to the selfe same place from whence it began to be moued.*



This definition also is of the same sort and condition, that the two definitions before geue were, namely, the definition of a Sphere and the definition of a Cone. For all are geuen by mouing of a superficies about a right line fixed, the one of a semicircle about his diameter, the other of a rectangle triangle about one of his sides. And this solide or body here defined is caused of the motion of a rectangle parallelograme hauing one of his sides containyng the right angle fixed from some one poynt till it returne to the same agayne where it began. As suppose $ABCD$ to be a rectangle parallelogramme, hauing his side AB fastned, about which imagine the whole parallelogramme to be turned, till it returne to the poynt where it began, then is that solide or body, by this motion described, a Cylinder: which because of his roundnes can not at full be described in a playne superficies, yet haue you for an example thereof a sufficient designation therof in the margent such as in a plaine may be. If you wil perfectly behold the forme of a cilinder. Consider a round piller that is perfectly round.



Ninetenth definition.

19 *The axe of a cilinder is that right line which abydeth fixed, about which the parallelogramme is moued. And the bases of the cilinder are the circles described of the two opposite sides which are moued about.*

Euen as in the description of a Sphere the line fastened was the axe of the Sphere produced: and in the description of a cone, the line fastened was the axe of the cone brought forth: so in this description of a cilinder the line abiding, which was fixed, about which the rectangle parallelogramme was moued is the axe of that cilinder. As in this example is the line AB . The bases of the cilinder &c. In the reuolution of a parallelogramme onely one side is fixed, therefore the three other sides are moued about: of which the two sides which with the axe make right angles, and which also are opposite sides, in their motion describe eche of them a circle, which two circles are called the bases of the cilinder. As ye see in the figure before put two circles described of the motiō of the two oppositelines AD and BC , which are the bases of the Cilinder.

A cylindricall superficies.

The other line of the rectangle parallelogramme moued, by his motion describeth the round superficies about the Cilinder. As the third line or side of a rectangle triangle by his motion described the round Conicall superficies about the Cone. And as the circūferēce of the semicircle described the round sphericall superficies about the Sphere. In this example it is the superficies described of the line DC .

Corollary.

By this definition it is playne that the two circles, or bases of a cilinder are euer equall and parallels: for that the lines moued which produced them remayned alwayes equall and parallels. Also the axe of a cilinder is euer an erected line vnto either of the bases. For with all the lines described in the bases, and touching it, it maketh right angles,

A rounde Columne or Sphere.

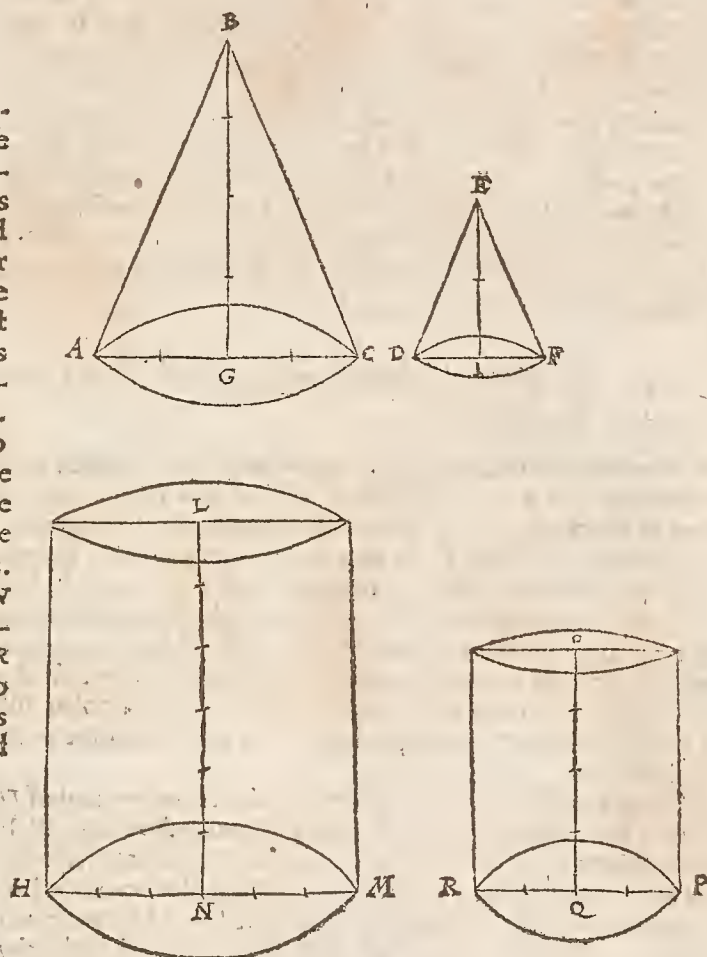
Campane, Vitellio, with other later writers, call this solide or body a round Columne or piller. And *Campane* addeth vnto this definition this, as a corollary. That of a round Columne, of a Sphere, and of

of a circle the cêtre is one and the selfe same. That is (as he him selfe declareth it & proueth the same) where the Columne, the Sphere, and the circle haue one diameter.

A Corollary added by Campanus.
Twenty diffinition.

20 Like cones and cilinders are those, whose axes and diameters of their bases are proportionall

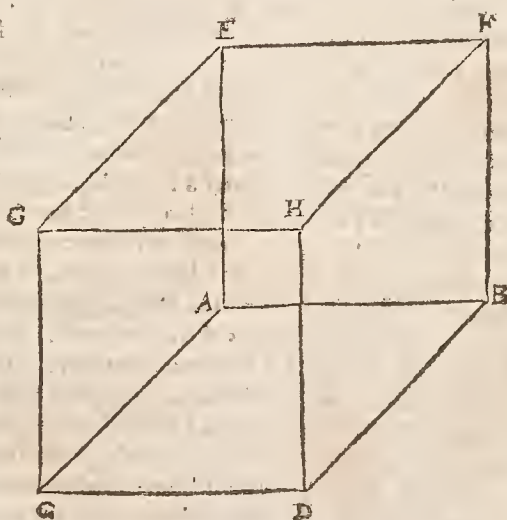
The similitude of cones and cilinders standeth in the proportion of those right lines, of which they haue their originall and spring. For by the diameters of their bases is had their length and breadth, and by their axe is had their heighth or deepenes. Wherefore to see whether they be like or vnlike, ye must compare their axes together, which is their depth, and also their diameters together, which is thier length & breadth. As if the axe BG of the cone ABC be to the axe EI of the cone DEF , as the diameter AC of the cone ABC is to the diameter DF of the cone DEF , then are the cones ABC and DEF like cones. Likewise in the cilinders. If the axe LN of the cilinder LMN haue that proportion to the axe OQ of the cilinder ROP , which the diameter HM hath to the diameter RP : then are the cilinders LMN and ROP like cilinders, and so of all others.



21 A Cube is a solide or bodely figure containned vnder sixe equall squares.

Twenty one diffinition.

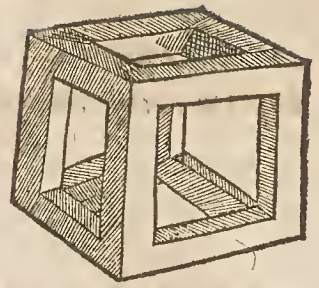
As is a dye which hath sixe sides, and eche of them is a full and perfect square, as limites or borders vnder which it is containned. And as ye may conceiue in a piece of timber containning a foote square euery way, or in any such like. So that a Cube is such a solide whose three dimensions are equall, the length is equall to the breadth thereof, and eche of them equall to the depth. Here is as it may be in a playne superficies set an image therof, in these two figures wherof the first is as it is commonly described in a playne, the second (which is in the beginning of the other side of this leafe) is drawn as it is described by arte vpo a playne superficies to shew somwhat bodilike. And in deede the latter descriptiō is for the sight better thē the first. But the first for the demōstrations of Euclides propositions in the five bookes following is of more vse, for that in it may be considered and sene



The eleventh Booke

all the fixe sides of the Cube. And so any lines or sections drawn in any one of the fixe sides. Which can not be so wel sene in the other figure described vpon a playnel. And as touching the first figure (which is set at the ende of the other side of this leafe) ye see that there are fixe parallelogrammes which ye must conceyue to be both equilater and rectangle, although in dede there can be in this description onely two of them rectangle, they may in dede be described al equilater. Now if ye imagine one of the fixe parallelogrammes, as in this example, the parallelogramme $ABCD$ to be the base lieng vpon a ground playne superficies. And so conceiue the parallelogramme $EFGH$ to be in the toppe ouer it, in such sort, that the lines AE, CG, DH , & BF may be erected perpendicularly from the pointes A, C, B, D , to the ground playne superficies or square $ABCD$. For by this imagination this figure wil shew vnto you bodilike. And this imagination perfectly had, wil make many of the propositions in these fise bookes following, in which are required to be described such like solides (although not all cubes) to be more plainly and easly conceiued.

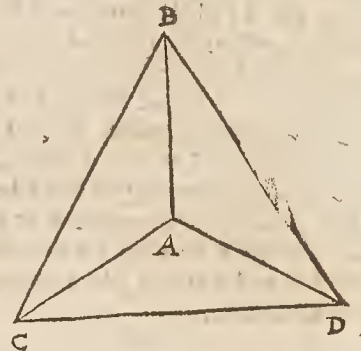
In many examples of the Greeke and also of the Latin, there is in this place set the diffinition of a Tetrahedron, which is thus.



Twenty two
diffinition.

22 *A Tetrahedron is a solide which is contained vnder fower triangles equall and equilater.*

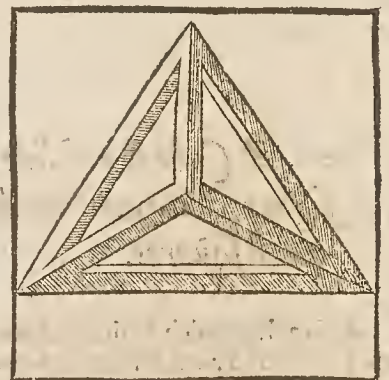
A forme of this solide ye may see in these two examples here set, whereof one is as it is commonly described in a playne. Neither is it hard to conceaue. For (as we before taught in a Pyramis) if ye imagine the triangle BCD to lie vpon a ground plaine superficies, and the point A to be pulled vp together with the lines AB, AC , and AD , ye shall perceauie the forme of the Tetrahedron to be contayned vnder 4. triangles, which ye must imagine to be al fower equilater and equiangle, though they can not so be drawn in a plaine. And a Tetrahedron thus described, is of more vse in these fise bookes following, then is the other, although the other appeare in forme to the eye more bodilike.



A Tetrahedron one of the fise regular bodyes.

Difference betwene a Tetrahedron and a Pyramis.

Why this definition is here left out both of *Campane* and of *Flussas*, I can not but maruell, considering that a Tetrahedron, is of all Philosophers counted one of the fise chiefe solides which are here defined of *Euclide*, which are called comonly regular bodies, without mencion of which, the entreatie of these should seeme much maimed: vnlesse they thought it sufficiently defined vnder the definition of a Pyramis, which plainly and generally taken, includeth in dede a Tetrahedron, although a Tetrahedron properly much differeth from a Pyramis, as a thing speciall or a particular, from a more generall. For so taking it, euery Tetrahedron is a Pyramis, but not euery Pyramis is a Tetrahedron. By the generall definition of a Pyramis, the superficieses of the sides may be as many in number as ye list, as 3. 4. 5. 6. or moe, according to the forme of the base, whereon it is set, whereof before in the definition of a Pyramis were examples geuen. But in a Tetrahedron the superficieses erected can be but three in number according to the base therof, which is euer a triangle. Againe, by the generall definition of a Pyramis, the superficieses erected may ascend as high as ye list, but in a Tetrahedron they must all be equall to the base. Wherefore a Pyramis may seeme to be more generall then a Tetrahedron, as before a Prisme seemed to be more generall then a Parallelipipedon, or a sided Columne: so that euery Parallelipipedon is a Prisme, but not euery Prisme is a Parallelipipedon. And euery axe in a Sphere is a diameter: but not euery diameter of a Sphere is the axe therof. So also noting well the definition of a Pyramis, euery Tetrahedron may be called a Pyramis, but not euery Pyramis a Tetrahedron. And in dede *Pfellus* in numbring of these fise solides or bodies, calleth a Tetrahedron a Pyramis in manifest wordes. This I say might make *Flussas* & others (as I thinke it did) to omitt the definition of a Tetrahedron in this place, as sufficiently comprehended withiu the definition of a Pyramis geuen before. But why then did he not count that definition of a Pyramis faultie, for that it extendeth it selfe to large, and comprehendeth vnder it a Tetrahedron (which differeth from a Pyramis by that it is contayned of equall triangles) as he not so aduisedly did before the definition of a Prisme.



Pfellus calleth a Tetrahedron a Pyramis.

Twenty three
definition.

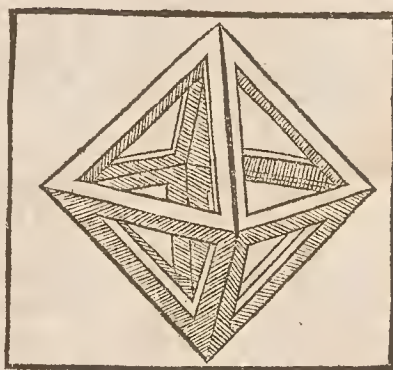
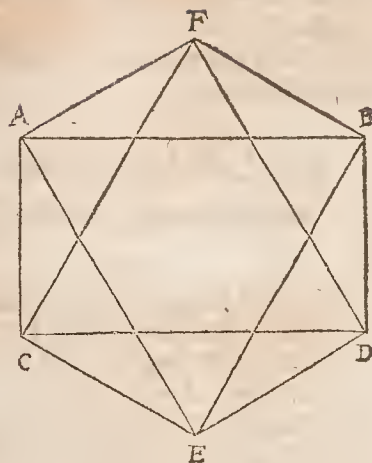
23 *An Octobedron is a solide or bodily figure cõtained vnder eight equall and equilater triangles.*

As

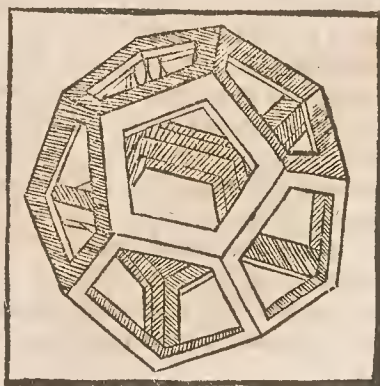
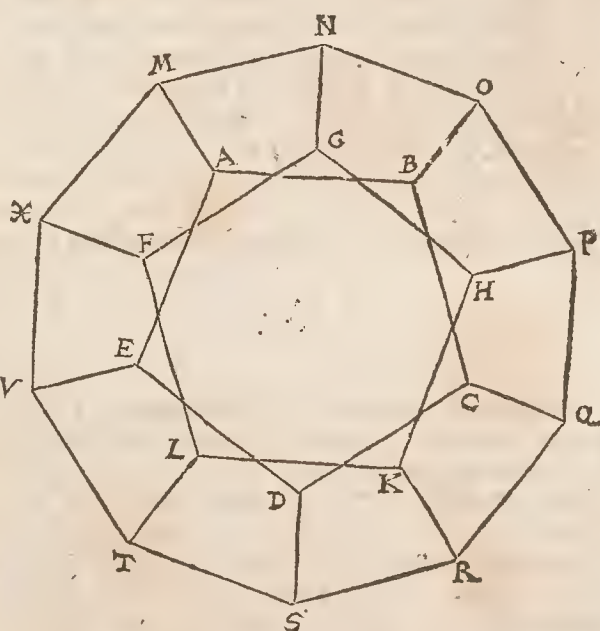
As a Cube is a solide figure containd vnder fixe superficiall figures of foure sides or squares which are equilater, equiangle, and equall the one to the other : so is an Octohedron a solide figure contained vnder eight triangles which are equilater and equall the one to the other . As ye may in these two figures here set beholde. Whereof the first is drawn according as this solide is commonly described vpon a plaine superficies . The second is drawn as it is described by arte vpon a plaine, to shewe bodilike . And in deede although the second appeare to the eye more bodilike, yet as I before noted in a Cube, for the vnderstanding of diuers Propositions in these fīue booke following, is the first description of more vse yea & of necessitie. For without it, ye can not cōceauē the draught of lines and sections in any one of the eight sides which are sometimes in the descriptions of some of those Propositions required. Wherefore to the consideration of this first description, imagine first that vpon the vpper face of the superficies of the parallelogramme *ABCD*, be described a Pyramis, hauing his fower triangles *AFB*, *AFC*, *CED*, and *DFB*, equilater and equiangle, and concurring in the point *F*. Thē cōceauē that on the lower face of the superficies of the former parallelogramme be described an other Pyramis, hauing his fower triangles *AEB*, *AEC*, *CED*, & *DEB*, equilater and equiangle, and concurring in the point *E*. For so although somewhat grossly by reason the triangles can not be described equilater, you may in a plaine perceauē the forme of this solide, and by that meanes conceauē any lines or sections required to be drawn in any of the sayd eight triangles which are the sides of that body.

24 *A Dodecahedron is a solide or bodily figure cōtained vnder twelue equall, equilater, and equiangle Pentagons.*

As a Cube, a Tetrahedron, and an Octohedron, are containd vnder equall plaine figures, a Cube vnder squares, the other two vnder triangles : so is this solide figure contained vnder twelue equilater, equiangle, and equall Pentagons, or figures of fīue sides . As in these two figures here set you may perceauē . Of which the first (which thing also was before noted of a Cube, a Tetrahedron, and an Octohedron) is the common description of it in a plaine, the other is the description of it by arte vpon a plaine to make it to appeare somewhat bodilike. The first description in deede is very obscure to conceauē, but yet of necessitie it must so, neyther can it otherwise, be in a plaine described to vnderstād those Propositions of *Euclide* in these fīue booke following which concerne the same. For in it although rudely, may you see all the twelue Pentagons, which should in deede be all equall, equilater, and equiangle . And now how you may somewhat conceauē the first figure described in the plaine to be a body . Imagine first the Pentagon *ABCDE* to be vpon a ground plaine superficies, then imagine the Pentagon *FGHKL* to be on high opposite vnto the Pentagon *ABCDE*. And betwene those two Pentagons there will be ten Pentagons pulled vp, fīue frō the fīue sides of the ground Pentagon, namely, from the side *AB* the Pentagon *ABONM*, from the side *BC* the Pentagon *BCQPO*, from the side *CD* the Pentagon *CDSRQ*, from the side *DE*, the Pentagon *DEVT*, from the side *EA* the Pentagon *EAMXV*, the other fīue Pentagons haue eche one of their sides common with one of the sides of the Pentagon *FGHKL*, which is opposite vnto the Pentagon in the ground superficies : namely, these are the other fīue Pentagons *FGNMX*, *GHPON*, *HKRQP*, *KLRST*, *LFXVT*. So



Twēty fower definition.



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So

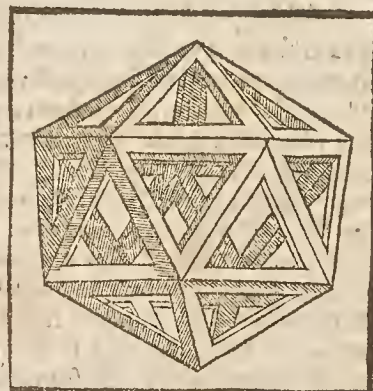
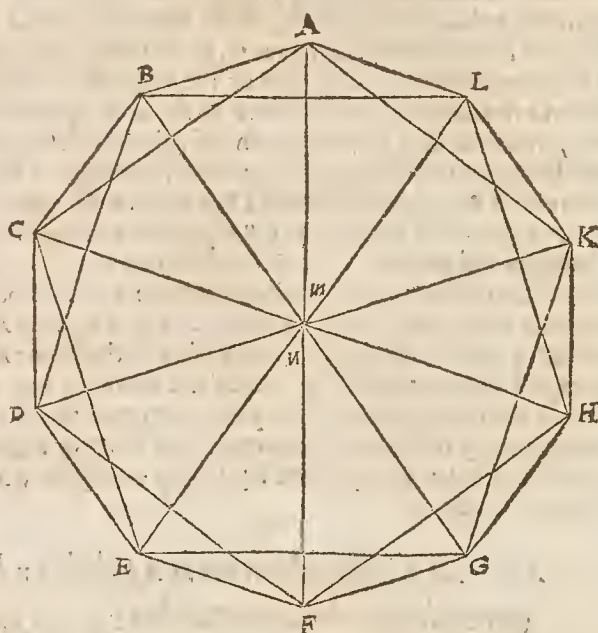
The eleventh Booke

So here you may behold twelue Pentagons, which if you imagine to be equall, equilater, & equiangle, and to be lifted vp, ye shall (although somewhat rudely) conceaue the bodily forme of a Pentagon. And some light it will geue to the vnderstanding of certaine Propositions of the five bookes following concerning the same.

Twenty five
definition.

25 *An Icosahedron is a solide or bodily figure contained vnder twentie equall and equilater triangles.*

As the solides before last mentioned are all described by the number and forme of the superficieses which containe them: so this body likewise is defined by that that it is containd of twentie triangles equall, equilater, and equiangle. And although this solide also be very hard to conceaue, as it is commonly described vpon a plaine (an example wherof you haue in the first figure here set): yet is it of necessitie that in that forme it be described, if we will vnderstand such descriptions as are set forth of *Euclide* touching that body in the five bookes following. Howbeit you may by it (although somewhat rudely) see the 20. triangles, which are imagined to be equall, equilater, and equiangle, if you consider five angles of five triangles to concurre together at a point. And forasmuch as there are in this solide 20. triangles, and enery triangle hath three angles, the concurse of the said triangles will be in twelue pointes. As in this example the pointes of the concurse are A, B, C, D, E, F, G, H, K, L, M, & N. Where note that in this plaine the two pointes M and N are but one point, yet must ye imagine one of those pointes to be erected vpward, and the other downward. Now the five triangles which concurre in the point M, are these, BMD, DMF, FMH, HML, and LMB: the five triangles which concurre in the point N, and are imagined to be erected downward, are these, ANC, CNE, ENG, GNK, and KNA: the other ten triangles which include this body, are these, ABC, BCD, CDE, DEF, EFG, FGH, GHK, HKL, KLA, LAB. The second figure here appeareth more bodilike vnto the eye.



Five regular
bodies.

These five solides now last defined, namely, a Cube, a Tetrahedron, an Octohedron, a Dodecahedron and an Icosahedron are called regular bodies. As in plaine superficieses, those are called regular figures, whose sides and angles are equal, as are equilater triangles, equilater pentagons, hexagons, & such lyke, so in solides such only are counted and called regular, which are comprehended vnder equal playne superficieses, which haue equal sides and equal angles, as all these five foresayd haue, as manifestly appeareth by their definitions, which were all geuen by this proprietie of equalitie of their superficieses, which haue also their sides and angles equall. And in all the course of nature there are no other bodies of this condition and perfection, but onely these five. Wherefore they haue euer of the auncient Philosophers bene had in great estimation and admiration, and haue bene thought worthy of much contemplacion, about which they haue bestowed most diligent study and endeouour to searche out the natures & properties of them. They are as it were the ende and perfection of all Geometry, for whose sake is written whatsoeuer is written in Geometry. They were (as men say) first inuented by the most witty *Pithagoras* then after ward set forth by the diuine *Plato*, and last of all meruelously taught and declared by the most excellent Philosopher *Euclide* in these bookes following, and euer since wonderfully embraced of all learned Philosophers. The knowledge of them containeth infinite secretes of nature. *Pithagoras*, *Timaeus* and *Plato*, by them searched out the composition of the world, with the harmony and preseruacion therof, and applied these five solides to the simple partes therof, the Pyramis, or Tetrahedron they ascribed to the fire, for that it ascendeth vpward according to the figure of the Pyramis. To the ayre they ascribed the Octohedron, for that through the subtle moisture which it hath, it extendeth it selfe enery way to the one side, and to the other, accordyng as that figure doth. Vnto the water they assigned the Icosahedron

The dignity of
these bodies.
A Tetrahe-
dron ascribed
vnto the fire.
An octohe-
dron ascribed
vnto the ayre.

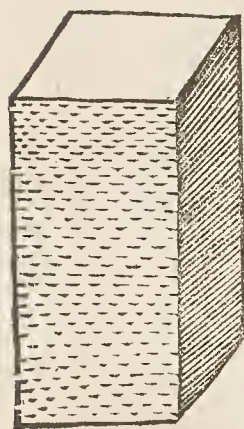
Icosahedron, for that it is continually flowing and mouing, and as it were making angles on euery side according to that figure. And to the earth they attributed a Cube, as to a thing stable, firme and sure as the figure signifieth. Last of all a Dodecahedron, for that it is made of Pentagōs, whose angles are more ample and large then the angles of the other bodies, and by that meanes draw more to roundnes, & to the forme and nature of a sphere, they assigned to a sphere, namely, to heauen. Who so will read *Plato* in his *Tineus*, shall read of these figures, and of their mutuall proportion, straunge matters, which here are not to be entreated of, this which is sayd, shall be sufficient for the knowledge of them, and for the declaration of their diffinitions.

An Icosahedron assigned vnto the water.
A cube assigned vnto the earth.
A dodecahedron assigned to heauen.

After all these diffinitions here set of *Euclide*, *Flusbas* hath added an other diffinition, which is of a Parallelipipedon, which bicause it hath not hitherto of *Euclide* in any place bene defined, and because it is very good and necessary to be had, I thought good not to omite it, thus it is,

A parallelipipedon is a solide figure comprehended vnder foure playne quadrangle figures, of which those which are opposite are parallels.

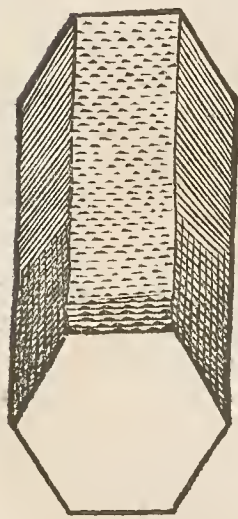
Diffinition of a parallelipipedon.



As in playne superficiees a parallelogramme is that which is contained vnder foure sides beyng lines, and whose opposite sides are equidistant and parallel lynes, so in solide figures a Parallelipipedon is that solide which is containd vnder foure quadrangle superficiees, whose opposite sides are also parallels, as it is easily to be sene and conceaued in a cube or die, all whose opposite sides are parallel superficiees, & so of others like, ye may also somewhat conceiue therof by the example in the margent.

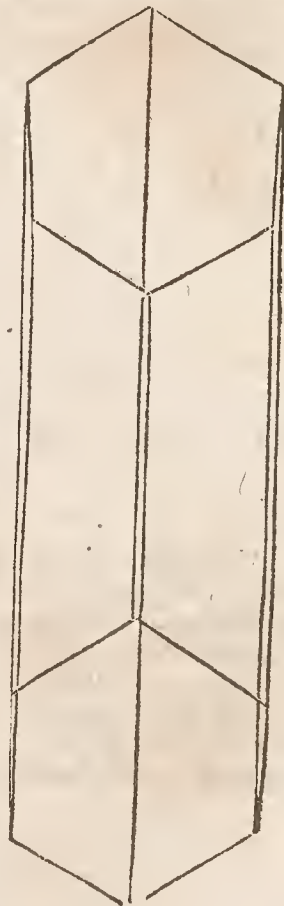
There is also in these bookes following, mencion made of solides, whose two bases are Polygonon figures, lyke, equall, equilater, and parallels, and the sides set vpon the bases are parallelogrammes: which kynde of solides Campano calleth sided Columnes (and which as was before noted, may be cōpredd vnder the definition of a Prisme) a forme wherof although grosely, behold in this example, whose bases are two like equall, equilater, equiangle, and parallel hexagons, and the sides set vpon those bases are sixe parallelogrammes: ye may better cōceiue the forme therof by the figure put vnder the figure of the parallelipipedon, which appeareth more bodilike. There may of these be infinite formes according to the diuersitie of their bases.

A sided Columne.



Because these fīue regular bodies here defined are not by these figures here set, so fully and liuely expressed, that the studious beholder can throughly according to their definitions conceyue them. I haue here geuen of them other descriptions drawn in a playne, by which ye may easily attayne to the knowledge of them. For if ye draw the like formes in matter that wil bow and geue place, as most aptly ye may do in fine pasted paper, such as pastwiues make womēs pastes of, & the with a knife cut euery line finely, not through, but halfe way only, if the ye bow and bende them accordingly, ye shall most plainly and manifestly see the formes and shapes of these bodies, euen as their definitions shew. And it shall be very necessary for you to haue store of that pasted paper by you, for so shal you vpon it describe

the formes of other bodies, as Prismes and Parallelipipedons, and such like set forth in these fīue bookes following, and see the very formes of those bodies there mēcioned: which will make these bokes concerning bodies, as easy vnto you as were the other bookes, whose figures you might plainly see vpon a playne superficies.

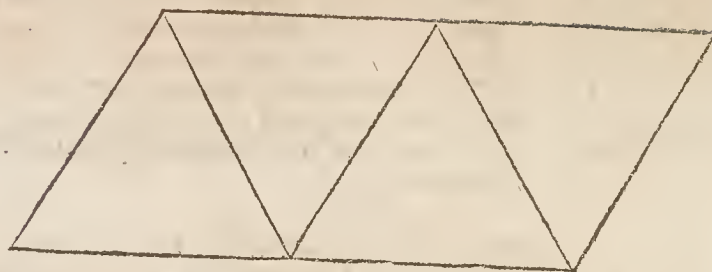


If

The eleuenth Booke

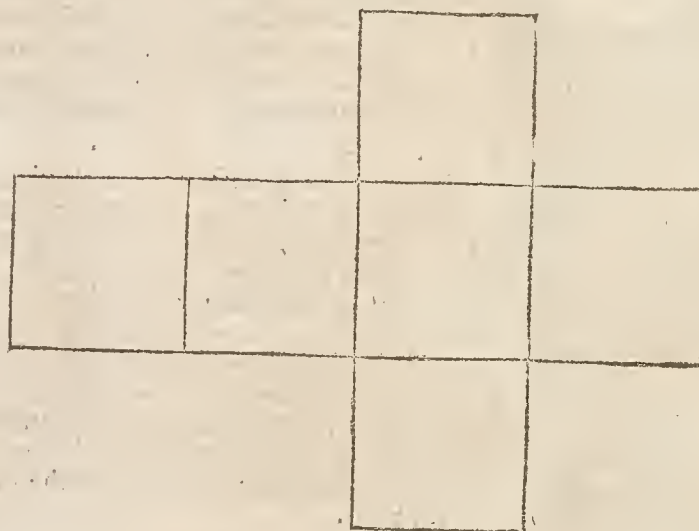
*A Tetrabe-
dron.*

If ye draw this figure consisting as ye see of fower equilater and equian-
gle triangles vpō pasted paper, or vp-
pon any other such like matter that
will bowe and geue place, and then
cut not through the paper, but as it
were halfe the thicknes of the paper,
the three lines contained within the
figure, and bowe & folde in the fower
triangles accordingly: they will close
together in such fort, that they will
make the perfecte forme of a Tetra-
hedron.



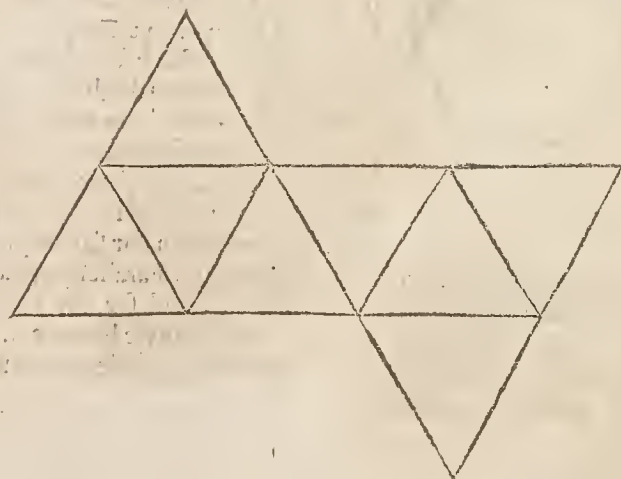
A cube.

This figure (consisting of
fixe equall squares) drawen vp-
on pasted paper, and the fixe
lines contained within the fi-
gure being cut finely halfe the
thicknes of the paper, or not
through, if then ye bowe and
folde accordingly the fixe e-
quall squares, they will so close
together, that they will cause
the perfecte forme of a Cube.



*An octobe-
dron.*

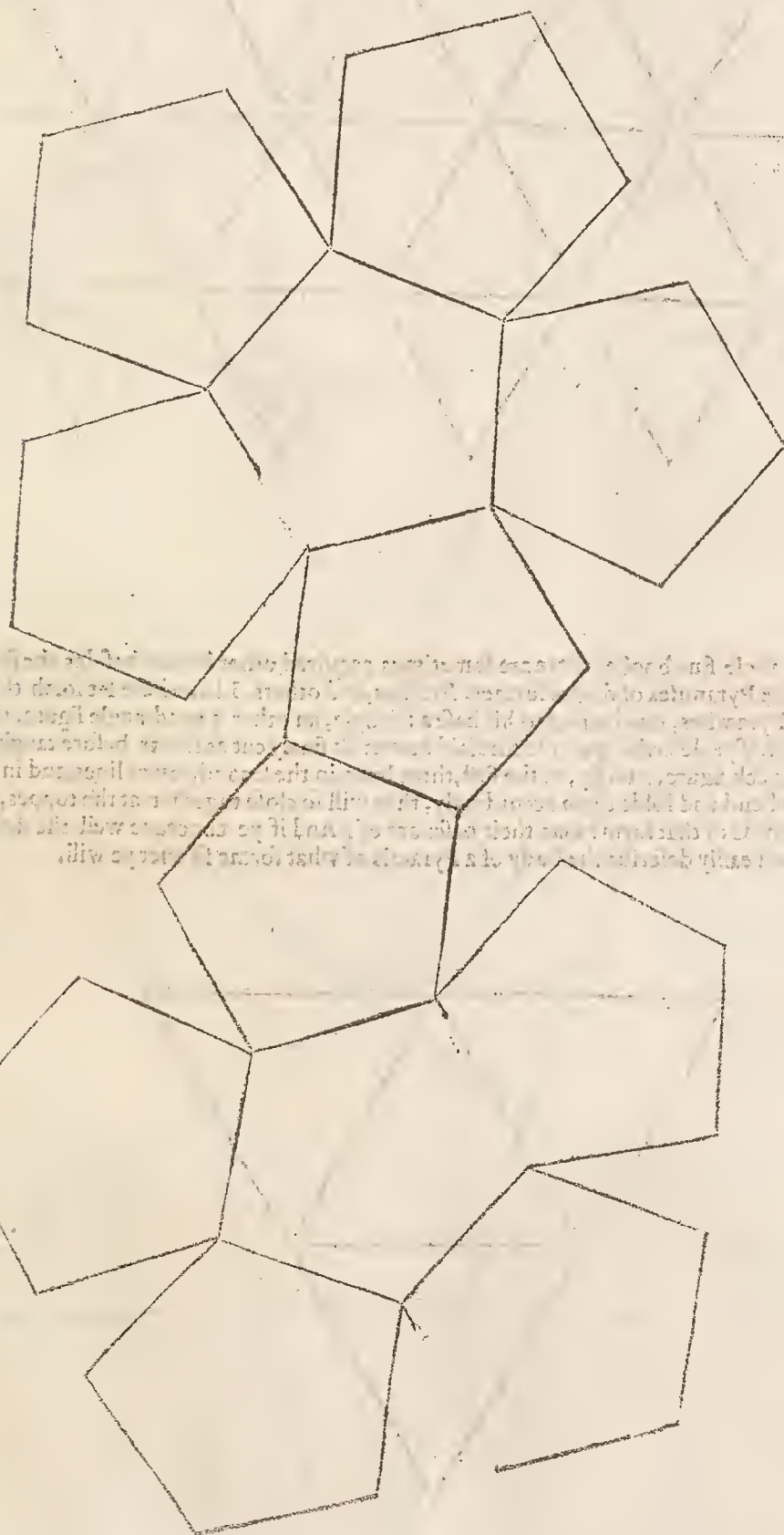
This figure (which consisteth of eight e-
quilater and equiangle triangles) drawen vp-
on the foresayd matter, and the seuen lines
contained within the figure being cut as be-
fore was taught, and the triangles bowed and
folded accordingly, they will close together
in such fort, that they will make the perfecte
forme of an Octohedron.



Describe

Describe this figure which consisteth of twelve equilater and equiangle Pentagons, vpon the fore-
said matter, and finely cut as before was taught the eleuen lines contained within the figure, and bow
and folde the Pentagons accordingly. And they will so close together, that they will make the very
forme of a Dodecahedron.

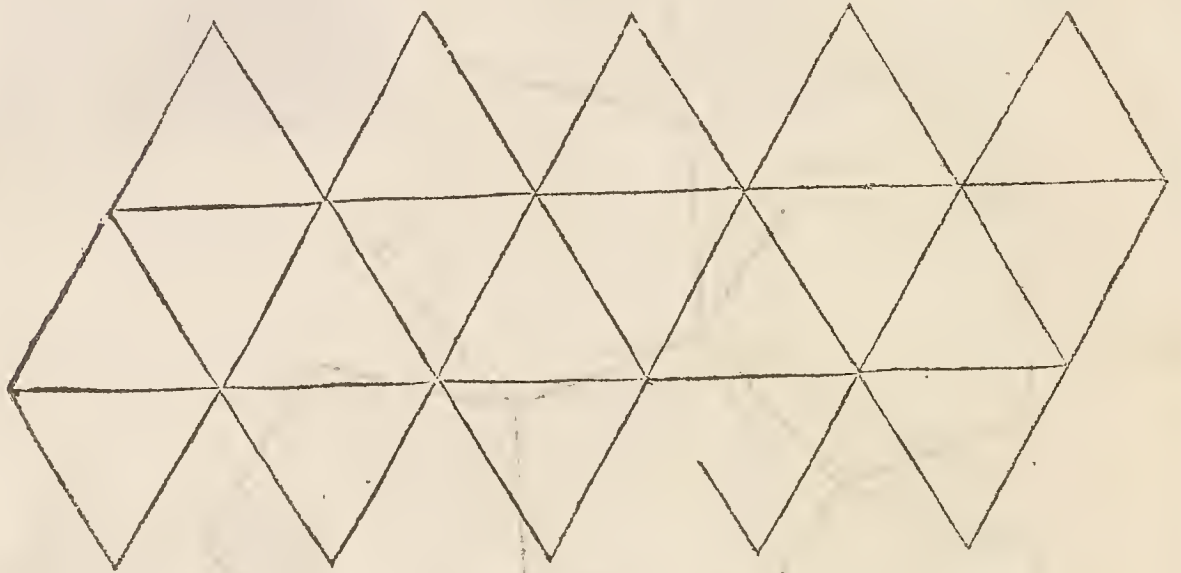
A Dodecabe-
dron.



The eleuenth Booke

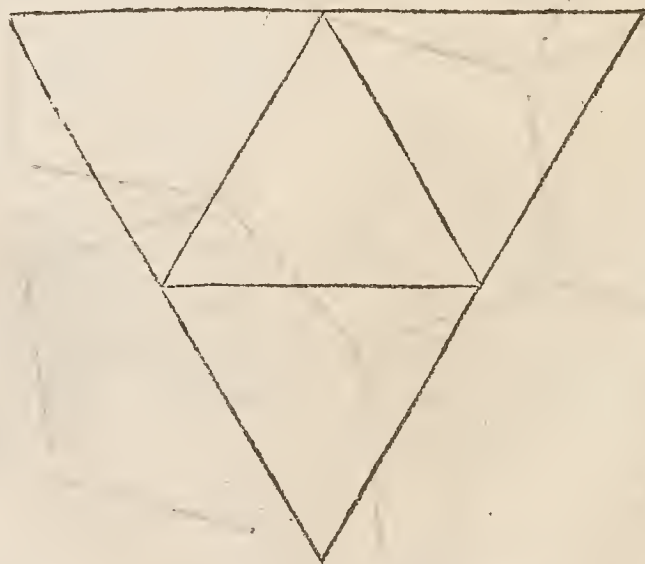
An Icosahedron.

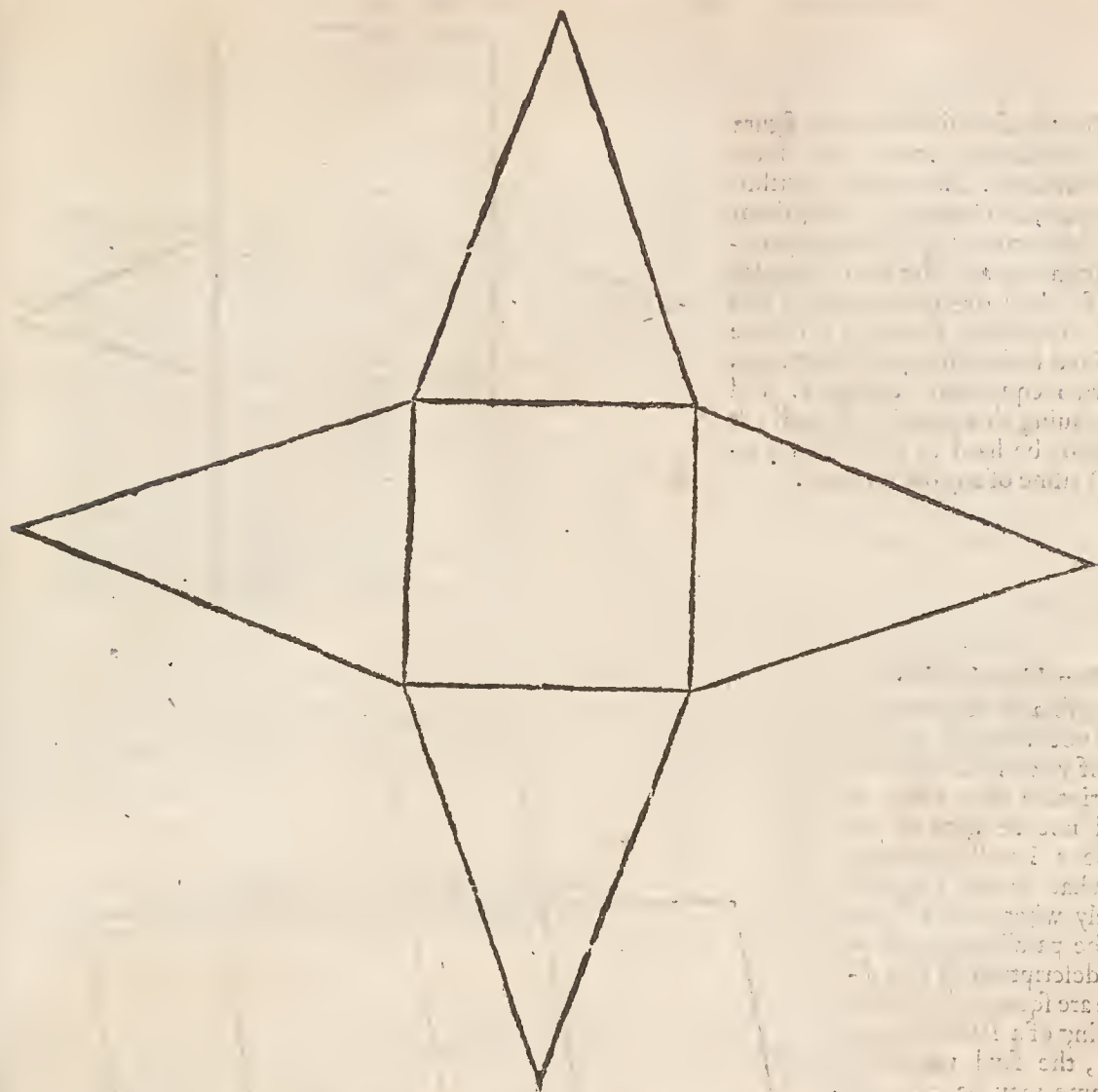
If ye describe this figure which consisteth of twentie equilater and equiangle triangles vpon the foresaid matter, and finely cut as before was shewed the ninetene lines which are containned within the figure, and then bowe and folde them accordingly, they will in such sort close together, that there will be made a perfecte forme of an Icosahedron.



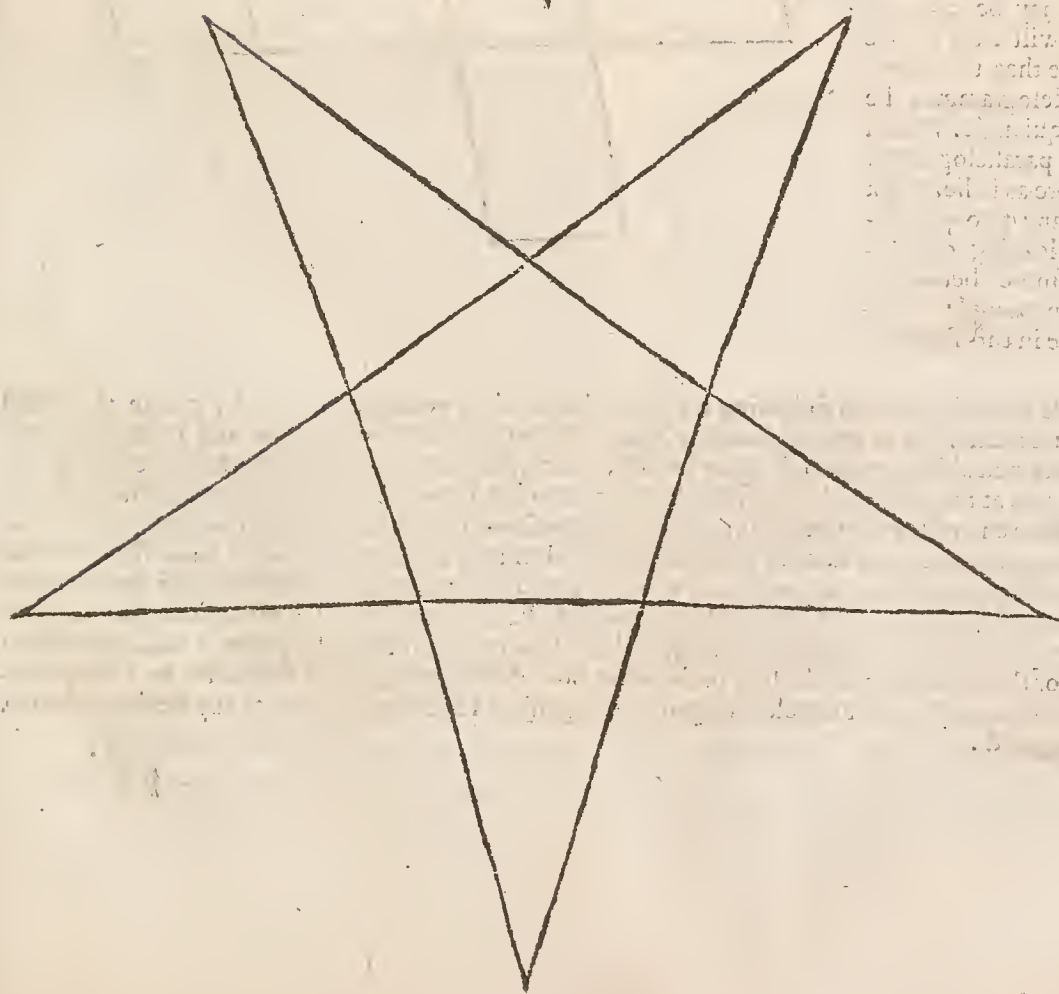
Because in these five bookes there are sometimes required other bodies besides the foresaid five regular bodies, as Pyramises of diuers formes, Prismes, and others, I haue here set forth three figures of three sundry Pyramises, one hauing to his base a triangle, an other a quadrangle figure, the other a Pentagon : which if ye describe vpon the foresaid matter & finely cut as it was before taught the lines contained within ech figure, namely, in the first, three lines, in the second, fower lines, and in the third, five lines, and so bend and folde them accordingly, they will so close together at the toppes, that they will make Pyramids of that forme that their bases are of. And if ye conceaue well the describing of these, ye may most easily describe the body of a Pyramis of what forme so euer ye will.

A triangled Pyramis.





The forme of
a quadrang-
led Pyramis.

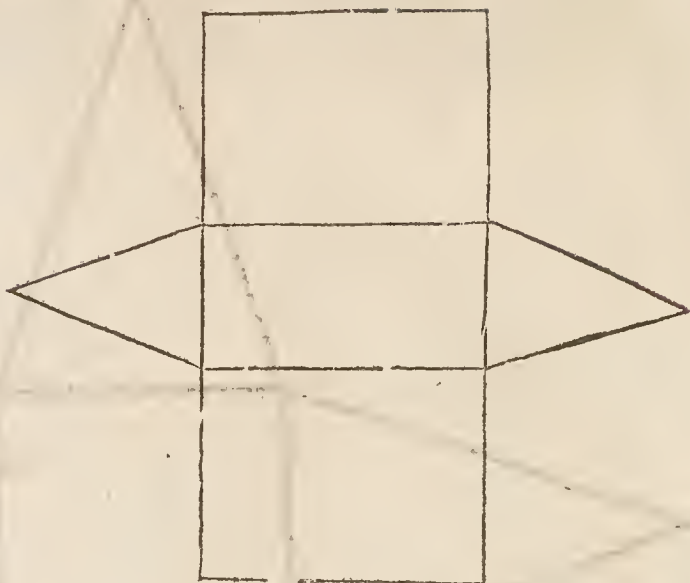


The forme of
a five angled
Pyramis.

The eleuenth Booke

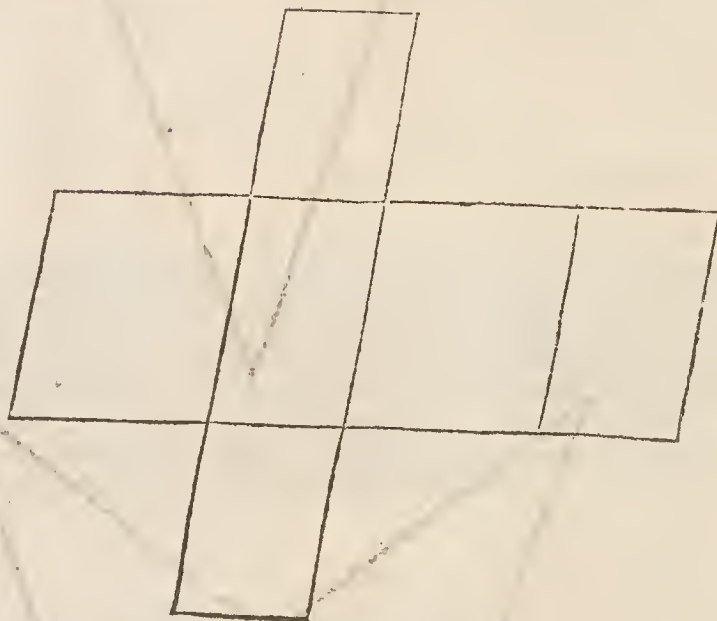
The forme of a prisme.

Likewise if ye describe this figure vpon the foresaid matter, and finely cutte the fower lines cõtained within the figure, and bowe and folde them together accordingly, the three parallelogrammes and the two triangles will so close together, that they will cause the perfecte forme of a Prisme cõtained vnder three parallelogrāmes and two equedistant triangles. And conceauing this description well, it shall not be hard to describe any other Prisme of any other forme.



The forme of a parallelipi- pedon.

Touching the description of Parallelipipedons I shall not neede to speake. For if ye consider well the description of a Cube, it shall not be hard to describe a Parallelipipedon of what forme ye will. Onely where as in a Cube all the parallelogrāmes in the description of that figure are squares, in the describing of a Parallelipipedon, the sayd parallelogramme may be of what forme ye will. So that ye take heede that the opposite parallelogrammes be equal & equiangle. Which opposite parallelogrāmes in the figure as it lieth in a plaine, is any two parallelogrammes leauing one parallelogramme betwene them. An example wherof beholde in this figure.



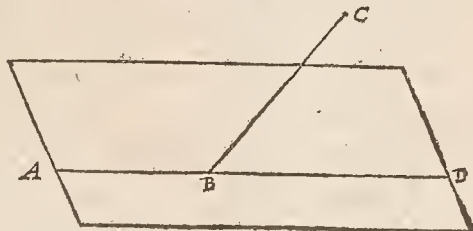
Because these fine bookes following are somewhat hard for young beginners, by reason they must in the figures described in a plaine imagine lines and superficieses to be eleuated and erected, the one to the other, and also conceaue solidos or bodies, which, for that they haue not hitherto bene acquainted with, will at the first sight be somewhat straunge vnto the, I haue for their more ease, in this eleuenth booke, at the end of the demonstration of euery Proposition either set new figures, if they concerne the eleuating or erecting of lines or superficieses, or els if they concerne bodies, I haue shewed how they shall describe bodies to be compared with the constructions and demonstrations of the Propositions to them belonging. And if they diligently weigh the maner obserued in this eleuenth booke touching the description of new figures agreeing with the figures described in the plaine, it shall not be hard for them of them selues to do the like in the other bookes following, when they come to a Proposition which concerneth either the eleuating or erecting of lines and superficieses, or any kindes of bodies to be imagined.

¶ The 1. Theoreme. The 1. Proposition.

That part of a right line should be in a ground playne superficies, & part eleuated vpward is impossible.



Or if it be possible, let part of the right line ABC , namely, the part AB be in a ground playne superficies, and the other part therof, namely, BC be eleuated vpwarde. And produce directly vpon the ground playne superficies the right line AB beyond the point B vnto the point D .



Wherefore vnto two right lines geuen ABC , and ABD , the line AB is a common section or part, which is impossible. For a right line can not touche a right line in mo pointes then one, vnlesse those right be exactly agreing and laid the one vpon the other. Wherefore that part of a right line should be in a ground playne superficies, and part eleuated vpward is impossible: which was required to be proued.

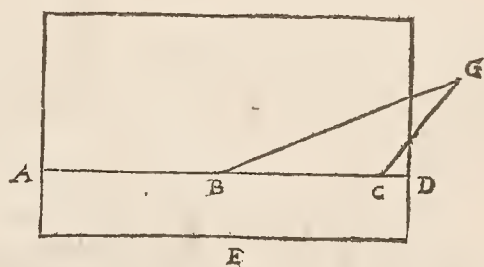
Demonstration leading to an impossibilitie.

This figure more plainly setteth forth the foresaid demonstration, if ye eleuate the superficies wherein is drawn the line BC .



An other demonstration after Flussas.

If it be possible let there be a right line ABG , whose part AB let be in the ground playne superficies AED , and let the rest therof BG be eleuated on high, that is, without the playne AED . Then I say that ABG is not one right line. For forasmuch as AED is a plaine superficies, produce directly & equally vpon the sayd playne AED the right lyne AB towards D , which by the 4. definition of the first shall be a right line. And from some one point of the right line ABD , namely, from C , draw vnto the point G a right lyne CG . Wherefore in the triangle BCG the outward angle ABG is equal to the two inward and opposite angles (by the 32. of the first) and therefore it is lesse then two right angles (by the 17. of the same) Wherefore the lyne ABG forasmuch as it maketh an angle, is not a right line. Wherefore that part of a right line should be in a ground playne superficies, and part eleuated vpward is impossible.



An other demonstration after Flussas.

If ye marke well the figure before added for the playner declaration of Euclides demonstration, it will not be hard for you to conceiue this figure which Flussas putteth for his demonstration: wherein is no difference but only the draught of the lyne GC .

¶ The 2. Theoreme. The 2. Proposition.

If two right line cut the one to the other, they are in one and the selfe same playne superficies: & euery triangle is in one & the selfe same superficies.

TT. ij.

Suppose

The eleuenth Booke



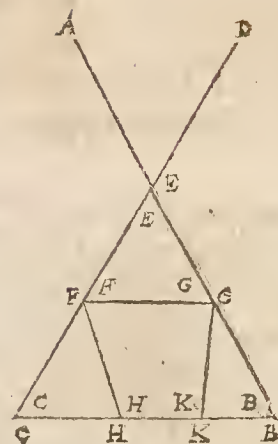
Suppose that these two right lines AB and CD doo cutte the one the other in the point E. Then I say that these lines AB and CD are in one and the selfe same superficies, and that euery triangle is in one &

Construction.

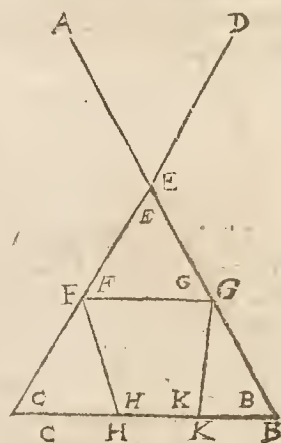
selfe same playne superficies. Take in the lines EC and EB points at all auentures, and let the same be F and G, and draw a right line from the poynt B to the point C, and an other from the point F to the point G. And draw the lines FH and GK. First I say

Demonstration leading to an impossibilitie.

that the triangle EBC is in one and the same ground superficies. For if part of the triangle EBC, namely the triangle FCH, or the triangle GBK be in the ground superficies, and the residue be in an other, then also part of one of the right lines EC or EB shall be in the ground superficies, and part in an other. So also if part of the triangle EBC, namely, the part EFG be in the ground superficies and the residue be in an other, then also one part of eche of the right lines EC and EB shall be in the ground superficies, & an other part in an other superficies, which (by the first of the eleuenth) is proued to be impossible. Wherefore the triangle EBC is in one and the selfe same playne superficies. For in what superficies the triangle BCE is, in the same also is either of the lines EC and EB, and in what superficies either of the lines EC and EB is, in the selfe same also are the lines AB and CD. Wherefore the right lines lines AB and CD are in one & the selfe same playne superficies, and euery triangle is in one & the selfe same playne superficies: which was required to be proued.



In this figure here set may ye more playnely conceaue the demonstration of the former proposition where ye may eleuate what part of the triangle ECB ye will, namely the part FCH or the part GBK, or finally the part FCGB as is required in the demonstration.



The 3. Theoreme.

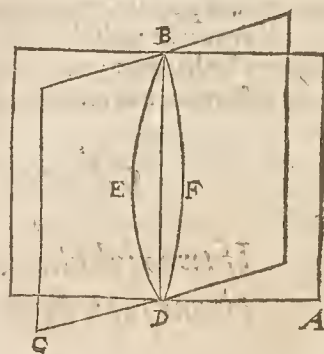
The 3. Proposition.

If two playne superficies cutte the one the other: their common section is a right line.



Suppose that these two superficieses AB & BC do cutte the one the other, and let their common section be the line DB. Then I say that DB is a right line. For if not, draw from the poynt D to the point B a right line DFB in the playne superficies AB, and likewise from the same poyntes draw an other right line DEB in the playne superficies BC. Now therfore two right lines DEB and DFB shall haue the selfe same endes, and therefore doo include a superficies which (by the last common sentence) is impossible. Wherefore the lines DEB and DFB are not right lines. In

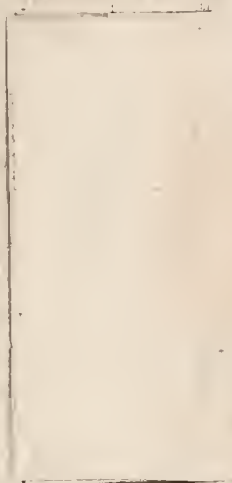
Demonstration leading to an impossibilitie.



like

like sort also may we proue that no other right line can be drawne from the poynt D to the point B besides the line DB which is the common section of the two superficieses AB and BC. If therefore two playne superficieses cutte the one the other, their common section is a right line: which was required to be demonstrated.

This figure here set, sheweth most playnely not onely this third proposition, but also the demonstration thereof, if ye eleuate the superficies AB, and so compare it with the demonstration.



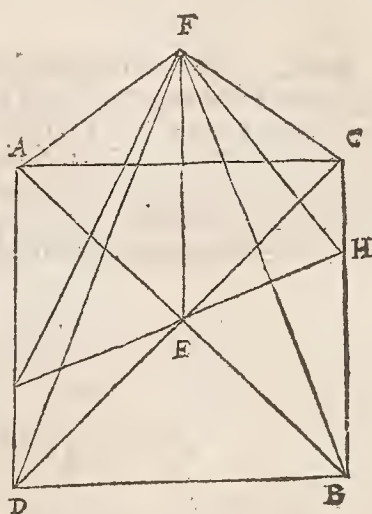
¶ The 4. Theoreme.

The 4. Proposition.

If from two right lines, cutting the one the other, at their common section, a right line be perpendicularly erected: the same shall also be perpendicularly erected from the playne superficies by the sayd two lines passing.



Suppose that there be two right lines AB and CD cutting the one the other in the poynt E. And from the poynt E let there be erected a right line EF perpendicularly to the sayd two right lines AB and CD: then I say that the right line EF, is also erected perpendiclar to the plaine superficies which passeth by the lines AB and CD. Let these lines AE, EB, EC, and ED be put equall the one to the other. And by the poynt E extend a right line at all adventures, and let the same be GEH. And drawe these right lines AD, CB, FA, FG, FD, FC, FH, and FB. And forasmuch as these two right lines AE & ED are equall to these two lines CE and EB, and they comprehend equall angles (by the 15. of the first): therefore (by the 4. of the first) the base AD is equall to the base CB, and the triangle AED is equall to the triangle CEB. Wherefore also the angle DAE, is equall to the angle ECB. But the angle AEG is equall to the angle BEH (by the 15. of the first). Wherefore there are two triangles AGE, and BEH hauing two angles of the one equall to two angles of the other, esche to his correspondent angle, and one side of the one equall to one side of the other, namely one of the sides which lye betwene the equall angles, namely, the side AE is equall to the side EB. Wherefore (by the 26. of the first) the sides remayning are equall to the sides remayning. Wherefore the side GE is equall to the side EH, and the side AG to the side BH. And forasmuch as the line AE is equall to the line EB, and the line FE is common to them both,



Construction.

Demonstration.

The eleuenth Booke



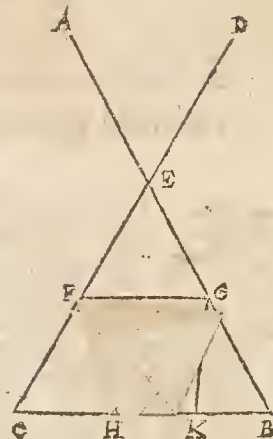
Suppose that these two right lines AB and CD doo cutte the one the other in the point E. Then I say that these lines AB and CD are in one and the selfe same superficies, and that euery triangle is in one &

Construction.

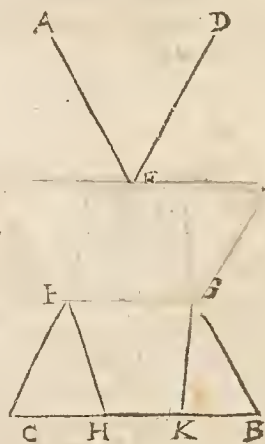
Take in the lines EC and EB points at all auentures, and let the same be F and G, and draw a right line from the poynt B to the point C, and an other from the point F to the point G. And draw the lines FH and GK. First I say that the triangle EBC is in one and the same ground superficies.

Demonstration leading to an impossibilitie.

For if part of the triangle EBC, namely the triangle FCH, or the triangle GBK be in the ground superficies, and the residue be in an other, then also part of one of the right lines EC or EB shall be in the ground superficies, and part in an other. So also if part of the triangle EBC, namely, the part EFG be in the ground superficies and the residue be in an other, then also one part of eche of the right lines EC and EB shall be in the ground superficies, & an other part in an other superficies, which (by the first of the eleuenth) is proued to be impossible. Wherefore the triangle EBC is in one and the selfe same playne superficies. For in what superficies the triangle BCE is, in the same also is either of the lines EC and EB, and in what superficies either of the lines EC and EB is, in the selfe same also are the lines AB and CD. Wherefore the right lines lines AB and CD are in one & the selfe same playne superficies, and euery triangle is in one & the selfe same playne superficies: which was required to be proued.



In this figure here set may ye more playnely conceaue the demonstration of the former proposition where ye may eleuate what part of the triangle ECB ye will, namely the part FCH or the part GBK, or finally the part FCGB as is required in the demonstration.



The 3. Theoreme.

The 3. Proposition.

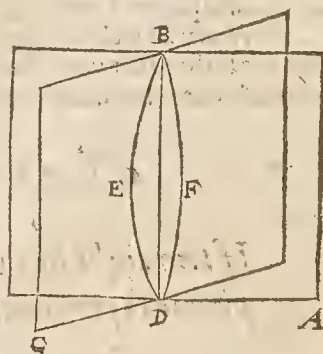
If two playne superficies cutte the one the other: their common section is a right line.



Suppose that these two superficieses AB & BC do cutte the one the other, and let their common section be the line DB. Then I say that DB is a right line. For if not, draw from the poynt D to the point

Demonstration leading to an impossibilitie.

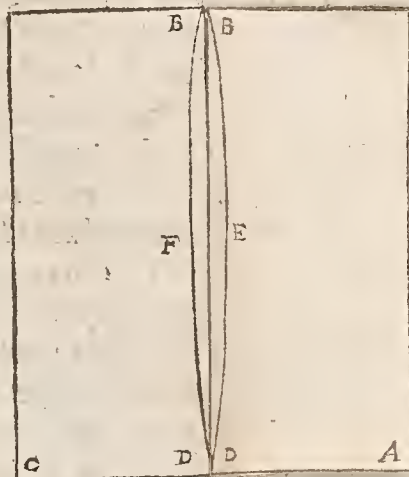
B a right line DFB in the playne superficies AB, and likewise from the same poyntes draw an other right line DEB in the playne superficies BC. Now therefore two right lines DEB and DFB shall haue the selfe same endes, and therefore doo include a superficies which (by the last common sentence) is impossible: Wherefore the lines DEB and DFB are not right lines. In



like

like sort also may we proue that no other right line can be drawne from the poynt D to the point B besides the line DB which is the common section of the two superficieses AB and BC. If therefore two playne superficieses cutte the one the other, their common section is a right line: which was required to be demonstrated.

This figure here set, sheweth most playnely not onely this third proposition, but also the demonstration thereof, if ye eleuate the superficies AB, and so compare it with the demonstration.



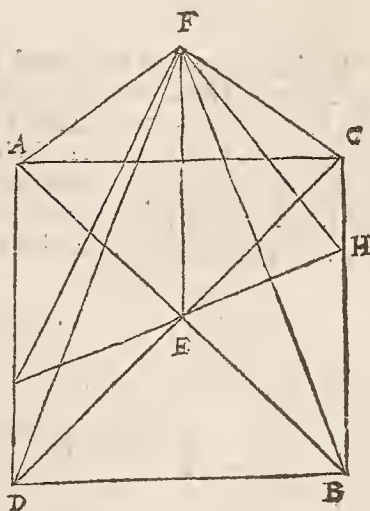
¶ The 4. Theoreme.

The 4. Proposition.

If from two right lines, cutting the one the other, at their common section, a right line be perpendicularly erected: the same shall also be perpendicularly erected from the playne superficies by the sayd two lines passing.



Suppose that there be two right lines AB and CD cutting the one the other in the poynt E. And from the poynt E let there be erected a right line EF perpendicularly to the sayd two right lines AB and CD: then I say that the right line EF, is also erected perpendiclar to the plaine superficies which passeth by the lines AB and CD. Let these lines AE, EB, EC, and ED be put equall the one to the other. And by the poynt E extend a right line at all adventures, and let the same be GEH. And drawe these right lines AD, CB, FA, FG, FD, FC, FH, and FB. And forasmuch as these two right lines AE & ED are equall to these two lines CE and EB, and they comprehend equall angles (by the 15. of the first): therefore (by the 4. of the first) the base AD is equall to the base CB, and the triangle AED is equall to the triangle CEB. Wherefore also the angle DAE, is equall to the angle ECB. But the angle AEG is equall to the angle BEH (by the 15 of the first). Wherefore there are two triangles AGE, and BEH having two angles of the one equall to two angles of the other, esche to his correspondent angle, and one side of the one equall to one side of the other, namely one of the sides which lye betwene the equall angles, namely, the side AE is equall to the side EB. Wherefore (by the 26. of the first) the sides remayning are equall to the sides remayning. Wherefore the side GE is equall to the side EH, and the side AG to the side BH. And forasmuch as the line AE is equall to the line EB, and the line FE is common to them both,



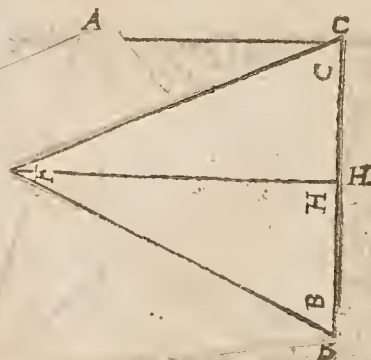
Construction.

Demonstration.

The eleuenth Booke

and maketh with them right angles, wherefore (by the fourth of the first) the base FA is equal to the base FB . And (by the same reason) the base FC is equal to the base FD . And forasmuch as the line AD is equal to the line BC , and the line FA is equal to the line FB as it hath bene proued. Therefore these two lines FA and AD are equal to these two lines FB & BC , the one to the other, & the base FD is equal to the base FC . Wherefore also the angle FAD is equal to the angle FCB . And againe forasmuch as it hath bene proued, that the line AG is equal to the line BH , but the line FA is equal to the line FB . Wherefore there are two lines FA and AG equal to two lines FB and BH and it is proued that the angle FAG is equal to the angle FBH : wherefore (by the 4. of the first) the base FG is equal to the base FH . Agayne forasmuch as it hath bene proued that the line GE is equal to the line EH , and the line EF is common to them both: wherefore these two lines GE and EF are equal to these two lines HE and EF , and the base FH is equal to the base FG : wherefore the angle GEF is equal to the angle HEF : Wherefore either of the angles GEF , and HEF is a right angle. Wherefore the line EF is erected, from the point E , perpendicularly to the line GH . In like sort may we proue, that the same line FE maketh right angles with all the right lines which are drawne vpon the ground playne superficies and touch the point B . But a right line is then erected perpendicularly to a plaine superficies, when it maketh right angles with all the lines which touch it, and are drawne vpon the ground playne superficies (by the 2. definition of the eleuenth). Wherefore the right line FE is erected perpendicularly to the ground playne superficies. And the ground plaine superficies is that which passeth by these right lines AB and CD . Wherefore the right line FE is erected perpendicularly to the playne superficies which passeth by the right lines AB and CD . If therefore from two right lines cutting the one the other and at their common section a right line be perpendicularly erected: it shall also be erected perpendicularly to the plaine superficies by the sayd two lines passing: which was required to be proued.

In this figure you may most evidently conceaue the proposition and demonstration, if ye erect perpendic-
ground playne superficies $ACBD$ the triang-
the triangles AFD , & CFB in such sort, that the
angle AFB may ioine & make one line with the line
angle AFD : and likewise that the line BF of the triangle
ioine & make one right line with the line BF of the triangle B .



¶ The 5. Theoreme.

The 5. Proposition.

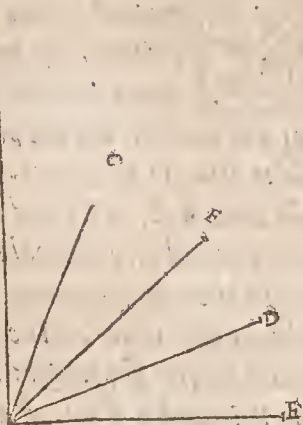
If vnto three right lines which touch the one the other, be erected a perpendicular line from the common point where those three lines touch: those three right lines are in one and the selfe same plaine superficies.



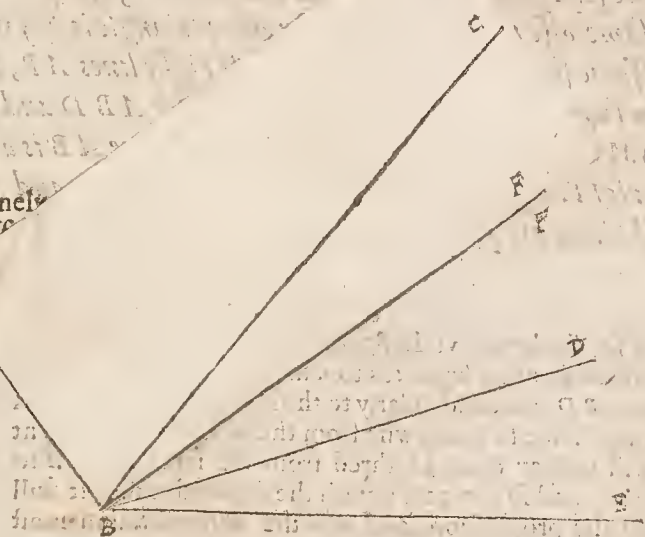
Suppose that vnto these three right lines BC , BD , and BE , touching the one the other in the poynt B , be erected perpendicularly from the poynt B , the line AB . Then I say, that those thre right lines BC , BD and BE , are in one & the selfe same plaine superficies. For if not, then if it be possible, let the lines BD & BE

BE be in the ground superficies, and let the line BC be erected upward (now the lines AB and BC are in one and the same playne superficies (by the 2. of the eleventh) for they touch the one the other in the point B). Extend the plaine superficies wherein the lines AB and BC are, and it shall make at the length a common section with the ground superficies, which common section shall be a right line (by the 3. of the eleventh): let that common section be the line BF. Wherefore the three right lines AB, BC, and BF are in one and the selfe same superficies, namely, in the superficies wherein the lines AB and BC are. And forasmuch as the right line AB is erected perpendicularly to either of these lines BD and BE, therefore the line AB is also (by the 4. of the eleventh) erected perpendicularly to the plaine superficies, wherein the lines BD and BE are. But the superficies wherein the lines BD and BE are is the ground superficies. Wherefore the line AB is erected perpendicularly to the ground plaine superficies. Wherefore (by the 2. definition of the eleventh) the line AB maketh right angles with all the lines which are drawne upon the ground superficies and touch it. But the line BF which is in the ground superficies doth touch it. Wherefore the angle ABF is a right angle. And it is supposed that the angle ABC is a right angle. Wherefore the angle ABF is equall to the angle ABC, and they are in one and the selfe same plaine superficies which is impossible. Wherefore the right line BC is not in an higher superficies. Wherefore the right lines BC, BD, BE are in one and the selfe same plaine superficies. If therefore unto three right lines touching the one the one the other, be erected a perpendicular line from the common point where those three lines touch: those three right lines are in one and the selfe same plaine superficies: which was required to be demonstrated.

Demonstration leading to an impossibility.



This figure here set more playnely declareth the demonstration of the former proposition, if ye erect perpendicularly unto the ground superficies wherein is drawne the line AB, and so compare it with the sayd demonstration.



The 6. Theoreme.

The 6. Proposition.

If two right lines be erected perpendicularly to one & the selfe same plaine superficies: those right lines are parallels the one to the other.

The eleuenth Booke

and maketh with them right angles, wherefore (by the fourth of the first) the base FA is equall to the base FB . And (by the same reason) the base FC is equall to the base FD . And forasmuch as the line AD is equall to the line BC , and the line FA is equal to the line FB as it hath bene proued. Therefore these two lines FA and AD are equall to these two lines FB & BC , the one to the other, & the base FD is equall to the base FC . Wherefore also the angle FAD is equall to the angle $FB C$. And againe forasmuch as it hath bene proued, that the line AG is equall to the line BH , but the line FA is equall to the line FB . Wherefore there are two lines FA and AG equall to two lines FB and BH and it is proued that the angle FAG is equall to the angle FBH : wherefore (by the 4. of the first) the base FG is equal to the base FH . Agayne forasmuch as it hath bene proued that the line GE is equal to the line EH , and the line EF is common to them both: wherefore these two lines GE and EF are equall to these two lines HE and EF , and the base FH is equall to the base FG : wherefore the angle GEF is equall to the angle HEF . Wherefore either of the angles GEF , and HEF is a right angle. Wherefore the line EF is erected, from the point E , perpendicularly to the line GH . In like sort may we proue, that the same line FE maketh right angles with all the right lines which are drawne vpon the ground playne superficies and touch the point B . But a right line is then erected perpendicularly to a plaine superficies, when it maketh right angles with all the lines which touch it, and are drawne vpon the ground playne superficies (by the 2. definition of the eleuenth). Wherefore the right line FE is erected perpendicularly to the ground playne superficies. And the ground plaine superficies is that which passeth by these right lines AB and CD . Wherefore the right line FE is erected perpendicularly to the playne superficies which passeth by the right lines AB and CD . If therefore from two right lines cutting the one the other and at their common section a right line be perpendicularly erected: it shall also be erected perpendicularly to the plaine superficies by the sayd two lines passing: which was required to be proued.

In this figure you may most evidently conceaue the proposition and demonstration, if ye erect perpendicularly from the point B the line FE to the ground playne superficies $ACBD$ the triangles AFD , & CFB in such sort, that the angle AFB may ioyne & make one line with the line AFD : and likewise that the line BF of the triangle CFB may ioyne & make one right line with the line BF of the triangle AFD .



¶ The s. Theoreme.

The s. Proposition.

If vnto three right lines which touch the one the other, be erected a perpendicular line from the common point where those three lines touch: those three right lines are in one and the selfe same plaine superficies.

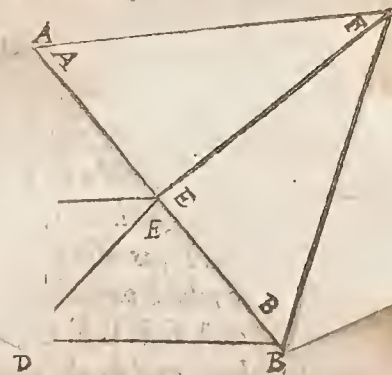


Suppose that vnto these three right lines BC , BD , and BE , touching the one the other in the poynt B , be erected perpendicularly from the poynt B , the line AB . Then I say, that those thre right lines BC , BD and BE , are in one & the selfe same plaine superficies. For if not, then if it be possible, let the lines BD & BE

The eleuenth Booke

and maketh with them right angles, wherefore (by the fourth of the first) the base FA is equal to the base FB . And (by the same reason) the base FC is equal to the base FD . And forasmuch as the line AD is equal to the line BC , and the line FA is equal to the line FB as it hath bene proued. Therefore these two lines FA and AD are equal to these two lines FB & BC , the one to the other, & the base FD is equal to the base FC . Wherefore also the angle FAD is equal to the angle FBC . And againe forasmuch as it hath bene proued, that the line AG is equal to the line BH , but the line FA is equal to the line FB . Wherefore there are two lines FA and AG equal to two lines FB and BH and it is proued that the angle FAG is equal to the angle FBH : wherefore (by the 4. of the first) the base FG is equal to the base FH . Agayne forasmuch as it hath bene proued that the line GE is equal to the line EH , and the line EF is common to them both: wherefore these two lines GE and EF are equal to these two lines HE and EF , and the base FH is equal to the base FG : wherefore the angle GEF is equal to the angle HEF . Wherefore either of the angles GEF , and HEF is a right angle. Wherefore the line EF is erected, from the point E , perpendicularly to the line GH . In like sort may we proue, that the same line FE maketh right angles with all the right lines which are drawne vpon the ground playne superficies and touch the point B . But a right line is then erected perpendicularly to a plaine superficies, when it maketh right angles with all the lines which touch it, and are drawne vpon the ground playne superficies (by the 2. definition of the eleuenth). Wherefore the right line FE is erected perpendicularly to the ground playne superficies. And the ground plaine superficies is that which passeth by these right lines AB and CD . Wherefore the right line FE is erected perpendicularly to the playne superficies which passeth by the right lines AB and CD . If therefore from two right lines cutting the one the other and at their common section a right line be perpendicularly erected: it shall also be erected perpendicularly to the plaine superficies by the sayd two lines passing: which was required to be proued.

In this figure you may most evidently conceaue the proposition and demonstration, if ye erect perpendic-
ground playne superficies $ACBD$ the trian-
the triangles AFD , & CFB in such sort, that the
angle AFB may ioine & make one line with the line
angle AFD : and likewise that the line BF of the triangle
ioine & make one right line with the line BF of the triangle B .



¶ The s. Theoreme.

The s. Proposition.

If vnto three right lines which touch the one the other, be erected a perpendicular line from the common point where those three lines touch: those three right lines are in one and the selfe same plaine superficies.

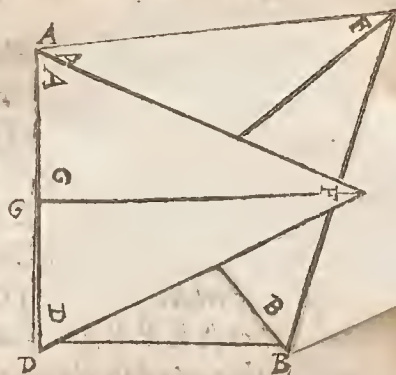


Suppose that vnto these three right lines BC , BD , and BE , touching the one the other in the poynt B , be erected perpendicularly from the poynt B , the line AB . Then I say, that those thre right lines BC , BD and BE , are in one & the selfe same plaine superficies. For if not, then if it be possible, let the lines BD &
 BE

The eleventh Booke

and maketh with them right angles, wherefore (by the fourth of the first) the base FA is equal to the base FB . And (by the same reason) the base FC is equal to the base FD . And forasmuch as the line AD is equal to the line BC , and the line FA is equal to the line FB as it hath bene proued. Therefore these two lines FA and AD are equal to these two lines FB & BC , the one to the other, & the base FD is equal to the base FC . Wherefore also the angle FAD is equal to the angle $FB C$. And againe forasmuch as it hath bene proued, that the line AG is equal to the line BH , but the line FA is equal to the line FB . Wherefore there are two lines FA and AG equal to two lines FB and BH and it is proued that the angle FAG is equal to the angle FBH : wherefore (by the 4. of the first) the base FG is equal to the base FH . Agayne forasmuch as it hath bene proued that the line GE is equal to the line EH , and the line EF is common to them both: wherefore these two lines GE and EF are equal to these two lines HE and EF , and the base FH is equal to the base FG : wherefore the angle GEF is equal to the angle HEF . Wherefore either of the angles GEF , and HEF is a right angle. Wherefore the line EF is erected, from the point E , perpendicularly to the line GH . In like sort may we proue, that the same line FE maketh right angles with all the right lines which are drawne vpon the ground playne superficies and touch the point B . But a right line is then erected perpendicularly to a plaine superficies, when it maketh right angles with all the lines which touch it, and are drawne vpon the ground playne superficies (by the 2. definition of the eleventh). Wherefore the right line FE is erected perpendicularly to the ground playne superficies. And the ground plaine superficies is that which passeth by these right lines AB and CD . Wherefore the right line FE is erected perpendicularly to the playne superficies which passeth by the right lines AB and CD . If therefore from two right lines cutting the one the other and at their common section a right line be perpendicularly erected: it shall also be erected perpendicularly to the plaine superficies by the sayd two lines passing: which was required to be proued.

In this figure you may most evidently conceaue the former proposition and demonstration, if ye erect perpendicularly vnto the ground playne superficies $ACBD$ the triangle AEB : and eleuate the triangles AFD , & CFB in such sort, that the line AE of the triangle AEB may ioyne & make one line with the line AF of the triangle AFD : and likewise that the line BE of the triangle AEB may ioyne & make one right line with the line BF of the triangle CFB .



¶ The s. Theoreme.

The s. Proposition.

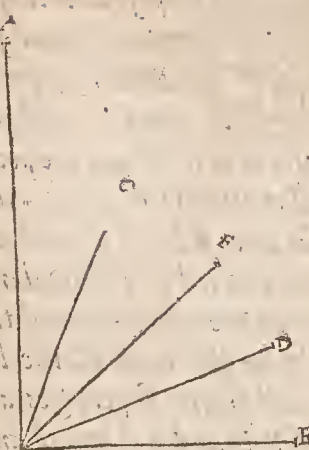
If vnto three right lines which touch the one the other, be erected a perpendicular line from the common point where those three lines touch: those three right lines are in one and the selfe same plaine superficies.



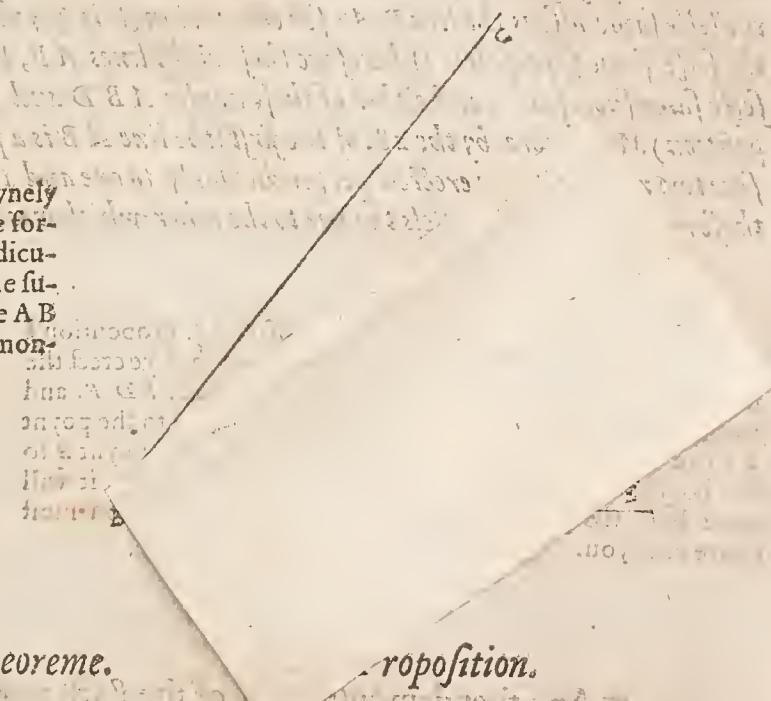
Suppose that vnto these three right lines BC , BD , and BE , touching the one the other in the poynt B , be erected perpendicularly from the poynt B , the line AB . Then I say, that those thre right lines BC , BD and BE , are in one & the selfe same plaine superficies. For if not, then if it be possible, let the lines BD & BE

BE. be in the ground superficies, and let the line BC be erected upward (now the lines AB and BC are in one and the same playne superficies (by the 2. of the eleuenth) for they touch the one the other in the point B). Extend the plaine superficies wherein the lines AB and BC are, and it shall make at the length a common section with the ground superficies, which common section shall be a right line (by the 3. of the eleuenth): let that common section be the line BF. Wherefore the three right lines AB, BC, and BF are in one and the selfe same superficies, namely, in the superficies wherein the lines AB and BC are. And forasmuch as the right line AB is erected perpendicularly to either of these lines BD and BE, therefore the line AB is also (by the 4. of the eleuenth) erected perpendicularly to the plaine superficies, wherein the lines BD and BE are. But the superficies wherein the lines BD and BE are is the ground superficies. Wherefore the line AB is erected perpendicularly to the ground plaine superficies. Wherefore (by the 2. definition of the eleuenth) the line AB maketh right angles with all the lines which are drawne vpon the ground superficies and touch it. But the line BF which is in the ground superficies doth touch it. Wherefore the angle ABF is a right angle. And it is supposed that the angle ABC is a right angle. Wherefore the angle ABF is equall to the angle ABC, and they are in one and the selfe same plaine superficies which is impossible. Wherefore the right line BC is not in an higher superficies. Wherefore the right lines BC, BD, BE are in one and the selfe same plaine superficies. If therefore vnto three right lines touching the one the one the other, be erected a perpendicular line from the common point where those three lines touch: those three right lines are in one and the selfe same plaine superficies: which was required to be demonstrated.

Demonstration leading to an impossibility.



This figure here set more playnely declareth the demonstration of the former proposition, if ye erect perpendicularly vnto the ground superficies, the superficies wherein is drawne the line AB and so compare it with the sayd demonstration.



The 6. Theoreme.

proposition.

If two right lines be erected perpendicularly to one & the selfe same plaine superficies: those right lines are parallels the one to the other.

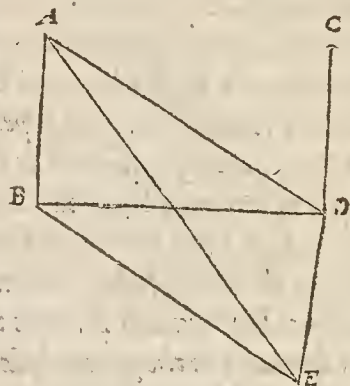
The eleuenth Booke

Construction.

* An Assumpe
as M. Dee
proueth it.
Demonstra-
tion.



Suppose that these two right lines AB and CD be erected perpendicularly to a ground plaine superficies. Then I say that the line AB is a parallel to the line CD . Let the pointes which those two right lines touch in the plaine superficies be B and D . And draw a right line from the point B to the point D . And (by the 11. of the first) from the point D , draw vnto the line BD in the ground superficies a perpendicular line DE . And (by the 2. of the first) put the line DE equall to the line AB . And draw these right lines BE , AE , and AD . And forasmuch as the line AB is erected perpendicularly to the ground superficies, therefore (by the 2. definition of the eleuenth) the line AB maketh right angles with all the lines which are drawne vpon the ground playne superficies and touch it. But either of these lines BD and BE which are in the ground superficies, touch the line AB , wherefore either of these angles ABD and ABE is a right angle: and by the same reason also either of the angles EDB , & EDC is a right angle. And forasmuch as the line AB is equall to the line DE , and the line BD is common to them both, therefore these two lines AB and BD , are equall to these two lines DE and DB , and they contayne right angles: wherefore (by the 4. of the first) the base AD is equall to the base BE . And forasmuch as the line AB is equall to the line DE , and the line AD to the line BE , therefore these two lines AB and BE are equall to these two lines ED and DA , and the line AE is a common base to them both. Wherefore the angle ABE is (by the 8. of the first) equal to the angle EDA . But the angle ABE is a right angle: wherefore also the angle EDA is a right angle: wherefore the line ED is erected perpendicularly to the line DA : and it is also erected perpendicularly to either of these lines BD and DC , wherefore the line ED is vnto these three right lines BD , DA , and DC erected perpendicularly from the poynt where these three right lines touch the one the other: wherefore (by the 5. of the eleuenth) these three right lines BD , DA , and DC are in one and the selfe same superficies. And in what superficies the lines BD and DA are, in the selfe same also is the line BA : for euery triangle is (by the 2. of the eleuenth) in one and the selfe same superficies. Wherefore these right lines AB , BD , and DC are in one and the selfe same superficies, and either of these angles ABD and DBC is a right angle (by supposition). Wherefore (by the 28. of the first) the line AB is a parallel to the line CD . If therefore two right lines be erected perpendicularly to one and the selfe same playne superficies, those right lines are parallels the one to the other: which was required to be proued.

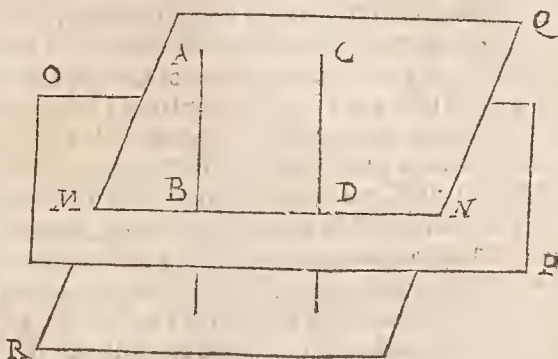


Here for the better vnderstanding of this 6. proposition I haue described an other figure: as touching which if ye erect the superficies ABD perpendicularly to the superficies BDE , and imagine only a line to be drawne from the poynt A to the poynt E (if ye will ye may extend a thred from the faide poynt A to the poynt E) and so compare it with the demonstration, it will make both the proposition, and also the demonstration most cleare vnto you.

¶ An other demonstration of the sixth proposition by M. Dee.

Suppose that the two right lines AB & CD be perpendicularly erected to one & the same playne superficies

perficies, namely the playne superficies $o p$. Then I say that $a b$ and $c d$ are parallels. Let the end points of the right lines $a b$ and $c d$, which touch the plaine superficies $o p$, be the poyntes B and D , frō B to D let a straight line be drawne (by the first petition) and (by the second petition) let the straight line $B D$ be extēded, as to the poynts M & N . Now forasmuch as the right line $a b$, from the poynt B produced, doth cutte the line $M N$ (by construction). Therefore (by the second proposition of this eleventh booke) the right lines $a b$ & $M N$ are in one plaine superficies. Which let be $Q R$, cutting the superficies $o p$ in the right line $M N$. By the same meanes may we conclude the right line $c d$ to be in one playne superficies with the right line $M N$. But the right line $M N$ (by supposition) is in the plaine superficies $Q R$: wherefore $c d$ is in the plaine superficies $Q R$. And $a b$ the right line was proued to be in the same plaine superficies $Q R$. Therefore $a b$ and $c d$ are in one playne superficies, namely $Q R$. And forasmuch as the lines $a b$ and $c d$ (by supposition) are perpendicular vpon the playne superficies $o p$, therefore (by the second definition of this booke) with all the right lines drawne in the superficies $o p$ and touching $a b$ and $c d$, the same perpēdiculars $a b$ and $c d$, do make right angles. But (by construction) $M N$, being drawne in the plaine superficies $o p$ toucheth the perpēdiculars $a b$ and $c d$ at the poyntes B and D . Therefore the perpēdiculars $a b$ and $c d$, make with the right line $M N$ two right angles namely $a b n$, and $c d m$: and $M N$ the right line is proued to be in the one and the same playne superficies, with the right lines $a b$ & $c d$: namely in the playne superficies $Q R$. Wherefore by the second part of the 28. proposition of the first booke, the right lines $a b$ and $c d$ are parallels. If therefore two right lines be erected perpēdicularly to one and the selfe same playne superficies those right lines are parallels the one to the other: which was required to be demonstrated.



A Corollary added by M. Dee.

Hereby it is euidēt that any two right lines perpēdicularly erected to one and the selfe same playne superficies, are also them selues in one and the same playne superficies, which is likewise perpēdicularly erected to the same playne superficies, vnto which the two right lines are perpēdicular.

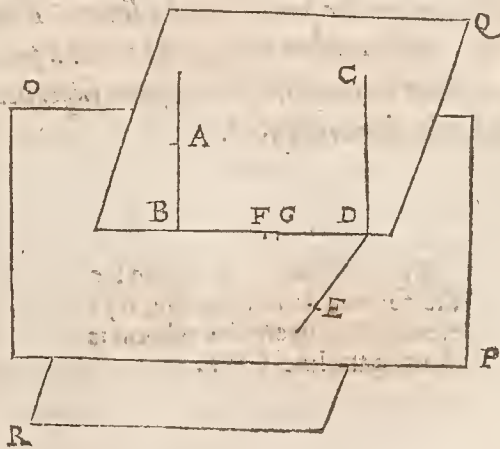
1.
2.

The first part hereof is proued by the former construction and demonstration, that the right lines $a b$ and $c d$ are in one and the same playne superficies $Q R$. The second part is also manifest (that is, that the playne superficies $Q R$ is perpēdicularly erected vpon the playne superficies $o p$) for that $a b$ and $c d$ being in the playne superficies $Q R$, are by supposition perpēdicular to the playne superficies $o p$: wherefore by the third definition of this booke $Q R$ is perpēdicularly erected to, or vpon $o p$: which was required to be proued.

1.
2.

Io. Dee his aduise vpon the Assumpt of the 6.

* As concerning the making of the line $D E$, equall to the right line $A B$, verely the second of the first, without some farther consideration, is not properly enough alledged. And no wonder it is, for that in the former bookes, whatsoeuer hath of lines bene spoken, the same hath alwayes bene imagined to be in one onely playne superficies considered or executed. But here the perpēdicular line $A B$, is not in the same playne superficies, that the right line $D E$ is. Therefore some other helpe must be put into the handes of young beginners, how to bring this problem to execution: which is this, most playne and brieve. Vnderstand that $B D$ the right line, is the common section of the playne superficies, wherein the perpēdiculars $A B$ and $C D$ are, & of the other playne superficies, to which they are perpēdiculars. The first of these (in my former demonstration of the 6), I noted by the playne superficies $Q R$: and the other, I noted by the plaine superficies $O P$. Wherefore $B D$ being a right line common to both the playne superficies $Q R$ & $O P$, therby the poynts B and D are cōmon to the playnes $Q R$ and $O P$. Now

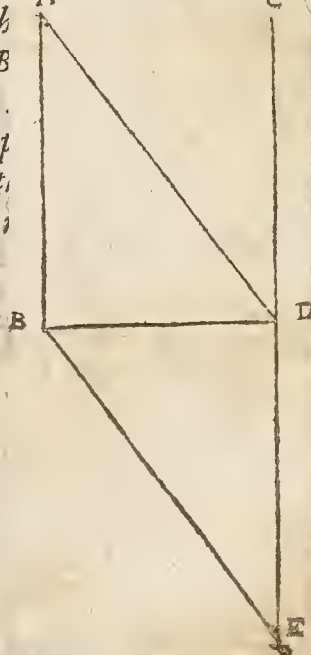
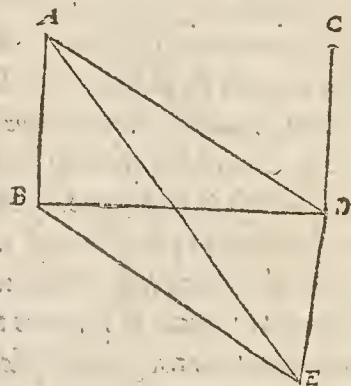


from

The eleventh Booke



Suppose that these two right lines AB and CD be erected perpendicularly to a ground plaine superficies. Then I say that the line AB is a parallel to the line CD . Let the pointes which those two right lines touch in the plaine superficies be B and D . And draw a right line from the point B to the point D . And (by the 11. of the first) from the point D , draw unto the line BD in the ground superficies a perpendicular line DE . And (by the 2. of the first)* put the line DE equall to the line AB . And draw these right lines BE , AE , and AD . And forasmuch as the line AB is erected perpendicularly to the ground superficies, therefore (by the 2. definition of the eleventh) the line AB maketh right angles with all the lines which are drawne upon the ground playne superficies and touch it. But either of these lines BD and BE which are in the ground superficies, touch the line AB , wherefore either of these angles ABD and ABE is a right angle: and by the same reason also either of the angles CDB , & CDE is a right angle. And forasmuch as the line AB is equall to the line DE , and the line BD is common to them both, therefore these two lines AB and BD , are equall to these two lines ED and DB , and they containe right angles: wherefore (by the 4. of the first) the base AD is equall to the base BE . And forasmuch as the line AB is equall to the line DE , and the line AD to the line BE , therefore these two lines AB and BE are equall to these two lines ED and DA , and the line AE is a common base to them both. Wherefore the angle ABE is (by the 8. of the first) equal to the angle EDA . But the angle ABE is a right angle: wherefore also the angle EDA is a right angle: wherefore the line ED is erected perpendicularly to the line DA : and it is also erected perpendicularly to either of these lines BD and DC , wherefore the line ED is unto these three right lines BD , DA , and DC erected perpendicularly from the poynt where these three right lines touch the one the other: wherefore (by the 5. of the eleventh) these three right lines BD , DA , and DC are in one and the selfe same superficies. And in what superficies the lines BD and DC are, in the selfe same also is the line BA : for every triangle is (by the 1. of the eleventh) in one and the selfe same superficies. Wherefore these right lines AB , B selfe same superficies, and either of these angles ABD and ABE is a right angle: wherefore (by the 28. of the first) the line AB is a parallel to the line CD . Therefore two right lines be erected perpendicularly to one and the other: those right lines are parallels the one to the other: which was to be proved.



Here for the better vnderstanding of this 6. proposition I haue described an other figure: as touching which if ye erect the superficies ABD perpendicularly to the superficies BDE , and imagine only a line to be drawne from the poynt A to the poynt E (if ye will ye may extend a thred from the saide poynt A to the poynt E) and so compare it with the demonstration, it will make both the proposition, and also the demonstration most cleare vnto you.

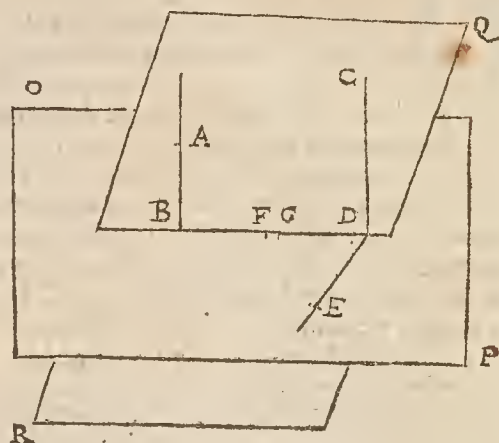
¶ An other demonstration of the sixth proposition by *M. Dee*.

Suppose that the two right lines AB & CD be perpendicularly erected to one & the same playne superficies

The eleventh Booke

from B D (sufficiently extended) cutte a right line equall to A B, (which suppose to be B F) by the third of the first, and orderly to B F make D E equall, by the 3. of the first, if D E be greater then B F. (Which alwayes you may cause so to be, by producing of D E sufficiently). Now forasmuch as B F by construction is cutte equall to A B, and D E also, by construction, put equall to B F, therefore by the 1. common sentence, D E is put equall to A B: which was required to be done.

In like sort, if D E were a line geuen to whome A B were to be cutte and made equall, first out of the line D B (sufficiently produced) cutting of D G, equall to D E by the third of the first: and by the same 3. cutting from B A (sufficiently produced) B A, equall to D G: then is it euident, that to the right line D E, the perpédicular line A B is put equall. And though this be easy to conceaue, yet I haue designed the figure accordingly, wherby you may instruct your imagination. Many such helps are in this booke requisite, as well to enforme the young students therewith, as also to matter the froward gaynesayer of our conclusion, or interrupter of our demonstrations course.



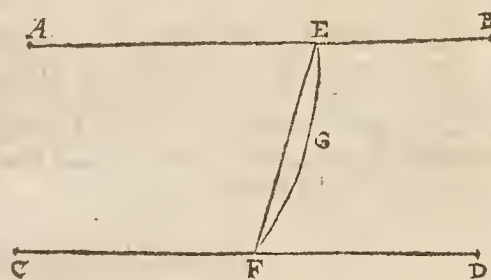
The 7. Theoreme.

The 7. Proposition.

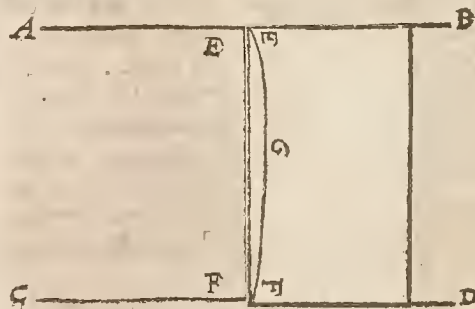
If there be two parallel right lines, and in either of them be taken a point at all aduentures: a right line drawn by the said pointes is in the self same superficies with the parallel right lines.

Demonstration leading to an impossibility.

Suppose that these two right lines A B and C D be parallels, and in either of the take a point at all aduentures, namely, E and F. Then I say, that a right line drawn from the point E to the point F, is in the self same plaine superficies that the parallel lines are. For if not, then if it be possible, let it be in an higher superficies, as the line E G F is, and draw the superficies wherein the line E G F is, & extend it, and it shall make a common section with the ground superficies, which section shall (by the 3. of the eleventh) be a right line: let that section be the right line E F. Wherefore two right lines E G F and E F include a superficies: which (by the last common sentence) is impossible.



Wherefore a right line drawn from the point E to the point F, is not in an higher superficies. Wherefore a right line drawn from the point E to the point F, is in the self same superficies wherein are the parallel right lines A B and C D. If therefore there be two parallel right lines, and in either of them be taken a point at all aduentures, a right line drawn by those pointes is in the self same plaine superficies with the parallel right lines: which was required to be demonstrated.



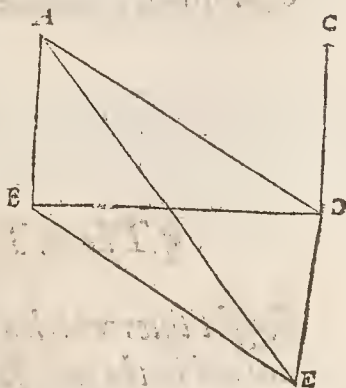
By this figure it is easie to see the former demonstration, if ye eleuate the superficies wherein is drawn the line E G F.

The 8. Theoreme. The 8. Proposition.

If there be two parallel right lines, of which one is erected perpendicularly to a round playne superficies: the other also is erected perpendicularly to the selfe same ground playne superficies.



Suppose that there be two parallel right lines AB and CD , and let one of them, namely, AB be erected perpendicularly to a ground superficies. Then I say that the line CD is also erected perpendicularly to the selfe same ground superficies. Let the lines AB and CD fall upon the ground superficies in the pointes B and D , and (by the first petition) draw a right line from the point B to the point D . And drawe (by the 11. of the first) in the ground superficies from the point D unto the line BD a perpendicular line DE , and (by the 2. of the first) put the line DE equall to the line AB , and draw a right line from the point B to the point E , and an other from the point A to the point E , and an other from the point A to the point D . And forasmuch as the line AB is erected perpendicularly to the ground superficies, therefore (by the 2. definition of the eleventh) the line AB is erected perpendicularly to all the right lines that are in the ground superficies and touche it. Wherfore, either of these angles ABD & ABE is a right angle. And forasmuch as upon these parallel lines AB and CD falleth a certaine right line BD , therefore (by the 29.



This proposition is as it were the converse of the sixth Construction.

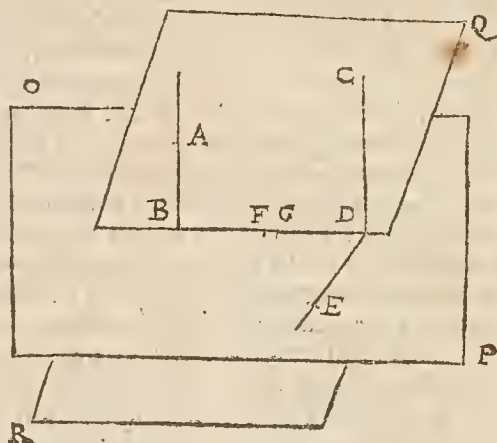
Demonstration.

of the first) the angles ABD and ADB are equal to two right angles. But the angle ABD is a right angle, wherfore also the angle ADB is a right angle. Wherfore the line CD is erected perpendicularly to the line BD . And forasmuch as the line AB is equall to the line DE , and the line BD is common to them both, therefore these two lines AB and BD are equal to these two lines ED and DB , and the angle ABD is equall to the angle EDB , for either of them is a right angle. Wherfore (by the 4. of the first) the base AD is equall to the base BE . And forasmuch as the line AB is equall to the line DE , and the line BE to the line AD , therefore these two lines AB and BE are equall to these two lines AD & DE , the one to the other, and the line AE is a common base to them both. Wherfore (by the 8. of the first) the angle ABE is equall to the angle ADE : but the angle ABE is a right angle, wherfore the angle EDA , is also a right angle. Wherfore the line ED is erected perpendicularly to the line AD , and it is also erected perpendicularly to the line DB . Wherfore the line ED is erected perpendicularly to the plaine superficies wherein the lines BD and BA are (by the 4. of this booke) Wherfore (by the 2. definition of the eleventh) the line ED is erected perpendicularly to all the right lines that touche it and are in the superficies wherein the lines BD and AD are. But in what superficies the lines BD and DA are, in the selfe same superficies is the line DC . For the line AD being drawn from two pointes taken in the parallel lines AB and CD is by the former proposition in the selfe same superficies with them. Now forasmuch as the lines AB and BD are in the superficies wherein the lines BD and DA are, but in what superficies the lines AB & BD are, in the same is the line DC . Wherfore the line ED is erected perpendicularly to the line DC . Wherfore also the line CD is erected perpendicularly to the line DE . And the line CD is erected perpendicularly to the line DB . For by the 29. of the first, the angle ADB being equall to the angle ABD is a right angle. Wherfore the line CD is from the point D erected perpendicularly to two right lines DE and DB cutting the one the other in the point D . Wherfore by the 4. of the eleventh, the line CD is erected perpendicularly to the plaine superficies, wherein are the lines DE and DB . But the

The eleventh Booke

from B D (sufficiently extended) cutte a right line equall to A B, (which suppose to be B F) by the third of the first, and orderly to B F make D E equall, by the 3. of the first, if D E be greater then B F. (Which alwayes you may cause so to be, by producing of D E sufficiently). Now forasmuch as B F by construction is cutte equall to A B, and D E also, by construction, put equall to B F, therefore by the 1. common sentence, D E is put equall to A B: which was required to be done.

In like sort, if D E were a line geuen to whome A B were to be cutte and made equall, first out of the line D B (sufficiently produced) cutting of D G, equall to D E by the third of the first: and by the same 3. cutting from B A (sufficiently produced) B A, equall to D G: then is it euident, that to the right line D E, the perpédicular line A B is put equall. And though this be easy to conceaue, yet I haue designed the figure accordingly, wherby you may instruct your imagination. Many such helpes are in this booke requisite, as well to enforme the young students therewith, as also to master the froward gaynesayer of our conclusion, or interrupter of our demonstrations course.



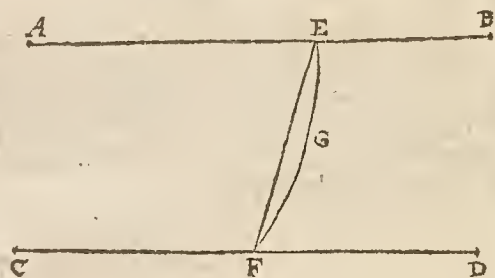
The 7. Theoreme.

The 7. Proposition.

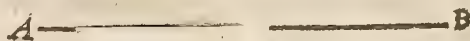
If there be two parallel right lines, and in either of them be taken a point at all adventures: a right line drawn by the said pointes is in the self same superficies with the parallel right lines.

Demonstration leading to an impossibility.

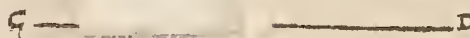
Suppose that these two right lines AB and CD be parallels, and in either of the take a point at all adventures, namely, E and F . Then I say, that a right line drawn from the point E to the point F , is in the selfe same plaine superficies that the parallel lines are. For if not, then if it be possible, let it be in an higher superficies, as the line EGF is, and draw the superficies wherein the line EGF is, & extend it, and it shall make a common section with the ground superficies, which section shall (by the 3. of the eleventh) be a right line: let that section be the right line EF . Wherefore two right lines EGF and EF include a superficies: which (by the last common sentence) is impossible.



Wherefore a right line drawn from the point E to the point F, is not in an higher superficies. Wherefore a right line drawn from the point E to the point F, is in the selfe same superficies wherein are the parallel right lines AB and CD. If therefore there be two parallel right lines, and in either of them be taken a point at all adventures, a right line drawn by those pointes is in the selfe same plaine superficies with the parallel right lines: which was required to be demonstrated.



By this figure it is easie to see the former demonstration, if ye eleuate the superficies wherein is drawn the line E G F.



The eleuenth Booke

the ground plaine superficies is that wherein are the lines DE and DB , to which superficies also the line AB is supposed to be erected perpendicularly. Wherefore the line CD is erected perpendicularly to the ground plaine superficies, wherunto the line AB is erected perpendicularly. If therefore there be two parallel right lines, of which one is erected perpendicularly to a ground plaine superficies, the other also is erected perpendicularly to the selfe same ground plaine superficies : which was required to be demonstrated.

This figure will more clearly set forth the former demonstration, if ye erect perpendicularly the superficies ABD to the superficies BDE , and imagine a lyne to be drawn from the point A to the point D , in steade wherof, as in the 6. proposition ye may extend a threede.

¶ The 9. Theoreme.

The 9. Proposition.

Right lines which are parallels to one and the selfe same right line, and are not in the selfe same superficies that it is in: are also parallels the one to the other.

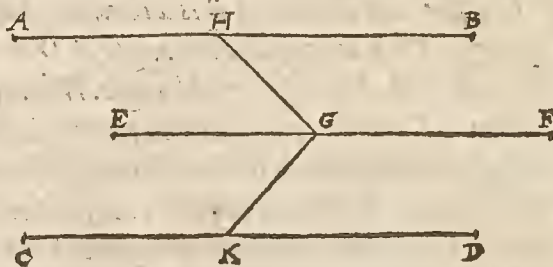
Construction.

Demonstration.



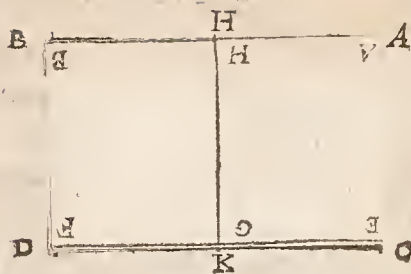
Suppose that either of these right lines AB and CD be a parallel to the line EF not being in the selfe same superficies with it. Then I say that the line AB is a parallel to the line CD . Take in the line EF a point at all adventures, and let the same be G . And from the point G raise up in the superficies wherein are the

lines EF and AB , unto the line EF a perpendicular line GH , and againe in the superficies wherein are the lines EF and CD , raise up from the same point G to the line EF a perpendicular line GK . And forasmuch as the line EF is erected perpendicularly to either of the lines GH and GK , therefore (by the 4. of the eleuenth) the line EF is erected perpendicularly to the superficies wherein the lines GH and GK are, but the line EF is a parallel line to the line AB . Wherefore (by the 8. of the eleuenth) the line AB is erected perpendicularly to the plaine superficies, wherein are the lines GH and GK . And by the same reason also the line CD is erected perpendicularly to the plaine superficies wherein are the lines GH & GK . Wherefore either of these lines AB and CD is erected perpendicularly to the plaine superficies, wherein the lines GH and GK are. But if two right lines be erected perpendicularly to one and the selfe same plaine superficies, those right lines are parallels the one to the other (by the 6. of the eleuenth) Wherefore the line AB is a parallel to the line CD . Wherefore right lines which are parallels to one & the selfe same right line, and are not in the selfe same superficies with it are also parallels the one to the other : which was required to be proued.



This

This figure more clearly manifesteth the former proposition and demonstration, if ye eleuate the superficieses AB EF and $CDEF$ that they may incline and concurre in the lyne EF .



¶ The 10. Theoreme.

The 10. Proposition.

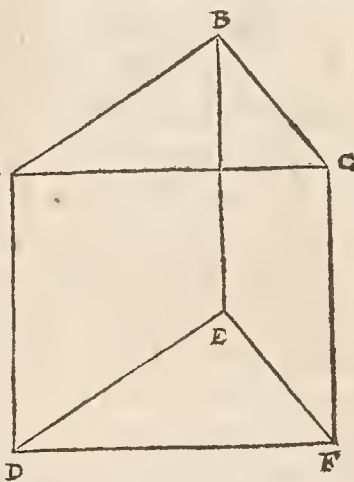
If two right lines touching the one the other be parallells to two other right lines touching the one the other, and not being in one and the selfe same superficies with the two first: those right lines cõtaine equall angles.



Suppose that these two right lines AB and BC touching the one the other, be parallells to these two lines DE and EF touching also the one the other, and not being in the selfe same superficies that the lines AB and BC are. The I say, that the angle ABC is equall to the angle DEF . For let the lines BA , BC ,

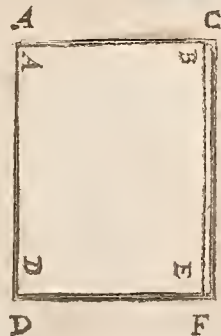
Construction.

ED , EF , be put equall the one to the other: and draw these right lines AD , CF , BE , AC , and DF . And forasmuch as the line BA is equall to the line ED , and also parallell unto it, therefore (by the 33. of the first) the line AD is equall and parallell to the line BE : and by the same reason also the line CF is equall & parallell to the line BE . Wherefore either of these lines AD and CF is equall & parallell to the line EB . But right lines which are parallells to one and the selfe same right line, and are not in the selfe same superficies with it, are also (by the 9. of the eleventh) parallells the one to the other. Wherefore the line AD is a parallell line to the line CF . And the lines AC and DF ioyn them together. Wherefore (by the 33. of the first) the line AC is equall and parallell to the line DF . And forasmuch as these two right lines AB & BC are equall to these two right lines DE and EF , and the base AC also is equall to the base DF : therefore (by the 8. of the first) the angle ABC is equall to the angle DEF . If therfore two right lines touching the one the other be parallells to two other right lines touching the one the other, and not being in one and the selfe same superficies with the two first: those right lines containe equall angles: which was required to be demonstrated.



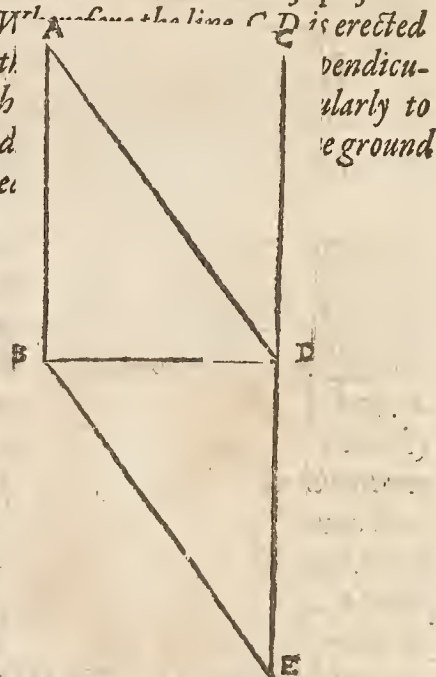
Demonstration.

This figure here set more plainly declareth the former Proposition and demonstration, if ye eleuate the superficieses $DABE$, and $FCBE$, till they concurre in the line FE .



The eleventh Booke

the ground plaine superficies is that wherein are the lines DE and DB , to which superficies also the line AB is supposed to be erected perpendicularly. Wherefore the line CD is erected perpendicularly to the ground plaine superficies, wherunto likewise. If therefore there be two parallel right lines, of which a ground plaine superficies, the other also is erected perpendicularly to the ground plaine superficies: which was required to be demonstrated.



This figure will more clearly set forth the former demonstration, if ye erect perpendicularly the superficies ABD to the superficies BDE , and imagine a lyne to be drawn from the point A to the point D , in steede whereof, as in the 6. proposition ye may extend a threede.

¶ The 9. Theoreme.

The 9. Proposition.

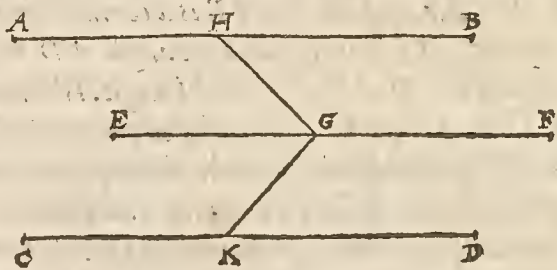
Right lines which are parallels to one and the selfe same right line, and are not in the selfe same superficies that it is in: are also parallels the one to the other.

Construction.



Demonstration.

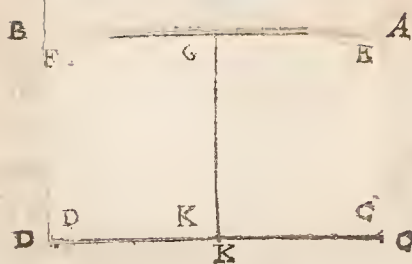
Suppose that either of these right lines AB and CD be a parallel to the line EF not being in the selfe same superficies with it. Then I say that the line AB is a parallel to the line CD . Take in the line EF a point at all adventures, and let the same be G . And from the point G raise up in the superficies wherein are the lines EF and AB , unto the line EF a perpendicular line GH , and againe in the superficies wherein are the lines EF and CD , raise up from the same point G to the line EF a perpendicular line GK . And forasmuch as the line EF is erected perpendicularly to either of the lines GH and GK , therefore (by the 4. of the eleventh) the line EF is erected perpendicularly to the superficies wherein the lines GH and GK are, but the line EF is a parallel line to the line AB . Wherefore (by the 8. of the eleventh) the line AB is erected perpendicularly to the plaine superficies, wherein are the lines GH and GK . And by the same reason also the line CD is erected perpendicularly to the plaine superficies wherein are the lines GH & GK . Wherefore either of these lines AB and CD is erected perpendicularly to the plaine superficies, wherein the lines GH and GK are. But if two right lines be erected perpendicularly to one and the selfe same plaine superficies, those right lines are parallels the one to the other (by the 6. of the eleventh) Wherefore the line AB is a parallel to the line CD . Wherefore right lines which are parallels to one & the selfe same right line, and are not in the selfe same superficies with it are also parallels the one to the other: which was required to be proved.



This

of Euclides Elementes.

This figure more clearly manifesteth the former proposition and demonstration, if ye eleuate the superficieses AB EF and $CDEF$ that they may incline and concurre in the lyne EF .



¶ The 10. Theoreme.

The 10. Proposition.

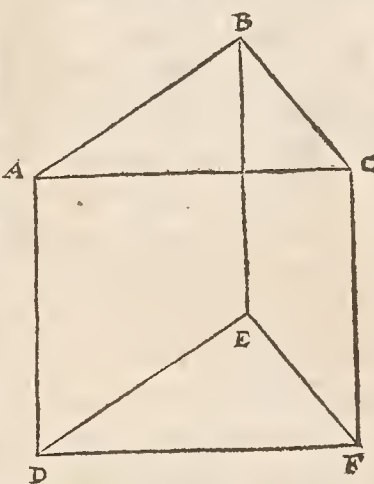
If two right lines touching the one the other be parallells to two other right lines touching the one the other, and not being in one and the selfe same superficies with the two first: those right lines cõtaine equall angles.



Suppose that these two right lines AB and BC touching the one the other, be parallells to these two lines DE and EF touching also the one the other, and not being in the selfe same superficies that the lines AB and BC are. The I say, that the angle ABC is equall to the angle DEF . For let the lines BA , BC ,

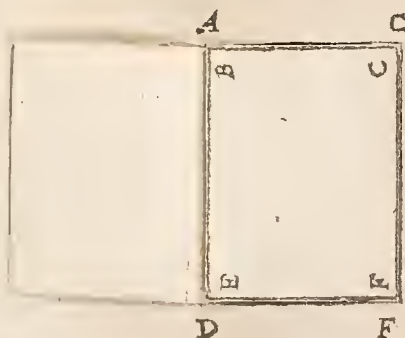
Construction.

ED , EF , be put equall the one to the other: and draw these right lines AD , CF , BE , AC , and DF . And forasmuch as the line BA is equall to the line ED , and also parallell unto it, therefore (by the 33. of the first) the line AD is equall and parallell to the line BE : and by the same reason also the line CF is equall & parallell to the line BE . Wherefore either of these lines AD and CF is equall & parallell to the line EB . But right lines which are parallells to one and the selfe same right line, and are not in the selfe same superficies with it, are also (by the 9. of the eleventh) parallells the one to the other. Wherefore the line AD is a parallell line to the line CF . And the lines AC and DF ioyn them together. Wherefore (by the 33. of the first) the line AC is equall and parallell to the line DF . And forasmuch as these two right lines AB & BC are equall to these two right lines DE and EF , and the base AC also is equall to the base DF : therefore (by the 8. of the first) the angle ABC is equall to the angle DEF . If therfore two right lines touching the one the other be parallells to two other right lines touching the one the other, and not being in one and the selfe same superficies with the two first: those right lines containe equall angles: which was required to be demonstrated.



Demonstration.

This figure here set more plainly declareth the former Proposition and demonstration, if ye eleuate the superficieses $DABE$, and $FCBE$, till they concurre in the line FE .



This figure more clearly manifesteth the former proposition and demonstration, if ye eleuate the superficieses $ABEF$ and $CDEF$ that they may incline and concurre in the lyne EF .

B

A

D

C

¶ The 10. Theoreme.

The 10.

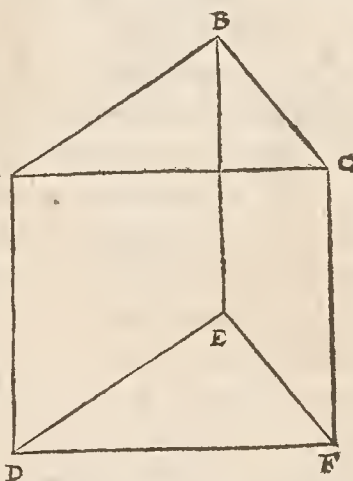
If two right lines touching the one the other
right lines touching the one the other, and not being in one and the selfe
same superficies with the two first: those right lines cõtaine equall angles.



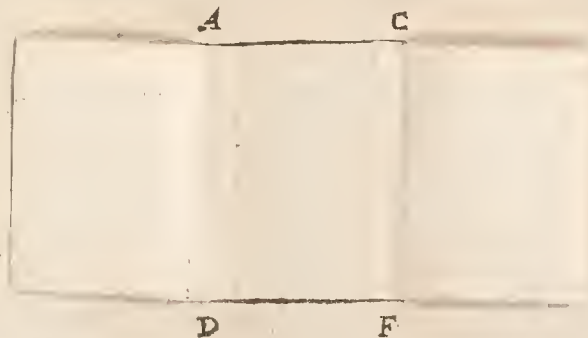
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Construction.

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fore either of these lines AD and CF is equall & parallell
to the line EB . But right lines which are parallells to one
and the selfe same right line, and are not in the selfe same su-
perficies with it, are also (by the 9. of the eleventh) parallells
the one to the other. Wherefore the line AD is a parallell
line to the line CF . And the lines AC and DF ioyn them
together. Wherefore (by the 33. of the first) the line AC is
equall and parallell to the line DF . And forasmuch as these two right lines AB & BC are
equall to these two right lines DE and EF , and the base AC also is equall to the base DF :
therefore (by the 8. of the first) the angle ABC is equall to the angle DEF . If therfore two
right lines touching the one the other be parallells to two other right lines touching the one
the other, and not being in one and the selfe same superficies with the two first: those right
lines containe equall angles: which was required to be demonstrated.



Demonstration.



This figure here set more
plainly declareth the former Pro-
position and demonstration, if ye
eleuate the superficieses $DABE,$
and $FCBE,$ till they concurre in
the line FE .

The eleventh Booke

¶ The 1. Probleme.

The 11. Proposition.

From a point geuen on high, to drawe vnto a ground plaine superficies a perpendicular right line.

Construction.
Two cases in
this propo-
sition.



The first case.

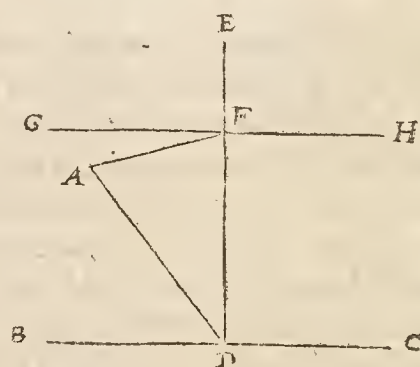
John Dee.

*This requireth the imagination of a plaine superficies passing by the pointe A, and the straight line B C. And so helpe your selfe in the lyke cases either Mathematically imagin-
ning, or Me-
chanically
practising.
Second case.
Demonstra-
tion.

Suppose that the point geuen on high be A, and suppose a ground plaine superficies, namely, B C G H. It is required from the point A to draw vnto the ground superficies a perpendicular line. Drawe in the ground superficies a right line at aduen-
tures, and let the same be B C. And (by the 12. of the first) from the point A draw vnto the line B C a perpendicular line A D. * Now if A D be a perpendicular line to the

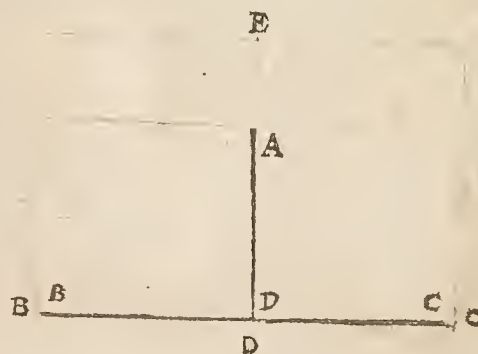
ground superficies, then is that done which was sought for. But if not, then (by the 11. of the first) from the point D raise vp in the ground superficies vnto the line B C a perpendicular line D E. And (by the 12. of the first) from the point A draw vnto the line D E a perpendicular line A F.

And by the point F draw (by the 31. of the first) vnto the line B C a parallell line F H. And extend the line F H from the point F to the point G. And forasmuch as the line B C is erected perpendicularly to either of these lines D E and D A, therefore (by the 4. of the eleventh) the line B C is erected perpendicularly to the superficies wherein the lines E D and A D are: and to the line B C the line G H is a parallell. But if there be two parallell right lines, of which one is erected perpendicularly to a certaine plaine superficies, the other also (by the 8. of the eleventh) is erected perpendicularly to the selfe same superficies.



Wherefore the line G H is erected perpendicularly to the plaine superficies wherein the lines E D and D A are. Wherefore also (by the 2. definition of the eleventh) the line G H is erected perpendicularly to all the right lines which touch it, and are in the plaine superficies wherein the lines E D and A D are. But the line A F toucheth it being in the superficies wherein the lines E D and A D are (by the 2. of this booke). Wherefore the line G H is erected perpendicularly to the line F A. Wherefore also the line F A is erected perpendicularly to the line G H: and the line A F is also erected perpendicularly to the line D E. Wherefore A F is erected perpendicularly to either of these lines H G and D E. But if a right line be erected perpendicularly from the common section of two right lines cutting the one the other, it shall also be erected perpendicularly to the plaine superficies of the said two lines (by the 4. of the eleventh). Wherefore the line A F is erected perpendicularly to that superficies wherein the lines E D and G H are. But the superficies wherein the lines E D and G H are, is the ground superficies. Wherefore the line A F is erected perpendicularly to the ground superficies. Wherefore from a point geuen on high, namely, fro the point A, is drawn to the ground superficies a perpendicular line: which was required to be done.

In this figure shall ye much more plainly see both the cases of this former demonstratiō. For as touching the first case, ye must erecte perpendicularly to the ground superficies, the superficies wherein is drawn the line A D, and compare it with the demonstration, and it will be clere vnto you. For the second case ye must erecte perpendicularly vnto the ground superficies the superficies wherein is drawn the line A F, and vnto it let the other superficies wherein is drawn the line A D, incline, so that the point A of the one may concurre with the point A of the other: and so with your figure thus ordered, compare it with the demonstration, and there will be in it no hardnes at all.



¶ The

¶ The 2. Probleme.

The 12. Proposition.

Vnto a playne superficies geuen, and from a poynt in it geuen, to rayse vp a perpendicular line.

Suppose that there be a ground playne superficies geuen, and let the poynt in it geuen be A. It is required from the point A to raise vp vnto the ground plaine superficies a perpendicular line. Vnderstand some certayne poynt on high, and let the same be B. And from the poynt B draw (by the 11. of the eleuenth) a perpendicular line to the ground superficies, and let the same be B C. And (by the 31. of the first) by the poynt A drawe vnto the line B C a parallel line D A. Now forasmuch as there are two parallel right lines A D and C B, and the one of them, namely, C B is erected perpendicularly to the ground superficies: wherefore the other line also, namely, A D, is erected perpendicularly to the same ground superficies (by the eight of the eleuenth). Wherefore vnto a playne superficies geuen, and from poynt in it geuen, namely, A, is rayssed up a perpendicular lyne A B as doone.

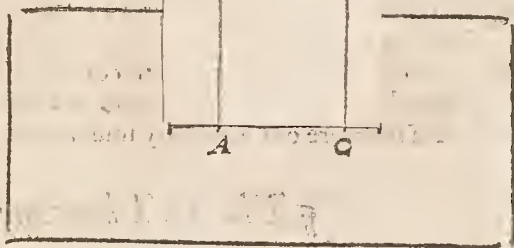


Construction.

Demonstration.

required to

In this second figure ye may consider playnely the demonstration of the former proposition if ye erect perpendicularly the superficies wherein are drawne the lines A D and C B.

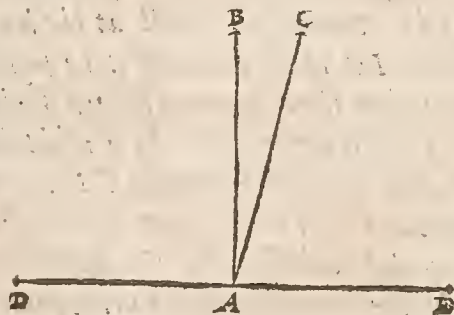


¶ The 11. Theoreme.

The 13. Proposition.

From one and the selfe poynt, and to one and the selfe same playne superficies, can not be erected two perpendicular right lines on one and the selfe same side.

EOr if it be possible from the poynt A let there be erected perpendicularly to one and the selfe same playne superficies two right lines A B and A C on one and the selfe same side. And extende the superficies wherein are the lines A B and A C: and it shall make at length a common section in the ground superficies which common section shall be a right line, and shall passe by the poynt A: let that common section be the line D A E. Wherefore (by the 3. of the eleuenth) the lines A B, A C, and D A E are in one and the selfe same playne superficies. And forasmuch as the line C A is erected perpendicularly to the ground superficies, therefore (by the



Demonstration leading to an impossibilitie. Note this maner of imagination Mathematicall.

VV.ij.

2.de-

The eleventh Booke

The 1. Probleme.

The 11. Propofition.

From a point geuen on high, to drawe vnto a ground plaine superficies a perpendicular right line.

Construction.
Two cases in
this propofition.



The first case.

John Dee.

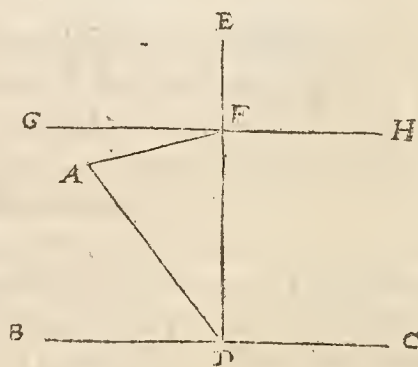
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of a plaine superficies passing by the
pointe A, and the straight line B C.
And so helpe your selfe in the lyke
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Second case.
Demonstra-
tion.

Suppose that the point geuen on high be A, and suppose a ground plaine superficies, namely, B C G H. It is required from the point A to draw vnto the ground superficies a perpendicular line. Drawe in the ground superficies a right line at aduen-
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B C a perpendicular line A D. * Now if A D be a perpendicular line to the

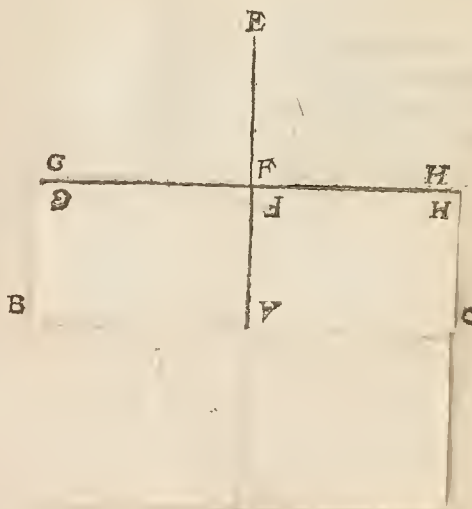
ground superficies, then is that done which was sought for. But if not, then (by the 11. of the first) from the point D raise up in the ground superficies vnto the line B C a perpendicular line D E. And (by the 12. of the first) from the point A draw vnto the line D E a perpendicular line A F.

And by the point F draw (by the 31. of the first) vnto the line B C a parallell line F H. And extend the line F H from the point F to the point G. And forasmuch as the line B C is erected perpendicularly to either of these lines D E and D A, therefore (by the 4. of the eleventh) the line B C is erected perpendicularly to the superficies wherein the lines E D and A D are: and to the line B C the line G H is a parallell. But if there be two parallell right lines, of which one is erected perpendicularly to a certaine plaine superficies, the other also (by the 8. of the eleventh) is erected perpendicularly to the selfe same superficies.



Wherefore the line G H is erected perpendicularly to the plaine superficies wherein the lines E D and D A are. Wherefore also (by the 2. definition of the eleventh) the line G H is erected perpendicularly to all the right lines which touch it, and are in the plaine superficies wherein the lines E D and A D are. But the line A F toucheth it being in the superficies wherein the lines E D and A D are (by the 2. of this booke). Wherefore the line G H is erected perpendicularly to the line F A. Wherefore also the line F A is erected perpendicularly to the line G H: and the line A F is also erected perpendicularly to the line D E. Wherefore A F is erected perpendicularly to either of these lines H G and D E. But if a right line be erected perpendicularly from the common section of two right lines cutting the one the other, it shall also be erected perpendicularly to the plaine superficies of the said two lines (by the 4. of the eleventh). Wherefore the line A F is erected perpendicularly to that superficies wherein the lines E D and G H are. But the superficies wherein the lines E D and G H are, is the ground superficies. Wherefore the line A F is erected perpendicularly to the ground superficies. Wherefore from a point geuen on high, namely, fro the point A, is drawn to the ground superficies a perpendicular line: which was required to be done.

In this figure shall ye much more plainly see both the cases of this former demonstratiō. For as touching the first case, ye must erecte perpendicularly to the ground superficies, the superficies wherein is drawn the line A D, and compare it with the demonstration, and it will be clere vnto you. For the second case ye must erecte perpendicularly vnto the ground superficies the superficies wherein is drawn the line A F, and vnto it let the other superficies wherein is drawn the line A D, incline, so that the point A of the one may concur with the point A of the other: and so with your figure thus ordered, compare it with the demonstration, and there will be in it no hardnes at all.



The

The eleventh Booke

The 1. Probleme.

The 11. Propofition.

From a point geuen on high, to drawe vnto a ground plaine superficies a perpendicular right line.

Construction.
Two cases in
this propo-
sition.



The first case.

John Dee.

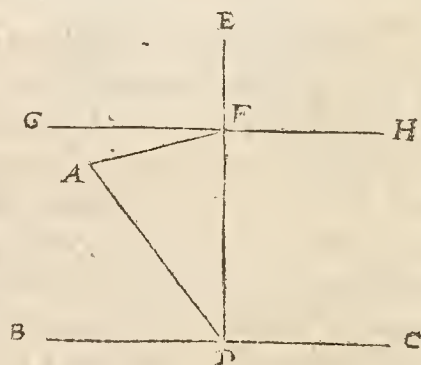
*This requireth the imagination of a plaine superficies passing by the pointe A, and the straight line B C. And so helpe your selfe in the lyke cases either Mathematically imagining, or Mechanically practising.
Second case.
Demonstration.

Suppose that the point geuen on high be A, and suppose a ground plaine superficies, namely, B C G H. It is required from the point A to draw vnto the ground superficies a perpendicular line. Drawe in the ground superficies a right line at aduen-
tures, and let the same be B C. And (by the 12. of the first) from the point A draw vnto the line

B C a perpendicular line A D. * Now if A D be a perpendicular line to the

ground superficies, then is that done which was sought for. But if not, then (by the 11. of the first) from the point D raise up in the ground superficies vnto the line B C a perpendicular line D E. And (by the 12. of the first) from the point A draw vnto the line D E a perpendicular line A F.

And by the point F draw (by the 31. of the first) vnto the line B C a parallell line F H. And extend the line F H from the point F to the point G. And forasmuch as the line B C is erected perpendicularly to either of these lines D E and D A, therefore (by the 4. of the eleventh) the line B C is erected perpendicularly to the superficies wherein the lines E D and A D are: and to the line B C the line G H is a parallell. But if there be two parallell right lines, of which one is erected perpendicularly to a certaine plaine superficies, the other also (by the 8. of the eleventh) is erected perpendicularly to the selfe same superficies.



Wherefore the line G H is erected perpendicularly to the plaine superficies wherein the lines E D and D A are. Wherefore also (by the 2. definition of the eleventh) the line G H is erected perpendicularly to all the right lines which touch it, and are in the plaine superficies wherein the lines E D and A D are. But the line A F toucheth it being in the superficies wherein the lines E D and A D are (by the 2. of this booke). Wherefore the line G H is erected perpendicularly to the line F A. Wherefore also the line F A is erected perpendicularly to the line G H: and the line A F is also erected perpendicularly to the line D E. Wherefore A F is erected perpendicularly to either of these lines H G and D E. But if a right line be erected perpendicularly from the common section of two right lines cutting the one the other, it shall also be erected perpendicularly to the plaine superficies of the said two lines (by the 4. of the eleventh). Wherefore the line A F is erected perpendicularly to that superficies wherein the lines E D and G H are. But the superficies wherein the lines E D and G H are, is the ground superficies. Wherefore the line A F is erected perpendicularly to the ground superficies. Wherefore from a point geuen on high, namely, fro the point A, is drawn to the ground superficies a perpendicular line: which was required to be done.

In this figure shall ye much more plainly see both the cases of this former demonstratiō. For as touching the first case, ye must erecte perpendicularly to the ground superficies, the superficies wherein is drawn the line A D, and compare it with the demonstration, and it will be clere vnto you. For the second case ye must erecte perpendicularly vnto the ground superficies the superficies wherein is drawn the line A F, and vnto it let the other superficies wherein is drawn the line A D, incline, so that the point A of the one may concur with the point A of the other: and so with your figure thus ordered, compare it with the demonstration, and there will be in it no hardnes at all.

The

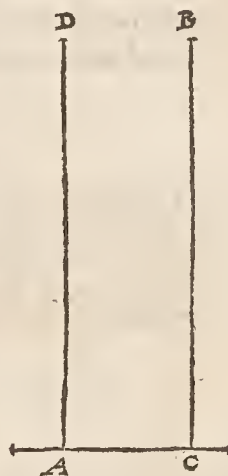
¶ The 2. Probleme.

The 12. Proposition.

Vnto a playne superficies geuen, and from a poynt in it geuen, to rayse vp a perpendicular line.



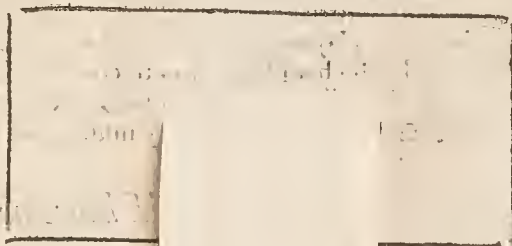
Vppose that there be a ground playne superficies geuen, and let the poynt in it geuen be A. It is required from the point A to raise vp vnto the ground plaine superficies a perpendicular line. Vnderstand some certayne poynt on high, and let the same be B. And from the poynt B draw (by the 11. of the eleuenth) a perpendicular line to the ground superficies, and let the same be B C. And (by the 31. of the first) by the poynt A drawe vnto the line B C a parallel line D A. Now forasmuch as there are two parallel right lines A D and C B, and the one of them, namely, C B is erected perpendicularly to the ground superficies: wherefore the other line also, namely, A D, is erected perpendicularly to the same ground superficies (by the eight of the eleuenth). Wherefore vnto a playne superficies geuen, and from a poynt in it geuen, namely, A, is rayssed vp a perpendicular lyne A D: which was required to be doone.



Construction.

Demonstration.

In this second figure ye may consider playnely the demonstration of the former proposition if ye erect perpendicularly the superficies wherein are drawne the lines A D and C B.



¶ The 11. Theoreme.

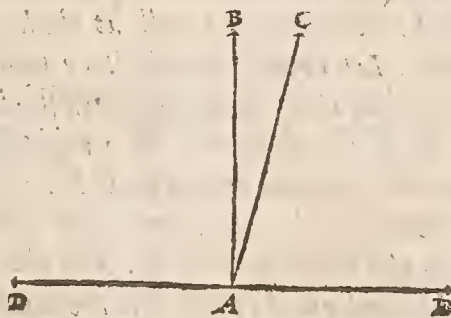
The 13. P

From one and the selfe poynt, and to one and the selfe same side, can not be erected two perpendicular right li

ie superfi-
the selfe



Or if it be possible from the poynt A let there be erected perpendicularly to one and the selfe same playne superficies two right lines A B and A C on one and the selfe same side. And extend the superficies wherein are the lines A B and A C: and it shall make at length a common section in the ground superficies which common section shall be a right line, and shall passe by the poynt A: let that common section be the line D A E. Wherefore (by the 3. of the eleuenth) the lines A B, A C, and D A E are in one and the selfe same playne superficies. And forasmuch as the line C A is erected perpendicularly to the ground superficies, therefore (by the



Demonstration leading to an impossibilitie. Note this maner of imagination Mathematicall.

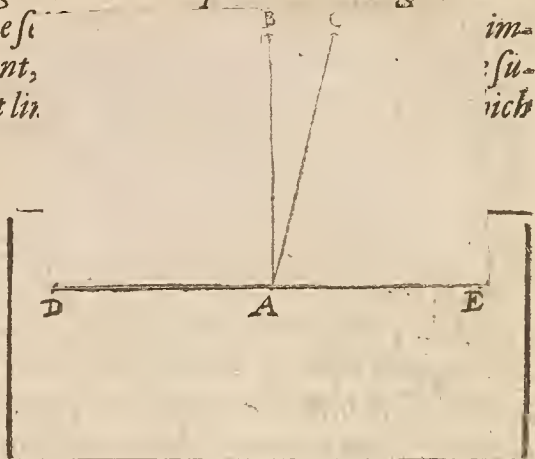
VV. ij.

2. de.

The eleuenth Booke

2. definition of the eleuenth) it maketh right angles with all the right lines that touch it, and are in the ground superficies. But the line DAE toucheth it, being in the ground superficies. Wherefore the angle CAE is a right angle, and by the same reason also the angle BAE is a right angle. Wherefore (by the 4 petition) the angle CAE is equall to the angle BAE the lesse to the more, both angles being in one & the selfe same poynnt, perfcies can not be erected two perpendicular right lin was required to be demonstrated.

In this figure if ye erect perpendicularly the superficies wherein are drawne the lines BA and CA to the ground superficies wherein is drawn the line DAE, and so compare it with the the demonstratiō of the former proposition it will be cleare vnto you.



M. Dee his annotation.

Euclides wordes in this 13. proposition admit two cases: one, if the poynnt assigned be in the playne superficies, (as cōmonly the demonstrations suppose) the other, if the poynnt assigned be any where with out the sayd playne superficies, to which, the perpendiculars fall, is considered. Contrary to either of which, if the aduersarie affirme, admitting from one poynnt two right lines, perpendiculars to one and the selfe same playne superficies, and on one and the same side thereof, by the 6. of the eleuenth he may be bridled: which will fore him to confesse his two perpendiculars to be also parallels. But by supposition agreed one, they concur at one and the same poynnt. which (by the definition of parallels) is impossible. Therefore our aduersary must recant and yelde to our proposition.

¶ The 12. Theoreme.

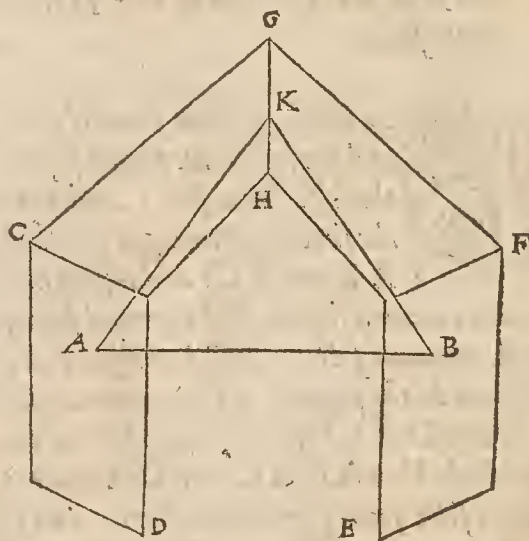
The 14. Proposition.

To whatsoeuer plaine superficies one and the selfe same right line is erected perpendicularly: those superficies are parallels the one to the other.



Suppose that a right line AB be erected perpendicularly to either of these plaine superficies CD and EF. Then I say, that these superficies CD and EF are parallels the one to the other. For if not, then if they be extended they will at the length meete. Let them meete, if it be possible. Now then their common section shall (by the 3. of the eleuenth) be a right line. Let that common section be GH. And in the line GH take a point at all adventures and let the same be K. And drawe a right line from the point A to the point K, and an other from the point B to the point K. And forasmuch as the line AB is e-

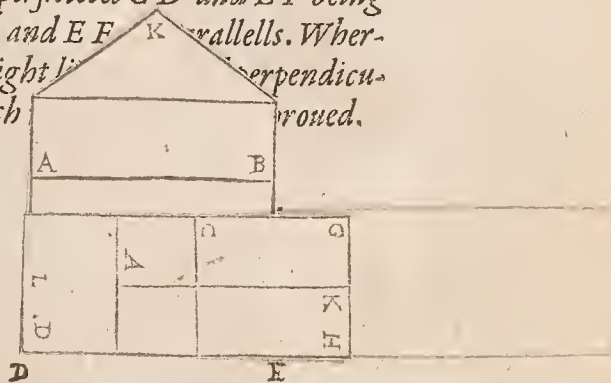
Demonstration leading to an impossibility.



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rected perpendicularly to the plaine superficies EF , therefore the line AB is also erected perpendicularly to the line BK which is in the extended superficies EF . Wherefore the angle ABK is a right angle. And by the same reason also the angle $B AK$ is a right angle. Wherefore in the triangle ABK , these two angles ABK & $B AK$, are equall to two right angles: which (by the 17. of the first) is impossible. Wherefore these superficieses CD and EF being extended meete not together. Wherefore the superficieses CD and EF are parallels. Wherefore to what soeuer plaine superficieses one and the selfe same right line is erected perpendicularly: those superficieses are parallels the one to the other: which

In this figure may ye plainly see the former demonstration if ye erecte the three superficieses, GD , GE , and KLM perpendicular to the ground plaine superficies: but yet in such sort that the two superficieses GD and GE may concur in the common line GKH , as is required in the demonstration.



A corollary added by Campane.

If a right line be erected perpendicularly to one of those superficies, it shall also be erected perpendicularly to the other.

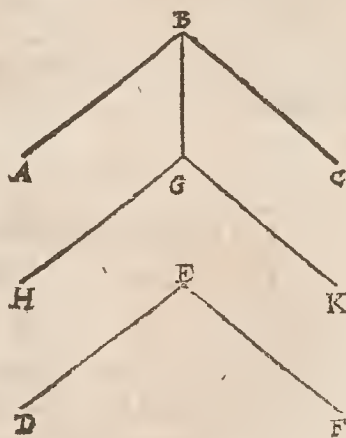
For if it should not be erected perpendicularly to the other, then it falling vpon that other shall make with some one line thereof an angle lesse then a right angle: which line should (by the 5. petition of the first) concur with some one line of that superficies whereunto it is perpendicular. So that those superficieses should not be parallels: which is contrary to the supposition. For they are supposed to be parallels.

¶ The 13. Theoreme.

The 15. Proposition.

If two right lines touching the one the other be parallels to two other right lines touching also the one the other and not being in the selfe same plaine superficies with the two first: the plaine superficieses extended by those right lines, are also parallels the one to the other.

Suppose that these two right lines AB and BC touching the one the other be parallels to these two right lines DE & EF touching also the one the other, and not being in the selfe same plaine superficies with the right lines AB and BC . Then I say, that the plaine superficieses by the lines AB and BC , and the lines DE and EF being extended, shall not meete together, that is, they are equedistant and parallels. From the point B draw (by the 11. of the eleuenth) a perpendicular line to the superficies wherein are the lines DE and EF , and let that perpendicular line be BG . And by the point G in the plaine superficies passing by DE , and EF , draw (by the 31. of the first) vnto the line ED a parallell line GH : and likewise by that point G drawe in the same superficies vnto the line EF a parallell line GK . And forasmuch as the line BG is erected perpendicularly to the superficies wherein are the lines DE and EF , therefore (by the 2. definition of the eleuenth) it is also erected perpendicularly to all the right lines which touch it, and are in the selfe same superficies wherein are the lines DE



Construction.

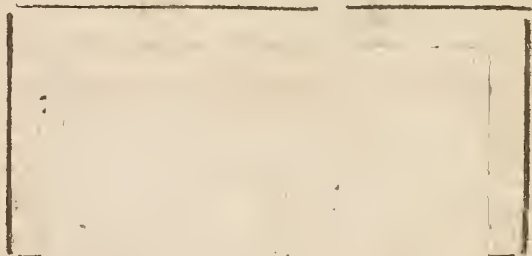
Demonstration.

VV. iij. and

The eleuenth Booke

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In this figure if ye erect perpendicularly the superficies wherein are drawne the lines BA and CA to the ground superficies wherein is drawn the line DAE , and so compare it with the demonstration of the former proposition it will be cleare vnto you.



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¶ The 12. Theoreme.

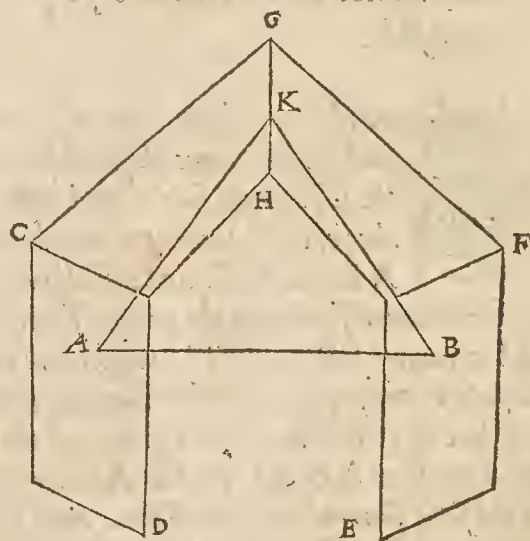
The 14. Proposition.

To whatsoeuer plaine superficies one and the selfe same right line is erected perpendicularly: those superficies are parallels the one to the other.



Suppose that a right line AB be erected perpendicularly to either of these plaine superficies CD and EF . Then I say, that these superficies CD and EF are parallels the one to the other. For if not, then if they be extended they will at the length meete. Let them meete, if it be possible. Now then their common section shall (by the 3. of the eleuenth) be a right line. Let that common section be GH . And in the line GH take a point at all adventures and let the same be K . And drawe a right line from the point A to the point K , and an other from the point B to the point K . And forasmuch as the line AB is e-

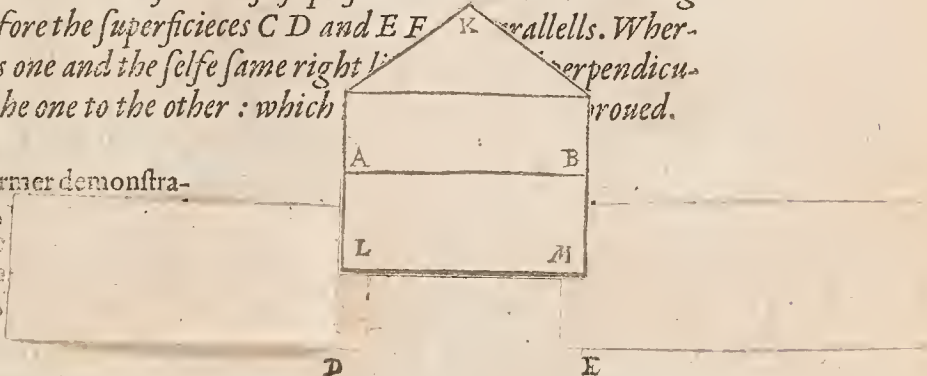
Demonstration leading to an impossibility.



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rected perpendicularly to the plaine superficies EF , therefore the line AB is also erected perpendicularly to the line BK which is in the extended superficies EF . Wherefore the angle ABK is a right angle. And by the same reason also the angle $B AK$ is a right angle. Wherefore in the triangle ABK , these two angles ABK & $B AK$, are equall to two right angles: which (by the 17. of the first) is impossible. Wherefore these superficieses CD and EF being extended meete not together. Wherefore the superficieses CD and EF are parallels. Wherefore to what soeuer plaine superficieses one and the selfe same right line is perpendicular: those superficieses are parallels the one to the other: which is demonstrated.

In this figure may ye plainly see the former demonstration if ye erecte the three superficieses, KLM perpendicular to the ground plaine yet in such sort that the two superficieses may concur in the common line GKH , the demonstration.



A corollary added by Campane.

If a right line be erected perpendicularly to one of those superficieses, it shall also be erected perpendicularly to the other.

For if it should not be erected perpendicularly to the other, then it falling vpon that other shall make with some one line thereof an angle lesse then a right angle: which line should (by the 5. petition of the first) concur with some one line of that superficies whereunto it is perpendicular. So that those superficieses should not be parallels: which is contrary to the supposition. For they are supposed to be parallels.

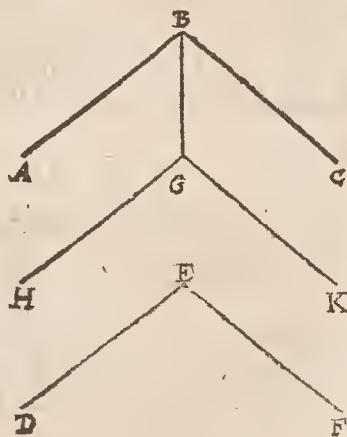
The 13. Theoreme.

The 15. Proposition.

If two right lines touching the one the other be parallels to two other right lines touching also the one the other and not being in the selfe same plaine superficies with the two first: the plaine superficieses extended by those right lines, are also parallels the one to the other.



Suppose that these two right lines AB and BC touching the one the other be parallels to these two right lines DE & EF touching also the one the other, and not being in the selfe same plaine superficies with the right lines AB and BC . Then I say, that the plaine superficieses by the lines AB and BC , and the lines DE and EF being extended, shall not meete together, that is, they are equidistant and parallels. From the point B draw (by the 11. of the eleuenth) a perpendicular line to the superficies wherein are the lines DE and EF , and let that perpendicular line be BG . And by the point G in the plaine superficies passing by DE , and EF , draw (by the 31. of the first) vnto the line ED a parallell line GH : and likewise by that point G drawe in the same superficies vnto the line EF a parallell line GK . And forasmuch as the line BG is erected perpendicularly to the superficies wherein are the lines DE and EF , therefore (by the 2. definition of the eleuenth) it is also erected perpendicularly to all the right lines which touch it, and are in the selfe same superficies wherein are the lines DE



Construction.

Demonstration.

VV. iij.

and

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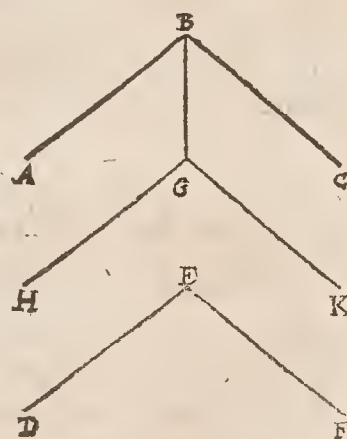
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If two right lines touching the one the other be parallels to two other right lines touching also the one the other and not being in the selfe same plaine superficies with the two first: the plaine superficieses extended by those right lines, are also parallells the one to the other.



Suppose that these two right lines AB and BC touching the one the other be parallells to these two right lines DE & EF touching also the one the other, and not being in the selfe same plaine superficies with the right lines AB and BC . Then I say, that the plaine superficieses by the lines AB and BC , and the lines DE and EF being extended, shall not meete together, that is, they are equidistant and parallels. From the point B draw (by the 11. of the eleuenth) a perpendicular line to the superficies wherein are the lines DE and EF , and let that perpendicular line be BG . And by the point G in the plaine superficies passing by DE , and EF , draw (by the 31. of the first) vnto the line ED a parallell line GH : and likewise by that point G drawe in the same superficies vnto the line EF a parallell line GK . And forasmuch as the line BG is erected perpendicularly to the superficies wherein are the lines DE and EF , therefore (by the 2. definition of the eleuenth) it is also erected perpendicularly to all the right lines which touch it, and are in the selfe same superficies wherein are the lines DE



Construction.

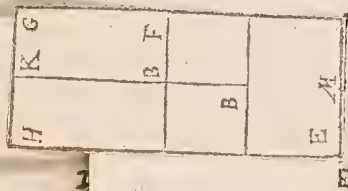
Demonstration.

VV. iij.

and

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A corollary added by Campane.

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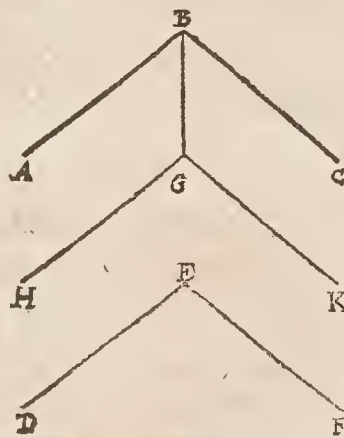
¶ The 13. Theoreme.

The 15. Proposition.

If two right lines touching the one the other be parallells to two other right lines touching also the one the other and not being in the selfe same plaine superficies with the two first: the plaine superficieses extended by those right lines, are also parallells the one to the other.



Suppose that these two right lines AB and BC touching the one the other be parallells to these two right lines DE & EF touching also the one the other, and not being in the selfe same plaine superficies with the right lines AB and BC . Then I say, that the plaine superficieses by the lines AB and BC , and the lines DE and EF being extended, shall not meete together, that is, they are equidistant and parallells. From the point B draw (by the 11. of the eleuenth) a perpendicular line to the superficies wherein are the lines DE and EF , and let that perpendicular line be BG . And by the point G in the plaine superficies passing by DE , and EF , draw (by the 31. of the first) vnto the line ED a parallell line GH : and likewise by that point G drawe in the same superficies vnto the line EF a parallell line GK . And forasmuch as the line BG is erected perpendicularly to the superficies wherein are the lines DE and EF , therefore (by the 2. definition of the eleuenth) it is also erected perpendicularly to all the right lines which touch it, and are in the selfe same superficies wherein are the lines DE



Construction.

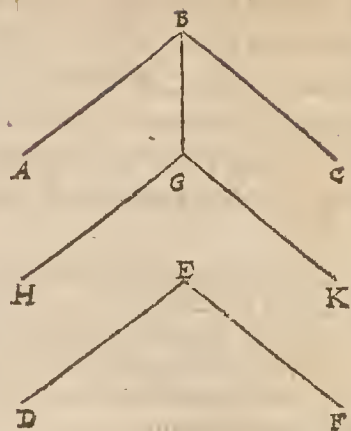
Demonstration.

VV. iij.

and

The eleuenth Booke

and EF . But either of these lines GH and GK touch it, and are also in the superficies wherein are the lines DE and EF , therefore either of these angles BGH , and BGK , is a right angle. And forasmuch as the line BA is a parallell to the line GH (that the lines GH and GK are parallells vnto the lines AB and BC it is manifest by the 9. of this booke): therefore (by the 29. of the first) the angles GBA and BGH are equall to two right angles. But the angle BGH is (by constructiō) a right angle, therefore also the angle GBA is a right angle: therefore the line GB is erected perpendicularly to the line BA . And by the same reason also may it be proued, that the line BG is erected perpendicularly to the line BC . Now forasmuch as the right line BG is erected perpendicularly to these two right lines BA and BC touching the one the other, therefore (by the 4. of the eleuenth) the line BG is erected perpendicularly to the superficies wherein are the lines BA and BC . And it is also erected perpendicularly to the superficies wherein are the lines GH and GK . But the superficies wherein are the lines GH and GK , is that superficies wherein are the lines DE and EF : wherefore the line BG is erected perpendicularly to the superficies wherein are the lines DE and EF . Wherefore the line BG is erected perpendicularly to the superficies wherein are the lines DE and EF , and to the superficies wherein are the lines AB and BC . But if one and the selfe same right line be erected perpendicularly to plaine superficieses, those superficieses are (by the 14. of the eleuenth) parallels the one to the other. Wherefore the superficies wherin are the lines AB and BC is a parallel to the superficies wherin are the lines DE and EF . If therefore two right lines touching the one the other be parallels to two other right lines touching also the one the other, and not being in the selfe same plaine superficies with the two first, the plaine superficieses extended by those right lines are also parallels the one to the other which was required to be demonstrated.



By this figure here put, ye may more clerely see the former 15. Proposition and also the demonstration thereof: if ye erecte perpendicularly vnto the superficies, the three superficieses ABC , KHL , and BML , and so compare it with the demonstration.



¶ A Corollary added by Fluffas.

Vnto a plaine superficies being geuen, to drawe by a point geuen without it, a parallel plaine superficies. Suppose as in the former description that the superficies geue be ABC , & let the point geue without it be G . Now then by the point G drawe (by the 31. of the first) vnto the lines AB and BC parallel lines GH and GK . And the superficies extended by the lines GH and GK shall be parallel vnto the superficies ABC , by this 15. Proposition.

The 14. Theoreme.

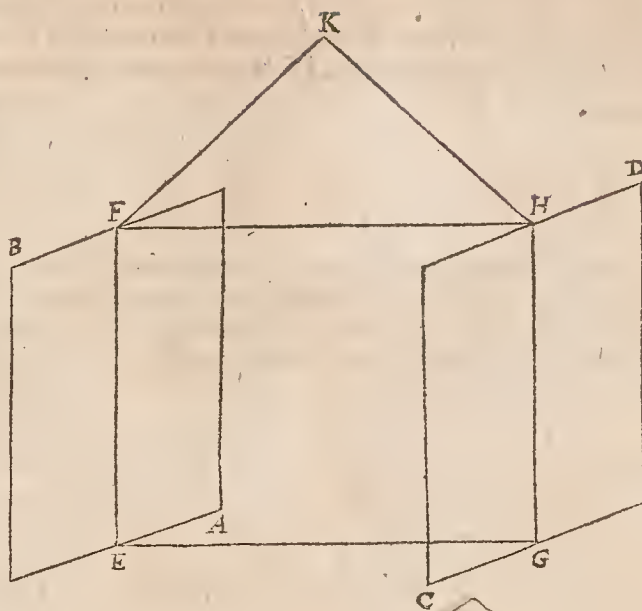
The 16. Proposition.

If two parallel playne superficieses be cut by some one playne superficies: their common sections are parallel lines.



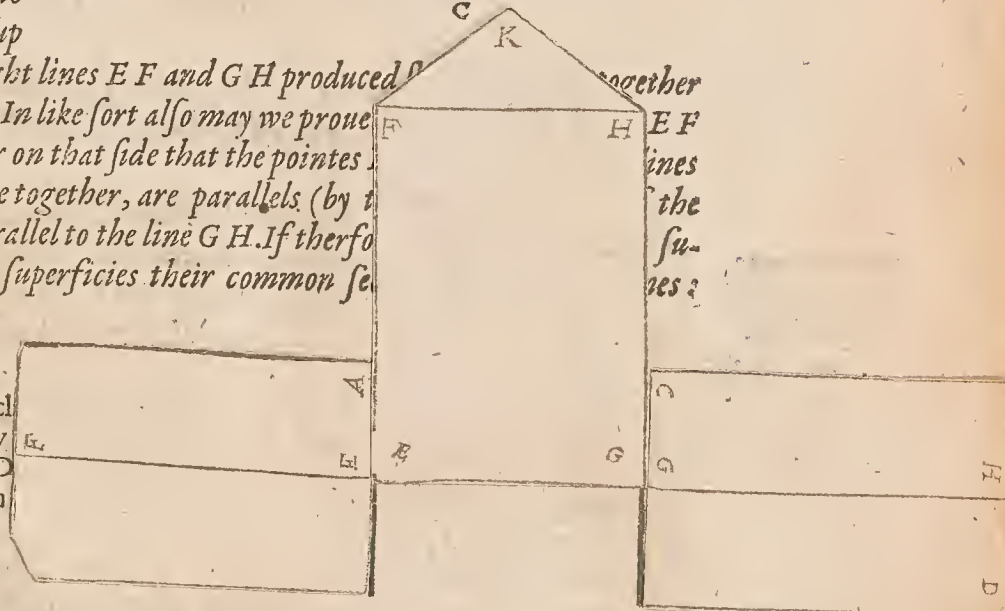
Suppose that these two plaine superficieses AB and CD be cut by this plaine superficies $EFGH$, and let their common sections be the right lines EF and GH . Then I say that the line EF is a parallel to the line GH . For if not, then the lines EF and GH being produced, shall at the length meete together either on the

the side that the pointes F, H are, or on the side that the pointes E, G are. First let them be produced on that side that the pointes F, H are, and let them mete in the point K. And forasmuch as the line E F K is in the superficies A B, therefore all the points which are in the line E F are in the superficies A B (by the first of this booke) But one of the pointes which are in the right line E F K is the point K, therefore the point K is in the superficies A B. And by the same reason also the point K is in the superficies C D. Wherefore the two superficieses A B and C D being produced do mete together; but by suppositio they mete not together, for they are supposed to be parallels. Wherefore the right lines E F and G H produced on that side that the pointes F, H are. In like sort also may we proue and G H produced meete not together on that side that the pointes which being produced on no side mete together, are parallels (by the first.) Wherefore the line E F is a parallel to the line G H. If therfore superficieses be cut by some one plaine superficies their common sections which was required to be proued.



Demonstration leading to an absurditie

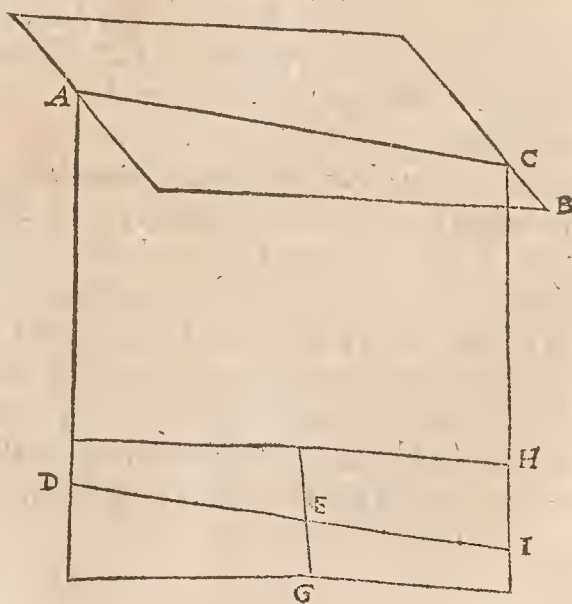
This figure here set more plainly declares demonstration, if ye erect perpendicularly superficieses the three superficieses A B, C D and so compare it with the demonstration



A Corollary added by Flusbas.

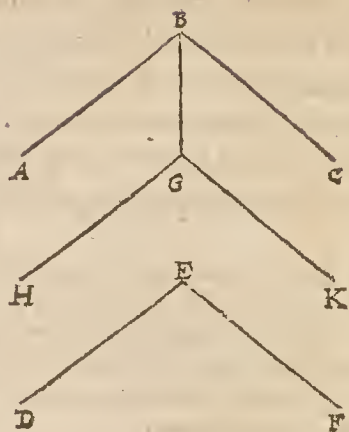
If two plaine superficieses be parallels to one and the selfe same playne superficies: they shall also be parallels the one to the other, or they shall make one and the selfe same plaine superficies.

For if the plaine superficieses D G and G H being parallels to one and the selfe same superficies, namely, to A B be not also parallels the one to the other, then being produced they shall concur (by the conuerse of the six definition of the eleuenth) Let them concur in the right line G E. Then I say that the superficieses G D and G H are in one and the selfe same playne superficies. Draw in the playne superficies A B a right line at all aduentures A C. And by that right lyne & the point E extende a playne superficies, cutting the two superficieses D G and G H by the right lines E D and E I. Wherefore the right lines A C and D E, also A C and E I are parallels by this proposition. But the lines D E and E I forasmuch as they concur in the point E are not parallels the one to the other. Wherefore the right lines D E and E I make directly one right line (by that which is added after the 30. propostion of the first.) And therfore the plaine superficieses D G and G H are in one and the selfe same playne superficies. For

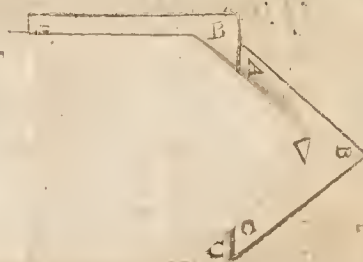


The eleuenth Booke

and EF . But either of these lines GH and GK touch it, and are also in the superficies wherein are the lines DE and EF , therefore either of these angles BGH , and BGK , is a right angle. And forasmuch as the line BA is a parallell to the line GH (that the lines GH and GK are parallells vnto the lines AB and BC it is manifest by the 9. of this booke): therefore (by the 29. of the first) the angles GBA and BGH are equall to two right angles. But the angle BGH is (by constructiō) a right angle, therefore also the angle GBA is a right angle: therefore the line GB is erected perpendicularly to the line BA . And by the same reason also may it be proued, that the line BG is erected perpendicularly to the line BC . Now forasmuch as the right line BG is erected perpendicularly to these two right lines BA and BC touching the one the other, therefore (by the 4. of the eleuenth) the line BG is erected perpendicularly to the superficies wherein are the lines BA and BC . And it is also erected perpendicularly to the superficies wherein are the lines GH and GK . But the superficies wherein are the lines GH and GK , is that superficies wherein are the lines DE and EF : wherefore the line BG is erected perpendicularly to the superficies wherein are the lines DE and EF . Wherefore the line BG is erected perpendicularly to the superficies wherein are the lines DE and EF , and to the superficies wherein are the lines AB and BC . But if one and the selfe same right line be erected perpendicularly to plaine superficieses, those superficieses are (by the 14. of the eleuenth) parallels the one to the other. Wherefore the superficieses wherin are the lines AB and BC is a parallel to the superficies wherin are the lines DE and EF . If therefore two right lines touching the one the other be parallels to two other right lines touching also the one the other, and not being in the selfe same plaine superficies with the two first, the plaine superficieses extended by those right lines are also parallels the one to the other which was required to be demonstrated.



By this figure here put, ye may more clerely see both the former 15. Proposition and also the demonstration thereof: if ye erecte perpendicularly vnto the ground superficies, the three superficieses ABC , KHE , and $LHBM$, and so compare it with the demonstration.



¶ A Corollary added by Fluffas.

Vnto a plaine superficies being geuen, to drawe by a point geuen without it, a parallel plaine superficies. Suppose as in the former description that the superficies geue be ABC , & let the point geue without it be G . Now then by the point G drawe (by the 31. of the first) vnto the lines AB and BC parallel lines GH and GK . And the superficies extended by the lines GH and GK shall be parallel vnto the superficies ABC , by this 15. Proposition.

The 14. Theoreme.

The 16. Proposition.

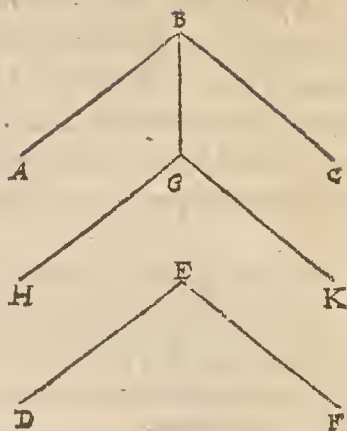
If two parallel playne superficieses be cut by some one playne superficies: their common sections are parallel lines.



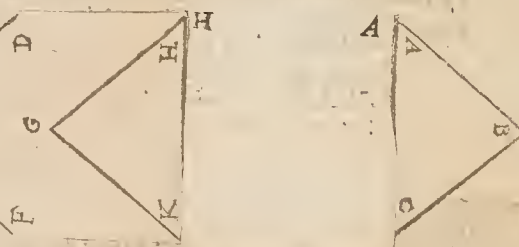
Suppose that these two plaine superficieses AB and CD be cut by this plaine superficies $EFGH$, and let their common sections be the right lines EF and GH . Then I say that the line EF is a parallel to the line GH . For if not, then the lines EF and GH being produced, shall at the length meete together either on the

The eleuenth Booke

and EF . But either of these lines GH and GK touch it, and are also in the superficies wherein are the lines DE and EF , therefore either of these angles BGH , and BGK , is a right angle. And forasmuch as the line BA is a parallell to the line GH (that the lines GH and GK are parallells vnto the lines AB and BC it is manifest by the 9. of this booke): therefore (by the 29. of the first) the angles GBA and BGH are equall to two right angles. But the angle BGH is (by constructiō) a right angle, therefore also the angle GBA is a right angle: therefore the line GB is erected perpendicularly to the line BA . And by the same reason also may it be proued, that the line BG is erected perpendicularly to the line BC . Now forasmuch as the right line BG is erected perpendicularly to these two right lines BA and BC touching the one the other, therefore (by the 4. of the eleuenth) the line BG is erected perpendicularly to the superficies wherein are the lines BA and BC . And it is also erected perpendicularly to the superficies wherein are the lines GH and GK . But the superficies wherein are the lines GH and GK , is that superficies wherein are the lines DE and EF : wherefore the line BG is erected perpendicularly to the superficies wherein are the lines DE and EF . Wherefore the line BG is erected perpendicularly to the superficies wherein are the lines DE and EF , and to the superficies wherein are the lines AB and BC . But if one and the selfe same right line be erected perpendicularly to plaine superficieses, those superficieses are (by the 14. of the eleuenth) parallels the one to the other. Wherefore the superficieses wherein are the lines AB and BC is a parallel to the superficieses wherein are the lines DE and EF . If therefore two right lines touching the one the other be parallels to two other right lines touching also the one the other, and not being in the selfe same plaine superficies with the two first, the plaine superficieses extended by those right lines are also parallels the one to the other which was required to be demonstrated.



By this figure here put, ye may more clerely see the former 15. Proposition and also the demonstration thereof: if ye erecte perpendicularly vnto the superficies, the three superficieses ABC , KHL HBM , and so compare it with the demonstration.



¶ A Corollary added by Fluffas.

Vnto a plaine superficies being geuen, to drawe by a point geuen without it, a parallel plaine superficies. Suppose as in the former description that the superficies geue be ABC , & let the point geue without it be G . Now then by the point G drawe (by the 31. of the first) vnto the lines AB and BC parallel lines GH and GK . And the superficieses extended by the lines GH and GK shall be parallel vnto the superficies ABC , by this 15. Proposition.

The 14. Theoreme.

The 16. Proposition.

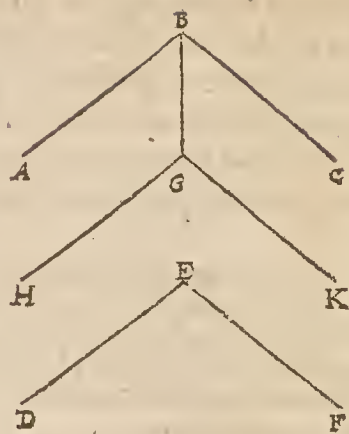
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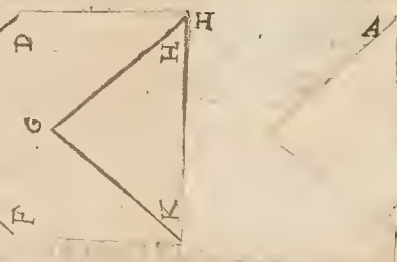
Suppose that these two plaine superficieses AB and CD be cut by this plaine superficies $EFGH$, and let their common sections be the right lines EF and GH . Then I say that the line EF is a parallel to the line GH . For if not, then the lines EF and GH being produced, shall at the length meete together either on the

The eleuenth Booke

and EF . But either of these lines GH and GK touch it, and are also in the superficies wherein are the lines DE and EF , therefore either of these angles BGH , and BGK , is a right angle. And forasmuch as the line BA is a parallell to the line GH (that the lines GH and GK are parallells vnto the lines AB and BC it is manifest by the 9. of this booke): therefore (by the 29. of the first) the angles GBA and BGH are equall to two right angles. But the angle BGH is (by constructiō) a right angle, therefore also the angle GBA is a right angle: therefore the line GB is erected perpendicularly to the line BA . And by the same reason also may it be proued, that the line BG is erected perpendicularly to the line BC . Now forasmuch as the right line BG is erected perpendicularly to these two right lines BA and BC touching the one the other, therefore (by the 4. of the eleuenth) the line BG is erected perpendicularly to the superficies wherein are the lines BA and BC . And it is also erected perpendicularly to the superficies wherein are the lines GH and GK . But the superficies wherein are the lines GH and GK , is that superficies wherein are the lines DE and EF : wherefore the line BG is erected perpendicularly to the superficies wherein are the lines DE and EF . Wherefore the line BG is erected perpendicularly to the superficies wherein are the lines DE and EF , and to the superficies wherein are the lines AB and BC . But if one and the selfe same right line be erected perpendicularly to plaine superficieses, those superficieses are (by the 14. of the eleuenth) parallels the one to the other. Wherefore the superficies wherin are the lines AB and BC is a parallel to the superficies wherin are the lines DE and EF . If therefore two right lines touching the one the other be parallels to two other right lines touching also the one the other, and not being in the selfe same plaine superficies with the two first, the plaine superficieses extended by those right lines are also parallels the one to the other which was required to be demonstrated.



By this figure here put, ye may more clerely see the former 15. Proposition and also the demonstration thereof: if ye erecte perpendicularly vnto the superficies, the three superficieses ABC , KHL , and HBM , and so compare it with the demonstration.



¶ A Corollary added by Fluffas.

Vnto a plaine superficies being geuen, to drawe by a point geuen without it, a parallel plaine superficies. Suppose as in the former description that the superficies geue be ABC , & let the point geue without it be G . Now then by the point G drawe (by the 31. of the first) vnto the lines AB and BC parallel lines GH and GK . And the superficies extended by the lines GH and GK shall be parallel vnto the superficies ABC , by this 15. Proposition.

The 14. Theoreme.

The 16. Proposition.

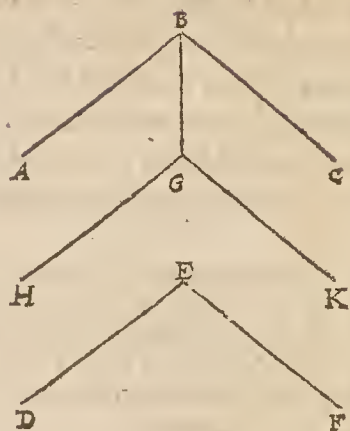
If two parallel playne superficieses be cut by some one playne superficies: their common sections are parallel lines.



Suppose that these two plaine superficieses AB and CD be cut by this plaine superficies $EFGH$, and let their common sections be the right lines EF and GH . Then I say that the line EF is a parallel to the line GH . For if not, then the lines EF and GH being produced, shall at the length meete together either on the

The eleventh Booke

and EF . But either of these lines GH and GK touch it, and are also in the superficies wherein are the lines DE and EF , therefore either of these angles BGH , and BGK , is a right angle. And forasmuch as the line BA is a parallell to the line GH (that the lines GH and GK are parallells vnto the lines AB and BC it is manifest by the 9. of this booke): therefore (by the 29. of the first) the angles GBA and BGH are equall to two right angles. But the angle BGH is (by constructiō) a right angle, therefore also the angle GBA is a right angle: therefore the line GB is erected perpendicularly to the line BA . And by the same reason also may it be proued, that the line GB is erected perpendicularly to the line BC . Now forasmuch as the right line GB is erected perpendicularly to these two right lines BA and BC touching the one the other, therefore (by the 4. of the eleventh) the line EG is erected perpendicularly to the superficies wherein are the lines BA and BC . And it is also erected perpendicularly to the superficies wherein are the lines GH and GK . But the superficies wherein are the lines GH and GK , is that superficies wherein are the lines DE and EF : wherefore the line GB is erected perpendicularly to the superficies wherein are the lines DE and EF . Wherefore the line GB is erected perpendicularly to the superficies wherein are the lines DE and EF , and to the superficies wherein are the lines AB and BC . But if one and the selfe same right line be erected perpendicularly to plaine superficieses, those superficieses are (by the 14. of the eleventh) parallels the one to the other. Wherefore the superficieses wherein are the lines AB and BC is a parallel to the superficieses wherein are the lines DE and EF . If therefore two right lines touching the one the other be parallels to two other right lines touching also the one the other, and not being in the selfe same plaine superficies with the two first, the plaine superficieses extended by those right lines are also parallels the one to the other which was required to be demonstrated.



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¶ A Corollary added by Flussas.

Vnto a plaine superficies being geuen, to drawe by a point geuen without it, a parallel plaine superficies. Suppose as in the former description that the superficies geue be ABC , & let the point geue without it be G . Now then by the point G drawe (by the 31. of the first) vnto the lines AB and BC parallel lines GH and GK . And the superficies extended by the lines GH and GK shall be parallel vnto the superficies ABC , by this 15. Proposition.

The 14. Theoreme.

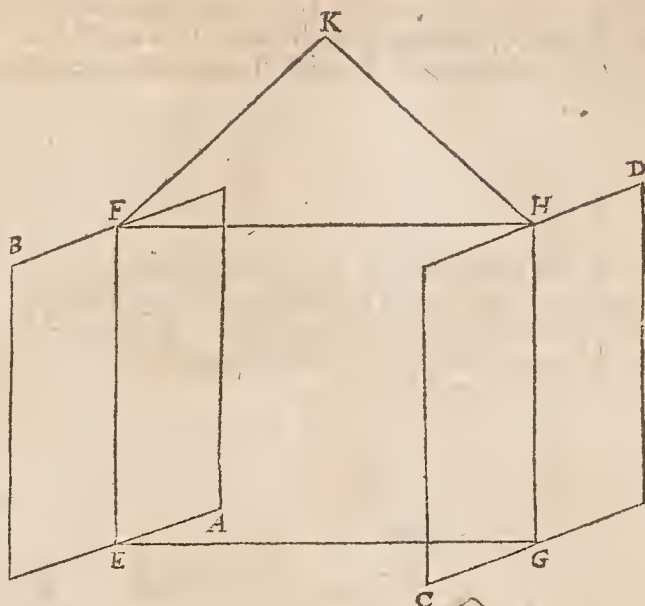
The 16. Proposition.

If two parallel playne superficieses be cut by some one playne superficies: their common sections are parallel lines.



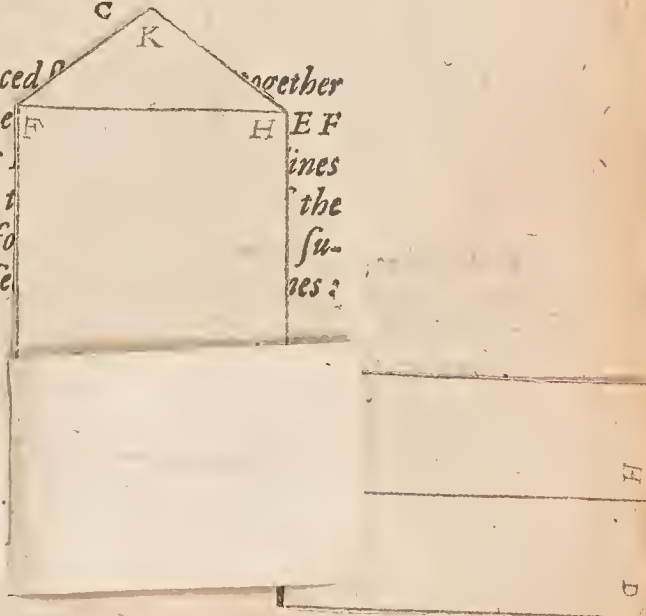
Vppose that these two plaine superficieses AB and CD be cut by this plaine superficies $EFGH$, and let their common sections be the right lines EF and GH . Then I say that the line EF is a parallel to the line GH . For if not, then the lines EF and GH being produced, shall at the length meete together either on the

the side that the pointes F, H are, or on the side that the pointes E, G are. First let them be produced on that side that the pointes F, H are, and let them mete in the point K. And forasmuch as the line E F K is in the superficies A B, therefore all the points which are in the line E F are in the superficies A B (by the first of this booke) But one of the pointes which are in the right line E F K is the point K, therefore the point K is in the superficies A B. And by the same reason also the point K is in the superficies C D. Wherefore the two superficieses A B and C D being produced do mete together, but by suppositiō they mete not together, for they are supposed to be parallels. Wherefore the right lines E F and G H produced on that side that the pointes F, H are. In like sort also may we proue and G H produced meete not together on that side that the pointes which being produced on no side mete together, are parallels (by the first.) Wherefore the line E F is a parallel to the line G H. If therfore superficieses be cut by some one plaine superficies their common section which was required to be proued.



Demonstration leading to an absurditie

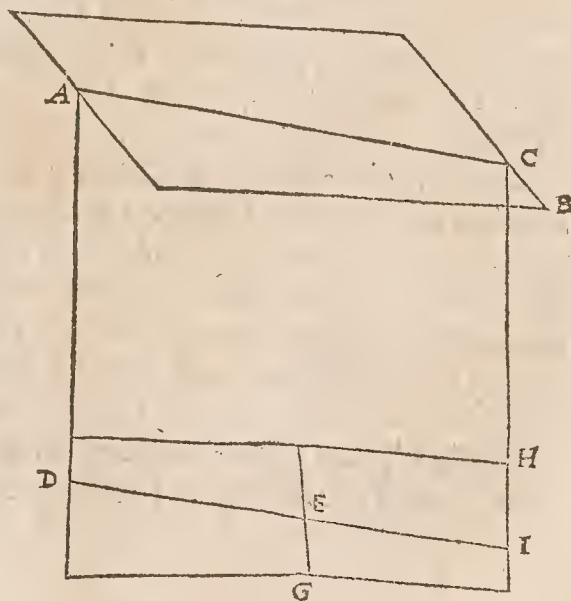
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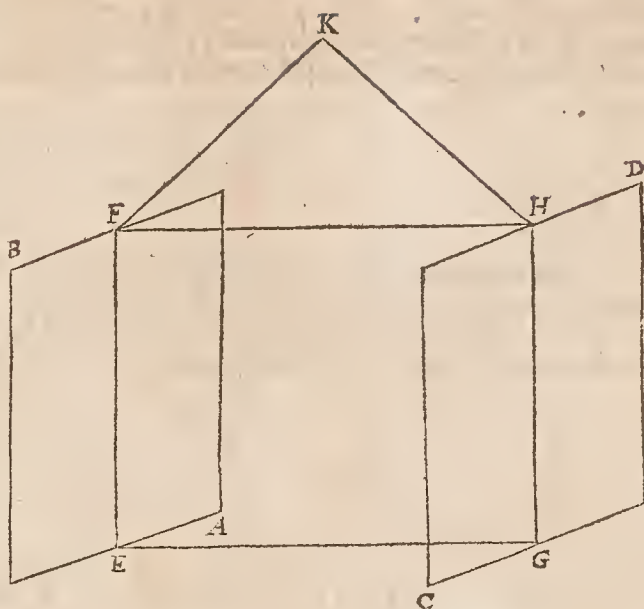
A Corollary added by Flußas.

If two plaine superficieses be parallels to one and the selfe same playne superficies: they shall also be parallels the one to the other, or they shall make one and the selfe same plaine superficieses.

For if the plaine superficieses D G and G H being parallels to one and the selfe same superficies, namely, to A B be not also parallels the one to the other, then being produced they shall concur (by the conuerse of the sixt definition of the eleuenth) Let them concur in the right line G E. Then I say that the superficieses G D and G H are in one and the selfe same playne superficies. Draw in the playne superficies A B a right line at all adventures A C. And by that right lyne & the point E extende a playne superficies, cutting the two superficieses D G and G H by the right lines E D and E I. Wherefore the right lines A C and D E, also A C and E I are parallels by this proposition. But the lines D E and E I forasmuch as they concur in the point E are not parallels the one to the other. Wherefore the right lines D E and E I make directly one right line (by that which is added after the 30. propoition of the first.) And therefore the plaine superficieses D G and G H are in one and the selfe same playne superficies. For

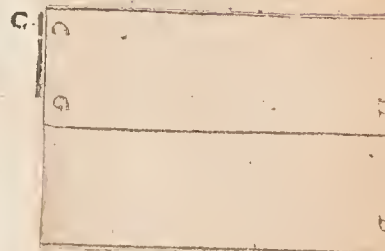
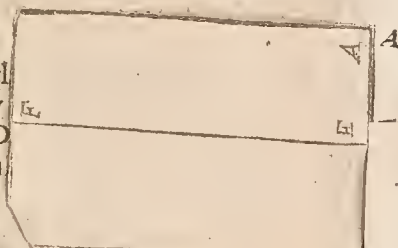


the side that the pointes F, H are, or on the side that the pointes E, G are. First let them be produced on that side that the pointes F, H are, and let them mete in the point K. And forasmuch as the line E F K is in the superficies A B, therefore all the points which are in the line E F are in the superficies A B (by the first of this booke) But one of the pointes which are in the right line E F K is the point K, therefore the point K is in the superficies A B. And by the same reason also the point K is in the superficies C D. Wherefore the two superficieses A B and C D being produced do mete together, but by suppositiō they mete not together, for they are supposed to be parallels. Wherefore the right lines E F and G H produced shall not meete together on that side that the pointes F, H are. In like sort also may we proue that the right lines E F and G H produced meete not together on that side that the pointes E, G are. But right lines which being produced on no side mete together, are parallels (by the last definition of the first.) Wherefore the line E F is a parallel to the line G H. If therefore two parallel plaine superficieses be cut by some one plaine superficies their common sections are parallel lines; which was required to be proued.



Demonstration leading to an absurditie

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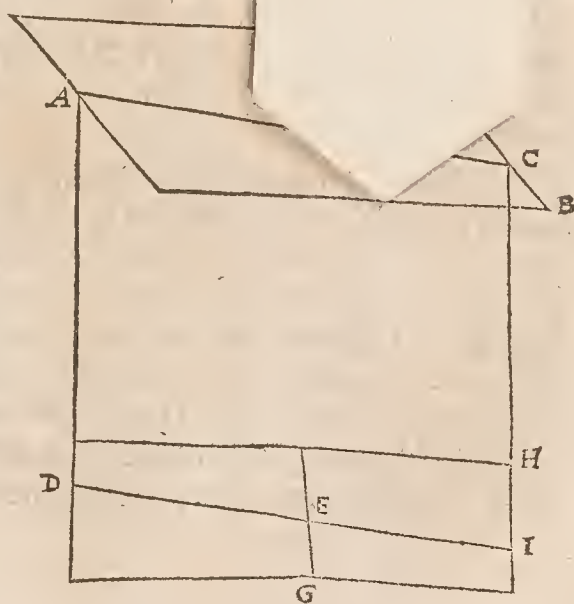


A Corollary added by Flusſas.

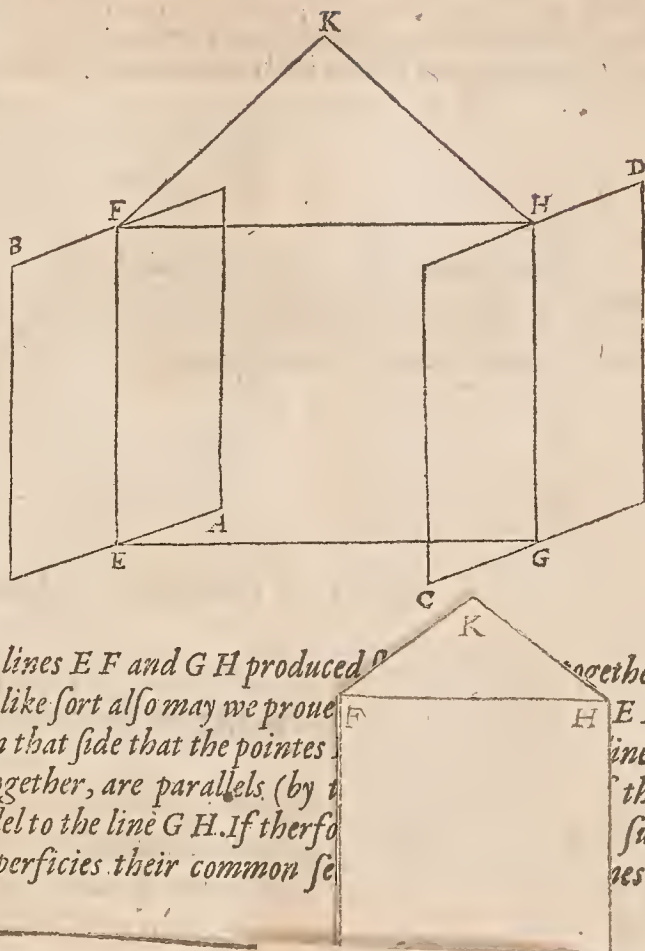
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so be

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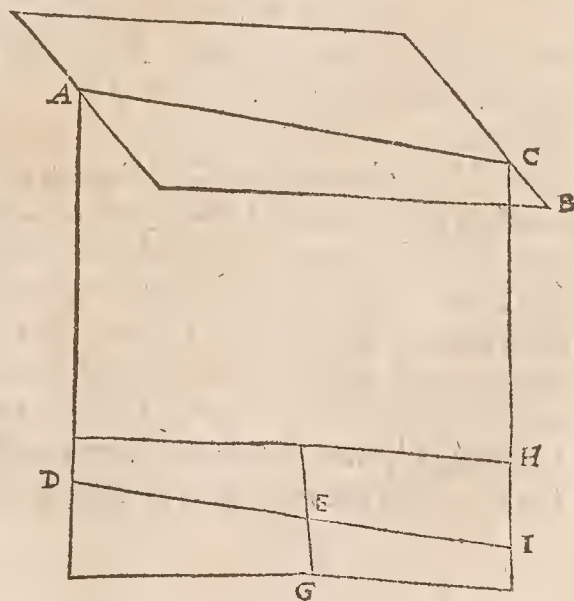
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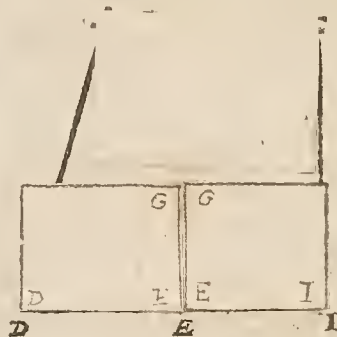
For if the plaine superficieses D G and G H being parallels to one and the selfe same superficies, namely, to A B be not also parallels the one to the other, then being produced they shall concur (by the conuerse of the fixt definition of the eleuenth) Let them concur in the right line G E. Then I say that the superficieses G D and G H are in one and the selfe same playne superficies. Draw in the playne superficieses A B a right line at all aduentures A C. And by that right lyne & the point E extende a playne superficies, cutting the two superficieses D G and G H by the right lines E D and E I. Wherefore the right lines A C and D E, also A C and E I are parallels by this proposition. But the lines D E and E I forasmuch as they concur in the point E are not parallels the one to the other. Wherefore the right lines D E and E I make directly one right line (by that which is added after the 30. propofition of the first.) And therefore the plaine superficieses D G and G H are in one and the selfe same playne superficies. For



The eleuenth Booke

if they be not, then part of the right line DI , namely, the part DE is in the playne superficies DG , and an other part therof, namely, EI is on high in an other superficies GH , which by the first of the eleueth is impossible. Wherefore the superficieses DG and GH are in one and the selfe same playne superficies. But if the superficieses DG and GH neuer concurre, then are they parallels by the 6. definition of the eleuenth.

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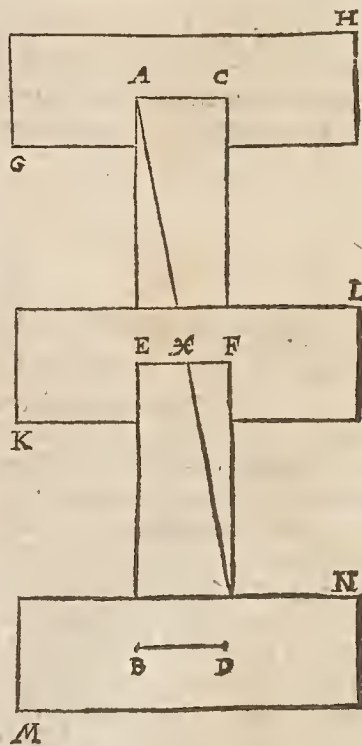
The 15. Theoreme. The 17. Proposition.

If two right lines be cut by playne superficieses being parallels: the partes of the lines deuided shall be proportionall.

In this proposition ye must vnderstand the proportionall partes or sections to be those which are contained betwene the parallel superficieses.
Construction.
Demonstration.



*V*ppose that these two right lines AB and CD be deuided by these plaine superficieses being parallels, namely, GH, KL, MN in the points A, E, B, C, F, D . The I say that as the right line AE is to the right line EB , so is the right line CF to the right line FD . Draw these right lines AC, BD and AD . And let the line AD and the superficieses KL concurre in the point X . And draw a right line from the point E to the point X and an other from the point X to the point F . And forasmuch as these two parallel superficieses KL and MN are cut by the superficieses $EBDX$, therefore their common sections which are the lines EX and BD , are (by the 16. of the eleuenth) parallels the one to the other. And by the same reason also forasmuch as the two parallel superficieses GH and KL be cut by the superficieses $AXFC$, their common sections AC and XF are (by the 16. of the eleuenth) parallels. And forasmuch as to one of the sides of the triangle ABD , namely, to the side BD is drawne a parallel line EX , therefore (by the 2. of the sixt) proportionally as the line AE is to the line EB , so is the line AX to the line XD . Againe forasmuch as to one of the sides of the triangle ADC , namely, to the side AC is drawen a parallel line XF , therefore by the 2. of the sixt, proportionally as the line AX is to the line XD , so is the line CF to the line FD . And it was proued that as the line AX is to the line XD , so is the line AE to the line EB , therefore also (by the 11. of the fift) as the line AE is to the line EB , so is the line CF to the line FD . If therefore two right lines be deuided by plaine superficieses being parallels, the partes of the lines deuided shall be proportionall: which was required to be demonstrated.



of Euclides Elementes.

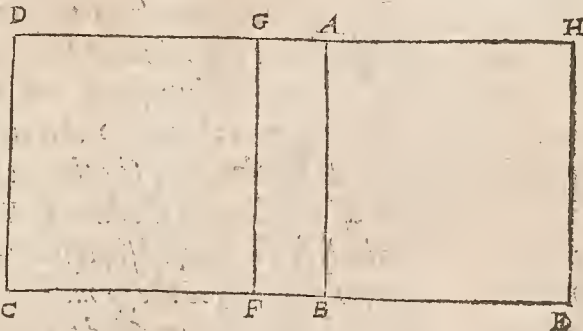
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If a right line be erected perpendicularly to a plaine superficies: all the superficieses extended by that right line, are erected perpendicularly to the selfe same plaine superficies.

Suppose that a right line $A B$ be erected perpendicularly to a ground superficies. The I say, that all the superficieses passing by the line $A B$, are erected perpendicularly to the ground superficies. Extend a superficies by the line $A B$, and let the same be $E D$, & let the common section of the plaine superficies and of the ground superficies be the right line $C E$. And take in the line $C E$ a point at all adventures, and let the same be F : and (by the 11. of the first) from the point F draw vnto the line $C E$ a perpendicular line in the superficies $D E$, and let the same be $F G$. And forasmuch as the line $A B$ is erected perpendicularly to the ground superficies, therefore (by the 2. definition of the eleventh) the line $A B$ is erected perpendicularly to all the right lines that are in the ground plaine superficies, and which touch it. Wherefore it is erected perpendicularly to the line $C E$. Wherefore the angle $A B E$ is a right angle. And the angle $G F B$ is also a right angle (by construction). Wherefore (by the 28. of the first) the line $A B$ is a parallel to the line $F G$. But the line $A B$ is erected perpendicularly to the ground superficies: wherefore (by the 8. of the eleventh) the line $F G$ is also erected perpendicularly to the ground superficies. And forasmuch as (by the 3. definition of the eleventh) a plaine superficies is then erected perpendicularly to a plaine superficies, when all the right lines drawn in one of the plaine superficieses vnto the common section of those two plaine superficieses making therewith right angles, do also make right angles with the other plaine superficies: and it is proued that the line $F G$ drawn in one of the plaine superficieses, namely, in $D E$, perpendicularly to the common section of the plaine superficieses, namely, to the line $C E$, is erected perpendicularly to the ground superficies: wherefore the plaine superficies $D E$ is erected perpendicularly to the ground superficies. In like sort also may we proue, that all the plaine superficieses which passe by the line $A B$, are erected perpendicularly to the ground superficies. If therefore a right line be erected perpendicularly to a plaine superficies all the superficieses passing by the right line, are erected perpendicularly to the selfe same plaine superficies: which was required to be demonstrated.



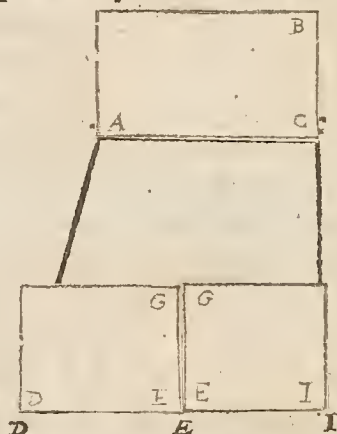
Construction.

Demonstration.

The eleuenth Booke

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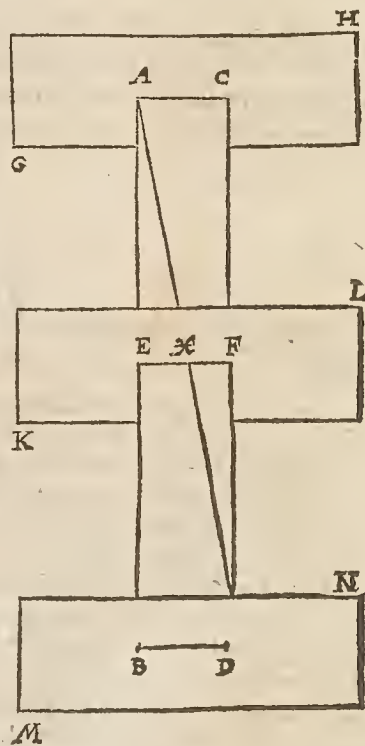
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If two right lines be cut by playne superficieses being parallels: the partes of the lines deuided shall be proportionall.

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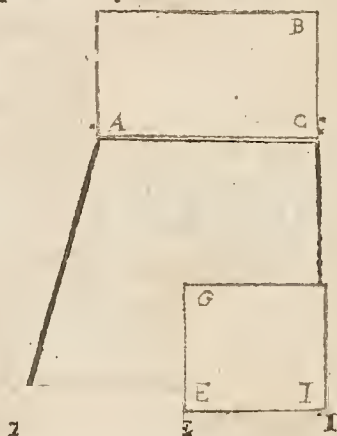


In

The eleuenth Booke

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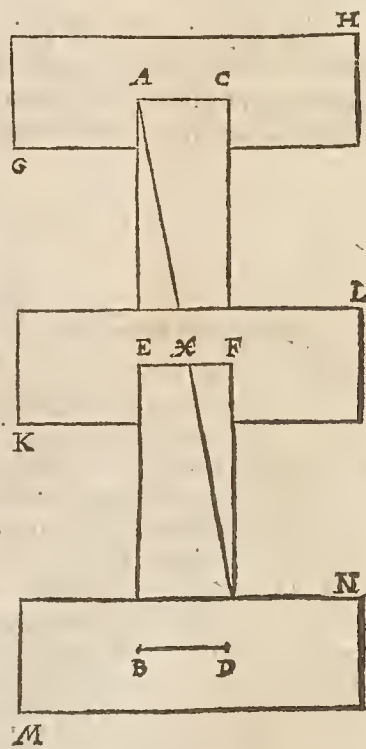
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Demonstration.



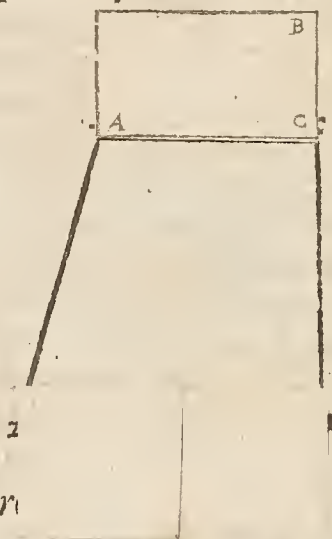
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The eleuenth Booke

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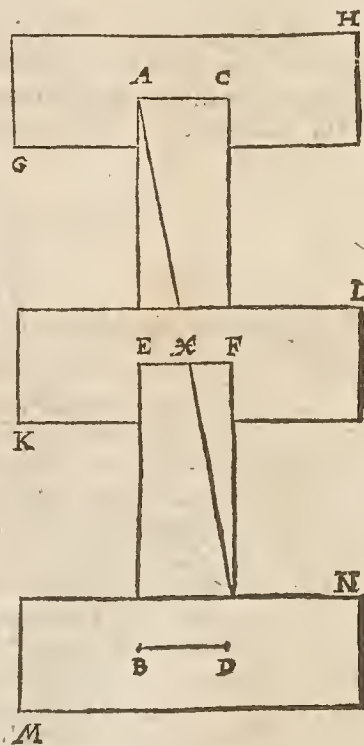
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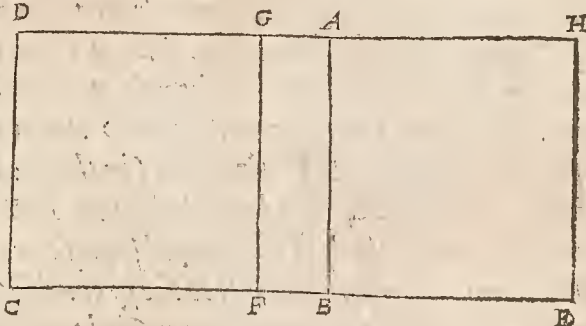


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And take in the line $C E$ a point at all adventures, and let the same be F : and (by the 11. of the first) from the point F draw vnto the line $C E$ a perpendicular line in the superficies $D E$, and let the same be $F G$. And forasmuch as the line $A B$ is erected perpendicularly to the ground superficies, therefore (by the 2. definition of the eleventh) the line $A B$ is erected perpendicu-



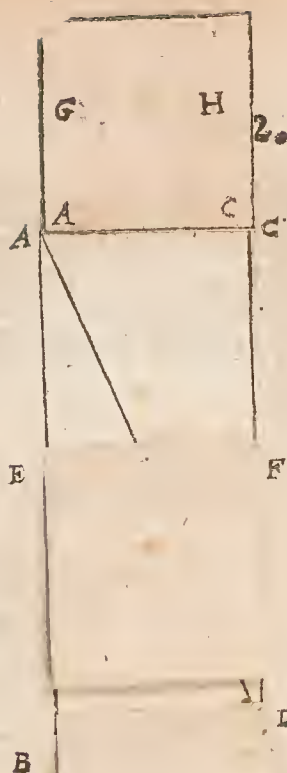
Construction.

larly to all the right lines that are in the ground plaine superficies, and which touch it. Wherefore it is erected perpendicularly to the line $C E$. Wherefore the angle $A B E$ is a right angle. And the angle $G F B$ is also a right angle (by construction). Wherefore (by the 28. of the first) the line $A B$ is a parallel to the line $F G$. But the line $A B$ is erected perpendicularly to the ground superficies: wherefore (by the 8. of the eleventh) the line $F G$ is also erected perpendicularly to the ground superficies. And forasmuch as (by the 3. definition of the eleventh) a plaine superficies is then erected perpendicularly to a plaine superficies, when all the right lines drawn in one of the plaine superficieses vnto the common section of those two plaine superficieses making therewith right angles, do also make right angles with the other plaine superficies: and it is proued that the line $F G$ drawn in one of the plaine superficieses, namely, in $D E$, perpendicularly to the common section of the plaine superficieses, namely, to the line $C E$, is erected perpendicularly to the ground superficies: wherefore the plaine superficies $D E$ is erected perpendicularly to the ground superficies. In like sort also may we proue, that all the plaine superficieses which passe by the line $A B$, are erected perpendicularly to the ground superficies. If therefore a right line be erected perpendicularly to a plaine superficies all the superficieses passing by the right line, are erected perpendicularly to the selfe same plaine superficies: which was required to be demonstrated.

Demonstration.

of Euclides Elementes.

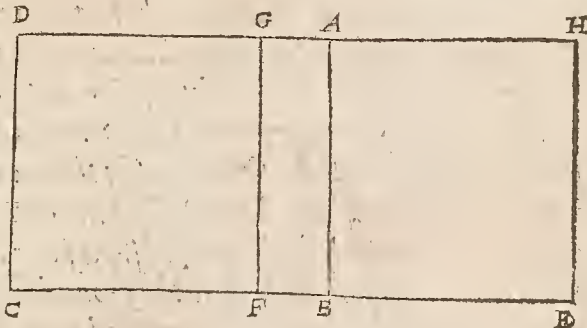
In this figure it is more easy to see the former demonstration, if ye erect perpendicularly vnto the ground superficies $A C B D$, the three superficieses, GH , KL , and MN , or if ye so erect them that they be equidistant one to the other.



¶ The 16. Theoreme. The 18. Proposition.

If a right line be erected perpendicularly to a plaine superficies: all the superficieses extended by that right line, are erected perpendicularly to the selfe same plaine superficies.

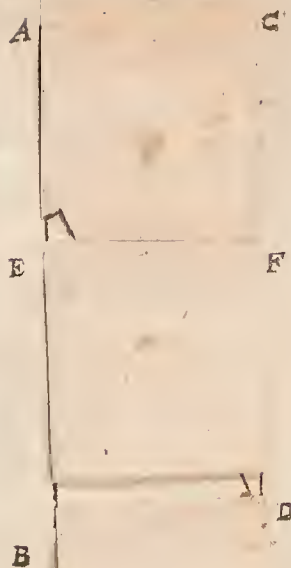
Suppose that a right line AB be erected perpendicularly to a ground superficies. I say, that all the superficieses passing by the line AB , are erected perpendicularly to the ground superficies. Extend a superficies by the line AB , and let the same be ED , & let the common section of the plaine superficies and of the ground superficies be the right line CE . And take in the line CE a point at all adventures, and let the same be F : and (by the 11. of the first) from the point F drawe vnto the line CE a perpendicular line in the superficies DE , and let the same be FG . And forasmuch as the line AB is erected perpendicularly to the ground superficies, therefore (by the 2. definition of the eleventh) the line AB is erected perpendicularly to all the right lines that are in the ground plaine superficies, and which touch it. Wherefore it is erected perpendicularly to the line CE . Wherefore the angle ABE is a right angle. And the angle GFB is also a right angle (by construction). Wherefore (by the 28. of the first) the line AB is a parallel to the line FG . But the line AB is erected perpendicularly to the ground superficies: wherefore (by the 8. of the eleventh) the line FG is also erected perpendicularly to the ground superficies. And forasmuch as (by the 3. definition of the eleventh) a plaine superficies is then erected perpendicularly to a plaine superficies, when all the right lines drawn in one of the plaine superficieses vnto the common section of those two plaine superficieses making therewith right angles, do also make right angles with the other plaine superficies: and it is proued that the line FG drawn in one of the plaine superficieses, namely, in DE , perpendicularly to the common section of the plaine superficieses, namely, to the line CE , is erected perpendicularly to the ground superficies: wherefore the plaine superficies DE is erected perpendicularly to the ground superficies. In like sort also may we proue, that all the plaine superficieses which passe by the line AB , are erected perpendicularly to the ground superficies. If therefore a right line be erected perpendicularly to a plaine superficies all the superficieses passing by the right line, are erected perpendicularly to the selfe same plaine superficies: which was required to be demonstrated.



Construction.

Demonstration.

In this figure it is more easy to see the former demonstration, if ye erect perpendicularly vnto the ground superficies $A C B D$, the three superficieses, $GH, K L$, and $M N$, or if ye so erect them that they be equidistant one to the other.

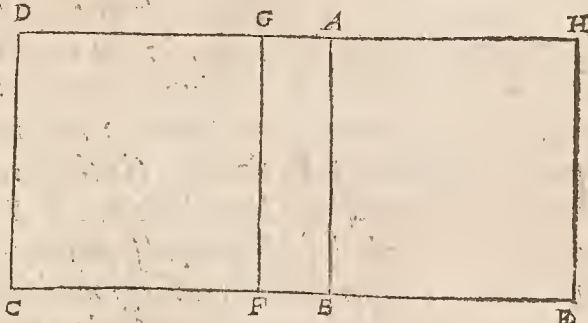


¶ The 16. Theoreme. The 18. Proposition.

If a right line be erected perpendicularly to a plaine superficies: all the superficieses extended by that right line, are erected perpendicularly to the selfe same plaine superficies.

Suppose that a right line AB be erected perpendicularly to a ground superficies. I say, that all the superficieses passing by the line AB , are erected perpendicularly to the ground superficies. Extend a superficies by the line AB , and let the same be ED , & let the common section of the plaine superficies and of the ground superficies be D .

the right line CE . And take in the line CE a point at all adventures, and let the same be F : and (by the 11. of the first) from the point F drawe vnto the line CE a perpendicular line in the superficies DE , and let the same be FG . And forasmuch as the line AB is erected perpendicularly to the ground superficies, therefore (by the 2. definition of the eleventh) the line AB is erected perpendicularly to all the right lines that are in the ground plaine superficies, and which touch it. Wherefore it is erected perpendicularly to the line CE . Wherefore the angle ABE is a right angle. And the angle GFB is also a right angle (by construction). Wherefore (by the 28. of the first) the line AB is a parallel to the line FG . But the line AB is erected perpendicularly to the ground superficies: wherefore (by the 8. of the eleventh) the line FG is also erected perpendicularly to the ground superficies. And forasmuch as (by the 3. definition of the eleventh) a plaine superficies is then erected perpendicularly to a plaine superficies, when all the right lines drawn in one of the plaine superficieses vnto the common section of those two plaine superficieses making therewith right angles, do also make right angles with the other plaine superficies: and it is proued that the line FG drawn in one of the plaine superficieses, namely, in DE , perpendicularly to the common section of the plaine superficieses, namely, to the line CE , is erected perpendicularly to the ground superficies: wherefore the plaine superficies DE is erected perpendicularly to the ground superficies. In like sort also may we proue, that all the plaine superficieses which passe by the line AB , are erected perpendicularly to the ground superficies. If therefore a right line be erected perpendicularly to a plaine superficies all the superficieses passing by the right line, are erected perpendicularly to the selfe same plaine superficies: which was required to be demonstrated.

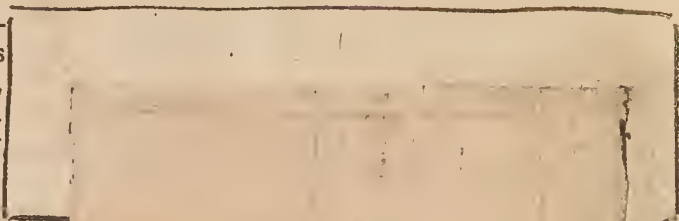


Construction.

Demonstration.

The eleventh Booke

In this figure here set ye may erect per-
pédiculary at your pleasure the superficies
wherin are drawn the lines DC , GF , AB ,
and HE , to the ground superficies wherein
is drawn the line $CFBE$, and so plainly
compare it with the demonstration before
put.

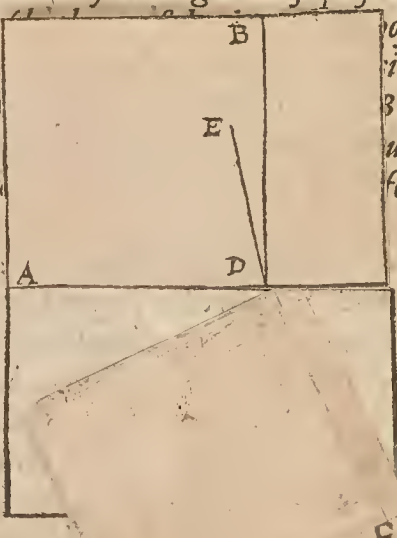
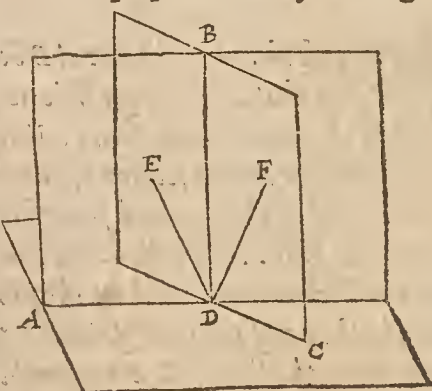


The 17. Theoreme.

If two plaine superficies cutting the one the other be erected perpendicu-
larly to any plaine superficies: their common section is also erected perpen-
dicularly to the selfe same plaine superficies.

Demonstrati-
on leading to
an impossibi-
lité.

Suppose that these two plaine superficies AB & BC cutting the one the other be e-
rected perpendicularly to a ground superficies, and let their common section be the
line BD . Then I say, that the line BD is erected perpendicularly to the ground
superficies. For if not, then (by the 11. of the first)
from the point D draw in the superficies AB unto
the right line DA a perpendicular line DE . And
in the superficies CB draw unto the line DC a per-
pendicular line DF . And forasmuch as the super-
ficies AB is erected perpendicularly to the ground
superficies, and in the plaine superficies AB unto the
common section of the plaine superficies and of the
ground superficies, namely, to the line DA is erected
a perpendicular line DE , therefore (by the converse
of the 3. definition of this booke) the line DE is e-
rected perpendicularly to the ground superficies. And in like sort may we proue, that the line
 DF is erected perpendicularly to the ground superficies. Wherefore from one and the selfe
same point, namely, from D , are erected perpendicularly to the ground superficies two right
lines both on one and the self same side: which is impossible. Where-
fore from the point D can not be erected perpendi-
cularly to the ground superficies any other
right lines besides BD , which is the common section
of the superficies AB and BC . If
therefore two plaine superficies cutting the one
plaine superficies, their common section is also er-
ected perpendicularly to the selfe same plaine
superficies: which was required to be proued.



Here haue I set an other figure which
will more plainly shewe vnto you the for-
mer demonstration, if ye erecte perpendi-
cularly to the ground superficies AC the
two superficies AB and BC which cut
the one the other in the line BD .

The 18. Theoreme.

The 20. Proposition.

If a solide angle be containned vnder three playne superficall angles: every
two

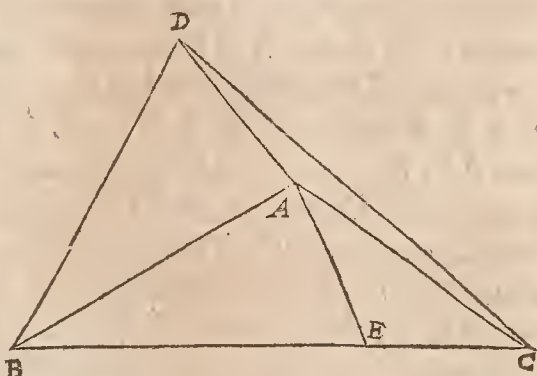
two of those three angles, which two so euer be taken, are greater then the third.



Suppose that the solide angle A be containned vnder three playne superficiall angles, that is, vnder BAC , CAD , and DAB . Then I say that two of these superficiall angles how so euer they be taken, are greater then the third. If the angles BAC , CAD , & DAB

be equall the one to the other, then is it manifest that two of them which two so euer be taken are greater then the third. But if not, let the angle BAC be the greater of the three angles. And vnto the right line AB and from the poynt A make in the playne superficies BAC vnto the angle DAB an equall angle BAE . And (by the 2. of the first) make the line AE equall to the line AD . Now a right line BE drawne by the poynt E , shall cut the right lines AB and

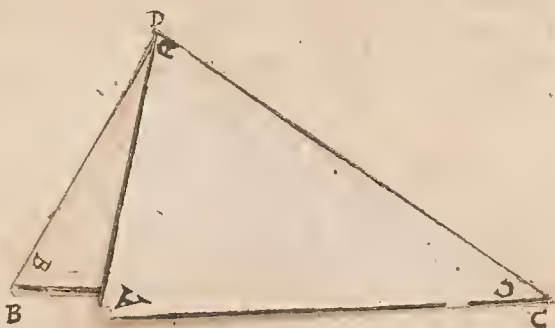
AC in the poyntes B and C : draw a right line from D to B , and an other from D to C . And forasmuch as the line DA is equall to the line AE , and the line AB is common to the both, therefore these two lines DA and AB are equall to these two lines AE and AB and the angle DAB is equall to the angle BAE . Wherefore (by the 4. of the first) the base DB is equall to the base BE . And forasmuch as these two lines DB and DC are greater then the line BC , of which the line DB is proued to be equall to the line BE . Wherefore the residue, namely, the line DC is greater then the residue, namely, then the line EC . And forasmuch as the line DA is equall to the line AE , and the line AC is common to them both, and the base DC is greater then the base EC , therefore the angle DAC is greater then the angle EAC . And it is proued that the angle DAB is equall to the angle BAE : wherefore the angles DAB and DAC are greater then the angle BAC . If therefore a solide angle be containned vnder three playne superficiall angles euery two of those three angles, which two so euer be taken are greater then the third: which was required to be proued.



26
Construction.

Demonstration.

In this figure ye may playnely behold the former demonstration, if ye eleuate the three triangles ABD , ABC and ACD in such sort that they may all meete together in the poynt A .



The 19. Theorcme.

The 21. Proposition.

Euery solide angle is comprehended vnder playne angles lesse then fower right angles.



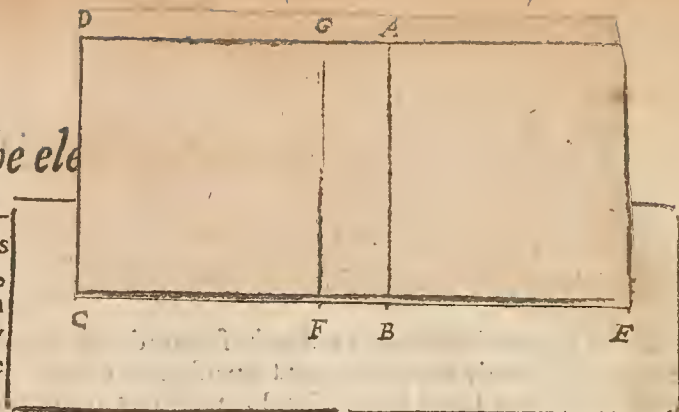
Suppose that A be a solide angle containned vnder these superficiall angles BAC , DAC and DAB . Then I say that the angles BAC , DAC and DAB are lesse then fower right angles. Take in euery one of these right lines AC , AB and

AD

Construction

The ele

In this figure here set ye may erect per-
pédiculary at your pleasure the superficies
wherin are drawn the lines DC, GF, AB ,
and HE , to the ground superficies wherin
is drawn the line $CFBE$, and so plainly
compare it with the demonstration before
put.



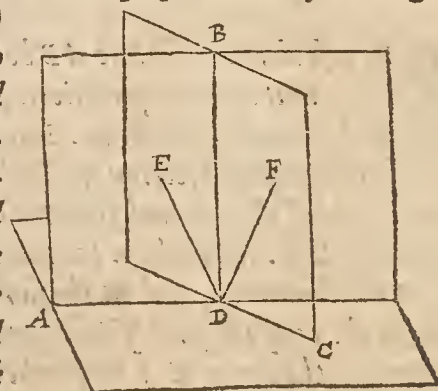
The 17. Theoreme.

The 19. Proposition.

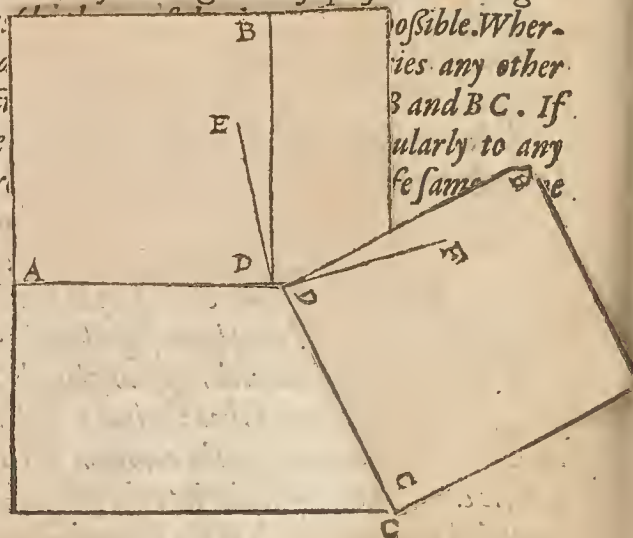
If two plaine superficiees cutting the one the other be erected perpendicu-
larly to any plaine superficies: their common section is also erected perpen-
dicularly to the selfe same plaine superficies.

Demonstrati-
on leading to
an impossibi-
lité.

Suppose that these two plaine superficiees AB & BC cutting the one the other be e-
rected perpendicularly to a ground superficies, and let their common section be the
line BD . Then I say, that the line BD is erected perpendicularly to the ground
superficies. For if not, then (by the 11. of the first)
from the point D draw in the superficies AB unto
the right line DA a perpendicular line DE . And
in the superficies CB draw unto the line DC a per-
pendicular line DF . And forasmuch as the super-
ficies AB is erected perpendicularly to the ground
superficies, and in the plaine superficies AB unto the
common section of the plaine superficies and of the
ground superficies, namely, to the line DA is erected
a perpendicular line DE , therefore (by the conuerse
of the 3. definition of this booke) the line DE is e-
rected perpendicularly to the ground superficies. And in like sort may we proue, that the line
 DF is erected perpendicularly to the ground superficies. Wherefore from one and the selfe
same point, namely, from D , are erected perpendicularly to the ground superficies two right
lines both on one and the self same side: which is impossible. Where-
fore from the point D can not be erected perpendi-
cularly to the ground superficies any other
right lines besides BD , which is the common section
of the two plaine superficies. Therefore two plaine superficiees cutting the one
the other in a line, their common section is also erected perpendicu-
larly to any plaine superficies: which was required to be proued.



Here haue I set an other figure which
will more plainly shewe unto you the for-
mer demonstration, if ye erecte perpendi-
cularly to the ground superficies AC the
two superficiees AB and BC which cut
the one the other in the line BD .



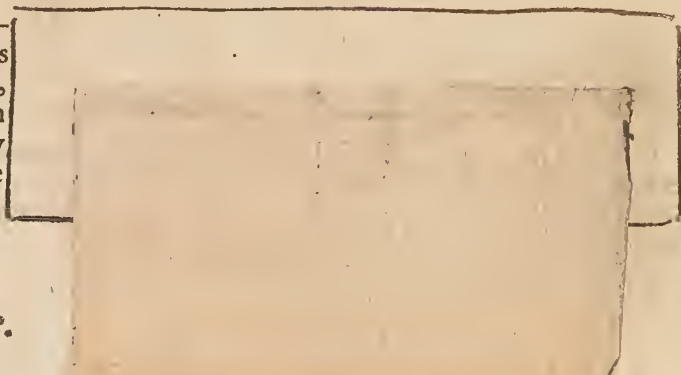
The 18. Theoreme.

The 20. Proposition.

If a solide angle be containd vnder three playne superficial angles: euery
two

The eleventh Booke

In this figure here set ye may erect perpendicularly at your pleasure the superficies wherein are drawn the lines DC , GF , AB , and HE , to the ground superficies wherein is drawn the line $C F B E$, and so plainly compare it with the demonstration before put.

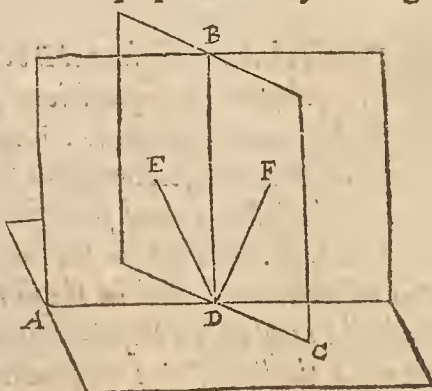


¶ The 17. Theoreme.

If two plaine superficies cutting the one the other be erected perpendicularly to any plaine superficies: their common section is also erected perpendicularly to the selfe same plaine superficies.

Demonstration leading to an impossibilitie.

Suppose that these two plaine superficies AB & BC cutting the one the other be erected perpendicularly to a ground superficies, and let their common section be the line BD . Then I say, that the line BD is erected perpendicularly to the ground superficies. For if not, then (by the 11. of the first) from the point D draw in the superficies AB unto the right line DA a perpendicular line DE . And in the superficies CB draw unto the line DC a perpendicular line DF . And forasmuch as the superficies AB is erected perpendicularly to the ground superficies, and in the plaine superficies AB unto the common section of the plaine superficies and of the ground superficies, namely, to the line DA is erected a perpendicular line DE , therefore (by the converse of the 3. definition of this booke) the line DE is erected perpendicularly to the ground superficies. And in like sort may we proue, that the line DF is erected perpendicularly to the ground superficies. Wherefore from one and the selfe same point, namely, from D , are erected perpendicularly to the ground superficies two right lines both on one and the self same side: which is (by the 15. of the eleventh) impossible. Wherefore from the point D can not be erected perpendicularly to the ground superficies any other right lines besides BD , which is the common section of the two superficieses AB and BC . If therefore two plaine superficies cutting the one the other be erected perpendicularly to any plaine superficies, their common section is also erected perpendicularly to the selfe same plaine superficies: which was required to be proued.



Here haue I set an other figure which will more plainly shewe vnto you the former demonstration, if ye erecte perpendicularly to the ground superficies AC the two superficieses AB and BC which cut the one the other in the line BD .

The 18. Theoreme.

The 20. Proposition.

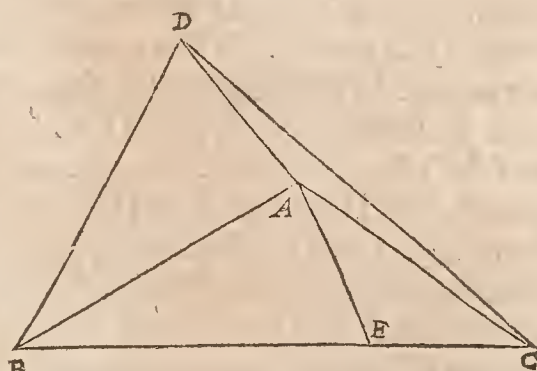
If a solide angle be containd vnder three playne superficiall angles: euery
two

two of those three angles, which two so euer be taken, are greater then the third.



Suppose that the solide angle A be containd vnder three playne superficiall angles, that is, vnder BAC, CAD, and DAB. Then I say that two of these superficiall angles how so euer they be taken, are greater then the third. If the angles BAC, CAD, & DAB

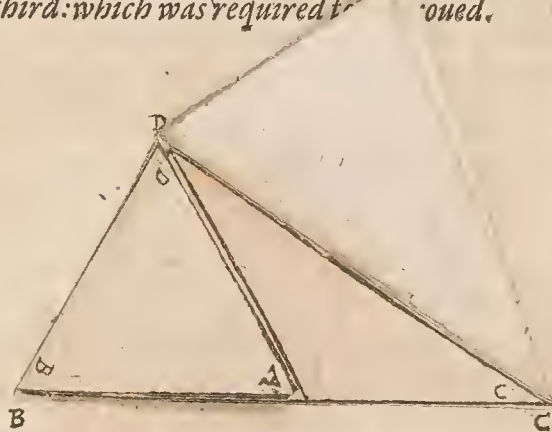
be equall the one to the other, then is it manifest that two of them which two so euer be taken are greater then the third. But if not, let the angle BAC be the greater of the three angles. And vnto the right line AB and from the poynt A make in the playne superficies BAC vnto the angle DAB an equall angle BAE. And (by the 2. of the first) make the line AE equall to the line AD. Now a right line BEC drawne by the poynt E, shall cut the right lines AB and AC in the poyntes B and C: draw a right line from D to B, and an other from D to C. And forasmuch as the line DA is equall to the line AE, and the line AB is common to the both, therefore these two lines DA and AB are equall to these two lines AB and AE and the angle DAB is equall to the angle BAE. Wherefore (by the 4. of the first) the base DB is equall to the base BE. And forasmuch as these two lines DB and DC are greater then the line BC, of which the line DB is proued to be equall to the line BE. Wherefore the residue, namely, the line DC is greater then the residue, namely, then the line EC. And forasmuch as the line DA is equall to the line AE, and the line AC is common to them both, and the base DC is greater then the base EC, therefore the angle DAC is greater then the angle EAC. And it is proued that the angle DAB is equall to the angle BAE: wherefore the angles DAB and DAC are greater then the angle BAC. If therefore a solide angle be containd vnder three playne superficiall angles euery two of those three angles, which two so euer be taken are greater then the third: which was required to be proued.



26
Construction.

Demonstration.

In this figure ye may playnely behold the former demonstration, if ye eleuate the three triangles AED, ABC and ADC in such sort that they may all meete together in the poynt A.



The 19. Theorome.

The 21. Proposition.

Euery solide angle is comprehended vnder playne angles lesse then fower right angles.



Suppose that A be a solide angle containd vnder these superficiall angles BAC, DAC and DAB. Then I say that the angles BAC, DAC and DAB are lesse then fower right angles. Take in euery one of these right lines ACAB and AD

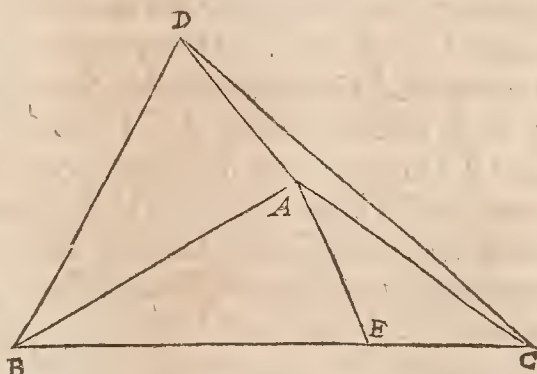
Construction

two of those three angles, which two so euer be taken, are greater then the third.



Suppose that the solide angle A be containd vnder three playne superficiall angles, that is, vnder BAC , CAD , and DAB . Then I say that two of these superficiall angles how so euer they be taken, are greater then the third. If the angles BAC , CAD , & DAB

be equall the one to the other, then is it manifest that two of them which two so euer be taken are greater then the third. But if not, let the angle BAC be the greater of the thre angles. And vnto the right line AB and from the poynt A make in the playne superficies BAC vnto the angle DAB an equall angle BAE . And (by the 2. of the first) make the line AE equall to the line AD . Now a right line BE drawne by the poynt E , shall cut the right lines AB and

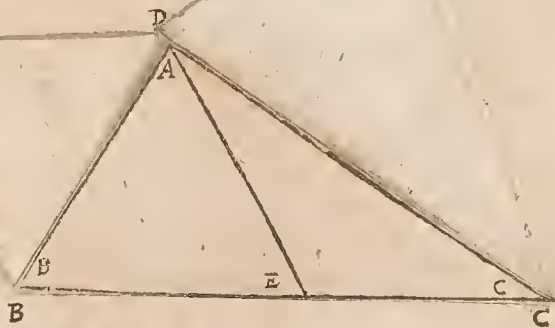


2.
Construction.

AC in the poyntes B and C : draw a right line from D to B , and an other from D to C . And forasmuch as the line DA is equall to the line AE , and the line AB is common to the both, therefore these two lines DA and AB are equall to these two lines AB and AE and the angle DAB is equall to the angle BAE . Wherefore (by the 4. of the first) the base DB is equall to the base BE . And forasmuch as these two lines DB and DC are greater then the line BC , of which the line DB is proued to be equall to the line BE . Wherefore the residue, namely, the line DC is greater then the residue, namely, then the line EC . And forasmuch as the line DA is equall to the line AE , and the line AC is common to them both, and the base DC is greater then the base EC , therefore the angle DAC is greater then the angle EAC . And it is proued that the angle DAB is equall to the angle BAE : wherefore the angles DAB and DAC are greater then the angle BAC . If therefore a solide angle be containd vnder three playne superficiall angles euery two of those three angles, which two so euer be taken are greater then the third: which was required to be proued.

Demonstration.

In this figure ye may playnely behold former demonstration, if ye eleuate the three triangles ABD , ABC and ACD in such sort that they may all meete together in the poynt A .



The 19. Theorème.

The 21. Proposition.

Euery solide angle is comprehended vnder playne angles lesse then fower right angles.



Suppose that A be a solide angle containd vnder these superficiall angles BAC , DAC and DAB . Then I say that the angles BAC , DAC and DAB are lesse then fower right angles. Take in euery one of these right lines AC AB and AD

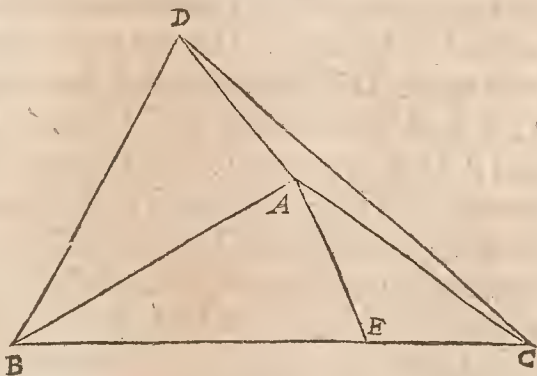
Construction

two of those three angles, which two so euer be taken, are greater then the third.



Suppose that the solide angle A be containd vnder three playne superficiall angles, that is, vnder BAC, CAD, and DAB. Then I say that two of these superficiall angles how so euer they be taken, are greater then the third. If the angles BAC, CAD, & DAB

be equall the one to the other, then is it manifest that two of them which two so euer be taken are greater then the third. But if not, let the angle BAC be the greater of the three angles. And vnto the right line AB and from the poynt A make in the playne superficies BAC vnto the angle DAB an equall angle BAE. And (by the 2. of the first) make the line AE equall to the line AD. Now a right line BEC drawne by the poynt E, shall cut the right lines AB and

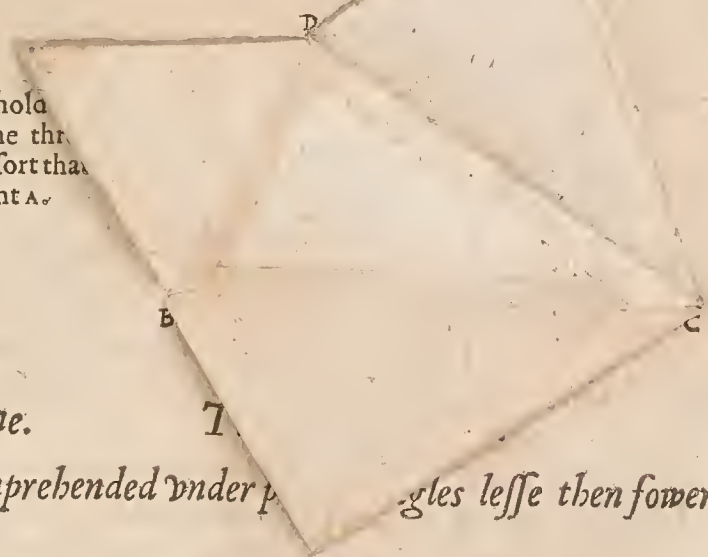


2.
Construction.

AC in the poyntes B and C: draw a right line from D to B, and an other from D to C. And forasmuch as the line DA is equall to the line AE, and the line AB is common to the both, therefore these two lines DA and AB are equall to these two lines AB and AE and the angle DAB is equall to the angle BAE. Wherefore (by the 4. of the first) the base DB is equall to the base BE. And forasmuch as these two lines DB and DC are greater then the line BC, of which the line DB is proued to be equall to the line BE. Wherefore the residue, namely, the line DC is greater then the residue, namely, then the line EC. And forasmuch as the line DA is equall to the line AE, and the line AC is common to them both, and the base DC is greater then the base EC, therefore the angle DAC is greater then the angle EAC. And it is proued that the angle DAB is equall to the angle BAE: wherefore the angles DAB and DAC are greater then the angle BAC. If therefore a solide angle be containd vnder three playne superficiall angles euery two of those three angles, which two so euer be taken are greater then the third: which was required to be proued.

Demonstration.

In this figure ye may playnely behold former demonstration, if ye eleuate the three triangles ABD, ABC and ACD in such sort that they may all meete together in the poynt A.



The 19. Theorème.

1

Euery solide angle is comprehended vnder p... angles lesse then fower right angles.



Suppose that A be a solide angle containd vnder these superficiall angles BAC, DAC and DAB. Then I say that the angles BAC, DAC and DAB are lesse then fower right angles. Take in euery one of these right lines AC AB and

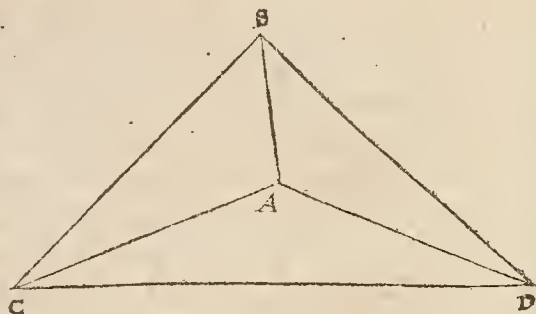
Construction

AD

The eleuenth Booke

Demonstration.

At a point at all adventures and let the same be B, C, D. And draw these right lines BC, CD and DB. And forasmuch as the angle B is a solide angle, for it is contained under three superficiall angles, that is, under CBA, ABD and CBD, therefore (by the 20. of the eleuenth) two of them which two so euer be taken are greater then the third. Wherefore the angles CBA and ABD are greater then the angle CBD: and by the same reason the angles BCA and ACD are greater then the angle BCD: and moreover the angles CDA and ADB are greater then the angle CDB. Wherefore these sixe angles CBA, ABD, BCA, ACD, CDA, and ADB are greater then these thre angles, namely, CBD, BCD, & CDB. But the three angles CBD, BDC, and BCD are equall to two right angles. Wherefore the sixe angles CBA, ABD, BCA, ACD, CDA, and ADB are greater then two right angles. And forasmuch as in euery one of these triangles ABC, and ABD and ACD three angles are equall two right angles (by the 32. of the first). Wherefore the nine angles of the thre triangles, that is, the angles CBA, ACB, BAC, ACD, DAC, CDA, ADB, DBA and BAD are equall to sixe right angles. Of which angles the sixe angles ABC, BCA, ACD, CDA, ADB and DBA are greater then two right angles. Wherefore the angles remayning, namely, the angles BAC, CAD and DAB which containe the solide angle are lesse then fower right angles. Wherefore euery solide angle is comprehended under playne angles lesse then fower right angles: which was required to be proued.



If ye will more fully see this demonstration compare it with the figure which I put for the better sight of the demonstration of the proposition next going before. Onely here is not required the draught of the line AE.

Although this demonstration of *Euclide* be here put for solide angles contained vnder three superficiall angles, yet after the like maner may you proceede if the solide angle be contained vnder superficiall angles how many so euer. As for example if it be contained vnder fower superficiall angles, if ye follow the former construction, the base will be a quadrangled figure, whose fower angles are equall to fower right angles: but the 8. angles at the bases of the 4. triangles set vpon this quadrangled figure may by the 20. proposition of this booke be proued to be greater then those 4. angles of the quadrangled figure: As we sawe by the discourse of the former demonstration. Wherefore those 8. angles are greater then fower right angles: but the 12. angles of those fower triangles are equall to 8. right angles. Wherefore the fower angles remayning at the toppe which make the solide angle are lesse then fower right angles. And obseruing this course ye may proceede infinitely.

¶ The 20. Theoreme.

The 22. Proposition.

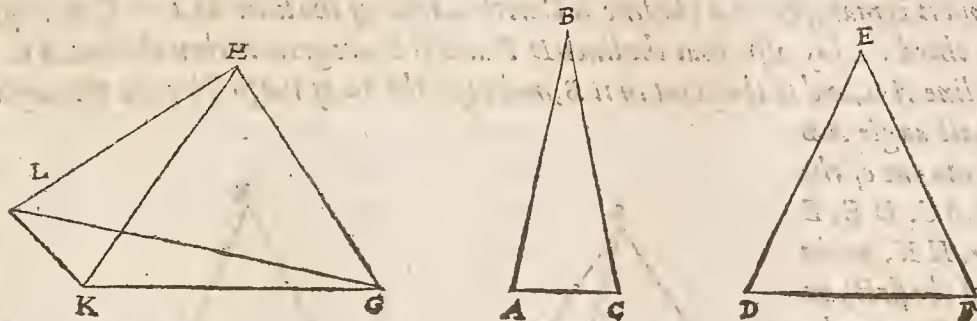
If there be three superficiall plaine angles of which two how soeuer they be taken, be greater then the third, and if the right lines also which containe those angles be equall: then of the lines coupling those equall right lines together, it is possible to make a triangle.



Suppose that there be thre superficial angles ABC, DEF, and GHK, of which let two, which two soeuer be taken, be greater then the third, that is, let the angles ABC, and DEF be greater then the angle GHK, and let the angles DEF and GHK be greater then the angle ABC: and moreover let the angles GHK and ABC be greater then the angle DEF. And let the right lines AB, BC, DE, EF, GH, and HK be equall the one to the other, and draw a right line from the point A to the point

point C, and an other from the point D to the point F, and moreover an other from the point G to the point K. Then I say that it is possible of three right lines equall to the lines AC, D F,

Two cases in
this proposi-
tion.



and G K, to make a triangle, that is, that two of the right lynes AC, D F, and G K, which two soeuer be taken, are greater then the third. Now if the angles ABC, D E F, and G H K be equall the one to the other, it is manifest that these right lines AC, D F, and G K being also (by the 4. of the first) equall the one to the other, it is possible of three right lines equall to the lines AC, D F, and G K to make a triangle. But if they be not equall, let them be vnequall. And (by the 23. of the first) vnto the right line H K, and at the point in it H, make vnto the angle ABC an equall angle K H L. And by the 2. of the first) to one of the lines A B, B C, D E, E F, G H, or H K make the line H L equal, & draw these right lines K L and G L. And forasmuch as these two lines A B and B C, are equall to these two lines K H and H L, and the angle B is equall to the angle K H L, therefore (by the 4. of the first) the base A C is equall to the base K L. And forasmuch as the angles A B C, and G H K are greater then the angle D E F, but the angle G H L is equall to the angles A B C, & G H K: therefore the angle G H L is greater then the angle D E F. And forasmuch as these two lines G H and H L are equall to these two lines D E and E F, and the angle G H L is greater then the angle D E F, therefore (by the 25. of the first) the base G L is greater the the base D F. But the lines G K and K L are greater then the line G L. Wherefore the lines G K & K L are much greater then the line D F. But the line K L is equal to the line A C. Wherefore the lines A C and G K are greater then the line D F. In like sort also may we proue that the lines A C and D F are greater then the line G K, and that the lines G K and D F are greater then the lynce A C. Wherefore it is possible to make a triangle of three lynes equall to the lines AC, D F, and G K: which was required to be demonstrated.

The first case.

Second case.
Construction.

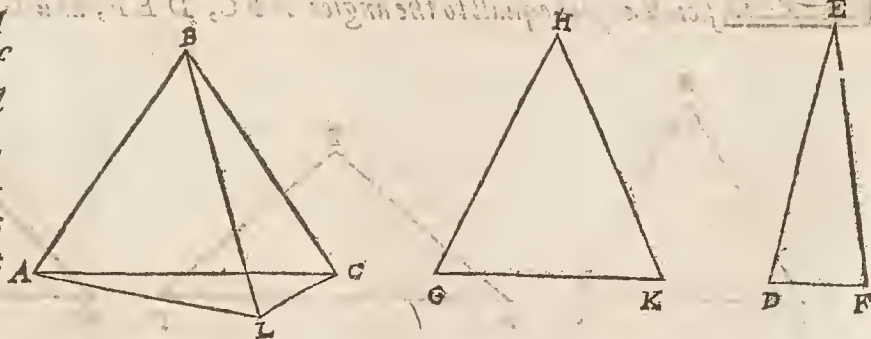
Demonstra-
tion.

An other demonstration.

Suppose that the three superficial angles be ABC, D E F, and G H K, of which angles two howsoeuer they be taken, are greater then the third. And let them be contained under these equall right lines A B, B C, D E, E F, G H, H K, which equall right lines let these lines A C, D F, and G K

An other de-
monstration.

ioyne together. Then I say that it is possible of three right lines equal to the lines AC, D F, and G K to make a triangle, which againe is as much to say, as that two of those lines which two soeuer be ta-



ken, are greater then the third. Now againe if the angles B, E, H, be equall, the lines also A C
XX.i. D F,

The eleuenth Booke

DF , and GK are equall, and so two of them shall be greater then the third. But if not, let the angles B, E, H , be vnequall, and let the angle B be greater then either of the angles E and H . Therefore (by the 24. of the first) the right line AC is greater then either of the lines DF & GK . And it is manifest that the line AC with either of the lines DF or GK is greater then the third. I say also that the lines DF and GK are greater then the line AC . Vnto the right line AB , and to the point in it B , make (by the 22. of the first) vnto the angle GH

Construction.

K an equall angle ABL , and vnto one of the lines AB, BC, DE, EF, GH or HK , make by the 2. of the first) an equall line BL . And draw a right line from the point A to y point L , and an other fro the point L to the point C .

Demonstration.

And forasmuch as these two lines AB & BL are equal to these two lines GH & HK the one to the other, and they containe equal angles: therefore (by the 4. of the first) the base AL is equall to the base GK . And forasmuch as the angles E and H be greater then the angle ABC , of which the angle GHK is equal to the angle ABL , therefore the angle remayning, namely, the angle E is greater then the angle LCB . And forasmuch as these two lines LB and BC , are equall to these two lines DE and EF the one to the other, and the angle DEF is greater then the angle LCB , therefore (by the 25. of the first) the base DF is greater then the base LC ; and it is proued that the line GK is equall to the line AL . Wherefore the lines DF & GK are greater then the lines AL & LC . But the lines AL and LC are greater then the line AC . Wherefore the lines DF & GK are much greater the the line AC . Wherefore two of these right lines AC, DF & GK which two soeuer be taken are greater then the third. Wherefore it is possible of three right lines equall to the lines AC, DF and GK to make a triangle: which was required to be demonstrated.

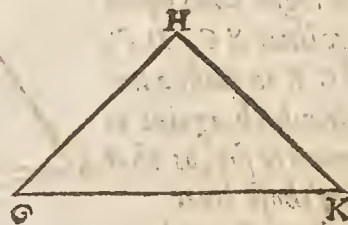
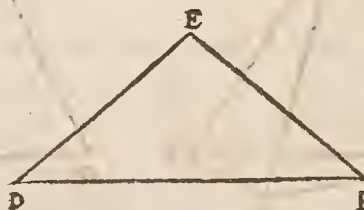
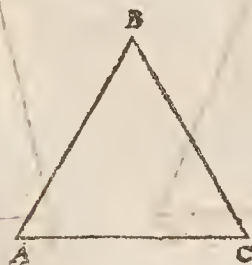
¶ The 3. Probleme.

The 23. Proposition.

Of three plaine superficiall angles, two of which how soeuer they be taken, are greater then the third, to make a solide angle: Now it is necessary that those three superficiall angles be lesse then fower right angles.



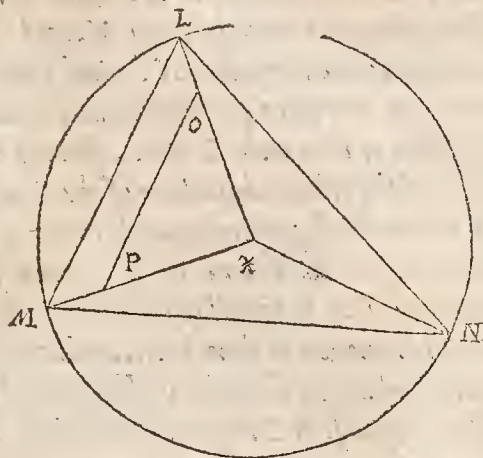
Suppose that the superficiall angles geuen be ABC, DEF, GHK : of which let two how soeuer they be taken, be greater then the third: and moreover, let those three angles be lesse then fower right angles. It is required of three superficiall angles equall to the angles ABC, DEF , and GHK , to make a solide



or bodily angle. Let the lines AB, BC, DE, EF, GH , and HK , be made equall: and drawe a right line from the point A to the point C , & an other from the point D to the point F , and an other from the point G to the point K . Now (by the 22. of the eleuenth) it is possible of three right lines equall to the right lines AC, DF , and GK , to make a triangle.

Construction.

Make such a triangle, and let the same be LMN , so that let the line AC be equall to the line LM , and the line DF to the line MN , and the line GK to the line LN . And (by the 5. of the fourth) about the triangle LMN describe a circle LMN , and take (by the 1. of the third) the centre of the same circle, which centre shall either be within the triangle LMN , or in one of the sides thereof, or without it. First let it be within the triangle, and let the same be the point X , & drawe these right lines LX, MX , and NX . Now I say, that the line AB is greater then the line LX . For if not, then the line AB is either equall to the line LX , or els it is lesse then it. First let it be equall. And forasmuch as



Three cases in this proposition.

The first case.

A necessary thing to be proued before he proceede any farther in the construction of the Probleme.

the line AB is equall to the line LX , but the line AB is equall to the line BC , therefore the line LX is equall to the line BC : and vnto the line LX the line XM is (by the 15. definition of the first) equall: wherefore these two lines AB and BC , are equall to these two lines LX and XM the one to the other: and the base AC is supposed to be equall to the base LM . Wherefore (by the 8. of the first) the angle ABC is equall to the angle LXM . And by the same reason also the angle DEF is equall to the angle MXN , and moreover the angle GHK to the angle NXL . Wherefore these three angles ABC, DEF , and GHK , are equall to these three angles LXM, MXN , & NXL . But the three angles LXM, MXN , and NXL , are equall to fower right angles (as it is manifest to see by the 13. of the first, if any one of these lines MX, LX , or NX , be extended on the side that the point X is). Wherefore the three angles ABC, DEF , and GHK , are also equall to fower right angles. But they are supposed to be lesse then fower right angles: which is impossible. Wherefore the line AB is not equall to the line LX . I say also that the line AB is not lesse then the line LX . For if it be possible, let it be lesse: and (by the 2. of the first) vnto the line AB put an equall line XO : and to the line BC put an equall line XP , and draw a right line from the point O to the point P . And forasmuch as the line AB is equall to the line BC , therefore also the line XO is equall to the line XP . Wherefore the residue OL is equall to the residue MP . Wherefore (by the 2. of the sixt) the line LM is a parallel to the line OP : and the triangle LMX is equiangle to the triangle OPX . Wherefore as the line XL is to the line LM , so is the line XO to the line OP . Wherefore alternately (by the 16. of the fift) as the line LX is to the line XO , so is the line LM to the line OP . But the line LX is greater then the line XO . Wherefore also the line LM is greater then the line OP . But the line LM is put to be equall to the line AC : wherefore also the line AC is greater then the line OP . Now forasmuch as these two right lines AB and BC are equall to these two right lines OX and XP , and the base AC is greater then the base OP , therefore (by the 25. of the first) the angle ABC is greater then the angle OPX . In like sort also may we proue, that the angle DEF is greater then the angle MXN , and that the angle GHK is greater then the angle NXL . Wherefore the three angles ABC, DEF , and GHK , are greater then the three angles LXM, MXN , and NXL . But the angles ABC, DEF , and GHK , are supposed to be lesse then fower right angles: wherefore much more are the angles LXM, MXN , & NXL lesse then fower right angles. But they are also equall to fower right angles: which is impossible. Wherefore the line AB is not lesse then the line LX : and it is also proued that it is not equall vnto it. Wherefore

XX. ij.

the

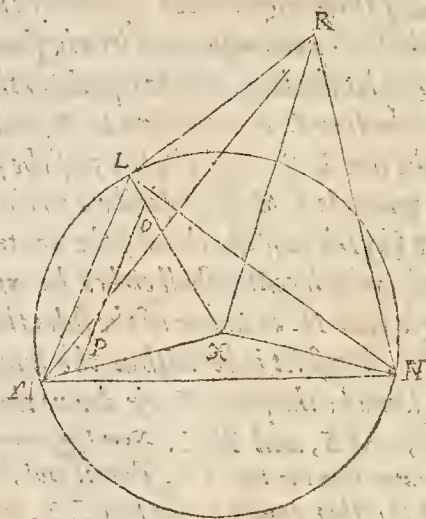
The eleventh Booke

the line AB is greater then the line LX .

* Which how to finde out is taught at the end of this demonstration, and also was taught in the assumpt put before the 14. proposition of the tenth booke.

Demonstration of the first case.

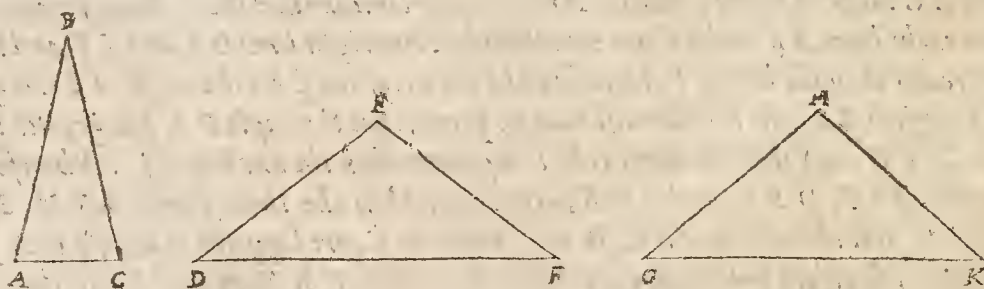
Now from the point X raise up vnto the plaine superficies of the circle LMN a perpendicular line XR (by the 12. of the eleventh). And vnto that which the square of the line AB exceedeth the square of the line LX * let the square of the line XR be equall. And draw a right line from the point R to the point L , and an other from the point R to the point M , and an other from the point R to the point N . And forasmuch as the line RX is erected perpendicularly to the plaine superficies of the circle LMN , therefore (by conuersion of the second definition of the eleventh) the line RX is erected perpendicularly to euery one of these lines LX , MX , and NX . And forasmuch as the line LX is equall to the line XM , & the line XR is common to them both, and is also erected perpendicularly to them both, therefore (by the 4. of the first) the base RL is equall to the base RM . And by the same reason also the line RN is equall to either of these lines RL and RM . Wherefore these three lines RL , RM , and RN , are equall the one to the other. And forasmuch as vnto that which the square of the line AB exceedeth the square of the line LX , the square of the line XR is supposed to be equall, therefore the square of the line AB is equall to the squares of the lines LX and XR . But vnto the squares of the lines LX and XR , the square of the line LR is (by the 47. of the first) equall, for the angle LXR is a right angle. Wherefore the square of the line AB is equall to the square of the line RL . Wherefore also the line AB is equall to the line RL . But vnto the line AB is equall euery one of these lines BC , DE , EF , GH , and HK , and vnto the line RL is equall either of these lines RM and RN . Wherefore euery one of these lines AB , BC , DE , EF , GH , and HK , is equall to euery one of these lines RL , RM , and RN . And forasmuch as these two lines RL and RM are equall to these two lines AB and BC , and the base LM is supposed to be equall to the base AC , therefore (by the 8. of the first) the angle LRM is equall to the angle ABC . And by the same reason also the MRN is equall to the angle DEF , and the angle LRN to the angle GHK . Wherefore of three superficial angles LRM , MRN , and LRN , which are equall to three superficial angles geue, namely, to the angles ABC , DEF , & GHK , is made a solide angle R , comprehended vnder the superficial angles LRM , MRN , and LRN ; which was required to be done.



Second case.

But now let the centre of the circle be in one of the sides of the triangle, let it be in the side MN , and let the centre be X . And draw a right line from the point L to the point X . I say againe, that the line AB is greater then the line LX . For if not, then AB is either equall to

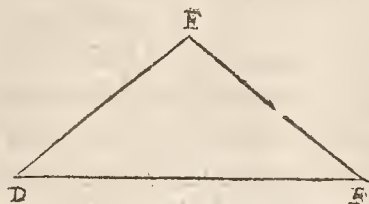
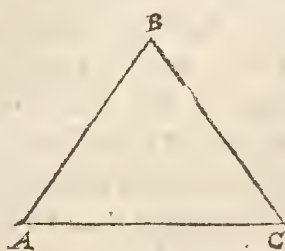
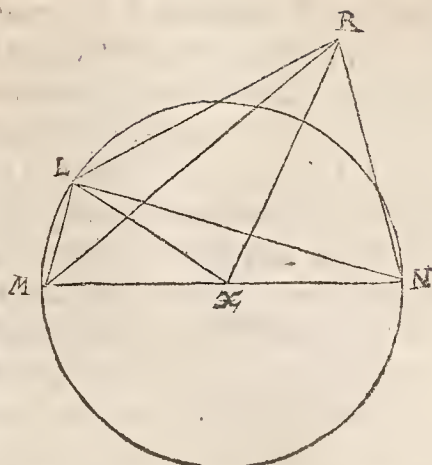
LX , or els it is lesse then it. First let it be equall. Now the these two lines AB and



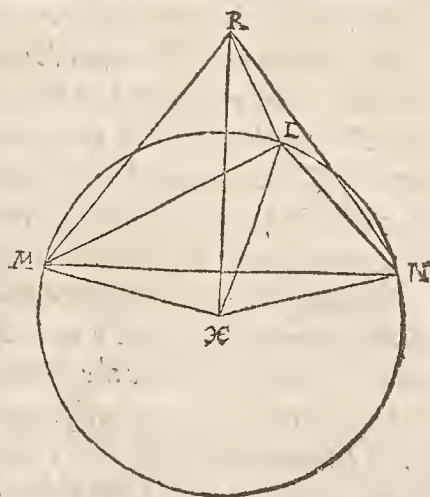
BC ,

BC , that is, DE & EF are equall to these two lines MX and XL , that is, to the line MN . But the line MN is supposed to be equall to the line DF . Wherefore also the lines DE and EF are equall to the line DF : which (by the 20. of the first) is impossible. Wherefore the line AB is not equall to the line LX . In like sort also may we proue, that it is not lesse. Wherefore the line AB is greater then the line LX . And now if as before vnto the plaine superficies of the circle be erected frō the point X a perpendicular line RX whose square let be equall vnto that which the square of the line AB exccedeth the square of the line LX , and if the rest of the construction and demonstration be obserued in this that was in the former case, then shall the Probleme be finished.

But now let the centre of the circle be without the triangle LMN , and let it be in the point X . And draw these right lines LX , MX , and NX . I say that in this case also the line AB is greater then the line LX . For if not, then is it either equall or lesse. First let it be e-



quall. Wherefore these two lines AB and BC are equall to these two lines MX and XL the one to the other, and the base AC is equall to the base ML . Wherefore (by the 8. of the first) the angle ABC is equall to the angle MXL : and by the same reason also the angle $G HK$ is equall to the angle LXN . Wherefore the whole angle MXN is equall to these two angles ABC and $G HK$. But the angles ABC and $G HK$ are greater then the angle DEF . Wherefore the angle MXN is greater then the angle DEF . And forasmuch as these two lines DE and EF are equall to these two lines MX and XN , and the base DF is equall to the base MN , therefore (by the 8. of the first) the angle MXN is equall to the angle DEF . And it is proued that it is also greater: which is impossible. Wherefore the line AB is not equall to the line LX . In like sort also may we proue, that it is not lesse. Wherefore the line AB is greater then the line LX .



And againe if vnto the plaine superficies of the circle be erected perpendicularly from the point X a line XR , whose square is equall to that which the square of the line AB exceedeth the square of the line LX , and the rest of the construction be done in this that was in the former cases, then shall the Probleme be finished.

I say moreouer, that the line AB is not lesse then the line LX . For if it be possible, let it be lesse. And vnto the line AB , put (by the 2. of the first) the line XO equall: and vnto the line BC put the line XP equall: And draw a right line from the point O to the point P .

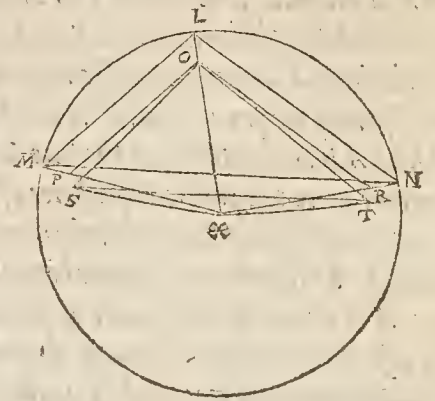
XX. iij.

An other demonstration to proue that the line AB is not lesse then the line LX .

And

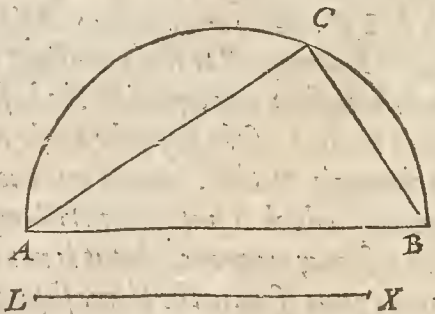
The eleventh Booke

And forasmuch as the line AB is equall to the line BC , therefore the line XO is equall to the line XP . Wherefore the residue OL is equall to the residue MP . Wherefore (by the 2. of the sixth) the line LM , is a parallel to the line PO . And the triangle LXM is equiangle to the triangle PXO . Wherefore (by the 6. of the sixth) as the line LX is to the line LM , so is the line XO to the line PO . Wherefore alternately (by the 16. of the fifth) as the line LX is to the line XO , so is the line LM to the line OP . But the line LX is greater then the line XO . Wherefore also the line LM is greater then the line OP . But the line LM is equall to the line AC . Wherefore also the line AC is greater then the line OP . Now forasmuch as these two lines AB and BC are equall to these two lines OX & XP the one to the other, and the base AC is greater then the base OP , therefore (by the 25. of the first) the angle ABC is greater then the angle OXP . And in like sort if we put the line XR equall to either of these lines XO or XP , and draw a right line from the point O to the point R , we may proue that the angle $G HK$ is greater then the angle OXR . Vnto the right line LX , and vnto the point in it X , make (by the 23. of the first) vnto the angle ABC an equall angle LXS : and vnto the angle $G HK$ make an equall angle LXT . And (by the second of the first) let either of these lines SX , and XT , be equall to the line OX . And draw these lines OS , OT , and ST . And forasmuch as these two lines AB and BC are equall to these two lines OX and XS , and the angle ABC is equall to the angle $OX S$, therefore (by the 4. of the first) the base AC , that is, LM , is equall to the base OS . And by the same reason also the line LN is equall to the line OT . And forasmuch as these two lines LM and LN are equall to these two lines SO and OT , and the angle MLN is greater then the angle SOT , therefore (by the 25. of the first) the base MN is greater then the base ST . But the line MN is equall to the line DF . Wherefore the line DF is greater then the line ST . Now forasmuch as these two lines DE and EF are equall to these two lines SX and XT , and the base DF is greater then the base ST , therefore (by the 25. of the first) the angle DEF is greater then the angle SXT . But the angle SXT is equall to the angles ABC and $G HK$. Wherefore also the angle DEF is greater then the angles ABC and $G HK$. But it is also lesse: which is impossible.

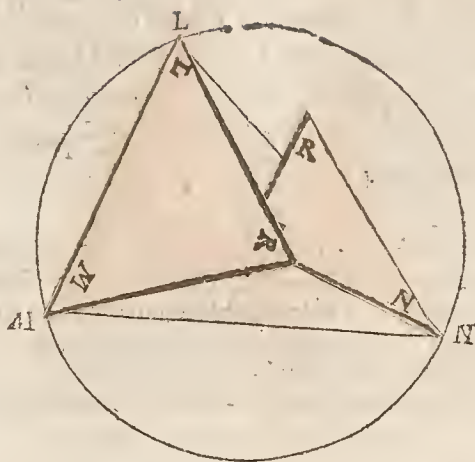


This was before taught in the tenth booke in the assumpt put before the 14. proposition.

But now let vs declare how to finde out the line XR , whose square shall be equall to that which the square of the line AB exceedeth the square of the line LX . Take the two right lines AB and LX , and let AB be the greater, and vpon AB describe a semicircle ACB , and from the poynt A apply into the semicircle a right line AC equall to the right line LX : and draw a right line from the poynt C to the poynt B . And forasmuch as in the semicircle ACB is an angle ACB , therefore (by the 31. of the third) the angle ACB is a right angle. Wherefore (by the 47. of the first) the square of the line AB is equall to the squares of the lines AC and CB : wherefore the square of the line AB is in power more then the square of the line AC by the square of the line CB : but the line AC is equall to the line LX , wherefore the square of the line AB is in power more then the square of the line LX by the square of the line CB . If therefore vnto the line CB we make the line XR equall, then is the square of the line AB greater then the square of the line LX by the square of the line XR : which was required to be doone.



In this figure may ye more fully see the construction and demonstration of the first case of the former 23. Proposition, if ye erect perpendicularly the triangle $x R N$, and vnto it bend the triangle $L M R$, that the angles R of eche may ioine together in the point R . And so fully vnderstanding this case, the other cases will not be hard to conceaue.



¶ The 21. Theoreme.

If a solide or body be containd vnder * sixe parallel playne superficieses, the opposite plaine superficieses of the same body are equall and parallelogrammes.

The 24. Proposition.

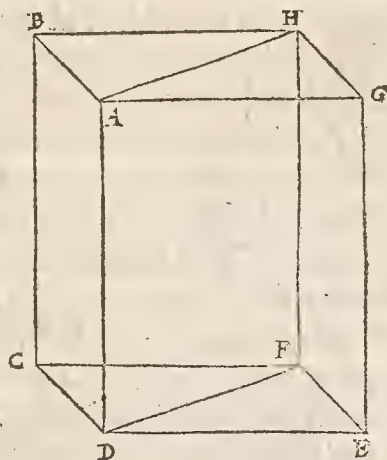
* M. Dec (to auoide canillation) addeth to Euclides proposition this worde sixe: whome I haue followed accordingly, and not Zamberts, in this.

This kinde of body mencioned in the proposition is called a Parallelipiped according to the diffinition before geuen thereof.



¶ Suppose that this solide body $C D H G$ be contained vnder these 6. parallel plaine superficieses, namely, $A C, G F, B G, C E, F B$, and $A E$. Then I say that the opposite superficieses of the same body, are equal and parallelogrammes, it is to wete,

the two opposites $A C$ and $G F$, and the two opposites $B G$ and $C E$, and the two opposites $F B$ and $A E$ to be equall, and al to be parallelogrammes. For forasmuch as two parallel plaine superficieses, that is, $B G$, and $C E$ are deni- ded by the plaine superficies $A C$, their common sections are (by the 16. of the eleuenth) parallels. Wherefore the line $A B$ is a parallel to the line $C D$. Again forasmuch as two parallel plaine superficieses $F B$ and $A E$ are deni- ded by the plaine superficies $A C$ their common sections are by the same proposition, parallels. Wherefore the lyne $A D$ is a parallel to the line $B C$. And it is also proued, that the line $A B$ is a parallel to the line $D C$. Wherefore the superficies $A C$ is a parallelogramme. In like sort also may we proue, that euery one of these superficieses $C E, G F, B G, F B$, and $A E$ are parallelo- grammes. Draw a right line from the point A , to the point H , and an other from the point D to the point F . And forasmuch as the line $A B$ is proued a parallel to the line $C D$, and the lyne $B H$ to the line $C F$, therefore these two right lines $A B$ and $B H$ touching the one the other, are parallels to these two right lines $D C$ and $C F$ touching also the one the other, and not being in one and the selfe same plaine superficies. Wherefore (by the 10. of the eleuenth) they compre- hend equall angles. Wherefore the angle $A B H$ is equall to the angle $D C F$. And forasmuch as these two lines $A B$ and $B H$ are * equall to these two lines $D C$ and $C F$, and the angle $A B H$ is proued equall to the angle $D C F$, therefore (by the 4. of the first) the base $A H$ is equall to the base $D F$, and the triangle $A B H$ is e- quall to the triangle $D C F$. And forasmuch as (by the 41. of the first) the parallelogramme $B G$ is double to the triangle $A B H$, and the parallelo- gramme $C E$ is also double to the triangle $D C F$, therefore the parallelo-



Demonstra-
tion that the
opposite sides
are parallelo-
grammes.

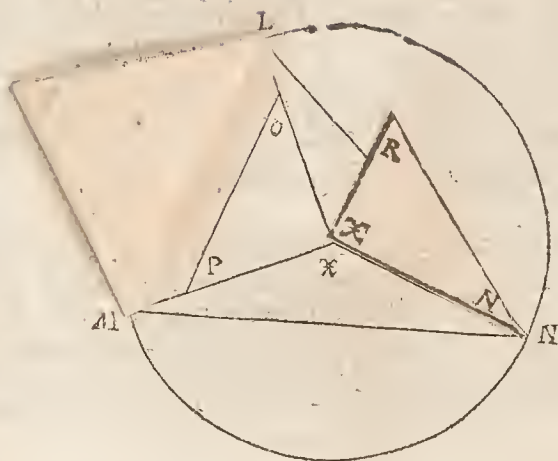
Demonstratiō
that the oppo-
site superficies
are equall.

* $A B$ is equall to $D C$, because the superficies $A C$ is proued a parallelograme, and by the same rea- son, is $B H$ equall to $C F$, because the superficies $F B$ is proued a parallelograme: therefore the 34. of the first is our prooffe.

XX.iiij.

gramme

In this figure may ye more fully see the construction and demonstration of the first case of the former 23. Proposition, if ye erect perpendicularly the triangle $x R N$, and vnto it bend the triangle $L M R$, that the angles R of eche may ioine together in the point R . And so fully vnderstanding this case, the other cases will not be hard to conceaue.



¶ The 21. Theoreme.

The 24. Proposition.

If a solide or body be containd vnder* sixe parallel playne superficieses, the opposite plaine superficieses of the same body are equall and parallelogrammes.

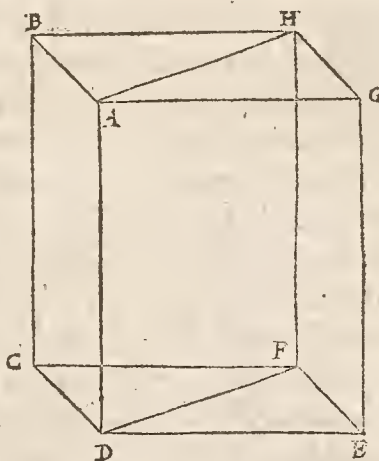
* M. Dee (to auoide cauillation) addeth to Euclides proposition this worde sixe: whome I haue followed accordingly, and not Zamberts, in this.

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Demonstration that the opposite sides are parallelogrammes.

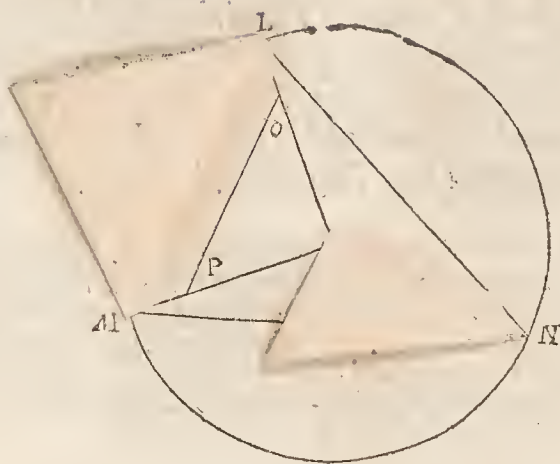
Demonstration that the opposite superficieses are equall.

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XX.iiij.

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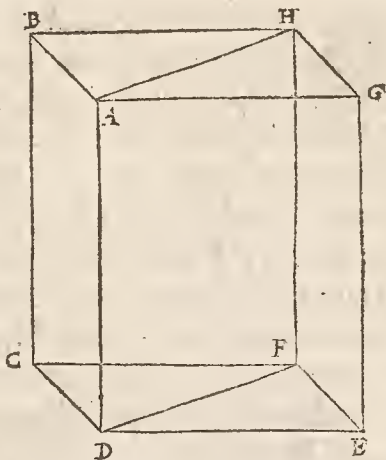
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Demonstration that the opposite sides are parallelogrammes.

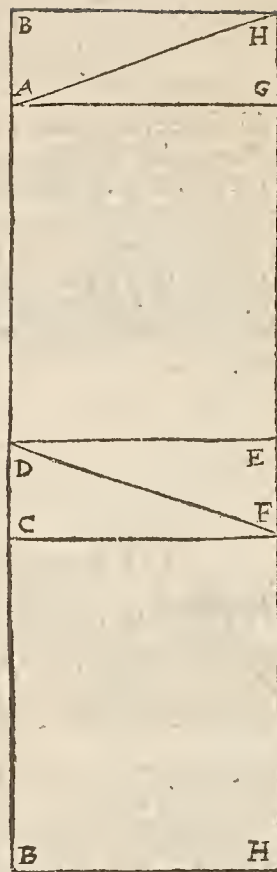
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The eleuenth Booke

gramme BG is equall to the parallelogramme CE . In like sort also may we proue that the parallelogramme AC is equall to the parallelogramme GF , and the parallelogramme AE to the parallelogramme FB . If therfore a solide or body be contained vnder sixe parallel plaine superficieses, the opposite plaine superficieses of the same body are equal & parallelogrammes: which was required to be demonstrated.

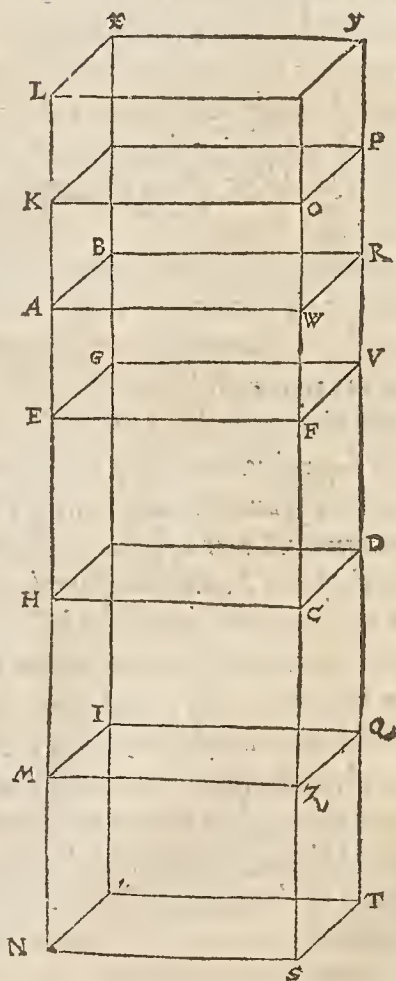
I haue for the better helpe of young beginners, described here an other figure whose forme if it be described vpon pasted paper with the letters placed in the same order that it is here, and then if ye cut finely these lines AG, DE and CF not through the paper, and folde it accordingly, compare it with the demonstration, and it will shake of all hardenes from it.



The 22. Theoreme. The 25. Proposition.

If a Parallelipipedō be cutte of a playne superficies beyng a parallel to the two opposite playne superficieses of the same body: then, as the base is to the base, so is the one solide to the other solide.

Suppose that this solide $ABCD$ being contained vnder parallel plaine superficieses (and therefore called a parallelipipedō) be cut of the plaine superficieses VE , being a parallel to the two opposite superficieses of the same body, namely, to the superficieses AR & DH . The I say that as the base $AEFW$ is to the base $EHCF$, so is the solide $ABFV$ to the solide $EGCD$. Extēd the line AH on either side, & put vnto y line EH as many equal lines as you wil, namely, HM , & MN : & likewise vnto the line AE , put as many equal lines as you will, namely, AK & KL , & make perfect these parallelogrāmes LO, KW, HZ , & MS , and likewise make perfect these solides or bodies LP, KR, DM , and MT . And forasmuch as these right lines, LK, KA , and AE are equall the one to the other, therfore these parallelogrammes LO, KW , and AF , are also (by the first of the sixt) equall the one to the other: and so also (by the same) are these parallelogrammes KX, KB , and AG equall the one to the other. And likewise (by the 24. of the eleuenth) are the parallelogrammes LY, KP , and AR , equal, for they are opposite the one to the other. And by the same reason also the parallelogrammes EC, HZ , and MS are equall the one to the other, and the parallelogrammes HG, HI , and IN are equal the one to the other. And more ouer the parallelogrammes DH, MQ , and NT are (by the 24. of the eleuenth) equall the one to the other, for they



are opposite: wherefore three plaine superficies of the solides LP , RK , and AV are equall to, three plaine superficies: but vnto eche of these three superficies are equall the three opposite superficies (by the 24. of the eleventh) Wherefore these three solides or bodies L , P , K , R , and AV , are equal the one to the other, by the 8. definition of the eleventh. And by the same reason also the three solides, ED , DM & MT are equal the one to the other. Wherefore how multiple $\propto$ the base LF is, to the base AF , so multiplex is the solide LV to the solide AV . And by the same reason also how multiplex the base NF is to the base FH , so multiplex is the solide NV to the solide HV : so that if the base LF be equall to the base NF , the solide also LV shall be equall to the solide VN , and if the base LF exceede the base NF , the solide also LV shall exceede the solide VN , and if the base LF be lesse then the base NF , the solide also LV shall be lesse then the solide VN (by the 1. and 14. of the fift.) Now then there are foure magnitudes, namely, the two bases AF and FH , and the two solides or bodies AV and HV , of which are take their equemultiplices, namely, the equemultiplices of the base AF , & of the solide AV , or the base LF , & the solide LV , & the equemultiplices of the base FH , & of the solide HV are the base NF , and the solide NV . And it is proued that if the base LF exceede the base NF , the solide also LV exceedeth the solide NV , & if it be equal, it is equal, and if it be lesse, it is lesse. Wherefore (by the 6. definitio of the fift) as the base AF is to the base FH , so is the solide AV to the solide HV . If therefore a parallelipipedon be cut of a playne superficies being a parallel to the two opposite playne superficieses of the same body, then, as the base is to the base, so is the one solide to the other solide: which was required to be proued.

I haue for the better sight of the cōstructiō & demōstratiō of the former 25. propositiō, here set another figure, whose forme if ye describe vpon pasted paper, and finely cut the three lines XI , BS , and TY , not through the paper but halfe way, and then fold it accordingly, and compare it with the construction and demōstration, you shall see that it will geue great light therunto.

L	b	y		
R	o	p		
A	v	r	B	A
E	F	V	G	E
H	C	D		H
M	z	e	I	M
N	S	T		N

Here *Flußas* addeth three Corollaries.

First Corollary.

If a Prisme be cutte of a playne superficies parallel to the opposite superficieses, the sections of the Prisme shall be the one to the other in that proportion, that the sections of the base are the one to the other.

For

The eleuenth Booke

For the sections of the bases, which bases (by the 11. definitiō of this booke) are parallelogrammes, shall likewise be parallelogrammes, by the 16. of this booke (when as the superficies which cutteth is parallel to the opposite superficieses) and shal also be equiangle. Wherefore if vnto the bases (by producing the right lines) be added like and equall bases, as was before shewed in a parallelipipedon, of those sections shalbe made as many like Prismes as ye will. And so by the same reason, namely, by the commō excessse, equalitie, or want of the multiplices of the bases & of the sections by the 5. definitiō of the fifth may be proued, that euery section of the Prisme multiplyed by any multiplication whatsoever, shall haue to any other section that proportion that the sections of the bases haue.

Second Corollary.

**Second Corol-
ry.**

*These solides
which he
speaketh of in
this Corollary
are of some
called solid
columnes.*

Solides whose two opposite superficieses are polygonon figures like equall and parallels, the other superficies, which of necessitie are parallelogrammes, being cutte of a playne superficies parallel to the two opposite superficieses: the sections of the base are the one to the other, as the sections of the solide are the one to the other.

Which thing is manifest, for such solides are deuided into Prismes, which haue one cōmon side, namely, the axe or right line, which is drawne by the centers of the opposite bases. Wherefore how many parallelogrammes or bases are set vpon the opposite polygonon figures, of so many Prismes shal the whole solide be cōposed. For those polygonon figures are deuided into so many like triangles by the 20. of the sixth, which describe Prismes. Which Prismes being cut by a superficies parallel to the opposite superficieses, the sectiōs of the bases shal (by the former Corollary) be proportional with the sectiōs of the Prismes. Wherefore by the 12. of the fifth, as the sections of the one are the one to the other, so are the sections of the whole the one to the other.

Of these solides there are infinite kindes, according to the varietie of the opposite and parallel polygonon figures, which polygonon figures doo alter the angles of the parallelogrammes set vpon them according to the diuersitie of their situation. But this is no let at all to this corollary, for that which we haue proued will alwayes follow. When as the superficieses which are set vpo the opposite like & equal polygonon and parallel superficieses are alwayes parallelogrammes.

Third Corollary.

**Third Corol-
lary.**

The foresayd solides composed of Prismes, being cutte by a playne superficies parallel to the opposite superficieses, are the one to the other as the beades or higher parts cutte are.

For it is proued that they are the one to the other as the bases are. Which bases forasmuch as they are parallelogrammes, are the one to the other as the right lines are vpon which they are set by, the first of the sixth, which right lines are the heddes or higher parts of the Prismes.

The 4. Probleme.

The 26. Proposition.

Vpon a right lyne geuen, and at a point in it geuen, to make a solide angle equall to a solide angle geuen.



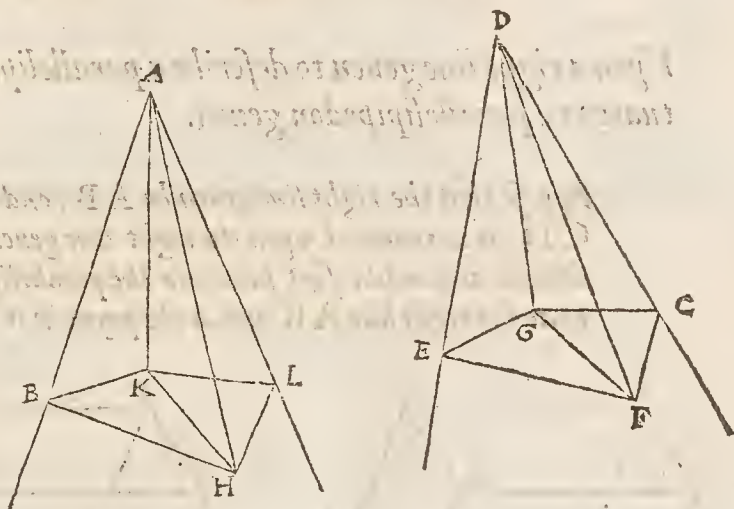
*V*ppose that the right line geuen be AB , and let the point in it geuen be A , and let the solide or bodily angle geuen be D being contained vnder these superficieses all angles EDC , EDF and FDC . It is required vpon the right line AB , & at the point in it geuen A to make a solide angle equall to the solide angle D .

Construction.

Take in the line DF a point at all aduentures, and let the same be F . And (by the 11. of the eleuenth) frō the point F . Draw vnto the superficies wherein are the lines ED & DC a perpendicular line FG , and let it fall vpon the plaine superficies in the point G , & draw a right line from the point D to the point G . And (by the 23. of the first) vnto the line AB , and at the point A make vnto the angle EDC an equall angle BAL , and vnto the angle EDG put the angle BAK equall: and by the 2. of the first, put the line AK equall to the line DG , and (by the 12. of the eleuenth) from the point K raise vp vnto the plaine superficies BAL a perpendicular line KH , and put the line KH equall to the line GF , and draw a right lyne from

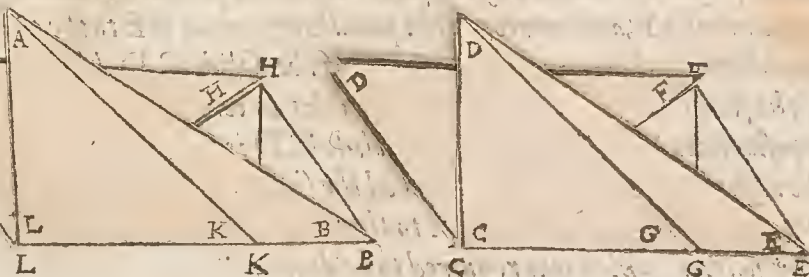
from the point *H* to the point *A*. Now I say that the solide angle *A* contained under the superficial angles *B A L*, *B A H*, and *H A L* is equall to the solide angle *D* contained under the superficial angles *E D C*, *E D F*, and *F D C*. Let the the lines *A B* and *D E* be put equall, and draw these right lines *H B*, *B K*, *F E*, and *E G*. And forasmuch as the line *F G* is erected perpendicularly to the ground superficies, therefore by the 2. definition of the eleventh, the line *F G* is also erected perpendicularly to all the right lines that are in the ground superficies and touche it. Wherfore either of these angles *F G D* and *F G E* is a right angle, and by the same reason also either of the angles *H K A* and *H K B* is a right angle. And forasmuch as these two lines *K A* & *A B* are equal to these two lines *G D* & *D E*, the one to the other, and they containe equall angles (by construction). Wherfore (by the 4. of the first) the base *K B* is equall to the base *E G*, and the line *K H* is equall to the line *G F*, and they cōprehēd right angles. Wherfore the line *B H* is equall to the line *F E*. Agayne, forasmuch as these two lines *A K* and *K H* are equal to these two lines *D G* and *G F*, and they containe right angles. Wherfore y^e base *A H* is (by the 4. of the first) equall to the base *D F*. And the line *A B* is equall to the line *D E*. Wherfore these

Demonstration.



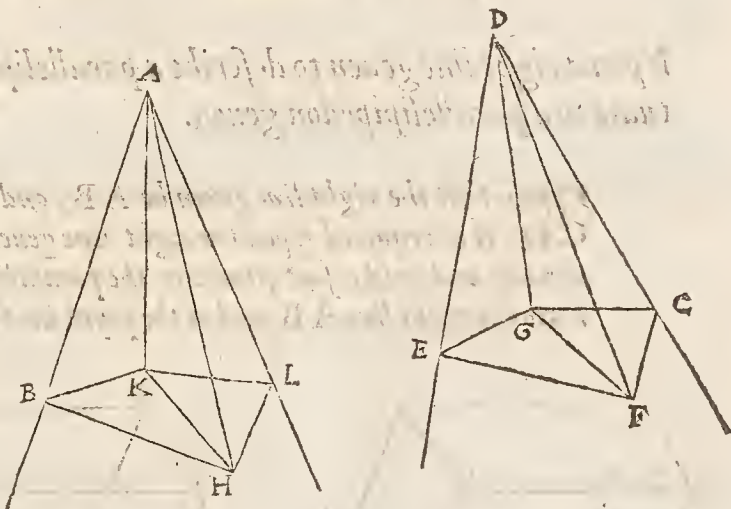
two lines *A B* and *A H* are equal to these two lines *F D* and *D E*, and the base *B H* is equall to the base *F E*. Wherfore (by the 8. of the first) the angle *B A H* is equall to the angle *E D F*. And by the same reason also the angle *H K L* is equall to the angle *F G C*. Wherfore if we put these lines *A L* and *D C* equall, and draw these right lines *K L*, *H L*, *G C*, and *F C*: forasmuch as the whole angle *B A L* is equall to the whole angle *E D C*, of which the angle *B A K* is supposed to be equall to the angle *E D G*, therefore the angle remayning, namely, *K A L* is equall to the angle remayning *G D C*. And forasmuch as these two lines *K A* and *A L* are equal to these two lines *G D* and *D C*, and they containe equall angles, therefore by the 4. of the first, the base *K L* is equall to the base *G C*, and the line *K H* is equall to the line *G F*, wherfore these two lines *L K* and *K H* are equal to these two lines *C G* and *G F*, and they cōtaineright angles. Wherfore the base *H L* is (by the 4. of the first) equal to the base *F C*. And forasmuch as these two lines *H A* and *A L* are equal to these two *F D* and *D C*, and the base *H L* is equall to the base *F C*, therefore (by the 8. of the first) the angle *H A L* is equall to the angle *F D C*, and by construction, the angle *B A L* is equall to the angle *E D C*. Wherfore vnto the right line geuen, and at the point in it geuen, namely, *A*, is made a solide angle equal to the solide angle geuen *D*: which was required to be done.

In these two figures here put, you may clearly conceiue the former construction and demonstration, if ye erect perpendicularly vnto the ground superficies the triangles *A L B* and *D C E*, & eleuate the triangles *A L H* and *D C F* that the lynes



from the point H to the point A . Now I say that the solide angle A contained under the superficial angles $B A L$, $B A H$, and $H A L$ is equall to the solide angle D contained under the superficial angles $E D C$, $E D F$, and $F D C$. Let the the lines $A B$ and $D E$ be put equall, and draw these right lines $H B$, $B K$, $F E$, and $E G$. And forasmuch as the line $F G$ is erected perpendicularly to the ground superficies, therefore by the 2. definition of the eleventh, the line $F G$ is also erected perpendicularly to all the right lines that are in the ground superficies and touche it. Wherefore either of these angles $F G D$ and $F G E$ is a right angle, and by the same reason also either of the angles $H K A$ and $H K B$ is a right angle. And forasmuch as these two lines $K A$ & $A B$ are equall to these two lines $G D$ & $D E$, the one to the other, and they containe equall angles (by construction). Wherefore (by the 4. of the first) the base $K B$ is equall to the base $E G$, and the line $K H$ is equall to the line $G F$, and they cōprehēd right angles. Wherefore the line $B H$ is equall to the line $F E$. Agayne, forasmuch as these two lines $A K$ and $K H$ are equal to these two lines $D G$ and $G F$, and they containe right angles. Wherefore $\hat{y}$ base $A H$ is (by the 4. of the first) equall to the base $D F$. And the line $A B$ is equall to the line $D E$. Wherefore these

Demonstration.



two lines $A B$ and $A H$ are equall to these two lines $F D$ and $D E$, and the base $B H$ is equall to the base $F E$. Wherefore (by the 8. of the first) the angle $B A H$ is equall to the angle $E D F$. And by the same reason also the angle $H K L$ is equall to the angle $F G C$. Wherefore if we put these lines $A L$ and $D C$ equall, and draw these right lines $K L$, $H L$, $G C$, and $F C$: forasmuch as the whole angle $B A L$ is equall to the whole angle $E D C$, of which the angle $B A K$ is supposed to be equall to the angle $E D G$, therefore the angle remayning, namely, $K A L$ is equall to the angle remayning $G D C$. And forasmuch as these two lines $K A$ and $A L$ are equall to these two lines $G D$ and $D C$, and they containe equall angles, therefore by the 4. of the first, the base $K L$ is equall to the base $G C$, and the line $K H$ is equall to the line $G F$, wherefore these two lines $L K$ and $K H$ are equall to these two lines $C G$ and $G F$, and they cōtaineright angles. Wherefore the base $H L$ is (by the 4. of the first) equal to the base $F C$. And forasmuch as these two lines $H A$ and $A L$ are equall to these two $F D$ and $D C$, and the base $H L$ is equall to the base $F C$, therefore (by the 8. of the first) the angle $H A L$ is equall to the angle $F D C$, and by construction, the angle $B A L$ is equall to the angle $E D C$. Wherefore vnto the right line geuen, and at the point in it geuen, namely, A , is made a solide angle equal to the solide angle geuen D : which was required to be done.

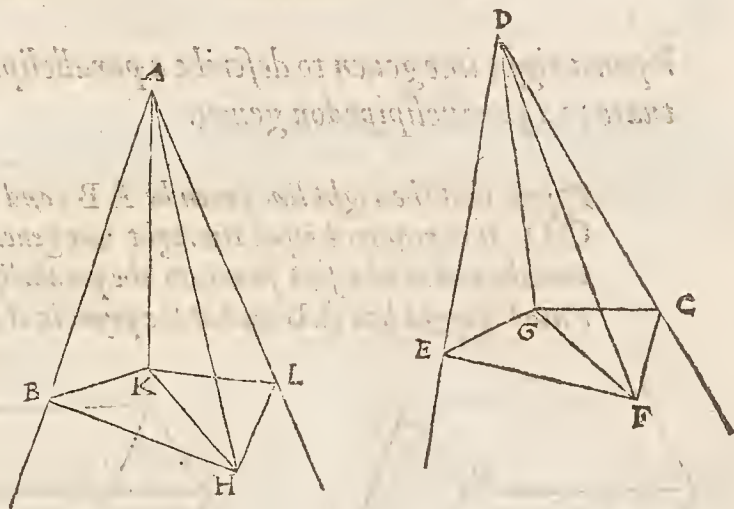
In these two sources here put, you may more clearly conceiue the former construction and demonstration, if ye erect perpendicularly vnto the ground superficies the triangles $A L B$ and $D C E$, & eleuate the triangles $A L H$ and $D C F$ that the lynes



from the point H to the point A . Now I say that the solide angle A contained under the superficial angles BAL , BAH , and HAL is equall to the solide angle D contained under the superficial angles EDC , EDF , and FDC . Let the the lines AB and DE be put equall, and draw these right lines HB , BK , FE , and EG . And forasmuch as the line FG is erected perpendicularly to the ground superficies, therefore by the 2. definition of the eleventh, the line FG is also erected perpendicularly to all the right lines that are in the ground superficies and touche it. Wherfore either of these angles FGD and FGE is a right angle; and by the same reason also either of the angles HKA and HKB is a right angle. And forasmuch as these two lines KA & AB are equall to these two lines GD & DE , the one to the other, and they containe equall

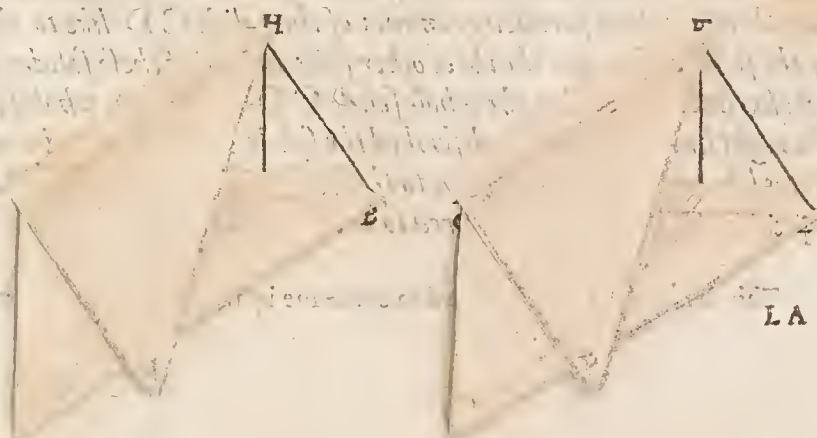
Demonstration.

angles (by construction). Wherfore (by the 4. of the first) the base KB is equall to the base EG , and the line KH is equall to the line GF , and they cōprehēd right angles. Wherfore the line BH is equall to the line FE . Agayne, forasmuch as these two lines AK and KH are equal to these two lines DG and GF , and they containe right angles. Wherfore by base AH is (by the 4. of the first) equall to the base DF . And the line AB is equall to the line DE . Wherfore these



two lines AB and AH are equall to these two lines FE and FD , and the base BH is equall to the base FE . Wherfore (by the 8. of the first) the angle BAH is equall to the angle EDF . And by the same reason also the angle HKL is equall to the angle FGC . Wherfore if we put these lines AL and DC equall, and draw these right lines KL , HL , GC , and FC : forasmuch as the whole angle BAL is equall to the whole angle EDC , of which the angle BAK is supposed to be equall to the angle EDG , therefore the angle remayning, namely, KAL is equall to the angle remayning GDC . And forasmuch as these two lines KA and AL are equall to these two lines GD and DC , and they containe equall angles, therefore by the 4. of the first, the base KL is equall to the base GC , and the line KH is equall to the line GF , wherfore these two lines LK and KH are equall to these two lines CG and GF , and they cōtain right angles. Wherfore the base HL is (by the 4. of the first) equal to the base FC . And forasmuch as these two lines HA and AL are equall to these two FD and DC , and the base HL is equall to the base FC , therefore (by the 8. of the first) the angle HAL is equall to the angle FDC , and by construction, the angle BAL is equall to the angle EDC . Wherfore vnto the right line geuen, and at the point in it geuen, namely, A , is made a solide angle equal to the solide angle geuen D : which was required to be done.

In these two figures here put, you may more clearly conceiue the former construction and demonstration, if ye erect perpendicularly vnto the ground superficies the triangles ALB and DCE , & eleuate the triangles ALH and DCF that the lynes



The eleventh Booke

LA and CD of them may exactly agree with the lines LA and CD of the triangles erected. For so ordering them, if ye compare the former construction and demonstration with them, they will be playne vnto you.

Although Euclides demonstration be touching solide angles which are contained only vnder three superficiall angles, that is, whose bases are triangles. yet by it may ye performe the Probleme touching solide angles contained vnder superficiall angles how many soeuer, that is, hauing to their bases any kinde of Polygonon figures. For euery Polygonon figure may by the 20. of the sixt, be resolued into like triangles. And so also shall the solide angle be deuided into so many solide angles as there be triangles in the base. Vnto euery one of which solide angles you may by this proposition describe an equall solide angle vpon a line geuen, and at a point in it geuen. And so at the length the whole solide angle after this maner described shall be equall to the solide angle geuen.

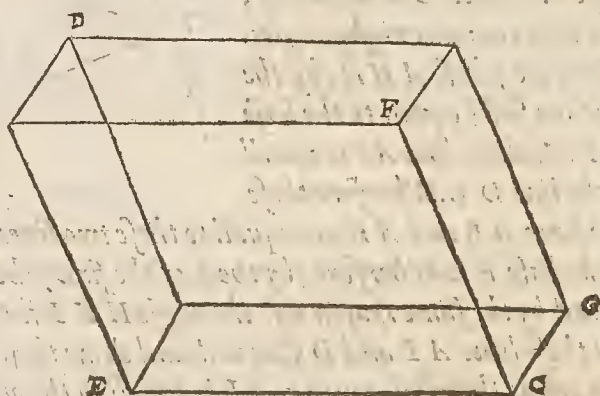
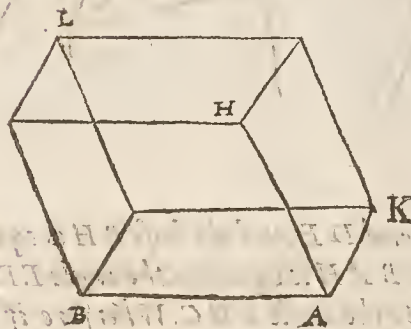
The 5. Theoreme.

The 27. Proposition.

Vpon a right line geuen to describe a parallelipipedon like and in like sort situate to a parallelipipedon geuen.

Vppose that the right line geuen be AB, and let the parallelipipedon geuen be CD. It is required vpon the right line geuen AB to describe a parallelipipedon like and in like sort situate to the parallelipipedon geuen, namely, to CD. Vnto the right line AB and at the poynt in it A describe (by the 26. of the ele-

Construction.

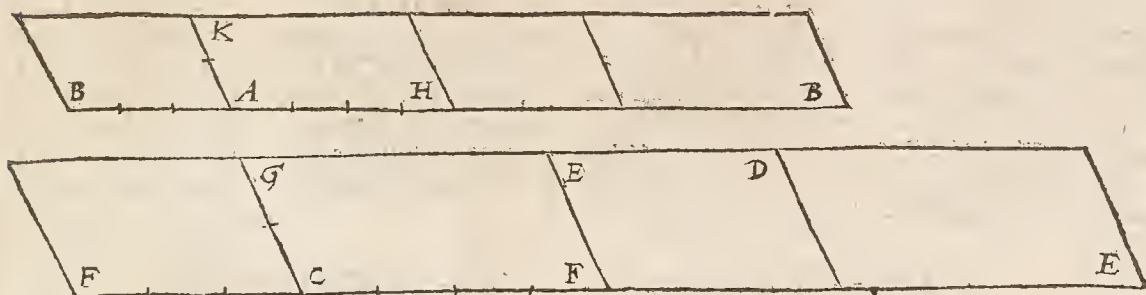


Demonstration.

uenth (a solide angle equall to the solide angle C, and let it be containd vnder these superficiall angles BAH, HAK, and KAB, so that let the angle BAH be equall to the angle ECF, and the angle BAK to the angle ECG, and moreover the angle KAH to the angle GCF. And as the line EC is to the line CG, so let the AB be to the line AK (by the 12. of the sixth) and as the line GC is to the line CF, so let the KA be to the line AH. Wherefore of equalitie (by the 22. of the fift) as the line EC is to the line CF, so is the line BA to the line AH. Make perfect the parallelogramme BH, and also the solide AL. Now for that as the line EC is to the line CG, so is the line BA to the line AK, therefore the sides which containe the equall angles, namely, the angles ECG and BAK are proportionall. wherefore (by the first definition of the sixth) the parallelogramme GE is like to the parallelogramme KB. And by the same reason the parallelogramme KH is like to the parallelogramme GF, and moreover the parallelogramme FE to the parallelogramme HB. Wherefore there are three parallelogrammes of the solide CD like to the three parallelogrammes of the solide AL. But the three other sides in eche of these solides are equall and like to their opposite sides. Wherefore the whole solide CD is like to the whole solide AL. Wherefore vpon the right line geuen AB is described the solide AL containd vnder parallel playne superficieses like and in like sort situate to the solide geue CD containd also vnder parallel playne superficieses: which was required to be doone.

This demonstration is not hard to conceaue by the former figure as it is described in a playne, if ye haue

that imagination of parallelipedons described in a playne which we before taught in the diffinition of a cube. Howbeit I haue here for the more ease of such as are not yet acquainted with solides, described



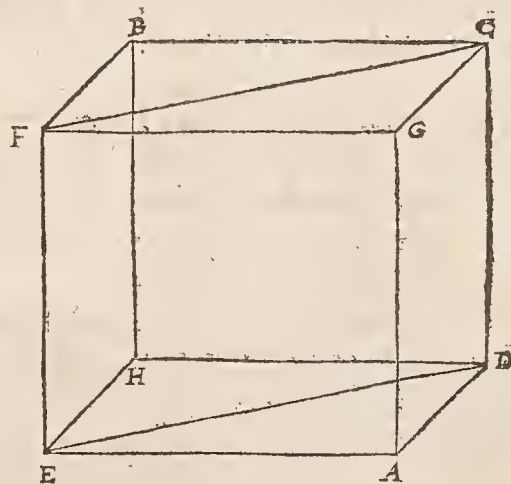
cribed two figures, whose formes first describe vpon pasted paper with the like letters noted in them, and then finely cutte the three middle lines of eche figure, and so fold them accordingly, and that doone compare them with the construction and demonstration of this 27. proposition, and they will be very easy to conceaue.

The 23. Theoreme.

The 28. Proposition.

If a parallelipedon be cutte by a playne superficies drawne by the diagonall lines of those playne superficiees which are opposite: that solide is by this playne superficies cutte into two equall partes.

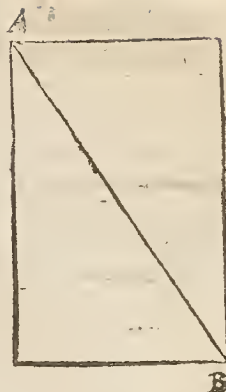
Suppose that the parallelipedon AB be cutte by the playne superficies CDEF drawne by the diagonall lines of y^e playne superficiees which are opposite, namely, of the superficiees CF and DE. Then I say that the parallelipedon AB is cutte into two equall partes by the superficies CDEF. For forasmuch as (by the 34. of the first) the triangle CGF is equall to the triangle CBF, and the triangle ADE to the triangle DEH, and the parallelogramme CA is equall to the parallelogramme BE, for they are opposite, and the parallelogramme GE is also equall to the parallelogramme CH, and the parallelogramme CE, is the common section by supposition: Wherefore the prisme contained vnder the two triangles CGF, and DAE, and vnder the three parallelogrammes GE, AC, and CE is (by the 8. definition of the eleuenth) equall to the prisme contained vnder the two triangles CBF and DEH and vnder the three parallelogrammes CH, BE, and CE. For they are cotayned vnder playne superficiees equall both in multitude and in magnitude. Wherefore the whole parallelipedon AB is cutte into two equall partes by the playne superficies CDEF: which was required to be demonstrated.



Demonstration.

The eleuenth Booke

A diagonall line is a right line which in angular figures is drawne from one angle and extended to his contrary angle as you see in the figure AB.



Describe for the better sight of this demonstration a figure vpon pasted paper like vnto that which you described for the 24. proposition onely altering the letters: namely, in steade of the letter A put the letter F, and in steade of the letter H the letter C: moreouer in steade of the letter C put the letter H, and the letter E for the letter D, and the letter A for the letter E, and finally put the letter D for the letter F. And your figure thus ordered compare it with the demonstration, only imagining a superficies to passe through the body by the diagonall lines C F and D E.

¶ The 24. Theoreme.

The 29. Proposition.

Parallelipedons consisting vpon one and the selfe same base, and vnder one and the selfe same altitude, whose* standing lines are in the selfe same right lines, are equall the one to the other.

* Looke at the end of the demonstration what is vnderstanded by standing lines.



Suppose that that these parallelipedons CM and CN doo consist vpon one and the selfe same base, namely AB, and let them be vnder one and the selfe same altitude, whose standing lines, that is, the fower sides of eche solide which fall vpon the base, as the lines AF, CD, BH, LM of the solide CM, and the

lines CE, BK, AG, and LN

of the solide CN, let be in the

selfe same right lines or paral-

lell lines FN, DK. Then I say

that the solide CM is equall to

the solide CN. For forasmuch

as either of these superficieses

CBDH, CBEK is a paralle-

logramme, therefore (by the 34.

of the first) the line CB is e-

quall to either of these lines D

H and EK. Wherefore also the

line DH is equall to the line

EK. Take away EH which is

common to them both, where-

fore the residue namely DE is equall to the residue HK. Wherefore also the triangle DCE

is equall to the triangle HKB. And the parallelogramme DG is equall to the parallelo-

gramme HN. And by the same reason the triangle AGF is equall to the triangle MLN,

and the parallelogramme CF is equall to the parallelogramme BM. But the parallelograme

CG is equall to the parallelogramme BN, by the 24. of the tenth for they are opposite the

one to the other. Wherefore the prisme containd vnder the two triangles FAG and DCE

and vnder the three parrallelogrames AD, DG, and CG is equall to the prisme cõtayned

vnder the two triangles MLN and HBK, and vnder the three parallelogrames, that is,

BM, NH, and BN. Put that solide common to them both, whose base is the parallelograme

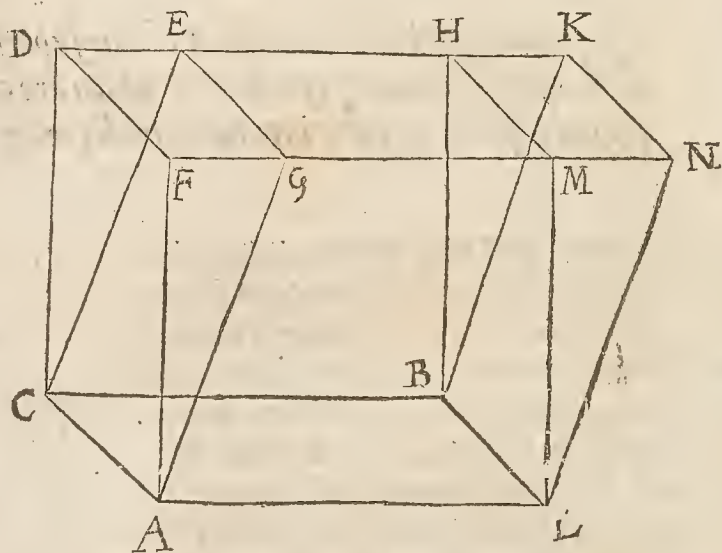
AB, and the opposite side vnto the base is the superficies GEHM. Wherefore the whole pa-

rallelipipedon CM is equall to the whole parallelipedon CN: Wherefore parallelipedons

consisting vpon one and the selfe same base, and vnder one and the selfe same altitude, whose

standing lines are in the selfe same right lines, are equall the one to the other: which was re-

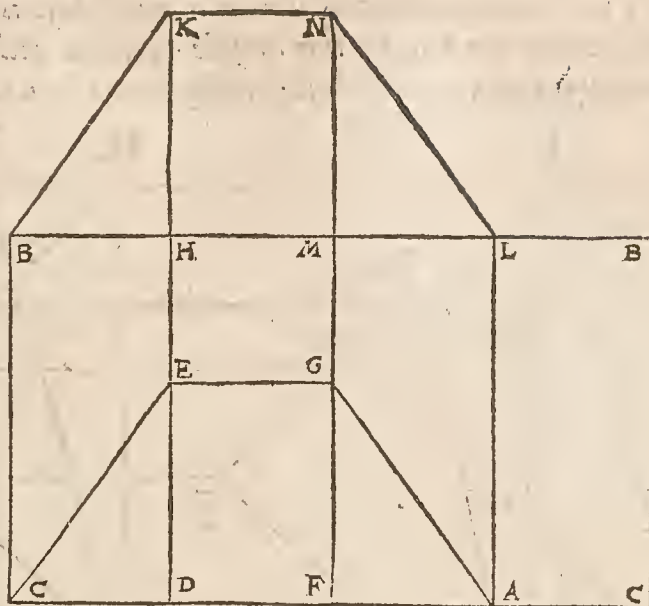
quired to be demonstrated.



John Dee his figure.
By this figure it appeareth why such Prisms were called wedges: of the very shape of a wedge, as is the solide DEFGA-C. &c.

Although

Although this demonstration be not hard to a good imagination to conceive by the former figure (which yet by *M. Dees* reforming is much better then the figure of this proposition commonly described in other copies both greake and lattin): yet for the ease of those which are young beginners in this matter of solides, I haue here set another figure whose forme if it be described vpon pasted paper, with the like letters to euery line as they be here put, and then if ye finely cut not thorough but as it were halfe way the three lines *L A, N M G F*, and *K H E D*, & so folde it accordingly, & compare it with the demonstratiō, it will geue great light thereunto.



Stāding lines are called those fower right lines of euery parallelipedon which ioyn together the angles of the vpper and nether bases of the same body. Which according to the diuersitie of the angles of the solides, may either be perpendicular vpon the base, or fall obliquely. And forasmuch as in this proposition and in the next proposition following, the solides compared together are supposed to haue one and the selfe same base, it is manifest that the standing lines are in respect of the lower base in the selfe same parallel lines, namely, in the two sides of the lower base. But because there are put two solides vpon one and the selfe same base, and vnder one and the selfe same altitude, the two vpper bases of the solides may be diuersly placed. For forasmuch as they are equall and like (by the 24. of this booke) either they may be placed betwene the selfe same parallel lines: and the standing lines are in either solide sayd to be in the selfe same parallel lines, or right lines: namely, when the two sides of eche of the vpper bases are containd in the selfe same parallel lines: but contrariwise if those two sides of the vpper bases be not containd in the selfe same parallel or right lines, neither shal the standing lines which are ioyned to those sides be sayd to be in the selfe same parallel or right lines. And therefore the standing lines are sayd to be in the selfe same right lines, when the sides of the vpper bases, at the least two of the sides are containd in the selfe same right lines: which thing is required in the supposition of this 29. proposition. But the standing lines are sayd not to be in the selfe same right lines, when none of the two sides of the vpper bases are containd in the selfe same right lines, which thing the next proposition following supposeth.

Stāding lines.

¶ The 28. Theoreme.

The 30. Proposition.

Parallelipipedons consisting vpon one and the selfe same base, and vnder the selfe same altitude, whose standing lines are not in the selfe same right lines, are equall the one to the other.

Suppose that these Parallelipipedons *C M* and *C N*, do consist vpon one and the selfe same base, namely, *A B*, and vnder one and the selfe same altitude, whose standing lines, namely, the lines *A F, C D, B H*, and *L M*, of the Parallelipipedon *C M*, and the standing lines *A G, C E, B K*, and *L N*, of the Parallelipipedon *C N*, let not be in the selfe same right lines. Then I say, that the Parallelipipedon *C M*, is equall to the Parallelipipedon *C N*. Forasmuch as the vpper superficiees *F H* & *G K*, of the former Parallelipipedons, are in one and the selfe same superficies (by reason they are supposed to be of one and the selfe same altitude): Extend the lines *F D* and *M H*, till they concur with the lines *N G* and *K E* (sufficiently both waies extended: for in diuers cases their concurse is diuers). Let *F D* extended, meete with *N G*, and cut it in the point *X*: and with *K E* in the point *P*. Let likewise *M H* extended, meete with *N G* (sufficiently produced) in the point *O*, and with *K E* in the point *R*. And drawe these right lines *A X, L O, C P*, and *B R*. Now (by the 29. of the eleuenth) the solide *C M*, whose base is the parallelogramme *A C B L*, and the parallelogramme opposite vn-

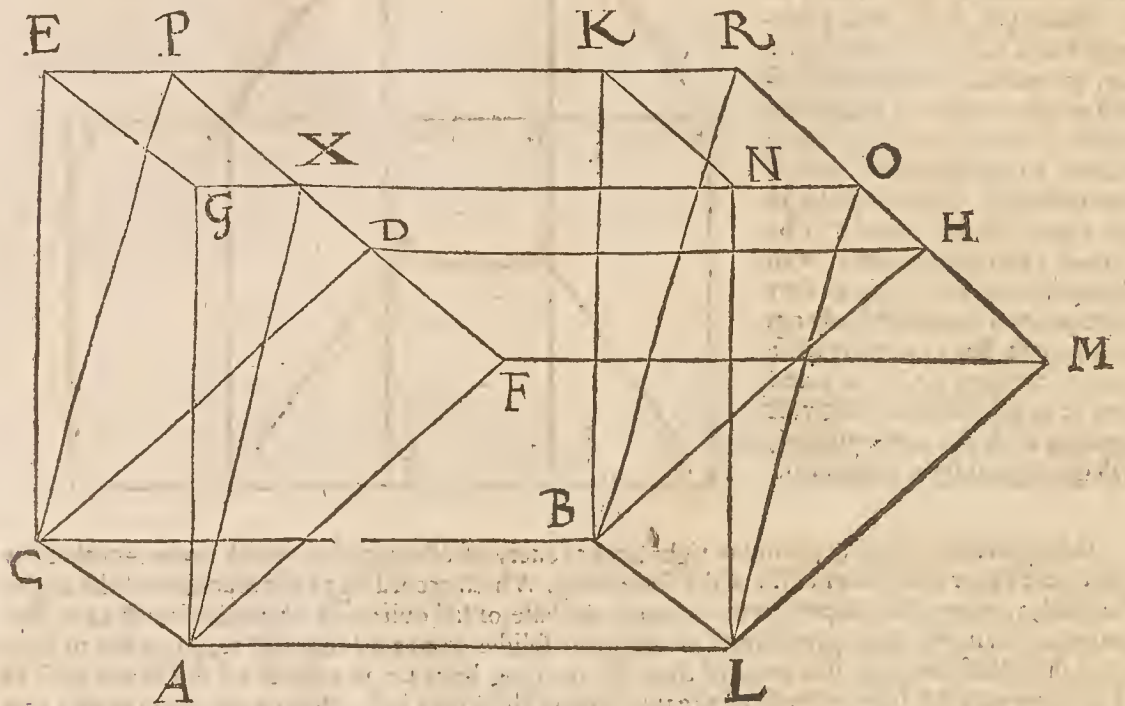
Construction.

The eleuenth Booke

Demonstration.

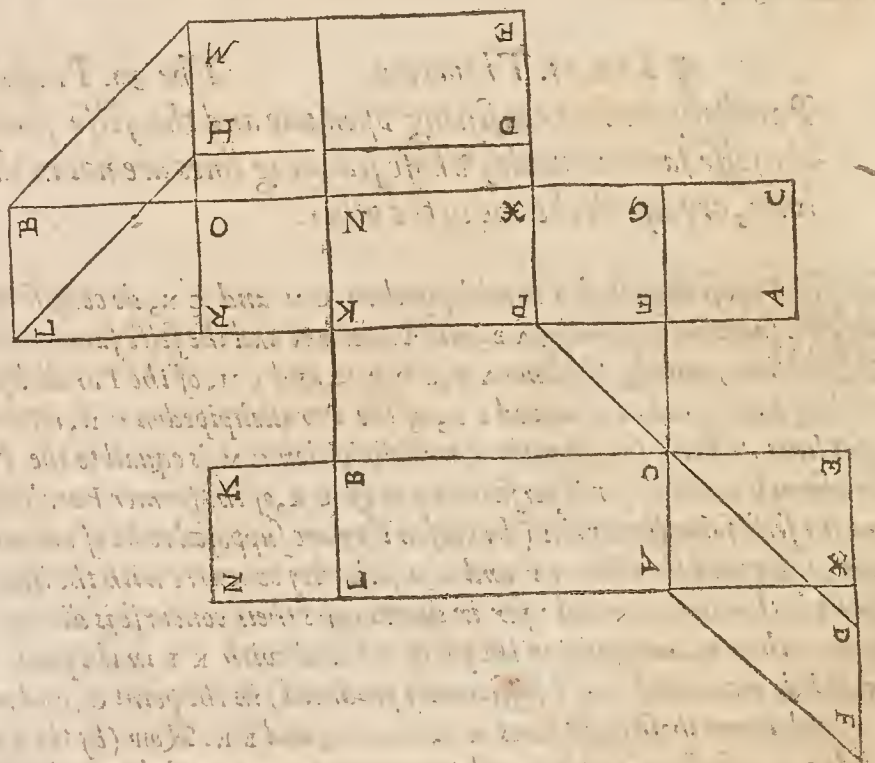
to it is $FDHM$, is equall to the solide CO , whose base is $ACBL$ and the opposite side the parallelogramme $XPRO$, for they consiste vpon one and the selfe same base, namely, vpon the parallelograme $ACBL$, whose standing lines $AF, AX, LM, LO, CD, CP, BH$, and BR ,

L. Dees figure.



are in the selfe same right lines FP and MR . But the solide CO , whose base is the parallelogramme $ACBL$, and the opposite superficies vnto it is $XPRO$, is equall to the solide CN , whose base is the parallelogramme $ACBL$, and the opposite superficies vnto it is the superficies $GEKN$, for they are vpon one and the selfe same base, namely, $ACBL$, and their standing lines $AG, AX, CB, CP, LN, LO, BK$, and BR , are in the selfe same right lines NX , and PK . Wherefore also the solide CM , is equall to the solide CN . Wherefore Parallelipipedons consisting vpon one and the selfe same base, and vnder the selfe same altitude, whose standing lines are not in the selfe same right lines, are equall the one to the other: which was required to be proued.

This demonstration is fomwhat harder then the former to conceaue by the figure described in the plaine (and yet before *M. Dee* inuented that figure which is placed for it, it was much harder) by reason one solide is contained in an other. And therefore for the clerer light therof, describe vpō pasted paper this figure here put with the like letters and finely cut the lines $AC, CB, EG, BL, EPK, ROHM$, and folde it accordingly that euery line may exactly agree with his correspondent lyne (which obseruing the letters of euery line



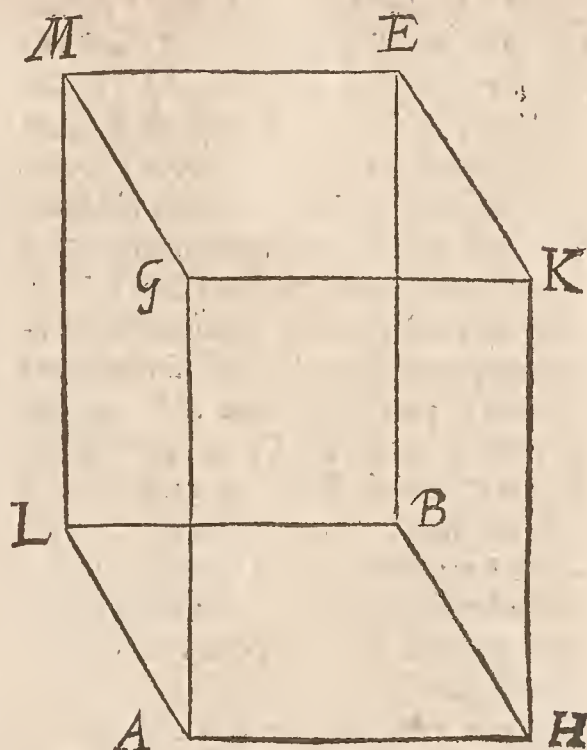
ye shall easily do) and so cōpare your figure with the demonstration, and it will make it very plaine vnto you.

The 26. Theoreme. The 31. Proposit.

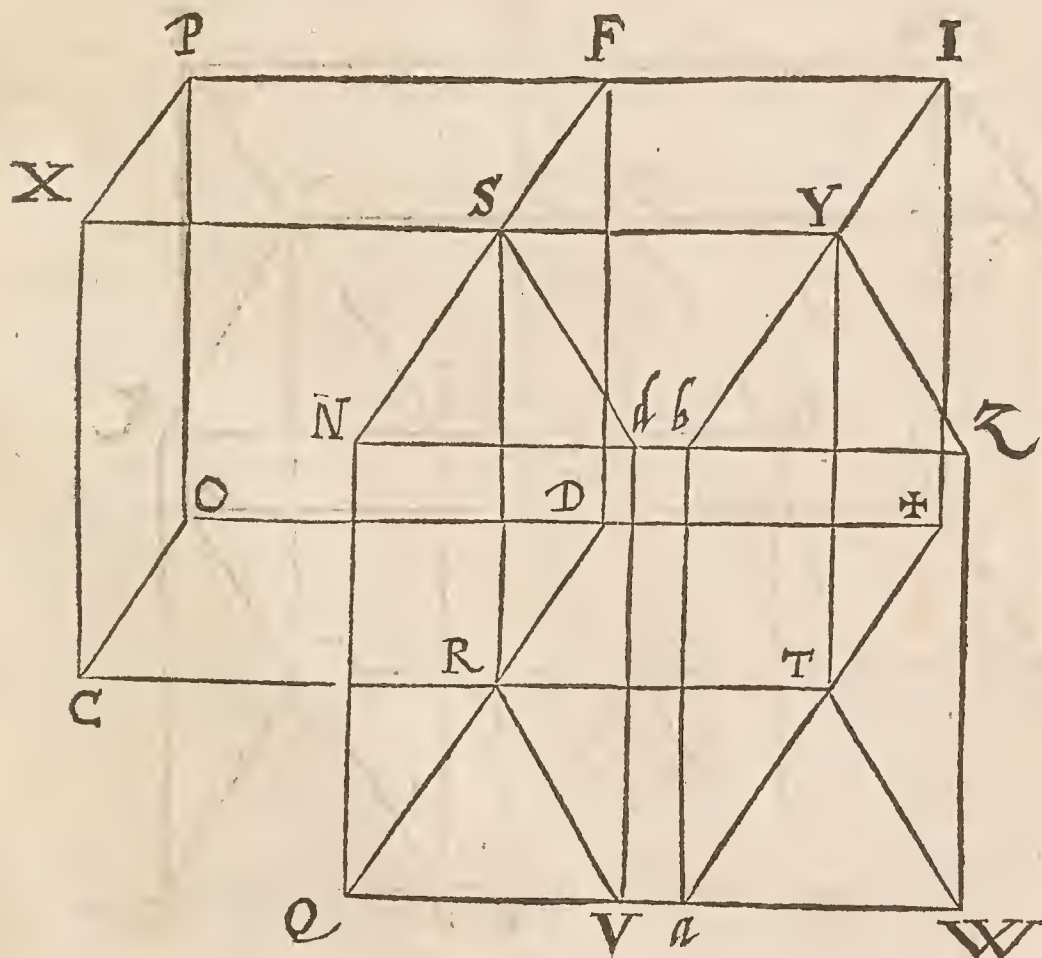
Parallelipipedons consisting vpon equall bases, and being vnder one and the selfe same altitude, are equall the one to the other.



Suppose that vpon these equall bases AB and CD do consist these parallelipipedons AE and CF , being vnder one & the selfe same altitude. The I say, that the solide AE is equall to the solide CF . First let the standing lines, namely, $HK, BE, AG, LM, OP, DF, CX$, and RS , be erected perpendicularly to the bases AB and CD , and let the angle ALB not be equall to the angle CRD . Extend the line CR to the point T . And (by the 23. of the first) vpon the right line RT , and at the point in it R , describe vn-



Two cases in this proposition.
The first case.
(construction.)

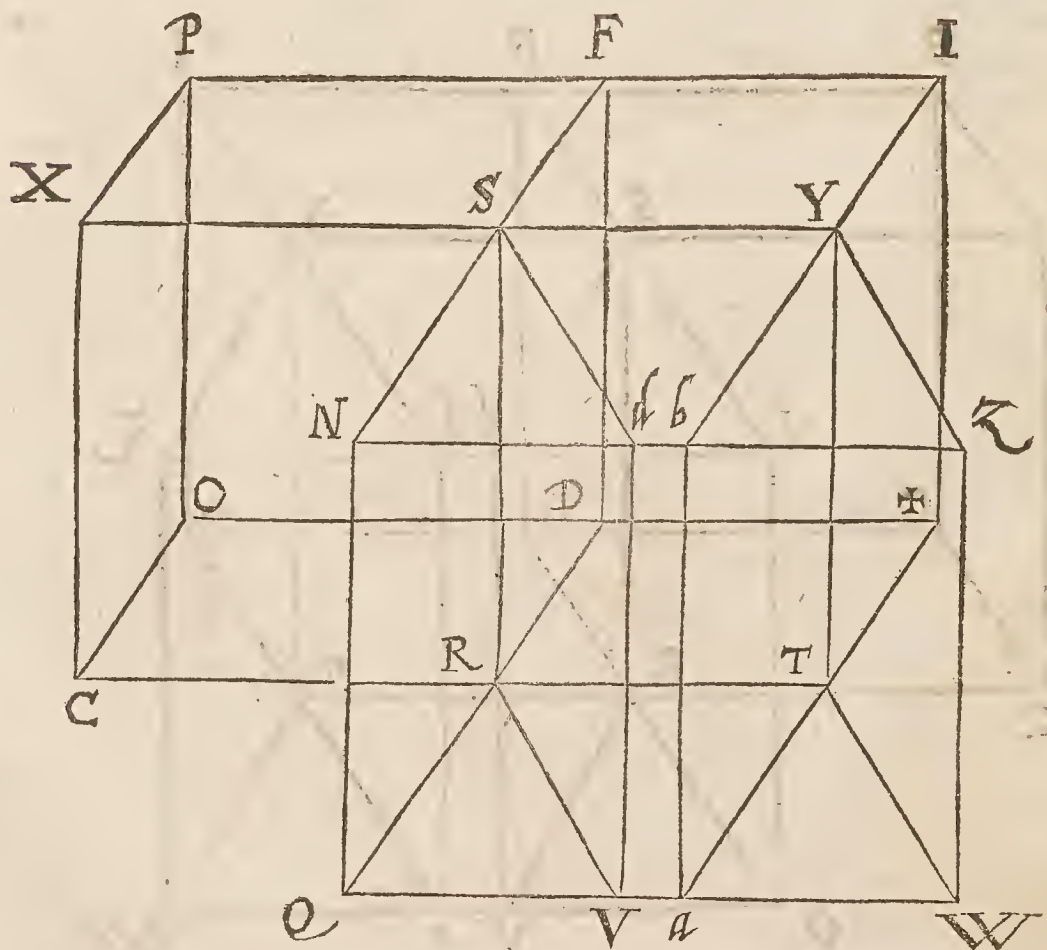
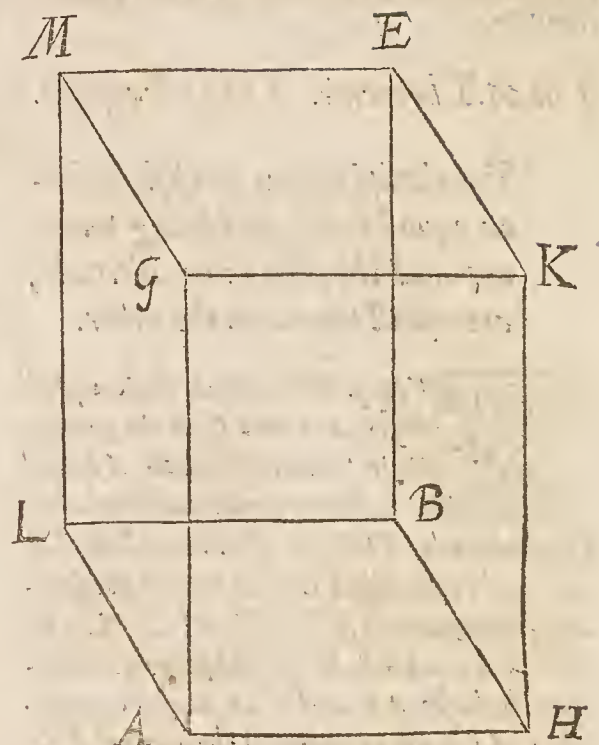


We are beholding to M. Dee for inuenting this figure, with other, which till his reforming were as much mishappen as this was, and so both in the Greeke and Latine copiest remains.

The eleventh Booke

Demonstration.

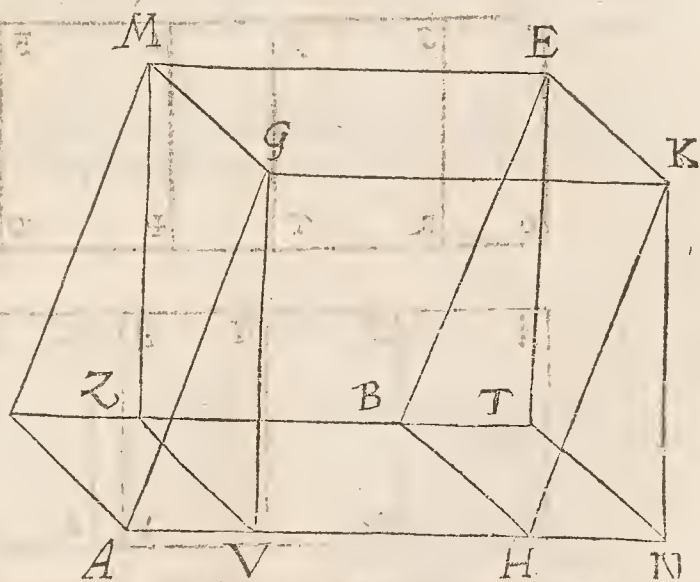
to the angle ALB an equall angle TRV . And (by the third of the first) put the line RT equall to the line LB , and the line RV equall to the line AL . And (by the 31. of the first) by the point V draw unto the line RT a parallel line VW : and make perfecte the base RW , and the solide TV . Now forasmuch as these two lines TR and RV are equall to these two lines BL and LA , and they containe equall angles, therefore the parallelogramme RW is equall and like to the parallelogramme AB . Againe, forasmuch as the line LB is equall to the line RT , and the line LM to the line RS (for the lines LM and RS are the altitudes of the Parallelipipedons AE and CF , which altitudes are supposed to be equall) and they containe right angles by supposition, therefore the parallelogramme RT is equall and like to the parallelogramme BM . And by the same reason also the parallelogramme LG is equall and like to the parallelogramme SV . Wherefore three parallelogrammes of



the solide AE are equall and like to the three parallelogrammes of the solide TV . But these three parallelogrammes are equall and like to the three opposite sides. Wherefore the whole Parallelipipedon AE is equall and like to the whole Parallelipipedon TV . Extend (by the second petition) the lines DR and WV , untill they concur, and let them concur in the point Q . And (by the 31. of the first) by the point T drawe unto the line RQ a parallel line Ta , and extend duely the lines Ta and DO untill they concur, and let them concur in the point $\star$. And make perfecte the solides QY and RI . Now the solide QY , whose ^{*} base is the parallelogramme RY , and the opposite side unto the base the parallelogramme Qb , is equall to the solide TV , whose base is the parallelogramme RY , and the opposite side unto the base the parallelogramme VZ . For they consist upon one and the selfe same base, namely, RY , and are under one and the selfe same altitude, and their standing lines, namely, RQ , RV , Ta , TW , SN , Sd , Tb , and TZ , are in the selfe same right lines, namely, QW , and NZ . But the solide TV is proued equall to the solide AE . Wherefore also the solide YQ is equall to the solide AE . Now forasmuch as the parallelogramme $RVWT$ is equall to the parallelogramme QT (by the 35. of the first) and the parallelogramme AB is equall to the parallelogramme RW : therefore the parallelogramme QT is equall to the parallelogramme AB , and the parallelogramme CD is equall to the parallelogramme AB (by supposition). Wherefore the parallelogramme CD is equall to the parallelogramme QT . And there is a certaine other superficies, namely, DT . Wherefore (by the 7. of the fift) as the base CD is to the base DT , so is the base QT to the base DT . And forasmuch as the whole Parallelipipedon CI is cut by the plaine superficies RF , which is a parallel to either of the opposite plaine superficieses, therefore as the base CD is to the base DT , so is the solide CF to the solide RI (by the 25. of the eleuenth). And by the same reason, also, forasmuch as the whole Parallelipipedon QI is cut by the plaine superficies RT , which is a parallel to either of the opposite plaine superficieses, therefore as the base QT is to the base DT , so is the solide QY to the solide RI . But as the base CD is to the base DT , so is the base QT to the base TD . Wherefore (by the 11. of the fift) as the solide CF is to the solide RI , so is the solide QY to the solide RI . Wherefore either of these solides CF and QY , haue to the solide RI one and the same proportion. Wherefore the solide CF is equall to the solide QY . But it is proued that the solide QY is equall to the solide AE . Wherefore also the solide CF is equall to the solide AE .

* Note now, how the base respectiue is taken: for so may alteration of respects alter the name of the bowndes eyther of solides or playnes.

But now suppose that the standing lines, namely, AG , HK , BE , LM , CX , OP , DF , and RS , be not erected perpendicularly to the bases AB and CD . Then also I say, that the solide AE is equall to the solide CF . Draw (by the 11. of the eleuenth) unto the ground plaine superficieses AB and CD from these pointes K , E , G , M , P , F , X , S , these perpendicular lines KN , L ET , GV , MZ , PW , FY , XQ , and SI . And draw these right lines NT , NV , ZV , ZT , WY , WQ , IQ , and IY . Now (by that which hath before bene proued in this 31. Proposition) the



Second case.

The eleventh Booke

* There you perceave how the base is diversly considered & chosen: as before weadvertised you.

solide K Z is equall to the solide P I, for they consist vpon equall

* bases; namely, K M, and P S,

and are vnder one and the selfe

same altitude, whose standing

lines also are erected perpendicu-

larly to the bases. But the solide

K Z is equall to the solide A E

(by the 29. of the eleventh): and

the solide P I is (by the same) e-

quall to the solide C F, for they

consist vpon one and the selfe

same base, and are vnder one

& the selfe same altitude, whose

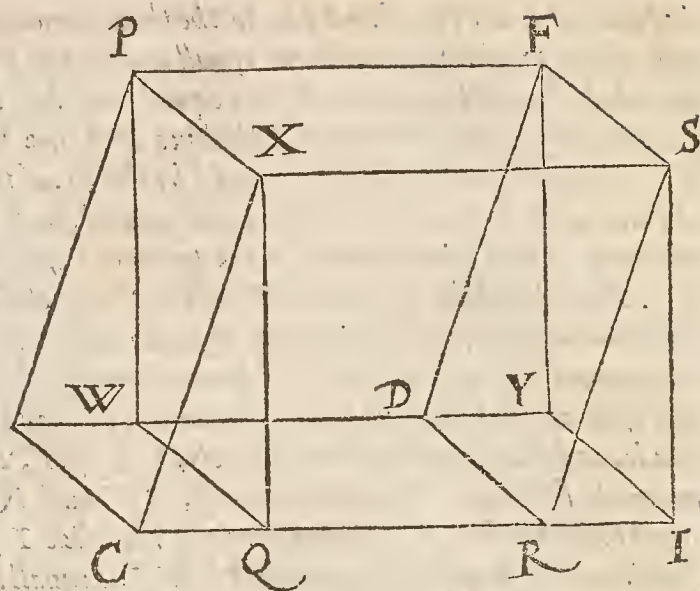
standing lines are vpon the selfe

same right lines. Wherefore also

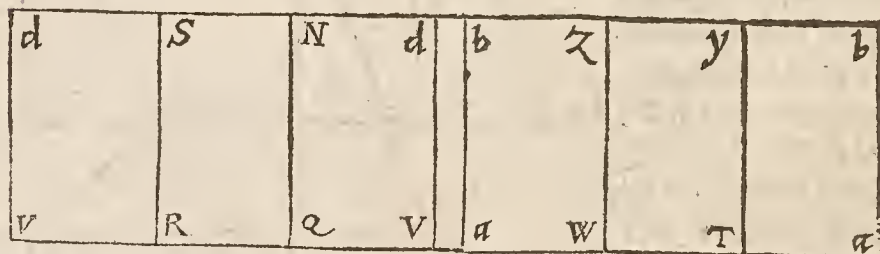
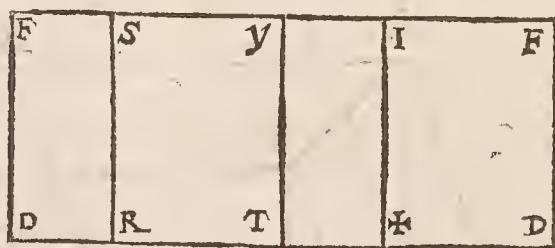
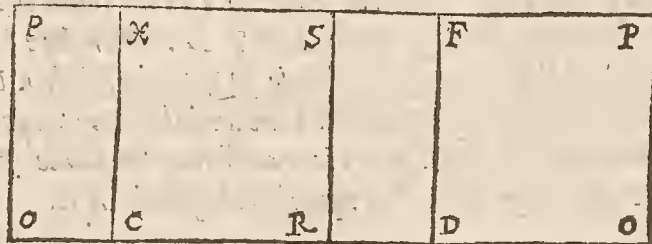
the solide A E is equall to the solide C F. Wherefore Parallelipipedons consisting vpon equall

bases and being vnder one and the selfe same altitude, are equall the one to the other: which

was required to be demonstrated.



The demonstration of the first case of this proposition is very hard to conceave by the figure described for it in a playne. And yet before M. Dee inuented that figure which we haue there placed for it, it was much harder. For both in the Greke and Lattin *Euclide*, it is very ill made, and it is in a maner impossible to conceave by it the construction and demonstration thereto appertayning. Wherefore I haue here described other figures, which first describe vpon pasted paper, or such like matter and then order them in maner following.



As touching the solide A E in the first case, I neede not to make any new description. For it is playne enough to conceave as it is there drawne. Although you may for your more ease of imagination describe of pasted paper a parallelipipedon hauing his sides equall with the sides of the parallelipipedon A E before described, and hauing also the sixe parallelogrammes thereof (contayned vnder those sides) equiangle

equiangle with the fixe parallelogrammes of that figure, ech side and eche angle equall to his correspondent side, and to his correspondent angle.

But concerning the other solide. When ye haue described these three figures vpon pasted paper: Where note for the proportion of eche line, to make your figure of pasted paper equall with the figure before described vpon the playne; let your lines OP, CX, RS, DF , &c. namely; the rest of the standing lines, of these figures, be equall to the standing lines OP, CX, RS, DF , &c. of that figure. Likewise let the lines OC, CR, RD, DO , &c. namely, the sides which cōtaine the bases of these figures be equal to the lines OC, CR, RD, DO , &c. namely, to the sides which cōtaine the bases of that figure. Moreover let the lines PX, XS, SF, FP , &c. namely, the rest of the lines which cōtaine the vpper superficieses of these figures, be equal to the lines PX, XS, SF, FP , &c. namely, to the rest of the lines which cōtaine the vpper superficieses of that figure (to haue described all those foresaid lines of these figures equal to all the lines of that figure, would haue required much more space then here can be spared: I haue made them equall onely to the halues of those lines, but by the example of these ye may, if ye will describe the like figures hauing their lines equall to the whole lines of the figure in the playne, eche line to his correspondent line). When I say ye haue as before is taught described these three figures, cut finely the lines XC, SR, FD of the first figure, and the lines SR, YR , and XT of the second figure: likewise the lines SR, NQ, ZV , and YR , of the third figure, and fold these figures accordingly, which ye can not chuse but doo if ye marke well the letters of euery line.

The three former figures being after this sort described, set them vpon this figure here described vpon a playne, as vpo their bases, namely, the lines $OC, CR, RD, DO: RT, T\text{X}, \text{X}D, DR: VR, RT, TW, Wa, VQ$, and QR of these three figures vpon the lines correspondent vnto them in this figure.

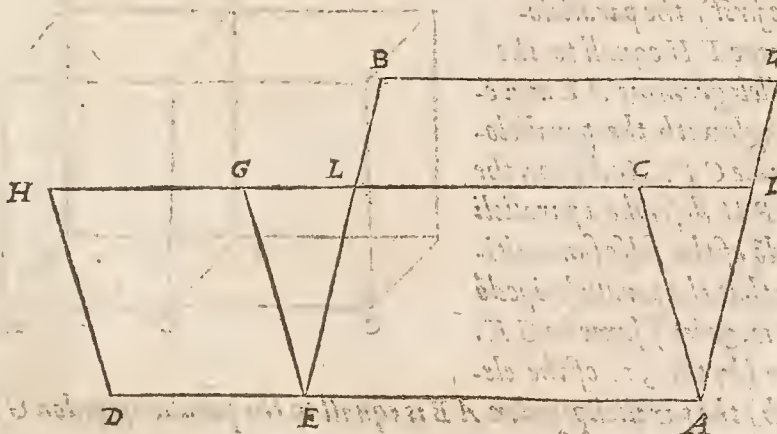
And they so standing compare them with the construction and demonstration of the first case, and they will geue great light vnto it. This also ye must note, that if ye make the lines of the foresayd three figures equall to the lines of the figure of the plaine described before in the demonstration of the first case: then must ye make a new base for them like vnto this, which is easy to doo, if ye draw a parallelogramme equall and like to the parallelogramme $OCT\text{X}$, and the cut of from the same a parallelogramme $DR T\text{X}$ equall and in like sort situate to the parallelogramme

$DR T\text{X}$ of that figure: & vpon the line RT describe two parallelogrammes, the one equall like, and in like sort situate to the parallelogramme $RTQa$ of that figure, and the other equall, like and in like sort situate to the parallelogramme $RTVW$ of the same figure. The lines of this base which I haue here put are equall onely to the halues of the lines of that figure in the demonstration.

As touching the second case ye neede no new figures, for it is playne to see by the figures (now reformed by *M. Dee*) described for it in the playne, especially if ye remember the forme of the figure of the 29. proposition of this booke. Only that which there ye conceaue to be the base, imagine here in both the figures of this second case to be the vpper superficies opposite to the base, and that which was there supposed to be the vpper superficies conceaue here to be the base. Ye may describe them vpon pasted paper for your better sight, taking hede ye note the letters rightly according as the construction requireth.

Flussas demonstrateth this proposition an other way taking onely the bases of the solides, and that after this maner.

Take equall bases (which yet for the surer vnderstanding let be ytterly vnlike) namely, $AEBF$ and $ADCH$, and let one of the sides of eche concurre in one & the same right line AED , & the bases being vpon one and the selfe same playne let there be supposed to be set vpon the parallelipipeds vnder one & the selfe same altitude. Then I say that the solide set vpo the base AB is equal to the solide set vpon the base AH . By the poynt E draw vnto the line AC a



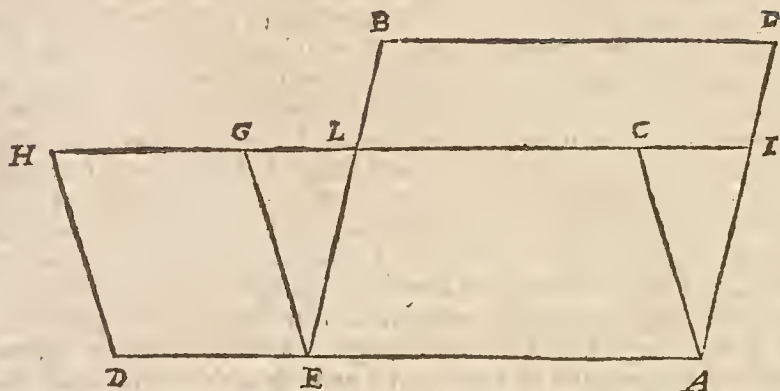
parallel

Construction.

Demonstration.

The eleuenth Booke

parallelline EG , which if it fall without the base AB , produce the right line HC to the poynt I . Now forasmuch as AB and AH are parallelogrāmes, therefore by the 24. of this booke, the triangles ACI and EGL shall be equaliter the one to the other: and by the 4. of the first, they shall be equiangle and equall: and by the first definition of the sixth, and fourth Proposition of the same, they shall be like. Wherefore Prisms erected vpon those triangles and vnder the same altitude that the solides AB and AH are, shall be equall and like, by the 8. definition of this booke. For they are containd vnder like playne superficieses equall both in multitude and magnitude. Adde the solide set vpon the base $ACLE$ common to them both. Wherefore the solide set vpon the base $AEGC$, is equall to the solide set vpon the base $AEIL$. And forasmuch as the superficieses $AEBF$, and $ADHC$ are equall (by supposition): and the part taken away AG is equall to the part taken away AL : therefore the residue BI shall be equall to the residue GD . Wherefore as AG is to GD as AL is to BI (namely, equals to equals), But as AG is to GD , so is the solide set vpon AG to the solide set vpon GD by the 25. of this booke, for it is cut by a playne superficies set vpon the line GE , which superficies is parallel to the opposite superficieses. Wherefore as AL is to BI , so is the solide set vpon AL to the solide set vpon BI . Wherefore (by the 11. of the fifth) as the solide set vpon AG (or vpon AL which is equall vnto it) is to the solide set vpon GD , so is the same solide set vpon AG or AL to the solide set vpon BI . Wherefore (by the 2. part of the 9. of the fifth) the solides set vpon GD and BI shall be equall. Vnto which solides if ye adde equall solides, namely, the solide set vpon AG to the solide set vpon GD , and the solide set vpon AL to the solide set vpon BI : the whole solides set vpon the base AH and vpon the base AB shall be equall. Wherefore Parallelipedons consistng vpon equall bases and being vnder one and the selfe same altitude are equall the one to the other: which was required to be proued.



The 27. Theoreme.

The 32. Proposition.

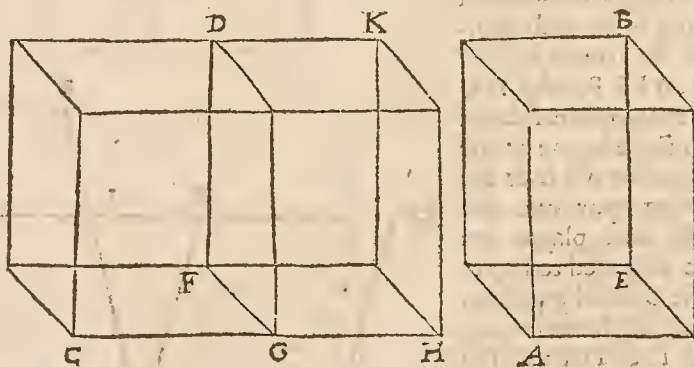
Parallelipedons being vnder one and the selfe same altitude, are in that proportion the one to the other that their bases are.



Suppose that these parallelipedons AB and CD be vnder one & the selfe same altitude. Then I say that those parallelipedons AB and CD are in that proportion the one to the other, that their bases are, that is, that as the base AE is to the base CF , so is the parallelipedon AB to the parallelipedon CD . Vpon the

Construction.

line FG describe (by the 45. of the first) the parallelogramme FH equall to the parallelogramme AE and equiangle with the parallelogramme CF . And vpon the base FH describe a parallelipedō of the selfe same altitude that the parallelipedō CD is, & let y same be GK .



Demonstration.

Now (by the 31. of the eleuenth) the parallelipedon AB is equall to the parallelipedon GK , for they consist vpon equall

quall bases, namely, AE and FH , and are under one and the selfe same altitude. And forasmuch as the parallelipipedon CK is cut by a plaine superficies DG , being parallel to either of the opposite plaine superficieses, therefore (by the 25. of the eleuenth) as the base HF is to the base FC , so is the parallelipipedon GK to parallelipipedon CD : but the base HF is equal to the base AE , and the parallelipipedon GK is proued equall to the parallelipipedon AB . Wherefore as the base AE is to the base CF , so is the parallelipipedon AB , to the parallelipipedon CD . Wherefore parallelipipedons being under one and the selfe same altitude, are in that proportion the one to the other that their bases are: which was required to be demonstrated.

I neede not to put any other figure for the declaration of this demonstration, for it is easie to see by the figure there described. Howbeit ye may for the more full sight thereof, describe solides of pasted paper according to the construction there set forth, which will not be hard for you to do, if ye remember the descriptions of such bodies before taught.

A Corollary added by *Fluſſas*.

Equall parallelipipedons contained under one and the selfe same altitude, haue also their bases equal. For if the bases should be vnequall, the parallelipipedons also should be vnequal by this 32. propositio. And equall parallelipipedons hauing equall bases, haue also one and the selfe same altitude. For if they should haue a greater altitude, they should exceede the equall parallelipipedons which haue the selfe same altitude: But if they should haue a lesse they should want so much of those selfe same equal parallelipipedons.

The 28. Theoreme.

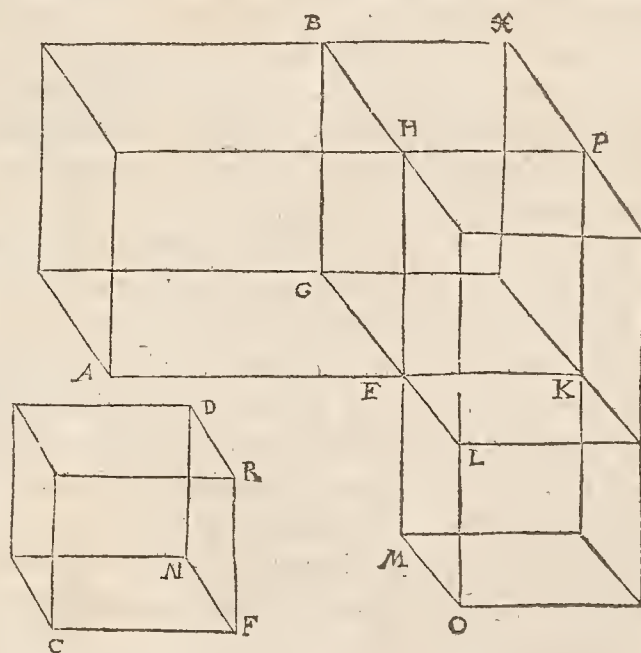
The 33. Proposition.

Like parallelipipedons are in treble proportion the one to the other of that in which their sides of like proportion are.

Suppose that these parallelipipedons AB and CD be like, & let the sides AE and CF be sides of like proportion. Then I say, the parallelipipedon AB is vnto the parallelipipedon CD in treble proportion of that in which the side AE is to the side CF . Extend the right lines AE , GE and HE to the pointes K , L , M . And

Construction.

(by the 2. of the first) vnto the line CF put the line EK equal, and vnto the line FN put the line EL equall, and moreouer vnto the line FR put the line EM equall, and make perfect the parallelogramme KL , and the parallelipipedon KO . Now forasmuch as these two lines EK and EL are equall to these two lines CF and FN , but the angle KEL is equall to the angle CFN (for the angle AEG is equall to the angle CFM by reason that the solides AB and CD are like). Wherefore the parallelogramme KL is equall and like to the parallelogramme CN , and by the same reason also the parallelogramme KM is equall and

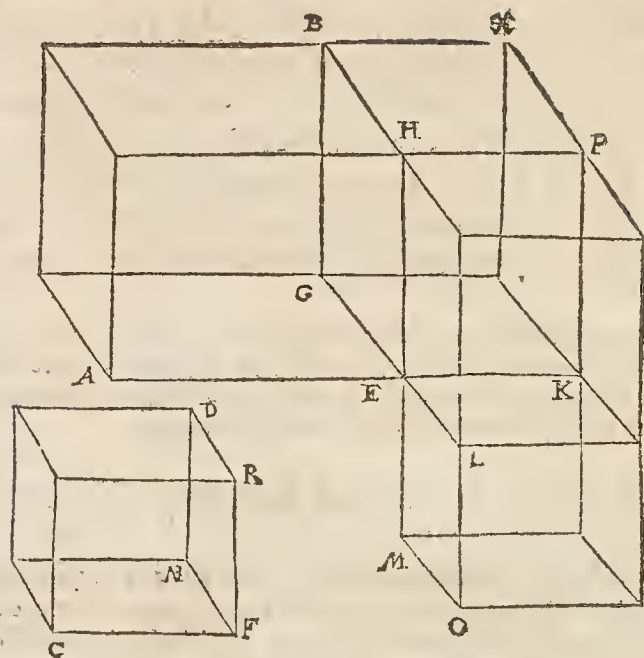


Demonstration.

like

The eleventh Booke

like to the parallelogramme C-
R, and moreover the parallelo-
gramme O E to the parallelo-
gramme F D. Wherefore three
parallelogrammes of the paralle-
lipipedon K O are like and equall
to three parallelogrammes of the
parallelipipedon C D: but those
three sides are equall and like to
the three opposite sides: wherefore
the whole parallelipipedon K O
is equal and like to the whole pa-
rallelipipedon C D. Make perfect
the parallelogramme G K. And
upon the bases G K and K L
make perfect to the altitude of
the parallelipipedon A B, the pa-
rallelipipedons E X & L P. And
forasmuch as by reason that the



parallelipipedons A B & C D are like, as the line A E is to the line C F, so is the line E G, to
the line F N, and the line E H to the line F R. But the line C F is equall to the line E K, and
the line F N to the line E L, and the line F R to the line E M, therefore as the line A E is to
the line E K, so is the line G E to the line E L, and the line H E to the line E M (by construc-
tion). But as the line A E is to the line E K, so is the parallelogramme A G to the parallelo-
gramme G K (by the first of the sixth). And as the line G E is to the line E L, so is the paral-
lelogramme G K to the parallelogramme K L. And moreover as the line H E is to the line E M,
so is the parallelogramme P E to the parallelogramme K M. Wherefore (by the 11. of the
fifth) as the parallelogramme A G is to the parallelogramme G K, so is the parallelogramme
G K to the parallelogramme K L, and the parallelogramme P E to the parallelogramme K M.
But as the parallelogramme A G is to the parallelogramme G K, so is the parallelipipedon
A B to the parallelipipedon E X, by the former proposition, and as the parallelogramme G K
is to the parallelogramme K L, so by the same is the parallelipipedon X E to the parallelipe-
don P L: and againe by the same, as the parallelogramme P E is to the parallelogramme K-
M, so is the parallelipipedon P L to the parallelipipedon K O. Wherefore as the parallelipipedo
A B is to the parallelipipedon E X, so is the parallelipipedon E X to the parallelipipedon P L,
and the parallelipipedon P L to the parallelipipedon K O. But if there be fower magnitudes
in* continuall proportion, the first shalbe vnto the fourth in treble propprtion that it is to the
second (by the 10. definition of the fifth). Wherefore the parallelipipedon A B is vnto the pa-
rallelipipedon K O in treble proportion that the parallelipipedon A B is to the parallelipipedon
E X. But as the parallelipipedon A B is to the parallelipipedo E X, so is the parallelogramme
A G to the parallelogramme G K, and the right line A E to the right line E K. Wherefore
also the parallelipipedon A B is to the parallelipipedon K O in treble proportio of that which
the line A E hath to the line E K. But the parallelipipedon K O is equall to the parallelipe-
pedon C D, and the right line E K to the right line C F. Wherefore the parallelipipedon A B
is to the parallelipipedon C D in treble proportion that the side of like proportion, namely, A-
E is to the side of like proportion, namely, to C F. Wherefore like parallelipipedons are in tre-
ble proportion the one to the other of that in which their sides of like proportion are: which
was required to be demonstrated.

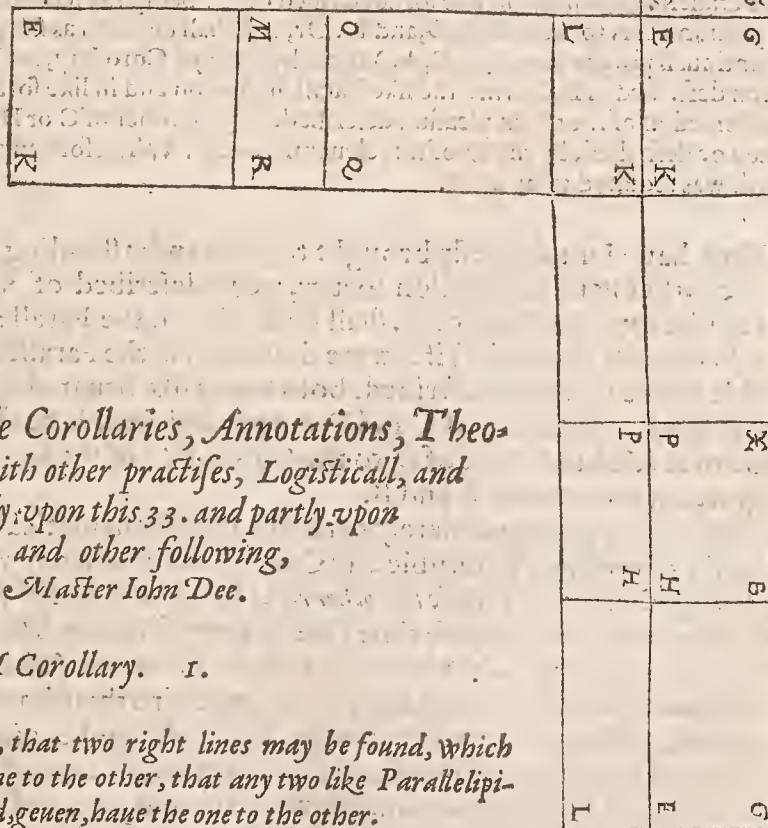
Because

¶ Corellary.

Hereby it is manifest, that if there be fower right lines in ^{*}continually proportion, as the first is to the fourth, so shall the Parallelipipedon described of the first line, be to the Parallelipipedon described of the second, both the Parallelipipedons being like and in like sort described.

For the first line is to the fourth in treble proportion that it is to the second: and it hath before bene proued that the Parallelipipedon described of the first, is to the Parallelipipedon described of the second, in the same proportion that the first line is to the fourth:

Because the one of the figures before, described in a plaine, pertayning to the demonstration of this 33. Proposition, is not altogether so easie to a younge beginner to conceaue, I haue here for the same described an other figure, which if ye first drawe vpon pasted paper, and afterward cut the lines & folde the sides accordingly, will agree with the construction & demonstration of the sayd Proposition. Howbeit this ye must note that ye must cut the lines O Q & M R on the contrary side to that which ye cut the other lines. For the solides which haue to their base the parallelogramme L K are set on vpward and the other downward: Ye may if ye thinke good describe after the same maner of pasted paper a body equall to the solide C D: though that be easie enough to conceaue by the figure thereof described in the plaine.



¶ Certaine most profitable Corollaries, Annotations, Theoremes, and Problemes, with other practises, Logisticall, and Mechanicall, partly vpon this 33. and partly vpon the 34. 36. and other following, added by Master Iohn Dee.

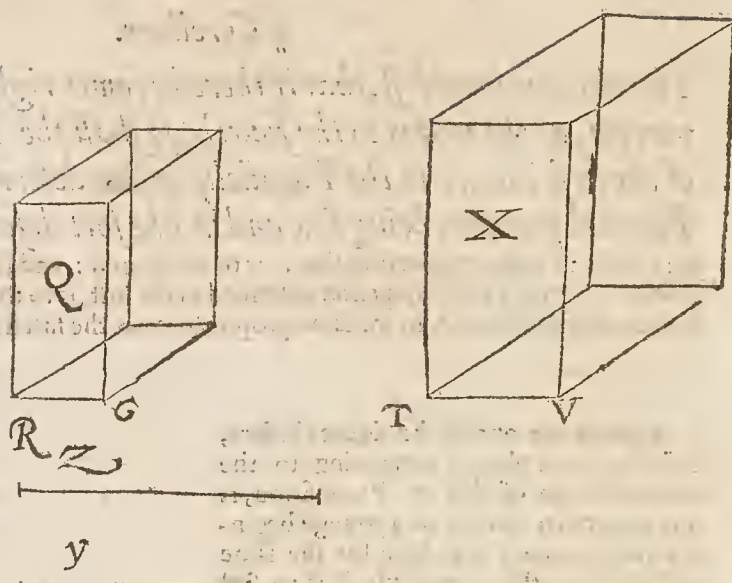
¶ A Corollary. 1.

1. Hereby it is manifest, that two right lines may be found, which shall haue that proportion, the one to the other, that any two like Parallelipipedons, and in like sort described, haue the one to the other.

Suppose Q and X to be two like Parallelipipedons, and in like sort described. Of Q take any of the three lines, of which it is produced: as namely, R G. Of X, take that right line of his production, which line is aunswerable to R G in proportion (which most aptly, after the Greke name, may be called AAa.j, called

The eleuenth Booke

called Omologall to R G) & let that be T V. By the 11. of the sixth, to R G. and T V, let the third line in proportion with them be founde, and let that be Y. By the same 11. of the sixth, to T V and Y, let the thirde right line be found, in the sayd proportion of T V to Y: & let that be Z. I say now that R G hath that proportion to Z, which Q hath to X. For by construction, we haue fower right lines in continuall proportion, namely, R G, T V, Y, and Z. Wherefore by *Euclides* Corollary, here before, R G is to Z, as Q is to X. Wherefore we haue found two right



lines hauing that proportion the one to the other, which any two like Parallelipipedons of like description, geuen, haue the one to the other : which was required to be done.

¶ A Corollary.

As a Conuerse, of my former Corollary, doth it followe : To finde two like Parallelipipedons of like description, which shall haue that proportion the one to the other, that any two right lines geuen, haue the one to the other.

Suppose the two right lines geuen to be A and B : Imagine of foure right lines in continuall proportion, A, to be the first, and B to be the fourth : (or contrariwise, B to be first, & A to be the fourth). The second and third are to be found, which may, betwene A & B, be two meanes in continuall proportion : as now, * suppose such two lines, found : and let them be C and D. Wherefore, by *Euclides* Corollary, as A is to B (if A were taken as first) so shall the Parallelipipedon described of A, be to the like Parallelipipedon and in like sort described of C : being the second of the fower lines in continuall proportion : it is to wete, A, C, D, and B. Or, if B shall be taken as first, (and that thus they are orderly in continuall proportion, B, D, C, A,) then, by the sayd Corollary, as B is to A, so shall the Parallelipipedon described of B, be vnto the like Parallelipipedon and in like sort described of D. And vnto a Parallelipipedon of A or B, at pleasure described, may an other of C or D be made like, and in like sort situated or described, by the 27. of this eleuenth booke. Wherefore any two right lines being geue, &c: which was required to be done.

* Note this famous Lemma.

The doubling of the Cube.

* Note.

Thus haue I most briefly brought to your vnderstanding, if (first) B were double to A, then what Parallelipipedon soeuer, were described of A, the like Parallelipipedon and in like sort described of C, shall be double to the Parallelipipedon described of A. And so likewise (secondly) if A were double to B, the Parallelipipedon of D, shoulde be double to the like, of B described, both being like situated. Wherefore if of A or B, were Cubes made, the Cubes of C and D are proued double to them : as that of C, to the Cube of A : and the Cube of D to the Cube of B ; in the second case. * And so of any proportion els betwene A and B.

Now also do you most clerely perceau the Mathematicall occasion, whereby (first of all men) *Hippocrates*, to double any Cube geuen, was led to the former Lemma: Betwene any two right lines geuen, to finde two other right lines, which shall be with the two first lines, in continuall proportion : After whose time (many yeares) diuine *Plato*, *Heron*, *Philo*, *Appollonius*, *Diocles*, *Pappus*, *Sporus*, *Menechmus*, *Archytas Tarentinus* (who made the wodden doue to flye) *Eratosthenes*, *Nicomedes*, with many other (to their immortall fame and renowme) published, * diuers their witty deuises, methods, and engines (which yet are extant) whereby to execute thys Problematicall Lemma. But notwithstanding all the trauailes of the foresayd Philosophers and Mathematiciens, yea and all others doinges and contriuinges (vnto this day) about the sayd Lemma, yet there remaineth sufficient matter, Mathematically so to demonstrate the same, that most exactly & readily, it may also be Mechanically practised : that who soeuer shall achieue that feate, shall not be counted a second *Archimedes*,

* All their wayes of executing the Lemma are in Eutocius commentaries & po Archimedes booke de Sphera & Cy-lindro: where Archimedes setteth the same Lemma.

medes, but rather a pereles Mathematicien, and Mathematicorum Princeps. I will sundry wayes (in my brieve additions and annotations vpon *Euclide*) excite you thereto, yea and bring before your eyes sundry new wayes, by me inuented: and, in this booke so placed, as matter thereof, to my inuentions appertayning, may geue occasion: Leauing the farther, full, & absolute my concluding of the Lemma, to an other place and time: which will, now, more cōpendiously be done: so great a part therof, being before hand in thys booke published.

Note what is yet lacking requisite to the doubling of the Cube.

¶ A Corollary added by *Flussas*.

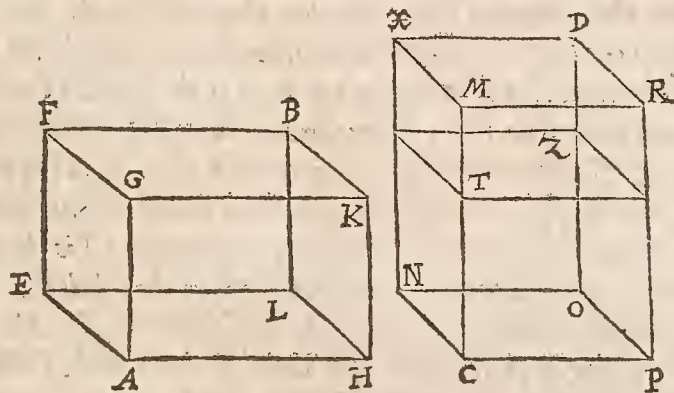
Parallelipedons consisting vpon equall bases, are in proportion the one to the other as their altitudes are. For if those altitudes be cut by a plaine superficies parallel to the bases: the sections shall be in proportion the one to the other as the sections of the bases cut, by the 25. of this booke. Which sections of the bases are the one to the other in that proportion that their sides or the altitudes of the solides are, by the first of the sixt. Wherefore the solides are the one to the other as their altitudes are. But if the bases be vnlike, the selfe same thing may be proued by the Corollary of the 25. of this booke, which by the 25. Proposition was proued in like bases.

¶ The 29. Theoreme.

The 34. Proposition.

In equall Parallelipedons the bases are reciprokall to their altitudes. And Parallelipedons whose bases are reciprokall to their altitudes, are equall the one to the other.

Suppose that these Parallelipedons AB & CD be equall the one to the other. Then I say, that the bases of the Parallelipedons AB and CD are reciprokall to their altitudes, that is, as the base EH is to the base NP , so is the altitude of the solide CD to the altitude of the solide AB . First let the standing lines AG, EF, LB, HK , of the solide AB , & the standing lines CM, NX, OD , and PR , of the solide CD , be erected perpendicularly to the bases EH & NP . The I say, that as the base EH is to the base NP , so is the line CM to the line AG . Now if the base EH be equal to the base NP , and the solide AB is equall to the solide CD , wherefore the line CM is equall to the line AG *. For if the bases EH and NP being equall, the altitudes AG and CM be not equall, neither also shall the solide AB be equall to the solide CD , but they are supposed to be equall. Wherefore the altitude CM is not vnequall to the altitude AG . Wherefore it is equall. And therefore as the base EH is to the base NP , so is the altitude CM to the altitude AG . Wherefore it is manifest, that the bases of the Parallelipedons AB and CD are reciprokall to their altitudes.



Two cases in the first part of this proposition. First case, which also may be two wayes. First way.

* This followeth also of the Corollary added of *Flussas* after the 33. proposition of this booke.

But now suppose that the base EH be not equall to the base NP . But let the base EH be the greater. Now the solide AB is equall to the solide CD . Wherefore also the altitude CM is greater then the altitude AG *. For if not, then againe are not the solides AB and CD equall: but they are by supposition equall. Wherefore (by the 2. of the first) put vnto the line AG an equall line CT . And vpon the base NP and the altitude being CT , make perfecte a solide cotained vnder parallel plaine superficieses, and let the same be CZ . And forasmuch

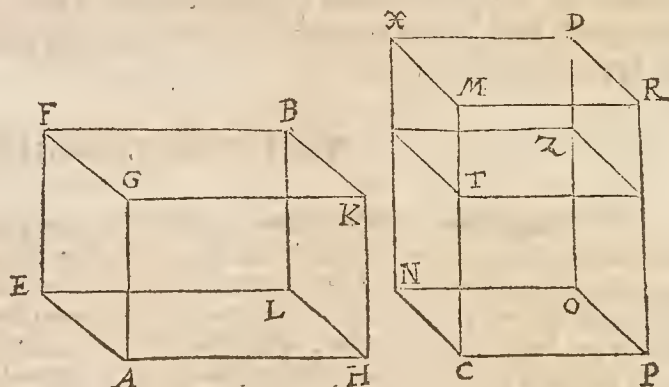
Second way.

* This also followeth of the former Corollary.

$AA.ij.$ as the

The eleventh Booke

as the solide AB is equall to the solide CD , and there is a certaine other solide, namely, CZ , but unto one and the selfe same magnitude equall magnitudes haue one and the selfe same proportion (by the 7. of the fift). Wherefore as the solide AB is to the solide CZ , so is the solide CD to the solide CZ . But as the solide AB is to the solide CZ , so is the base EH to the base NP (by the 32. of the eleventh): for the solides AB and CZ are vnder equall altitudes. And as the solide CD is to the solide CZ , so is the base MP to the base PT , and the line MC to the line CT . Wherefore (by the 11. of the fift) as the base EH is to the base NP , so is the line CM to the line CT . But the line CT is equall to the line AG . Wherefore (by the 7. of the fift) as the base EH is to the base NP , so is the altitude CM to the altitude AG . Wherefore in these Parallelipipedons AB and CD the bases are reciprokall to their altitudes.



The conuerse
of both the
partes of the
first case.

But now againe suppose that the bases of the Parallelipipedons AB and CD be reciprokall to their altitudes, that is, as the base EH is to the base NP , so let the altitude of the solide CD be to the altitude of the solide AB . Then I say, that the solide AB is equall to the solide CD . For againe let the standing lines be erected perpendicularly to their bases.

1. And now if the base EH be equall to the base NP : but as the base EH is to the base NP , so is the altitude of the solide CD to the altitude of the solide AB . Wherefore the altitude of the solide CD is equall to the altitude of the solide AB . But Parallelipipedons consisting vpon equall bases and vnder one and the selfe same altitude, are (by the 31. of the eleventh) equall the one to the other. Wherefore the solide AB is equall to the solide CD .

2. But now suppose that the base EH be not equall to the base NP : but let the base EH be the greater. Wherefore also the altitude of the solide CD , that is, the line CM is greater then the altitude of the solide AB , that is, then the line AG . Put againe (by the 3. of the first) the line CT equall to the line AG , and make perfecte the solide CZ . Now for that as the base EH is to the base NP , so is the line MC to the line AG . But the line AG is equall to the line CT . Wherefore as the base EH is to the base NP , so is the line CM to the line CT . But as the base EH is to the base NP , so (by the 32. of the eleventh) is the solide AB to the solide CZ , for the solides AB and CZ are vnder equall altitudes. And as the line CM is to the line CT , so (by the 1. of the sixt) is the base MP to the base PT , and (by the 32. of the eleventh) the solide CD to the solide CZ . Wherefore also (by the 11. and 9. of the fift) as the solide AB is to the solide CZ , so is the solide CD to the solide CZ . Wherefore either of these solides AB and CD haue to the solide CZ one and the same proportion. Wherefore (by the 7. of the fift) the solide AB is equall to the solide CD : which was required to be demonstrated.

Second case.

Construction.

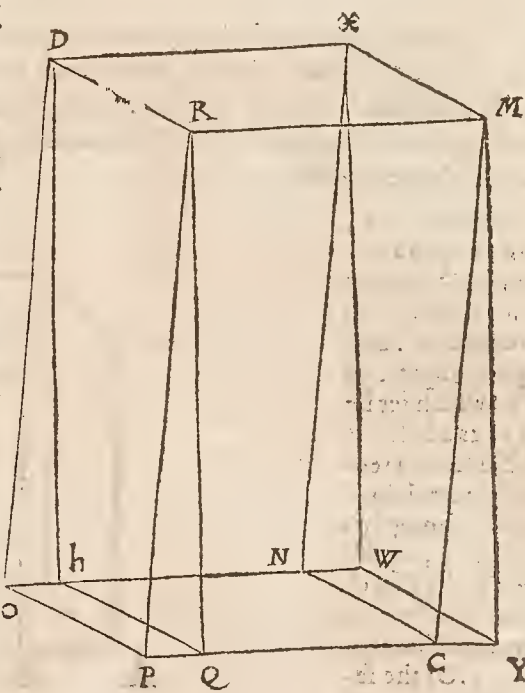
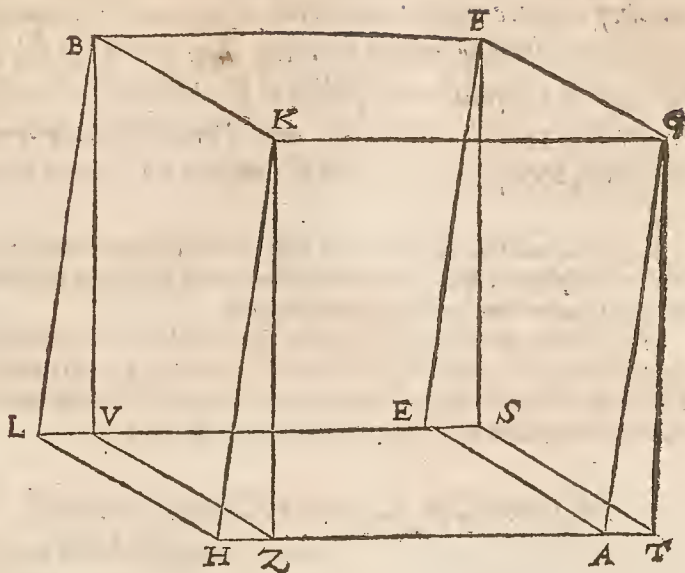
But now suppose that the standing lines, namely, $FE, BL, GA, KH, XN, DO, MC$, and RP , be not erected perpendicularly to their bases. And (by the 11. of the eleventh) from the pointes F, G, B, K, X, M, D, R , draw vnto the plaine superficies of the bases EH and NP perpendicular lines, and let those perpendicular lines light vpo the pointes S, T, V, Z, W, Y, d , and Q , and make perfecte the Parallelipipedons FZ , and XQ . I say that, euen in this case also, if the solides AB and CD be equall, their bases are reciprokall to their altitudes, that is, as the base EH is to the base NP , so is the altitude of the solide CD to the altitude of the solide AB . For forasmuch as the solide AB is equall to the solide CD

CD, but the solide AB is equall to the solide BT (by the 20. of the eleuenth) for they are vpon one and the selfe same base, namely, the parallelogramme KF, and vnder one and the selfe same altitude, whose standing lines are in the selfe same right lines, namely, HZ AT, and LV ES: and the solide CD is by the same reason equall to the solide DY, for they both consist vpon one and the selfe same base, namely, the parallelogramme XR, and are vnder one and the selfe same altitude, whose standing lines are in the selfe same right lines, namely, PQ CY, and OH NW. Wherefore the solide BT is equall to the solide DY. But in equall Parallelipipedons, whose altitudes are erected perpendicularly to their bases, their bases are reciprokall to their altitudes (by the first part of this Proposition). Wherefore as the base FK is to the base XR, so is the altitude of the solide DY to the altitude of the solide BT. But the base FK is equall to the base EH, and the base XR to the base NP. Wherefore as the base EH is to the base NP, so is the altitude of the solide DY to the altitude of the solide BT. But the altitudes of the solides DY & BT, and of the solides DC & BA are one and the selfe same. Wherefore as the base EH is to the base NP, so is the altitude of the solide CD to the altitude of the solide AB. Wherefore the bases of the Parallelipipedons AB and CD are reciprokall to their altitudes.

Again suppose that the bases of the Parallelipipedons AB and CD be reciprokall to their altitudes, that is, as the base EH is to the base NP, so let the altitude of the solide CD be to the altitude of the solide AB. Then I say, that the solide AB is equall to the solide CD. For the same order of construction remayning, for that as the base EH is to the base NP, so is the altitude of the solide CD to the altitude of the solide AB: but the base EH is equall to the parallelogramme FK, and the base NP to the parallelogramme XR: wherefore as the base FK is to the base XR, so is the altitude of the solide CD to the altitude of the solide AB. But the altitudes of the solides AB and BT are equall, and so also are the altitudes of the solides DC and DY. Wherefore as the base FK is to the base XR, so is the altitude of the solide DY to the altitude of the solide BT. Wherefore the bases of the Parallelipipedons BT and DY are reciprokall to their altitudes. But Parallelipipedons whose altitudes are erected perpendicularly to their bases, and whose bases are reciprokall to their altitudes, are equall the one to the other (by this Proposition). Wherefore the solide BT is equall to the solide DY. But the solide BT is equall to the solide BA (by the 29. of the eleuenth) for they consist vpon one and the selfe same base, namely, FK, and are vnder one and the selfe same altitude, whose

standing

Demonstration.



The conuerse of the second case.

standing

The eleventh Booke

The generall
conclusion.

standing lines are in the selfe same right lines. And the solide DY is equall to the solide DC , for they consiste vpon one and the selfe same base, namely, XR , and are vnder one and the selfe same altitude, whose standing lines are in the selfe same right lines. Wherefore also the solide AB is equall to the solide CD . Wherefore in equall Parallelipipeds, the bases are reciprokall to their altitudes. And Parallelipipeds whose bases are reciprokall with their altitudes, are equall the one to the other: which was required to be proued.

The demonstration of the first case of this Proposition is easie to conceaue by the figure as it is described in the plaine. But ye may for your more full sight describe Parallelipipeds of pasted paper, according as the construction teacheth you.

And for the second case, if ye remeber well the forme of the figures which you made for the second case of the 3rd. Proposition: and describe the like for this, taking heede to the letters that ye place them like as the construction in this case requireth, ye shall most easly by them come to the full vnderstanding of the construction and demonstration of the said case.

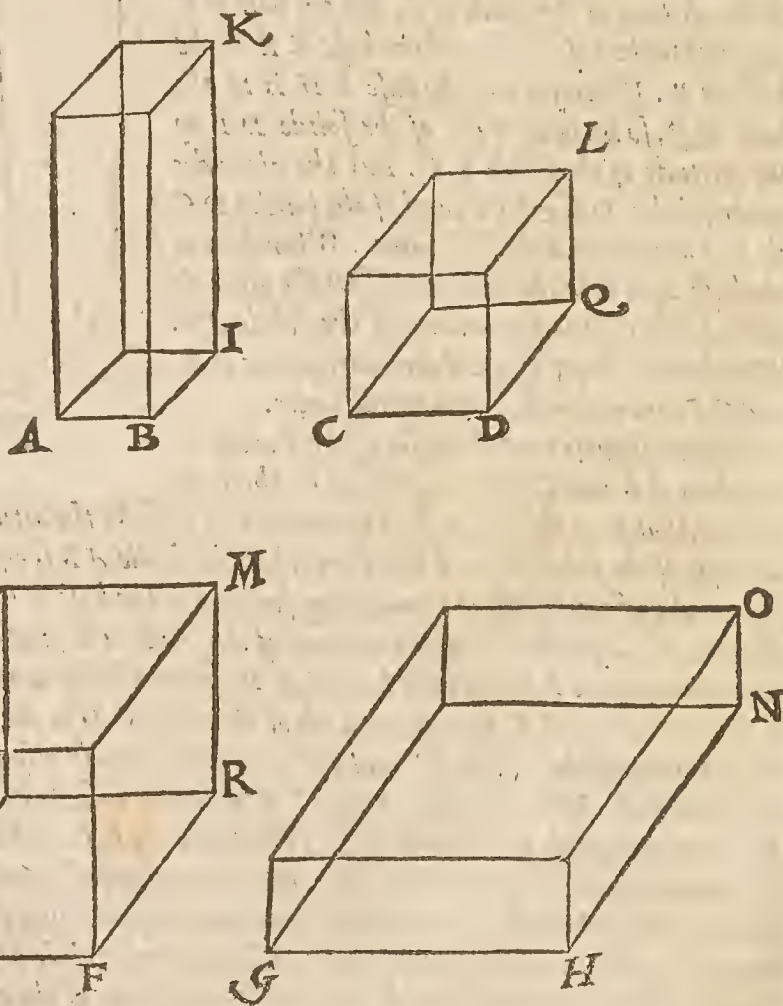
*M. Iohn Dee, his sundry Inuentions and Annotacions, very necessary,
here to be added and considered.*

A Theoreme.

If fower right lines be in continuall proportion, and vpon the square of the first, as a base, be erected a rectangle parallelipedon, whose heith is the fourth line: that rectangle parallelipedon is equal to the Cube made of the second line. And if vpon the square of the fourth line, as a base, be erected a rectangle parallelipedon, whose heith is the first line, that parallelipedon is equall to the Cube made of the third line.

Construction.

Suppose AB , CD , EF , and GH to be fower right lines in continuall proportion: and vpon the square of AB (which let be AI) as a base, let be erected a rectangle parallelipedon, hauing his heith IK , equall to GH , the fourth line. And let that parallelipedon be AIK . Of the second line CD , let a Cube be made: whose square base, let be noted with CQ : and let his heith be noted by QL : & let the whole Cube be signified by CL . I say that AIK is equall to CL . Let the like construction be for the cube of the third line: that is, vpon the square of GH (which suppose to be GN) let a rectangle parallelipedon be e-



rected

rected, hauyng his heith NO , equall to AB , the first line : which parallelipedon let be noted with GO . And suppose the cube of the third line (EF) to be EM : whose square base, let be noted by ER : and hys heith by RM . I say now (secondly) that GO is equal to EM . For the first part consider, that AI (the square base of AK) is to CQ , the square base of CL , as AB is to the third line EF , by the 2. Corollary of the 20. of the sixth. But as AB , is to EF , so (by alternate proportion) is CD to GH , to CD . The cubes roote, is QL , the same cubes heith equall: and to GH is IK (by construction) equall: wherefore, as AI is to CQ , so is QL to IK . The bases therefore and heithes of AK and CL , are reciprocally in proportion: wherefore by the second part of this 34. proposition, AK and CL are equall. For prooffe of the second part of my theoreme, I say, that as AB , CD , EF , and GH , are in continuall proportion forward, so are they backward in continuall proportion, as by the fourth of the fift may be proued. Wherefore now considering GH to be as first, and so AB to be the fourth: the square base GN , is to the square base ER , as GH is to CD , by the 2. corollary of the 20. of the sixth: But as GH is to CD , so is EF to AB , by alternate proportion: to the Cubik roote EF , is RM (the heith of the same Cube EM) equall. And to AB , is the heith NO equall, by construction: wherefore as GN is to ER , so is RM to NO . Therefore by the second part of this 34. proposition, GO is equall to EM . If fowre right lines (therefore) be in continuall proportion &c. as in the proposition: which was required to be demonstrated.

*Demonstratio
of the first
part.*

*Demonstratio
of the second
part.*

A Corollary logistical.

Of my former Theoreme it followeth: Any two numbers being geuen, betwene which two we would haue two other numbers, middle, in continuall proportion: That if we multiply the square of the first number geuen, by the other geuen number (as if it were the fourth): the roote Cubik of that of come or product, shall be the second number sought: And farther, if we multiply the square of the other number geuen, by the first geuen number, the roote Cubike of that of come shall be the thirde number sought.

*To finde two
middle propor
tionals be
twene two
numbers geue.*

For (by my Theoreme) those rectangle parallelipedons made of the squares of the first & fourth, multiplied by the fourth & the first, accordingly, are equall to the Cubes made of the second & third numbers: Which we make our two middle proportionals. Wherefore of those parallelipedons (as Cubes) the Cubik rootes, by good and vsuall arte sought and found, geue the very two middle numbers desired. And where those numbers, are not by logistical consideration accounted Cubik numbers, ye may use the logistical secret of approaching nere to the precise verytpe: so that therof most easily you shall perceauie, that your fayle is of the sence neuer to be perceaued: it is to wete, as in a lyne of an inch long, not to want or excede the thousand thousand part: or farther you may (infinitely approche at pleasure. O Mechanicall friend, be of good comfort, put to thy hand: Labor improbus, omnia vincit.

*Note the prac
tise of app
ching to pre
cienes in Cu
bik rootes.*

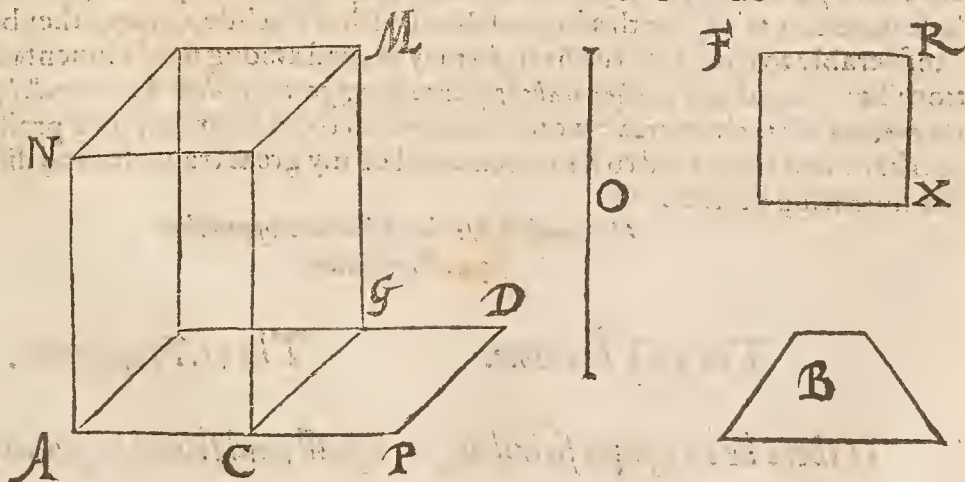
A Probleme. I.

Vppon a right lined playne superficies geuen, to apply a rectangle parallelipedon geuen.

Or we may thus expresse the same thing.

Vppon a right lined playne superficies geuen, to erect a rectangle parallelipedon, equall to a rectangle parallelipedon geuen.

Suppose the right lined playne superficies geuen to be B : and the rectangle parallelipedon geuen to be AM . Vppon B , as a base must AM be applied: that is, a rectangle parallelipedon must be erected vppon B , as a base, which shall be equall to AM . By the laste of the second, to the right lynd figure B , let an equall square be made: which suppose to be FRX . *Produce one side of the base of the pa-



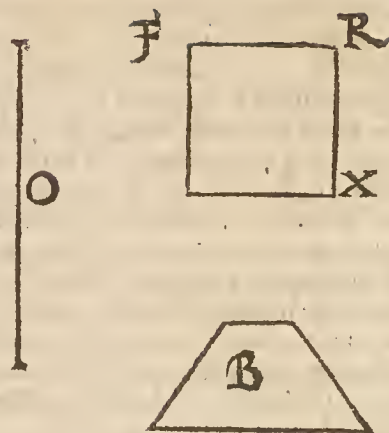
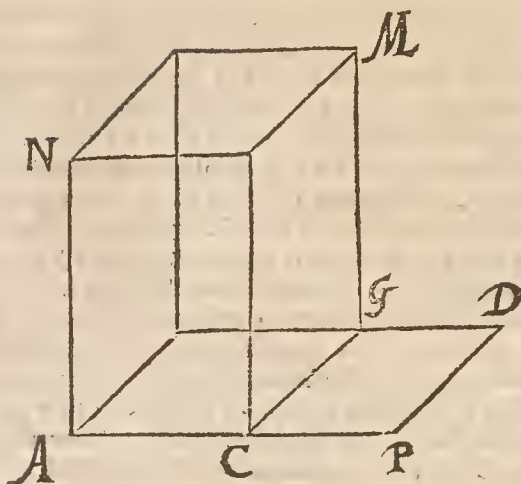
AAa.iiij.

parallelipedon

** This is the
way to apply
any square
geuen, to a
line also geue,
sufficiently ex
tended.*

The eleuenth Booke

parallepipeds
AM, which
let be AC,
extended to
the point P.
Let the o-
ther side of
the sayde
base, con-
curring with
AC, be CG.
As CG is to
FR (the side
of the square
FRX) so let
the same FR
be to a line
cut off from



CP sufficiently extended: by the 11. of the sixth: and let that third proportionall line be CP. Let the rectangle parallelogramme be made perfect, as CD. It is euident, that CD, is equall to the square FRX by the 17. of the sixth: and by construction FRX, is equall to B. Wherefore CD, is equall to B. By the 12. of the sixth, as CP, is to AC, so let AN (the heith of AM) be to the right line O. I say that a solide perpendicularly erected vppon the base B, hauinge the heith of the line O, is equall to the parallelipedon AM. For CD is to AG, as CP is to AC by the firste of the sixth, and B is proued equall to CD: Wherefore by the 7. of the fifth, B is to AG as CP is to AC: But as CP is to AC, so is AN to O, by construction: Wherefore B is to AG as AN is to O. So than the bases B and AG are reciprocally in proportion with the heithes AN and O. By this 34 therefore, a solide erected perpendicularly vppon B as a base, hauing the height O, is equall to AM. Wherefore vppon a right lyned playn superficies geuen, we haue applied a rectangle parallelipedon geuen: Which was requisite to be donne.

A Probleme 2.

A rectangle parallelipedon being geuen to make an other equall to it of any heith assigned.

Suppose the rectangle parallelipedon geuen to be A, and the heith assigned to be the right line B: Now must we make a rectangle parallelipedon, equal to A: Whose heith must be equall to B. According to the manner before vsed, we must frame our construction to a reciprokall proportio betwene the bases and heithes. Which will be done if, as the heith assigned beareth it selfe in proportion to the heith of the parallelipedon giuen: so, one of the sides of the base of the parallelipedon giuen, be to a fourth line, by the 12. of the sixth found. For that line founde, and the other side of the base of the geuen parallelipedon, contayne a parallelogramme, which doth serue for the base, (which onely, we wanted) to vse with our giuen heith: and so is the Probleme to be executed.

Note.

Euclide in the 27. of this eleuenth hath taught, how, of a right line geue, to describe a parallepipeds, like, & likewise situated, to a parallepipeds geue: I haue also added, How, to a parallepipeds geuen, an other may be made equall, vppon any right lined base geuen, or of any heith assigned: But if either Euclide, or any other before our time (answerably to the 25. of the sixth, in playns) had among solids inuented this proposition: Two vnequall and vnlke parallelipedons being geuen, to describe a parallelipedon equall to the one, and like to the other, we would haue geuen them their deserued praise: and I would also haue ben right glad to haue ben eased of my great trauayles and discourses about the inuenting thereof.

A probleme
worth the
searching
for.

Here ende I. Dee his additions vppon this
34. Proposition.

The 30. Theoreme.

The 35. Proposition.

If there be two superficial angles equall, and from the pointes of those angles

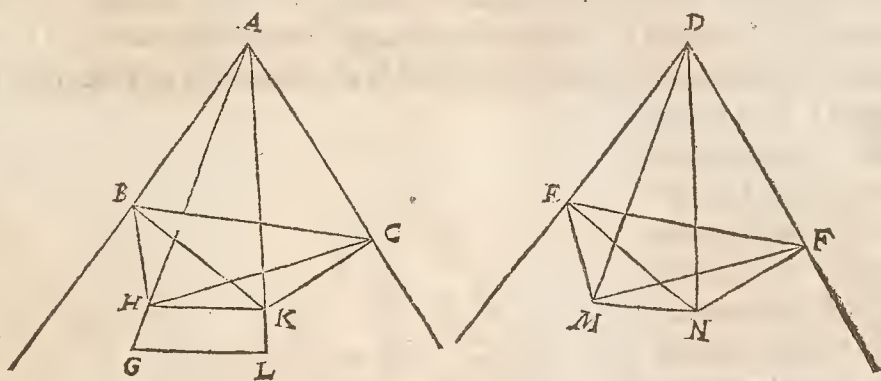
gles be eleuated on high right lines, comprehending together with those right lines which containe the superficiall angles, equall angles, eche to his corespondent angle, and if in eche of the eleuated lines be take a point at all auentures, and from those pointes be drawen perpendicular lines to the ground playne superficieses in which are the angles geuen at the beginning, and from the pointes which are by those perpendicular lines made in the two playne superficieses be ioyned to those angles which were put at the beginning right lines: those right lines together with the lines eleuated on high shall contayne equall angles.



Suppose that these two rectiline superficiall angles BAC , and EDF be equall the one to the other: and from the pointes A and D let there be eleuated upward these right lines AG and DM , comprehending together with the lines put at the beginninge equall angles, ech to his correspondent angle, that is, the angle $MD E$ to the angle GAB , and the angle $MD F$ to the angle GAC , and take in the lines AG and DM pointes at all auentures and let the same be G and M . And (by the 11. of the eleuenth) from the pointes G and M draw vnto the ground playne superficieses wherein are the angles BAC and E

Construction.

DF perpendicular lines GL and MN and let them fall in the sayd playne superficieses in the pointes N and L , and drawe a right line from the point L to the point A and an other from the pointe N to the

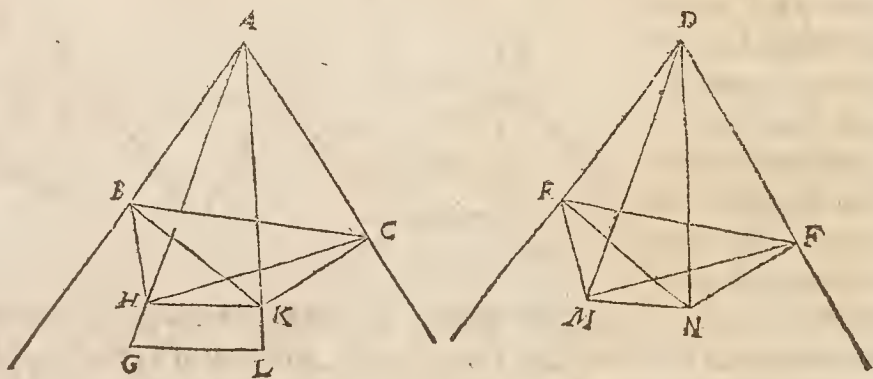


pointe D . Then I say that the angle GAL is equall to the angle MDN . Fro the greater of the two lines AG and DM , (which let be AG) cut of by the 3. of the first the line AH equall vnto the line DM . And (by the 31. of the first) by the point H , drawe vnto the line GL a parallel line, and let the same be HK . Now the line GL is erected perpendicularly to the grounde playne superficies BAL : Wherefore also (by the 8. of the eleuenth) the line HK is erected perpendicularly to the same grounde plaine superficies BAC . Drawe (by the 12. of the first) fro the pointes K and N vnto the right lines $AB, AC, DF, \& DE$ perpendiclar right lines, and let the same be KC, NF, KB, NE . And drawe these right lines HC, CB, MF, FE . Now forasmuch as (by the 47. of the first) the square of the line HA is equall to the squares of the lines HK and KA , but vnto the square of the line KA are equall the squares of the lines KC and CA : Wherefore the square of the line HA is equall to the squares of the lines HK, KC and CA . But by the same vnto the squares of the lines HK and KC is equall the square of the line HC : Wherefore the square of the line HA is equall to the squares of the lines HC and CA : wherefore the angle HCA is (by the 48. of the first) a right angle. And by the same reason also the angle $MF D$ is a right angle. Wherefore the angle HCA is equall to the angle $MF D$. But the angle HAC is (by suppositiō) equal to the angle $MD F$. Wherefore there are two triangles $MD F$ and HAC hauing two angles of the one equall to twoo angles of the other, eche to his correspondent angle, and one side of the one equall to one side of the other, namely, that side which subtendeth one of the equall angles,

Demonstration.

The eleventh Booke

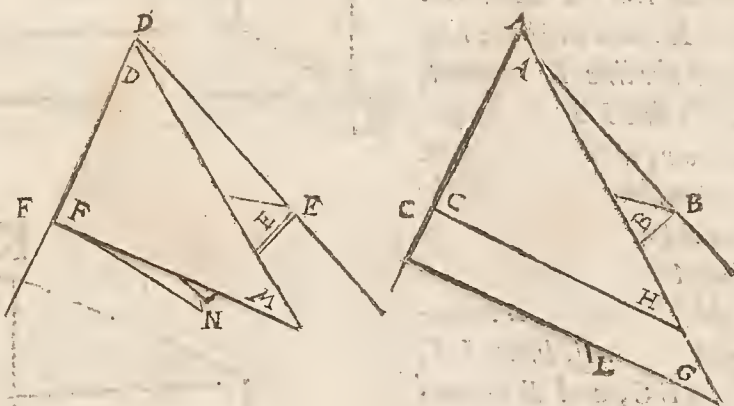
angles, that is, the side HA is equall to the side DM by construction. Wherefore the sides remayning are (by the 26. of the first) equall to the sides remayning. Wherefore the side AC is equall to the side DF . In like sort may we proue that the side AB is equall to the side DE , if ye drawe a right line from the point H to the point B , and an other from the point M to the point E . For forasmuch as the square of the line AH is (by the 47. of the first) equall to the squares of the lines AK and KH , and (by the same) unto the square of the line AK are equall the squares of the lines AB and BK . Wherefore the squares of the lines AB , BK , and KH are equall to the square of the line AH . But unto the squares of the lines BK and KH is equall the square of the line BH (by the 47. of the first) for the angle HKB is a right angle, for that the line HK is erected perpendicularly to the ground playne superficies: Wherefore the square of the line AH is equall to the squares of the lines AB and BH . Wherefore (by the 48. of the first) the angle ABH is a right angle. And by the same reason the angle DEM is a right angle. Now the angle BAH is equall to the angle EDM , for it is so supposed, and the line AH is equall to the line DM . Wherefore (by the 26. of the first) the line AB is equall to the line DE . Now forasmuch as the line AC is equall to the line DF , and the line AB to the line DE , therefore these two lines AC and AB are equall to these two lines FD and DE . But the angle also CAB is by supposition equall to the angle FDE . Wherefore (by the 4. of the first) the base BC is equall to the base EF , and the triangle to the triangle, and the rest of the angles to the reste of the angles. Wherefore the angle ACB is equall to the angle DFE . And the right angle ACK is equal to the right angle DFN . Wherefore the angle remayning, namely, BCK , is equall to the angle remayning, namely, to EFN . And by the same reason also the angle CBK is equal to the angle $FE N$. Wherefore there are two triangles BCK , & EFN , hauing two angles of the one equal to two angles of the other, eche to his correspondent angle, and one side of the one equall to one side of the other, namely, that



side that lieth betwene the equall angles, that is the side BC is equall to the side EF : Wherefore (by the 26. of the first) the sides remaining are equall to the sides remayning. Wherefore the side CK is equall to the side FN : but the side AC is equall to the side DF . Wherefore these two sides AC and CK are equall to these two sides DF and FN , and they containe equall angles: Wherefore (by the 4. of the first) the base AK is equall to the base DN . And forasmuch as the line AH is equall to the line DM , therefore the square of the line AH is equall to the square of the line DM . But unto the square of the line AH are equall the squares of the lines AK and KH (by the 47. of the first) for the angle AKH is a right angle. And to the square of the line DM are equall the squares of the lines DN and NM , for the angle DNM is a right angle. Wherefore the squares of the lines AK and KH are equall to the squares of the lines DN and NM : of which two, the square of the line AK is equall to the square of the line DN (for the line AK is proued equall to the line DN). Wherefore the residue, namely, the square of the line KH is equal to the residue, namely, to the square of the line NM . Wherefore the line HK is equall to the line MN . And forasmuch as these two lines HA and AK are equall to these two lines MD and DN , the one to the other, and the base

base $H K$ is equall to the base $M N$: therefore (by the 8. of the first) the angle $H A K$ is equall to the angle $M D N$. If therefore there be two superficiall angles equall, and frō the pointes of those angles be eleuated on high right lines, comprehending together with those right lines which were put at the beginning, equall angles, ech to his coreispondent angle, and if in ech of the erected lines be taken a point at all aduentures, and from those pointes be drawn perpendicular lines to the plaine superficieses in which are the angles geuen at the beginning, and from the pointes which are by the perpendicular lines made in the two plaine superficieses be ioyned right lines to those angles which were put at the beginning, those right lines shall together with the lines eleuated on high make equall angles: which was required to be proued.

Because the figures of the former demonstration are somewhat hard to conceaue as they are there drawn in a plaine, by reason of the lines that are imagined to be eleuated on high, I haue here set o^r ther figures, wherein you must erecte perpendicularly to the ground superficieses the two triangles $B H K$, and $E M N$, and then eleuate the triangles $D F M$, & $A C H$, in such sort that the angles M and H of these triangles, may concurre with the angles M and H of the o^ther erected triangles. And then imagining only a line to be drawn from the point G of the line $A G$ to the point L in the ground superficies, compare it with the former construction & demonstration, and it will make it very easye to conceaue.



¶ Corollary.

By this it is manifest, that if there be two rectiline superficiall angles equall, and vpon those angles be eleuated on high equall right lines contayning together with the right lines put at the beginning equall angles: perpendicular lines drawn from those eleuated lines to the ground plaine superficieses wherein are the angles put at the beginning, are equall the one to the other. For it is manifest, that the perpendicular lines $H K$, & $M N$, which are drawn from the endes of the equall eleuated lines $A H$, and $D M$, to the ground superficieses, are equall.

¶ The 31. Theoreme.

The 36. Proposition.

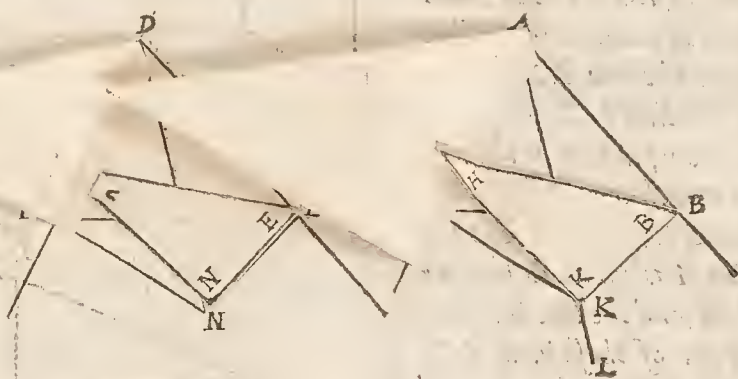
If there be three right lines proportionall: a Parallelipipedon described of those three right lines, is equall to the Parallelipipedon described of the middle line, so that it consiste of equall sides, and also be equiangle to the foresayd Parallelipipedon.



Vppose that these three lines A, B, C , be proportionall, as A is to B , so let B be to C . Then I say, that the Parallelipipedon made of the lines A, B, C , is equall to the Parallelipipedon made of the line B , so that the solide made of the line B consist of equall

base $H K$ is equall to the base $M N$: therefore (by the 8. of the first) the angle $H A K$ is equall to the angle $M D N$. If therefore there be two superficiall angles equall, and fro the pointes of those angles be eleuated on high right lines, comprehending together with those right lines which were put at the beginning, equall angles, ech to his corespondent angle, and if in ech of the erected lines be taken a point at all aduentures, and from those pointes be drawen perpendicular lines to the plaine superficieses in which are the angles geuen at the beginning, and from the pointes which are by the perpendicular lines made in the two plaine superficieses be ioyned right lines to those angles which were put at the beginning, those right lines shall together with the lines eleuated on high make equall angles: which was required to be proued.

Because the figures of the former demonstration are somewhat hard to conceaue as they are there drawn in a plaine, by reason of the lines that are imagined to be eleuated on high, I haue here set o-
ther figures, wherein you must erecte perpendicularly to the ground superficieses the two triangles $B H K$, and $E M N$, and then eleuate the triangles $D F M$, & $A C H$, in such sort that the angles M and H of these triangles, with the angles M and H of the erected triangles. And then imagining only a line to be drawn from the point G of the line $A G$ to the point L in the ground superficies, compare it with the former construction & demonstration, and it will make it very easye to conceaue.



¶ Corollary.

By this it is manifest, that if there be two rectiline superficiall angles equall, and vpon those angles be eleuated on high equall right lines containing together with the right lines put at the beginning equall angles: perpendicular lines drawen from those eleuated lines to the ground plaine superficieses wherein are the angles put at the beginning, are equall the one to the other. For it is manifest, that the perpendicular lines $H K$, & $M N$, which are drawen from the endes of the equall eleuated lines $A H$, and $D M$, to the ground superficieses, are equall.

¶ The 31. Theoreme.

The 36. Proposition.

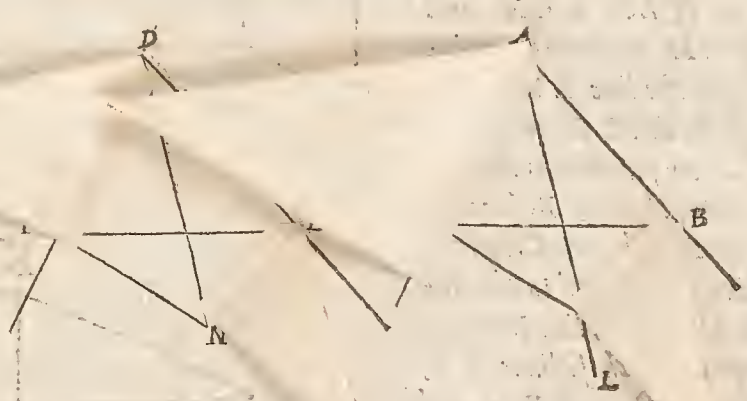
If there be three right lines proportionall: a Parallelipipedon described of those three right lines, is equall to the Parallelipipedon described of the middle line, so that it consiste of equall sides, and also be equiangle to the foresayd Parallelipipedon.



Suppose that these three lines A, B, C , be proportionall, as A is to B , so let B be to C . Then I say, that the Parallelipipedon made of the lines A, B, C , is equall to the Parallelipipedon made of the line B , so that the solide made of the line B consist of equall

base $H K$ is equall to the base $M N$: therefore (by the 8. of the first) the angle $H A K$ is equall to the angle $M D N$. If therefore there be two superficiall angles equall, and fro the pointes of those angles be eleuated on high right lines, comprehending together with those right lines which were put at the beginning, equall angles, ech to his correspondent angle, and if in ech of the erected lines be taken a point at all aduentures, and from those pointes be drawen perpendicular lines to the plaine superficieses in which are the angles geuen at the beginning, and from the pointes which are by the perpendicular lines made in the two plaine superficieses be ioyned right lines to those angles which were put at the beginning, those right lines shall together with the lines eleuated on high make equall angles: which was required to be proued.

Because the figures of the former demonstration are somewhat hard to conceaue as they are there drawn in a plaine, by reason of the lines that are imagined to be eleuated on high, I haue here set o-
ther figures, wherein you must erecte perpendicularly to the ground superficieses the two triangles $B H K$, and $E M N$, and then eleuate the triangles $D F M$, & $A C H$, in such sort that the angles M and H of these triangles, with the angles M and H of the erected triangles. And then imagining only a line to be drawn from the point G of the line $A G$ to the point L in the ground superficies, compare it with the former construction & demonstration, and it will make it very easye to conceaue.



¶ Corollary.

By this it is manifest, that if there be two rectiline superficiall angles equall, and vpon those angles be eleuated on high equall right lines containing together with the right lines put at the beginning equall angles: perpendicular lines drawen from those eleuated lines to the ground plaine superficieses wherein are the angles put at the beginning, are equall the one to the other. For it is manifest, that the perpendicular lines $H K$, & $M N$, which are drawen from the endes of the equall eleuated lines $A H$, and $D M$, to the ground superficieses, are equall.

¶ The 31. Theoreme.

The 36. Proposition.

If there be three right lines proportionall: a Parallelipipedon described of those three right lines, is equall to the Parallelipipedon described of the middle line, so that it consiste of equall sides, and also be equiangle to the foresayd Parallelipipedon.



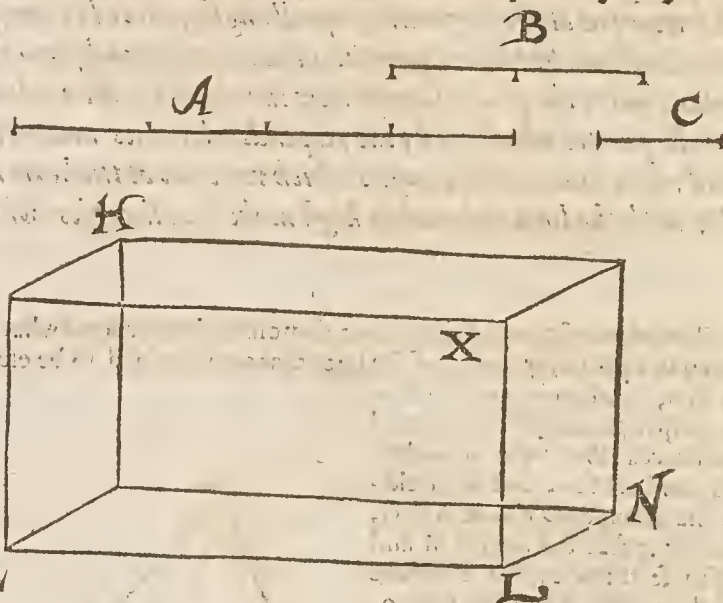
Suppose that these three lines A, B, C , be proportionall, as A is to B , so let B be to C . Then I say, that the Parallelipipedon made of the lines A, B, C , is equall to the Parallelipipedon made of the line B , so that the solide made of the line B consist of equall

The eleventh Booke

Construction.

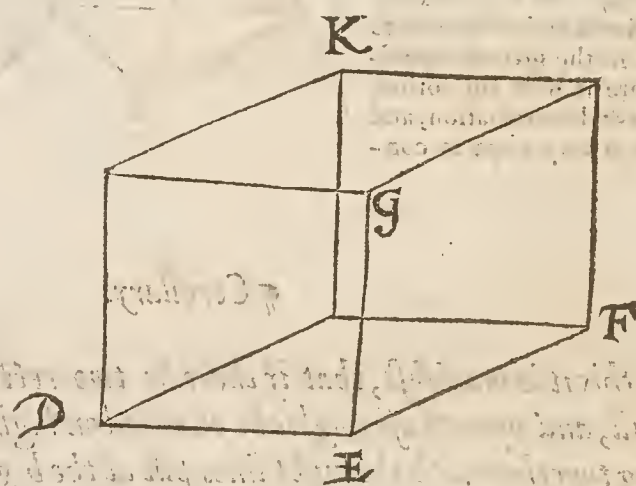
quall sides, and be also equiangle to the solide made of the lines A, B, C . Describe (by the 23. of the eleventh) a solide angle E contained vnder three superficiall angles, that is, DEG , GEF , and FED : and (by the 3. of the first) put vnto the line B euery one of these lines $DE, GE, \& EF$, equall:

and make perfecte the solide EK . And vnto the line A let the line LM be equall. And (by the 26. of the eleventh) vnto the right line LM , and at the point in it L , describe vnto the solide angle E an equall solide angle, contained vnder these plaine superficiall angles NLX , XLM , and NLM , and vnto the line B put the line LX equall, & the line LN to the line



Demonstration.

Now for that as the line A is to the line B , so is the line B to the line C : but the line A is equall to the line LM , and the line B to euery one of these lines LX , EF , EG , and ED , and the line C to the line LN . Wherefore as LM is to EF , so is DE to LN : So then the sides about the equal angles MLN , & DEF , are reciprokall: Wherefore (by the 14. of the sixth) the parallelogramme MN



is equall to the parallelogramme DEF . And forasmuch as two plaine superficiall angles, namely, DEF and NLM are equall the one to the other, and vpon them are erected vprward equall right lines, LX and EG , comprehending with the right lines put at the beginning equall angles the one to the other. Wherefore* perpendicular lines drawn from the pointes X and G to the plaine superficieses wherein are the angles NLM , and DEF , are (by the Corollary of the former Proposition) equall the one to the other: and those perpendiculars are the altitudes of the Parallelipipedons LH and EK , by the 4. definition of the sixth. Wherefore the solides LH and EK , are vnder one and the selfe same altitude. But Parallelipipedons consisting vpon equall bases, and being vnder one and the selfe same altitude, are (by the 31. of the eleventh) equall the one to the other. Wherefore the solide LH is equall to the solide EK . But the solide LH is described of the lines A, B, C , and the solide EK is described of the line B . Wherefore the Parallelipipedon described of the lines A, B, C , is equall to the Parallelipipedon made of the line B , which consisteth of equall sides, and is also equiangle to the foresaid Parallelipipedon. If therfore there be three right lines proportionall, a Parallelipipedon described of those three right lines is equall to the Parallelipipedon described of the middle line, so that it consist of equall sides, and also be equiangle to the foresaid Parallelipipedon: which was required to be proued.

* It is euident that those perpendiculars are all one with the standing lines of the solides, if their solide angles be made of superficiall right angles onely.

The construction and demonstration of this Proposition, and of the next Proposition following, may easily be conceived and understood by the figures described in the plaine belonging to them. But ye may for the more full sight of them, describe such bodies of pasted paper, having their sides proportionall, as is required in the Propositions.

¶ *New inventions (coincident) added by Master Iohn Dee.*

✱ *A Corollary. I.*

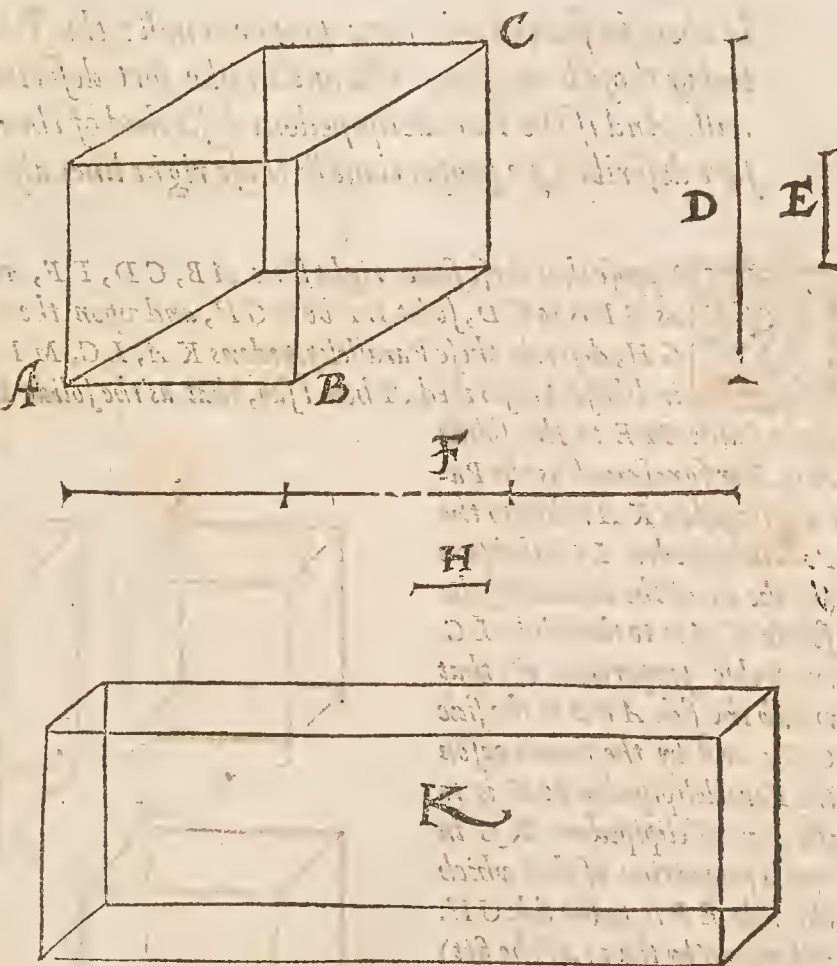
Hereby it is evident, that if three right lines be proportionall: the Cube produced of the middle line, is equall to the rectangle Parallelipipedon made of those three lines.

For a Cube is a Parallelipipedon of equall sides: and also rectangled: as we suppose the Parallelipipedon, made of the three lines to be likewise rectangled.

¶ *A Probleme. I.*

A Cube being given, to find three right lines proportionall, in any proportion given between two right lines: of which three lines, the rectangle Parallelipipedon produced, shall be equall to the Cube given.

Suppose AC to be the Cube given: whose roote, suppose to be AB. Let the proportion given, be that which is between the two right lines D and E. I say now, three right lines are to be found, proportionall, in the proportion of D to E, of which, the rectangle Parallelipipedon produced, shall be equall to AC. By the 12. of the sixth let a line be found, which to AB hath that proportion that D hath to E. Let that line be F: and by the same 12. of the sixth, let an other line be found, to which, AB, hath that proportion that D hath to E: and let that line found be H. Let a rectangle Parallelipipedon mathematically be produced of the three right lines F, AB, and H, which suppose to be K: I say now, that F, AB, and H, are three right lines found proportionall in the proportion of D to E, of which, the rectangle Parallelipipedon K, produced, is equall to AC the Cube given. First it is evident that F, AB, and



H, are proportionall in the proportion of D to E. For, by construction, as D is to E, so is F to AB: and by construction likewise, as D is to E, so is AB to H. Wherefore F is to AB, and AB is to H, as D is to E. So then it is manifest, F, AB, and H, to be proportionall in the proportion of D to E, and AB to be the middle line. By my former Corollary, therefore, the rectangle parallelipipedon made of F, AB, and H, is equall to the Cube made of AB. But AC, is (by supposition) the Cube made of AB: and of the three lines F, AB, and H, the rectangle parallelipipedon produced, is K, by construction: Wherefore, K, is equall to AC: A Cube being given, therefore, three right lines are found, proportionall in

BBb.j.

any

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any proportion geue betwene two right lines, of which three right lines the rectangle parallelipipedon produced, is equall to the Cube geuen. Which ought to be done.

* A Probleme. 2.

A rectangle Parallelipipedon being geuen, to finde three right lines proportionall: of the Which, the rectangle Parallelipipedon produced, is equall to the rectangle Parallelipipedon geuen.

Doubling of the Cube. &c.

** Demonstrati-
on of possibilitie
in the Probleme.*

*An other argu-
ment to comfort
the studious.*

Listen to this new deuise, you couragious Mathematiciens: consider, how nere this creepeth to the famous Probleme of doubling the Cube. What hope may (in maner) any young beginner cōceiue, by one meanes or other, at one time or other, to execute this Probleme? * Seing to a Cube may infinitely infinite Parallelipipedons be found equall: all which Parallelipipedons shall be produced of three right lines proportionall, by the former Probleme: but to any rectangle Parallelipipedon geuen, some one Cube is equall: as is easie to demonstrate: We can not doubt, but vnto our rectangle Parallelipipedon geuen, many other rectangle Parallelipipedons are also equall, hauing their three lines of production, proportionall. In the former Probleme, infinitely infinite Parallelipipedons may be found of three proportionall lines produced, equall to the Cube geuen: it is to wete, the three lines to be of all proportions, that a man can deuise betwene two right lines: and here any one will serue: where also is infinite varietie: though all of one quantitie: as before in the Cube. I leaue as now, with thys marke here set vp to shoote at. Hit it who can.

¶ The 32. Theoreme.

The 37. Proposition.

If there be fower right lines proportionall: the Parallelipipedons described of those lines, being like and in like sort described, shall be proportionall. And if the Parallelipipedons described of them, being like and in like sort described, be proportionall: those right lines also shall be proportionall.



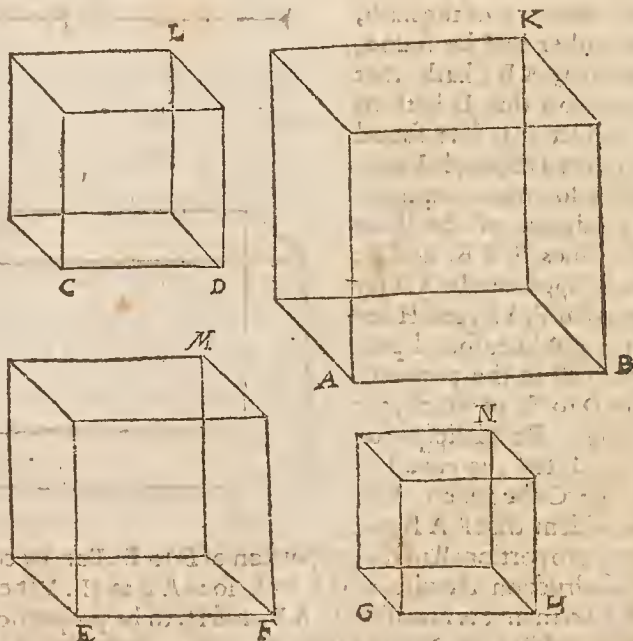
Suppose that these fower right lines AB, CD, EF, and GH, be proportionall, as AB is to CD, so let EF be to GH, and vpon the lines AB, CD, EF, and GH, describe these Parallelipipedons KA, LC, ME, and NG, being like and in like sort described. Then I say, that as the solide KA is to the solide LC, so

is the solide ME to the solide

*Demonstrati-
on of the first
part.*

NG. For forasmuch as the Parallelipipedon KA is like to the Parallelipipedon LC: therefore (by the 33. of the eleuenth) the solide KA is to the solide LC in treble proportion of that which the side AB is to the side CD: and by the same reason the Parallelipipedon ME is to the Parallelipipedon NG in treble proportion of that which the side EF is to the side GH. Wherefore (by the 11. of the fift) as the Parallelipipedon KA is to the Parallelipipedon LC, so is the Parallelipipedon ME to the Parallelipipedon NG.

But now suppose, that as



the

the Parallelipipedon LC , so is the Parallelipipedon ME to the Parallelipipedon NG . Then I say, that as the right line AB is to the right line CD , so is the right line EF to the right line GH . For againe forasmuch as the solide KA is to the solide LC in treble proportion of that which the side AB is to the side CD , and the solide ME also is to the solide NG in treble proportion of that which the line EF is to the line GH , and as the solide KA is to the solide LC , so is the solide ME to the solide NG . Wherefore also as the line AB is to the line CD , so is the line EF to the line GH . If therefore there be fower right lines proportionall: the Parallelipipedons described of those lines, being like & in like sort described, shall be proportionall. And if the Parallelipipedons described of them, and being like and in like sort described, be proportionall: those right lines also shall be proportionall. which was required to be proued.

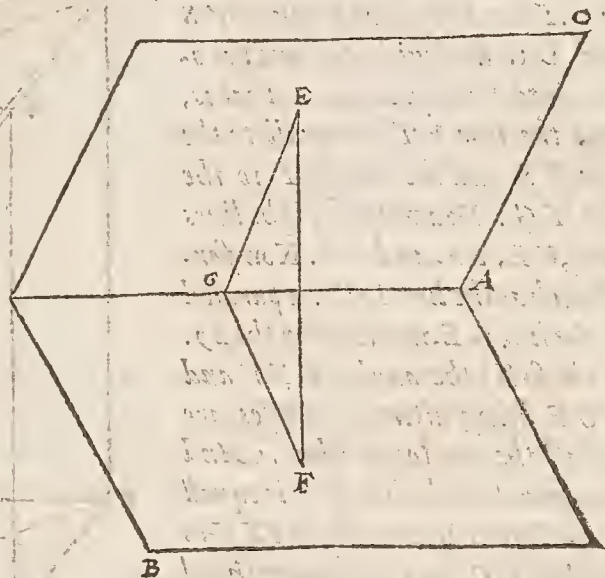
Demonstration of the second part, which is the conuerse of the first part.

¶ The 33. Theoreme.

The 38. Proposition.

If a plaine superficies be erected perpendicularly to a plaine superficies, and from a point taken in one of the plaine superficies be drawn to the other plaine superficies, a perpendicular line: that perpendicular line shall fall vpon the common section of those plaine superficies.

Suppose that the plaine superficies CD be erected perpendicularly to the plaine superficies AB , and let their common section be the line DA : and in the superficies CD take a point at all aduentures, and let the same be E . Then I say, that a perpendicular line drawn from the point E to the plaine superficies AB , shall fall vpon the right line DA . For if not, then let it fall without the line DA , as the line EF doth, and let it fall vpon the plaine superficies AB in the point F . And (by the 12. of the first) from the point F draw vnto the line DA , being in the superficies AB a perpendicular line FG , which line also is erected perpendicularly to the plaine superficies CD : by the third diffinitio: by reason we presuppose CD and AB to be perpendicularly erected ech to other. Draw a right line from the point E to the point G . And forasmuch as the line FG is erected perpendicularly to the plaine superficies CD , and the line EG toucheth it being in the superficies CD . Wherefore the angle FGE is (by the 2. definition of the eleuenth) a right angle. But the line EF is also erected perpendicularly to the superficies AB : wherefore the angle EFG is a right angle. Now therefore two angles of the triangle EFG , are equall to two right angles: which (by the 17. of the first) is impossible. Wherefore a perpendicular line drawn fro the point E to the superficies AB , falleth not without the line DA . Wherefore it falleth vpon the line DA : which was required to be proued.



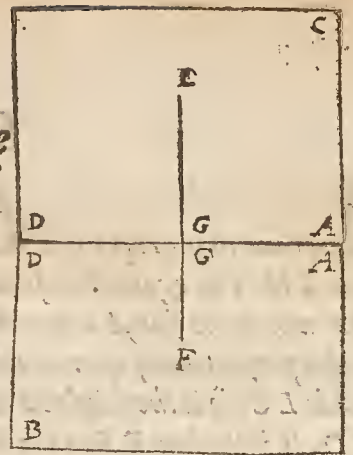
Demonstration leading to an impossibility.

¶ Note.

Campane maketh this as a Corollary, following vpon the 13: and very well, with small ayde of other Propositions, he proueth it: whose demonstratio there, Flussas hath in this place, and none other: though he sayth that Campane of such a Propositio, as of Euclides, maketh no mention.

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In this figure ye may more fully see the former Proposition and demonstration if ye erecte perpendicularly vnto the ground plaine superficies A B the superficies C D, and imagine a line to be extended from the point E to the point F, in steade whereof ye may extend if ye will a thred.



The 34. Theoreme.

The 39. Proposition.

If the opposite sides of a Parallelipipedon be deuided into two equall partes, and by their common sections be extended plaine superficieses: the common section of those plaine superficieses, and the diameter of the Parallelipipedon shall deuide the one the other into two equall partes.



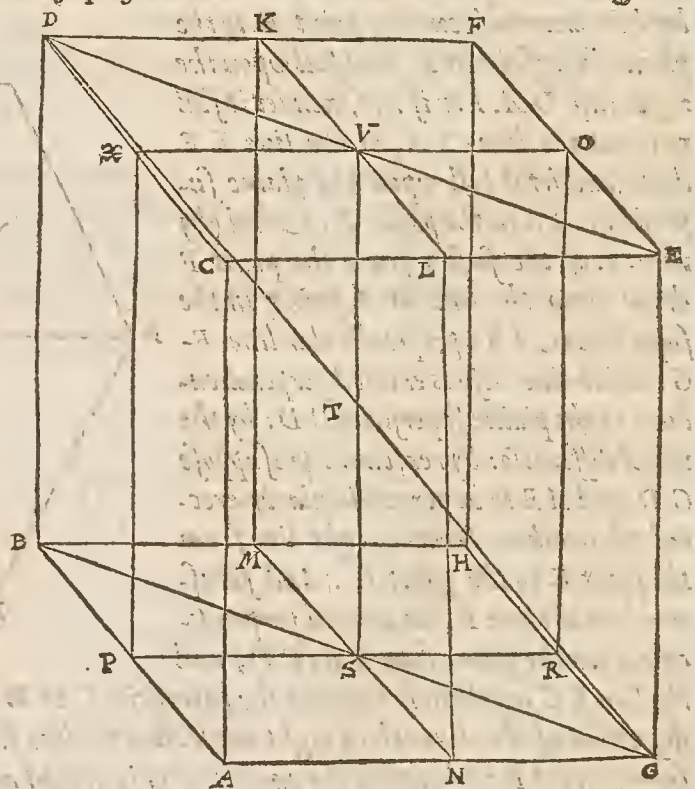
Suppose that A F be a Parallelipipedon, and let the opposite sides thereof C F and A H be deuided into two equall partes in the pointes K, L, M, N, and likewise let the opposite sides A D and G F be deuided into two equall partes in the pointes X, P, O, R, and by those sections extend these two plaine superficieses K N & X R, and let the common section of those plaine superficieses be the line V S, and let the diagonall line of the solide A B be the line

Construction.

D G. Then I say, that the lines V S and D G do deuide the one the other into two equall partes, that is, that the line V T is equall to the line T S, and the line D T to the line T G. Drawe these right lines D V, V E, B S, and S G. Now forasmuch as the line D X is a parallel to the line O E, therefore (by the 29. of the first) the angles D X V and V O E being alternate angles, are equall the one to the other. And forasmuch as the line D X is equall to the line O E, and the line X V to the line V O, and they comprehend equall angles: Wherefore the base D V is equall to the base V E (by the 4. of the first): and the triangle D X V is equall to the triangle V O E, and the rest of the angles to

Demonstration.

the rest of the angles. Wherefore the angle X V D is equall to the angle O V E. Wherefore D V E is one right line, and by the same reason B S G is also one right line, and the line B S is equall to the line S G. And forasmuch as the line C A is equall to the line D B, and is vnto it a parallel, but the line C A is equall to the line G E, and is vnto it also a parallel: wherefore (by the first common sentence) the line D B is equall to the line G E, & is also a parallel vnto it: but the right lines D E and B G do ioine these parallel lines together: Wherefore (by the 33. of the first) the line D E is a parallel vnto the line B G. And in either of these lines



are taken pointes at all aduentures, namely, D, V, G, S , and a right line is drawen from the point D to the point G , and an other from the point V to the point S . Wherefore (by the 7. of the eleuenth) the lines DG and VS are in one and the selfe same plaine superficies. And forasmuch as the line DE is a parallel to the line BG , therefore (by the 24. of the first) the angle EDT is equall to the angle BGT , for they are alternate angles, and likewise the angle DTV is equall to the angle GTS . Now then there are two triangles, that is, DTV and GTS , hauing two angles of the one equall to two angles of the other, and one side of the one equall to one side of the other, namely, the side which subtendeth the equall angles, that is, the side DV to the side GS , for they are the halfe of the lines DE and BG : Wherefore the sides remayning are equall to the sides remayning. Wherefore the line DT is equall to the line TS , and the line VT to the line TS . If therefore the opposite sides of a Parallelipipedon be deuided into two equall partes, and by their sections be extended plaine superficieses, the common section of those plaine superficieses, and the diameter of the Parallelipipedon, do deuide the one the other into two equall partes: which was required to be demonstrated.

A Corollary added by *Fluſſas*.

Euery playne superficies extended by the center of a parallelipipedon, diuideth that solide into two equall partes: and so doth not any other playne superficies not extended by the center.

For euery playne extended by the center, cutteth the diameter of the parallelipipedon in the center into two equall partes. For it is proued, that playne superficieses which cutte the solide into two equall partes, do cut the dimetient into two equall partes in the center. Wherefore all the lines drawen by the center in that playne superficies shall make angles with the dimetient. And forasmuch as the diameter falleth vpon the parallel right lines of the solide, which describe the opposite sides of the sayde solide, or vpon the parallel playne superficieses of the solide, which make angles at the endes of the diameter: the triangles containd vnder the diameter, and the right line extended in that playne by the center, and the right line, which being drawen in the opposite superficieses of the solide, ioyneth together the endes of the foresayde right lines, namely, the ende of the diameter, and the ende of the line drawen by the center in the superficies extended by the center, shall alwayes be equall, and equiangle, by the 26. of the first. For the opposite right lines drawen by the opposite playne superficieses of the solide do make equall angles with the diameter, forasmuch as they are parallel lines, by the 16. of this booke. But the angles at the ceter are equall, by the 15. of the first, for they are head angles: & one side is equall to one side, namely, halfe the dimetient. Wherefore the triangles containd vnder euery right line drawen by the center of the parallelipipedon in the superficies, which is extended also by the sayd center, and the diameter thereof, whose endes are the angles of the solide, are equall, equilateral, & equiangle (by the 26. of the first). Wherefore it followeth that the playne superficies which cutteth the parallelipipedon, doth make the partes of the bases on the opposite side, equall, and equiangle, and therefore like, and equall both in multitude, and in magnitude: wherefore the two solide sections of that solide, shall be equall and like, by the 8. diffinition of this booke. And now that no other playne superficies, besides that which is extended by the center, deuiddeth the parallelipipedon into two equall partes, it is manifest: if vnto the playne superficies which is not extended by the center, we extend by the center a parallel playne superficies (by the Corollary of the 15. of this booke). For forasmuch as that superficies which is extended by the center, doth deuide the parallelipipedon into two equall partes: it is manifest, that the other playne superficies (which is a parallel to the superficies which deuiddeth the solide into two equall partes) is in one of the equall partes of the solide: wherefore seing that the whole is euer greater then his partes, it must nedes be that one of these sections is lesse then the halfe of the solide, and therefore the other is greater.

For the better vnderstanding of this former proposition, & also of this Corollary added by *Fluſſas*, it shall be very nedefull for you to describe of pasted paper or such like matter a parallelipipedon or a Cube, and to deuide all the parallelogrames therof into two equall parts, by drawing by the ceters of the sayd parallelogrammes (which centers are the poyns made by the cutting of diagonall lines drawen frō the opposite angles of the sayd parallelogrames) lines parallels to the sides of the parallelogrames: as in the former figure described in a plaine ye may see, are the fixe parallelogrames DE, EH, HA, AD, DH , and CG , whom these parallel lines drawen by the ceters of the sayd parallelogrames, namely, XO, OR, PR , and PX , do deuide into two equall partes: by which fower lines ye must imagine a playne superficies to be extended, also these parallel lynes KL, LN, NM , and MK , by which fower lines likewise ye must imagine a playne superficies to be extended ye: may if ye will put within your body made thus of pasted paper, two superficieses made also of the sayd paper, hauing to their limites lines equall to the foresayde parallel lines: which superficieses must also be deuided into two equall partes by parallel

The eleventh Booke

In this figure ye may more fully see the former Proposition and demonstration if ye erecte perpendicularly vnto the ground plaine superficies A B the superficies C D, and imagine a line to be extended from the point E to the point F, in steade whereof ye may extend if ye will a thred.

The 34. Theoreme.

The 39. Proposition.

If the opposite sides of a Parallelepipedon be deuided into two equall partes, and by their common sections be extended plaine superficieses: the commo section of those plaine superficieses, and the diameter of the Parallelepipedon shall deuide the one the other into two equall partes.



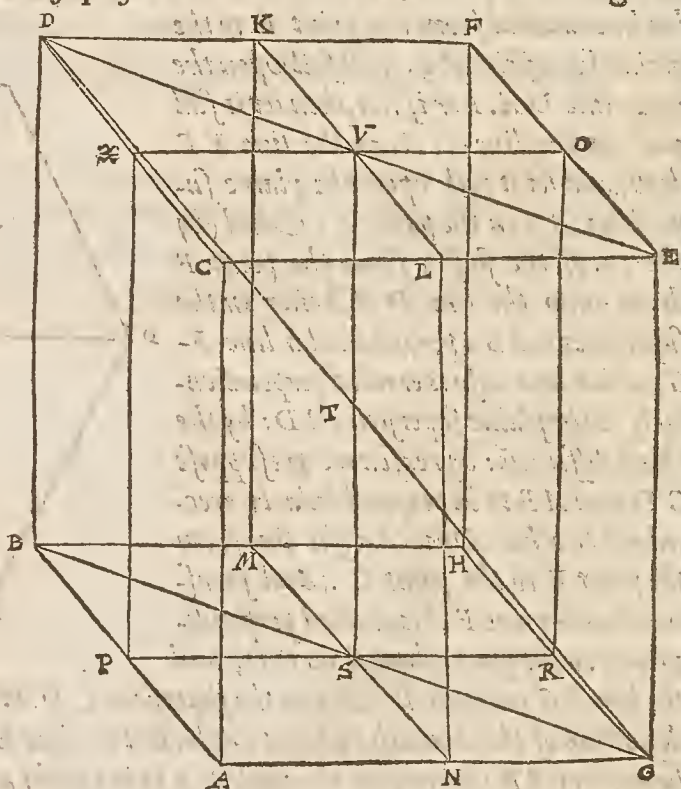
Suppose that A F be a Parallelepipedon, and let the opposite sides thereof C F and A H be deuided into two equall partes in the pointes K, L, M, N, and likewise let the opposite sides A D and G F be deuided into two equall partes in the pointes X, P, O, R, and by those sections extend these two plaine superficieses K N & X R, and let the common section of those plaine superficieses be the line V S, and let the diagonall line of the solide A B be the line D G. Then I say, that the lines V S and D G do deuide the one the other into two equall partes, that is,

Construction.

that the line V T is equall to the line T S, and the line D T to the line T G. Drawe these right lines D V, V E, B S, and S G. Now forasmuch as the line D X is a parallel to the line O E, therefore (by the 29. of the first) the angles D X V and V O E being alternate angles, are equall the one to the other. And forasmuch as the line D X is equall to the line O E, and the line X V to the line V O, and they comprehend equall angles: Wherefore the base D V is equall to the base V E (by the 4. of the first) and the triangle D X V is equall to the triangle V O E, and the rest of the angles to

Demonstration.

the rest of the angles. Wherefore the angle X V D is equall to the angle O V E. Wherefore D V E is one right line, and by the same reason B S G is also one right line, and the line B S is equall to the line S G. And forasmuch as the line C A is equall to the line D B, and is vnto it a parallel, but the line C A is equall to the line G E, and is vnto it also a parallel: wherefore (by the first common sentence) the line D B is equall to the line G E, & is also a parallel vnto it: but the right lines D E and B G do ioyne these parallel lines together: Wherefore (by the 33. of the first) the line D E is a parallel vnto the line B G. And in either of these lines are



The eleuenth Booke

lines drawn by their centers, and must cut the one the other by these parallel lines. And for the diameter of this body, extēd a thred from one angle in the base of the solide to his opposite angle, which shall passe by the center of the parallelipedon, as doth the line D G in the figure before described in the playne. And draw in the base and the opposite superficies vnto it, Diagonall lines, from the angles from which is extended the diameter of the solide: as in the former description are the lines B G and D E. And when you haue thus described this body, compare it with the former demonstration, and it will make it very playne vnto you, so your letters agrée with the letters of the figure described in the booke. And this description will playnely set forth vnto you the corollary following that proposition. For where as to the vnderstanding of the demonstration of the proposition the superficieses put within the body were extended by parallelynes drawn by the cēters of the bases of the parallelipedon: to the vnderstanding of the sayd Corollary, ye may extēde a superficies by any other lines drawn in the sayd bases, so that yet it passe through the middest of the thred, which is supposed to be the center of the parallelipedon.

¶ The 35. Theoreme.

The 40. Proposition.

If there be two Prismes vnder equall altitudes, & the one haue to his base a parallelogramme, and the other a triangle, and if the parallelogramme be double to the triangle: those Prismes are equall the one to the other.



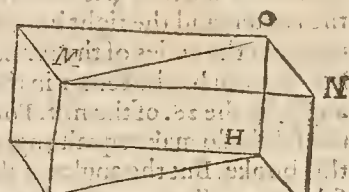
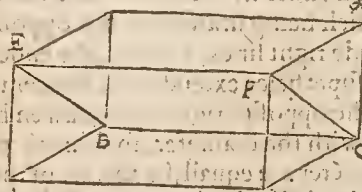
Suppose that these two Prismes ABCDEF, GHKMON, be vnder equall altitudes, and let the one haue to his base the parallelogramme AC, and the other the triangle GHK, and let the parallelogramme AC be double to the triangle GHK. Then I say, that the Prisme ABCDEF is equall to the Prisme

Construction.

GHKMON. Make perfecte the Parallelipipedons AX & GO.

Demonstration.

And forasmuch as the parallelogramme AC is double to the triangle GHK, but the parallelogramme GH is also (by the 41. of the first) double to the triangle GHK, wherefore the parallelogramme AC is equall to the parallelogramme GH. But Parallelipipedons consisting vpon equall bases and vnder one and



the selfe same altitude, are equall the one to the other (by the 31. of the eleuenth). Wherefore the solide AX is equall to the solide GO. But the halfe of the solide AX is the Prisme ABCDEF, and the halfe of the solide GO is the Prisme GHKMON. Wherefore the Prisme ABCDEF is equall to the Prisme GHKMON. If therefore there be two Prismes vnder equall altitudes, and the one haue to his base a parallelogramme, & the other a triangle, and if the parallelogramme be double to the triangle: those Prismes are equall the one to the other: which was required to be proued.

This Proposition and the demonstration thereof are not hard to conceaue by the former figures: but ye may for your fuller vnderstanding of the take two equall Parallelipipedons equilater and equiangle the one to the other described of pasted paper or such like matter, and in the base of the one Parallelipipedon draw a diagonall line, and draw an other diagonall line in the vpper superficies opposite vnto the said diagonall line drawn in the base. And in one of the parallelogrammes which are set vpon the base of the other Parallelipipedon draw a diagonall line, and drawe an other diagonall line in the parallelogramme opposite to the same. For so if ye extend plaine superficieses by those diagonall lines there will be made two Prismes in ech body. Ye must take heede that ye put for the bases of ech of these Parallelipipedons equall parallelogrames. And then note the with letters according to the letters of the figures before described in the plaine. And cōpare the with the demonstration, and they will make both it and the Proposition very clere vnto you. They will also geue great light to the Corollary following added by *Flussas*.

By

A Corollary added by Fluxus.

By this and the former propositions it is manifest, that Prismes and solides* containd under two polygon on figures equall, like, and parallels, and the rest parallelogrammes: may be compared the one to the other after the selfe same maner that parallelipedons are.

* Which of some are called sided Columnes.

For forasmuch as (by this proposition and by the second Corollary of the 25. of this booke) it is manifest, that euery parallelipedon may be resolued into two like, and equal Prismes, of one and the same altitude, whose base shalbe one and the selfe same with the base of the parallelipedon or the halfe thereof, which Prismes also shalbe containd vnder the selfe same sides with the parallelipedon, the sayde sides beyng also sides of like proportion: I say that Prismes may be compared together after the like maner that their Parallelipedons are. For if we would deuide a Prisme like vnto his solide by the 25. of this booke, ye shall finde in the Corollaries of the 25. proposition, that that which is set forth touching a parallelipedon, followeth not onely in a Prisme, but also in any sided columnne whose opposite bases are equall, and like, and his sides parallelogrammes.

If it be required by the 27. proposition vpon a right line geuen to describe a Prisme like and in like sorte situate to a Prisme geuen: describe first the whole parallelipedon whereof the prisme geuen is the halfe (which thing ye see by this 40. proposition may be done). And vnto that parallelipedon describe vpon the right line geuen by the sayd 27. proposition an other parallelipedon like: and the halfe thereof shalbe the prisme which ye seeke for, namely, shalbe a prisme described vpon the right line geuen, and like vnto the prisme geuen.

In deede Prismes can not be cut according to the 28. proposition. For that in their opposite sides can be drawen no diagonall lines: howbeit by that 28. proposition those Prismes are manifestly confirmed to be equall and like, which are the halues of one and the selfe same parallelipedon.

And as touching the 29. proposition, and the three following it, which proueth that parallelipedons vnder one and the selfe same altitude, and vpon equall bases, or the selfe same bases, are equal: or if they be vnder one and the selfe same altitude, they are in proportion the one to the other, as their bases are: to apply these comparisons vnto Prismes, it is to be required, that the bases of the Prismes compared together, be either all parallelogrammes, or all triangles. For so one and the selfe altitude remayning, the comparison of thinges equall is euer one and the selfe same, and the halues of the bases are euer the one to the other in the same proportion, that their wholes are. Wherefore Prismes which are the halues of the parallelipedons, and which haue the same proportion the one to the other that the whole parallelipedons haue, which are vnder one and the selfe same altitude: must needs cause that their bases being the halues of the bases of the parallelipedons are in the same proportio the one to the other, that their whole parallelipedons are. If therefore the whole parallelipedons be in the proportion of the whole bases, their halues also (which are Prismes) shalbe in the proportion either of the wholes if their bases be parallelogrammes, or of the halues if they be triangles, which is euer all one by the 15. of the 5. ueth.

And forasmuch as by the 33. proposition, like parallelipedons which are the doubles of their Prismes are in treble proportion the one to the other that their sides of like proportion are, it is manifest, that Prismes being their halues (which haue the one to the other the same proportion that their wholes haue, by the 15 of the 5. ueth) and hauing the selfe same sides that their parallelipedons haue, are the one to the other in treble proportion of that which the sides of like proportion are.

And for that Prismes are the one to the other in the same proportion that their parallelipedons are, and the bases of the Prismes (being all either triangles or parallelogrammes) are the one to the other in the same proportion that the bases of the parallelipedons are, whose altitudes also are alwayes equall, we may by the 34. proposition conclude, that the bases of the prismes and the bases of the parallelipedons their doubles (being ech the one to the other in one and the selfe same proportion) are to the altitudes, in the same proportion that the bases of the double solides, namely, of the parallelipedons are. For if the bases of the equall parallelipedons be reciprokall with their altitudes, then their halues which are Prismes shall haue their bases reciprokall with their altitudes.

By the 36. proposition we may conclude, that if there be three right lines proportionall, the angle of a Prisme made of these three lines (being common with the angle of his parallelipedon which is double) doth make a prisme, which is equall to the Prisme described of the middle line, and containyng the like angle, consistyng also of equall sides. For as in the parallelipedon, so also in the Prisme, this one thing is required, namely, that the three dimensions of the proportionall lines do make an angle like vnto the angle containd of the middle line taken three tymes. Now then if the solide angle of the Prisme be made of those three right lines, there shall of them be made an angle like to the angle of the parallelipedon which is double vnto it. Wherefore it followeth of necessitie, that the Prismes which are alwayes the halues of the Parallelipedons, are equiangle the one to the other, as also are their doubles, although they be not equilater: and therefore those halues of equall solides are equall the one to the other: namely, that which is described of the middle proportionall line is equall to that which is described of the three proportionall lines.

By the 37. proposition also we may conclude the same touching Prismes which was concluded BBb.iiij. touching

The eleuenth Booke

touching Parallelipipeds. For forasmuch as Prismes, described like & in like sort of the lines geuen, are the halues of the Parallelipipeds which are like and in like sort described, it followeth that these Prismes haue the one to the other the same proportion that the solides which are their doubles haue. And therefore if the lines which describe them be porportionall, they shalbe proportionall, and so conuerſedly according to the rule of the sayd 37. proposition.

But forasmuch as the 39. proposition supposeth the opposite superficiall sides of the solide to be parallelogrammes, and the same solide to haue one diameter, which thinges a Prisme can not haue, therefore this proposition can by no meanes be applyed to Prismes.

But as touching solides whose bases are two like, equall, and parallel polygon figures, and their sides are parallelogrammes, forasmuch as by the second Corollary of the 25. of this booke it hath bene declared, that such solides are composed of Prismes, it may easely be proued that their nature is such, as is the nature of the Prismes, whereof they are composed. Wherefore a parallelipipidon being by the 27. proposition of this booke described, there may also be described the halfe thereof, which is a Prisme: and by the description of Prismes, there may be composed a solide like vnto a solide geuen composed of Prismes.

So that it is manifest, that that which the 29. 30. 31. 32. 33. 34. and 37. propositions set forth touching parallelipipeds, may well be applyed also to these kyndes of solides.

Sided Co-
lumnar.

The end of the eleuenth booke of Euclides Elementes.



The twelueeth booke of Eu- clides Elementes.



IN THIS TVVELVETH BOOKE, EVCLIDE setteth forth the passions and proprieties of Pyramids, Prismes, Cones, Cylinders, and Spheres. And compareth Pyramids, first to Pyramids, then to Prismes: so likewise doth he Cones, and Cylinders. And lastly he compareth Spheres the one to the other. But before he goeth to the treatie of those bodies, he proueth that, like Polygonon figures inscribed in circles, and also the circles the selues are in proportion the one to the other, as the squares of the diameters of those circles are. Because that was necessary to be proued, for the confirmation of certayne passions and proprieties of those bodies.

¶ The 1. Theoreme.

The 1. Proposition.

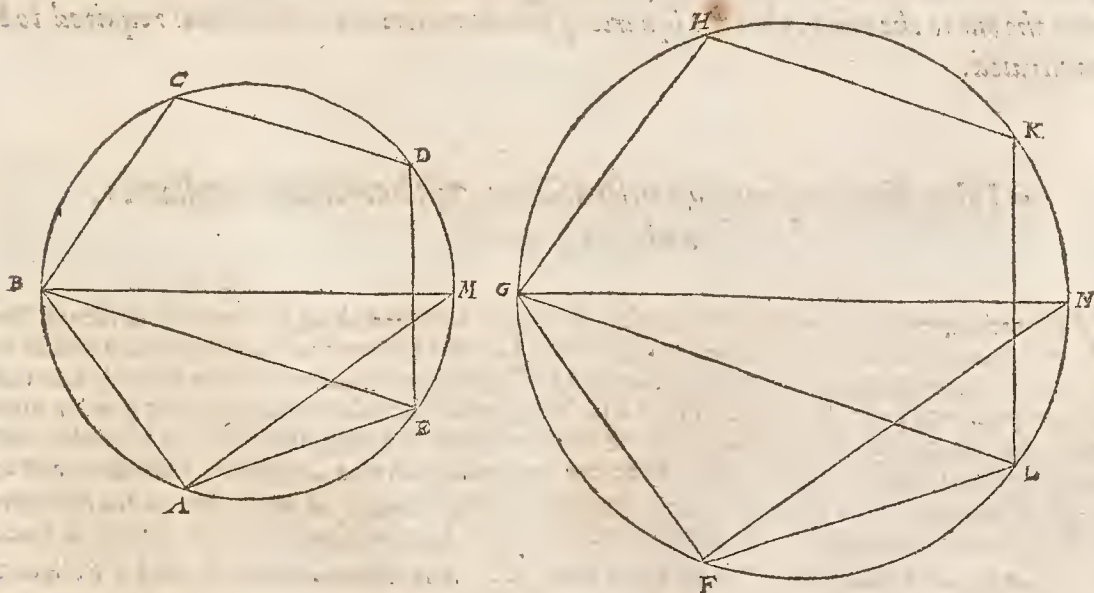
Like Polygonon figures described in circles: are in that proportion the one to the other, that the squares of their diameters are.



Suppose that there be two circles $ABCDE$, and $FGHKL$, and in them let there be described like Polygonon figures, namely, $ABCDE$, and $FGHKL$, and let the diameters of the circles be BM , and GN . Then I say, that as the square of the line BM is to the square of the line GN , so is the Polygonon figure $ABCDE$ to the Polygonon figure $FGHKL$. Drawe these right lines BE , AM , GL , and FN . And forasmuch as the Polygonon figure $ABCDE$ is like to the Polygonon figure $FGHKL$, therefore the angle BAE is equall to the angle

Construction.

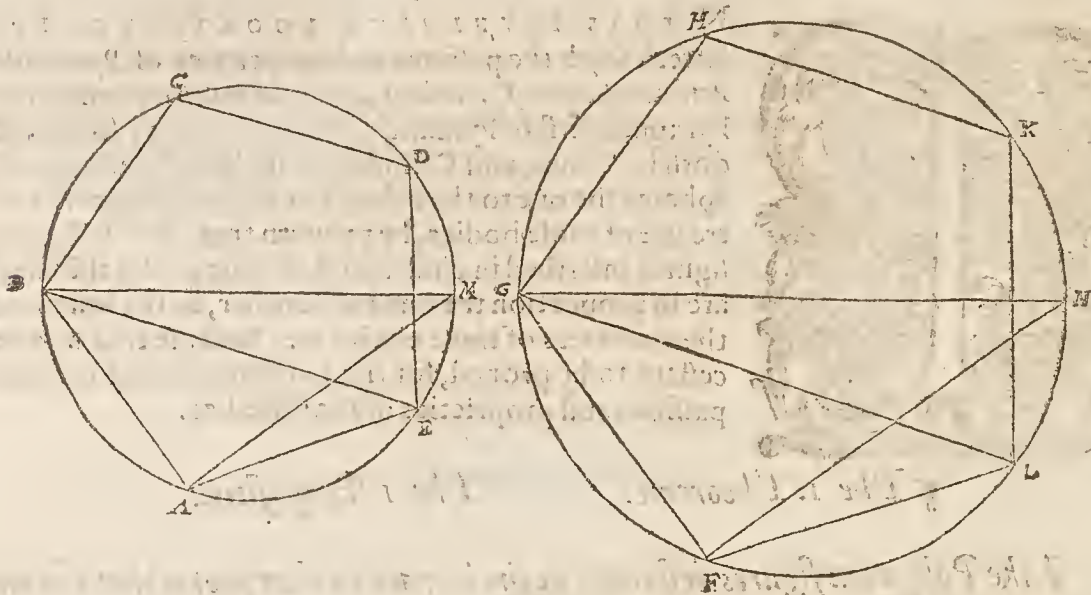
Demonstration.



GFL , and as the line BA is to the line AE , so is the line GF to the line FL (by the definition of like Polygonon figures). Now therefore there are two triangles BAE and GFL , hauing

The twelvethe Booke

having one angle of the one equall to one angle of the other, namely, the angle $B A E$ equall to the angle $G F L$, and the sides about the equall angles are proportionall. Wherefore (by the first definition of the sixth) the triangle $A B E$ is equiangle to the triangle $F G L$. Wherefore the angle $A E B$ is equall to the angle $F L G$. But (by the 21. of the third) the angle $A E B$ is equall to the angle $A M B$, for they consist upon one and the selfe same circumference: and



by the same reason the angle $F L G$ is equall to the angle $F N G$. Wherefore the angle $A M B$ is equall to the angle $F N G$. And the right angle $B A M$, is (by the 4. petition) equall to the right angle $G F N$. Wherefore the angle remayning, is equall to the angle remayning. Wherefore the triangle $A M B$ is equiangle to the triangle $F N G$. Wherefore proportionally as the line $B M$ is to the line $G N$, so is the line $B A$ to the line $G F$. But the square of the line $B M$ is to the square of the line $G N$ in double proportion of that which the line $B M$ is to the line $G N$ (by the Corollary of the 20. of the sixth). And the Polygonon figure $A B C D E$ is to the Polygonon figure $F G H K L$ in double proportion of that which the line $B A$ is to the line $G F$ (by the 20. of the sixth). Wherefore (by the 11. of the fifth) as the square of the line $B M$ is to the square of the line $G N$, so is the Polygonon figure $A B C D E$, to the Polygonon figure $F G H K L$. Wherefore like Polygonon figures described in circles, are in that proportion the one to the other, that the squares of the diameters are: which was required to be demonstrated.

¶ Iohn Dee his fruitfull instructions, with certaine Corollaries,
and their great vse.

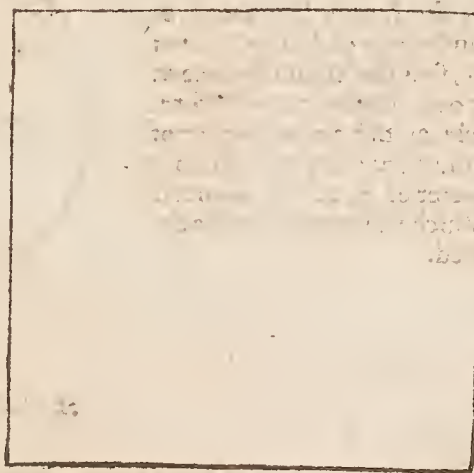
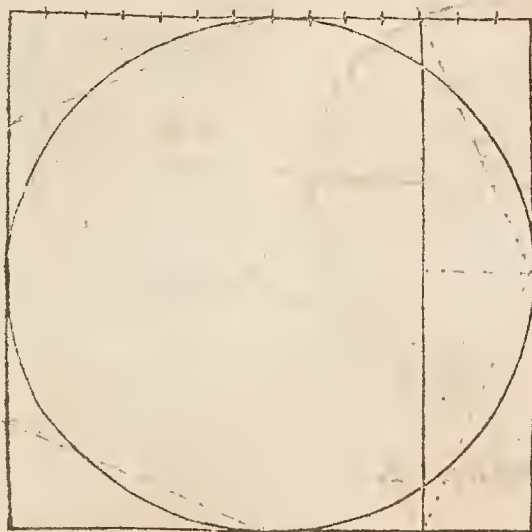
Who can not easily perceave, what occasion and ayde, *Archimedes* had, by these first & second Propositions, to finde the nere Area, or Content of a circle: betwene a Polygonon figure within the circle, and the like about the same circle, described? Whose precise quantities are most easily known: being comprehended of right lines. Where also (to auoyde all occasions of error) it is good in numbers, not hauing precise square rootes, to vse the Logistickall processe, according to the rules, with $\sqrt{8}12$, $\sqrt{8}19$, and so, of such like. Who can not readily fall into *Archimedes* reckoning and account, by his method? To finde the proportion of the circumference of any circle to his diameter, to be almost triple, and one seuenth of the diameter: but to be more then triple and ten one & seuentithes: that is to be lesse then $3\frac{1}{7}$ and more then $3\frac{10}{71}$. And where *Archimedes* vsed a Polygonon figure of 96. sides: he that, for exercise sake, or for earnest desire of a more nerenes, will vse Polygonon figures of 384. sides (or more) may well trauaile therein, till either wearines cause him stay, or els he finde his labour fruitles. In deede *Archimedes* concluded proportion, of the circumference to the diameter,

meter, hath hitherto serued the vulgare and Mechanicall workemen : wherewith, who so is not contented, let his owne Methodicall trauaile satisfie his desire : or let him procure other therto . For, narrower termes (of greater and lesse) found, and appointed to the circumference, will also winne to the Area of the circle a nearer quantitie : seing, it is well demōstrated of *Archimedes*, that a triangle rectangle, of whose two sides (contayning the right angle) one is equall to the semidiameter of the circle, and the other to the circumference of the same, is equall to the Area of that circle . Vpon which two Theoremes, it followeth, that the square made of the diameter, is in that proportion to the circle (very neare) in which, 14, is to 11. Wherefore euery circle is $\frac{11}{14}$ eleuen fowertenthes (well neare) of the square about him described . The one side, then, of that square, deuide into 14. equall partes : and from that point which endeth the eleuenth part, drawe to the opposite side, a line, parallel to the other sides, and so make perfecte the parallelogramme . Then, by the last Proposition of the second booke, vnto that parallelogramme (whose one side hath those 11. equall partes), make a square equall . Then is it euident, that square to be equall to the circle, about which the first square is described. As ye may here beholde in these figures .

2.

3.

The squaring of the circle.



Gentle frend, the great desire, which I haue, that both with pleasure and also profite, thou mayest spend thy time in these excellent studies, doth cause me here to furnishe thee somewhat (extraordinarily) about the circle : not onely by pointing vnto thee, the wellspring of *Archimedes*, his so much wondered at, and iustly commended trauaile (in the former 3. Theoremes, here repeated), but also to make thee more apt, to vnderstand and practise this and other bookes following , where, vse of the circle may be had in any consideration : as in Cones, Cylinders, and Spheres, &c.

¶ A Corollary. 1.

By *Archimedes* second Theoreme (as I haue here alleaged them) it is manifest , that a parallelogramme contained, either vnder the semidiameter and halfe the circumference, or vnder the halfe semidiameter and Whole circumference of any circle, is equall to the circle : by the 41. of the first : and first of the sixt.

¶ A Corollary. 2.

Likewise it is euident, that the parallelogramme contayned vnder the semidiameter, and halfe of any portion of the circumference of a circle geuen, is equall to that sector of the same circle, to which the whole portion of the circumference geuen, doth belong . Or you may vse the halfe semidiameter, and the whole portion of the circumference : as sides of the said parallelogramme.

The farther winning, and inferring, I committe to your skill, care, and studye. But in an other sort will I geue you newe ayde, and instruction here.

¶ A Theoreme.

The twelue th Booke

¶ Theoreme.

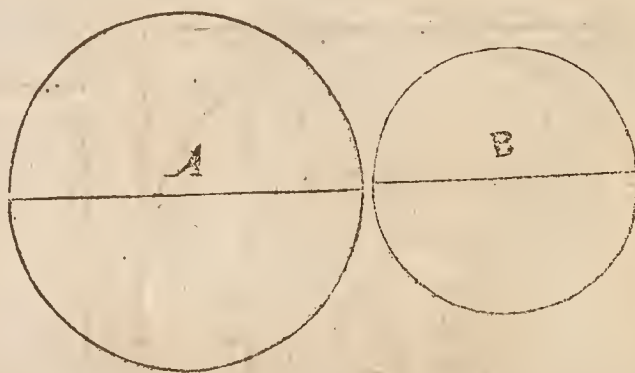
Of all circles, the circumferences to their owne diameters, haue one and the same proportion: in what one circle soeuer, they are assigned.

That is (as *Archimedes* hath demonstrated) almost, as 22. to 7: or nearer, if nearer be found: vn- till the very precise proportion be demonstrated. Which, what soeuer it be, in all circumferences to their proper diameters, will be demonstrated one and the same.

✱ A Corollary. 1.

Wherefore if two circles be propounded, which suppose to be *A* and *B*, as the circumference of *A* is to the circumference of *B*, so is the diameter of *A* to the diameter of *B*.

For by the former Theoreme, as the circumference of *A*, is to his own diameter, so is the circumference of *B*, to his own diameter: Wherefore, alternately, as the circumference of *A*, is to the circumference of *B*, so is the diameter of *A*, to the diameter of *B*: Which was required to be demonstrated.



✱ A Corollary. 2.

It is now then euident, that we can geue two circles whose circumferences one to the other, shall haue any proportion geuen in two right lines.

The great Mechanicall vse (besides Mathematicall considerations) which, these two Corollaries may haue in Wheelles of Milles, Clockes, Cranes, and other engines for water workes, and for warres, and many other purposes, the earnest and wittie Mechanicien will soone bout out, & gladly practise.

¶ John Dee.

¶ The 2. Theoreme. The 2. Proposition.

Circles are in that proportion the one to the other, that the squares of their diameters are.

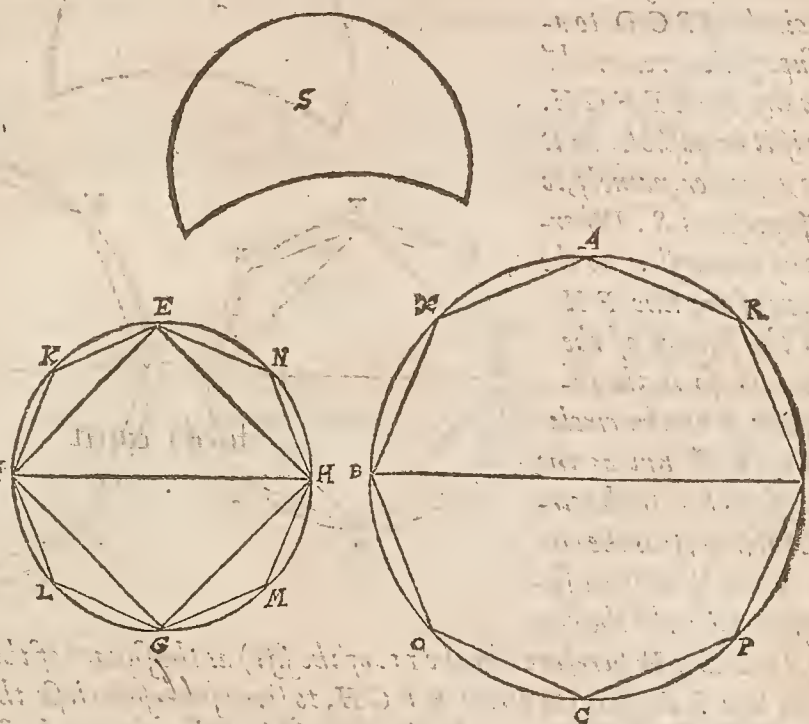


Suppose that there be two circles *ABCD*, and *EFGH*, and let their diameters be *BD* and *FH*. Then I say, that as the square of the line *DB* is to the square of the line *FH*, so is the circle *ABCD* to the circle *EFGH*. For if the circle *ABCD* be not vnto the circle *EFGH*, as the square of the line *BD* is to the square of the line *FH*: then the square of the line *BD* shall be to the square of the line *FH*, as the circle *ABCD* is to a superficies, either lesse then the circle *EFGH*, or greater. First let the square of the line *BD* be to the square of the line *FH*, as the circle *ABCD* is to a superficies lesse then the circle *EFGH*, namely, to the superficies *S*. Describe (by the 6. of the fourth) in the circle *EFGH* a square *EFGH*. Now this square thus described is greater then the halfe of the circle *EFGH*. For if by the pointes *E*, *F*, *G*, *H*, we drawe right lines touching

Demonstration leading to an impossibilitie. Two cases in this proposition. The first case.

touching the circle, the square $EFGH$, is the halfe of the square described about the circle, but the square described about the circle, is greater then the circle. Wherefore the square $EFGH$, which is inscribed in the circle, is greater then the halfe of the circle $EFGH$. Deuide the circumferences EF , FG , GH , and HE , into two equall partes in the pointes K , L , M , N . And drawe these right lines EK , KF , FL , LG , GM , MH , HN , and NE . Wherefore euery one of these triangles EKF , FLG , GMH , and HNE , is greater then the halfe of the segmēt of the circle which is described about it. For if by the pointes K , L , M , N , be drawen lines touching the circle, and then be made perfecte the parallelogrames made of the right lines EF , FG , GH , & HE , euery one of the triangles EKF , FLG , GMH , & HNE , is the halfe of the parallelograme which is described about it (by the 41. of the first): but the segmēt described about it is lesse then the parallelogramme. Wherefore euery one of these triangles EKF , FLG , GMH , and HNE , is greater then the halfe of the segmēt of the circle which is described about it. Now then deuiding the circumferences remaining into two equall partes, and drawing right lines from the pointes where those diuisions are made, & so continually doing this, we shall at the length (by the 1. of the tenth) leaue certaine segmentes of the circle, which shall be lesse then the ex-
 cesse, wherby the circle $EFGH$ exceedeth the superficies S . For it hath bene proued in the first Proposition of the tenth booke, that two vnequall magnitudes being geuen, if from the greater be taken away more then the halfe, and likewise againe from the residue more then the halfe, and so continually, there shall at the length be left a certaine magnitude which shall be lesse then the lesse magnitude geuen. Let there be such segmentes left, & let the segmentes of the circle $EFGH$, namely, which are made by the lines EK , KF , FL , LG , GM , MH , HN , and NE , be lesse then the ex-
 cesse, wherby the circle $EFGH$ exceedeth the superficies S . Wherefore the residue, namely, the Polygonon figure $EKF L G M H N$, is greater then the superficies S . Inscribe in the circle $ABCD$ a Polygonon figure like to the Polygonon figure $EKF L G M H N$, and let the same be $AXBOCPDR$. Wherefore (by the Proposition next going before) as the square of the line BD is to the square of the line FH , so is the Polygonon figure $AXBOCPDR$ to the Polygonon figure $EKF L G M H N$. But as the square of the line BD is to the square of the line FG , so is the circle $ABCD$ supposed to be to the superficies S . Wherefore (by the 11. of the fift) as the circle $ABCD$ is to the superficies S , so is the Polygonon figure $AXBOCPDR$ to the Polygonon figure $EKF L G M H N$. Wherefore alternately (by the 16. of the fift) as the circle $ABCD$ is to the Polygonon figure described in it, so is the superficies S to the Polygonon figure $EKF L G M H N$. But the circle $ABCD$ is greater then the Polygonon figure described in it. Wherefore also the superficies S is greater then the Polygonon figure $EKF L G M H N$: but it is also lesse: which is impossible. Wherefore as the square of the line BD is to the square of the line FH ,

That a square within any circle described is bigger than halfe the circle. That the 1/2 of isosceles triangles, without the square, are greater then halfe the segmēts wherin they are.



so is not the circle $ABCD$ to any superficies lesse then the circles $EFGH$.
Second case. In li ke sort also may wproue, that as the square of the line FH is to the square of the line

BD , so is not the circle $EFGH$ to any superficies lesse then the circle $ABCD$. I say, namely,

that as the square of the line BD is to the square of the line FH , so is not

the circle $ABCD$ to any superficies greater the then the circle $EFGH$.

For if it be possible, let it be to a greater, namely, to the superficies S . Wherefore by conuerſion, as the

square of the line FH is to the square of the line BD , so is the su-

perſicies S to the circle $ABCD$. * But as the

superſicies S is to the circle $ABCD$, so is the circle $EFGH$ to some su-

perſicies lesse the the circle $ABCD$. Wherefore (by the 11. of the fiſt) as the square of the line FH is to the square

of the line BD , so is the circle $EFGH$, to some superficies lesse then the circle $ABCD$:

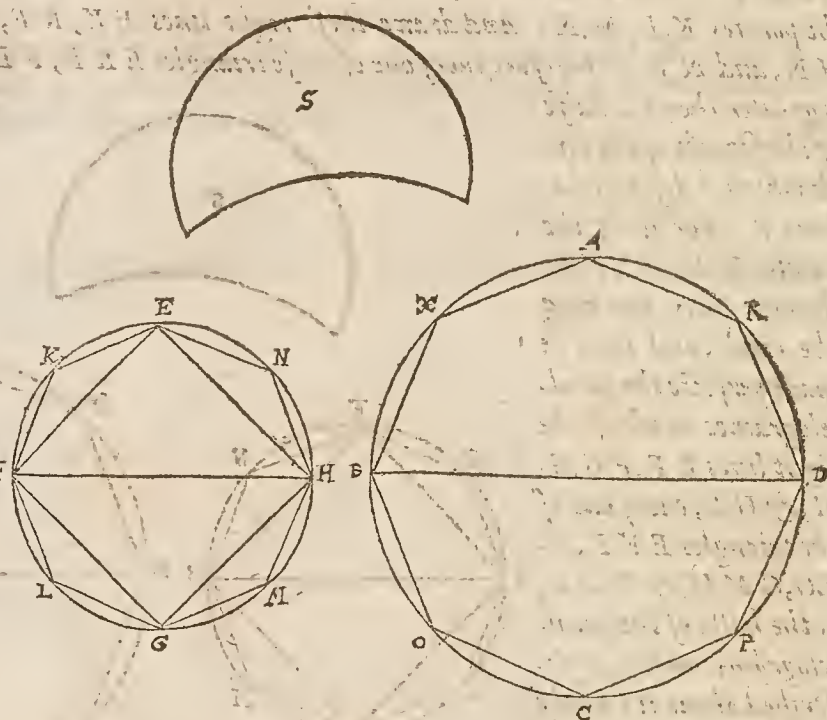
which is in the first caſe proued to be impossible. Wherefore as the square of the line BD is to the square of the line FH , so is not the circle $ABCD$ to any superficies greater then the

circle $EFGH$. And it is also proued that it is not, to any lesse. Wherefore as the square of the line BD is to the square of the line FH , so is the circle $ABCD$ to the circle $EFGH$.

Wherefore circles are in that proportion the one to the other, that the squares of their diame-

ters are : which was required to be proued.

* This Aſſumpt.
 is afterward
 at the end of
 the demōſtra-
 tion proued.



¶ An Aſſumpt.

I ſay now, that the superficies S being greater then the circle $EFGH$, as the superficies S is to the circle $ABCD$,

so is the circle $EFGH$ to some superficies lesse then the circle $ABCD$.

For, as the superficies S is to the circle $ABCD$, so let the circle $EFGH$ be to the superficies T . Now I ſay, that the superficies T is lesse then the circle $ABCD$.

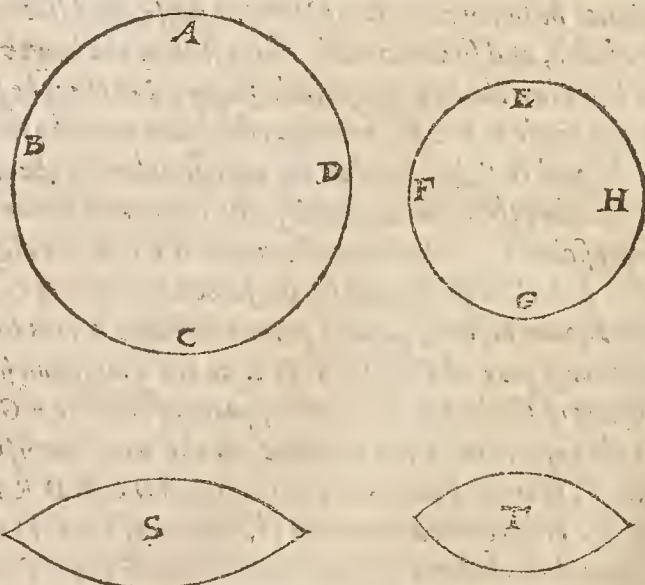
For for that as the superficies S is to the circle $ABCD$, so is the circle $EFGH$ to the superficies T , there-

fore alternately (by the 16. of the fiſt) as the superficies S is to the circle $EFGH$, so is the circle $ABCD$ to the su-

perſicies T . But the superficies S is greater then the circle $EFGH$ (by ſuppoſition). Wherefore also the circle $ABCD$ is greater

then the superficies T (by the 14. of the fiſt). Wherefore as the superficies S is to the circle

$ABCD$,



$ABCD$,

ABCD, so is the circle EFGH to some superficies lesse then the circle ABCD: which was required to be demonstrated.

¶ A Corollary added by Flusbas.

Circles haue the one to the other, that proportion, that like Polygonon figures and in like sort described in them haue. For, it was by the first Proposition proued, that the Polygonon figures haue that proportio the one to the other, that the squares of the diameters haue, which proportion likewise, by this Proposition, the circles haue.

¶ Very needefull Problemes and Corollaries by Master Ihon Dee inuented: whose wonderfull use also, he partly declareth.

A Probleme. 1.

Two circles being geue: to finde two right lines, which haue the same proportion, one to the other, that the geuen circles haue, one to the other.

Suppose A and B, to be the diameters of two circles geuen: I say that two right lines are to be founde, hauing that proportio, that the circle of A hath to the circle of B. Let to A & B (by the 11. of the sixth) a third proportionall line be found, which suppose to be C. I say now that A hath to C, that proportion which the circle of A hath to the circle of B. For forasmuch as A, B, and C, are (by construction) three proportionall lines, the square of A is to the square of B, as A is to C, (by the Corollary of the 20. of the sixth): but as the square of the line A is to the square of the line B, so is the circle whose diameter is the line A, to the circle whose diameter is the line B, by this second of the eleueth. Wherefore the circles of the lines A and B, are in the proportion of the right lines A and C. Therefore two circles being geuen, we haue found two right lines hauing the same proportion betwene the, that the circles geuen, haue one to the other: which ought to be done.

Construction.

Demonstration.

A Probleme. 2.

Two circles being geuen, and a right line: to finde an other right line, to which the line geue shall haue that proportion, which the one circle hath to the other.

Suppose two circles geue: which let be A & B, & a right line geue, which let be C: I say that an other right line is to be founde, to which the line C shall haue that proportion that the circle A, hath to the circle B. As the diameter of the circle A, is to the diameter of the circle B, so let the line C be to a fourth line, (by the 12. of the sixth) let that fourth line be D. And, by the 11. of the sixth, let a thirde line proportionall be found, to the lines C & D, which let be E: I say now, that the line C hath to the line E, that proportion which the circle A, hath to the circle B. For (by construction) the lines C, D, and E, are proportionall: therefore the square of C, is to the square of D, as C is to E, by the Corollary of the 20. of the sixth. But by construction, as the diameter of the circle A, is to the diameter of the circle B, so is C, to D: wherefore as the square of the diameter of the circle A, is to the square of the diameter of the circle B, so is the square of the line C to the square of the line D, by the 22. of the sixth. But as the square of the diameter of A, the circle, is to the square of the diameter of the circle B, so is the circle A, to the circle B, by the second of the twelfth: wherefore by 11. of the sixth, as the circle A, is to

Construction

Demonstration.

The twelue th Booke

the circle B, so is the square of the line C, to the square of the line D. But it is proued, that as the square of the line C, is to the square of the line D, so is the line C to the line E. Wherefore by the 11. of the fifteth, as the circle A, is to the circle B, so is the line C to the line E. Two circles being geuen therefore, and a right line, we haue found a right line, to which the right line geuen, hath that proportion, which the one circle hath to the other. Which ought to be done.

Note.

The difference betwene this Probleme, and that next before, is this: there, although we had two circles geuen, and two lines were found in that proportion the one to the other, in which the geuen circles were, : and here likewise are two circles geue, and two lines also are had in the same proportio, that the geue circles are: yet there we tooke at pleasure the first of the two lines, wherunto we framed the second proportionally, to the circles geuen. But here the first of the two lines, is assigned, poynted, and determined to vs: and not our choyse to be had therein, as was in the former Probleme.

*Diffreence
betwene the
first probleme
and the se-
cond.*

A Probleme. 3.

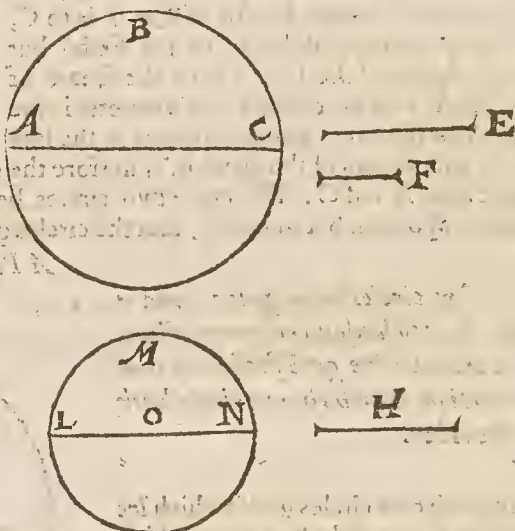
A circle being geuen, to finde an other circle, to which the geuen circle is in any proportion geue in two right lines.

Suppose the circle A B C geuen, and therefore his semidiameter is geuen: whereby his diameter also is geuen: which diameter let be A C. Let the proportion geuen, be that which is betwene E, F, two right lines. I say, a circle is to be found, vnto which A B C hath that proportion that E hath to F.

Construction.

As E is to F, so let A C the diameter, be to an other right line, by the 12. of the sixth. Which line suppose to be H. Betwene A C and H, finde a middle proportionall line, by the 13. of the sixth: which let be L N. By the 10. of the first, deuide L N, into two equall partes: and let that be done in the point O. Now vpon O, (O being made the center) describe a circle: which let be L M N. I say that A B C, is to L M N, as E is to F. For feing that A C, L N, and H, are three right lines in continuall proportion (by construction) therefore (by the Corollary of the 20. of the sixth) as A C is to H, so is the square of A C to the square of L N. But A C is to H, as E is to F, by construction. Wherefore the square of A C is to the square of L N, as E is to F: but as the square of the diameter A C, is to the square of the diameter L N, so is the circle A B C to the circle L M N, by this 2. of the twelue th, wherefore by the 11. of the fifteth, the circle A B C is to the circle L M N, as E is to F. A circle being geuen (therefore) an other circle is founde, to which the geuen circle is in any proportion geuen betwene two right lines: which ought to be done.

Demonstration.



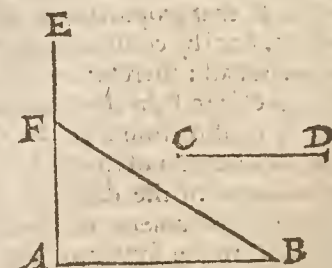
A Probleme. 4.

Two circles being geuen, to finde one circle equall to them both.

Suppose the two circles geue, haue their diameters A B & C D. I say that a circle must be found equall to the two circles whose diameters are A B and C D: vnto the line A B, at the point A, erect a perpendicular line A E: from which (sufficiently produced) cut a line equall to C D, which let be A F. By the first petition draw from F to B a right line: so is F A B, made a triangle rectangle. I say now that a circle whose diameter is F B, is equal to the two circles whose diameters are A B and C D. For by the 47. of the first, the square of F B is equall to the squares of A B & A F. Which A F is (by construction) equall to C D: wherefore the square of F B is equall to the squares of A B and C D. But circles are one to an other, as the squares of their diameters are one to the other, by this second of the twelue th. Therefore the circle whose diameter is F B is equall to the circles whose diameters are A B and C D. Therefore two circles being geuen we haue found a circle equall to them both. Which was required to be done.

Construction.

Demonstration.



A Corollary

✱ A Corollary. 1.

Hereby it is made eident, that in all triangles rectangle, the circles, semicircles, quadrants, or any other portions of circles described upon the subtendent line, is equall to the two circles, semicircles, quadrants, or any two other like portions of circles, described on the two lines comprehending the right angle, like to like being compared.

For like partes haue that proportion betwene them selues, that their whole magnitudes haue, of which they are like partes, by the 15. of the fifth. But of the whole circles, in the former problem it is eident: and therefore in the fornamed like portions of circles, it is a true consequent.

✱ A Corollary. 2.

By the former probleme, it is also manifest, vnto circles three, fower, five, or to how many so euer one will geue, one circle may be geuen equall.

For if first, to any two, by the former probleme, you finde one equall, and then vnto your found circle and the third of the geuen circles, as two geuen circles, finde one other circle equall, and then to that second found circle; and to the fourth of the first geuen circles, as two circles, one new circle be found equall; and so proceede till you haue once cuppled orderly, euery one of your propoised circles (except the first and second already doone) with the new circle thus found: for so the last found circle is equall to all the first geuen circles. If ye doubt, or sufficiently vnderstand me not: helpe your selfe by the discourse and demonstration of the last proposition in the second booke, and also of the 31. in the sixth booke.

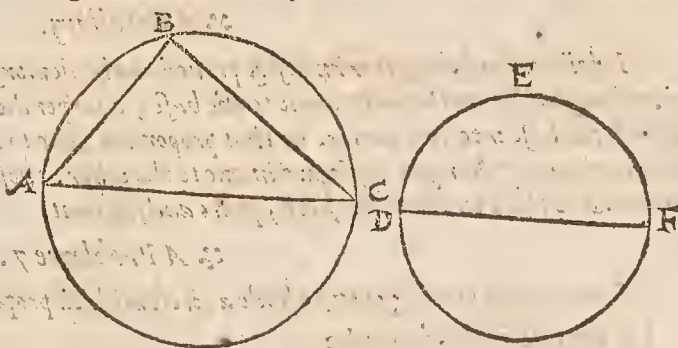
¶ A Probleme 5.

Two vnequall circles being geuen, to finde a circle equall to the excesse of the greater to the lesse.

Suppose the two vnequall circles geue, to be A B C & D E F, & let A B C be the greater: whose diameter suppose to be A C: & the diameter of D E F suppose to be D F. I say a circle must be found equal to that excesse in magnitude, by which A B C is greater the D E F. By the first of the fourth, in the circle A B C, Apply a right line equall to D F: whose one end let be at C, and the other let be at B. Fro B to A draw a right line. By the 30. of the third, it may appeare, that A B C is a right angle: and thereby A B C, the triangle is rectangled: wherefore by the first of the two corollaries, here before, the circle A B C is equall to the circle D E F, (For B C by construction is equall to D F) and more ouer to the circle whose diameter is A B. That circle therefore whose diameter is A B, is the circle conteyning the magnitude, by which A B C is greater then D E F. Wherefore two vnequall circles being geuen, we haue found a circle equall to the excesse of the greater to the lesse: which ought to be doone.

Construction.

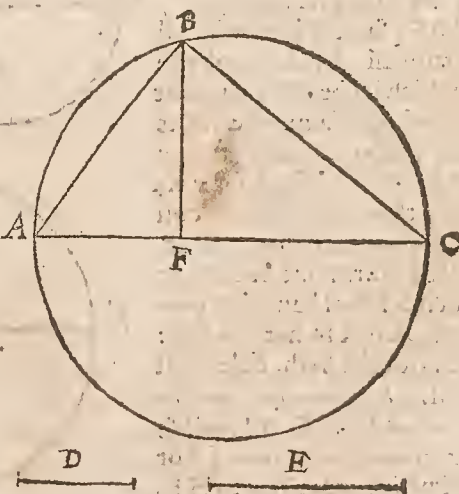
Demonstration.



A Probleme. 6.

A Circle being geuen to finde two Circles equall to the same: which found Circles, shall haue the one to the other, any proportion geuen in two right lines.

Suppose A B C, a circle geuen: and the proportion geuen, let it be that, which is betwene the two right lines D and E. I say, that two circles are to be found equall to A B C: and with al, one to the other, in the proportio of D to E. Let the diameter of A B C be A C. As D is to E, so let A C be deuided, by the 10. of the sixth, in the poynt F. At F, to the line A C let a perpédiculaire be drawne F B, and let it mete the circúferéce at the poynt B. From the poynt B to the points A and C, let right lines be drawne: B A and B C. I say that the circles whose diametes are the lines B A and B C are equall to the circle A B C: and that those circles hauing to their diameters the lines B A and B C are one to the other in the proportion of the line D to the line E. For, first that they are equal, it is e-



Construction.

The twelvethe Booke

ident: by reason that ABC is a triangle rectangle: wherefore by the 47. of the first the squares of BA and BC are equall to the square of AC : And so by this second it is manifest, the two circles to be equall to the circle ABC . Secondly as D is to E , so is AF to FC : by construction.

And as the line AF is to the line FC , so is the square of the line

BA to the square of the line BC . [Which thing, we will

briefely prove thus. The parallelogramme containd

vnder AC and AF , is equall to the square of BA : by the

Lemma after the 32. of the tenth booke: and by the same

Lemma or Assumpt, the parallelogramme containd vnder

AC and FC , is equall to the square of the line BC .

Wherefore as the first parallelogramme hath it selfe to the

second: so hath the square of BA (equall to the first pa-

rallelogramme) it selfe, to the square of BC , equall to the

second parallelogramme. But both the parallelogrammes

have one heighth, namely, the line AC : and bases, the

lines AF and FC : wherefore as AF is to FC , so is the

parallelogramme containd vnder AC , AF , to the parallelogramme containd vnder

AC , FC , by the first of the sixth. And therefore as AF is to FC , so is the square of BA to the square of BC .] And

as the square of BA is to the square of BC : so is the circle whose diameter is BA , to the circle whose

diameter is BC , by this second of the twelfth. Wherefore the circle whose diameter is BA , is to the

circle whose diameter is BC , as D is to E . And before we proued them equall to the circle ABC .

Wherefore a circle being geuen, we haue found two circles equall to the same: which haue the one

to the other any proportion geuen in two right lines. Which ought to be done.

Note.

Here may you perceiue an other way how to execute my first probleme, for if you make a right angle conteyned of the diameters geuen, as in this figure suppose them BA and BC : and then subtend the right angle with the line AC : and from the right angle, let fall a line perpendicular to the base AC : that perpendicular at the point of his fall, diuideth AC into AF and FC , of the proportion required.

✱ A Corollary.

It followeth of thinges manifestly proued in the demonstration of this probleme, that in a triangle rectangle, if from the right angle to the base, a perpendicular be let fall: the same perpendicular cutteth the base into two partes, in that proportion, one to the other, that the squares of the right lines, conteyning the right angle, are in, one to the other: those on the one side the perpendicular, being compared to those on the other, both square and segment.

✱ A Probleme 7.

Betweene two circles geuen, to finde a circle middell proportionall.

Let the two circles geuen, be ACD and BEF : I say, that a circle is to be found, which betwene ACD and BEF is middell proportionall. Let the diameter of ACD , be AD , and of BEF , let BF be the diameter: betwene AD and BF , finde a line middell proportionall, by the 13. of the sixth: which let be HK : I say that a circle, whose diameter is HK is middell proportionall betwene ACD and BEF . To AD , HK , and BF , (three right lines in continuall proportion, by construction) let a fourth line be found: to which BF shal haue that proportion, that AD hath to HK : by the 12. of the sixth, & let that

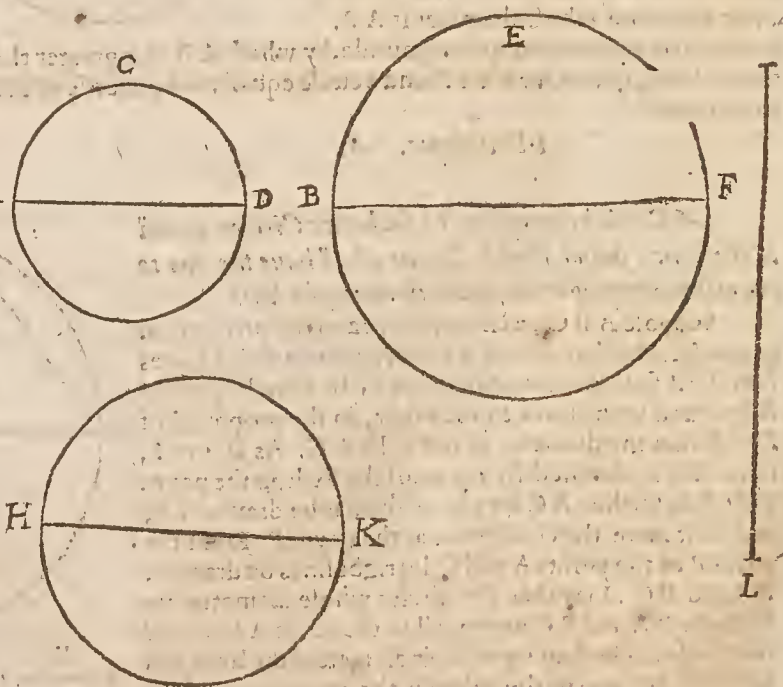
line be L . It is manifest that the

four lines AD , HK , BF , and L ,

are in continuall proportion. [For

by construction, as AD is to HK ,

so is BF to L . And by constructi-



on, as $A D$ is to $H K$, so is $H K$ to $B F$: wherefore $H K$ is to $B F$, as $B F$ is to L : by the 11. of the fifth, wherefore the 4. lines are in continuall proportion.] Wherefore as the first is to the third, that is $A D$ to $B F$, so is the square of the first to the square of the second: that is, the square of $A D$, to the square of $H K$: by the corollary of the 20. of the sixth. And by the same corollary, as $H K$ is to L , so is the square of $H K$ to the square of $B F$. But by alternate proportion, the line $A D$ is to $B F$, as $H K$ is to L : wherefore the square of $A D$ is to the square of $H K$, as the square of $H K$ is to the square of $B F$. Wherefore the square of $H K$ is middell proportionall, betwene the square of $A D$ and the square of $B F$. But as the squares are one to the other, so are the circles (whose diameters produce the same squares) one to the other, by this second of the twelfth: wherefore the circle whose diameter is the line $H K$, is middell proportional, betwene the circles whose diameters are the lines $A D$ and $B F$. Wherefore betwene two circles geuen, we haue found a circle middell proportionall: which was requisite to be doone.

¶ A Corollary.

Hereby it is manifest, three lines or more, being in continuall proportion, that the circles hauing those lines to their diameters, are also in continuall proportion.

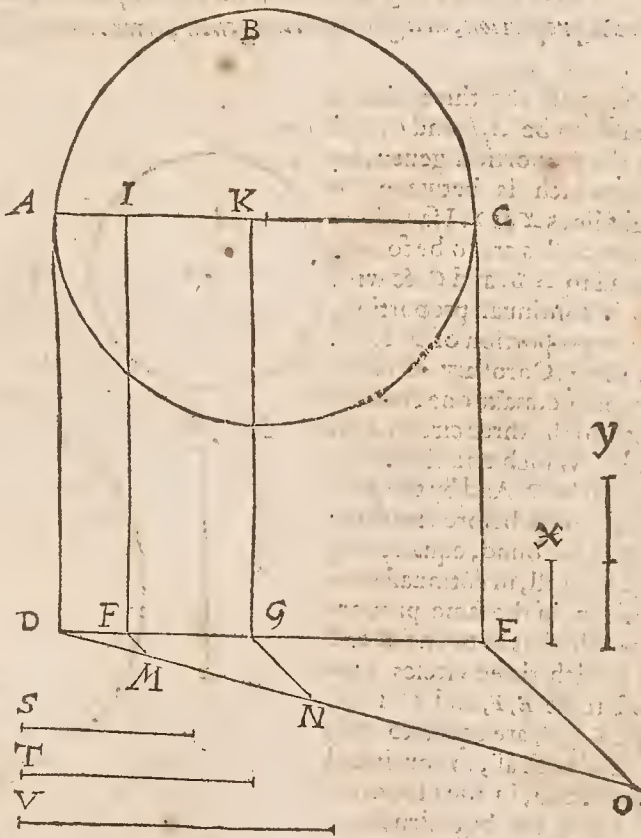
As of three, our demonstration hath already proued: so of fower, will the proufe go forward: if you adde a fifth line in continuall proportion to the fower geuen: as we did to the three, adde the fourth: namely, the line L . And so, if you haue 6, by putting to one more, the demonstration will be easie and plaine. And so of as many as you will.

* A Probleme. 8.

To a circle being geuen, to findethree circles equall: which three circles shall be in continuall proportion, in any proportion geuen betwene two right lines.

Suppose the circle geuen to be $A B C$: and the proportion geuen to be that which is betwene the lines X and Y . I say, that three circles are to be geuen, which three, together, shall be equall to the circle $A B C$: and withall in continuall proportion, in the same proportion which is betwene the right lines X and Y . Let the diameter of $A B C$, be $A C$. Of $A C$, make a square: by the 46. of the first: which

which let be $A C D E$. From the point D drawe a line, sufficiently long (any way, * without the square): which let be $D O$. At the point D , and from the line $D O$, cut a line equall to X : which let be $D M$. At the point M , and from the line $M O$, cut a line equall to Y : which let be $M N$. At the point N , to the two lines $D M$ and $M N$, set a third line proportionall, by the 12. of the sixth: which let be $N O$. From E (one of the angles of the square $A C D E$, next to D) draw a right line to O : making perfecte the triangle $D E O$. Now from the pointes M and N , drawe lines, to the side $D E$, parallel to the side $E O$: by the 31. of the first: which let be $M F$ and $N G$. Wherefore, by the 2. of the sixth, the side $D E$, is proportionally cut in the pointes F and G , as $D O$ is cut in the pointes M and N : therefore, as $D M$ is to $M N$, so is $D F$ to $F G$: and as $M N$ is to $N O$, so is $F G$ to $G E$. Wherefore, seing $D M$, $M N$, and $N O$, are, by construction, continually proportioned, in the proportion of X to Y : So likewise, are $D F$, $F G$, and $G E$, in continuall proportion, in the proportion of X to Y , by the 11. of the fifth. From the pointes F and G , to the opposite side $A C$, let right lines be drawn parallel to the other sides: which



lines, suppose to be $F I$ and $G K$: making thereby, three parallelogrammes $D I, F K$, and $G C$, equall to the whole square $A C D E$. Which three parallelogrammes, by the first of the sixth, are one to another, as their bases, $D F$, $F G$, and $G E$, are. But $D F$, $F G$, and $G E$, were proued to be in continuall proportion, in the proportion of X to Y : Wherefore, the three parallelogrammes $D I, F K$, and $G C$, by the 11. of the fifth, are also in continuall proportion, and in the same, which X is in, to Y . Let three squares be made, equall to the three parallelogrammes $D I, F K$, and $G C$: by the last of the second: Let the sides of those squares be, orderly, S , T , and V . Forasmuch as, it was last concluded that the three paral-

* Though I say, without the square, yet you must thinke, that it may be also within the square: & that diuersly. Wherefore this Probleme may haue diuerse cases, so, but briefly, to a-void all, may thus be said: cut any side of that square into 3. parts, in the proportion of X to Y .

Note the maner of the drift in this demonstration and construction, mixtly: and with no determination to the construction: as commonly is in problems which is here of me so vsed, for an example to young students of variety in art.

The twelue Book

leogrammes, D , E , F , and G (which are equall to the square $ACDE$) are also in continuall proportion, in the proportion of X to T , therefore their equalls, namely, the three squares of S , T , & V , are also equall to the whole square $ACDE$, and in continuall proportion, in the proportion of X to T . Wherefore the three circles, whose diameters are S , T , and V , are equall to the circle, whose diameter is AC , the side of the square $ACDE$, and also in continuall proportion, in the proportion of X to T : by this second of the twelfth. But, by construction, AC is the diameter of the circle ABC . Wherefore we haue found three circles, equall to ABC : namely, the circle, whose diameter is S : and the circle, whose diameter is T : and the circle, whose diameter is V : which three circles, also, are in continuall proportion, in the proportion of X to T . Wherefore to a circle being geuen, we haue found three circles equall in any proportion, geuen, betwene two right lines: which was requisite to be done.

¶ A Corollary.

Hereby, it is euident, that a circle geuen, we may finde circles 4, 5, 6, 10, 20, 100, 1000, or how many soeuer shall be appointed, being in continuall proportion, in any proportion, geuen betwene two right lines: which circles, all together, shall be equall to the circle geuen.

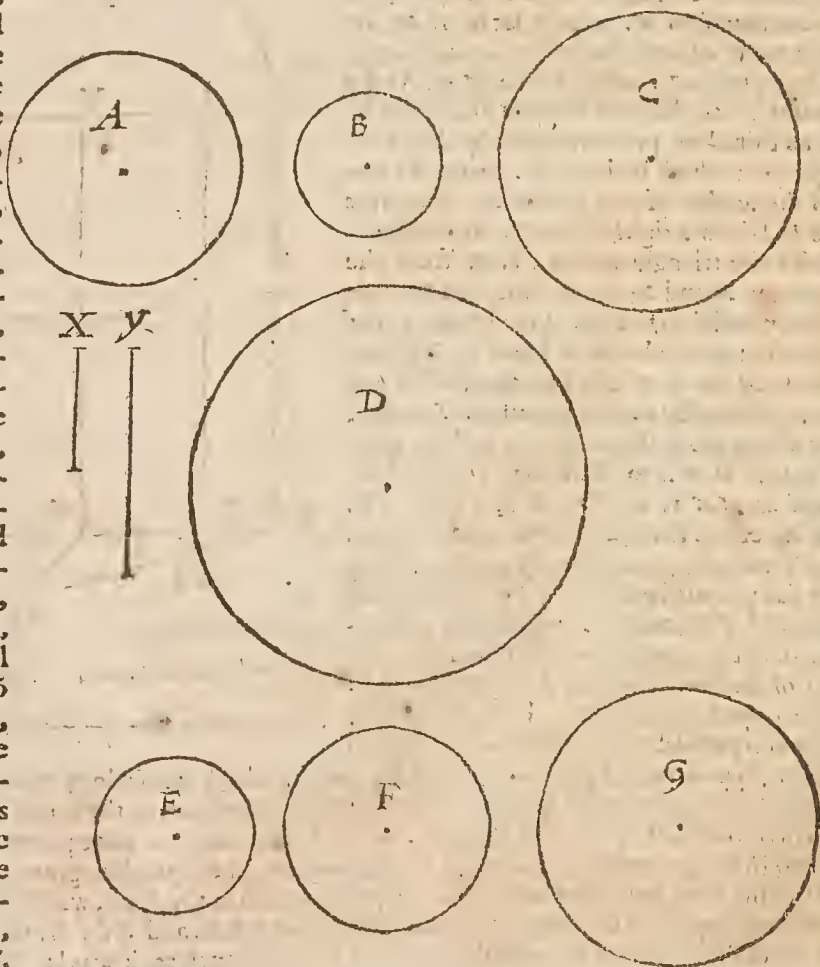
For, euermore deuiding the one side of the chiefe square (which is made of the diameter of the circle geuen) into so many partes, as circles are to be made: so that betwene those partes be continued the proportion geuen betwene two right lines: and from the pointes of those diuisions, drawe parallels, perpendiculars to the other side of the said chiefe square: making so many parallelogrammes of the chiefe square, as are circles to be made: and to those parallelogrammes (orderly) making equall squares: it is manifest that the sides of those squares, are the diameters of the circles required to be made.

* A Probleme. 9.

Three circles being geuen, to finde three equall to them: which three found circles shall be in continuall proportion, in any proportion geuen betwene two right lines.

Suppose the three circles geuen to be A , B , and C , and let the proportion geuen, be that which is betwene the right lines X & T . I say, three other circles are to be found, equall to A , B , and C , & with all, in continuall proportion, in the proportion of X to T .

By the 2. Corollary of my 4. Probleme, make one circle equall to the three circles A , B , and C . Which one circle suppose to be D : And by the probleme next before, let three circles be found, equall to D , and with all, in continuall proportion, in the same proportion which is betwene X and T . Which three circles, suppose to be E , F , and G . I say, that E , F , G , are equall to A , B , C : and with all, in continuall proportion, in the proportion of X to T . For, by construction, the circle D is equall to the circles A , B , & C : and by construction likewise, the circles E , F , and G , are equall to the same circle D : Wherefore the three circles E , F , & G , are equall to the three circles A , B , & C : and by construction E , F , and



Construction.

Demonstration.

and G , are in continuall proportion, in the proportion of the line X to the line T . Wherefore E , F , and G , are equall to A , B , and C : and in continuall proportion, in the proportion of X to T . Three circles, therefore, being geuen, we haue found three circles, equall to them, and also in continuall proportion, in any proportion geuen, betwene two right lines. Which was requisite to be done.

¶ A Corollary. 1.

It is hereby very manifest, that vnto 4. 5. 6. 10. 20. 100. or how many circles soeuer, shall be geuen, we may finde 3. 4. 5. 8. 10. or how many soeuer, shall be appointed: which, all together, shall be equall to the circles geuen, how many soeuer they are: and with all, our found circles, to be in continuall proportion, in any proportion assigned betwene two right lines geuen.

For, enermore, by the Corollary of the 4. Probleme, reduce all your circles to one: and by the Corollary of my 8. Probleme, make as many circles as you are appointed, equall to the circles geuen, and continuall in proportion, in the same, wherein, the two right lines geuen, are: And so haue you performed, the thing required.

Note.

What incredible fruite in the Science of proportions may hereby grow, no mans tounge can sufficiently expresse. And sory I am, that vtterly leysure is taken from me, somewhat to specifie in particular hereof.

¶ The key of one of the chiefe treasure houses, belonging to the State
Mathematicall.

That, which in these 9. Problemes, is said of circles, is much more sayd of squares, by whose meanes, circles, are thus handled. And therefore seing to all Polygonon right lined figures, equall squares may be made, by the last of the second: and contrariwise, to any square, a right lined figure may be made equall, and withall, like to any right lined figure geuen, by the 25. of the sixth. And fourthly, seing vpon the said plaine figures, as vpo bases, may Prismes, Parallelipedons, Pyramids, sided Columnes, Cones, and Cylinders, be reared: which being * all of one height, shall haue that proportion, one to the other, that their bases haue, one to the other. And fiftly, seing Spheres, Cones, and Cylinders are one to other in certaine knowen proportions: and so may be made, one to the other in any proportion assigned. And sixtly, seing vnder euery one of the kindes of figures, both plaine, and solide, infinite cases may chaunce, by the ayde of these Problemes, to be soluted and executed: How infinite (then) vpon infinite, is the number of practises, either Mathematicall, or Mechanicall, to be performed, of comparisons betwene diuers kindes, of plaines to plaines, and solides to solides?

Farthermore, to speake of playne superficiall figures, in respect of the contēt, or Area of the circle, fundry mixt line figures, Anular and Lunular figures: and also of circles to be geuen equall to the sayd vnused figures: and in all proportions els: and euermore thinking of solides, (like high) set vpon any of those vnused figures, (O Lord) in cōsideration of al the premisses, how infinite, how straunge and incredible speculations and practises, may (by the ayde and direction of these few problemes), fall redily into the imaginations and handes of them, that will bring their minde and intent wholly and fixedly to such mathematicall discourses? In these Elementes, I entend but to geue to young beginners some light, aide, and courage to exercise their owne witts, and talent, in this most pleasant and profitable science. All things may not, neither yet can, in euery place be sayd. *Opportunitie, and Sufficiency, best are to be allowed.*

1.
2.
3.
4.
* Note and
5. remember
one both in
6. these solids.

¶ The 3. Theoreme.

The 3. Proposition.

Euery Pyramis hauing a triangle to his base: may be deuided into two Pyramids equall and like the one to the other, and also like to the whole, hauing also triangles to their bases, and into two equall prismes: and those two prismes are greater then the halfe of the whole Pyramis.



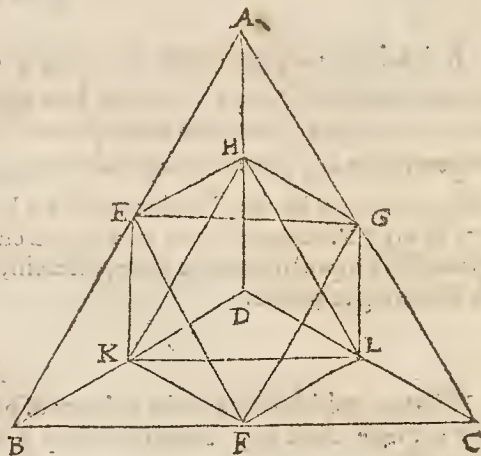
Vppose that there be a Pyramis, whose base let be the triangle ABC , and his toppe the point D . Then I say, that the Pyramis $ABCD$ may be deuided into two Pyramids equall and like the one to the other, and also like to the whole, hauing also triangles to their bases, and into two equall prismes, and those two prismes

The twelue th Booke

Construction. *prismes are greater then the halfe of the whole Pyramis. Denide (by the 10. of the first) the lines $AB, BC, CA, AD, BD, \& CD$, into two equall partes in the pointes E, F, G, H, K , and L . And drawe these right lines $EH, EG, GH, HK, KL, LH, EK, KF$, and FG . Now*

Demonstration of the first part, namely, that the whole Pyramis is deni- ded into two Pyramis-e- quall and like the one to the other, and also to the whole, & hauing tri angles to their bases.

forasmuch as the line AE is equall to the line EB , and the line AH to the line HD : therefore (by the 2. of the sixth) the line EH is a parallel to the line DB . And by the same reason, the line HK is a parallel to the line AB : Wherefore $HEKB$ is a parallelogramme. Wherefore the line HK is equall to the line EB . But the line EB is equall to the line AE : Wherefore the line AE is equall to the line HK . And the line AH is equall to the line HD . Now therefore there are two lines AE and AH , equall to two lines KH and HD , the one to the other, and the angle EAH is (by the 29. of the first) equall to the angle KHD : Wherefore (by the 4. of the first) the base EH is equall to the base KD . Wherefore the triangle AEH , is equall and like to the triangle HKD . And by the same reason also the triangle AHG , is equall and like to the triangle $HL D$. And forasmuch as two right lines EH & HG touching the one the other, are parallels to two right lines KD and DL touching also the one the other, and not being in one and the selfe same plaine superficies with the two first: those lines (by the 10. of the eleuenth) containe equall angles. Wherefore the angle EHG is equall to the angle KDL . And forasmuch as two right lines EH and HG , are equall to two right lines KD & DL , the one to the other, and the angle EHG is (by the 10. of the eleuenth) equall to the angle KDL , therefore (by the 4. of the first) the base EG is equall to the base LK . Wherefore the triangle EHG is equall and like to the triangle KDL . And by the same reason also the triangle AEG is equall and like to the triangle HKL . Wherefore the Pyramis, whose base is the triangle AEG , and toppe the point H , is equall and like to the Pyramis, whose base is the triangle HKL , and toppe the point D .



And forasmuch as to one of the sides of the triangle ADB , namely, to the side AB , is drawn a parallel line HK , therefore the whole triangle ADB is equiangle to the triangle DHK , and their sides are proportionall (by the Corollary of the 2. of the sixth). Wherefore the triangle ADB is like to the triangle DHK . And by the same reason also the triangle DBC is like to the triangle DKL , and the triangle ADC to the triangle DHL . And forasmuch as two right lines BA and AC touching the one the other, are parallels to two right lines KH and HL , touching also the one the other, but not being in one and the selfe same superficies with the two first lines, therefore (by the 10. of the eleuenth) they containe equall angles. Wherefore the angle BAC is equall to the angle KHL . And as the line BA is to the line AC , so is the line KH to the line HL . Wherefore the triangle ABC is like to the triangle KHL . Wherefore the whole Pyramis whose base is the triangle ABC & top the point D , is like to the pyramis whose base is the triangle HKL , and toppe the point D . But the pyramis whose base is the triangle HKL and toppe the point D , is proued to be like to the pyramis whose base is the triangle AEG and toppe the poynt H . Wherefore also the pyramis whose base is the triangle ABC , and toppe the poynt D , is like to the pyramis whose base is the triangle AEG and toppe the poynt H (by the 21. of the sixth). Wherefore either of these pyramids $AEGH$ and $HKL D$ is like to the whole pyramis $ABCD$.

The conclusi- on of the first part.

Demonstratiō of the second part, namely,

[And forasmuch as the lines BE, KH , are parallel lines and equall, as it hath bene proued, therefore the right lines BK and EH , which ioine them together, are equall and parallels, by the 33. of the first. Againe forasmuch as the lines BE and FG are parallel lines and equall, therefore the lines EG and

and B F, which ioynē them together, are also equall and parallels: and by the same reason forasmuch as the lines F G and K H are equall parallels, the lines F K and G H, which ioynē them together, are also equall parallels. Wherefore B E H K, B E G F, and K H G F, are parallelogrammes. And forasmuch as their opposite sides are equall, by the 34. of the first: therefore the triangles E H G, & B K F, are equi-angle, by the 8. of the first: and therefore, by the 4. of the same, they are equall: and moreover, by the 15. of the eleuenth, their superficieses are parallels. Wherefore the solide B K F E H G is a Prisme, by the 11. definition of the eleuenth. Likewise forasmuch as the sides of the triangle H K L are equall and parallels to the sides of the triangle G F C, as it hath before bene proued: It is manifest, that C F K L, F K H G, and C L H G, are parallelogrammes, by the 33. of the first. Wherefore the whole solide K L H F C G, is a Prisme, by the 11. definition of the eleuenth, and is containēd vnder the sayd parallelogrammes C F K L, F K H G, and C L H G, and the two triangles H K L and G F C, which are opposite and parallels.]

And forasmuch as the line B F is equall to the line F C, therefore (by the 41. of the first) the parallelogramme E B F G is double to the triangle G F C. And forasmuch as if there be two Prismes of equall altitudes, and the one haue to his base a parallelogramme, and the other a triangle, and if the parallelogramme be double to the triangle, those Prismes are (by the 40. of the eleuenth) equall the one to the other: therefore the Prisme containēd vnder the two triangles B K F and E H G, and vnder the three parallelogrammes E B F G, E B K H, and K H F G, is equall to the Prisme containēd vnder the two triangles G F C, and H K L, and vnder the three parallelogrammes K F C L, L C G H, and H K F G.

And it is manifest, that both these Prismes, of which the base of one is the parallelogramme E B F G, and the opposite vnto it the line K H, and the base of the other is the triangle G F C, and the opposite side vnto it the triangle K L H, are greater then both these Pyramids, whose bases are the triangles A G E, and H K L, and toppes the pointes H & D. For if we drawe these right lines E F and E K, the Prisme whose base is the parallelogramme E B F G, and the opposite vnto it the right line H K, is greater then the Pyramis whose base is the triangle E B F, & toppe the point K. But the Pyramis whose base is the triangle E B F, and toppe the point K, is equall to the Pyramis whose base is the triangle A E G and toppe the point H, for they are containēd vnder equall and like plaine superficieses. Wherefore also the Prisme whose base is the parallelogramme E B F G and the opposite vnto it the right line H K, is greater then the Pyramis, whose base is the triangle A E G, and toppe the point H. But the prisme whose base is the parallelogramme E B F G, and the opposite vnto it the right line H K, is equall to the prisme, whose base is the triangle G F C, and the opposite side vnto it the triangle H K L: And the Pyramis whose base is the triangle A E G, and toppe the point H, is equall to the Pyramis, whose base is the triangle H K L, and toppe the point D. Wherefore the foresaid two prismes are greater then the foresaid two Pyramids, whose bases are the triangles A E G, H K L, and toppes the pointes H and D. Wherefore the whole Pyramis whose base is the triangle A B C, and toppe the point D, is deuīded into two Pyramids equall and like the one to the other, and like also vnto the whole Pyramis, hauing also triangles to their bases, and into two equall prismes, and the two prismes are greater then halfe of the whole Pyramis: which was required to be demonstrated.

that it is deuīded moreover into two equal Prismes.

Conclusion of the second part.

Demonstratiō of the last part that the two Prismes are greater then the halfe of the whole Pyramis.

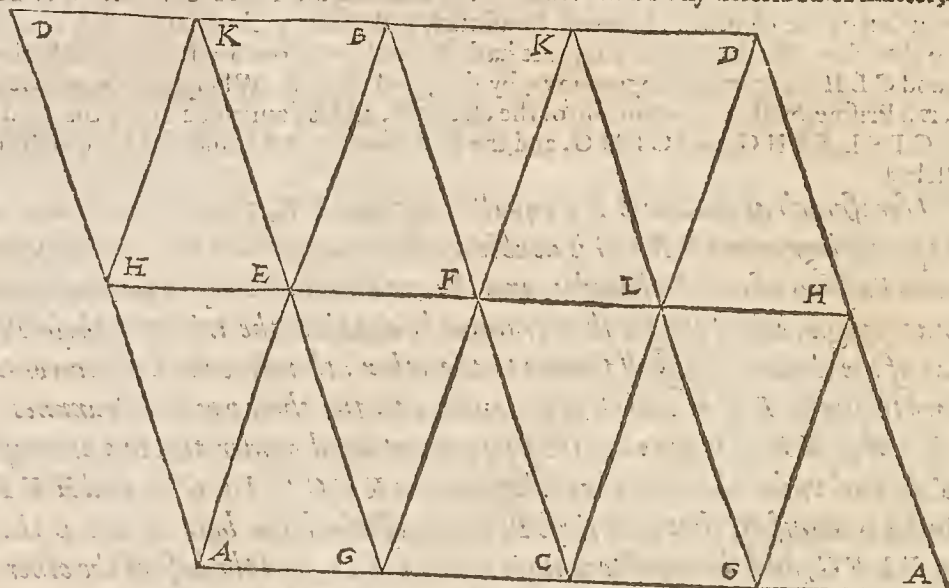
Conclusion of the last part.

Conclusion of the whole proposition.

If ye will with diligence reade these fower bookes following of Euclide, which concerne bodies, and clearly see the demonstrations in them conteyned, it shall be requisite for you when you come to any proposition, which concerneth a body or bodies, whether they be regular or not, first to describe of pasted paper (according as I taught you in the end of the definitions of the eleuenth booke) such a body or bodies, as are there required, and hauing your body, or bodies thus described, when you haue noted it with letters according to the figure set forth vpō a plaine in the propositiō, follow the construction required in the proposition. As for example, in this third propositiō it is sayd that, *Euery pyramis hauing a triangle to his base, may be deuīded into two pyramids.* &c. Here first describe a pyramis of pasted paper hauing his base triangled (it skilleth not whether it be equilater, or equiangled, or not, only in this proposition is required that the base be a triangle. Then for that the proposition supposeth the base of the pyramis to be the triangle A B C, note the base of your pyramis which you haue described with the letters A B C, and the toppe of your pyramis with the letter D: For so is required in the proposition. And thus haue you your body ordered ready to the construction. Now in the construction it is required that ye deuīde the lines, A B, B C, C A. &c, namely, the fixe lines which are the sids of the fower triangles con-
tayning

The twelueeth Booke

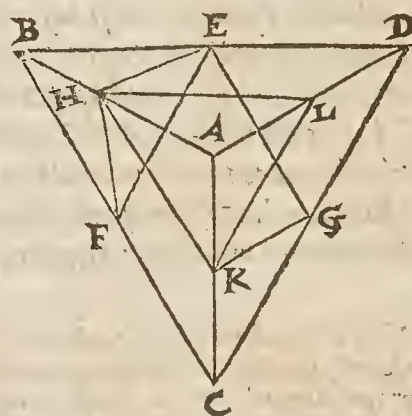
tryning the piramis, into two equall partes in the poyntet E, F, G , &c. That is, ye must deuide the line AB of your pyramis into two equall partes, and note the poynt of the deuision with the letter E , and so the line BC being deuided into two equall partes, note the poynt of the deuision with the letter F . And so the rest, and this order follow ye as touching the rest of the construction there put, and when ye haue finished the construction, compare your body thus described with the demonstration: and it will make it very playne and easy to be vnderstaded. Whereas without such a body described of matter, it is hard



For young beginners (vnlesse they haue a very deepe imagination) fully to conceaue the demonstration by the figue as it is described in a plaine. Here for the better declaration of that which I haue sayd, haue I set a figure, whose forme if ye describe vpon pasted paper noted with the like letters, and cut the lines BA, DA, DC , and folde it accordingly, it will make a Pyramis described according to the construction required in the proposition. And this order follow ye as touching all other propositions which concerne bodyes.

¶ An other demonstration after *Campane* of the 3. proposition.

Suppose that there be a Pyramis $ABCD$ hauing to his base the triangle BCD , and let his toppe be the solide angle A : from which let there be drawne three subtended lines AB, AC , and AD to the three angles of the base, and deuide all the sides of the base into two equall partes in the three poyntes E, F, G : deuide also the three subtended lines AB, AC , and AD in two equall partes in the three points H, K, L . And draw in the base these two lines EF and EG : So shall the base of the pyramis be deuided into three superficiesces: whereof two are the two triangles BEF , and EGD , which are like both the one to the other, and also to the whole base, by the 2. part of the second of the sixth, & by the definition of like superficiesces, & they are equal the one to the other, by the 8. of the first: the third superficies is a quadrangled parallelogramme, namely, $EFGC$: which is double to the triangle EGD , by the 40. and 41. of the first. Now then agayne from the poynt H draw vnto the points E and F these two subtendent lines HE and HF : draw also a subtended line from the poynt K to the poynt G . And draw these lines HK, KL , and LH . Wherefore the whole pyramis $ABCD$ is deuided into two pyramids, which are $HBEF$, and $AHKL$, and into two prismes of which the one is $EHFGKC$, and is set vpon the quadrangled base $EFGC$: the other is $EGDHLK$, and hath to his base the triangle EGD . Now as touching the two pyramids $HBEF$ and $AHKL$, that they are equall the one to the other, and also that they are like both the one to the other and also to the whole, it is manifest by the definition of equall and like bodyes, and by the 10. of the eleuenth, and by 2. part of the second of the sixth. And that the two Prismes are equall it is manifest by the last of the eleuenth. And now that both the prismes taken together are greater then the halfe of the whole pyramis, hereby it is manifest, for that either of them may be deuided into two pyramids, of which the one is a triangular pyramis equall to one of the two pyramids into which together with the two prismes is deuided the whole pyramis, and the other is a quadrangled pyramis double to the other pyramis. Wherefore it is playne that the two prismes taken together are three quarters of the whole pyramis



pyramis deuider. But if ye are desirous to know the proportiō between them, reade the 6. of this booke. But now here to this purpose it shall be sufficient to know, that the two prismes taken together do exceede in quantity the two partial pyramids taken together, into which together with the two prismes the whole pyramis was deuider.

The 4. Theoreme.

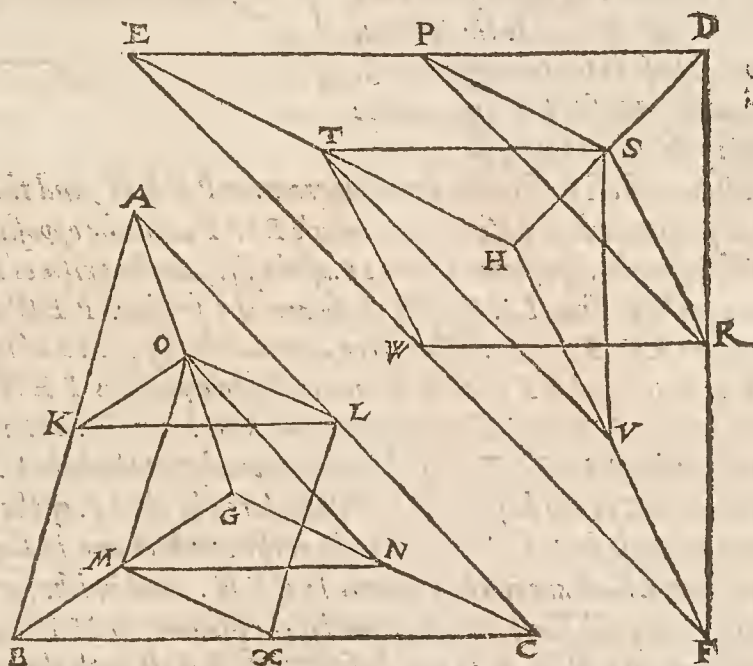
The 4. Proposition.

If there be two Pyramids vnder equall altitudes, hauing triangles to their bases, and either of those Pyramids be deuider into two Pyramids equall the one to the other, and like vnto the whole, and into two equall Prismes, and againe if in either of the Pyramids made of the two first Pyramids be still obserued the same order and maner: then as the base of the one Pyramis is to the base of the other Pyramis, so are all the Prismes which are in the one Pyramis to all the Prismes which are in the other, being equall in multitude with them.



Suppose that there be two Pyramids vnder equall altitudes, hauing triangles to their bases, namely, ABC , and DEF , and hauing to their toppes the pointes G and H . And let either of these pyramids be diuider into two pyramids equall the one to the other, and like vnto the whole, and into two equall prismes (according to the methode of the former Proposition). And againe, let either of those pyramids so made of the two first pyramids, be imagined to be after the same order deuider, and so do continually. Then I say, that as the base ABC is to the base DEF , so are all the prismes which

are in the pyramis $ABCG$, to all the prismes which are in the pyramis $DEFH$ being equall in multitude with them. For forasmuch as the line BX is equall to the line XC , and the line AL to the line LC : (For as we saw in the construction pertayning to the former Proposition, al the fixe sides of the whole pyramids, are ech deuider into two equall parts, the like of which construction is in this proposition also supposed): therefore the line XL is a parallel to the line AB , & the triangle ABC , is like to the triangle LXC , (by the Corollary of the second of the sixth): and by the same reason the tri-



angle DEF is like to the triangle RWF . And forasmuch as the line BC is double to the line CX , and the line FE to the line FW : therefore as the line BC is to the line CX , so is the line FE to the line FW . And vpon the lines BC and CX are described rectiline figures like and in like sort set, namely, the triangles ABC and LXC , and vpon the lines EF and FW are also described rectiline figures, like and in like sort set, namely, the triangles DEF & RWF . But if there be fower right lines proportionall, the rectiline figures described of them being like und in like sort set, shall also be proportionall (by the 22. of the sixt). Wherefore as the

DDd.i.

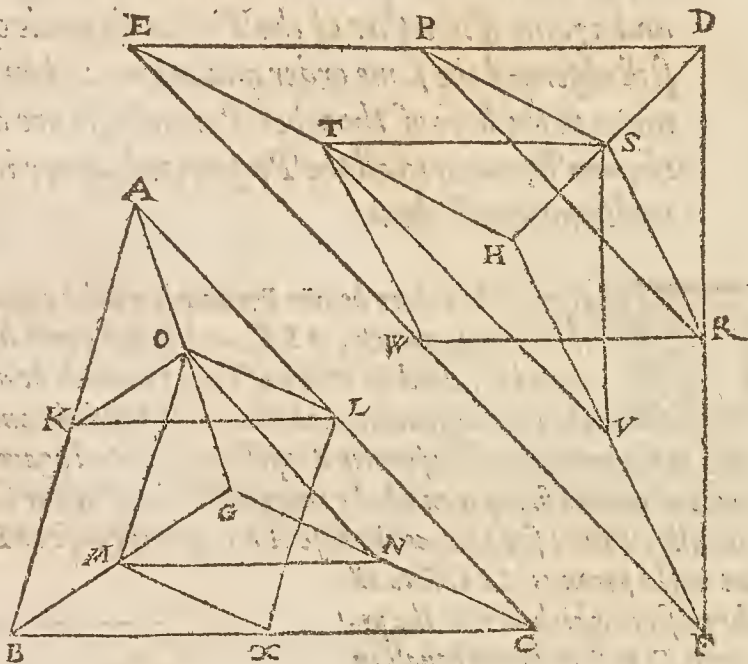
triangle

The twelueeth Booke

An Assumpt.

triangle ABC is to the triangle LXC , so is the triangle DEF to the triangle RWF . Wherefore alternately (by the 16. of the fifth) as the triangle ABC is to the triangle DEF , so is the triangle LXC to the triangle RWF . * But as the triangle LXC is to the triangle RWF , so is the prisme whose base is the triangle LXC , and the opposite side vnto it the triangle OMN , to the prisme whose base is the triangle RWF , and the opposite side vnto it the triangle STV (by the Corollary of the 40. of the eleuenth). For these prismes are vnder one & the selfe same altitude, namely, vnder the halfe of the altitude of the whole Pyramids, which Pyramids are supposed to be vnder one and the selfe same altitude: this is also proued in the Assumpt following). Wherefore (by the 11. of the fifth) as the triangle ABC is to

the triangle DEF , so is the prisme whose base is the triangle LXC , and the opposite side vnto it the triangle OMN , to the prisme whose base is the triangle RWF , and the opposite side vnto it the triangle STV . And forasmuch as there are two prismes in the pyramis $ABCG$ equal the one to the other, & two prismes also in the pyramis $DEFH$ equall the one to the other: therefore as the prisme, whose base is the parallelograme $BK-LX$, and the opposite side vnto it the line MO , is to the prisme, whose base is the triangle LXC , and the opposite side vnto it the triangle OMN , so is the



prisme, whose base is the parallelogramme $PERW$, and the opposite vnto it the line ST , to the prisme, whose base is the triangle RWF , and the opposite side vnto it the triangle STV . Wherefore by composition (by the 18. of the fifth) as the prismes $KBXLMO$, & $LXCMNO$, are to the prisme $LXCMNO$, so are the prismes $PEWRST$, and $RWFSTV$, to the prisme $RWFSTV$. Wherefore alternately (by the 16. of the fifth) as the two prismes $KBXLMO$, and $LXCMNO$, are to the two prismes $PEWRST$, and $RWFSTV$, so is the prisme $LXCMNO$ to the prisme $RWFSTV$. But as the prisme $LXCMNO$ is to the prisme $RWFSTV$, so haue we proued that the base LXC is to the base RWF , and the base ABC to the base DEF . Wherefore (by the 16. of the fifth) as the triangle ABC is to the triangle DEF , so are both the prismes which are in the pyramis $ABCG$, to both the prismes which are in the pyramis $DEFH$. And in like sort if we diuide the other pyramids after the selfe same maner, namely, the pyramis $OMNG$, and the pyramis $STVH$: as the base OMN is to the base STV , so shall both the prismes that are in the pyramis $OMNG$, be to both the prismes which are in the pyramis $STVH$. But as the base OMN is to the base STV , so is the base ABC to the base DEF . Wherefore (by the 11. of the fifth) as the base ABC is to the base DEF , so are the two prismes that are in the pyramis $ABCG$, to the two prismes that are in the pyramis $DEFH$, and the two prismes that are in the pyramis $OMNG$, to the two prismes that are in the pyramis $STVH$, and the fower prismes to the fower prismes. And so also shall it followe in the prismes made by diuiding the two pyramids $AKLO$, and $DPRS$, and of all the other pyramids in generall, being equall in multitude.

And that as the triangle LXC is to the triangle RWF , so is the prisme whose base is the triangle LXC and the opposite side OMN , to the prisme whose base is the triangle RWF and the opposite side the triangle STV , may thus be proued. For in the selfe same construction, imagine perpendicular lines to be drawne from the poyntes G and H to the two playne superficieses wherein are the triangles ABC and DEF . Now those perpendicular lines shall be equall the one to the other, for that the two pyramids are supposed to be of equall altitude. And forasmuch as two right lines, namely, GC , and the perpendicular line drawne from the poynt G , are denided by two parallel playne superficieses, namely, ABC and OMN , therefore (by the 17. of the eleuenth) the partes of the lines denided are proportionall. But the line GC is by the playne superficies OMN denided into two equall partes in the poynt N . Wherefore also the perpendicular line drawne from the poynt G to the playne superficies wherein is the triangle ABC , is denided into two equall partes by the superficies OMN : & by the same reason also the perpendicular line which is drawne from the poynt H to the playne superficies DEF is denided into two equall partes, by the playne superficies STV . And the perpendiculars drawne from the poyntes G and H to the playne superficieses ABC and DEF are equal. Wherefore also the perpendicular lines which are drawne from the triangles OMN and STV to the playne superficieses ABC and DEF are equal the one to the other. Wherefore also the prismes whose bases are the triangles LXC and RWF and the opposite sides the triangles OMN and STV , are of equall altitude. Wherefore also the parallelipedons which are described of the forsayd prismes and being equall in altitude with them, are the one to the other, as the base of the one is to the base of the other. Wherefore also as the halfe of the bases of those parallelipedons, namely, as the base LXC is to the base RWF , so are the forsayd prismes the one to the other. If therfore there be two pyramids vnder equall altitudes, hauing triangles to their bases and either of those pyramids be denided into two pyramids equall the one to the other and like vnto the whole, and into two equall prismes, and agayne if in either of the pyramids made of the two first pyramids be still obserued the same order and maner: then as the base of the one pyramis is to the base of the other pyramis, so all the prismes which are in the one pyramis to all the prismes which are in the other being equall in multitude: which was required to be proued.

An Assumpt.

Conclusion of the whole.

The 5. Theoreme.

The 5. Proposition.

Pyramids consisting vnder one and the selfe same altitude, hauing triangles to their bases: are in that proportion the one to the other that their bases are.



Suppose that these two Pyramids, whose bases are the triangles ABC & DEF , and toppes the pointes G and H , be vnder equall altitudes. Then I say, that as the base ABC is to the base DEF , so is the pyramis $ABCG$ to the pyramis $DEFH$. For if the pyramis $ABCG$ be not to the pyramis $DEFH$, as the base ABC is to the base DEF , then as the base ABC is to the base DEF , so is the pyramis $ABCG$ to a solide, either lesse then the pyramis $DEFH$, or greater. First let it be to some lesse, and let the same be X . And (by the 3. of the twelfth) let the pyramis $DEFH$ be denided into two pyramids equall the one to the other, and like vnto the whole, and into two equall prismes. Now the two prismes are greater then the halfe of the whole pyramis.

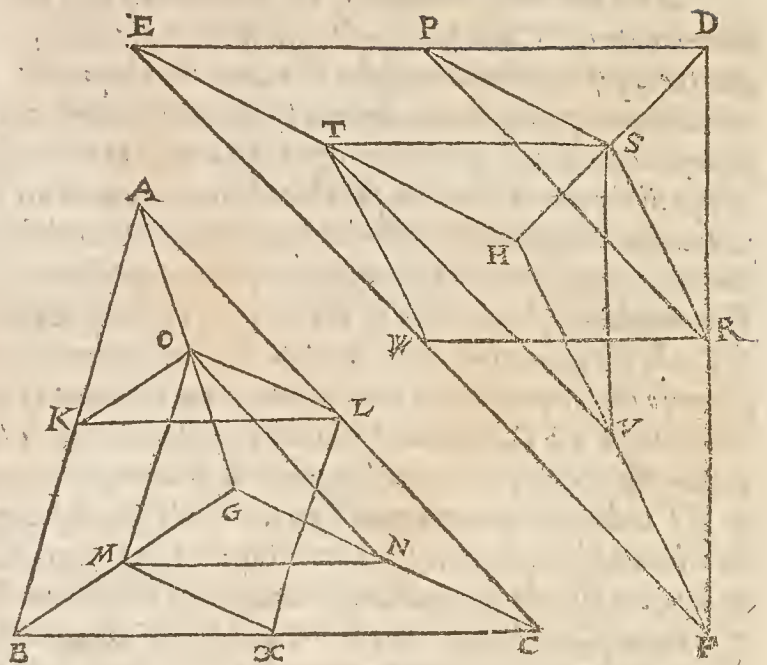
Demonstration leading to an impossibilitie.

DDd.ij.

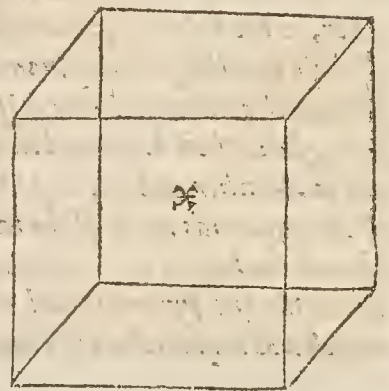
And

The twelueeth Booke

And againe (by the same) let the pyramids which are made of the diuision, be in like sort diuided, and do this continually, vntill there remaine some pyramids made of the pyramis $DEFH$, which are lesse then the excesse, whereby the pyramis $DEFH$ exceedeth the solide X . Let such pyramids be taken, and for example sake, let those pyramids be $DPRS$, & $STVH$. Wherefore the prisms remayning which are in the pyramis $DEFH$, are greater then the solide X . Deuide (by the Proposition next going before) the pyramis $ABCG$ in like sort, & as many times as the pyramis $DEFH$ is deuided.



Wherefore (by the same) as the base ABC is to the base DEF , so are all the prisms which are in the pyramis $ABCG$, to all the prisms which are in the pyramis $DEFH$. But as the base ABC is to the base DEF , so is the pyramis $ABCG$ to the solide X . Wherefore (by the 11. of the fift) as the pyramis $ABCG$, is to the solide X , so are the prisms which are in the pyramis $ABCG$, to the prisms which are in the pyramis $DEFH$. Wherefore alternately (by the 16. of the fift) as the pyramis $ABCG$ is to the prisms which are in it, so is the solide X , to the prisms which are in the pyramis $DEFH$. But the pyramis $ABCG$ is greater then the prisms which are in it. Wherefore also the solide X is greater then the prisms which are in the pyramis $DEFH$ (by the 14. of the fift). But it is supposed to be lesse: which is impossible. Wherefore as the base ABC is to the base DEF , so is not the pyramis $ABCG$ to any solide lesse then the pyramis $DEFH$.



I say moreover, that as the base ABC is to the base DEF , so is not the pyramis $ABCG$, to any solide greater then the pyramis $DEFH$. For if it be possible, let it be vnto some greater, namely, to the solide X . Wherefore (by conuersion, by the Corollary of the 4. of the fift) as the base DEF is to the base ABC , so is the solide X to the pyramis $ABCG$. But as the solide X is to the pyramis $ABCG$, so is the pyramis $DEFH$ to some solide lesse then the pyramis $ABCG$,* as we haue before proued: Wherefore also (by the 11. of the fift) as the base DEF is to the base ABC , so is the pyramis $DEFH$, to some solide lesse then the Pyramis $ABCG$: which thing we haue proued to be impossible. Wherefore as the base ABC is to the base DEF , so is not the pyramis $ABCG$ to any solide greater then the pyramis $DEFH$: and it is also proued that it is not in that proportion to any lesse then the pyramis $DEFH$. Wherefore as the base ABC is to the base DEF , so is the pyramis $ABCG$ to the pyramis $DEFH$. Wherefore pyramids consisting vnder one and the selfe same altitude, and hauing triangles to their bases, are in that proportion the one to the other, that their bases are: which was required to be demonstrated.

* In the Assumpt following the second proposition of this booke.

¶ The 6. Theoreme.

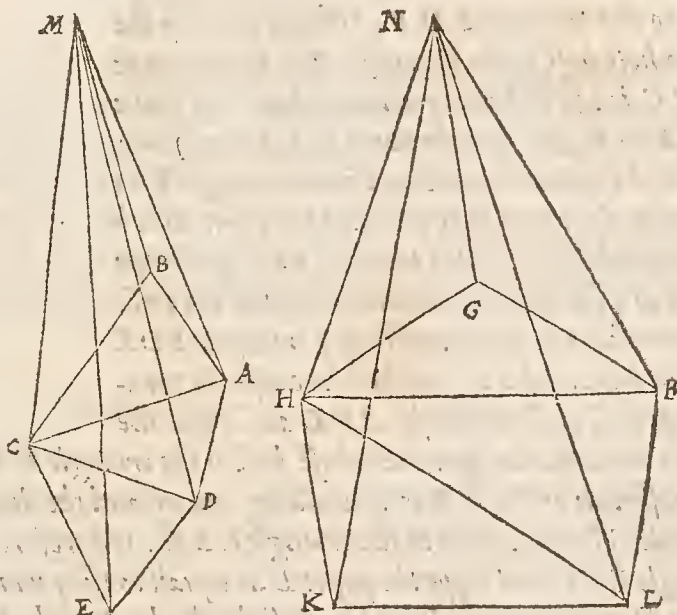
The 6. Proposition.

Pyramids consisting vnder one and the selfe same altitude, and hauing Polygonon figures to their bases: are in that proportion the one to the other, that their bases are.



Suppose that there be two Pyramids, hauing to their bases these Polygonon figures $ABCE D$, and $FGHKL$, and let their toppes be the pointes M and N , which let be of one and the selfe same altitude. Then I say, that as the base $ABCE D$ is to the base $FGHKL$, so is the pyramis $ABCE D M$, to the pyramis $FGH-$

$KL N$. Diuide the base $ABCE D$ into these triangles ABC , ACD , & CDE , and likewise the base $FGHKL$ into these triangles FGH , FHL , and HKL . And imagine that vpon euery one of those triangles be set a pyramis of equall altitude with the two pyramids put at the beginning. And for that as the triangle ABC is to the triangle ADC , so is the pyramis $ABCM$ to the pyramis $ADCM$ (by the 5. of this booke). Wherefore, by composition (by the 18. of the fift) as the



Construction.

Demonstration.

fower sided figure $ABCE D$ is to the triangle ACD , so is the pyramis $ABCE D M$ to the pyramis $ACD M$. But as the triangle ACD is to the triangle CDE , so is the pyramis $ACD M$ to the pyramis $CDE M$. Wherefore of equalitie (by the 22. of the fift) as the base $ABCE D$ is to the base CDE , so is the pyramis $ABCE D M$ to the pyramis $CDE M$. Wherefore againe by composition (by the 18. of the fift) as the base $ABCE D$ is to the base CDE , so is the pyramis $ABCE D M$ to the pyramis $CDE M$. And by the same reason also as the base $FGHKL$ is to the base HKL , so is the pyramis $FGHKL N$ to the pyramis $HKL N$. And forasmuch as there are two pyramids $CDE M$ and $HKL N$, hauing triangles to their bases, and being vnder one and the selfe same altitude, therefore (by the 5. of the twelfth) as the base CDE is to the base HKL , so is the pyramis $CDE M$ to the pyramis $HKL N$. Now for that as the base $ABCE D$ is to the base CDE , so is the pyramis $ABCE D M$ to the pyramis $CDE M$. But as the base CDE is to the base HKL , so is the pyramis $CDE M$ to the pyramis $HKL N$. Wherefore of equalitie (by the 22. of the fift) as the base $ABCE D$ is to the base HKL , so is the pyramis $ABCE D M$ to the pyramis $HKL N$. But also as the base HKL is the base $FGHKL$, so is the Pyramis $HKL N$ to the pyramis $FGHKL N$. Wherefore againe of equalitie (by the 22. of the fift) as the base $ABCE D$ is to the base $FGHKL$, so is the pyramis $ABCE D M$ to the pyramis $FGHKL N$. Wherefore pyramids consisting vnder one and the selfe same altitude, and hauing Polygonon figures to their bases, are in that proportion the one to the other, that their bases are: which was required to be proued.

The twelue Book

The 7. Theoreme.

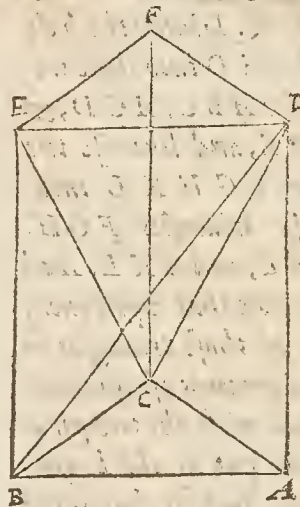
The 7. Proposition.

Euery prisme hauing a triangle to his base, may be deuided into three pyramids equall the one to the other, hauing also triangles to their bases.

Demonstration.



Suppose that $AB C D E F$ be a prisme, hauing to his base the triangle $A B C$, and the opposite side vnto it, the triangle $D E F$. Then I say that the prisme $A B C D E F$, may be deuided into three pyramids equall the one to the other, and hauing triangles to their bases. Draw these right lines $B D$, $E C$, and $C D$. And forasmuch as $A B E D$ is a parallelogramme, and his diameter is the line $B D$, therefore the triangle $A B D$ is equall to the triangle $E D B$. Wherefore also the pyramid whose base is the triangle $A B D$, and toppe the poynt C , is equall to the pyramid whose base is the triangle $E D B$, & toppe the poynt C , by the 5. of this booke. But the pyramid whose base is the triangle $E D B$, and toppe the poynt C , is one and the same which the pyramid whose base is the triangle $E B C$, and toppe the poynt D , for they are comprehended of the selfe same playne superficieses, namely, of 3 triangles $B D E$, $D E C$, $D B C$, and $E B C$. Wherefore also the pyramid whose base is the triangle $A B D$ and toppe the poynt C , is equall to the pyramid whose base is the triangle $E B C$ and toppe the poynt D . Againe forasmuch as $B C F E$ is a parallelogramme, and the diameter thereof is $E C$, therefore the triangle $E C F$ is equall to the triangle $C B E$. Wherefore also the pyramid whose base is the triangle $E B C$ and toppe the poynt D , is equall to the pyramid, whose base is the triangle $E C F$ and toppe the poynt D , by the 5. of this booke. But the pyramid whose base is the triangle $B E C$ and toppe the poynt D , is proued to be equall to the pyramid whose base is the triangle $A B D$, and toppe the poynt C . Wherefore also the pyramid whose base is the triangle $C E F$ and toppe the poynt D , is equall to the pyramid whose base is the triangle $A B D$ & toppe the poynt C . Wherefore the prisme $A B C D E F$ is deuided into three equall pyramids hauing triangles to their bases. And forasmuch as the pyramid whose base is the triangle $A B D$ and toppe the poynt C , is one & the selfe same with the pyramid whose base is the triangle $C A B$ & toppe the poynt D (for they are containd vnder the selfe same playne superficieses) but it hath bene proued that the pyramid whose base is the triangle $A B D$ and toppe the poynt C , is the third pyramid of the prisme whose base is the triangle $A B C$ and the opposite side vnto it, the triangle $D E F$. Wherefore the pyramid whose base is the triangle $A B C$ and toppe the poynt D is the third pyramid of the prisme, whose base is the triangle $A B C$, and opposite side the triangle $D E F$. Wherefore euery prisme hauing a triangle to his base, may be deuided into three pyramids equall the one to the other, hauing also triangles to their bases: which was required to be proued.



¶ Corollary.

Hereby it is manifest that euery pyramid is the third part of a prisme hauing one and the same base with it and also being vnder the selfe same altitude with it. For if the base of the prisme be any other rectiline figure the a triangle, that also may be deuided into prismes which shal haue triangles to their bases.

Here

Here Campane and Flußas adde certayne Corollaryes.

First Corollary.

Every Prisme is treble to the Pyramis, which hath the selfe same triangle to his base that the Prisme hath, and the selfe same altitude. As it is manifest by this proposition, where the Prisme is deuided into three equall Pyramids, of which, two are vpon one and the selfe same base, and vnder one and the selfe same altitude. But if the Prisme haue to his base a parallelogramme, and if the Pyramis haue to his base the halfe of the same parallelogramme, and their altitudes be equall, then agayne the Pyramis shalbe the third part of the Prisme. For it was manifest, by the 40. of the eleueth, that Prismes, being vnder equall altitudes, and the one hauing to his base a triangle, and the other a parallelogramme double to the same triangle, are equall the one to the other. Wherof followeth the former conclusion.

Second Corollary.

If there be many Prismes vnder one and the same altitude, and hauing triangles to their bases, and if the triangular bases be so ioyned together vpon one and the same playne, that they compose a Polygonon figure: A pyramis set vpon that base being a Polygonon figure, and vnder the same altitude, is the third part of that solide, which is composed of all the Prismes added together.

For forasmuch as euery one of the Prismes which hath to his base a triangle, to euery one of the Pyramids set vpon the same base (the altitude being alwayes one and the same) is treble, it is manifest by the 12. of the fiieth, that all the Prismes are to all the Pyramids treble. Wherefore Parallelepipedons are treble to Pyramids set vpon the selfe same base with them, and vnder the same altitude, for that they contayne two Prismes.

Third Corollary.

If two Prismes being vnder one and the selfe same altitude, haue to their bases either, both triangles, or both parallelogrammes, the Prismes are the one to the other, as their bases are.

For forasmuch as those Prismes are equemultiplises vnto the Pyramids vpon the selfe same bases, and vnder the same altitude, which Pyramids are in proportion as their bases, it is manifest (by the 15. of the fift), that the Prismes are in the proportion of the bases. For by the former Corollary, the Prismes are treble to the Pyramids set vpon the triangular bases.

Fourth Corollary.

Prismes are in sesquialtera proportion to Pyramids which haue the selfe same quadrangled base that the Prismes haue, and are vnder the selfe same altitude.

For, that Pyramis containeth two Pyramids set vpon a triangular base of the same Prisme, for it is proued, that that Prisme is treble to the Pyramis which is set vpon the halfe of his quadrangled base, vnto which the other which is set vpon the whole base is double, by the sixth of this booke.

Fiuenth Corollary.

Wherefore we may in like sort conclude, that solides mencioned in the second Corollary (which solids Campane calleth sided Columnes) being vnder one and the selfe same altitude, are in proportion the one to the other, as their bases, which are polygonon figures.

For they are in the proportion of the Pyramids or Prismes, set vpon the selfe same bases, and vnder the selfe same altitude, that is, they are in the proportiō of the bases of the sayde Pyramids or Prismes. For those solids may be deuided into Prismes hauing the selfe same altitude, when as their opposite bases may be deuided into triangles, by the 20 of the sixth. Vpon which triangles Prismes beyng set, are in proportion as their bases.

By this 7. Proposition it playnely appeareth that Euclide, as it was before noted in the diffinitions, vnder the diffinition of a Prisme, comprehended also those kinds of solids, which Campane calleth sided Columnes. For in that he sayth, Every Prisme hauing a triangle to his base, may be deuided. &c. he neded not (taking a Prisme in that sense which Campane and most men take it) to haue added that particule, hauing to his base a triangle. For by their sense, there is no Prisme, but it may haue to his base a triangle: and so it may seeme that Euclide ought without exception haue sayd, that, euery prisme whatsoeuer, may

Note:

Sided Columnes (some time called prismes) are treble to pyramids, hauing one base and equal heith with them.

Note: Parallelepipedons treble to pyramids of one base and heith with them.

The twelue Booke

be deuided into three pyramids equall the one to the other, hauing also triangles to their bases. For so do *Campane* and *Flussas* put the proposition, leauing out the former particle *hauing to his base a triangle*, which yet is red in the Greeke cople, & not left out by any other interpreters knowne abroad except by *Campane* and *Flussas*. Yea and the Corollary following of this proposition added by *Theon* or *Euclide*, and ameded by *M. Dee* semeth to confirme this sence.

Of this, is it made manifest, that every pyramid is the third part of the prisme, hauing the same base with it, and equall altitude. For, and if the base of the prisme haue any other right lined figure (then a triangle) and also the superficies opposite to the base, the same figure: that prisme may be deuided into prismes, hauing triangled bases: and the superficieses to those bases opposite, also triangled a like and equally.

For there, as we see are put these wordes, *For and if the base of the prisme be any other right lined figure. &c.* whereof a man may well inferre that the base may be any other rectiline figure whatsoever, & not only a triangle or a parallelogramme, and it is true also in that sence, as it is plaine to see by the second corollary added out of *Flussas*, which corollary, as also the first of his corollaries, is in a maner all one with the Corollary added by *Theon* or *Euclide*. Farther *Theon* in the demonstration of the 10. proposition of this booke (as we shall afterward see) most playnely calleth not onely sided columnes prismes, but also parallelepipedons. And although the 40. proposition of the eleuenth booke may seme hereunto to be a let. For that it can be vnderstanded of those prismes onely which haue triangles to their like, equall, opposite, and parallel sides, or but of some sided columnes, and not of all: yet may that let be thus remoued away, to say that *Euclide* in that proposition vsed *genus pro specie*, that is, the generall word for some special kinde therof: which thing also is not rare, not only with him, but also with other learned philosophers. Thus much I thought good by the way to note in farther defence of *Euclids* definition of a Prisme.

The 8. Theoreme.

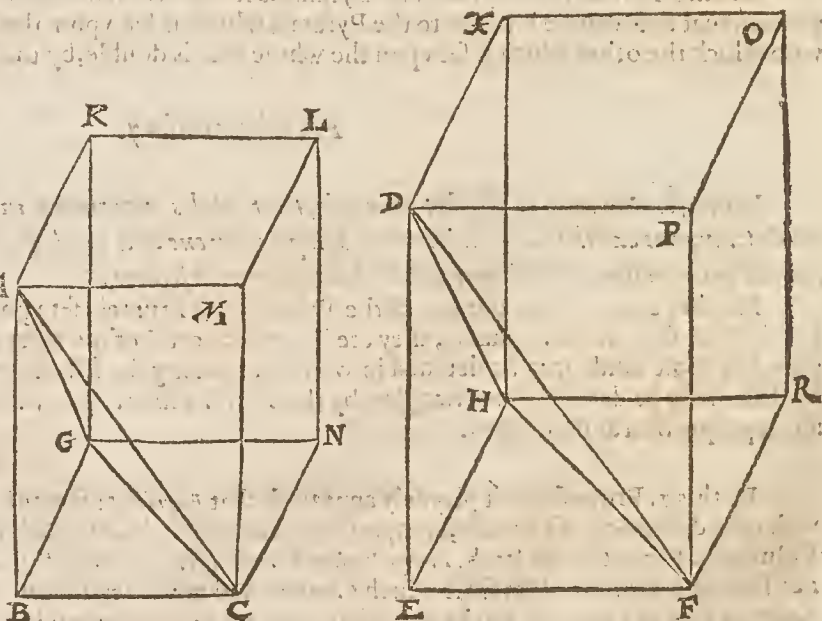
The 8. Proposition.

Pyramids being like & hauing triangles to their bases, are in treble proportion the one to the other, of that in which their sides of like proportion are.



Suppose that these pyramids whose bases are the triangles $GB C$ and $H E F$ and toppes, the poyntes A and D be like, and in like sort described, and let $A B$ and $D E$ be sides of like proportion. Then I say that the pyramid $A B C G$ is to the pyramid $D E F H$ in treble proportio, of that in which the side $A B$ is to the side

$D E$. Make perfect the parallelepipedons, namely, the solides $B C K L$ & $E F X O$. And forasmuch as the pyramid $A B C G$ is like to the pyramid $D E F H$, therefore the angle $A B C$ is equall to the angle $D E F$, & the angle $G B C$ to the angle $H E F$, and moreover the angle $A B G$ to the angle $D E H$, and as the line $A B$ is to the line $D E$, so is the line $B C$ to the line $E F$, and the line $B G$ to the line $E H$. And for that as the line $A B$ is to the line $D E$, so is the line $B C$ to the line $E F$, and the sides about the equall angles, are proportionall, therefore the parallelogramme $B M$ is like to the parallelogramme $E P$: and by the same reason the parallelogramme $B N$ is like



Construction.
Demonstration.

to the parallelogramme ER , and the parallelogramme BK is like unto the parallelogramme EX . Wherefore the three parallelogrammes BM , KB and BN are like to the three parallelogrammes EP , EX , and ER . But the three parallelogrammes BM , KB , and BN are equall and like to the three opposite parallelogrammes, and the three parallelogrammes EP , EX , and ER are equall and like to the three opposite parallelogrammes. Wherefore the parallelepipedons $BCKL$ and $EFXO$ are comprehended under playne superficieses like and equall in multitude. Wherefore the solide $BCKL$ is like to the solide $EFXO$. But like parallelepipedons are (by the 33. of the eleventh) in treble proportion the one to the other of that in which side of like proportion is to side of like proportion. Wherefore the solide $BCKL$ is to the solide $EFXO$ in treble proportion of that in which the side of like proportion AB is to the side of like proportion DE . But as the solide $BCKL$ is to the solide $EFXO$, so is the pyramis $ABCG$ to the pyramis $DEFH$ (by the 15. of the fifth) for that the pyramis is the sixth part of this solide: for the prisme, being the halfe of the parallelepipedon is treble to the pyramis. Wherefore the pyramis $ABCG$ is to the pyramis $DEFH$ in treble proportion of that in which the side AB is to the side DE . Which was required to be proued.

Corollary.

Hereby it is manifest that like pyramids hauing to their bases polygon figures, are in treble proportion the one to the other, of that in which side of like proportion, is to side of like proportion.

For if they be deuided into pyramids hauing triangles to their bases (for like polygon figures are deuided into like triangles, and equal in multitude, and the sides are of like proportion) as one of the pyramids of the one, hauing a triangle to his base, is to one of the pyramids of the other, hauing also a triangle to his base, so also are all the pyramids of the one pyramis hauing triangles to their bases to all the pyramids of the other pyramis hauing also triangles to their bases. That is, the pyramis hauing to his base a polygon figure, to the pyramis hauing also to his base a polygon figure. But a pyramis hauing a triangle to his base, is to a pyramis hauing also a triangle to his base, & being like unto it, in treble proportion of that in which side of like proportion is to side of like proportion. Wherefore a pyramis hauing to his base a polygon figure, is to a pyramis hauing also a polygon figure to his base, the sayd pyramids being like the one to the other, in treble proportion of that in which side of like proportion is to side of like proportion. Likewise Prismes and sided columnes, being set vpon the bases of those pyramids, and vnder the same altitude (forasmuch as they are equemultiples vnto the pyramids, namely, triples, by the corollary of the 7. of this booke) shal haue the former proportion that the pyramids haue, by the 15. of the fifth, and therefore they shall be in treble proportion of that in which the sides of like proportion are.

An addition
by Campane
and Flussas.

¶ The 9. Theoreme.

The 9. Proposition.

In equall pyramids hauing triangles to their bases, the bases are reciprokall to their altitudes. And pyramids hauing triangles to their bases, whose bases are reciprokall to their altitudes, are equall the one to the other.



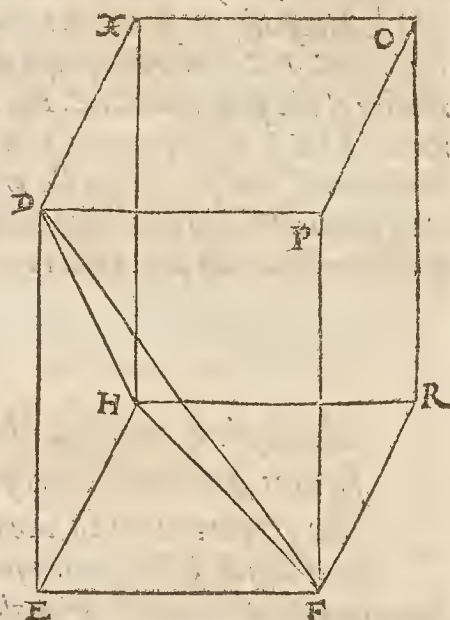
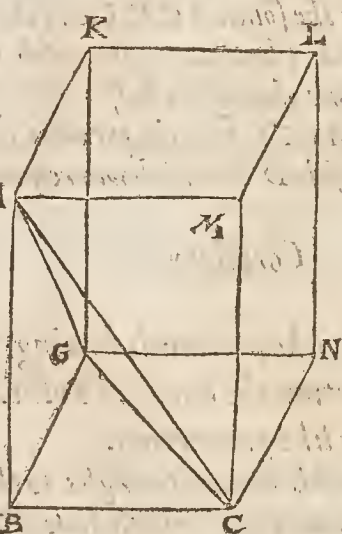
Suppose that $BCGA$ and $EFHD$ be equall pyramids, hauing to their bases the triangles BCG and EFH , and the tops the pointes A and D . Then I say that the bases of the two pyramids $BCGA$ and $EFHD$ are reciprokall to their altitudes: that is, as the base BCG is to the base EFH , so is the altitude of the pyramis $EFHD$ to the altitude of the pyramis $BCGA$. Make perfect the parallelepipedons, namely,

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Demonstration of the first part.

namely, $BGM L$ and $EHP O$. And forasmuch as the pyramid $BCGA$ is equal to the pyramid $EFHD$, & the solide $BGM L$ is sextuple to the pyramid $BCGA$. (For the parallelepipedon is duple to the Prisme set upon the base of the Pyramid, & the Prisme is triple to the pyramid): and likewise the solide $EHP O$ is sextuple to the pyramid $EFHD$. Wherefore the solide $BGM L$ is equal to the solide $EHP O$. But in equall parallelepipedons, the bases are (by the 34. of the eleventh) reciprokall to their altitudes.

Wherefore as the base BN is to the base $E Q$, so is the altitude of the solide $EHP O$, to the altitude of the solide $BGM L$. But as the base BN is to the base $E Q$, so is the triangle GBC to the triangle HEF (by the 15. of the fifth, for the triangles GBC & HEF are the halves of the parallelogrammes BN and $E Q$). Wherefore (by the 11. of the fifth) as the triangle GBC is to the triangle HEF , so is the altitude of the solide $EHP O$ to the altitude of the solide $BGM L$.



But the altitude of the solide $EHP O$ is one and the same with the altitude of the pyramid $EFHD$, and the altitude of the solide $BGM L$ is one and the same with the altitude of the pyramid $BCGA$. Wherefore as the base GBC is to the base HEF , so is the altitude of the pyramid $EFHD$ to the altitude of the pyramid $BCGA$. Wherefore the bases of the two pyramids $BCGA$ and $EFHD$ are reciprokall to their altitudes.

Demonstration of the second part, which is the converse of the first.

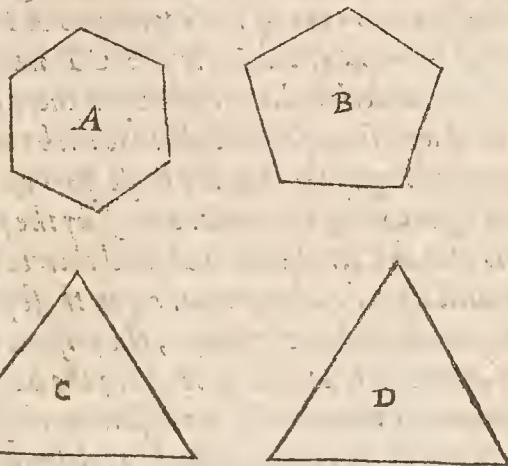
But now suppose that the bases of the pyramids $BCGA$ and $EFHD$, be reciprokall to their altitudes; that is, as the base GBC is to the base HEF , so let the altitude of the pyramid $EFHD$ be to the altitude of the pyramid $BCGA$. Then I say that the pyramid $BCGA$ is equal to the pyramid $EFHD$. For (the selfe same order of construction remaining), for that as the base GBC is to the base HEF , so is the altitude of the pyramid $EFHD$ to the altitude of the pyramid $BCGA$. But as the base GBC is to the base HEF , so is the parallelogramme GC to the parallelogramme HF . Wherefore (by the 11. of the fifth) as the parallelogramme GC is to the parallelogramme HF , so is the altitude of the pyramid $EFHD$ to the altitude of the pyramid $BCGA$. But the altitude of the pyramid $EFHD$ and of the solide $EHP O$, is one and the selfe same, and the altitude of the pyramid $BCGA$ and of the solide $BGM L$, is also one and the same. Wherefore as the base GC is to the base HF , so is the altitude of the solide $EHP O$ to the altitude of the solide $BGM L$. But parallelepipedons, whose bases are reciprokall to their altitudes are (by the 34. of the eleventh) equall the one to the other. Wherefore the parallelepipedon $BGM L$ is equal to the parallelepipedon $EHP O$. But the pyramid $BCGA$ is the sixth part of the solide $BGM L$, and likewise the pyramid $EFHD$ is the sixth part of the solide $EHP O$. Wherefore the pyramid $BCGA$ is equal to the pyramid $EFHD$. Wherefore in equall pyramids hauing triangles to their bases, the bases are reciprokall to their altitudes. And pyramids hauing triangles to their bases, whose bases are reciprokall to their altitudes, are equall the one to the other: which was required to be demonstrated.

A Corollary

A Corollary added by Campane and Fluffas.

Hereby it is manifest that equall pyramids hauing to their bases Poligonon figures, haue their bases reciprokall with their altitudes. And Pyramids whose bases being poligonon figures are reciprokall with their altitudes, are equall the one to the other.

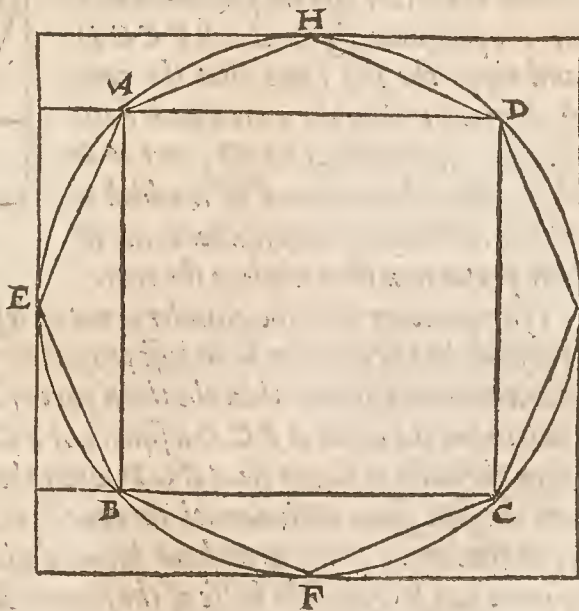
Suppose that vpon the poligonon figures A and B, be set equall pyramids. Then I say that their bases A and B are reciprokall with their altitudes. Describe by the 25. of the sixth, triangles equall to the bases A and B. Which let be C and D. Vpon which let there be set pyramids equall in altitude with the pyramids A and B. Wherefore the pyramids C and D, being set vpon bases equall with the bases of the pyramids A and B, and hauing also their altitudes equall with the altitudes of the sayd pyramids A and B, shall be equall by the 6. of this booke. Wherefore by the first part of this proposition, the bases of the pyramids, C to D are reciprokall with the altitudes of D to C. But in what proportion are the bases C to D, in the same are the bases A to B, forasmuch as they are equall. And in what proportion are the altitudes of D to C, in the same are the altitudes of B to A, which altitudes are likewise equall. Wherefore by the 11. of the fifth, in what proportion the bases A to B are, in the same reciprokally are the altitudes of the pyramids B to A. In like sort by the second part of this proposition may be proued the conuerse of this corollary. The same thing followeth also in a Prisme, and in a sided colume, as hath before at large bene declared in the corollary of the 40. proposition of the 11. booke. For those solides are in proportio the one to the other, as the pyramids or parallelipedons, for they are either partes of equemultiplices or equemultiplices to partes.



The 10. Theoreme. The 10. Proposition.

Euery cone is the third part of a cilinder, hauing one and the selfe same base and one and the selfe same altitude with it.

Suppose that there be a cone hauing to his base the circle $ABCD$, and let there be a cilinder hauing the selfe same base, and also the same altitude that the cone hath. Then I say that the cone is the third part of the cilinder, that is, that the cilinder is in treble proportion to the cone. For if the cilinder be not in treble proportion to the cone, then the cilinder is either in greater proportion then triple to the cone, or els in lesse. First let it be in greater then triple. And describe (by the 6. of the fourth) in the circle $ABCD$ a square $ABCD$. Now the square $ABCD$, is greater then the halfe of the circle $ABCD$. For if about the circle $ABCD$, we describe a square, the square described in the circle $ABCD$ is the halfe of the square described about the circle. And let there be Parallelipedon prismes described vpon those squares, equall in altitude with the cilinder. But prismes are in that proportion the one to the other, that their bases are (by the 32. of the eleuenth, and 5. Corollary of the 7. of this booke). Wherefore the prisme described vpon the square $ABCD$ is the halfe of the prisme described vpon the square that is described about the circle. Now the clinder is lesse then the



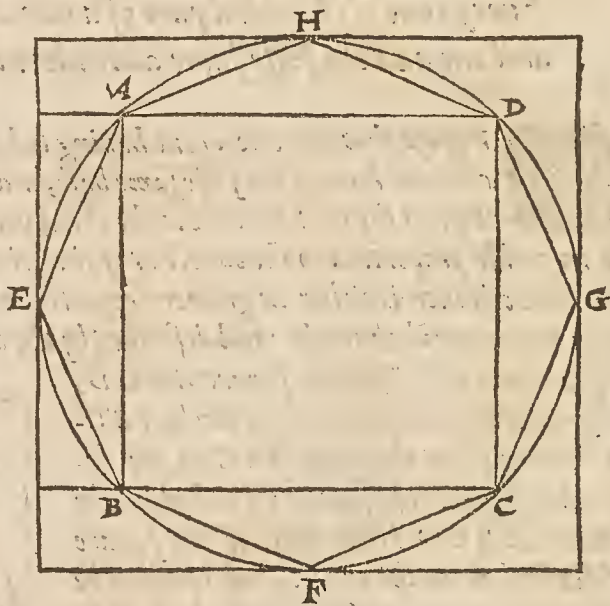
Construction.
1.

Parallelipipedons called Prismes.

prisme

The twelue Book

prisme which is made of the square described about the circle $ABCD$, being equal in altitude with it, for it containeth it. Wherefore the prisme described upon the square $ABCD$ and being equall in altitude with the cylinder, is greater then half the cylinder. Deuide (by the 30. of the third) the circumferences AB , BC , CD and DA into two equall parts in the points E , F , G , H , And draw these right lines AE , EB , BF , FC , CG , GD , DH & HA . Wherefore euery one of these triangles AEB , BFC , CGD and DHA is greater then halfe of that segment of the circle $ABCD$ which is described about it, as we haue before in the 2. proposition declared. Describe vpon euery one of these triangles AEB , BFC , CGD , and DHA a prisme of equall altitude with the cylinder. Wherefore euery one of these prismes so described is greater then the halfe part of the segment of the cylinder that is set vpon the sayd segments of the circle. For if by the pointes E , F , G , H , be drawen parallell lines to the lines AB , BC , CD and DA , and then be made perfect the parallelogrammes made by those parallell lines, and moreouer vpon those parallelogrammes be erected parallelipipedons equall in altitude with the cylinder, the prismes which are described vpon eche of the triangles AEB , BFC , CGD , and DHA are the halfes of euery one of those parallelipipedons. And the segments of the cylinder are lesse then those parallelipipedons so described. Wherefore also euery one of the prismes which are described vpon the triangles AEB , BFC , CGD and DHA is greater then the halfe of the segment of the cylinder set vpon the sayd segment. Now therefore deuiding euery one of the circumferences remaining into two equall partes, and drawing right lines, and raising vp vpon euery one of these triangles prismes equall in altitude with the cylinder, and doing this continually, we shall at the length (by the first of the tenth) leaue certaine segments of the cylinder which shalbe lesse then the excessse whereby the cylinder exceedeth the cone more then thrise. Let those segments be AE , EB , BFC , CGD , DH and HA . Wherefore the prisme remayning, whose base is the polygonon figure $AEBFCGDH$, and altitude the selfe same that the cylinder hath, is greater then the cone taken three tymes.* But the prisme whose base is the polygonon figure $AEBFCGDH$ and altitude the selfe same that the cylinder hath, is treble to the pyramis whose base is the polygonon figure $AEBFCGDH$ and altitude the selfe same that the cone hath, by the corollary of the 3. of this booke. Wherefore also the pyramis whose base is the polygonon figure $AEBFCGDH$ and toppe the selfe same that the cone hath, is greater then the cone which hath to his base the circle $ABCD$. But it is also lesse, for it is containd of it which is impossible. Wherefore the cylinder is not in greater proportion then triple to the cone.



* By this it is manifest that Euclide comprehended sided Columnes also vnder the name of a Prisme.

I say moreouer that the cylinder is not in lesse proportion then triple to the cone. For if it be possible let the cylinder be in lesse proportion then triple to the cone. Wherefore by conuerſion, the cone is greater then the third part of the cylinder. Describe now (by the sixth of the fourth) in the circle $ABCD$ a square $ABCD$. Wherefore the square $ABCD$ is greater then the halfe of the circle $ABCD$ vpon the square $ABCD$ describe a pyramis hauing one & the selfe same altitude with the cone. Wherefore the pyramis so described is greater the halfe of the cone. (For if as we haue before declared we describe a square about the circle, the square $ABCD$ is the halfe of the square described about the circle, and if vpon the squares be described parallelipipedons equall in altitude with the cone, which solides are also called

called prismes, the prisme or parallelipipedon described vpon the square $ABCD$ is the halfe of the prisme which is described vpon the square described about the circle, for they are the one to the other in that proportiō that their bases are (by the 32. of the eleuēth, & 5. corollary of the 7. of this booke.) Wherefore also their third parts are in the self same proportion (by the 15. of the first). Wherefore the pyramis whose base is the square $ABCD$ is the halfe of the pyramis set vpon the square described about the circle. But the pyramis set vpon the square described about the circle is greater then the cone whome it comprehendeth. Wherefore the pyramis whose base is the square $ABCD$, and altitude the self same that the cone hath, is greater then the halfe of the cone.) Deuide (by the 30. of the third) euery one of the circumferences AB , BC , CD , and DA into two equall partes in the pointes E , F , G , and H : and drawe these right lines AE , EB , BF , FC , CG , GD , DH , and HA . Wherefore euery one of these triangles AEB , BFC , CGD , and DHA is greater then the halfe part of the segment of the circle described about it. Vppon euery one of these triangles AEB , BFC , CGD , and DHA describe a pyramis of equall altitude with the cone: and after the same maner euery one of those pyramids so described is greater then the halfe part of the segment of the cone set vpon the segment of the circle. Now therefore diuiding (by the 30. of the third) the circumferences remaining into two equall parts, & drawing right lines & raysing up vpon euery one of those triangles a pyramis of equall altitude with the cone, and doing this continually, we shal at the length (by the first of the tenth) leaue certayne segmentes of the cone, which shalbe lesse then the excesse whereby the cone exceedeth the third part of the cylinder. Let those segmentes be AE , EB , BF , FC , CG , GD , DH , and HA . Wherefore the pyramis remayning, whose base is the polygonō figure $AEBFCGDH$ and altitude the self same with the cone, is greater then the third part of the cylinder. But the pyramis whose base is the polygonon figure $AEBFCGDH$ and altitude the self same with the cone, is the third part of the prisme whose base is the polygonō figure $AEBFCGDH$ and altitude the self same with the cylinder. Wherefore * the prisme whose base is the polygonon figure $AEBFCGDH$, and altitude the self same with the cylinder, is greater then the cylinder whose base is the circle $ABCD$. But it is also lesse, for it is contayned of it, which is impossible. Wherefore the cylinder is not in lesse proportion to the cone then in treble proportion. And it is proued that it is not in greater proportion to the cone then in treble proportion, wherefore the cone is the third part of the cylinder. Wherefore euery cone is the third part of a cylinder, hauing one & the self same base, and one and the selfe same altitude with it: which was required to be demonstrated.

* A prisme hauing for his base a polygonon figure as we haue often before noted vnto you.

¶ Added by M. Iohn Dee.

¶ A Theoreme. 1.

The superficies of euery vpright Cylinder, except his bases, is equall to that circle whose semidiameter is middell proportionall betwene the side of the Cylinder, and the diameter of his base.

¶ A Theoreme. 2.

The superficies of euery vpright, or Isosceles Cone, except the base, is equall to that circle, whose semidiameter is middell proportionall betwene the side of that Cone, and the semidiameter of the circle: which is the base of the Cone.

My entent in additions is not to amend *Euclides* Method, (which nedeth little adding or none at all). But my desire is somewhat to furnish you, toward a more general art Mathematical the *Euclides* Elemēts, (remayning in the termes in which they are written) can sufficiently helpe you vnto. And though *Euclides* Elementes with my Additions, run not in one Methodicall race toward my marke: yet in the meane space my Additions either geue light, where they are annexed to *Euclides* matter, or geue some ready ayde, and shew the way to dilate your discourfes Mathematicall, or to inuent and practise thinges

Note: M. Dee his chiefe purpose in his additions.

The twelvethe Booke

Mechanically. And (in deede) if more ley for had happened, many more straunge matters Mathematicall had, (according to my purpose generall) bene presently published to your knowledge : but want of due leasour causeth you to want, that, which my good will toward you, most hartely doth wish you.

As concerning the two Theoremes here annexed, their veritie ; is by *Archimedes*, in his booke of the Sphere and Cylinder manifestly demonstrated, and at large : you may therefore boldly trust to them, and vse them, as suppositions, in any your purposes: till you haue also their demonstrations. But if you well remember my instructions vpon the first proposition of this booke, and my other additions vpon the second, with the suppositions how a Cylinder and a Cone are Mathematically produced : you will not neede *Archimedes* demonstration: nor yet be vtterly ignoraunt of the solide quantities of this Cylinder and Cone here compared: (the diameter of their base, and heith being knowne in any measure) neither can their croked superficies remayne vnmeasured. Whereof vndoubtedly great pleasure and commoditie may grow to the sincere student, and precise practiser.

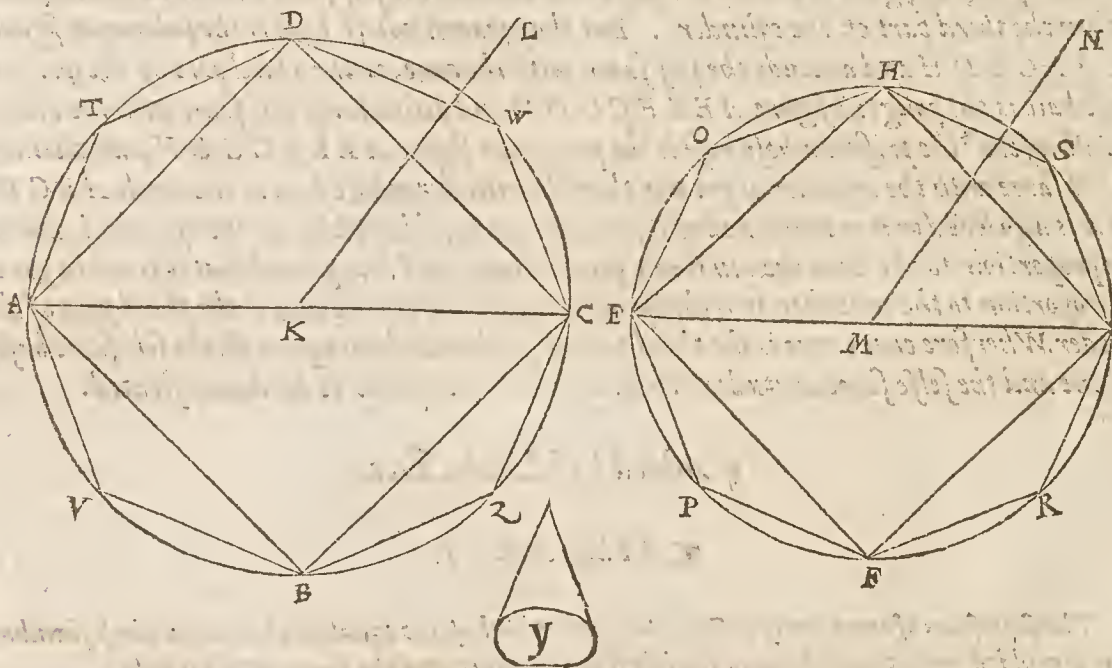
The 11. Theoreme.

The 11. Proposition.

Cones and Cylinders being vnder one and the selfe same altitude, are in that proportion, the one other that their bases are.

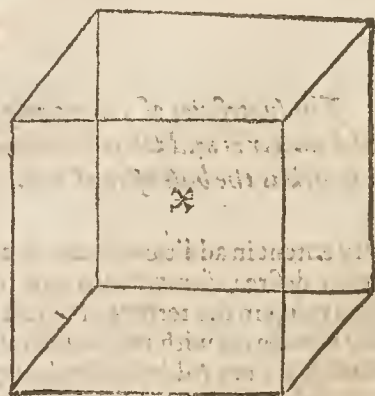
Let there be take cones & cylindres vnder one and the selfe same altitude whose bases let be the circles *ABCD* and *EFGH*, and axes the lines *KL* & *MN*, and let the diameters of their bases be *AC* and *EG*. The I say that as the circle *ABCD* is to the circle *EFGH*, so is the cone *AL* to the cone *EN*, and also the cylinder *AL* to the cylinder *EN*. For if the cone *AL* be not to the cone *EN* as the circle *ABCD* is to the circle *EFGH*, then as the circle *ABCD* is to the circle *E*

Demonstration as touching Cones.



First case. 1.

F *G* *H*, so is the cone *AL* to some solide either lesse then the cone *EN*, or greater. First let it be vnto a lesse, namely, to the solide *X*. And vnto that which the solide *X* is lesse then the cone *EN*, let the solide *Y* be equall. Wherefore the cone *EN* is equal to the solides *X* & *Y*. Describe (by the 6. of the fourth) in the circle *EFGH* a square *EFGH*. Wherefore the square is greater then halfe the circle. Rayse vp vpon the square *EFGH* a pyramis of equall altitude with the cone. Wherefore the pyramis, so rayssed vp is greater then halfe of the cone. For if we describe a square about the circle, and vpon that square rayse vp a pyramis of equall al-



itude

itude with the cone, the pyramis which is set upon the square described with in the circle is the halfe of the pyramis set upon the square described about the circle, for they are in proportion the one to the other as their bases. But the cone is lesse then the pyramis which is set upon the square described about the circle. Wherefore the pyramis whose base is the square $EFGH$ and top one and the same with the cone is greater then halfe of the cone: deuide (by the 30. of the third) the circumferences EF, FG, GH and HE into two equal partes in the points O, P, R, S , and draw these right lines $HO, OE, EP, PF, FR, RG, GS$, and SH . Wherefore euery one of these triangles HOE, EPF, FRG , and GSH is greater then halfe the segmēt of the circle described about the. Rayse vp vpon euery one of the triangles HOE, EPF, FRG and GSH a pyramis equall in altitude with the cone. Wherefore euery one of the pyramids so rayssed vp is greater then the halfe part of the segment of the cone described about it. Now then deuiding (by the 30. of the third) the circumferences remayning into two equall partes, and drawing right lines, and rayssing vp vpon euery one of those triangles a pyramis of equall altitude with the cone, and thus doing continually, we shall at length by the first of the tenth leue certayne segmentes of the cone which shall be lesse then the solide X . Let those segmentes be HOE, EPF, FRG , and GSH . Wherefore the pyramis remayning whose base is the polygonon figure $HOEPFRGS$ & top the selfe same with the cone, is greater then the solide X . Inscribe in the circle $ABCD$ a polygonon figure like and in like sort situate to the polygonon figure $HOEPFRGS$, and let the same be $DTAVBZCW$, and vpon it rayse a pyramis of equall altitude with the cone AL . Now for that as the square of the line AC is to the square of the line EG , so is the polygonon figure $DTAVBZCW$ to the polygonon figure $HOEPFRGS$ (by the first of this booke): But as the square of the line AC is to the square of the line EG , so is the circle $ABCD$ to the circle $EFGH$ (by the second of this booke). Wherefore (by the 11. of the fift) as the circle $ABCD$ is to the circle $EFGH$, so is the polygonon figure $DTAVBZCW$ to the polygonon figure $HOEPFRGS$ (this foloweth also of the corollary of the 2. of this booke): moreouer as the circle $ABCD$ is to the circle $EFGH$, so is the cone AL to the solide X . And as the polygonon figure $DTAVBZCW$ is to the polygonon figure $HOEPFRGS$, so is the pyramis whose base is the polygonon figure $DTAVBZCW$, and toppe the poynt L , to the pyramis whose base is the polygonon figure $HOEPFRGS$, and toppe the poynt N . Wherefore (by the 11. of the fift) as the cone AL is to the solide X , so is the pyramis whose base is the polygonon figure $DTAVBZCW$, and toppe the poynt L , to the pyramis whose base is the polygonon figure $HOEPFRGS$, and toppe the poynt N . Wherefore alternately (by the 16. of the fift) as the cone AL is to the pyramis which is in it, so is the solide X , to the pyramis which is in the cone EN . But the cone AL , is greater then the pyramis which is in it. Wherefore also the solide X is greater then the pyramis which is in the cone EN . But it is also lesse by construction. Which is impossible. Wherefore as the circle $ABCD$, is to the circle $EFGH$, so is not the cone AL to any solide lesse then the cone EN .

In like sorte also may we proue, that as the circle $EFGH$, is to the circle $ABCD$, so is not the cone EN to any solide lesse then the cone AL . Now I say that as the circle $ABCD$, is to the circle $EFGH$, so is not the cone AL to any solide greater then the cone EN . For if it be possible let it be vnto a greater, namely to the solide X . Wherefore by conuersion, as the circle $EFGH$, is to the circle $ABCD$, so is the solide X to the cone AL : but as the solide X is to the cone AL , so is the cone EN , to some solide lesse then the cone AL (as we may see by the assumpt put after the second of this booke): Wherefore (by the 11. of the fift) as the circle $EFGH$ is to the circle $ABCD$, so is the cone EN to some solide lesse then the cone AL , which we haue proued to be impossible. Wherefore as the circle $ABCD$, is to the circle $EFGH$, so is not the cone AL to any solide greater then the cone EN . And it is also proued that it is not to any lesse. Wherefore as the circle $ABCD$, is to the circle $EFGH$, so is the cone AL to the cone EN .

But as the cone is to the cone, so is the cylinder to the cylinder, (by the 15. of the fift) for

The twelue th Booke

Demonstration touching cylinders.

the one is in treble proportion to the other. Wherefore by the 11. of the fifth as the circle $A B C D$ is to the circle $E F G H$, so are the cylinders which are set upon them the one to the other, the said cylinders being under equall altitudes with the cones. Cones therefore and cylinders, being under one & the self same altitude, are in that proportion the one to the other, that their bases are: which was required to be demonstrated.

¶ The 12. Theoreme.

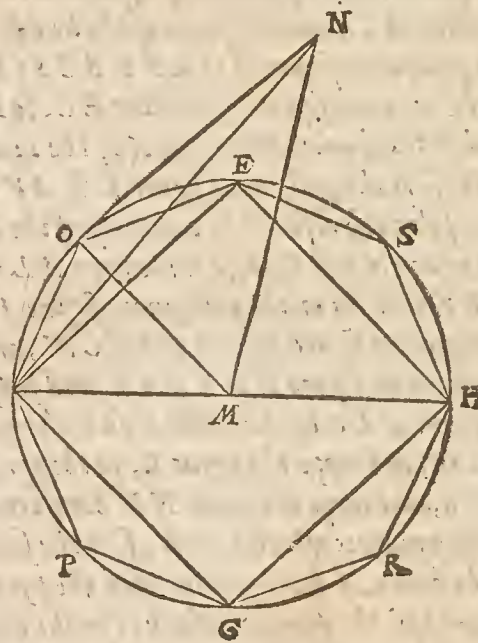
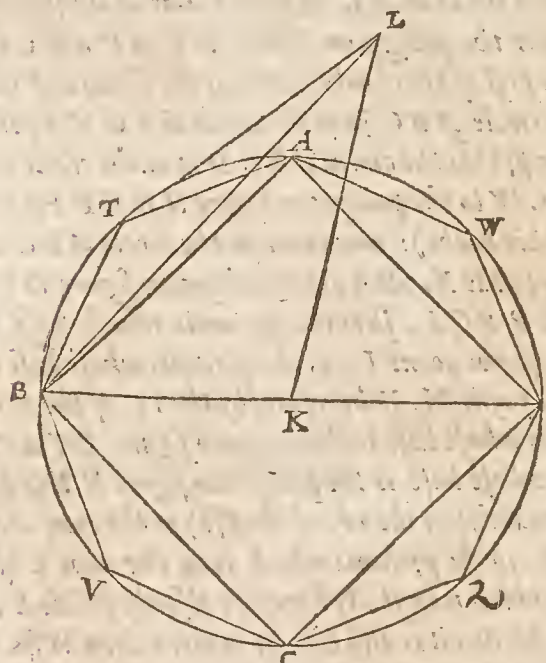
The 12. Proposition.

Like Cones and Cylinders, are in treble proportion of that in which the diameters of their bases are.

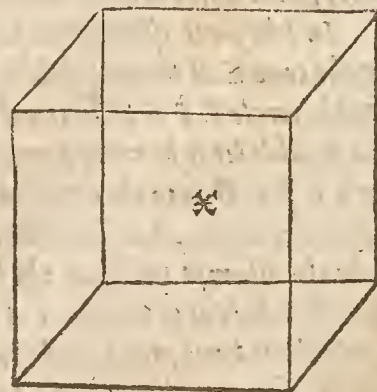


Suppose that these Cones and Cylinders, whose bases are the circles $A B C D$, and $E F G H$, and diameters of their bases $B D$ and $F H$, be like, and let the axes of the Cones or Cylinders be $K L$ and $M N$, and let the centres of their bases be the pointes K and M . Then I say, that the cone, whose base is the circle $A B C D$, & toppe the point L , is to the Cone, whose base is the circle $E F G H$, and toppe the point N , in treble proportion of that in which the diameter $B D$ is to the diameter $F H$. For if the cone $A B C D L$ be not to the cone $E F G H N$ in treble proportion of that in which the diameter

First part which concerneth cones.



$B D$ is to the diameter $F H$, the cone $A B C D L$ shall be in treble proportion of that in which the diameter $B D$ is to the diameter $F H$, either to some solide lesse then the cone $E F G H N$, or to some solide greater. First let it be unto a lesse, namely, to the solide X . Describe (by the 6. of the fourth) in the circle $E F G H$, a square $E F G H$. Wherefore the square $E F G H$ is greater then the halfe of the circle $E F G H$. Raise up from the square $E F G H$ a pyramis of equall altitude with the cone. Wherefore the pyramis so raised up, is greater then the halfe part of the cone. Diuide now (by the 30. of the third) the circumferences $E F, F G, G H$, & $H E$, into two equall partes in the pointes O, P, R, S , and drawe these right lines $E O, O F, F P, P G, G R, R H, H S$, and $S E$. Wherefore euery one of these



First case. Construction.

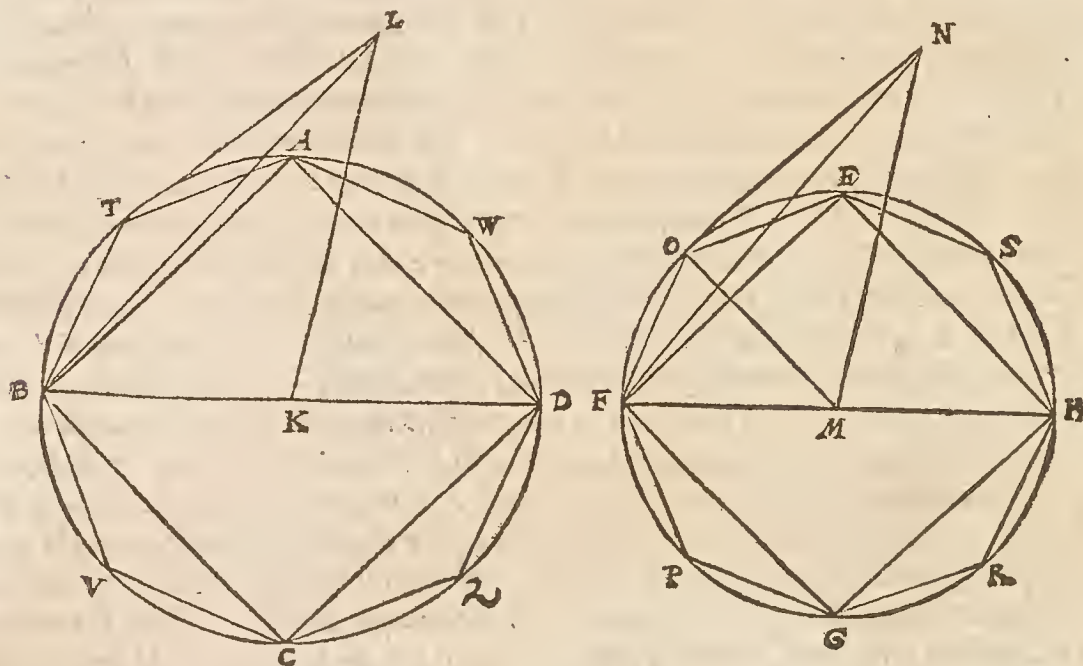
triangles

triangles $EO F$, $F P G$, $G R H$, and $H S E$, is greater then the halfe of the segment of the circle $E F G H$, described about ech of them. Erecte upon euery one of these triangles $EO F$, $F P G$, $G R H$, and $H S E$, a pyramis, hauing one and the selfe same altitude with the cone. Wherefore euery one of the pyramids so raised vp, is greater then the halfe of the segment of the cone described about them. Now therefore diuiding the circumferences remayning into two equall partes, and drawing right lines, and raising vp upon euery one of the triangles a pyramis, hauing one and the selfe same altitude with the cone, and, thus doing continually, we shall at the length (by the 1. of the tenth) leaue certaine segmentes of the cone, which shall be lesse then the excesse whereby the cone $E F G H N$ exceedeth the solide X . Let those segmentes left be EO , OF , FP , PG , GR , RH , HS , and SE . Wherefore the pyramis remayning, whose base is the Polygonon figure $EO F P G R H S$, and toppe the point N , is greater then the solide X . Describe (by the 18. of the sixth) in the circle $A B C D$, vnto the Polygonon figure $EO F P G R H S$, a Polygonon figure like and in like sort situate, and let the same be $AT B V C Z D W$, and from it raise vp a pyramis, hauing one and the selfe same altitude with the cone $A B C D L$. And let one of the triangles comprehending the pyramis, whose base is the Polygonon figure $AT B V C Z D W$, and toppe the point L , be $L B T$, and let also one of the triangles comprehending the pyramis, whose base is the Polygonon figure $EO F P G R H S$, and toppe the point N , be $N F O$, and drawe these right lines $K T$ and $M O$. And forasmuch as the cone $A B C D L$ is like to the cone $E F G H N$, therefore (by the 20. definition of the eleuenth) as the diameter $B D$ is to the diameter $F H$, so is the axe $K L$ to the axe $M N$: But as the diameter $B D$ is to the diameter $F H$, so (by the 15. of the fift) is the semidiameter $B K$ to the semidiameter $F M$. Wherefore (by the 11. of the fift) as $B K$ is to $F M$, so is $K L$ to $M N$. Wherefore alternately also (by the 16. of the fift) as $B K$ is to $K L$, so is $F M$ to $M N$. Wherefore the sides about the equall angles $B K L$ and $F M N$ (which angles are equall, for that they are right angles (by the 18. definition of the eleuenth) are proportionall. Wherefore (by the 1. definition of the sixth) the triangle $B K L$ is like to the triangle $F M N$. Againe for that as $B K$ is to $K T$, so is $F M$ to $M O$, and they comprehend equall angles, namely, $B K T$ and $F M O$, for what part the angle $B K T$ is of those fower right angles which are made at the centre K , the selfe same part is the angle $F M O$ of the fower right angles which are made at the centre M : forasmuch therefore as the sides about the equall angles are proportionall, the triangle $B K T$ is like to the triangle $F M O$. Againe, forasmuch as it was proued, that as $B K$ is to $K L$, so is $F M$ to $N M$, but $B K$ is equall to $K T$, and $F M$ to $M O$, therefore as $T K$ is to $K L$, so is $O M$ to $M N$. Wherefore the sides about the equall angles $T K L$ and $O M N$ (which angles are equall, for that they are right angles) are proportionall. Wherefore the triangle $L K T$ is like to the triangle $M N O$. And for that (by the 6. of the sixth) and by reason of the likenes of the triangles $L K B$ and $N M F$, as $L B$ is to $B K$, so is $N F$ to $F M$, and againe by reason of the likenes of the triangles $B K T$ and $F M O$, as $K B$ is to $B T$, so is $M F$ to $F O$, therefore of equalitie (by the 22. of the fift) as $L B$ is to $B T$, so is $N F$ to $F O$. Againe for that by reason of the likenes of the triangles $L K T$ and $N O M$, as $L T$ is to $T K$, so is $N O$ to $O M$, and by reason of the likenes of the triangles $T K B$ & $O M F$, as $K T$ is to $T B$, so is $M O$ to $O F$: therefore of equalitie (by the 22. of the fift) as $L T$ is to $T B$, so is $N O$ to $O F$. And it was proued that as $T B$ is to $B L$, so is $O F$ to $F N$. Wherefore againe of equalitie, as $T L$ is to $L B$, so is $O N$ to $N F$. Wherefore the sides of the triangles $L T B$ & $N O F$, are proportionall. Wherefore (by the 5. of the sixth) the triangles $L T B$ and $N O F$, are equiangle. Wherefore also they are like. Wherefore the pyramis, whose base is the triangle $B K T$, and toppe the point L , is like vnto the pyramis, whose base is the triangle $F M O$, and toppe the point N . For they are comprehended vnder like plaine superficieses, and equall in multitude. But pyramids being like, and hauing triangles to their bases, are (by the 8. of the twelfth) in treble proportion the one to the other. of that in which sides of like proportion are. Wherefore the pyramis

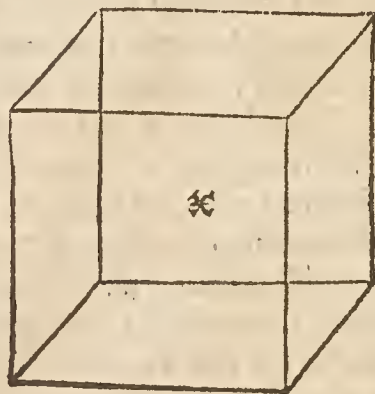
Demonstration leading to an impossibility.

The twelvethe Booke

$BKT L$ is to the pyramis $FMON$, in treble proportion of that in which the line BK is to the line FM . And in like sort if we draw right lines from the pointes A, W, D, Z, C , and V , to the point K , and likewise from the pointes E, S, H, R, G , and P , to the point M , and raise up upon the triangles pyramids, hauing the selfe same altitudes with the cones, we may proue that euery one of those pyramids of one & the selfe same order, is to euery one of the pyramids of the selfe same order, in treble proportion of that in which the side of like proportion BK is to side of like proportion FM , that is, of that which the line BD hath to the line FH . But as one of the antecedentes is to one of the consequentes, so are all the antecedentes to all the consequentes (by the 12. of the fift). Wherefore as the pyramis $BKT L$ is to the pyramis $FMON$, so is the whole pyramis, whose base is the Polygonon figure $ATBVCZDW$, and toppe the point L , to the whole pyramis, whose base is the Polygonon figure $EOFPGRHS$, and toppe the point N . Wherefore the pyramis, whose base is the Polygonon figure $ATBVCZDW$, and toppe the point L , is to the pyramis, whose base is the Polygonon figure $EOFPGRHS$, and toppe the point N , in treble proportion of that in which the line BD is to the line FH .



And it is supposed also that the cone, whose base is the circle $ABCD$, and toppe the point L , is to the solide X , in treble proportion of that in which the line BD is to the line FH . Wherefore as the cone, whose base is the circle $ABCD$, and toppe the point L , is to the solide X , so is the pyramis, whose base is the Polygonon figure $ATBVCZDW$, & toppe the point L , to the pyramis, whose base is the Polygonon figure $EOFPGRHS$, and toppe the point N . Wherefore alternately (by the 16. of the fift) as the cone, whose base is the circle $ABCD$, & toppe the point L , is to the pyramis which is in it, whose base is the Polygonon figure $ATBVCZDW$, & toppe the point L , so is the solide X , to the pyramis, whose base is the Polygonon figure $EOFPGRHS$, and toppe the point N . But the foresaid cone is greater then the pyramis which is in it, for it containeth it. Wherefore the solide X is greater then the pyramis, whose base is the Polygonon figure $EOFPGRHS$, and toppe the point N . But it was supposed to be lesse: which is impossible. Wherefore the cone $ABCDL$ is not to any solide lesse then the cone $EFGHN$, in treble proportion of that in which the diameter



diameter BD is to the diameter FH . In like sort also may we proue, that the cone $EFGHN$ is not to any solide lesse then the cone $ABCDL$, in treble proportion of that in which FH is to BD .

Now also I say, that the cone $ABCDL$ is not to any solide greater then the cone $EFGHN$, in treble proportion of that in which the diameter BD is to the diameter FH . For if it be possible, let it be to a greater, namely, to the solide X . Wherefore by conuersion (by the Corollary of the 4. of the fift) the solide X is to the cone $ABCDL$, in treble proportion of that in which the diameter FH is to the diameter BD . But as the solide X is to the cone $ABCDL$, so is the cone $EFGHN$ to some solide lesse then the cone $ABCDL$ (as it is easie to see by the Assumpt put after the 2. Proposition). Wherefore also the cone $EFGHN$ is unto some solide lesse then the cone $ABCDL$, in treble proportion of that in which the diameter FH is to the diameter BD : which is proued to be impossible. Wherefore the cone $ABCDL$ is not to any solide greater then the cone $EFGHN$ in treble proportion of that in which the diameter BD is to the diameter FH , and it is also proued that it is not to any lesse. Wherefore the cone $ABCD$ is to the cone $EFGHN$ in treble proportion of that in which the diameter BD is to the diameter FH .

Second case.

But as cone is to cone, so is cylinder to cylinder (by the 15. of the fift) for the cylinder is triple to the cone which is described on the one and selfe same base, and hauing one and the selfe same altitude with the cone. For it is proued (by the 10. of the twelfth) that euery cone is the third part of a cylinder, hauing one and the selfe same base with it, and one & the selfe same altitude. Wherefore the cylinder is unto the cylinder in treble proportion of that in which the diameter BD is to the diameter FH . Wherefore like cones & cylinders are in treble proportion of that in which the diameters of their bases are: which was required to be proued.

Second part which concerneth Cylinders.

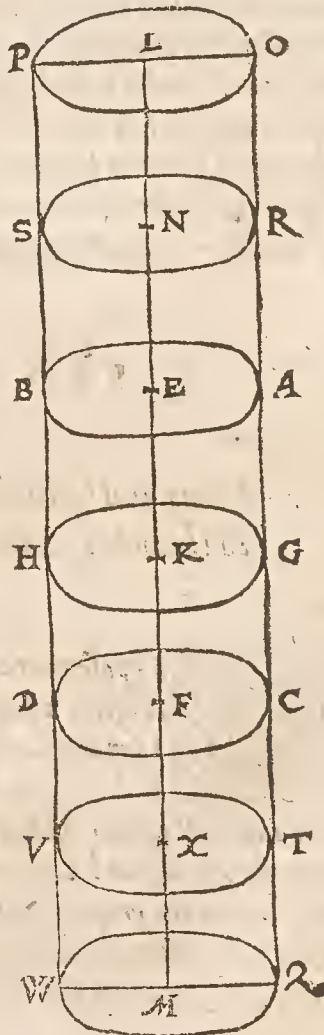
The 13. Theoreme.

The 13. Proposition.

If a Cylinder be diuided by a playne superficies being a parallell to the two opposite playne superficieses: then as the one Cylinder is to the other Cylinder, so is the axe of the one to the axe of the other.



Suppose that there be a cylinder AD , whose axe let be EF , and let the opposite bases be the circles AB , and CD , and let AD be diuided by the superficieses GH being a parallell to the two opposite playne superficieses, AB , and CD . Then I say that as the cylinder BG is to the cylinder GD , so is the axe EK to the axe KF . Extend the axe EF on either side to the pointes L & M . And unto the axe EK put as many axes equall as you will, namely, EN , & NL , & likewise unto the axe FK put as many axes equal as you wil, namely, FX & XM . And by the pointes L , N , & X , M , extēde playne superficieses parallels to the two superficieses AB , & CD (by the corollary of the 15. of the eleuēth): & in the plaine superficieses thus extended by the pointes L , N , X , M , imagine to be drawne these circles, namely, OP , RS , TV , and ZW , hauing to their centers the pointes L , N , M , X , and let them be equal to either of the circles AB , and CD : and upon those circles imagine these cylinders PR , RB , DT , TW to be

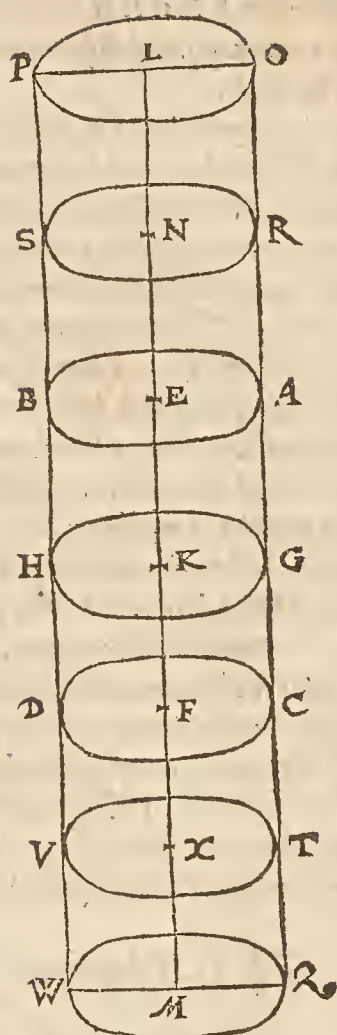


Construction.

The twelue th Booke

Demonstration.

be set . Now forasmuch as the axes LN , NE , and EK are equall the one to the other : Therefore the cylinders PR , RB , and BG are (by the 11. of the twelue th) in proportion the one to the other as their bases are. But the bases are equall. Wherefore also the cylinders PR , RB , and BG are equall the one to the other. And forasmuch as the axes LN , NE , and EK are equal the one to the other, and the cylinders PR , RB , and BG are also equall the one to the other, and the multitude of the axes LN , NE , and EK is equall to the multitude of the cylinders PR , RB , and BG : therefore how multiplex the whole axe KL is to the axe EK , so multiplex is the whole cylinder PG to the cylinder BG . And (by the same reason) also how multiplex the whole axe MK is the axe KF , so multiplex is the whole cylinder WG to the cylinder GD . Wherefore if the axe KL be equal the axe KM , the cylinder PG is equall to cylinder GW . And if the axe KL be greater then the axe KM , the cylinder PG is greater then the cylinder GW . And if it be lesse it is lesse. Now therefore there are foure magnitudes, namely, the two axes EK , and KF , and the two cylinders BG , and GD , and vnto the axe EK , and to the cylinder BG , namely, to the first and the third are taken equemultiples, namely, the axe KL , and the cylinder PG . And likewise vnto the axe KF , and vnto the cylinder GD , namely, the second and the fourth, are taken other equemultiples, namely, the axe KM , and the cylinder GW . And it is proued that if the axe KL , excede the axe KM , the cylinder PG excedeth the cylinder GW and that if it be equall it is equall, and if it be lesse it is lesse. Wherefore (by the 6. definition of the fift) as the axe EK is to the axe KF , so is the cylinder BG to the cylinder GD . If therefore a cylinder be diuided by a plaine superficies being a parallel to the two opposite plaine superficieses: then as the one cylinder is to the other cylinder, so is the axe of the one to the axe of the other : which was required to be proued.



¶ The 14. Theoreme.

The 14. Proposition.

Cones and Cylinders consisting vpon equall bases, are in proportion the one to the other as their altitudes.

Construction.

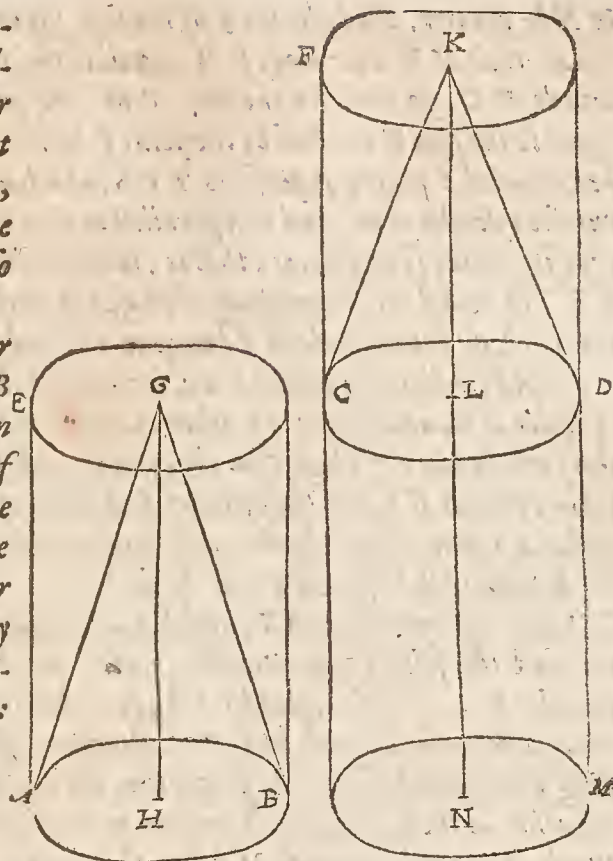


Demonstration touching Cylinders.

Suppose that the cylinders FD , and EB , and the cones AGB , and CKD , do consist vpon equall bases, namely, vpon the circles AB , and CD . Then I say that as the cylinder EB is to the cylinder FD , so is the axe GH to the axe KL . Extend the axe KL directly to the poynte N , and vnto the axe GH , put the axe LN equall: and about the axe LN imagine a cylinder CM . Now forasmuch as the cylinders EB , and CM , are vnder equall altitudes, therefore (by the 11. of the twelue th) they are in the proportion the one to the other as their bases are. But the bases are equall the one to the other. Wherefore also the cylinders EB , and CM are equall the one to the other. And forasmuch as the whole cylinder FM , is diuided by a playne superficies CD being a parallel

a parallell to either of the opposite plaine superficieses: therefore (by the 13. of the twelue) as the cylinder CM is to the cylinder FD , so is the axe LN to the axe LK . But the cylinder CM is equal to the cylinder EB , and the axe LN to the axe GH . Wherefore as the cylinder EB is to the cylinder FD , so is the axe GH to the axe LK .

But as the cylinder EB is to the cylinder FD , so (by the 15. of the fift) is the cone ABG to the cone CDK , for the cylinders are in treble proportion to the cones (by the 10. of the twelue). Wherefore (by the 11. of the fift) as the axe GH is to the axe LK , so is the cone ABG to the cone CDK , & the cylinder EB to the cylinder FD . Wherefore cones & cylinders consisting upon equal bases are in proportion the one to the other as their altitudes: which was required to be demonstrated.



Demonstration touching Cones.

¶ The 15. Theoreme.

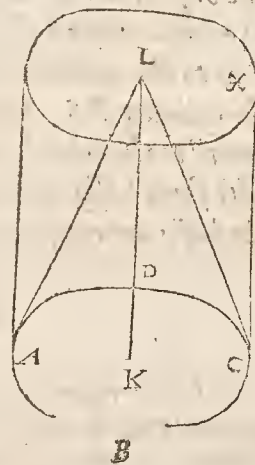
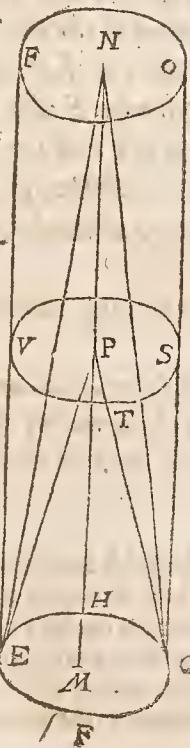
The 15. Proposition.

In equall Cones and Cylinders, the bases are reciprokall to their altitudes. And cones and Cylinders whose bases are reciprokall to their altitudes, are equall the one to the other.



Suppose that these cones ACL , EGN or these cylinders AX , EO , whose bases are the circles $ABCD$, $EFGH$, and axes KL , and MN (which axes are also the altitudes of the cones & cylinders) be equall the one to the other. The I say that the bases of the cylinders XA & EO are reciprokall to their altitudes, that is, that as the base $ABCD$ is to the base $EFGH$, so the altitude MN to the altitude KL . For the altitude KL is either equall to the altitude MN or not. First let it be equall. But the cylinder AX is equal to the cylinder EO . But cones and cylinders consisting under one and the selfe same altitude, are in proportion the one to the other as their bases are (by the 11. of the twelue). Wherefore the base $ABCD$ is equall to the base $EFGH$. Wherefore also they are reciprokall: as the base $ABCD$ is to the base $EFGH$, so is the altitude MN to the altitude KL .

But now suppose that the altitude LK be not equall to the altitude MN , but let the altitude



First part of the proposition demonstrated touching Cones. Two cases in this proposition. The first case.

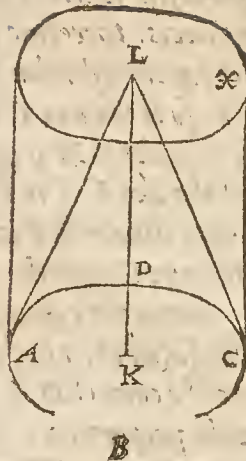
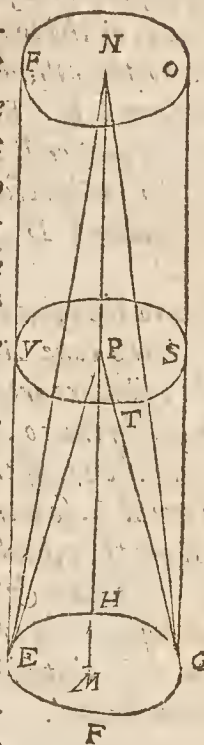
Second case. Construction.

MN

The twelue Booke

Demonstration touching cylinders.

M N be greater. And (by the 3. of the first) from the altitude *M N* take away *P M* equall to the altitude *K L*, so that let the line *P M*, be put equal to the line *K L*. And by the point *P* let there be extended a playne superficies *T V S*, which let cut the cylinder *E O*, and be a parallell to the two opposite playne superficieses, that is, to the circles *E F G H*, and *R O*. And making the base the circle *E F G H*, & the altitude *M P* imagine a cylinder *E S*. And for that the cylinder *A X* is equall to the cylinder *E O*, and there is an other cylinder *E S*, therfore (by the 7. of the fift) as the cylinder *A X* is to the cylinder *E S*, so is the cylinder *E O* to the cylinder *E S*. But as the cylinder *A X* is to the cylinder *E S*, so is the base *A B C D* to the base *E F G H*. For the cylinders *A X*, and *E S* are vnder one and the selfe same altitude. And as the cylinder *E O*, is to the cylinder *E S*, so is the altitude *M N* to the altitude *M P*. For cylinders cōsisting vpon equall bases are in proportion the one to the other as their altitudes. Wherefore as the base *A B C D* is to the base *E F G H*, so is the altitude *M N* to the altitude *M P*. But the altitude *P M* is equall to the altitude *K L*. Wherefore as the base *A B C D* is to the base *E F G H*, so is the altitude *M N* to the altitude *K L*. Wherefore in the equall cylinders *A X*, and *E O* the bases are reciprokall to their altitudes. But now suppose that the bases of the cylinders *A X*, and *E O* be reciprokall to their altitudes, that is, as the base *A B C D* is to the base *E F G H*, so is the altitude *M N* to the altitude *K L*. Then I say that the cylinder *A X* is equall to the cylinder *E O*. For the selfe same order of constructiō remayning, for that as the base *A B C D* is to the base *E F G H*, so is the altitude *M N* to the altitude *K L*, but the altitude *K L* is equall to the altitude *P M*. Wherefore as the base *A B C D* is to the base *E F G H*, so is the altitude *M N* to the altitude *P M*. But as the base *A B C D* is to the base *E F G H*, so is the cylinder *A X* to the cylinder *E S*, for they are vnder equall altitudes: and as the altitude *M N* is to the altitude *P M*, so is the cylinder *E O* to the cylinder *E S* (by the 14. of the twelue). Wherefore also as the cylinder *A X* is to the cylinder *E S*, so is the cylinder *E O* to the cylinder *E S*. Wherefore the cylinder *A X* is equall to the cylinder *E O* (by the 9. of the fift). And so also is it in the cones which haue the selfe same bases and altitudes with the cylinders. Wherefore in equall cones and cylinders, the bases are reciprokall to their altitudes &c. which was required to be demonstrated.



Second part demonstrated.

A Corrollary added by Campane and Flussas.

Hitherto hath bene shewed the passions and proprieties of cones and cylinders whose altitudes fall perpendicularly vpon the bases. Now will we declare that cones and cylinders whose altitudes fall obliquely vpon their bases haue also the selfe same passions and proprieties which the foresayd cones and cylinders haue.

Forasmuch as in the tenth of this booke it was sayd, that euery Cone is the third part of a cylinder hauing one, and the selfe same base, & one & the selfe same altitude with it, which thing was demonstrated by a cylinder geuen, whose base is cut by a square inscribed in it, and vpon the sides of the square are described Isosceles triangles, making a polygonon figure, and againe vpon the sides of this polygonon figure are infinitely after the same maner described other Isosceles triangles taking away more the halfe, as hath oftentimes bene declared: therfore it is manifest, that the solides set vpon these bases, being vnder

vnder the same altitude that the cilinder inclined is, and being also included in the same cilinder, do take away more then the halfe of the cilinder, and also more then the halfe of the residue, as it hath bene proued in erected cylinders. For these inclined solides being vnder equall altitudes and vpon equall bases with the erected solides are equall to the erected solides by the corollary of the 40. of the eleuenth. Wherefore they also in like sort as the erected, take away more then the halfe. If therefore we compare the inclined cilinder, to a cone set vpon the selfe same base, and hauing his altitude erected, and reason by an argument leading to an impossibilitie by the demonstration of the tenth of this booke, we may proue that the sided solide included in the inclined cylinder is greater then the triple of his pyramis, and it is also equall to the same which is impossible. And this is the first case, wherein it was proued that the cilinder not being equall to the triple of the cone is not greater then the triple of the same: And as touching the second case, we may after the same maner conclude that that sided solide containd in the cylinder is greater then the cylinder: which is very absurd. Wherefore if the cylinder be neither greater then the triple of the cone, nor lesse, it must needs be equall to the same. The demonstration of these inclined cylinders most playnely followeth the demonstration of the erected cylinders: for it hath already bene proued, that pyramids, and sided solides (which are also called generally Prismes) being set vpon equall bases and vnder one and the selfe same altitude, whether the altitude be erected or inclined, are equall the one to the other, namely, are in proportion as their bases are by the 6. of this booke. Wherefore a cylinder inclined shall be triple to euery cone (although also the cone be erected) set vpon one and the same base with it, and being vnder the same altitude. But the cilinder erected was the triple of the same cone by the tenth of this booke. Wherefore the cilinder inclined is equall to the cilinder erected being both set vpon one and the selfe same base, and hauing one and the selfe same altitude. The same also cometh to passe in cones, which are the third partes of equall cylinders, & therefore are equall the one to the other.

Wherefore according to the eleuenth of this booke it followeth, that cylinders and cones inclined or erected, being vnder one and the selfe same altitude, are in proportion the one to the other as their bases are. For forasmuch as the erected are in proportion as their bases are, and to the erected cylinders the inclined are equall: therefore they also shall be in proportion as their bases are.

And therefore by the 12. of this booke like cones and cylinders being inclined are in triple proportion of that in which the diameters of the bases are. For forasmuch as they are equall to the erected which haue the proportion by the 12. of this booke, and their bases also are equall with the bases of the erected, therefore they also shall haue the same proportion.

Wherefore it followeth by the 13. of this booke, that a cylinder inclined, being cut by a playne superficies parallel to the opposite playne superficieses thereof shall be cut according to the proportions of the axes. For suppose that vpon one and the selfe same base be set an erected cylinder and an inclined cylinder, being both vnder one and the selfe same altitude, which let be cut by a playne superficies parallel to the opposite bases. Now it is manifest that the sections of the one cylinder are equall to the sections of the other cylinder, for they are set vpon equall bases, and vnder one and the selfe same altitude, namely, betwene the parallel playne superficieses. And their axes also are by those parallel playne superficieses cut proportionally by the 17. of the eleuenth. Wherefore the inclined cylinders (being equall to the erected cylinders) shall haue the proportion of their axes, as also haue the erected. For in ech, the proportion of the axes is one and the same.

Wherefore inclined Cones and Cylinders, being set vpon equall bases, shall by the 14. of this booke be in proportion as their altitudes are. For forasmuch as the inclined are equall to the erected which haue the selfe same bases and altitude, and the erected are in proportion as their altitudes: therefore the inclined shall be in proportion the one to the other as the selfe same altitudes which are common to ech, namely, to the inclined and to the erected.

And therefore in equall cones and cylinders whether they be inclined or erected, the bases shall be reciprokally proportionall with the altitudes, and contrariwise by the 15. of this booke. For forasmuch as we haue oftentimes shewed that the inclined cones and cylinders are equall to the erected, hauing the selfe same bases and altitudes with them, and the erected vnto whome the inclined are equall, haue their bases reciprokally proportionally with their altitudes, therefore it followeth, that the inclined (being equall to the erected) haue also their bases and altitudes (which are common to eche) reciprokally proportionall. Likewise if their altitudes & bases be reciprokally proportionall, they theselues also shall be equall, for that they are equall to the erected cylinders and cones set vpon the same bases and being vnder the same altitude, which erected cylinders are equall the one to the other by the same 15. of this booke. Wherefore we may conclude, that those passions & proprietes which in this twelfth booke haue bene proued to be in cones and cylinders whose altitudes are erected perpendicularly to the bases, are also in those cones and cylinders whose altitudes are set obliquely vpon their bases. Howbeit this is to be noted that such inclined cones or cylinders are not perfect round as are the erected:
for

The twelue Booke

so that if they be cut by a playne superficies passing at right angles with their altitude, this section is a Conicall section, which is called *Ellipsis*, and shall not describe in their superficies a circle as it doth in erected cylinders & cones, but a certaine figure, whose lesse diameter is in cylinders equall to the diameter of the base: that is, is one and the same with it. And the same thing happeneth also in cones inclined, being cut after the same maner.

The 1. Probleme.

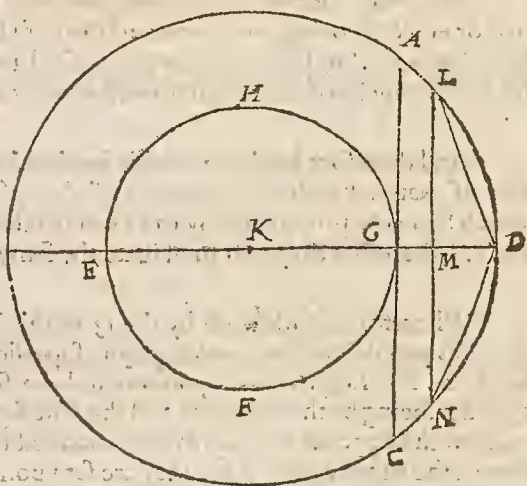
The 16. Proposition.

Two circles hauing both one and the selfe same centre being geuen, to inscribe in the greater circle a polygonon figure, which shall consist of equall and euen sides, and shall not touch the superficies of the lesse circle.



Construction.

Suppose that there be two circles $A B C D$, and $E F G H$ hauing one & the selfe same centre, namely, K . It is required in the greater circle which let be $A B C D$ to inscribe a polygonon figure which shalbe of equal and euen sides and not touch the circle $E F G H$. Drawe by the centre K a right line $B D$. And (by the 11. of the first) from the point G rayse vp vnto the right line $B D$ a perpendicular line $A G$, and extend it to the point C . Wherefore the line $A C$ toucheth the circle $E F G H$ (by the 15. of the third). Now therefore if (by the 30. of the third) we diuide the circumference $B A D$ into two equall partes, and againe the halfe of that into two equal partes, and thus do continually, we shall (by the corollary of the 1. of the tenth) at the length leaue a certayne circumference lesse then the circumference $A D$. Let the circumference left be $L D$. And from the point L . Drawe (by the 12. of the first) vnto the line $B D$ a perpendicular line $L M$, and extend it to the point N . And draw these right lines $L D$ and $D N$. And forasmuch as the angles $D M L$, and $D M N$ are right angles, therefore (by the 3. of the third) the right line $B D$ diuideth the right line $L N$ into two equall parts in the pointe M . Wherefore (by the 4. of the first) the rest of the sides of the triangles $D M L$, and $D M N$, namely, the lines $D L$, and $D N$ shalbe equall. And forasmuch as the line $A C$ is a parallell to the $L N$ (by the 28. of the first): But $A C$ toucheth the circle $E F G H$, wherfore the line $L M$ toucheth not the circle $E F G H$, and much lesse do the lines $L D$, and $D N$ touch the circle $E F G H$. If therefore there be applied right lines equall to the line $L D$ continually into the circle $A B C D$ (by the 1. of the fourth) there shalbe described in the circle $A B C D$ a polygonon figure which shalbe of equall, and ^{*} euen sides, and shall not touch the lesse circle, namely, $E F G H$: (by the 14. of the third or by the 29.) which was required to be done.



I. Dec.

* Note this $L M$, because of $K Z$ in the next proposition, and here the point M : for the point Z in the next demonstration.

* For that the sections were made by the number two: that is, by taking halues, and of the residue the halfe: and so, to $L D$, being an halfe, and a residue which shall be a comon measure backe againe, to make sides of the Polygonon figure.

¶ Corollary.

Hereby it is manifest that a perpendicular line drawn from the poynt L to the line $B D$ toucheth not one of the circles.

¶ An Assumpt added by Flussas.

If a Sphere be cut of a playne superficies, the common section of the superficieses, shall be the circumference of a circle.

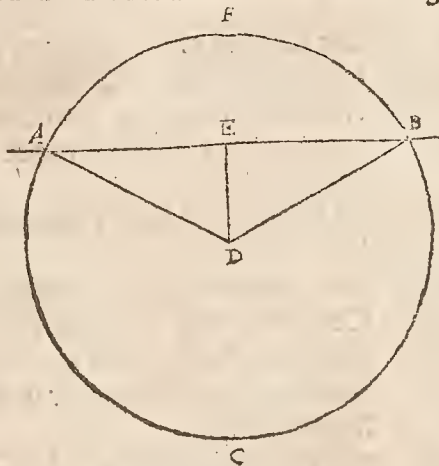
Suppose

Suppose that the sphere ABC be cut by the playne superficies AEB , and let the centre of the sphere be the poynt D . And from the poynt D , let there be drawne vnto the playne superficies AEB a perpendicular line (by the 11. of the eleuenth) which let be the line DE . And from the poynt E draw in the playne superficies AEB vnto the common section of the sayd superficies and the sphere, lines how many so euer, namely, EA and EB . And draw these lines DA and DB . Now forasmuch as the right angles DEA and DEB are equall (for the line DE is erected perpendicularly to the playne superficies). And the right lines DA & DB which subtend those angles are by the 12. definirion of the eleuenth equall: which right lines moreouer (by the 47. of the first) do contayne in power the squares of the lines DE , EA , and DE , EB : if therfore from the squares of the lines DE , EA , and DE , EB ye take away the square of the line DE which is cōmon vnto them, the residue namely the squares of the lines EA & EB shall be equal. Wherefore also the lines EA and EB are equall. And by the same reason may we proue, that all the right lines drawne from the poynt E to the line which is the cōmon section of the superficies of sphere and of the playne superficies, are equall. Wherefore that line shall be the circumference of a circle, by the 15. definirion of the first.

* But if it happen the plaine superficies which cutteth the sphere, to passe by the centre of the sphere, the right lines drawne from the centre of the sphere to their common section, shall be equall by the 12. definirion of the eleuenth. For that common section is in the superficies of the sphere. Wherefore of necessitie the playne superficies comprehended vnder that line of the common section shall be a circle, and his centre shall be one and the same with the centre of the sphere.

Construction.

Demonstration.



* The circles so made: or so considered in the sphere, are called the greatest circles: All other, not hauing the center of the sphere, to be their center also, are called lesse circles. Note these descriptions.

John Dee.

Euclide hath among the definition of solides omitted certayne, which were easy to conceaue by a kinde of Analogie. As a segment of a sphere, a sector of a sphere, the vertex, or toppe of the segment of a sphere: with such like. But that (if neede be) some farther light may be geue, in this figure next before, vnderstand a segment of the sphere ABC to be that part of the sphere contayned between the circle AB (whose center is E) and the sphericall superficies AFB . To which (being a lesse segment) adde the cone ADB (whose base is the former circle: and toppe the center of the sphere) and you haue $DAFB$ a sector of a sphere, or solide sector (as I call it). DE extended to F , sheweth the top or vertex of the segment, to be the poynt F : and EF is the altitude of the segment sphericall. Of segmentes, some are greater the the halfe sphere, some are lesse. As before ABF is lesse, the remanent, ABC is a segment greater then the halfe sphere.

¶ A Corollary added by the same Flussas.

By the foresayd assumpt it is manifest, that if from the centre of a sphere the lines drawne perpendicularly vnto the circles which cutte the sphere, be equall: those circles are equall. And the perpendicular lines so drawne fall vpon the centres of the same circles.

For the line which is drawne frō the centre of the sphere to the circumference, containeth in power, the power of the perpendicular line, and the power of the line which ioyneth together the endes of those lines. Wherefore frō that square or power of the line from the center of the sphere to the circumference or cōmon sectiō drawne, which is the semidiameter of the sphere, taking away the power of the perpendicular, which is cōmon to them round about, it followeth, that the residues how many so euer they be, be equall powers, and therefore the lines are equall the one to the other. Wherefore they will describe equall circles, by the first definirion of the third. And vpon their centers fall the perpendicular lines by the 9. of the third.

* And those circles vpon which falleth the greater perpendicular lines are the lesse circles. For the powers of the lines drawne from the centre of the sphere to the circumference being alwayes one and equall, to the powers of the perpendicular lines and also to the powers of the lines drawne from the centres of the circles to their circumference, the greater that the powers of the perpendicular lines taken away from the power contayning them both are, the lesse are the powers and therefore the lines remayning, which are the semidiameters of the circles and therefore the lesse are the circles which they describe. Wherefore if the circles be equall, the perpendicular lines falling from the centre of the sphere vpon the, shall also be equall. For if they should be greater or lesse, the circles should be vnequall as it is before manifest. But we suppose the perpendiculars to be equall. * Also the perpendicular lines falling vpon those bases are the least of all, that are drawne from the centre of the sphere: for the other drawne

* An other Corollary.

* An other Corollary.

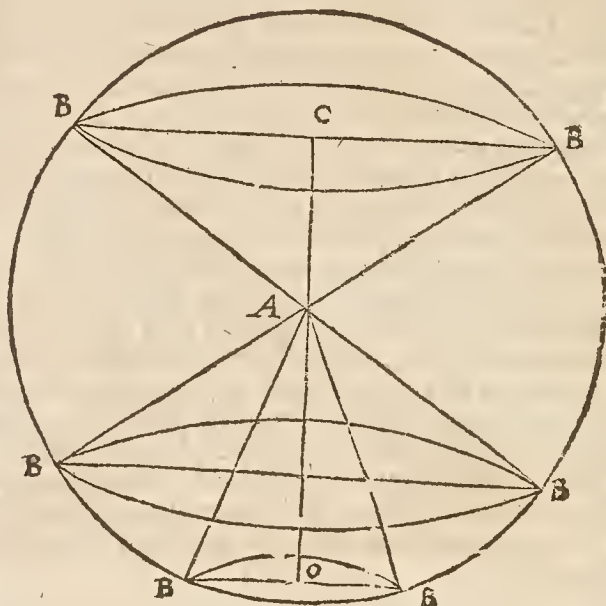
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from

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from the centre of the sphere to the circumference of the circles, are in power equall both to the powers of the perpendiculars and to the powers of the lines ioyning these perpendiculars and these subtendent lines together : making triangles rectangleround about : as most easily you may conceaue of the figure here annexed.

- A the Center of the Sphere.
- AB the lines from the Center of the Sphere to the Circumference of the Circles made by the Section.
- BCB the Diameters of Circles made by the Sections.
- AC the perpendiculars from the Center of the Sphere to the Circles: whose diameters BCB are one both sides or in any situation els.
- CB the Semidiametr^s of the Circles made by the Sections.
- AO a perpendicular longe then AC: and therefore the Semidiameter OB is lesse.
- ACB, & AOB triangles rectangle.



¶ The 2. Probleme.

The 17. Proposition.

Two spheres consisting both about one & the selfe same cētre, being geuē, to inscribe in the greater sphere a solide of many sides (which is called a Polyhedron) which shall not touch the superficies of the lesse sphere.

Construction.



* This is also proued in the Assumpt before added out of Flus-
sas.

Note what a greater or greatest circle in a Spere is. First part of the Construc-
tion.

Suppose that there be two spheres about one & the selfe same cētre, namely, about A. It is required in the greater sphere to inscribe a Polyhedron, or a solide of many sides, which shal not with his superficies touch the superficies of the lesse sphere. Let the spheres be cut by some one plaine superficies passing by the center A. Then shall their sectiōs be circles.* [For (by the 12. definition of the eleuenth) the Diameter remaining fixed, and the semicircle being turned round about maketh a sphere. Wherefore in what positiō so euer you imagine the semicircle to be, the playne superficies which passeth by it shal make in the superficies of y^e sphere a circle. And it is manifest that is also a greater circle, for the diameter of the sphere which is also the diameter of the semicircle, and therefore also of the circle is (by the 15. of the third) greater then all the right lines drawne in the circle or sphere] which circles shall haue both one center being both also in that one playne superficies, by which the spheres were cut. Suppose that that section or circle in the greater sphere be BCDE, and in the lesse, be the circle FGHI. Drawe the diameters of those two circles in such sorte that they make right angles, and let those diameters be BD, and CE. And let the line AG, being part of the line AB, be the semidiameter of the lesse sphere and circle, as AB is the semidiameter of the greater sphere and greater circle: both the spheres and circles hauing one and the same center. Now two circles that is, BCDE, and FGHI consisting both about one, and the selfe same centre being geuen, let there be described (by the proposition nexte going before) in the greater circle BCDE a polygonon figure consisting of equall and euen sides, not touching the lesse circle FGHI. And let the sides of that figure in the fourth part of the circle, namely, in BE, be BK, KL, LM, and ME. And draw a right line from the points

point K to the point A, and extend it to the point N. And (by the 12. of the eleventh) from the A raise up to the superficies of the circle B C D E a perpendicular line A X, and let it light upon the superficies of the greater sphere in the point X. And by the line A X, and by either of these lines B D, and K N extend plane superficieses. Now by that which was before spoke, those plane superficieses shall in the superficies of the sphere make two greater circles. Let their semicircles consisting upon the diameters B D, and K N be B X D, and K X N. And for as-

Note.
* You know full well, that in the superficies of the sphere, only the circumferences of the circles are: but by those circumferences the limitation and assigning of circles is used: and so, the circumference of a circle, usually called a circle, which in this place can not offend.

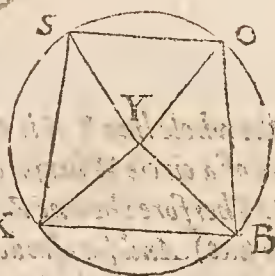
This figure is restored by M. Dee his diligence. For in the greeke and Latine Euclides, the line G L, the line A G, and the line K Z, (in which three lines the chiefe pinch of both the demonstrations doth stand), are contructively drawn: as by comparing, the studious may perceive.

Note.
You must imagine the right line A X, to be perpendicular upon the diameters B D and C E: though here A C the semidiater, seeme to be part of A X. And so in other pointes, in this figure, and many other, strengthen your imagination, according to the tenor of constructions: though in the delineation in plane, sense be not satisfied.

much as the line X A is erected perpendicularly to the plane superficies of the circle B C D E. Therefore all the plane superficieses which are drawne by the line X A are erected perpendicularly to the superficies of the circle B C D E (by the 18. of the eleventh). Wherefore the semicircles B X D, and K X N are erected perpendicularly to the plane superficies of the circle B C D E. And forasmuch as the semicircles B E D, B X D, and K X N are equal, K for they consist upon equal diameters B D, and K N: therefore also the fourth parts or quarters of those circles, namely, B E, B X, and K X are equal the one to the other. Wherefore how many sides of a polygonon figure there are in the fourth parte or quarter B E, so many also are there in the other fourth partes or quarters B X, and K X, equal to the right lines B K, K L, L M, and M E. Let those sides be described, and let them be B O, O P, P R, R X, K S, S T, T V, and V X: and draw these right lines S O, T P, and V R. And from the pointes O and S. Draw to the plane superficies of the circle B C D E perpendicular lines, which perpendicular lines will fall upon the common sections of the plane superficieses, namely, upon the lines B D, and K N, (by the 38. of the eleventh)

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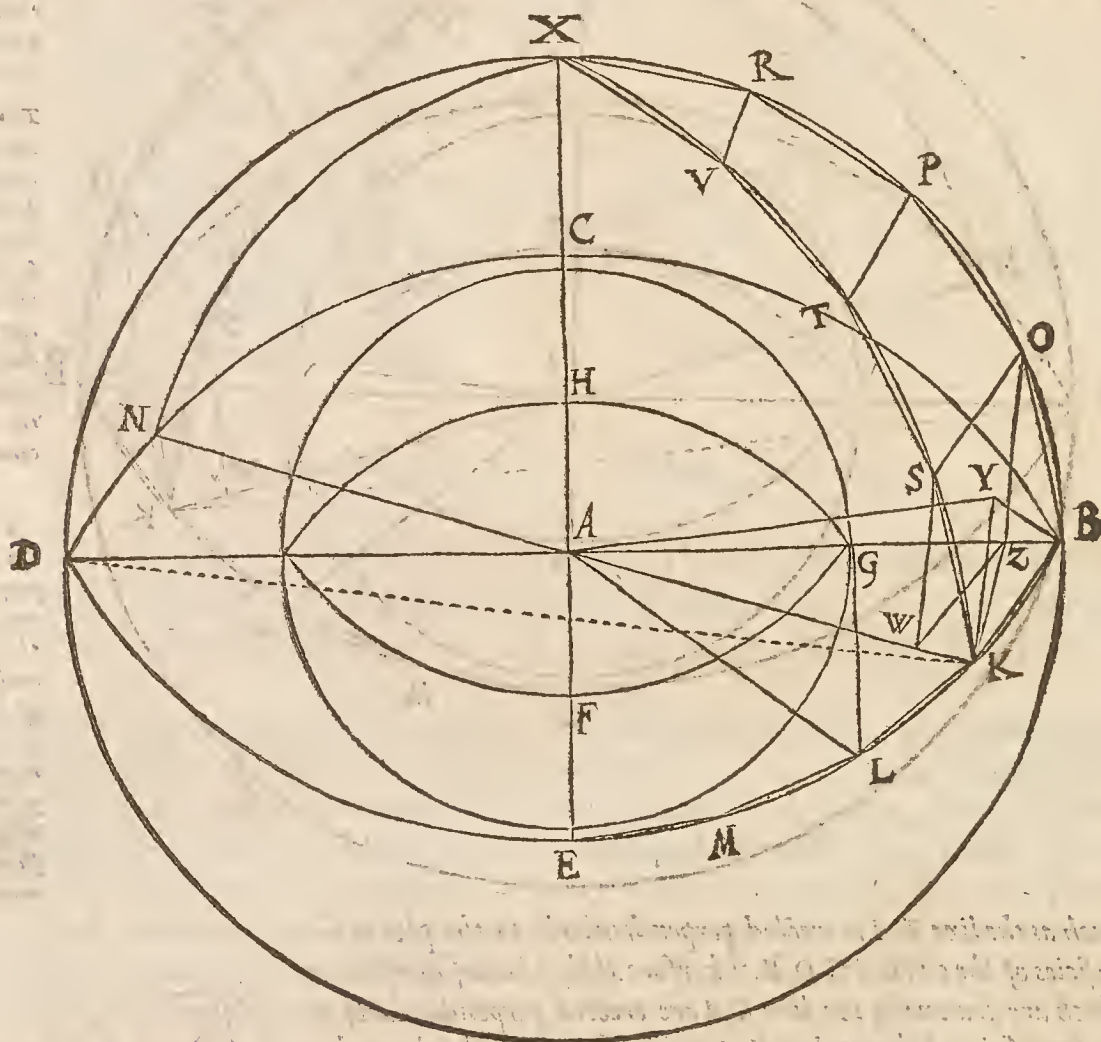
Note.
B O equal to B K, in respect of M. Dee his demonstration following.



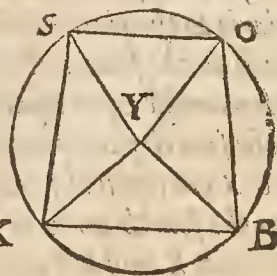
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† Note
this point
Z that you may
the better gra-
destand it.
See his demo-
stration.

eleuenth) for that the playne superficieses of the semicircles $B X D$, $K X N$ are erected perpendicularly to the playne superficieses of the circle $B C D E$. Let those perpendicular lines be $G Z$, † and $S W$. And drawe a right line from the point Z to the point W . And forasmuch as in the equall semicircles $B X D$, and $K X N$ the right lines $B O$, and $K S$ are equall, from the ends wherof are drawne perpendicular lines $O Z$, and $S W$, therefore (by the corollary of the 35. of the eleuenth) the line $O Z$ is equall to the line $S W$, & the line $B Z$ is equal to the line $K W$. [Flussas proueth this an other way thus: Forasmuch as in the triangles $S W K$, and $O Z B$, the two angles $S W K$, and $O Z B$ are equal, for that by construction they are right angles, and by the 27. of third the angles $W K S$, and $Z B O$ are equall, for they subtend equal circumter ences $S X N$ and $O X D$, and the side $S K$ is equall to the side $O B$ as it hath before ben proued. Wherefore (by the 26. of the first) the other sides & angles are equall, namely, the line $O Z$ to the line $S W$, and the line $B Z$ to the line $K W$,]

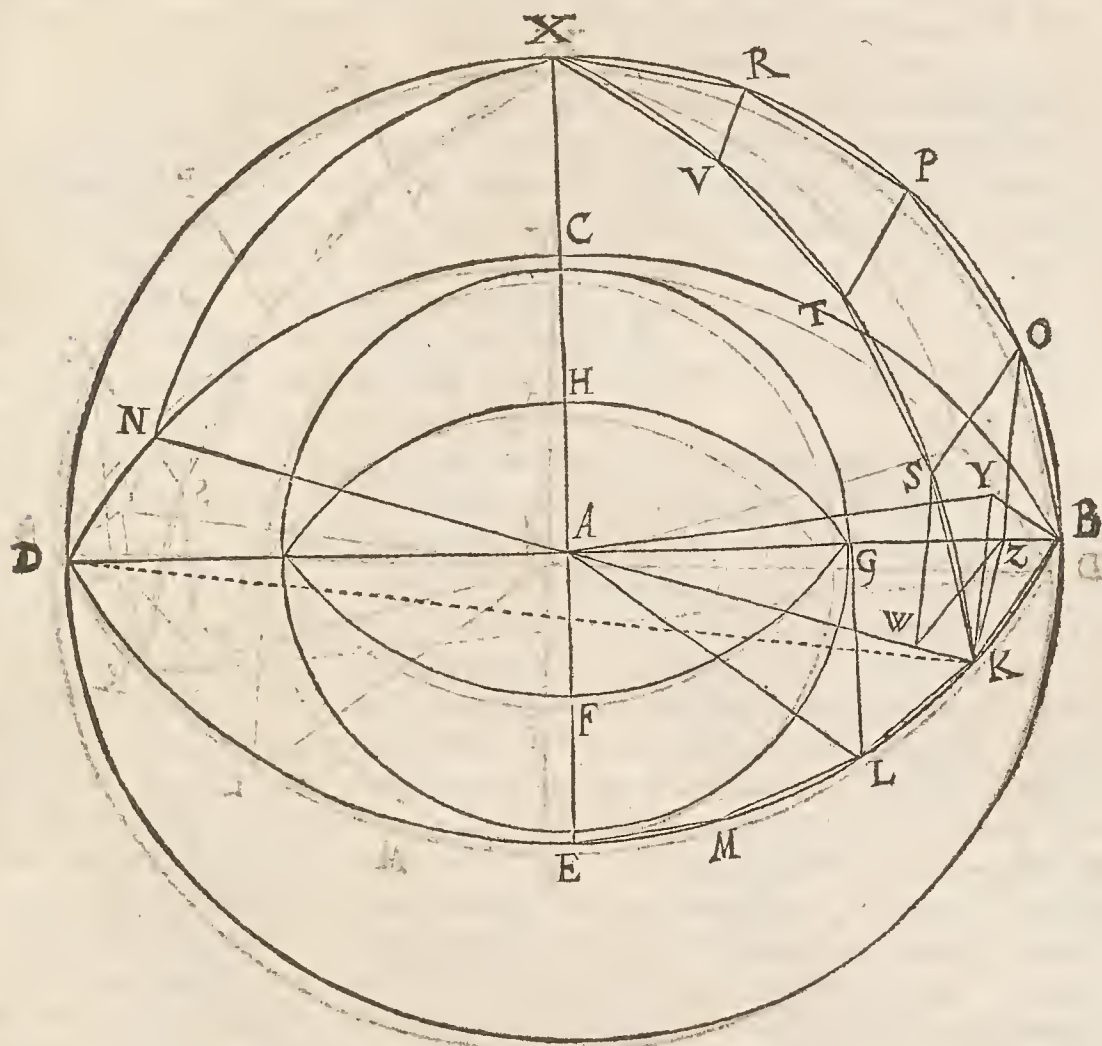


But the whole line $B A$ is equall to the whole line $K A$: by the definition of a circle. Wherefore the residue $Z A$ is equall to the residue $W A$. Wherefore the line $Z W$ is a parallel to the line $B K$ (by the 2. of the sixth). And forasmuch as either of these lines $O Z$, & $S W$ is erected perpendicularly to the playne superficieses of the circle $B C D E$, therefore the line $O Z$ is a parallell to the line $S W$, (by the 6. of the eleuenth) and it is proued that it is also equal vnto it. Wherefore the lines $W Z$ and $S O$ are also equall and parallels (by the 7. of the eleuenth, and the 33. of the first, and by the 3. of the first) And forasmuch as $W Z$ is a parallell to $S O$: But $Z W$ is a parallell is to $K B$: Wherefore $S O$ is also a parallell to $K B$, (by the 9. of the eleuenth). And the lines $B O$, and $K S$ do knit them together. Wherefore the fower sided figure $B O K S$ is in one and the selfe same playne superficieses.



For

For (by the 7. of the eleuenth) if there be any two parallell right lines, and if in either of them be taken a point at allauentures, a right line drawn by these points is in one, & the selfe same playne superficies with the parallels. And by the same reason also euery one of the fower sided figures $S O P T$, and $T P R V$ is in one and the selfe same playne superficies. And the triangle $V R X$ is also in one and the selfe same plaine superficies (by the 2. of the eleuenth). Now if we imagine right lines drawne fro the pointes O, S, P, T, R, V , to the point A , there shalbe descri-



bed a Polyhedron or a solide figure of many sides, betwene the circūferences $B X$ and $K X$ composed of pyramids, whose bases are the fower sided figures $B K O S$, $S O P T$, $T P R V$, and the triangle $V R X$, and toppe the point A . And if in euery one of the sides $K L$, $L M$ and $M E$ we vse the selfe same construction that we did in $B K$, and moreouer in the other three quadrants or quarters, and also in the other halfe of the sphere, there shall then be made K a Polyhedron or solide figure consisting of many sides described in the sphere, which Polyhedron is made of the pyramids whose bases are the foresayd fower sided figures, and the triangle $V R X$, and others which are in the selfe same order with them: and common toppe to them all in the point A . Now I say that the foresayd polihedron solide of many sides toucheth not the superficies of the lesse sphere in which is the circle $F G H$. Draw (by the 11. of the eleuenth) fro the poynt A to the playne superficies of the fower sided figure $K B O S$ a perpendicular line $A T$, and let it fall vpon the playne superficies in the point T . And drawe these right lines $B T$ & $T K$. And forasmuch as the line $A T$ is erected perpendicularly to the playne superficies of $B K O S$, therefore

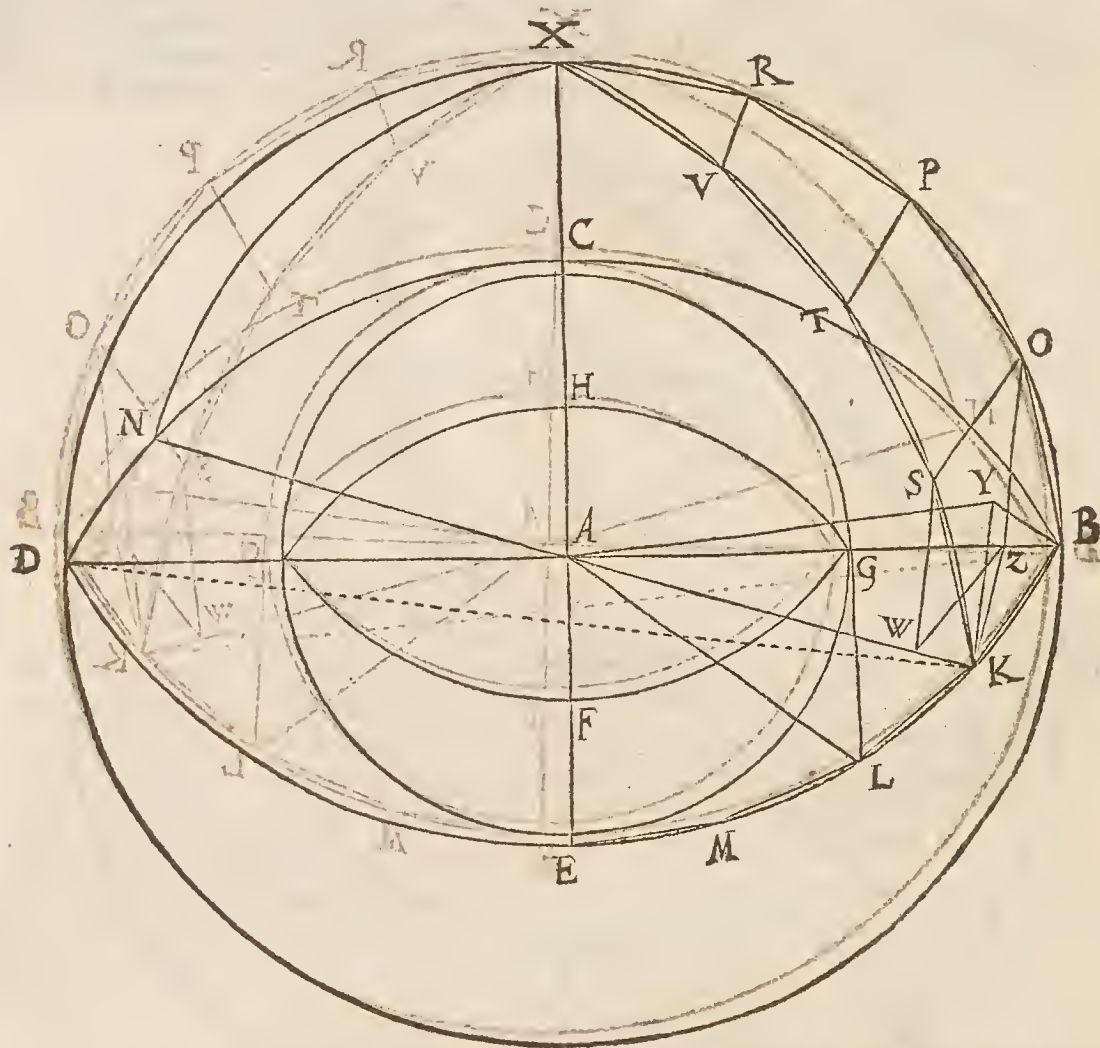


Second part
of the con-
struction.

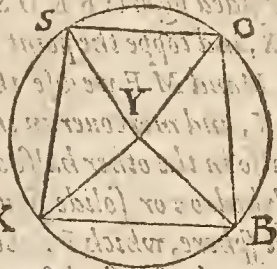
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Second part
of the demon-
stration.

therefore the same AT is erected perpendicularly to all the right lines that touch it, and are in the plaine superficies of the fower sided figure (by the 2. definition of the eleuenth). Wherefore the line AT is erected perpendicularly to either of these lines BT , and TK . And forasmuch as (by the 13. definition of the first) the line AB is equall to the line AK , therefore the square of the line AB is equall to the square of the line AK . And to the square of the line AB are equall the squares of the lines AT , and TB (by the 47. of the first) for the angle B .

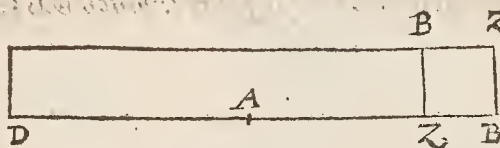


TA is a right angle. And to the square of the line AK are equal the squares of the lines AT and TK . Wherefore the squares of the lines AT , and TB are equal to the squares of the lines AT , and TK : take away the square of the line AT which is common to them both. Wherefore the residue, namely, the square of the BT is equall to the residue, namely, to the square of the line TK . Wherefore the line BT is equall to the line TK . In like sort also K may we proue that right lines drawne from the point T to the pointes O , and S , are equall to either of the lines BT , and TK . Wherefore making the center the poynt T , and the space either the line BT or the line TK describe a circle, and it shal passe by the poyntes O , and S , and the fower sided figure $KBO S$ shalbe inscribed in the circle. And forasmuch as the line KB is greater then the line WZ (by the 2. of the sixt, because AK is greater then AW), but the line WZ is equall to the line SO . Wherefore the line BK is greater then the line SO . But the line BK is equal to either of



of the lines $K S$, and $B O$ by construction. Wherefore either of the lines $K S$, and $B O$ is greater then the line $S O$. And forasmuch as in the circle is a fower sided figure $K B O S$, and the sides $B K$, $B O$ and $K S$ are equal, and the side $O S$ is lesse then any one of them, and the line $B Y$ is drawne from the centre of the circle: therefore the square of the line $K B$ is greater then the double of the square of the line $B Y$ (by the 12. of the second) (for that it subtendeth an angle greater then a right angle containd of the two equall lines $B Y$, and $Y K$, which angle $B Y K$ is an obtuse angle. For the 4. angles at the ceter Y are equal to 4. right angles: of which three, namely, the angles $B Y K$, $K Y S$, and $B Y O$ are equall by the 4. of the first, and the fourth namely, the angle $S Y O$ is lesse then any of those three angles, by the 25. of the first.)

Drawe (by the 12. of the first) from the point K to the line $B Z$, a perpendicular line $K Z$. And forasmuch as the line $B D$ is lesse then the double to the line $D Z$ (for the line $B D$ is double to the line $D A$, which is lesse then the line $D Z$): but as the line $B D$ is to the line $D Z$, so is the parallelogramme contained under the lines $D B$ and $B Z$, to the parallelogramme contained under the lines $D Z$ and $Z B$ (by the 1. of the sixt): therefore if ye describe vpon the line $B Z$ a square, and making perfecte the parallelogramme contained under the lines $Z D$ and $Z B$, that which is contained under the lines



$D B$ & $B Z$, shall be lesse then the double to that which is contained under the lines $D Z$ and $Z B$. And if ye drawe a right line from the point K to the point D , that which is contained under the lines $D B$ and $B Z$, is equall to the square of the line $B K$ (by the Corollary of the 8. of the sixt) for the angle $B K D$ is a right angle, by the 31. of the third, for it is in the semicircle $B E D$: and that which is contained under the lines $D Z$ and $Z B$, is equall to the square of the line $K Z$ (by the same Corollary). Wherefore the square of the line $K B$, is lesse then the double to the square of the line $K Z$. But the square of the line $K B$ is greater then the double to the square of the line $B Y$, as before hath bene proued. Wherefore the square of the line $K Z$, is greater then the square of the line $B Y$ (by the 10. of the fift). And forasmuch as (by the 15. definition of the first) the line $B A$ is equall to the line $K A$: therefore also the square of the line $B A$ is equall to the square of the line $K A$. But (by the 47. of the first) vnto the square of the line $A B$, are equall the squares of the lines $B Y$ & $Y A$: (for the angle $B Y A$ is by construction, a right angle). And (by the same reason) to the square of the line $K A$, are equall the squares of the lines $K Z$ and $Z A$: (for the angle $K Z A$ is also by construction, a right angle). Wherefore the squares of the lines $B Y$ and $Y A$, are equall to the squares of the lines $K Z$ and $Z A$: Of which the square of the line $K Z$ is greater then the square of the line $B Y$, as hath before bene proued. Wherefore the residue, namely, the square of the line $Z A$, is lesse then the square of the line $Y A$. Wherefore the line $Y A$ is greater then the line $A Z$. * Wherefore the line $A Y$ is much greater then the line $A G$. But the line $A Y$ falleth vpon one of the bases of the Polihedron, and the line $A G$ falleth vpon the superficies of the lesse sphere. Wherefore the Polihedron toucheth not the superficies of the lesse sphere.

✠ Which of necessity shall fall vpon Z , as M. Dee proueth it: and his proue is set after at this marke ✠ following.

I. Dee.

* But $A Z$ is greater then $A G$, as in the former proposition, $K M$ was euident to be greater then $K G$: so may it also be made manifest that $K Z$ doth neyther touch nor cut the circle $F G H$.

Another proue that the line $A Y$ is greater then the line $A G$.

An other and more ready demonstration to proue that the line $A Y$ is greater then the line $A G$. Raise vp (by the 11. of the first) from the poynt G to the line $A G$ a perpendicular line $G L$. And draw a right line from the point A to the poynt L . Now the deuiding (by the 30. of the third) the circumference $E B$ into halues, & agayne that halfe into halues, & thus doing continually, we shall at the length by the corollary of the first of the tenth, leaue a certayne circumference which shall be lesse then the circumference of the circle $B C D$ which is subtended of a line equall to the line $G L$. Let the circumference left be $K B$. Wherefore also the right line $K B$ is lesse then the right line $G L$. And forasmuch as the fower sided figure $B K O S$ is

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in a circle, and the lines OB , BK , and KS are equall, and the line OS is lesse, therefore the angle BYK is an obtuse angle. Wherefore the line BK is greater then the line BY . But the line GL is greater then the line KB . Wherefore the line GL is much greater then the line BY . Wherefore also the square of the line GL is greater then square of the line BY . And forasmuch as (by the 15. definition of the first) the line AL is equall to the line AB , therefore the square of the line AL is equall to the square of the line AB . But unto the square of the line AL are equall the squares of the lines AG and GL , and to the square of the line AB are equall the squares of the lines BY and YA . Wherefore the squares of the lines AG and GL are equall to the squares of the lines BY and YA , of which the square of the line BY is lesse then the square of the line GL . Wherefore the residue, namely, the square of the line YA is greater then the square of the line AG . Wherefore also the line YA is greater then the line AG . Wherefore two spheres consisting both about one and the selfe same center, being geuen, there is inscribed in the greater sphere a polihedron or solide of many sides which toucheth not the superficies of the lesse sphere: which was required to be done.

¶ Corollary.

And if in the other sphere, namely, in the lesse sphere be inscribed a Polihedron or solide of many sides like to the polihedron inscribed in the sphere $BCDE$, then the polihedron inscribed in the sphere $BCDE$ is to the polihedron inscribed in the other sphere in treble proportion of that in which the diameter of the sphere $BCDE$ is to the diameter of the other sphere. For those solides being deuided into pyramids equall in number and equall in order, the pyramids shall be like. But like pyramids are (by the 8. of the twelfth) the one to the other in treble proportion of that in which side of like proportion is to side of like proportion. Wherefore the pyramis whose base is the fower sided figure $KBO S$ and toppe the poynt A is to that pyramis which is of like order in the other sphere, in treble proportion of that in which side of like proportiō is to side of like proportiō, that is of that in which the side AB which is drawne frō the cēter of the sphere which is about the cēter A , is to the side which is drawn frō the cēter of the other sphere. And in like sort also euery one of y pyramids which is in y sphere which is about the cētre A is to euery one of the pyramids of the selfe same order in the other sphere in treble proportiō of that in which the side AB is to the side which is drawne from the center of the other sphere. But as one of the antecedentes is to one of the consequentes, so are all the antecedents to all the consequentes by the 12. of the fifth. Wherefore the whole polihedron solide of many sides which is in the sphere which is about the center A , is to the whole polihedron or solide of many sides which is in the other sphere, in treble proportion of that in which the side AB is to the side which is drawne from the center of the other sphere, that is, of that which the diameter BD is to the diameter of the other sphere, by the 15. of the fifth: which was required to be demonstrated.

¶ Mast

The twelueh Booke

¶ Master Dee his aduise and demonstration, reforming a great errour
in the designation of the former figure of Euclides second Probleme: with
two Corollaries (by him inferred) vpon his said demonstration.

✱ A Theoreme.

✱ That a right line drawen from the point K, perpendicularly vpon the line BZ, doth fall vpon
the point Z: We will, thus, make eident.

BY the premisses, it is manifest, that the point Z is that point where a right line from the point O, being perpendicularly let fall to the circle BCDE, doth touch the same circle. Which point Z also is proued to be in the right line BAD, the common section of two circles cutting eche other: being one to the other perpendicularly erected. These things, with other, before demonstrated, I here make my suppositions. Consider now the two triangles rectangles OZB and KZB: Of which, the angle OZB is equall to the angle KZB. For, by construction, they are both right angles: * and the angle ZBO is equall to the angle ZBK. For, if from D to K you imagine a right line: and the like from D to O: you haue two triangles in equall semicircles, rectangles, namely, DKB and DOB: which haue the diameter BD common: and BK, the Chord, equall to BO the Chord, by construction. Wherefore (by the 47. of the first) the third side, namely, DK, is equall to the third, namely, DO: Wherefore (by the 5. of the first) the angle ZBO, which is DOB, is equall to ZBK, which is DBK. (For the line ZB, by construction, is part of DB). And seing two angles of OZB, are proued equall, to two angles of KZB, of necessitye the third, namely, ZOB, is equall to the third, namely, ZKB, by the 32. of the first. Wherefore the two triangles rectangles OZB and KZB, are proued equiangled. By the fourth, therefore, of the sixth, their sides are proportionall: therefore by the premisses, proued, as BO is to BK, so is OZ to KZ, and the third line, which subtendeth the angle ZOB, to the third line which subtendeth the angle ZKB. But, by construction, BO is equall to BK: therefore OZ is equall to KZ: And the third also is equall to the third: Wherefore the point Z, in respect of the two triangles rectangles, OZB and KZB, determineth one and the same magnitude, in the line BZ. Which can not be: if any other point, in the line BZ, were assigned, nearer, or farther of, from the point B. One onely poynt therefore, is that, at which the two perpendiculars KZ and OZ fall: But, by construction, OZ falleth at Z the point, and therefore at the same Z, doth the perpendicular, drawen from K, fall likewise: Which was required to be demonstrated.

Although a briefe monition, mought herein haue serued for the pregnant or the humble learner, yet for them that are well pleased to haue thinges made plaine, with many wordes, and for the stiffnecked busie body, it was necessary, with my controlment of other, to annexe the cause & reason therof, both inuincible and also eident.

✱ A Corollary. 1.

Hereby it is manifest, that two equall circles cutting one the other by the whole diameter, if from one and the same end of their common diameter, equall portions of their circumferences be taken: and from the pointes ending those equall portions, two perpendiculars be let downe to their common diameter, those perpendiculars shall fall vpon one and the same point of their common diameter.

2.

Secondly it followeth that those perpendiculars are equall.

¶ Note.

From circles in our first supposition eche to other perpendicularly erected, we procede and inferre now these Corollaries, whether they be perpendicularly erected or no: by reason the demonstration hath a like force, vpon our suppositions here vsed.

¶ The 16. Theoreme.

The 18. Proposition.

Spheres are in treble proportion the one to the other of that in which their diameters are.

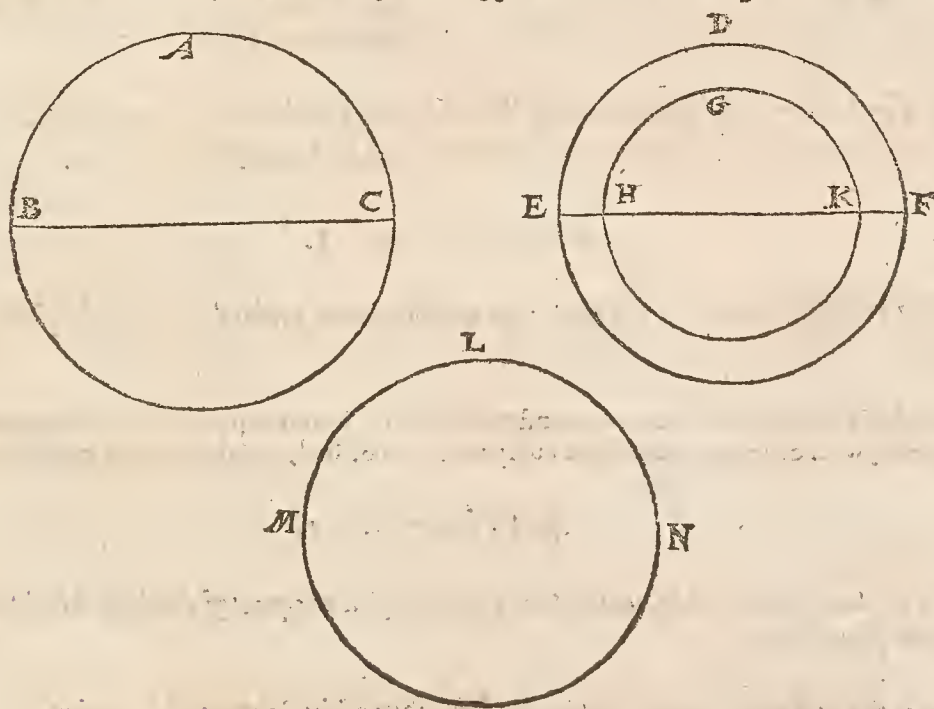
Suppose



Suppose that there be two spheres ABC and DEF , and let their diameters be BC and EF . Then I say that the sphere ABC is to the sphere DEF in treble proportion of that in which the diameter BC is to the diameter EF . For if not, then the sphere ABC is in treble proportion of that in which BC is to EF , either to some sphere lesse then the sphere DEF , or to some sphere greater. First let it be unto a lesse, namely, to $G HK$. And imagine that the spheres DEF and $G HK$ be both about one and the selfe same centre. And (by the proposition next going before) describe in the greater sphere DEF a polihedron or a solide of many sides not touching the superficies of the lesse sphere $G HK$. And suppose also that in the sphere ABC be inscribed a polihedron like to the polihedron which is in the sphere DEF . Wherefore (by the corollary of the same) the polihedron which is in the sphere ABC , is to the polihedron which is in the sphere DEF in treble proportion of that in which the diameter BC is to the diameter EF . But by supposition the sphere ABC is to the sphere $G HK$ in treble proportion of that in which the diameter BC is to the diameter EF . Wherefore as the sphere ABC is to the sphere $G HK$, so is the polihedron which is described in the sphere ABC to the polihedron which is described in the sphere DEF by the 11. of the fift. Wherefore alternately (by the 16. of the fift) as the sphere ABC is to the polihedron which is described in it, so is the sphere $G HK$ to the polihedron which is in

Two cases in this proposition.

The first case. Demonstration leading to an impossibility.



the sphere DEF . But the sphere ABC is greater then the polihedron which is described in it. Wherefore also the sphere $G HK$ is greater then the polihedron which is in the sphere DEF by the 14. of the fift. But it is also lesse, for it is containd in it, which impossible. Wherefore the sphere ABC is not in treble proportion of that in which the diameter BC , is to the diameter EF , to any sphere lesse then the sphere DEF . In like sort also may we proue that the sphere DEF is not in treble proportion of that in which the diameter EF is to the diameter BC , to any sphere lesse then the sphere ABC . Now I say that the sphere ABC is not in treble proportion of that in which the diameter BC is to the diameter EF , to any sphere greater then the sphere DEF . For if it be possible, let it be to a greater, namely, to $L MN$. Wherefore by conuersion the sphere $L MN$ is to the sphere ABC in treble proportion of that in which the diameter EF is to the diameter BC . But as the sphere $L MN$ is to the sphere ABC , so is the sphere DEF to some sphere lesse then the sphere ABC ,* as it hath before bene proued, for the sphere $L MN$ is greater then the sphere DEF . Wherefore the sphere DEF is in treble proportion of that in which

Second case.

* As it is easie to gather by the assumption after the second of this booke.

The twelvethe Booke

which the diameter EF is to the diameter BC to some sphere lesse then the sphere ABC , which is proued to be impossible. Wherefore the sphere ABC is not in treble proportion of that in which BE is to EF to any sphere greater then the sphere DEF . And it is also proued that it is not to any lesse. Wherefore the sphere ABC is to the sphere DEF in treble proportion of that in which the diameter BC is to the diameter EF : which was required to be demonstrated.

A Corollary added by *Flussas*.

Hereby it is manifest, that spheres are the one to the other, as like Polihedrons and in like sort described in them are: namely, each are in triple proportion of that in which the diameters.

✱ A Corollary added by *M. Dee*.

It is then euident, how to geue two right lines, hauing that proportion betwene them, which, any two spheres geuen, haue the one to the other.

Notes: a generall rule.

For, if to their diameters, as to the first and second lines (offower in continuall proportion) you adioyne a third and a fourth line in continuall proportion (as I haue taught before): The first and fourth lines, shall aunswere the Probleme. How generall this rule is, in any two like solides, with their correspondent (or Omologall) lines, I neede not, with more wordes, declare.

¶ Certaine Theoremes and Problemes (whose vse is manifolde, in Spheres, Cones, Cylinders, and other solides) added by *Ioh. Dee*.

✱ A Theoreme. 1.

The whole superficies of any Sphere, is quadrupla, to the greatest circle, in the same Sphere containd.

It is needeles to bring *Archimedes* demonstration hereof, into this place: seing his booke of the Sphere and Cylinder, with other his workes, are euery where to be had, and the demonstration thereof, easie.

✱ A Theoreme. 2.

Euery Sphere, is quadrupla, to that Cone, whose base is the greatest circle, & height, the semidiameter of the same sphere.

This is the 32. Proposition of *Archimedes* first booke of the Sphere and Cylinder.

✱ A Probleme. 1.

A Sphere being geuen, to make an upright Cone, equall to the same: or in any other proportion, geuen, betwene two right lines.

Construction.

Suppose the Sphere geuen, to be A , his diameter being BC , and center D : with a line equall to the semidiameter BD (which let be NO) describe a circle NRP : whose diameter let be NP , and center O , it is euident, that NRP is equall to the greatest circle in A , containd. At the center O , let a perpendicular be reared equall to BD (the semidiameter of A) which suppose to be OQ : It is now plaine that to the Cone, whose base is the circle NRP , and height OQ , the Sphere A , is quadrupla: by the 2. Theoreme here, and by construction. Take a line equall to NP , which let be FE : and with the semidiameter FE (making the point F center) describe a circle: which suppose to be EKG , and diameter EG . At the center F , reare a line perpendicular to EKG , by the 12. of the eleuenth: and make it equall to OQ . Let that line be FL . I say that the Cone, whose base is the circle EKG , and height the line FL , is equall to A . For seing FE , the semidiameter of EKG , is equall to NP (the diameter of NRP) by construction: EG , the diameter of EKG , shall be double to NP . Wherefore the square of EG , is quadrupla to the square of NP : by the 4. of the second. But as the square of EG , is to the square

Demonstration.

square of $N P$, so is the circle $E K G$ to the circle $N R P$, by the 2. of this twelfth. Wherefore the circle $E K G$, is quadruple to the circle $N R P$. And $F L$, the height is (by construction) equall to $O Q$ the height. Wherefore the cone, whose base is the circle $E K G$, and height $F L$, is quadruple to the cone, whose base is $N R P$, and the height $O Q$: by the 11. of this twelfth. But vnto the same cone whose base is $N R P$, and height $O Q$, the Sphere A is likewise proued quadruple. Wherefore the cone whose base is $E K G$ and height $F L$, is equall to the Sphere A : by the 7. of the fifth. To a Sphere being geuen therefore, we haue made an vpright cone equall.

And as concerning the other part of this Probleme, it is now easie to execute, and that two wayes: I meane to A the sphere geuen, to make an vpright cone in any proportion geuen between two right lines. For, let the proportion geuen, be that which is betwene X and Y . By the order of my additions, vpon the 2. of this twelfth booke: to the circle $E K G$ make an other circle in that proportion that X is to Y : which let be Z . Vpon the center of Z , reare a line perpendicular and equall to $F L$. I say that the cone, whose base is Z , and the height equall to $F L$, is to A , in the proportion of X to Y . For the cone vpon Z , by construction, hath height equall to the height of the cone $E K G$: and Z , by construction, is to $E K G$, as X is to Y : Wherefore, by the 11. of this twelfth, the cone vpon Z , is to the cone $E K G$, as X is to Y . But the cone $E K G$ is proued equall to the sphere A . Wherefore the cone vpon Z , is to A , as X is to Y , by the 7. of the fifth. To a Sphere geuen therefore, we haue made a cone in any proportion geuen, between two right lines. Secondly, as X is to Y , so to $F L$, let there be a fourth line, by the 12. of the sixth: and suppose it to be W . I say that a cone, whose base is equall to $E K G$, and height the line W , is to A , as X is to Y . For, by the 11. of this twelfth, cones being set on equall bases, are one to the other, as their heights are: But, by construction, the height W , is to the height $F L$, as X is to Y . Wherefore the cone which hath his base equall to $E K G$, and height the line W , is to the cone $E K G$, as X is to Y . And it is proued, that to the cone $E K G$, the Sphere A is equall: Wherefore, by the 7. of the fifth, the cone, whose base is equall to $E K G$, and height the line W , is to A , as X is to Y . Therefore a Sphere being geuen, we haue made an vpright cone, in any proportion geuen between two right lines. And before, we made an vpright cone, equall to the Sphere geuen: Wherefore a Sphere being geuen, we haue made an vpright cone, equall to the same, or in any other proportion, geuen between two right lines. I call that an vpright cone, whose axe is perpendicular to his base.

The second part of the Probleme two wayes executed.

1.

2.

An vpright Cone.

¶ A Corollary.

Of the first part of the demonstration, it is euident: A Sphere being propounded, that a Cone, whose base hath his semidiameter, equall to the diameter thereof, and height equall to the semidiameter of the same Sphere, is equall to that sphere propounded.

¶ A Probleme.

A Sphere being geuen, and a circle, to reare an vpright Cone, vpon that circle (as a base) equall to the Sphere geuen: or in any proportion between two right lines assigned.

Suppose the Sphere geuen, to be Q : and the circle geuen to be C : By the first Probleme make an vpright cone equall to Q the Sphere geuen: which cone suppose to be A : and (by the 2. Probleme of my additions vpon the second of this twelfth booke) as C the circle geuen, is to the base of A , so let

G G g. j. the

The tweluth Booke

the height of A, be to a line found : which let be D. Then it is evident, that the cone, which hath for his base C, the circle geuen, and height the line D, last found, shall be equall to Q the Sphere geuen : which cone let be F. For, by construction, F hath his base and height in reciprokall proportion with the cone A, made equall to Q the Sphere geuen : Wherefore by the 15. of this twelfth, and 7. of the fifth, this vp-right cone F, reared vpon C, the circle geuen, is equall to Q, the Sphere geuen : which thing the Probleme first required.

The second part of the Probleme.

And the second part of this Probleme is thus performed. Suppose the proportion geuen to be that which is betwene X & Y. Then, as X is to Y, so let an other right line found, be to the height of F : which line let be G. For this G, the found height (by construction) being to the height of F, as X is to Y, doth cause this cone (which let be M) vpon C, the circle geue (or an other to it equall) duely reared, to be vnto the cone F, as X is to Y, by the 14. of this twelfth. But F is proued equall to the Sphere geuen : Wherefore M, is to the Sphere geuen, as X is to Y. And M, is reared vpon the circle geuen : or his equall. Wherefore, a Sphere being geuen, & a circle, we haue reared an vp-right cone, vpon that geuen circle (as a base) equall to the Sphere geuen : or in any proportion, betwene two right lines assigned : which was required to be done.

¶ A Probleme.

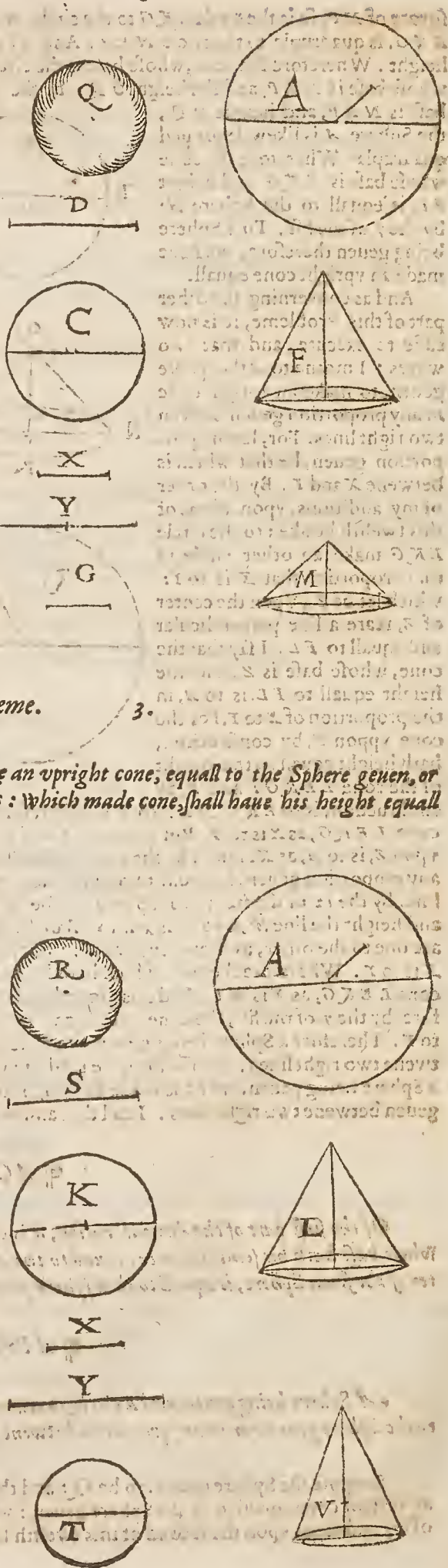
A Sphere being geuen, and a right line, to make an vp-right cone, equall to the Sphere geuen, or in any other proportion geuen betwene two right lines : which made cone, shall haue his height equall to the right line geuen.

Suppose the Sphere geuen, to be R : and the right line geuen, to be S. To R the Sphere geuen, make an vp-right cone, equall : by the first Probleme : which cone suppose to be A. Then as S, the line geuen, is to the height of A, so let the base of A, be to an other circle, which let be K, by my additions, vpon the second Proposition of this twelfth booke. I say that an vp-right cone, hauing his height, equall to S, the right line geuen, and his base K, is equall to the Sphere geuen. Let this cone be noted by L : for by construction, thys cone L, and A, haue theyr heightes and bases reciprokall in proportiō : Wherefore this cone L, and the cone A, are equall, by the 15. of the twelfth. But A is equall to the Sphere geuen by construction. Wherefore L is equall to the Sphere geuen. And the height of L, is equall to the right line geuen, by construction : which ought to be done.

The second part of the Probleme.

For the second part : finde a circle, which shall haue to the base of L, any proportion appointed in right lines : as the proportion of X to Y : which, by my additions, vpo the second of this booke, ye haue learned to do. Then, with the height, equall to the height of L reared vpon this last found circle, which let be T, as a base, you shall satisfie the Probleme. Let that Cone be V. For this last cone V, is to L, as his base is to the base of L, by the 11. of this twelfth. But L is proued equall to the Sphere geuen : Wherefore by the 7. of the fift, this last cone V, hath to R,

the



the Sphere geuen, that proportion which is betwene X and Y assigned : and forasmuch as the height of this cone V, is equall to the height of L : and the height of L, equall to S, the right line geuen (by construction) : it is euident, that a Sphere being geuen, & a right line, we haue made an vpright cone, equall to the Sphere geuen, or in any other proportion geuen betwene two right lines : which made cones, haue their height equall to the right line geuen : which ought to be done.

Vu willing I am to vse thus many wordes, in matters so plaine and easie. But this (I thinke) can not hinder them, that by nature are not so quicke of inuention, as to lead euery thing, generally spoken, to a particular execution.

¶ A Theoreme. 3.

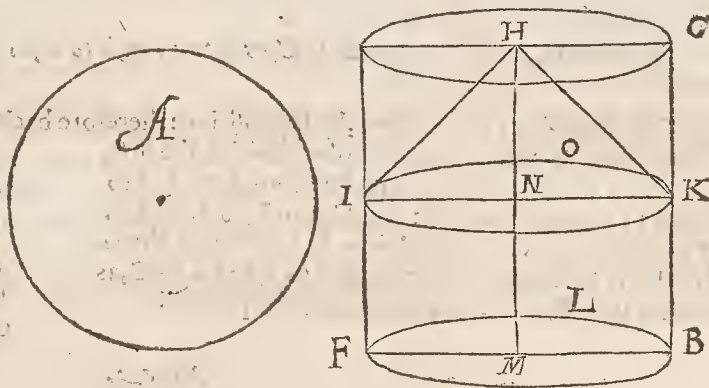
Euery Cylinder which hath his base, the greatest Circle in a Sphere, & heith equall to the diameter of that Sphere, is Sesquialtera to that Sphere. Also the superficies of that Cylinder, with his two bases is Sesquialtera to the superficies of the Sphere: and without his two bases, is equall to the superficies of that Sphere

Suppose, a sphere to be signified by A, and an vpright cylinder hauing to his base a circle equall to the greatest circle in A contained, and his heith equall to the diameter of A, let be signified by F G. I say that F G, is sesquialter to A: Secondly I say that the croked cylindricall superficies of F G, together with the superficieses of his two opposit bases, is sesquialtera to the whole superficies sphericall of A. Thirdly I say that the cylindricall superficies of F G, omitting his two opposite bases, is equall to the superficies of the spere A. Let the base of F G, be the circle F L B : whose center, suppose M, and diameter F B. And the axe of the same F G, let be, M H. Which is his heith (for we suppose the cylinder to be vpright) : and suppose H, to be his toppe or vertex. Forasmuch as, by supposition M H is equall to the diameter of A. Let M H be deuided into two equall partes in the point N, by a playne superficies passing by the point N, and being parallell to the opposit bases of F G. By the thirteenth of this twelfth booke, it then foloweth, that the cylinder F G, is also deuided into two equall parts: being cylinders: which two equall cylinders let be I G, and F K: the axe of I G suppose to be H N : and of F K the axe to be N M. And for that, F G, is an vpright cylinder, and at the poynt N, cut by a playne Superficies parallell to his opposite bases, the common section of that playne superficies and the cylinder F G, must be a * circle, equall, to his base F L B, and haue his center, the point N. Which circle, let be I O K: And feing that F L B is, by supposition, equall to the greatest circle in A, I O K, also, shall be equall to the greatest circle, in A, contained: Also, by reason M H, is by supposition, equal to the diameter of A : and N H, by construction, half of M H, it is manifest that N H is equall to the semidiameter of A.

If, therefore, you suppose a cone to haue the circle I O K to his base : and N H to his heith, the sphere A, shall be to that Cone, quadrupla, by the 2. Theoreme. Let that cone be H I O K. Wherefore A, is quadrupla to H I O K: And the Cylinder I G hauing the same base, with H I O K (the circle I O K) and the same heith, (the right line N H) is triple to the cone H I O K: by the 10. of this twelfth booke. But to I G, the whole cylinder F G, is double, as is proued: Wherefore F G, is triple and triple, to the cone H I O K, that is, sextuple. And A is proued quadrupla to the same H I O K. Wherefore F G is to H I O K, as 6. to 1: and A, is to H I O K, as 4. to 1: * Therefore F G is to A, as 6, to 4: which in the least termes, is, as 3 to 2. but 3 to 2, is the termes of sesquialtera proportion. Wherefore the cylinder F G, is to A sesquialtera in proportion. Secondly, forasmuch as the superficies of a cylinder (his two opposite bases excepted) is equall to that circle whose semidiameter is middell proportionall betwene the side of the cylinder, and the diameter of his base: (as vnto the 10. of this booke, I haue added.) But of F G, the side B G, being parallell and equall to the axe M H, must also be equall to the diameter of A. And the base F L B, being (by supposition) equall to the greatest circle in A contained, must haue his diameter (F B) equal to the sayd diameter of A. The middle proportional therfore betwene B G and F B, being equall eche to other, shall be a line, equall to either of them.

[As if you set B G and F B together, as one line, and vpon that line composed, as a diameter make a semicircle: and from the center, to the circumference draw a line perpendicular to the sayd diameter: by the 13. of the sixth, that perpendicular, is middell proportional betwene F B and B G, the semidiameters: and he him selfe also a semidiameter: and therefore by the definition of a circle, equall to F B, and likewise, to B G.] And a circle, hauing his semidiameter, equall to the diameter F B, is quadruple to the circle F L B. [For the square of euery whole line, is quadruple to the square of his halfe line, as may

GGg. ij. be



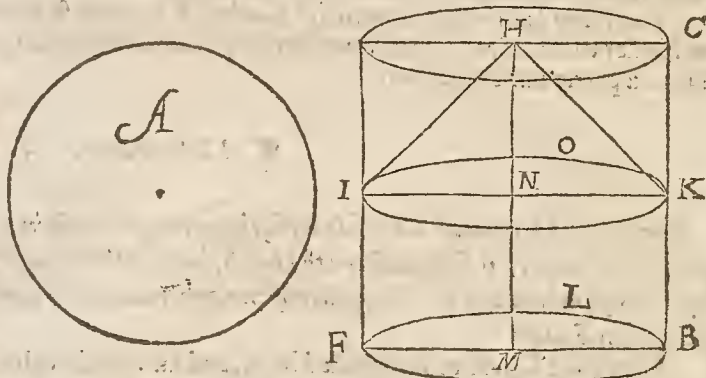
* This may easily be demonstrated, as in the 17. proposition the section of a sphere was proued to be a circle.

* For taking away all doubt, this, as a Lemma, afterward is demonstrated.

A Lemma (as it were) presently demonstrated.

The twelvethe Booke

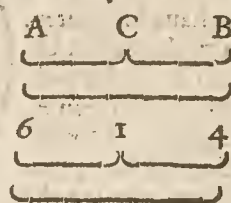
be proued by the 4. of the second: and by the second of this twelfth, circles are one to the other, as the squares of their diameters, are.] Wherefore the superficies cylindricall of FG , alone, is quadrupla to his base FLB . But if a certayne quantity be dupla to one thing, and an other, quadrupla to the same one thing, those two quantities together are sextupla to the same one thing. Therefore seing the base, opposite to FLB , (being equall to FLB) added to FLB , maketh that cōpound, double to FLB : that double added to the cylindricall superficies of FG , doth make a superficies sextupla to FLB . And the superficies of A , is quadrupla to the same FLB , by the first Theoreme. Therefore the cylindricall superficies of FG , with the superficieses of his two bases is to the superficies FLB , as 6 to 1. and the superficies of A to FLB , is as 4 to 1. Wherefore the cylindricall superficies of FG , & his two bases, together, are to the superficies of A , as 6 to 4: that is, in the smallest termes, as 3 to 2. Which is proper to sesquialtera proportion. Thirdly, it is already made euident that the superficies cylindricall, of FG (onely by it self) is quadrupla to FLB . And also it is proued, that the superficies of the sphere A , is quadrupla to the same FLB . Wherefore by the 7. of the fifth, the cylindricall superficies of FG , is equall to the superficies of A . Therefore, euery cylinder, which hath his base the greatest circle in a sphere, and heith equal to the diameter of that sphere, is sesquialtera to that spere: Also the superficies of that cylinder with his two bases, is sesquialtera, to the superficies of the sphere: and without his two bases is equall to the superficies of the sphere: which was to be demonstrated.



The Lemma.

If A be to C , as 6, to 1: and B , to C , as 4 to 1: A , is to B , as 6, to 4.

For, seing, B is to C , as 4 to 1, by supposition: therefore backward, by the 4. of the fifth, C is to B , as 1, to 4. Imagine now two orders of quantities, the first, A , C , and B the second, 6, 1, and 4. Forasmuch as, A , is to C , as 6, to 1, by supposition: and C is to B , as 1, to 4, as we haue proued: wherefore, A is to B , as 6 to 4, by the 22 of the fifth. Therefore, if A be to C as 6 to 1, and B to C , as 4 to 1: A is to B , as 6, to 4. which was to be proued.



Note.

Sleight things (some times) lacking euidēt proufe, brede doubt or ignorance. And, I nede not warne you, how generall, this demonstration is: for if you put in the place of 6 and 4, any other numbers, the like manner of conclusion will follow. So likewise, in place of 1. any other one number may be, as, if A be to C as 6 to 5: and B vnto C , be as 7 to 5: A , shall be to B , as 6 to 7. &c.

A Probleme. 4.

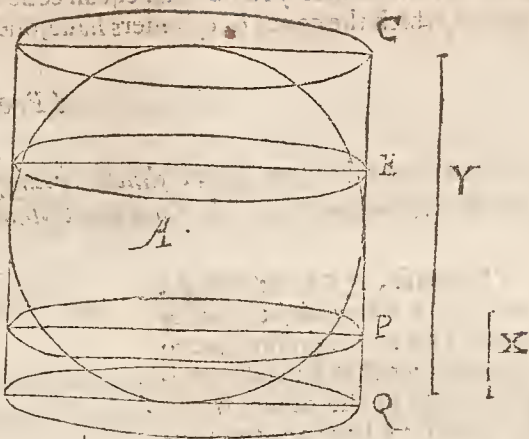
To a Sphere geuen, to make a cylinder equall, or in any proportion geuen betwene two right lines.

Construction.

Suppose the geuen Sphere to be A : and the proportion geuen to be that betwene X and Y . I say that a cylinder is to be made, equall to A : or, els in the same proportion to A , that is betwene X to Y . Let a cylinder be made (such one as the Theoreme next before supposed) that shall haue his base equall to the greatest circle in A , and height equall to the diameter of A : Let that cylinder be the vpright cylinder BC . Let the one side of BC , be the right line QC . Deuide QC into three equal parts: of which, let QE containe two, and let the third part be CE . By the point E suppose a plaine (parallel to the bases of BC) to passe through the cylinder BC , cutting the same by the circle DE . I say that the cylinder BE is equall to the Sphere A . For seing BC , being an vpright cylinder, is cut by a plaine, parallel to his bases, by construction: therefore as the cylinder DC , is to the cylinder BE , so is the axe of DC , to the axe of BE , by the 13. of this twelfth. Wherefore as the axe is to the axe, so is cylinder to cylinder. But axe is to axe, as side to side, namely, CE to QE , because the axe is parallel to any side of an

Demonstration.

of an vpright cylinder: by the definition of a cylinder. And the circle of the section, is parallel to the bases, by construction. Wherefore in the parallelogramme (made of the axe, and of two semidiameters, on one side parallels, one to the other, being coupled together by a line drawn betwene their endes in their circumferences; which line is the side QC) it is euident, that the axe of BC is cut in like proportion, that the side QC is cut. Wherefore the cylinder DC , is to the cylinder BE , as EC is to QE . Wherefore, by composition, the cylinders DC and BE , that is, whole BC , are to the cylinder BE , as CE and QE (the whole right line QC) are to QE . But by construction, QC is of 3. such partes, as QE containeth 2. Wherefore the cylinder BC , is of 3. such partes, as BE containeth 2. Wherefore BC the cylinder, is to BE , as 3. to 2: which is sesquialtera proportion. But by the former Theoreme, BC is sesquialtera to the Sphere A : Wherefore, by the 7. of the fift, BE is equall to A . Therefore to a Sphere geuen, we haue made a cylinder equall.



Or thus more briefly omitting all cutting of the Cylinder.

Forasmuch as BC is an vpright Cylinder: his sides are equall to his axe or heith: therefore the two cylinders, whereof one hath the heith QC and the other the heith QE , hauing both their bases, the greatest circle in the Sphere A , are one to the other as QC is to QE , by the 14. of this twelfth, but QC is to QE , as 3. to 2, by construction: and 3. to 2. is in Sesquialtera proportion: therefore the cylinder BC hauing his heith QC , & his base the greatest circle in A conteyned, is Sesquialtera to the cylinder which hath his base the greatest circle in A conteyned, and heith the line QE . But by the former Theoreme, BC , is also Sesquialtera to A : wherefore the cylinder hauing the base BQ (which by supposition, is equal to the greatest circle in A conteyned) and heith, QE , is equall to the sphere A , by the 7. of the fift. And now it can not be hard, to geue a cylinder, to A , in that proportion, which is betwene X and Y . For let the side QE , be to QP , as Y is to X , by the 12. of the sixt. Therefore backward, QP is to QE as X to Y . Wherefore the cylinder hauing the base the greatest circle in A and heith the line QP , is to the cylinder hauing the same base, and heith the line QE , as X is to Y , by the 14. of this twelfth: but the cylinder hauing the heith QE , & his base the greatest circle in A , conteyned, is proued equall to the Sphere A . Wherefore by the 7. of the fift, the cylinder whose heith is QP and base the greatest circle in A , conteyned, is to the sphere A , as X to Y . Therefore to a sphere geuen: we haue made a cylinder, in any proportion geuen betwene two right lines, and also, before we haue to a sphere geuen, made a cylinder equall. Therefore to a sphere geuen, we haue made a cylinder equall, or in any proportion geuen betwene two right lines.

The second
part of the
Probleme.

A Probleme. 5.

A Sphere being geuen, and a circle, upon that circle as a base, to reere a cylinder, equall to the sphere geuen: or in any proportion geuen betwene two right lines.

A Probleme. 6.

A Sphere being geuen, and a right line, to make a cylinder, equall to the sphere geuen, or in any other proportion, betwene two right lines, geuen.

In this 5. and 6. probleme, first make a cylinder equall to the sphere geuen, by the 4. probleme: and then by the order of the 2. and 3. problemes, in cones, execute these accordingly in cylinders.

A Probleme. 7.

Two unlike Cones or Cylinders, being geuen, to finde two right lines, which haue the same proportion one to the other, that the two geuen cones or cylinders haue one to the other.

Vpon one of their bases reere a cone (if cones be compared) or a cylinder (if cylinders be compared) equall to the other: by the order of the second and third problemes: and the heith of the cone or cylinder,

GGg. iij.

The twelvethe Booke

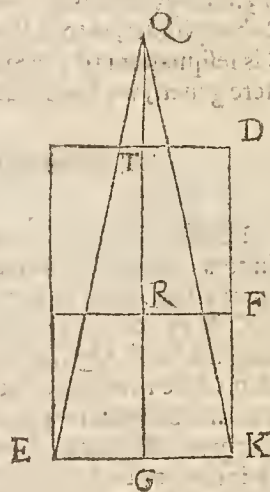
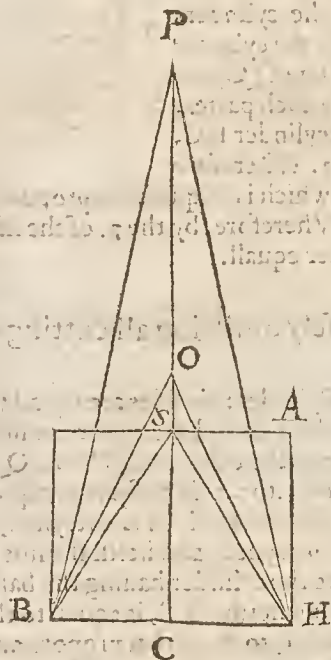
linder, on whose base you rered an equall cone or cylinder, with the new heith found, haue that proportion, which the cones or cylinders haue, one to the other, by the 14. of this twelfth booke.

A Probleme. 8.

An vpright Cone, and Cylinder, being geuen: to finde two right lines, hauing that proportion, the one to the other, which the Cone and Cylinder haue, one to the other.

Construction.

Suppose, QEK an vpright cone and AB an vpright cylinder geuen. I say two right lines are to be geue which shall haue that proportion one to the other, which QEK and AB haue one to the other. Vpon the base BH , erecte a Cone, equall to QEK : by the order of the secod probleme: which let be OBH , and his heith let be OC , and let the heith of AB , (the cylinder), be CS , produce CS to P : so that CP , be tripla to CS , & make perfect the cone PBH . I say that PC and OC haue that proportion, which AB hath to QEK . For, by construction, OBH is equall to QEK : and PBH is equal to AB , as we will proue. (Assumptiue). And PBH , and OBH are vpon one base, namely BH : wherefore by the 14. of this twelfth, PBH and OBH are one to the other as their heithes PC , and OC , are one to the other. Wherefore the



Demonstration.

cylinder and cone equall to PBH and OBH are as PC is to OC : by the 7. of the fifth. But AB the cylinder QEK the cone, are equall to PBH and OBH : by construction: wherefore AB the cylinder, is to QEK the cone, as PC is to OC . Wherefore we haue found two right lines hauing that proportion that a cone and a cylinder geuen, haue one to the other. Which thing we may execute vpon the base of the cone geuen, as we did vpon the base of the cylinder geuen, on this maner. Vpon the base of the cone QEK , which base, let be EK , erect a cylinder, equall to AB , by the order of my second probleme. Which cylinder let be ED , and GT his heith, and let the heith of the cone QEK , be QG . Take the line GR , the third part of QG , (by the 9. of the sixth): and with a playne passing by R , parallel to EK , cut of the cylinder EF : which shall be equal to the cone QEK , by the assumpt following: I say now, that AB , the cylinder, is to QEK the cone, as GT is to GR . For the cylinder ED is to the cylinder EF , as GT is to GR , by the 14. of the twelfth: and to ED is the cylinder AB equall: by construction: and to EF , we haue proued the cone QEK , equall, wherefore by the 7. of the fifth, AB is to QEK , as GT is to GR . Wherefore an vpright cone, & a cylinder being geuen, we haue found two right lines hauing the same proportion betwene them, which the cone and the cylinder, haue one to the other: which was requisite to be done.

An other way of executing this probleme.

An assumpt.

If a cone, and a cylinder, being both on one base, are equall one to the other: the heith of the cone is tripla to the heith of the cylinder. And if a cone and a cylinder being both on one base, the heith of the cone be tripla to the heith of the cylinder, the cone and the cylinder are equall.

We will vse the cylinder AB & the cone PBH in the former probleme: with their base & heithes so noted as before. I say if PBH be equall to AB , that CP the heith of PBH , is tripla to CS the heith of the cylinder AB . Suppose vpon the base BH , a cone to be rered of the heith of CS , which let be SBH : it is manifest that AB is tripla to that cone SBH , by the 10. of this twelfth. Wherefore a cone equal to AB the cylinder is tripla to SBH the cone, by the 7. of the fifth, but PBH is supposed equall to AB . Therefore PBH is tripla to SBH , therefore the heith of PBH shall be tripla to the heith of SBH by the 14. of the twelfth. But the heith of PBH is CP : and of SBH , the heith is CS : wherefore CP is tripla to CS . And CS is the heith of the cylinder AB by supposition. Therefore a cone and a cylinder, being

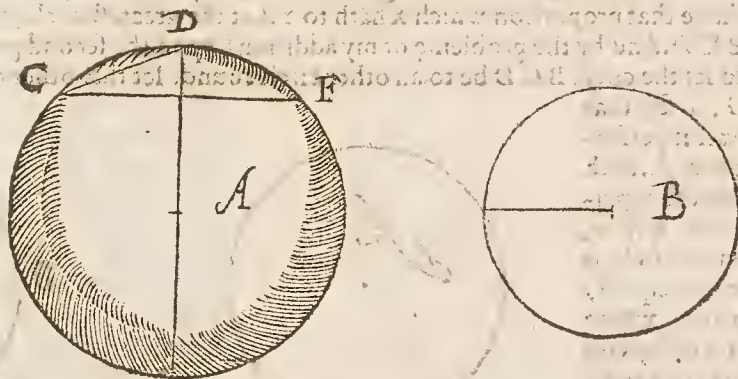
being both on one base, and equall, the heith of the cone is tripla to the heith of the cylinder. And the second part, as easely may be confirmed. For if AB a cylinder, and PBH a cone, haue one base, as the circle about BH : and the heith of PBH be tripla to the heith of AB , I say that PBH , and AB are equal. The heith of AB let be (as afore) CS : and of PBH , the heith, let be CP : of the heith CS , imagine a Cone vpon the same base BH : by the 10. of this twelfth, AB shall be tripla to that cone. And the cone PBH hauing heith CP , (by supposition) tripla to CS , shall also be tripla to that cone SBH : by the 14. of this twelfth. Wherefore by the 7. of the fifth AB and PBH are equall. Therefore, if a cone and a cylinder being both on one base, the heith of the cone be tripla to the heith of the cylinder, the cone and the cylinder are equall. So haue we demonstrated both partes: as was required.

The conuerse of the assupt.

¶ A Theoreme. 4.

The superficies of the segment or portion of any sphere, is equall to the circle, whose semidiameter, is equall to that right line which is drawne from the toppe of that segment to the circumference of the circle, which is the base of that portion or segment.

As in the Sphere A , a Segment being cut off by the circle, whose diameter is CE : & the same circle being the base of the Segment, whose top also is D : the croked superficies sphericall of the same Segment, is equall to a circle whose Semidiameter is equall to the right line DC . As is the circle B .



This hath Archimedes demonstrated in this first booke of the Sphere and Cylinder, in his 40. and 41. propositions: and I remitte them thether, that will herein demonstratiuely be certified: I would with all Mathematiciens, as well of verities easy, as of verities rare and obscure, to seeke the causes demonstratiue, the finall fruite thereof, is perfection in this art.

* Note.

Besides all other vses and commodities, that are of the Croked superficieses of the Cone, Cylinder, and Sphere, so easely and certaynely, of vs to be dealt with all: this is not the least, that a notable Error, which among Sophisticall brablers, and vngeometricall Masters and Doctors, hath a long time bene vpholden: may most euidently, hereby be confuted, and vtterly rooted out of all mens fantasies for euer. The Error is this, *Curui ad rectum, nulla est proportio*, that is: Betwene croked and straight, is no proportion. This error, in lines, superficieses, and solides, may with more true demonstrations be overthrowne, then the fauourers of that fond fantasie are able, with argument, either probable or Sophisticall to make shew or pretence to the contrary. In lines, I omitte (as now) Archimedes two wayes, for the finding of the proportion of the circles circumference to a straight line. I meane, by the inscription and circumscription of like polygon figures, and that other, by spirall lines. And I omitte likewise (as now) in solides, of a parallelipipidon, equall to a Sphere, Cone, or Cylinder: or any segment or sector of the sayd solides. And onely, here require you to consider in this twelfth booke, the wayes brought to your knowledge, how to the croked superficieses of a cone and cylinder, and of a sphere, (the whole, any segment or sector thereof) a playne and straight superficies may be geuen equall: Namely, a Circle to be geuen equall, to any of the sayd croked superficieses assigned, and geuen. And then farther by my Additions vpon the second proposition, you haue meanes to proceede in all proportions, that any man can in right lines geue, or assigne. Therefore, *Curui ad rectum, proportio omnimoda potest dari*. One thing it is, to demonstrate, that betwene a croked line and a straight: or a croked superficies and a playne or straight superficies, &c. there is proportion. And an other thing it is, to demonstrate a particular and speciall kinde of proportion, being betwene a croked superficies and a straight or playne superficies. For this also confirmeth the first. This short warning will cause you to auoyde the sayd error, and make you also hable to cure them, which are infected therewith.

A great error commonly maintained.

” Betwene
” straight
” and croked
” all maner
” of propor-
” tio may be
” geuen.

* A Theoreme. 5.

Any two Spheres being geuen, as the Sphericall Superficies of the one, is to the Sphericall Superficies

The twelvetb Booke

ficies of the other: So is the greatest Circle conteyned in that one, to the greatest Circle conteyned in that other. And as greatest Circle, is to greatest Circle, so is Sphericall superficies, to Sphericall superficies.

For the superficies of every sphere is quadrupla to his greatest circle, by my first Theoreme; wherefore, of two geuen spheres, as the sphericall superficies of the one, is to his greatest circle, so is the sphericall superficies of the other, to his greatest circle: therefore by alternate proportion, as sphericall superficies is to sphericall superficies, so is greatest circle to greatest circle. And therefore also as greatest circle, is to greatest circle, so is sphericall superficies, to sphericall superficies: which was to be demonstrated.

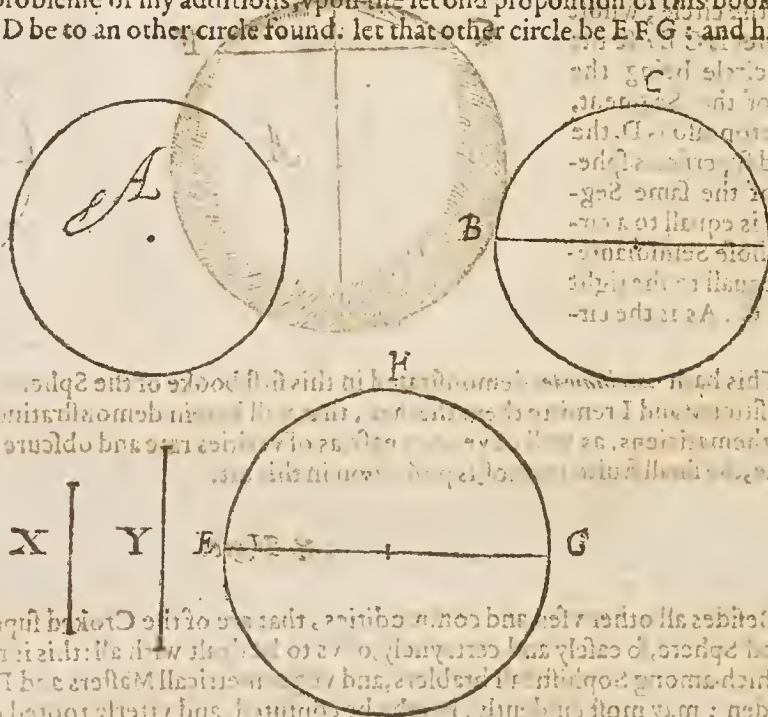
A Probleme. 9.

A Sphere being geuen, to geue an other Sphere, to whose Sphericall superficies, the superficies Sphericall of the Sphere geuen shall haue any proportion, betwene two right lines geuen.

Construction.

Demonstration.

Suppose A, to be a sphere geuen, and the proportion geuen, to be that, which is betwene the right lines X and Y. I say that a sphere is to be geuen to whose sphericall superficies, the superficies sphericall of A, shall haue that proportion which X hath to Y. Let the greatest circle, conteyned in A the sphere be the circle B C D. And by the probleme of my additions, vpon the second proposition of this booke, as X is to Y, so let the circle B C D be to an other circle found: let that other circle be E F G: and his diameter E G. I say that the sphericall superficies of the sphere A, hath to the sphericall superficies of the sphere, whose greatest circle is E F G, (or his equall) that proportion, which X hath to Y. For (by construction) B C D is to E F G, as X is to Y: and by the theoreme next before, as B C D is to E F G, so is the sphericall superficies of A (whose greatest circle is B C D, by supposition) to the sphericall superficies, of the sphere, whose greatest circle is E F G: wherefore, by the 11. of the fifth as X is to Y: So is the sphericall superficies of A, to the sphericall superficies of the sphere, whose greatest circle is E F G: wherefore, the sphere whose diameter is E G, (the diameter also of E F G) is the sphere, to whose sphericall superficies, the sphericall superficies of the sphere A, hath that proportion which X hath to Y. A sphere being geuen therefore, we haue geuen an other sphere, to whose sphericall superficies, the superficies sphericall of the sphere geuen hath any proportion geuen, betwene two right lines: which ought to be done.



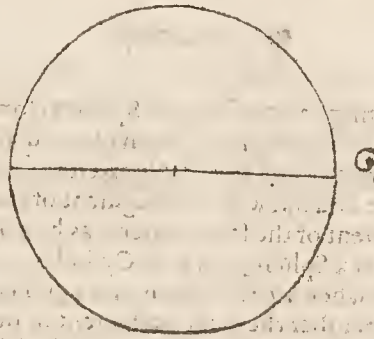
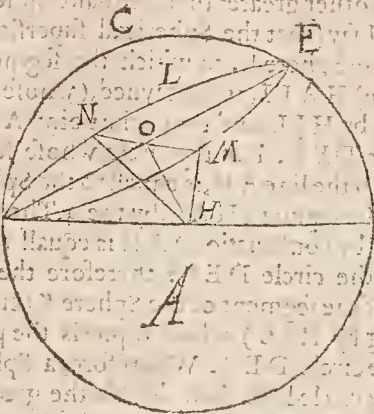
A Probleme. 10.

A Sphere being geuen, and a Circle lesse then the greatest Circle, in the same Sphere conteyned, to coapt in the Sphere geuen, a Circle equall to the Circle geuen.

The diffinitio of a circle coapted in a sphere. Construction.

Suppose A, to be the sphere geuen: and the circle geuen lesse then the greatest circle in A conteyned, to be F K G. I say, that in the Sphere A, a circle, equall to the circle F K G, is to be coapted: First vnderstand, what we meane here, by coapting of a circle in a Sphere. We say, that circle to be coapted in a Sphere, whose whole circumferéce is in the superficies of the same Sphere. Let the greatest circle in the Sphere A conteyned, be the circle B C D. Whose diameter suppose to be B D, and of the circle F K G, let F G be the diameter. By the 1. of the fourth, let a line equall to F G, be coapted in the circle B C D. Which line coapted, let be B E. And by the line B E, suppose a playne to passe, cutting the Sphere A, and to be

to be perpendicularly erected to the superficies of $B C D$. Seing that the portion of the playne remayning in the sphere, is called their common section: the sayd section shall be a circle, as before is proved. And the common section of the sayd playne, and the greatest circle $B C D$, (which is $B E$ by supposition) shall be the diameter of the same circle, as we will proue. For, let that circle be $B L E M$. Let the center of the sphere A , be the point H : which H , is also the ceter of the circle $B C D$, because $B C D$ is the greatest circle in A conteyned. From H , the center of the sphere A , let a line perpendicularly be let fall to the circle $B L E M$. Let that line be $H O$: and it is euident that $H O$ shall fall vpon the common section $B E$, by the 38. of the eleuenth. And it deuideth $B E$, into two equall parts, by the second part of the third proposition of the third booke: by which poynt O , all other lines drawne in the circle $B L E M$, are, at the same poynt O , deuided into two equall parts. As if from the poynt M , by the poynt O , a right line be drawne one the other side coming to the circumference, at the poynt N : it is manifest that $N O M$ is deuided into two equall partes at the poynt O : by reason, if from the center H , to the poyntes N and M , right lines be drawne, $H N$ and $H M$, the squares of $H M$, and $H N$ are equall: for that all the semidiameters of the sphere are equal: and therefore their squares are equall one to the other: and the square of the perpendicular $H O$, is common: wherefore the square of the third line $M O$ is equall to the square of the third line $N O$: and therefore the line $M O$ to the line $N O$. So therefore is $N M$ equally deuided at the poynt O . And so may be proued of all other right lines, drawne in the circle $B L E M$, passing by the poynt O , to the circumference one both sides. Wherefore O is the center of the circle $B L E M$: and therefore $B E$ passing by the poynt O is the diameter of the circle $B L E M$. Which circle (I say) is equal to $F K G$: for by construction $B E$ is equall to $F G$: and $B E$ is proued the diameter of $B L E M$, and $F G$ is by supposition the diameter of the circle $F K G$: wherefore $B L E M$ is equall to $F K G$ the circle geuen: and $B L E M$ is in A the sphere geuen. Wherefore we haue in a sphere geuen coapted a circle equal to a circle geuen: which was to be done.



Demonstration.

This is manifest: if you consider the two triangles right-angles, $H O M$ and $H O N$: and then with all, vse the 47. of the first of Euclide.

* A Corollary.

Besides our principall purpose, in this Probleme, euidently demonstrated, this is also made manifest: that if the greatest circle in a Sphere, be cut by an other circle, erected vpon him at right angles, that the other circle is cut by the center, and that their common section is the diameter of that other circle: and therefore that other circle deuided is into two equall partes.

* A Probleme. II.

A Sphere being geuen, and a circle, lesse then double the greatest circle in the same Sphere contained, to cut of, a segment of the same Sphere, whose Sphericall superficies, shall be equall to the circle geuen.

Suppose K to be a Sphere geuen, whose greatest circle let be $A B C$: and the circle geuen, suppose to be $D E F$. I say, that a segment of the Sphere K , is to be cut of, so great, that his Sphericall superficies, shall be equall to the circle $D E F$. Let the diameter of the circle $A B C$, be the line $A B$. At the point A , in the circle $A B C$, coapt a right line equall to the semidiameter of the circle $D E F$ (by the first of the fourth). Which line suppose to be $A H$. From the point H , to the diameter $A B$, let a perpendicular line be drawn: which suppose to be $H I$. Produce $H I$ to the other side of the circumference, and let it come to the circumference at the point L . By the right line $H I L$ (perpendicular to $A B$) suppose a plaine superficies to passe, perpendicularly erected vpon the circle $A B C$: and by this

Construction.

HHh.

playne

The twelue th Booke

plaine superficies, the Sphere to be cut into two segmentes : one lesse then the halfe Sphere, namely, H A - L I : and the other greater then the halfe Sphere, namely, H B L I. I say, that the Sphericall superficies of the segment of the Sphere K, in which the segment (of the greatest circle) H A L I, is containd, (whose base is the circle passing by H I L, and toppe the point A) is equall to the circle D E F. For the circle, whose semidiameter is equall to the line A H, is equall to the Sphericall superficies of the segment H A L, by the 4. Theoreme here added. And (by construction) A H is equall to the semidiameter of the circle D E F : therefore the Sphericall superficies of the segment of the Sphere K (cut of by the circle passing by H I L) whose toppe is the point A, is equall to the circle D E F. Wherefore, a Sphere being geuen, and a circle lesse then double the greatest circle in the same Sphere, we haue cut of, a segment of the same Sphere, whose Sphericall superficies, is equall to the circle geuen : which was requisite to be done.

Demonstratio

¶ *An aduise.*

In noting or signifying of Spheres, sometimes we vse by one and the same circle, in plaine designed, to represent a Sphere and also the greatest circle in the same contained : and likewise, by a segment of that circle, signifie a segment of the same Sphere, as by a straight line, we often signifie the circle, which is the base of a segment of a Sphere, Cone, or Cylinder : and so in such like. Wherin, consider our suppositions : and take heede when we shift from one signification to an other, in one and the same designation : and withall remember the principall intent of our drift : and such light thinges, can not either trouble or offend thee. Compendiousnes and artificiall custome, procureth such meanes : sufficient, to stirre vp imagination Mathematicall : or to informe the practiser Mechanicall.

¶ *A Probleme. 12.*

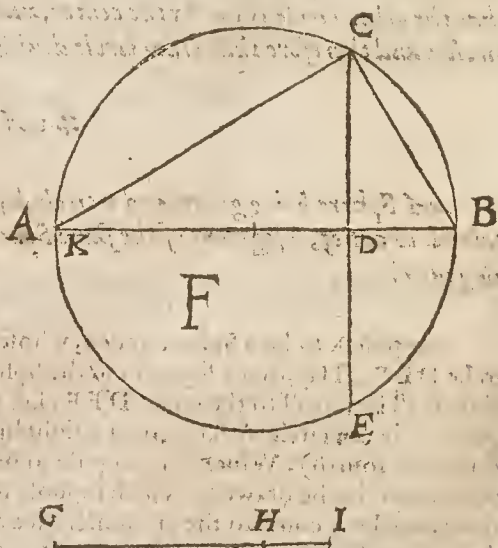
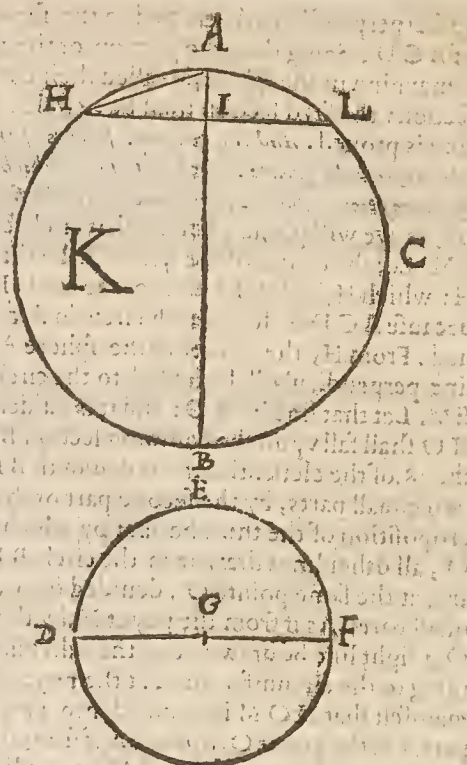
To cut a Sphere geuen, into two such segmentes, that the Sphericall superficieses of the segmentes shall haue one to the other, any proportion geuen betwene two right lines.

Constructio.

Suppose F to be a Sphere geuen : and the proportion geuen, to be that, which is betwene G H and H I. I say, that the Sphere F, is to be cut into two such segmentes, that the Sphericall superficieses of those segmentes, shall haue that proportion, one to the other, which, the right line G H, hath to the right line H I. Suppose A B C E to be a greatest circle, in the Sphere F, contained : and his diameter, to be A B. Deuide A B into two such partes, as G I is diuided into, in the point H (by the 10. of the sixth) Let those partes be A D, and D B. So that, as G H is to H I, so is A D to D B. By the point D, let a plaine superficies passe, cutting the Sphere F, and the diameter A B : So, that vnto that cutting plaine, the diameter A B, be perpendicular : and the Sphere also thereby deuided into two segmentes, whose comon base suppose to be the circle C E, hauing the center, the point D : and the toppe of the one to be the point A, and the toppe of the other to be the point B : and the segmentes them selues, to be noted by E A C, and E B C : Drawe from the two toppes, A and B, to C (a point in the circumference of their common base) two right lines A C and B C : I say now, that the Sphericall superficieses of the segment E A C, hath to the Sphericall superficieses of the segment E B C, the same proportion, which G H hath to H I. For, forasmuch as circles haue that proportion, one to the other, that the squares of their diameters haue one to the other (by the 2. of this twelfth). And the squares of the semidiameters, haue the same proportion one to the other, which the squares of the diameters haue. [For like partes haue

Demonstratio

This in maner of a Lemma, is presently proved.



that

that proportion one to the other, that the whole magnitudes, whose like partes they are, haue the one to the other: by the 15. of the fift. But the square of euery diameter is quadruple to the square of his semidiameter: as hath often before, bene proued: therefore, circles haue one to another, that proportion, that the squares of their semidiameters haue one to the other] . Wherefore, seing A C and B C are semidiameters of two circles, whereof eche is equall to the Sphericall superficies of the segmentes, betwene whose toppes and circumference of their base, they are drawn: by the 4. Theoreme of these additions: it followeth that both those circles, whose semidiameters they are: and also those Sphericall superficieses, which are equall to those circles, haue the one to the other, the same proportion, which the square of A C hath to the square of B C. But A C is drawn betwene the circumference of the base, and toppe of the segment Sphericall, E A C, by construction: and likewise B C is drawn betwene the toppe, and circumference of the base, of the Sphericall segment E B C, by construction: Wherefore the Sphericall superficies of the segment E A C, is to the Sphericall superficies of the segment E B C, as the square of A C is to the square of B C. But the square of A C is to the square of B C, as A D is to D B: by the Corollary of the Probleme of my additions vpon the second of this twelfth: And A D is to D B, as G H is to H I: by construction. Wherefore the Sphericall superficies of the segment E A C, is to the Sphericall superficies of the segment E B C, as G H is to H I. We haue therefore, cut the Sphere geuen, into two such segmentes, that the Sphericall superficieses of the segmentes, haue one to the other any proportion geuen betwene two right lines: which was to be done.

¶ A Corollary. 1.

Here it appeareth demonstrated, that, circles are one to the other, as the squares of their semidiameters are, one to the other.

Wherby (as occasion shall serue) you may, by force of the former argument, vse other like partes of the diameter, as well as halues.

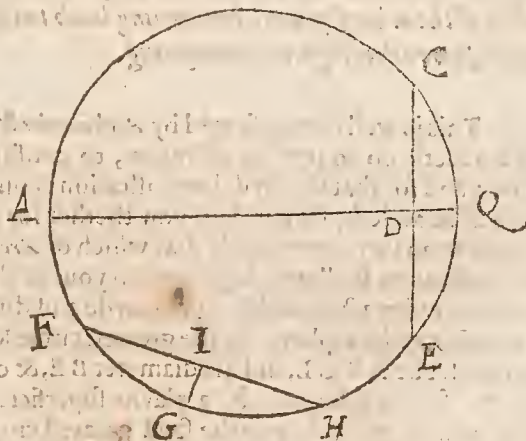
¶ A Corollary. 2.

It is also euident, that the Sphericall superficieses of the two segmentes of any Sphere, to whose common base, the diameter (passing to their two toppes.) is perpendicular, haue that proportion the one to the other, that the portions of the sayd diameter, haue the one to the other: that superficies and that portion of the diameter, on the one side of the common base, being compared to that superficies, and that portion of the diameter, on the other side of the common base.

¶ A Corollary. 3.

It likewise euidently followeth, that the two Sphericall superficieses of two segmentes of a Sphere: which two segmentes are equall to the Sphere, are in that proportion the one to the other, that their axes (perpendicularly erected to their bases) are in, one to the other: where soeuer in the Sphere those segmentes be taken.

I say that the Sphericall superficies of the segment C A E, and the Sphericall superficies of the segment F G H, hauing their axes A D and G I (perpendicular to their bases) : are in proportion one to the other, as A D is to G I: if the segment of the Sphere containing C A E with (the segment of the same Sphere) F G H, be equall to the whole Sphere. For seing the diameter (or axe) A D, extended to the other pole or toppe, opposite to A (which opposite toppe, let be Q) doth make with the segment C A E, the complement of the whole Sphere: and by supposition, the segment F G H, with the segment C A E, are equall to the whole Sphere: Wherefore from equall, taking C A E (the segment common) remaineth the segment C Q E, equall to the segment F G H. And thereby, Axe, Base, Solitie, and superficies Sphericall of the segment F G H, must (of necessitie) be equall to



Note here of
Axe, base, &
Soliditie, more
then I neede to
bring any fur
ther prooffe
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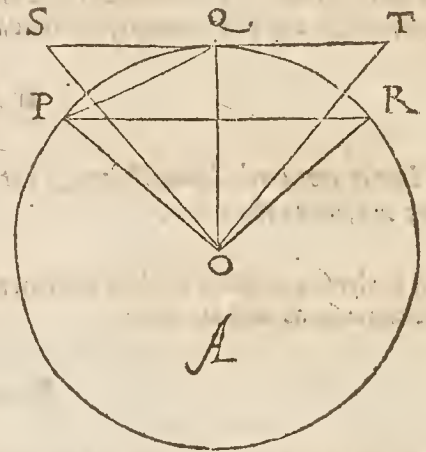
The twelueth Booke

the Axe, Base, Soliditie, and superficies Sphericall of the segment C Q E : Wherefore, by the second Corollary here, and the 7. of the fift, our conclusion is inferred, the superficies Sphericall, of the segment C A E, to be, to the superficies Sphericall of the segment F G H, as A D is to G I.

A Theoreme. 6.

To any solide sector of a Sphere, that vpright Cone is equall, whose base is equall to the conuex Sphericall superficies of that sector, and heith equall to the semidiameter of the same Sphere.

Hereof the demonstration in respect of the premises : and the common argument of inscription and circumscription of figures is easy : and neuerthelesse, if your owne witte will not helpe you sufficiently : you may take helpe at *Archimedes* hand, in his first booke & last proposition of the sphere and cylinder. Whether if ye haue recourse, you shall perceauie how your Theoreme here amendeth the common translation there : and also our delineation geueth more liuely shew of the chiefe circumstances necessary to the construction, then there you shall finde. Of the sphere here imagined to be A, we note a solide sector by the letters P Q R O. So that P Q R doth signifie the sphericall superficies, to that solide sector belonging : (which is also common to the segment of the same sphere I' R Q) and therefore a line drawne from the toppe of that segment; (which toppe suppose to be Q,) is the semidiameter of the circle, which is equall to the sphericall superficies of the sayd solide sector, or segment : as before is taught. Let that line be Q P. By Q draw a line contingent : which let be S Q T. At the poynt Q from the line Q S, cut a line equall to P Q which let be S Q. And vnto S Q, make Q T equall, then draw the right lines O S O T and O Q. About which O Q (as an axe fastened) if you imagine the triangle O S T, to make an



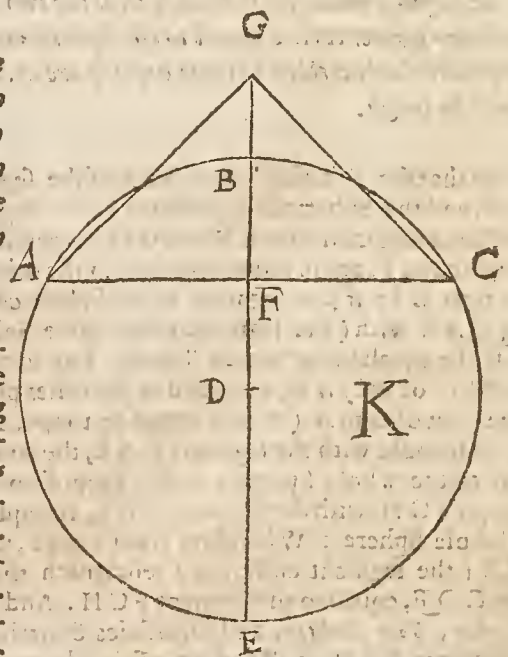
Note.

* I say halfe a circular reuolution: for that sufficeth in the whole diameter ST, to describe a circle by: if it be moued about his center Q &c.

halfe circular reuolution, you shall haue the vpright cone O S T : (whose heith is O Q, the semidiameter of the sphere, and base the circle, whose diameter is S T,) equall to the solide sector P Q R O.

A Theoreme. 7.

To any segment, or portion of a Sphere, that cone is equall, which hath that circle to his base, which is the base of the segmēt, and heith, a right line, which vnto the heith of the segmēt hath that proportiō, which the semidiameter of the Sphere, together with the heith of the other segment remayning hath to the heith of the same other segment remayning.



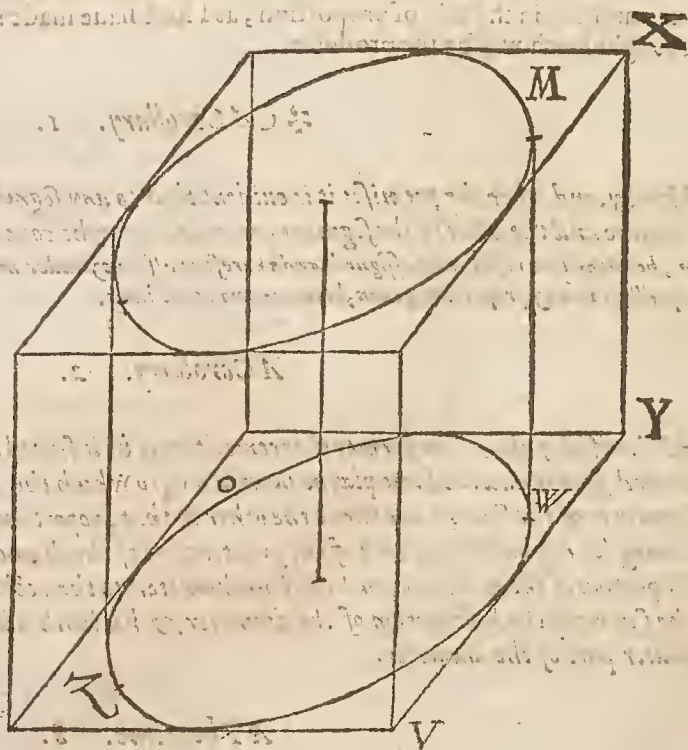
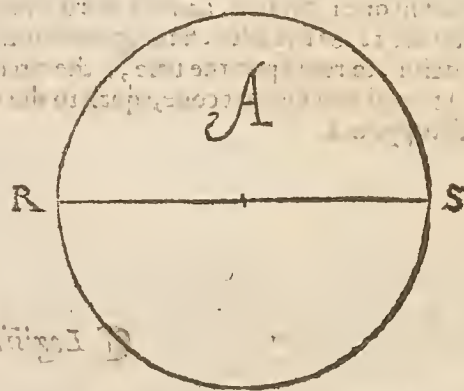
This is well demonstrated by *Archimedes* & therefore nedeth no inuention of myne, to confirme the same : and for that the sayd demonstration is ouer long here to be added, I will refere you thether for the demonstration : and here supply that which to *Archimedes* demonstration shall geue light, and to your farther speculation and practise, shal be a great ayde and direction. Suppose K to be a sphere : & the greatest circle K in cōteyned, let be A B C E, and his diameter B E, & ceter D. Let the sphere K, be cutte by a playne superficies, perpendicularly erected vpon the sayd greatest circle A B C E : let the section be the circle about A C : And let the

Lib. 2. prop. 2.
de Sphæra &
Cylindro.

The twelueeth Booke

As vpon the first and second propositions of this booke, I began my additions with the circle (being the chiefe among playne figures) and therein brought manifold considerations, about circles: as of the proportion betwene their circumferences and their diameters: of the content or Area of circles: of the proportion of circles to the squares described of their diameters: & of circles to be geuen in al proportions, to other circles: with diuerse other most necessary problemes (whose vse is partly there specified): So haue I in the end of this booke, added some such Problemes & Theoremes, about the sphere (being among solides the chiefe) as of the same, either in it selfe considered, or to cone and cylinder, compared (by reason of superficies, or soliditie, in the hole, or in part:) such certaine knowledge demonstration may arise, and such mechanical exercise thereby be deuised, that (sure I am) to the sincere & true student great light, ayde, and comfortable courage (farther to wade) will enter into his harr: and to the Mechanicall, witty, and industrious deuiser, new maner of inuentions, & executions in his workes will (with small trauayle for fete application) come to his perceiuraunce and vnderstanding. Therefore, euen as manifolde speculations & practises may be had with the circle, his quantitie being not knowne in any kinde of smallest certayne measure: So likewise of the sphere many Problemes may be executed and his precise quantitie, in certayne measure, not determined, or knowne: yet, because, both one of the first (humane) occasiōs of inuenting and stablishing this Arte, was measuring of the earth (and therefore called Geometria, that is, Earthmeasuring), and also the chiefe and generall end (in deede) is measure: and measure requireth a determination of quantitie in a certayne measure by nūber expressed: It was nedefull for Mechanicall earthmeasures, not to be ignorant of the measure and contents of the circle, neither of the sphere his measure and quantitie, as neere as sense can imagine or wish. And (in very deede) the quantitie and measure of the circle, being knowne, maketh not onely, the cone and cylinder, but also the sphere his quantitie to be as precisely knowne, and certayne. Therefore seing in respect of the circles quantitie (by *Archimedes* specified) this Theoreme is noted vnto you: I wil, by order, vpon that (as a supposition) inferre the conclusion of this our Theoremes.

Suppose a sphere to be signified by A: whose diameter let be, RS. To RS, let a line equall be taken, which let be TV: of TV, by the 46. of the first, describe a square. Let that square be TY. With in TY let a circle be inscribed: by the 2. of the fourth, which circle suppose to be OZW. That OZW is equall to the greatest circle in the sphere A conteyned, it is euident by the diameter, equal to TV: if vpon the square TY, as a base; be erected, a parallelepipedō rectāgle, whose heith is equall to TV, it is euident that that parallelepipedon is a cube. Which let be done: and that cube produced, let be noted by TX. Likewise, if vpon the circle OZW, as a base, and of the heith equall to the line TV, a cylinder be erected, it is manifest that the cylinder hath his base equall to the greatest circle, in the sphere A, conteined: & heith, a line equall to the diameter of the same sphere A. Which cylinder let be produced and noted by ZM. I say now that the sphere A, is to the cube TX, (in manner) as the number 11. is to the number 21. For seing the cube TX, was produced of his base, (the square TY), being brought into the heith of a line equall to TV: & likewise seing the cylinder ZM,



is produced, of his base (the circle O Z W) being brought into a line, equal to the said T V: it followeth, seeing their heithes is all one, that the cube T X shall be to the cylinder Z M, as the base of T X (which is the square T Y) is to the base of Z M, that is the circle O Z W. But the square T Y, is to the circle O Z W, as the number 14, is to the number 11. (in maner), by Archimedes demonstratio: wherefore, the cube T X is to the cylinder Z M, as the number 14, is to the number 11. (well nere). And by my third Theoreme (here added) the cylinder Z M, is to the Sphere A, in lesquialtera proportion: that is, as 3, to 2. Wherefore the cylinder Z M, hauing the same 11. equall partes (which he conteyneth in respect of the cube T X, being 14, of the same partes) deuided into 3. equall portions, euery one of those portions is $3\frac{2}{3}$. And allowing to the Sphere A, two of those portions: it is euident, that the Sphere A shall be $7\frac{1}{3}$ such partes as are 14. in the cube T X: and 11. in the cylinder Z M. Wherefore the Sphere A, is to the cube T X, as $7\frac{1}{3}$ to 14. The fraction being reduced, maketh $\frac{22}{21}$; and the number 14. being brought to the same name, and denomination of thirds, maketh $\frac{42}{21}$. Put away now theyr common denominator: and then remayneth, for the Sphere A, 22. such partes, as the cube T X hath 42. And then depresseing them, to the smallest termes: for the Sphere A, you shall haue 11. such partes as the cube T X conteyneth 21. Wherefore euery Sphere, to the cube made of his diameter is, as 11. to 21. which was requisite to be demonstrated.

Note.

Wherefore if you deuide the one side (as T Q) of the cube T X into 21. equall partes, and where 11. partes do end, reckening from T, suppose the point P: and by that point P, imagine a plaine (passing parallel to the opposite bases) to cut the cube T X: and therby, the cube T X, to be deuided into two rectangle parallelipedons, namely, T N, and P X: It is manifest, * T N, to be equall to the Sphere A, by construction: and the 7. of the fift.

Note.

Secondly, the whole quantitie, of the Sphere A, being contained in the rectangle parallelipedon T N, you may easilie transforme the same quantitie, into other parallelipedons rectangles, of what height, and of what parallelogramme base you list: by my first and second Problemes vpon the 34. of this booke. And the like may you do, to any assigned part of the Sphere A: by the like meanes deuiding the parallelipedon T N: as the part assigned doth require. As if a third, fourth, fifth, or sixth, part of the Sphere A, were to be had in a parallelipedon, of any parallelogramme base assigned, or of any heith assigned: then deuiding T P, into so many partes (as into 4. if a fourth part be, to be transformed: or into five, if a fifth part, be to be transformed &c.) and then proceede, as you did with cutting of T N, from T X. And that I say of parallelipedons, may in like sort (by my sayd two problemes, added to the 34. of this booke) be done in any sided columnes, pyramids, and prismes: so that in pyramids and some prismes you vse the cautions necessary, in respect of their quantites, compared to the quantites of bodyes hauing parallel, equall, and opposite bases: whose partes those pyramids or prismes are: as before in their propositions, is by Euclide demonstrated. And finally, seeing, in these present additions, you haue the wayes and orders how to geue to a Sphere, or any segment of the same, Cones, or Cylinders equall, or in any proportion betwene two right lines, geuen: with many other most necessary speculations and practises about the Sphere: I trust that I haue sufficiently fraughted your imagination, for your honest and profitable studie herein, and also geuen you ready matter, wherewith to stop the mouthes of the malycious, ignorant,

HHh.iiij.

* A rectangle parallelipedon geuen, equall to a Sphere geuen;

To a Sphere, or to any part of a Sphere assigned: as a third, fourth, fifth &c. to geue a parallelipedon equall.
Sided Columnes
Pyramids, and prismes to be geuen equall to a Sphere, or to any certayne part thereof.
To a Sphere or any segment, or sector of the same, to geue a cone or cylinder equall or in any proportion assigned.

Farther use
of Sphericall
Geometrie.

rant, and arrogant, despisers of the most excellent discourses, traauayles, and inuentions mathematicall. Seing aswel the heauenly spheres, & sterres their sphericall soliditie, with their conuexe spherical superficies, to the earth at all times respecting, and their distances from the earth, as also the whole earthly Sphere and globe it selfe, and infinite other cases, concerning Spheres or globes, may hereby with as much ease and certainty be determined of, as of the quantitie of any bowle, ball, or bullet, which we may gripe in our handes (reason, and experience, being our witnesses): and without these aydes, such thinges of importance neuer hablé of ys, certainly to be knowne, or attained vnto.

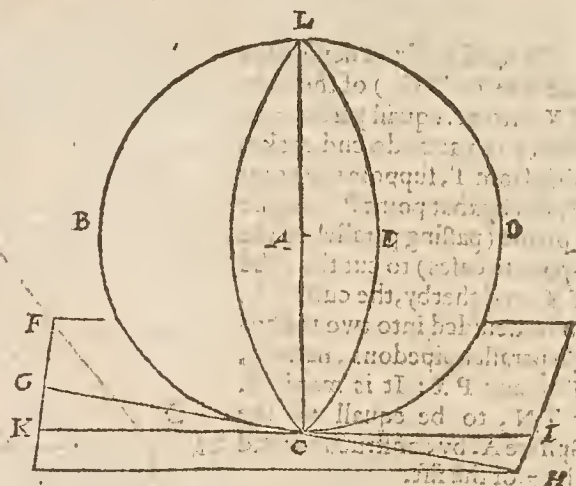
Here ende *M. Iohn Dee* his additions vpon
the last proposition of the twelfth booke.

A proposition added by *Flusbas*.

If a Sphere touche a playne superficies, a right line drawne from the center to the touche, shall be erected perpendicularly to the playne superficies.

Suppose that there be a Sphere B C D L: whose centre let be the poynt A. And let the playne superficies G C I touch the Spere in the poynt C, and extend a right line from the centre A to the poynt

C. Then I say that the line A C is erected perpendicularly to the playne G C I. Let the sphere be cutte by playne superficieses passing by the right line L A C: which playnes let be A B C D L and A C E L, which let cut the playne G C I by the right lines G C H and K C I. Now it is manifest (by the assumpt put before the 17. of this booke) that the two sections of the sphere shall be circles, hauing to their diameter the line L A C, which is also the diameter of the sphere. Wherefore the right lines G C H and K C I which are drawne in the playne G C I, do at the poynt C fall without the circles B C D L and E C L. Wherefore they touch the circles in the poynt C, by the second definition of the third. Wherefore the right line L A C maketh right angles with the lines G C H and K C I by the 16. of the third. Wherefore by the 4. of the eleuenth the right line A C is erected perpendicularly to the playne superficieses G C I wherein are drawne the lines G C H and K C I. If therefore a Sphere touch a playne superficies, a right line drawne from the centre to the touche, shall be erected perpendicularly to the playne superficies: which was required to be proued.



The ende of the twelfth booke
of Euclides Elementes.



The thirteenth booke of Euclides Elementes.



IN THIS THIRTEENTH BOOKE are set forth certayne most wonderfull and excellent passions of a lyne deuided by an extreme and meane proportion: a matter vndoubtedly of great and infinite vse in Geometry, as ye shall both in thys booke, and in the other bookes following most euidently perceauce. It teacheth moreouer the composition of the fiue regular solides, and how to inscribe them in a Sphere geuen, and also setteth forth certayne comparisons of the sayd bodyes both the one to the other, and also to the Sphere, wherein they are described.

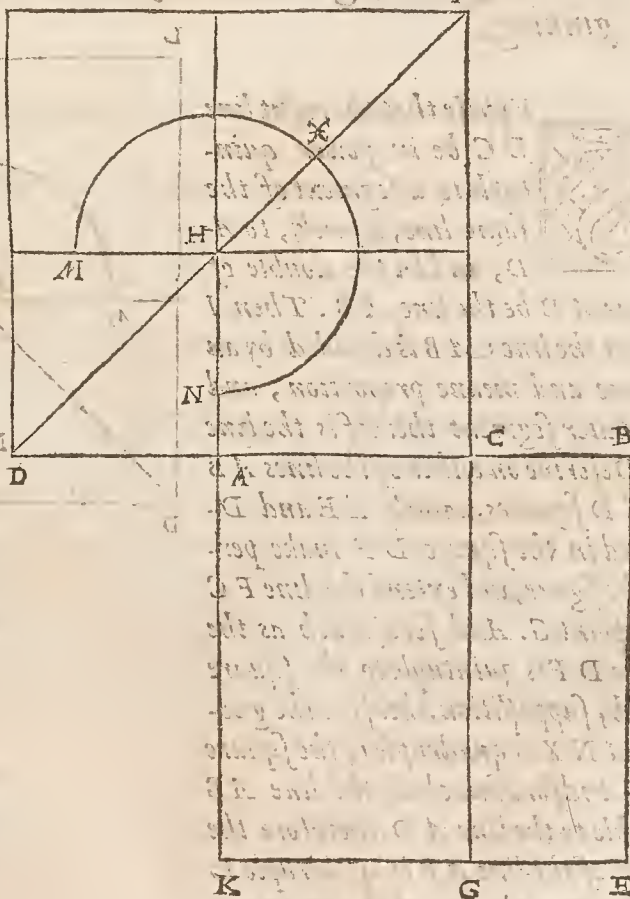
*The argument
of the thir-
teenth booke.*

The 1. Theoreme. The 1. Proposition.

If a right line be deuided by an extreme and meane proportion, and to the greater segment, be added the halfe of the whole line: the square made of those two lines added together shalbe quintuple to the square made of the halfe of the whole lyne.



Suppose that the right line AB be deuided by an extreme and meane proportion in the point C . And let the greater segment thereof, be AC . And vnto AC , adde directly a right line AD , and let AD be equall to the halfe of the line AB . Then I say that the square of the line CD is quintuple to the square of the line DA . Describe (by the 46. of the first) vpon the lines AB and DC squares, namely, AE & DF . And in the square DF , describe and make complete the figure. And extend the line FC , to the point G . And forasmuch as the line AB is deuided by an extreme and meane proportion in the point C , therefore that which is containd vnder the lines AB and BC is equall to the square of the line AC . But that which is containd vnder the lines AB and BC , is the parallelogramme CE , and the square of



Construction.

*Demonstra-
tion.*

The thirteenth Booke

the line AC is the square HF . Wherefore the parallelogramme CE is equall to the square HF . And forasmuch as the line BA , is double to the line AD , by construction: but the lyne BA is equall to the line KA , and the line AD , to the lyne AH : therefore also, the lyne KA , is double to the line AH . But as the lyne KA is to the line AH , so is the parallelogramme CK to the parallelogramme CH . Wherefore the parallelogramme CK is double to the parallelogramme CH . And the parallelogrammes LH and CH are double to the parallelogramme CH (for supplementes of parallelogrammes are by the 43. of the first equall the one to the other). Wherefore the parallelogramme CK is equall to the parallelogrammes LH & CH . And it is proued that the parallelogramme CE is equall to the square HF . Wherefore the whole square AE is equall to the gnomon MXN . And forasmuch as the line BA , is double to the line AD , therefore the square of the line BA is, by the 20. of the sixth, quadruple to the square of the line DA , that is, the square AE to the square DH . But the square AE is equall to the gnomon MXN , wherefore the gnomon MXN , is also quadruple to the square DH . Wherefore the whole square DF is quintuple to the square DH . But the square DF , is the square of the line CD , and the square DH is the square of the line DA . Wherefore the square of the line CD , is quintuple to the square of the line DA . If therefore a right line be deuided by an extreame and meane proportion, and to the greater segment, be added the halfe of the whole line: the square made of those two lines added together shalbe quintuple to the square made of the halfe of the whole line: Which was required to be demonstrated.

This proposition is an other way demonstrated after the fiftieth proposition of this booke.

The 2. Theoreme. The 2. Proposition.

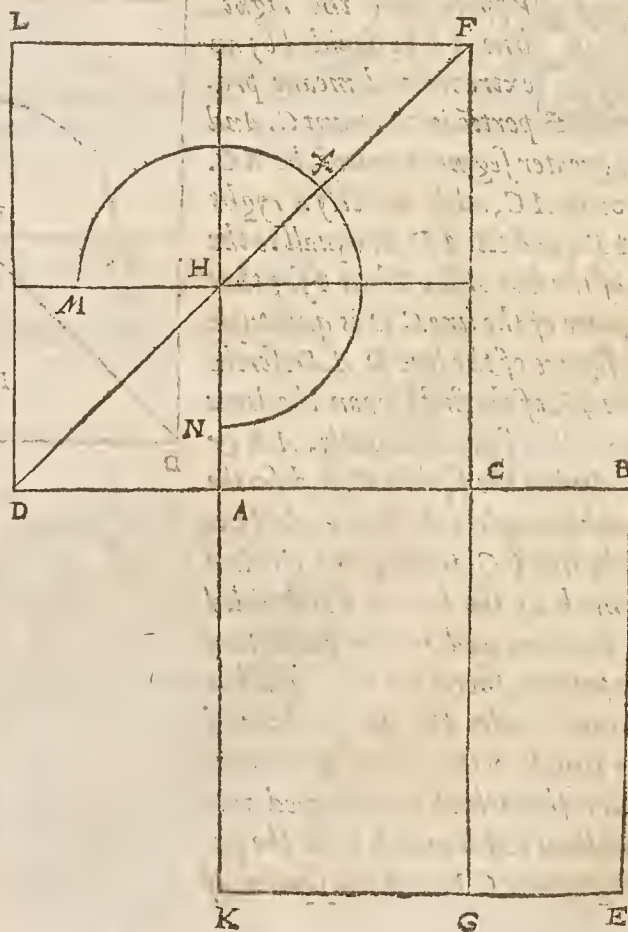
If a right line, be in power quintuple to a segment of the same line: the double of the sayd segment is deuided by an extreame and meane proportion, and the greater segment thereof is the other part of the line geuen at the beginning.

This proposition is the conuerse of the former.



Suppose that the right line DC be in power quintuple to a segment of the same line, namely, to AD , and let the double of

the line AD be the line AB . Then I say that the line AB is deuided by an extreme and meane proportion, and the greater segment thereof is the lyne AC . Describe on either of the lines AB and CD squares, namely AE and DF . And in the square DF make perfect the figure, and extend the line FC to the point G . And forasmuch as the square DF is quintuple to the square DH , by supposition, therefore the gnomon MXN is quadruple to the square DH . And forasmuch as the line AB is double to the line AD , therefore the square of the line AB is quadruple to the square of the line AD (by the 20.



Construction.

Demonstration.

of

of the sixt), that is, the square AE to the square DH . And it is proued that the gnomon MNX is quadruple to the square DH . Wherefore the gnomon MNX is equall to the square AE . And forasmuch as the line AB is double to the line AD , but the line AB is equall to the line AK , and the line AD to the line AH : therefore the line AK is double to the line AH : wherefore also (by the first of the sixth) the parallelogramme AG is double to the parallelogramme CH . But the parallelogrammes LH and CH are double to the parallelogramme CH (by the 43. of the first): wherefore the parallelogramme AG is equall to the parallelogrammes LH and CH . And it is proued that the whole gnomon MNX is equall to the whole square AE . Wherefore the residue HF is equall to the parallelogramme CE . And CE is that which is contained vnder the lines AB and CB , for the line AB is equall to the line BE , and HF is the square made of the line AC . Wherefore that which is containd vnder the lines AB and BC , is equall to the square of the line AC . Wherefore as the line AB is to the line AC , so is the line AC to the line CB . * But the line AB is greater then the line AC , wherefore the line AC is greater then the line CB . Wherefore the line AB is deuided by an extreme and meane proportion, and the greater segment thereof is the line AC . If therfore a right line be in power quintuple to a segment of the same line, the double of the sayd segment is deuided by an extreame & meane proportion, and the greater segment thereof is the other part of the line giuen at the beginning: Which was required to be proued.

* An Assumpt.

* Now, that the double of the line AD (that is AB) is greater then the line AC may thus be proued. For if not, then if it be possible let the line AC be double to the line AD , wherefore the square of the line AC is quadruple to the square of the line AD . Wherefore the squares of the lines AC and AD are quintuple to the square of the line AD . And it is supposed that the square of the line DC is quintuple to the square of the line AD , wherefore the square of the line DC is equall to the squares of the lines AC and AD : which is impossible (by the 4. of the second). Wherefore the line AC is not double to the line AD . In like sorte also may we proue that the double of the line AD is not lesse then the line AC , for this is much more absurd: wherefore the double of the line AD is greater thē the line AC : which was required to be proued.

* The Assumpt. proued.

This proposition also is an other way demonstrated after the fiftieth proposition of this booke.

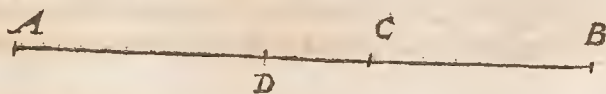
Two Theoremes, (in Euclides Method necessary) added by M. Dee.

A Theoreme. I.

A right line can be deuided by an extreame and meane proportion, but in one onely poynt.

Suppose a line diuided by extreame and meane proportion, to be AB . And let the greater segment be AC . I say, that AB can not be deuided by the sayd proportion, in any other point then in the point C . If an aduersary would contend that it may, in like sort, be deuided in an other point: let his other point, be supposed to be D : making AD , the greater segment of his imagined diuision. Which AD , also, let be lesse then our AC : for the first discourse. Now, forasmuch as by our aduersaries opinion, AD , is the greater segment, of his diuided line: the parallelogramme conteyned vnder AB , and DB , is equall to the square of AD , by the third definition and 17. proposition of the sixth Booke. And by the same definition and proposition, the parallelogramme vnder AB , and CB , conteyned, is equall to the square of our greater segment AC . Wherefore, as the parallelogramme, vnder AB , and DB , is to the square of AD : so is the parallelogramme, vnder AB , and CB , to the square of AC . For proportion of equality, is concluded in them both. But, forasmuch as DB , is (by *supposition) greater thē CB , the parallelogramme vnder AB , and DB , is greater then the parallelogramme vnder AC ,

* Because AC is supposed greater then AD : therefore his residue is lesse, then the residue of AD , by the common sentence. Wherefore, by the supposition: DB is greater then CB .



III. ij.

and

The thirteenth Booke

and CB: by the first of the sixth (for AB is their equall heith.) Wherefore, the square of AD, shalbe greater then the square of AC: by the 14. of the fifth. But the line AD, is lesse then the line AC, by supposition: wherefore the square of AD

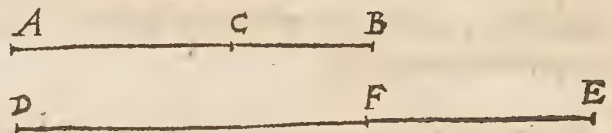
is lesse then the square of AC. And it is concluded also to be greater then the square of AC: Wherefore the square of AD, is both greater, then the square of AC: and also lesse. Which is a thing impossible. The square therefore of AD, is not equall to the parallelogramme vnder AB, and DB. And therefore by the third definition of the sixth, AB is not deuided by an extreame and meane proportion, in the point D: as our aduersary imagined. And (Secondly) in like sort will the inconueniency fall out: if we assigne AD, our aduersaries greater segment, to be greater then our AC. Therefore seing neither on the one side of our point C: neither on the other side of the same point C, any point can be had, at which the line AB can be deuided by an extreame and meane proportion, it followeth of necessitie, that AB can be deuided by an extreame and meane proportion in the point C, onely. Therefore, a right line can be deuided by an extreame and meane proportion, but in one, onely point: which was requisite to be demonstrated.



A Theoreme. 2.

What right line so euer, being deuided into two partes, hath those his two partes, proportionall, to the two segmentes of a line deuided by extreame and meane proportion: is also it selfe deuided by an extreame and meane proportion: and those his two partes, are his two segments, of the sayd proportion.

Suppose, AB, to be a line deuided by an extreame and meane proportion in the point C, and AC to be the greater segment. Suppose also the right line DE, to be deuided into two partes, in the point F: and that the part DF, is to FE, as the segment AC, is to CB: or DF, to be, to AC, as FE is to CB. For so these partes are proportionall, to the sayd segmentes. I say now, that DE is also deuided by an extreame and meane proportion in the point F. And that DF, FE, are his segmentes of the sayd proportion. For, seing, as AC, is to CB: so is DF, to FE: (by supposition). Therefore, as AC, and CB (which is AB) are to CB: so is DF, and FE, (which is DE) to FE: by the 18. of the fifth. Wherefore (alternately) as AB is to DE: so is CB, to FE. And therefore, the residue AC, is to the residue DF, as AB is to DE, by the fifth of the fifth. And then alternately, AC is to AB, as DE, is to DF. Now therefore backward, AB is to AC, as DE is to DF. But as AB is to AC, so is AC to CB: by the third definition of the sixth booke. Wherefore DE is to DF, as AC is to CB: by the 11. of the fifth. And by supposition, as AC is to CB, so is DF to FE: wherefore by the 11. of the fifth, as DE is to DF: so is DF to FE. Wherefore by the 3. definition of the sixth, DE is deuided by an extreame and meane proportion, in the point F. Wherefore DF, and FE are the segmentes of the sayd proportion. Therefore, what right line so euer, being deuided into two partes, hath those his two partes, proportionall to the two segmentes of a line deuided by extreame and meane proportion: is also it selfe deuided by an extreme and meane proportion, and those his two partes are his two segmentes, of the sayd proportion: which was requisite to be demonstrated.



Note.

Many wayes, these two Theoremes, may be demonstrated: which I leaue to the exercise of young studentes. But vtterly to want these two Theoremes, and their demonstrations: in so principall a line, or rather the chiefe pillar of *Euclides* Geometricall pallace, was hetherto, (and so would remayne) a great disgrace. Also I thinke it good to note vnto you, what we meane, by *one onely poyn*t. We meane, that the quantities of the two segmentes, can not be altered, the whole line being once geuen. And though, from either end of the whole line, the greater segment may begin: And so as it were the point of section may seeme to be altered: yet with vs, that is no alteration: forasmuch as the quantities of the segmentes, remayne all one. I meane, the quantitie of the greater segment, is all one: at which end so euer it be taken: And therefore, likewise the quantitie of the lesse segment is all one. &c. The like consideration may be had in *Euclides* tenth booke, in the Binomiall lines, &c.

Iohn Dee. 1569. Decemb. 18.

The 3.

*The chiefe line
in all Euclides
Geometrie.
What is ment
here by, A secti-
on in one onely
point.*

The 3. Theoreme. The 3. Proposition.

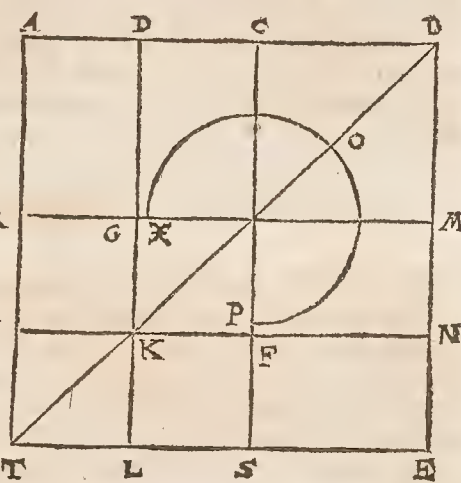
If a right line be deuided by an extreme and meane proportion, and to the lesse segment be added the halfe of the gerater segment: the square made of those two lines added together, is quintuple to the square made of the halfe line of the greater segment.



Suppose that the right line AB be deuided by an extreme and meane proportion in the point C . And let the greater segment thereof be AC . And deuide AC into two equall partes in the point D . Then I say that the square of the line BD , is quintuple to the square of the line DC . Describe (by the 46. of the first) vpon

Construction.

the line AB a square AE . And describe and make perfect the figure (that is diuide the lyne AT like vnto the diuision of the line AB , by the 10. of the sixth, in the pointes R, H , by which pointes drawe (by the 31. of the first) vnto the line AB parallel lines RM and HN . So likewise draw by the pointes D, C , vnto the line BE these parallel lines DL and CS , & draw the diameter BT). And forasmuch as the line AC is double to the line DC , therefore the square of AC , is quadruple to the square of DC , by the 20. of the sixth, that is, the square RS to the square FG . And forasmuch as that which is containd vnder the lines AB and BC is equall to the square of the line AC , and that which is containd vnder the lines AB and BC is equall to the parallelogramme CE , &



Demonstration.

the square of the line AC is the square RS : wherefore the parallelogramme CE is equall to the square RS . But the square RS is quadruple to the square FG : wherefore the parallelogramme CE also is quadruple to the square FG . Agayne forasmuch as the line AD is equall to the line DC , therefore the line HK is equall the line KF , wherefore also the square GF is equall to the square HL : wherefore the line GK is equall to the line KL , that is, the line MN to the line NE : wherefore the parallelogramme MF is equall to the parallelogramme FE . But the parallelogramme MF is equall to the parallelogramme CG , wherefore the parallelogramme CG is also equal to the parallelogramme FE . Put the parallelogramme CN , common to the both: * Wherefore the gnomon XOP is equall to the parallelogramme CE . But the parallelogramme CE is proued to be quadruple to GF the square, wherefore the gnomon XOP is quadruple to the square GF . Wherefore the square DN is quintuple to the square FG . And DN is the square of the line DB , and GF the square of the line DC . Wherefore the square of the line DB is quintuple to the square of the line DC . If therefore a right line be deuided by an extreme and meane proportion, and to the lesse segment be added the halfe of the greater segment: the square made of those two lines added together, is quintuple to the square made of the halfe line of the greater segment: Which was required to be demonstrated.

* Note, how CE and the gnomon XOP , are proued equall, for it serueth in the conuerse demonstrated by M. Dee, here next after.

Ye shall finde this proposition an other way demonstrated after the fiftieth proposition of this booke.

Here foloweth M. Dee, his additions.

¶ A Theoreme. I.

If a right line, geuen, be quintuple in power, to the powre of a segment of him self: the double of that
segment
Ili. iij.

The thirteenth Booke

This proposition, is the conuerse of the former.

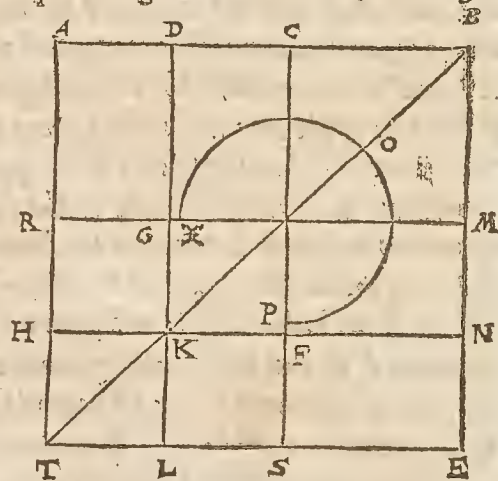
segment, and the other part remayning, of the first geuen line, make a line, diuided by extreme and meane proportion: and that double of the segment, is the greater part thereof.

** As we have noted the place of the peculier profe there in the demonstration of the 3.*

Forasmuch as, this, is the conuerse of Euclides third proposition: we will vse the same suppositions and constructions there specified: so farre, as they shall serue our purpose. Beginning therefore at the conclusion, we must infer the part of the proposition, before graunted. It was concluded, that the square of the line DB , is quintuple, to the square of the line DC , his owne segment. Therefore DN (the square of DB) is quintuple, to GF , (the square of DC). But the square of AC (the double of DC) which is RS , is quadruple to GF , (by the second Corollary of the 20. of the sixth): and therefore RS , with GF , are quintuple to GF : and so it is euident, that the square DN , is equall to the square RS , together with the square GF . Wherefore, from those two equalles, taking, the square GF , (common to them both): remayneth the square RS , equall to the Gnomon XOP . But to the Gnomon XOP , the parallelogramme CE , is equall: Wherefore the square of the line AC , which is RS , is equall to the parallelogramme CE . Which parallelogramme is contained vnder BE , (equall to AB): and CB , the part remayning of the first line geuen: which was DB . And the line AB , is made of the double of the segment DC , and of CB , the other part of the line DB , first geuen. Wherefore the double of the segment DC , with CB , the part remayning (which altogether, is the whole line AB) is to AC , (the double of the segment DC) as that same, AC , is to CB : by the second part of the 16. of the sixth. Therefore by the 3. definition of the sixth booke, the whole line AB , is deuided by an extreme and meane proportion, & AC , (the double of the segment DC) being middell proportionall, is the greater part thereof. Wherefore, if a right line, be quintuple in power, &c. (as in the proposition) which was to be demonstrated.

Or, thus it may be demonstrated.

Forasmuch as the square, DN is quintuple to the square GF , (I meane the square of DB the line geuen, to the square of DC the segment): And the same square DN , is equall to the parallelogramme vnder AB , CB , with the square made of the line DC : by the sixth of the second: (for vnto the line AC , equally deuided: the line, CB , is, as it were adioyned). Wherefore the parallelogramme vnder AB , CB , together with the square of DC , which is GF , is quintuple to the square GF , made of the line DC . Taking then, that square GF , from the parallelogramme vnder AB , CB : that parallelogramme (vnder AB , CB) remayning alone, is but quadruple to the sayd square of the line DC . But, (by the 4. of the second, or the second Corollary of the 20. of the sixth) RS , the square of the line AC , is quadruple to the same square GF : Wherefore by the 7. of the fifth, the square of the line AC , is equall to the parallelogramme vnder AB , CB , and so, by the second part of the 16. of the sixth: AB , AC , and CB , are three lines in continuall proportion. And seing AB is greater then AC , the same AC , the double of the line DC , shall be greater then the part CB , remayning: Wherefore by the 3. definition of the sixth, AB , (composed or made of the double of DC , and the other part of DB remaining) is deuided by an extreme and middell proportion: and also his greater segment is AC the double of the segment DC . Wherefore, If a right line be quintuple in power &c. as in the proposition: which was to be demonstrated.

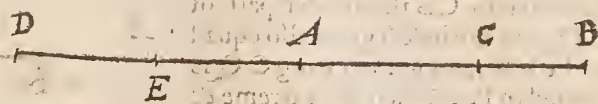


* A Theoreme. 2.

If a right line, deuided by an extreme and meane proportion, be geuen, and to the great segment thereof, be directly adioyned a line equal to the whole line geuen, that adioyned line, and the said greater segment, do make a line diuided by extreme and meane proportion, whose greater segment is the line adioyned.

Suppose the line geuen, deuided by extreame and meane proportion, to be AB deuided in the point C , and his greater segment, let be AC : vnto AC directly adioyne a line equall to AB : let that be AD : I say, that AD , together with AC , (that is DC) is a deuided by extreme and middell proportion, whose greater segment is AD , the line adioyned. Deuide AD , equally in the point E . Now, forasmuch as AE , is the halfe of AD , (by construction,) it is also, the halfe of AB (equall, to AD , by construction): Wherefore by the 1. of the thirteenth, the square of the line composed of AC and AE (which line is EC) is quintuple to the square of the line AE . Wherefore the double of AE , and the line AC composed

composed, (as in one right line) is a line divided by extreme and meane proportion, by the conuerse of this third (by me demonstrated) : and the double of A E, is the greater segment. But D C is the line composed of the double of A E, & the line A C: and with all, A D is the double of A E. Wherefore, D C, is a line divided by extreme and meane proportion, and A D, is his greater segment. If a right line, therefore, divided by extreme and meane proportion, be geuen, and to the greater segment thereof, be directly adioyned a line equall to the whole line geuen, that adioyned line, and the sayd greater segment, do make a line divided by extreame and meane proportion, whose greater segment, is the line adioyned : Which was required to be demonstrated.



Two other brieue demonstrations of the same.

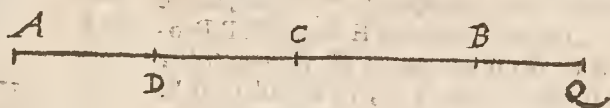
Forasmuch as, A D is to A C: as A B, is to A C (because A D is equall to A B, by construction) : but as A B is to A C, so is A C to C B: by supposition. Therefore by the 11. of the fifth, as A C, is to C B, so is A D to A C. * Wherefore, as A C, and C B, (which is A B) is to C B: so is A D, and A C (which is D C) to A C. Therefore, euerfeldy, as A B, is to A C: so is D C to A D. And it is proued, A D, to be to A C: as A C is to C B. Wherefore as A B is to A C, and A C, to C B: so is D C, to A D, and A D, to A C. But A B, A C, and C B are in continuall proportion, by supposition: Wherefore D C, A D, and A C, are in continuall proportion. Wherefore, by the 3. definition of the sixth booke, D C, is divided by extreme and middell proportion, and his greatest segment, is A D. Which was to be demonstrated. Note from the marke *, how this hath two demonstrations. One I haue set in the margent by.

* Therefore, by my second Theoreme added upon the second proposition, D C is divided by extreme and meane proportion in the point A. And because A C is bigger then C B: therefore D A is greater then A C: wherefore if a right line E C, as in the proposition. Which was to be demonstrated.

¶ A Corollary. 1.

Upon Euclides third proposition demonstrated, it is made euident: that, of a line divided by extreame and meane proportion, if you produce the lesse segment, equally to the length of the greater: the line thereby adioyned, together with the sayd lesse segment, make a new line divided by extreame and middle proportion: Whose lesse segment, is the line adioyned.

For, if A B, be divided by extreme and middell proportion in the point C, A C, being the greater segment, and C B be produced, from the poynt B, making a line, with C B, equall to A C, which let be C Q: and the line thereby adioyned, let be B Q: I say that C Q, is a line also divided by an extreame and meane proportion, in the point B: and that B Q (the line adioyned) is the lesse segment. For by the thirde, it is proued, that halfe A C, (which, let be, C D) with C B, as one line, composed, hath his powre or square, quintuple to the powre of the segment C D: Wherefore, by the second of this booke, the double of C D, is divided by extreme and middell proportion: and the greater segment thereof, shalbe C B. But, by construction, C Q, is the double of C D, for it is equall to A C. Wherefore C Q is divided by extreme and middle proportion, in the point B: and the greater segment thereof shalbe, C B. Wherefore B Q, is the lesse segment, which is the line adioyned. Therefore, a line being divided, by extreme and middell proportion, if the lesse segment, be produced equally to the length of the greater segment, the line thereby adioyned together with the sayd lesse segment, make a new line divided, by extreme & meane proportion, whose lesse segment, is the line adioyned. Which was to be demonstrated.



¶ A Corollary. 2.

If, from the greater segment, of a line divided, by extreme and middle proportion, a line, equall to the lesse segment, be cut of: the greater segment, thereby, is also divided by extreme and meane proportion, whose greater segment, shall be now that part of it, which is cut of.

For, taking from A C, a line equall to C B: let A R remayne. I say, that A C, is divided by an extreame and meane proportion in the point R: and that C R, the line cut of, is the greater segment. For it is proued in the former Corollary that C Q is divided by extreme and meane proportion in the point B. But A C, is equall to C Q, by construction: and C R is equall to C B by construction: Wherefore the

The thirteenth Booke

residue, A R is equal to B Q the residue. Seing therefore the whole A C is equal to the whole C Q: and the greater part of A C, which is C R is equal to C B the greater part of C Q: and the lesse segmēt also equal to the lesse: and withall seing C Q is proued to be diuided by extreme & meane proportion in the point B, it foloweth of necessity, that A C is diuided by extreme and meane proportion in the point R. And seing C B, is the greater segment of C Q: C R shall be the greater segment of A C. Which was to be demonstrated.

* *A Corollary.*

*It is evident thereby, a line being divided by extreme and meane proportion, that the line where-
by the greater segment exceedeth the lesse, together with the lesse segment, do make a line divided by
extreme and meane proportion: whose lesse segment, is the sayd line of excessse, or difference betwene
the segments.*

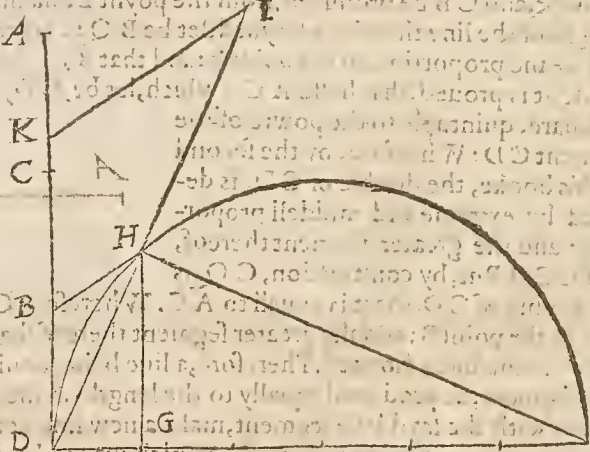
John Dee.

¶ Two new wayes, to deuide any right line geuen by an extreme and meane proportion: demonstrated and added by M. Dee.

A Probleme.

To divide by an extreme and meane proportion, any right line given, in length and position.

Suppose a line geuen in length and position, to be A B. I say that A B is to be deuided by an extreme, and meane proportion: Deuide A B into two equall parts as in the point C. Produce A B directly, from the point B, to the point D: making B D, equal to B C. To the line A D, and at the point D, let a line be drawn *perpendicular: by the 11. of the first, which let be D F: (of what length you will). From D F, and at the point D, cut off the sixth parte of D F: by the 9. of the sixth. And let that sixth part, be the line D G. Vppon D F, as a diameter, describe a semicircle: which let be D H F. From the point G, ere a line perpendicular to D F, which suppose to be G H: and let it come to the circumference of D H F, in the point H. Draw right lines, H D, and H F. Produce D H, from the point H, so long, till a line adioyned with D H, be equall to H F, which let be D I, equall to H F. From the point H, to the point B, (the one ende of our line geuen) let a right line be drawn: as H B. From the point I, let a line be drawn, to the line A B: so that it be also parallel to the line H B. Which parallel line suppose to be I K: cutting the line A B, at the point K. I say that A B, is deuided by an extreme & meane proportion, in the point K. For the triangle D K I, hauing H B, parallel to I K, hath his sides D K and D I, cut proportionally, by the 2. of the sixth. Wherefore as L H is to H D: so is K B, to B D. And therefore compoundingly, (by the 18. of the sixth) as D I, is to D H: so is D K to D B. But by construction D I is equall to H F: wherefore by the 7. of the fifth, D I is to D H, as H F is to D H. Wherefore by the 11. of the fifth, D K is to D B, as H F is to D H. Wherefore the square of D K is to the square of D B, as the square of H F, is to the square of D H: by the 22. of the sixth. But the square of H F, is to the square of D H: as the line G F is to the line G D: by my corollary vpon the 6. probleme of my additions to the second proposition of the twelfth. Wherefore by the 11. of the fifth, the square of D K is to the square of D B, as the line G F is to the line G D. But by construction, G F is quintuple to G D. Wherefore the square of D K is quintuple to the square of D B: and therefore, the double of D B, is deuided by an extreme and meane proportiō, and B K is the greater segment therof, by the 2. of this thirteenth. Wherefore seing A B is the double of D B by construction: the line A B is deuided by an extreme and meane proportion:



Construcciones

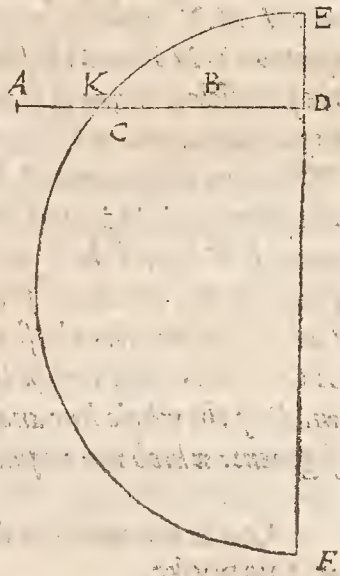
* I thought I say,
perpendicular:
yet you may
perceive how in-
finite other po-
sitions will serve:
so that D I and
A D make an
angle for a tri-
angle to have
his sides propor-
tionally cut.
&c.

Demonstration

proportion: and his greater segment, is the line BK . Wherefore AB is deuided by an extreme and meane proportion, in the point K . We haue therefore deuided by extreme and meane proportion any line geuen in length and position. Which was requisite to be done.

The second way to execute this problem.

Suppose the line geuen to be AB . Deuide AB into two equall parts: as suppose it to be done in the point C . Produce AB from the point B : adioyning a line equall to BC , which let be BD . To the right line AD , and at the point D , erect a perpendicular line equall to BD , let that be DE . Produce ED fro the point D to the point F : making DF to contayne fife such equall partes, as DE is one. Now vpon EF as a diameter, describe a semicircle which let be EKF : and let the point where the circumference of EKF , doth cut the line AB , be the point K . I say that AB , is deuided in the point K , by an extreme and meane proportion. For by the 13. of the sixth ED, DK , & DF , are three lines in continuall proportion, (DK being the middle proportionall). Wherefore by the corollary of the 20. of the sixth, as ED is to DF , so is the square of ED , to the square of DK , but by construction, ED is subquintuple to DF . Wherefore the square of ED , is subquintuple to the square of DK . And therefore the square of DK , is quintuple to the square of ED . And BD is equall to ED ; by construction, therefore the square of DK , is quintuple to the square of BD . Wherefore the double of BD , is deuided by an extreme and meane proportion: whose greater segment is BK : by the second of this thirteenth. But by construction, AB , is the double of BD . Wherefore AB , is deuided by extreme and meane proportion, and his greater segment, is BK : and thereby, K , the point of the diuision. We haue therefore deuided by extreme and meane proportion, any right line geuen, in length and position. Which was to be done.



Note. The first way is more exact than the second.

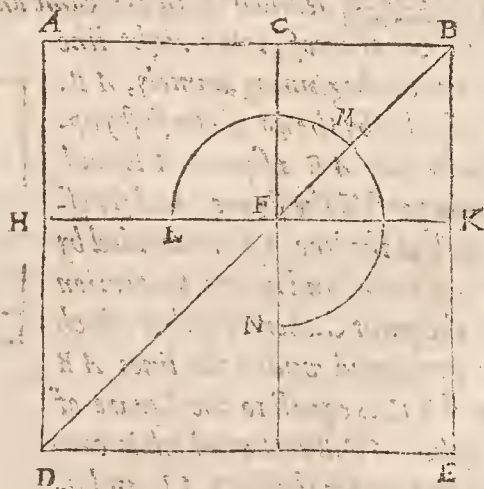
Ech of these wayes, may well be executed: But in the first, you haue this auantage: that the diameter is taken at pleasure. Which in the second way, is euer iust thrise so long, as the line geuen to be deuided.

* Iohn Dee.

The 4. Theoreme. **The 4. Proposition.**

If a right line be deuided by an extreame and meane proportion: the squares made of the whole line and of the lesse segmēt, are treble to the square made of the greater segment.

Suppose that the right line AB , be deuided by an extreame & meane proportio, in the point C . And let the greater segment thereof be AC . Then I say, that the squares made of the lines AB , and BC , are treble to the square of the line AC . Describe (by the 46. of the first) vpon the line AB , a square $ADEB$. And make perfect the figure. Now forasmuch as the line AB , is deuided by an extreame and meane proportion, in the point C : and the greater segmēt thereof, is the line AC , therefore that which is containd vnder the lines AB and BC is equall to the square of the line AC . But that which is containd vnder the lines AB and CB is the parallelo-



Demonstration.

The thirteenth Booke

Looke for an other demonstration of this proposition after the fifth proposition of this booke.

The 5. Theoreme.

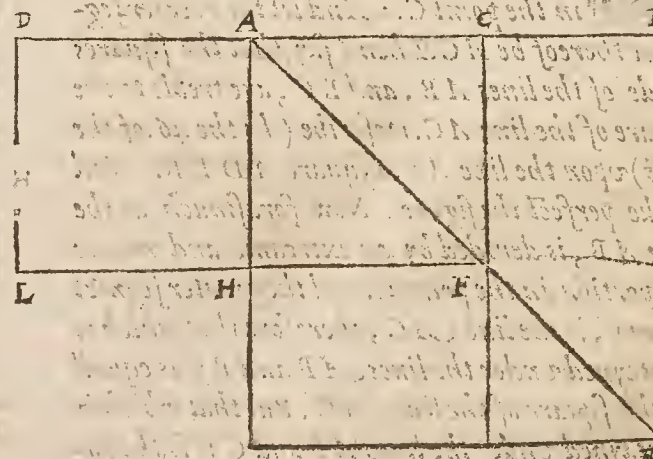
The 5. Proposition.

1. Dec.

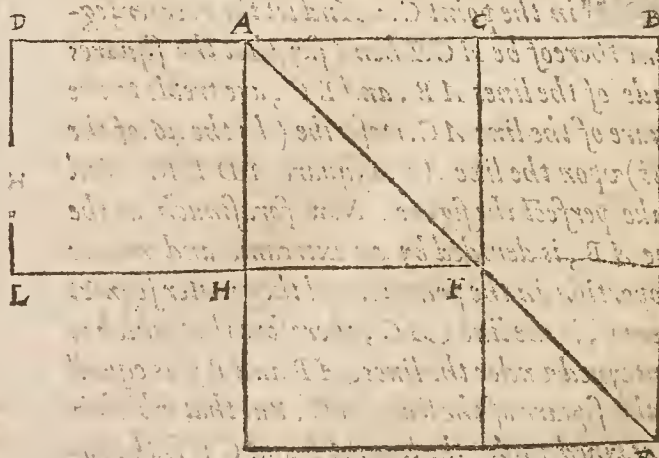
This is manifest evident of my second Theorem, added to the third proposition. For to add to a whole line, a line equal to the greater segment: Eg to add to the greater segment a line equal to the whole line, is all one thing in the line produced. By the whole line, I mean the line divided by extreme and mean proportion.

If a right line be deuided by an extreame and meane proportion, and vnto it be added a right line, equall to the greater segment, the whole right line is deuided by an extreame and meane proportion, and the greater segment thereof, is the right line geuen at the beginning.

 Suppose that the right line AB be divided by an extreme and mean proportion in the point C , and let the greater segment thereof be AC . And unto the line AB , add the line AD equall to the line AC . Then I say that the line DB is divided by an extreme and mean proportion in the point A : and the greater segment thereof, is the right line put at the beginning, namely, AB . Describe (by the 46. of the first) up-on the line AB a square AE , and make perfect the figure. And forasmuch as the line AB , is divided by an extreme and mean proportion in the point C , therefore that which is contained under the lines AB and BC is equall to the square of the line AC . But that which is contained under the lines AB and $B-$



Cis



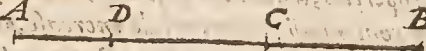
C is the parallelogramme C E, and the square of the line A C is the square C H. Wherefore the parallelogramme C E is equall to the square C H. But vnto the square C H is equall the square D H, by the first of the sixth: and vnto the parallelogramme C E, is equall the parallelogramme H E. Wherefore the parallelogramme D H is equall to the parallelogramme H E. Adde the parallelogramme H B, common to them both. Wherefore the whole parallelogramme D K is equall to the whole square A E. And the parallelogramme D K, is that which is containd vnder the lines B D and D A, for the line A D is equall to the line D L, & the square A E is the square of the line A B. Wherefore that which is containd vnder the lines A D and D B is equall to the square of the line A B. Wherefore as the line D B is to the line B A, so is the line B A to the line A D, by the 17. of the sixth. But the line D B is greater then the line B A. Wherefore the line B A is greater then the line A D. Wherefore the line B D is deuided by an extreame and meane proportion in the point A, and his greater segment is the line A B. If therefore a right line be deuided by an extreame and meane proportion, and vnto it be added a right line, equall to the greater segment: the whole right line is deuided by an extreame and meane proportion, and the greater segment therof, is the right line geuen at the beginning: which was required to be demonstrated.

This proposition is agayne afterward demonstrated.

A Corollary added by Campane.

Hereby it is manifest that if from the greater segment of a line deuided by an extreame & meane proportion, be taken away the lesse segment: the sayd greater segment shall be deuided by an extreame and meane proportion, and the greater segment thereof, shall be the line taken away.

As let the line A B, be deuided by an extreame and meane proportion, in the point C. And let the greater segment, be the line A C. From A C take C B: making the residue A D. I say that A D is also deuided by an extreame and meane proportion in the point D, and that his greater portion is D C. For, by the definition (of a line so deuided) A B, is to A C, as A C is to C B. But as A C is to C B, so is A C to D C, by the 7. of the fifth (for D C, by construction is equall to C B) wherefore, by the 11. of the fifth, as A B is to A C, so is A C to D C: and therefore by the 19. of the fifth, as A B is to A C, so is the residue C B, to the residue A D. But C B is to A D, as D C is to A D (by the 7. of the fifth) for D C is by construction equall to C B. Wherefore, A C is to D C, as D C is to A D, and so, by the definition of a line deuided by an extreame and meane proportion, it appeareth, A C in the point D, to be deuided, by an extreame and meane proportion: which was to be proued.



This is before demonstrated most evidently and briefly by M. Dee, after the 3. proposition.

Two Corollaries (added by M. Dee) following chiefly vpon the veritie, and demonstration of his Additions, vnto the 3. porposition annexed, and partly vpon this fifth, by Euclide demonstrated.

A Corollary. I.

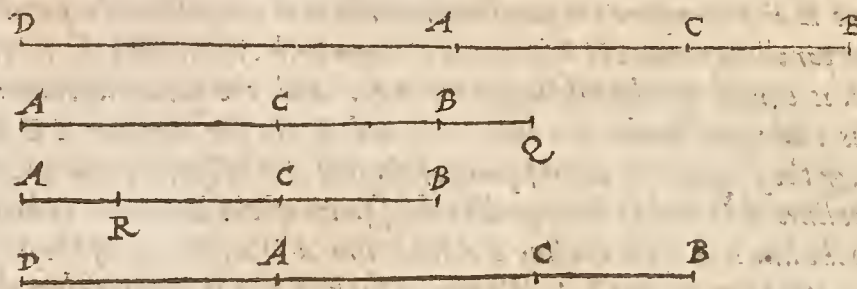
As any line being deuided by an extreame and middle proportion, doth gene vs three right lines, in continuall proportion: So either by adioyning directly to the greater segment, a line equall to the first whole line: or (secondly) by producing the lesse segment, equally to the length of the greater segment: or (thirdly) by cutting from the greater segment, a part equal to the lesse segment: or (fourthly) by adioyning directly to the greater segment, a line equall to the same segment: it is manifest, that in euery of these fower wayes, we haue two lines deuided by an extreame and meane proportion: (it is to wete, one geuen, and the other made) and with all, in euery way, we haue fower lines in continuall proportion.

Of these two lines, (by extreame and meane proportion deuided,) their demonstrations, are after Euclides 3. proposition added: and here in this fifth by Euclide proued. But of the fower lines in continuall proportion, seing, the demonstration is most easy for any man to frame. I will here, but note the lines

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Note 4. 1.
Proportionall
lines.

vnto you: as in euery of the fower places, the constructions haue them lettred and specified. As in my first way added after the third proposition, D C, A D, A C, and C B: are fower lines in continuall propor-



2. tion. And in the second way, A B, C Q, (equall to A C) C B, and B Q: are fower lines in continuall proportion. And in the third way, A B, A C, C R, (equall to C B) and A R, are the fower lines in continuall proportion. And in the fourth way (by Euclide declared) D B, A B, A C (equall to A D) and C B, are fower lines in continuall proportion. So, that in the first way you haue A D, and A C, middle proportionalls betwene D C, and C B. In the second way, you haue C Q, and C B, betwene A B, and B Q. In the third way, you haue A C, C R, betwene A B and A R: and in the fourth way, you haue A B, and A C, betwene D B, and C B.

Note
two mid-
dle propor-
tionalls.

A Corollary. 2.

Note 4. Wayes
of progression, in
the proportion
of a line diuided
by extreme and
middle propor-
tion.

It is also manifest, that you may by any of the fower wayes, here specified, proceede infinitely, in the proportion of a line deuised by extreme & middle proportion: And in the first, and fourth wayes, encreasing continually the quantities of the lines made: but in the second and third wayes, diminishing continually the quantities of the sayd whole lines, made (and thereby their segmentes). And yet, neuerthelesse reteyning in euery line made (by any of the wayes) and in his segmentes, all, and the same properties, which the first line, and his segmentes haue. After which rate of Progression, as the termes in continuall proportion do encrease, and are moe in number: So, likewise, do the middle proportionalls, (accordingly) become moe: But euer fewer in number, by two, then the termes of the Progression are.

What Resolution is.

What resolution
and composition
is, hath before
bene taught in
the beginning of
the first booke.

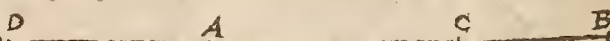
Resolution is the assumption or taking of the thing which is to be proued, as graunted, and by thinges which necessarily follow it, to passe vnto some truth graunted.

What Composition is.

Composition, is an assumption or taking of a thing graunted, and by thinges which of necessity follow it, to passe vnto the finding out of the thing sought or to be proued.

Resolution of the first Theoreme.

Suppose that a certaine right line A B, be diuided by an extreme & meane proportion in the point C, & let the greater segmet therof be A C, vnto which adde a line equal to the halfe of the line A B, and let that line be A D. Then I say that the square of the line C D is quintuple to the square of A D. For forasmuch as the square of the line C D is quintuple to the square of A D: but the square of the line C D is (by the 4. of the second) equall to that which is composed of the squares of the lines C A, & A D, together with that



which

which is containned under the lines $C A$, and $A D$ twise. Wherefore that which is composed of the squares of the lines $C A$, & $A D$, together with that which is cotained under the lines $C A$, & $A D$ twise is quintuple to the square of the line $A D$. Wherefore, that which is composed of the square of the line $C A$ together with that, which is containned under the lines $C A$, and $A D$ twise is quadruple to the square of the line $A D$. But unto that, which is containned under the lines $C A$, and $A D$, twise, is equall that which is containned under the lines $C A$, and $A B$, for the line $A B$ is double to the line $A D$. And unto the square of the line $A C$, is equall that which is cotained under the lines $A B$ & $B C$, for the line $A B$, is by supposition diuided by an extreme and meane proportion in the point C . Wherefore, that, which is containned under the lines $A B$, and $A C$, together with that which is containned under the lines $A B$, and $B C$, is quadruple to the square of the line $A D$. But that, which is composed of that which is containned under the lines $A B$, and $A C$, together with that which is containned under the lines $A B$, and $B C$, is the square of the line $A B$ (by the 2. of the second). Wherefore the square of the line $A B$ is quadruple to the square of the line $A D$. And so is it in deede: for the line $A B$ is double to the line $A D$: as was at the first supposed.

Composition of the first Theoreme.

Now forasmuch as the square of the line $A B$ is quadruple to the square of the line $A D$, but the square of the line $A B$, is that which is containned under the lines $A B$, and $A C$, together with that, which is containned under the lines $B A$, and $B C$. Wherefore that which is containned under the lines $B A$, and $A C$, together with that which is containned under the lines $B A$, and $B C$ is quadruple to the square of the line $A D$. But that which is containned under the lines $B A$, and $A C$, is equall to that which is containned under the lines $D A$, and $A C$ twise (by the 1. of the sixth), and that which is containned under the lines $A B$, and $B C$ is equall to the square of the line $A C$, by the definition of a line diuided by extreme and meane proportion. Wherefore the square of the line $A C$, together with that, which is containned under the lines $D A$, and $A C$ twise, is quadruple to the square of the line $D A$. Wherefore that which is composed of the squares of the lines $D A$, and $A C$, together with that which is containned under the lines $D A$, and $A C$ twise, is quintuple to the square of the line $D A$. But that which is composed of the squares of the lines $D A$, and $A C$, together with that which is containned under the lines $D A$, and $A C$ twise, is equall to the square of the line $C D$ (by the 4. of the second). Wherefore the square of the line $C D$ is quintuple to the square of the line $A D$: which was required to be demonstrated.

Resolution of the 2. Theoreme.

Suppose that a certayne right line, $C D$, be quintuple to a segmēt of the same line, namely, to $D A$ and let the double of the line $D A$, be $A B$. The I say that the line $A B$ is diuided by an extreme and meane proportion in the point C : and the greater segmēt therof is $A C$, which is the rest of the right line put at the beginning. For forasmuch as the line $A B$ is diuided by an extreame and meane proportion in the poynt C , and the greater segment thereof is the line $A C$, therefore that which is contained under the lines $A B$, and $B C$, is equall to the square of the line $A C$. But that which is containned under the lines $B A$, and $A C$, is equal to that which is containned under the lines $D A$, and $A C$ twise: for the line $B A$, is double to the line $A D$. Wherefore that which is containned under the lines $A B$, and $B C$ together with that which is cotained under the lines $B A$, and $A C$, which is the square of the line $A B$ (by the 2. of the second) is equall to



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that which is containned under γ lines DA , & AC , twice together with the square of the line AC . But the square of the line AB , is quadruple to the square of the line DA . Wherefore that which is containned under the lines DA , and AC , twice, together with the square of the line AC , is quadruple to the square of the line AD . Wherefore the squares of the lines DA , and AC , together with that which is containned under the lines DA , and AC , twice, which is the square of the line DC , are quintuple to the square of the line DA . And so are they in deede by supposition.

Composition of the 2. Theoreme.

Now forasmuch as the square of the line CD is quintuple to the square of the line DA , But the square of the line CD is that which is composed of the squares of γ lines DA , & AC , together with that which is containned under the lines DA , & AC , twice. Wherefore γ squares of the line DA , & AC together with that which is containned under the lines DA , & AC , twice, are quintuple to the square of the line DA . Wherefore, by diuision, that which is containned under the lines DA , and AC , twice together with the square of the line CA , is quadruple to the square of the line AD . And the square of the line AB , is quadruple to the square of the line AD . Wherefore that which is containned under the lines DA , and AC twice, which is that, which is containned under the lines BA , and AC once, together with the square of the line AC , is equall to the square of the line AB . But the square of BA , is that which is containned under the lines BA , and AC , together with that which is containned under the lines BA , and BC . Wherefore that which is containned under the lines BA , & AC , together with that which is containned under the lines AB , & BC , is equall to that which is containned under the lines BA , and AC together with the square of the line AC . Now then taking away that which is common to them both, namely, that which is containned under the lines BA , and AC , the residue, namely, that which is containned under the lines AB , BC is equall to the square of the line AC . Wherefore as the line BA , is to the line AC , so is the line AC to the line CB . But the line BA is greater then the line AC , wherefore the line AC also is greater then the line CB . Wherefore the line AB is divided by an extreame, and meane proportion in the point C , and the greater segment thereof is the line AC , which was required to be demonstrated.

Resolution of the 3. Theoreme.

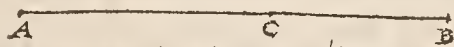
Suppose that a certayne right line AB , be divided by an extreame, and meane proportion in the point C : and let the greater segment thereof be the line AC , and let the halfe of the line AC , be the line CD . Then I say that the square of the line BD is quintuple to the square of the line CD . For forasmuch as the square of the line BD , is quintuple to the square of the line CD . But the square of the line DB , is that which is containned under the lines AB , and BC , together with the square of the line DC (by the 6. of the second). Wherefore that which is containned under the lines AB , and BC , together with the square of the line DC , is quintuple to the square of the line DC . Wherefore, that which is $\overbrace{A}^{\quad} \overbrace{D}^{\quad} \overbrace{C}^{\quad} \overbrace{B}^{\quad}$ containned under the lines AB , and BC , is quadruple to the square of the line DC . But vnto that which is containned under the lines AB , and BC , is equall the square of the line AC : for the line AB , is divided by an extreame and meane proportion in the point C . Wherefore the square of the line AC is quadruple to the square of the line DC : and so is it in deede, for the line AC is double to the line DC .

Composition of the 3. Theoreme.

Forasmuch as the line AC is double to the line DC , therefore the square of the line AC is quadruple to the square of the line DC (by the 20. of the sixth). But unto the square of the line AC , is equall that which is containned under the lines AB , and BC , by supposition: wherefore that which is containned under the lines AB , and BC , is quadruple to the square of the line DC . Wherefore, that which is containned under AB , and BC , together with the square the line DC , which is the square of the line DB (by the 6. of the second) is quintuple to the square of the line DC : which was required to be demonstrated.

Resolution of the 4. Theoreme.

Suppose that a certayne right line AB , be divided by an extreme and meane proportion in the point C . And let the greater segment thereof be AC . Then I say that the squares of the lines AB , and BC , are treble to the square of the line AC . For forasmuch as the squares of the lines AB , and BC , are treble to the square of the line AC , but the squares of the lines AB , and BC , are that which is containned under AB , and BC , twise together with the square of the line AC (by the 7. of the second). Wherefore that which is containned under the lines AB , and BC , twise, together with the square of the line AC , is treble to the square of the line AC . Wherefore, that which is containned under the lines AB , & BC , twise, is double to the square of the line AC . Wherefore that which is containned under the lines AB , and BC , once, is equall to the square of the line AC . And so it is in deede. For the line AB is divided by an extreme, and meane proportion in the point C .

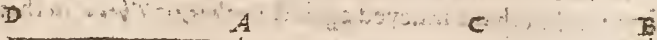


Composition of the 4. Theoreme.

Forasmuch therefore as the line AB , is divided by an extreme and meane proportion in the point C , and the greater segment thereof is the line AC , therefore that which is containned under the lines AB , and BC is equall to the square of the line AC . Wherefore that which is containned under the lines AB , and BC twise is double to the square of AC . Wherefore that which is containned under the lines AB , and BC , twise, together with the square of the line AC , is treble to the square of the line AC . But that which is containned under the lines AB , and BC , twise, together with the square of the line AC , is the squares of the lines AB , and BC (by the 7. of the second). Wherefore the squares of the lines AB , and BC , are treble to the square of the line AC : which was required to be demonstrated.

Resolution of the 5. Theoreme.

Suppose that a certayne right line AB , be divided by an extreme and meane proportion in the point C . And let the greater segment thereof be the line AC . And unto the line AB , adde a line equall to the line AC , and let the same be AD . Then I say that the line DB , is divided by an extreme and meane proportion in the point A . And the greater segment thereof is the line AB . For forasmuch as the line DB is divided by an extreme & meane proportion in the point A , and the greater segment thereof is the line AB , therefore as the line DB , is to the line BA , so is the line BA , to the line AD : but the line AD , is equall to the line AC : wherefore as



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the line DB , is to the line BA , so is the line BA to the line AC . Wherefore by conuersion as the line BD is to the line DA , so is the line AB to the line BC (by the corollary of the 19. of the fifth): wherefore by diuision, by the 17. of the fifth, as the line BA , is to the line AD , so is the line AC , to the line CB . But the line AD is equall to the line AC . Wherefore as the line BA , is to the line AC , so is the line AC to the line CB . And so it is indeede, for the line AB is, by supposition, diuided by an extreme and meane proportion in the point C .

Composition of the 5. Theoreme.

Now forasmuch as the line AB , is diuided by an extreme and meane proportion in the point C : therefore as the line BA is to the line AC , so is the line AC to the line CB : but the line AC is equall to the line AD . Wherefore as the line BA is to the line AD , so is the line AC to the line CB . Wherefore by composition (by the 18. of the fifth) as the line BD is to the line DA , so is the line AB to the line BC . Wherefore by conuersion (by the corollary of the 19. of the fifth) as the line DB is to the line BA , so is the line BA to the line AC : but the line AC is equall to the line AD . Wherefore as the line DB is to the line BA , so is the line BA to the line AC . Wherefore the line DB , is diuided by an extreme and meane proportion in the point A : and his greater segment is the line AB : which was required to be demonstrated.

An Aduise, by Iohn Dee, added.

SEing, it is doubteles, that this parcel of Resolution and Composition, is not of Euclides doying: it can not iustly be imputed to Euclide, that he hath, thereby, eyther superfluitie or any part disproportioned in his whole Composition Elementall. And though, for one thing, one good demonstration well suffiseth: for stablishing of the veritie: yet, of one thing diuersly demonstrated: to the diligent examiner of the diuerse meanes, by which, that varietie ariseth, doth grow good occasions of inuventing demonstrations, where matter is more straunge, harde, and barren. Also, though resolution were not in all Euclide before vsed: yet thanks are to be geuen to the Greke Scholie writter, who did leaue both the definition, and also, so short and easy examples of a Method, so auncient, and so profitable. The antiquity of it, is aboue 2000. yeares: it is to wete, euer since Plato his time, and the profite, therof so great, that thus I finde in the Greeke recorded. *Μεθοδοί δὲ ἑμῶς παραδίδονται: καλλίστη μὲν ἡ διὰ τῆς ἀναλύσεως, ἐκ ἀρχῆς ὁμολογεῖσθαι, ἀνάγκη τὸ ἐπισκοπεῖν: ἢ καὶ ὁ Πλάτων (ὡς φασι) Λεοδάμαντι παρέδωκεν. ἀφ' ἧς καὶ ἐκείνος πολλὰ καὶ γεωμετρικὰ ἐπεὶ ἰσορῆται γενέσθαι. Proclus hauing spoken of some by nature, excellent in inuventing demonstrations, pithy and breif sayeth: Yet are there Methods geuen [for that purpose]. And in dede, that, the best, which, by Resolution, reduceth the thing inquired of, to an vndoubted principle. Which Method, Plato, taught Leodamas (as is reported) And he is registred, thereby, to haue bene the inuenter of many things in Geometry.

And, verely, in Problemes, it is the chief ayde for winning and ording a demonstration: first by Supposition, of the thing inquired of, to be done: by due and orderly Resolution to bring it to a stay, at an vndoubted Veritie. In which point of Art, great abundance of examples, are to be seen, in that excellent and mighty Mathematicien, Archimedes: & in his expositor, Eutocius, in Menæchmus likewise: and in Diocles booke, de Pyrijs: and in many other. And now, for as much as, our Euclide in the last six Propositions of this thirteenth booke propoundeth, and concludeth those Problemes, which were the ende, Scope, and principall purpose, to which all the premisses of the 12. bookes, and the rest of this thirteenth, are directed and ordered: It shall be artificially done, and to a great commodity, by Resolution, backward, from these 6. Problemes, to returne to the first definition of the first booke: I meane, to the definition of a point. Which, is nothing hard to do. And I do counsaile all such, as desire to attaine to the profound knowledge of Geometrie, Arithmetike, or any braunche of the sciences Mathematicall, so by Resolution, (discreatly and aduisedly) to resolue, vnlose, vnioynt and disseauer euery part of any worke Mathematicall, that, thereby, aswell, the due placing of euery verity, and his prooffe: as also, what is either superfluous, or wanting, may euidently appeare. For so to inuent, & there with to order their writings, was the custome of them, who in the old time, were most excellent. And (for my part) in writing any Mathematicall conclusion, which requireth great discourse, at length haue found, (by experience) the commoditie of it, such: that to do other wayes, were to me a confusion, and an vnmethodicall heaping of matter together: besides the difficulty of inuventing the matter to be disposed and ordred. I haue occasion, thus to geue you friendly aduise, for your behofe: because some, of late, haue inueyed against Euclide, or Theon in this place, otherwise than I would wish they had.

* Proclus in
the Greeke:
in the 58.
page.

The 6. Theoreme.

The 6. Proposition

If a rationall right line be diuided by an extreme and meane proportion: eyther of the segments, is an irrationall line of that kinde, which is called a residuall line.



Suppose that AB , beyng a rationall line be deuied by an extreme and meane proportion in the point C , and let the greater segment thereof be AC . Then I say that eyther of the lines AC , and CB , is an irrationall line of that kinde, which is called a residuall. Extend the line AB , to the point D : and let the line

Construction.

AD , be equall to halfe of the line AB .

Now foras much as the right line AB , is diuided by an extreme & meane proportion in the point C , and vnto the greater



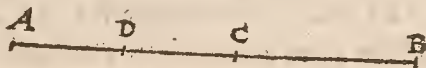
Demonstration.

segment AC is added a line AD , equall to the halfe of the right line AB : therefore (by the 1. of the thirteenth) the square of the line CD , is quintuple to the square of the line AD . Wherefore the square of the line CD , hath to the square of the line AD , that proportion that nūber hath to nūber. Wherefore the square of the line CD is commensurable to the square of the line AD . But the square of the line DA is rationall, for the line DA is rationall, forasmuch as it is the halfe of the rationall line AB . Wherefore the square of the line CD , is rationall. Wherefore also the line CD , is rationall. And forasmuch as the square of the line CD , hath not to the square of the line AD , that proportion that a square number hath to a square number, therefore (by the 9. of the tenth) the line CD , is incommensurable in length, to the line AD . Wherefore the lines CD , and DA are rationall commensurable in power only. Wherefore the line AC is a residuall line, by the 73. of the tenth. Againē, forasmuch as the line AB , is deuied by an extreme and meane proportion, and the greater segment thereof is AC , therefore that which is cotayned vnder the lines AB , & BC , is equall to the square of the line AC . Wherefore the square of the line AC , applyed to the rationall line AB , maketh the bredth BC . But the square of a residuall line, applyed to a rationall line maketh the bredth a first residuall line (by the 97. of the tenth). Wherefore the line CB , is a first residuall line. And it is proued that the line AC , is also a residuall line. If therefore a rationall right line be diuided by an extreme and meane proportion, either of the segments, is an irrationall line of that kinde, which is called a residuall line, which was required to be demonstrated.

¶ A Corollary added by Campane.

Hereby it is manifest, that if the greater segment be a rationall line: the lesse segment shalbe a residuall line.

For if the greater segment AC , of the right line ACB be diuided into two equall partes in the point D , the square of the line DB , shalbe quintuple to the square of the line DC , (by the 3. of this booke.) And forasmuch as the line CD , (beyng the halfe of the rationall line supposed AC) is rationall by the 6. diffinition of the tenth: And vnto the square of the line DC , the square of the line DB is commensurable, (for it is quintuple vnto it) wherefore the square of the line DB , is rationall. Wherefore also the line DB is rationall. And forasmuch as the squares of the lines DB & DC , are not in proportion, as a square nūber is to a square number: therefore the lines DB and DC are incommensurable in length (by the 9. of the tenth.) Wherefore they are commensurable in power only: Wherefore by the 73. of the tenth, the line BC , which is the lesse segment, is a residuall line.



The 7. Theoreme.

The 7. Proposition.

If an equilater Pētagon haue three of his angles whether they follow in order,

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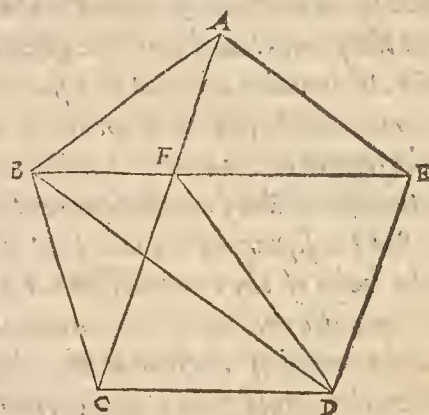
The thirteenth Booke

der, or not in order, equall the one to the other: that Pentagon shalbe equi-
angle.

Two cases in
this proposi-
tion.
Construction.
The first case.
Demonstra-
tion.



Suppose that $ABCDE$, be an equilater pentagon. And let the angles of the
sayd Pentagon, namely, first, three angles folowing in order, which are at the
points A, B, C , be equal the one to the other. Then I say that the Pentagon $ABCDE$
is equiangle. Draw these right lines AC, BE , and FD . Now forasmuch as
these two lines CB , and BA , are equal to these two lines BA , and AE , the one to the other,
and the angle CBA is equall to the angle BAE : therefore (by the 4. of the first) the base AC
is equall to the base BE , and the triangle ABC is equall to the triangle ABE , and the rest
of the angles are equal to the rest of the angles, vnder
which are subtended equall sides. Wherefore the an-
gle BCA is equall to the angle BEA , and the angle
 ABE to the angle CAB . Wherefore also the side AF ,
is equall to the side BF (by the 6. of the first). And
it was proued that the whole line AC is equal to the
whole line BE . Wherefore the residue CF is equall to
the residue FE . And the line CD , is equall to the line
 DE . Wherefore these two lines FC , and CD are e-
quall to hese two lines FE , and ED , and the base FD ,
is common to them both. Wherefore the angle FC
 D , is equall to the angle FED (by the 8. of the
first). And it is proued that the angle BCA , is equal
to the angle BEA . Wherefore the whole angle BCD is equall to the whole angle AED . But
the angle BCD , is supposed to be equall to the angles A , and B . Wherefore the angle AED ,
is equall to the angles A and B . In like sort also may we proue that the angle CDE , is
equall to the angles A , and B . Wherefore the Pentagon $ABCDE$ is equiangle.



The second
case.

But now suppose that three angles, which folow not in order, be equal the one to the o-
ther, namely, let the angles A, C, D , be equall. Then I say that in this case also the Pentagon
 $ABCDE$ is equiangle. Draw a right line from the point B , to the point D . Now forasmuch
as these two lines BA , and AE , are equal to these two lines BC , and CD , and they compre-
hende equall angles, therefore (by the 4. of the first) the base BE , is equall to the base BD .
And the triangle ABE , is equall to the triangle BDC , and the rest of the angles are equall
to the rest of the angles, vnder which are subtended equall sides: wherefore the angle AEB ,
is equall to the angle CDB . And the angle BED , is equall to the angle BDE , (by the 5. of
the first) for the side BE , is equall to the side BD . Wherefore the whole angle AED , is equal
to the whole angle CDE . But the angle CDE , is supposed to be equall to the angles A , and
 C . Wherefore the AED , is equall to the angles A , and C . And by the same reason also the an-
gle ABC , is equall to the angles A , C , and D . Wherefore the Pentagon $ABCDE$ is equian-
gle. If therefore an equilater Pentagon haue three of his angles, whither they folow in order,
or not in order, equall the one to the other: that Pentagon shalbe equiangle: which was re-
quired to be proued.

The 8. Probleme.

The 8. Proposition.

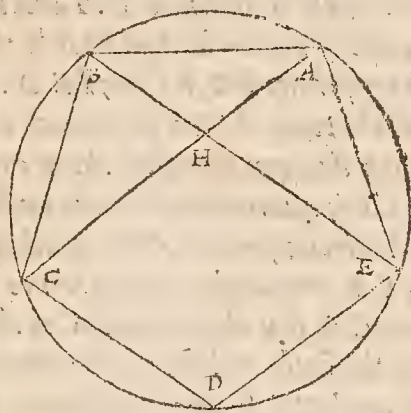
If in an equilater & equiangle Pentagon two right lines do subtend two of
the angles folowing in order: those lines doo diuide the one the other by an
extreme and meane proportion: and the greater segments of those lines are
each equall to the side of the Pentagon.

Suppose



Vppose that $ABCDE$ be an equilater and equiangle Pentagon. And let two right lines AC , and BE , subtend the two angles A , and B , which follow in order. And let them cut the one the other in the point H . Then I say that either of those lines is divided by an extreme & meane proportiō in the point H : And that eche of the greater segments of those lines are equal to the side of the Pentagon. Circumscribe (by the 14. of the fourth) about the Pentagon $ABCDE$, a circle $ABCDE$. And forasmuch as these two right lines EA , and AB , are equal to these two right lines AB , and BC , and they contayne equall angles: therefore (by the 4. of the first) the base BE , is equal to the base AC : and the triangle ABE , is equall to the triangle ABC , and the angles remayning, are equal to the angles remayning, the one to the other, vnder which are subtended equall sides. Wherefore the angle BAC , is equall to the angle ABE . Wherefore the angle AHE is double to the angle BAH , (by the 32. of the first) for it is an outward angle of the triangle ABH . And the angle EAC is double to the angle BAC (by the last of the sixth). For the circumference EDC is double to the circumference CB . Wherefore the angle $HA E$ is equall to the angle AHE . Wherefore also the right line HE , is (by the 6. of the first) equall to the right line EA , that is to the line AB . And forasmuch as the right line BA is equall to the right line AE , therefore the angle ABE is equall to the angle AEB . But it is proued that the angle ABE is equal to the angle BAH : wherefore also the angle BEA is equall to the angle BAH . And in the two triangles ABE , and ABH , the angle ABE is common to them both, wherefore the angle remayning, namely, BAE is equall to the angle remayning, namely, to AHB (by the corollary of the 32. of the first). Wherefore the triangle ABE , is equiangle to the triangle ABH . Wherefore proportionally as the line EB , is to the line BA , so is the line AB to the line BH (by the 4. of the sixth). But the line BA is equall to the line EH . Wherefore as the line BE is to the line EH , so is the line EH to the line HB . But the line BE is greater then the line BA : wherefore the line EH , also is greater then the line HB . Wherefore the line BE , is divided by an extreme and meane proportion in the point H (by the 3. diffinition of the sixth) and his greater segment EH is equal to the side of the Pentagon. In like sort also may we proue that the line AC is divided by an extreme and meane proportion in the point H , and that his greater segment CH , is equal to the side of the Pentagon. (For the whole line AC is equall to the whole line BE , and it hath bene proued that the parts taken away BH , and AH are equall. wherefore the residue CH is equall to the residue EH (by the 19. of the fifth). If therefore in an equilater and equiangle Pentagon two right lines do subtend two of the angles following in order: those lines doo diuide the one the other by an extreme and meane proportion: and the greater segments of those lines are eche equal to the side of the Pentagon: which was required to be demonstrated.

Construction
Demonstration.



¶ The 9. Theoreme.

The 9. Proposition.

If the side of an equilater hexagon, and the side of an equilater decagon or teangled figure, which both are inscribed in one & the selfe same circle, be added together: the whole right line made of them is a line diuided by an extreame and meane proportion, and the greater segment of the same is

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the

The thirteenth Booke

the side of the hexagon.

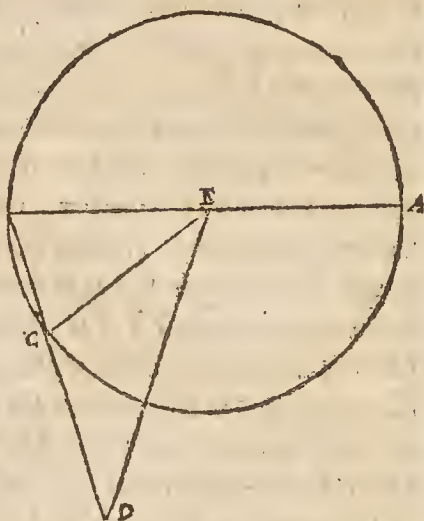


Suppose that there be a circle ABC . And let the side of a decagon or tenangled figure inscribed in the circle ABC , be BC , and let the side of an hexagon or six angled figure inscribed in the same circle, be CD . And let the lines BC and CD be so ioyned together directly that they both make one right line, namely, BD .

Construction.

Demonstration.

Then I say that the line BD is divided by an extreame and meane proportion in the point C : and that the greater segmēt thereof is the line CD . Take (by the 1. of the third) the centre of the circle. And let it be the point E : and draw these right lines EB , EC , and ED . And extend the line BE to the point A . Now forasmuch as BC is the side of an equilater decagon, therefore the circumference or semicircle ACB is quintuple to the circumference CB . Wherefore the circumference AC is quadruple to the circumference CB . But as the circumference AC is to the circumference CB , so is the angle AEC to the angle CEB , by the last of the sixth. Wherefore the angle AEC is quadruple to the angle CEB . And forasmuch as the angle ECB is equall to the angle ECB (by the 5. of the first,) for the line



EB is equall to the line EC , by the diffinition of a circle, therefore the angle AEC is double to the angle ECB , by the 32. of the first. And forasmuch as the right line EC is equall to the right line CD , by the corollary of the 15. of the fourth (for either of them is equall to the side of the hexagon inscribed in the circle ABC) therefore the angle CED is equall to the angle CDE : wherefore the angle ECB is double to the angle EDC , by the 32. of the first. But it is proued that the angle AEC is double to the angle ECB , wherefore the angle AEC is quadruple to the angle EDC . And it is proued that the angle AEC is quadruple to the angle ECB . Wherefore the angle EDC is equall to the angle ECB . And the angle EBD is common to the two triangles ECB and EDC : wherefore the angle remayning BED is equall to the angle remayning ECB , by the corollary of the 32. of the first. Wherefore the triangle EBD is equiangle to the triangle ECB . Wherefore, by the 4. of the sixth, proportionally, as the line BD is to the line BE , so is the line BE to the line BC . But the line EB is equall to the line CD . Wherefore as the line BD is to the DC , so is the line DC to the line CB . But the line BD is greater then the line DC : wherefore also the line DC is greater then the line CB . Wherefore the right line BD is divided by an extreame and meane proportion in the point C : and his greater segment is DC . If therefore the side of an equilater hexagon, and the side of an equilater decagon or tenangled figure, which both are inscribed in one and the selfe same circle, be added together: the whole right line made of them, is a line divided by an extreame and meane proportion, and the greater segment of the same is the side of the hexagon: which was required to be proued.

A Corollary added by Flussas.

This Corollary is the 3. proposition of the 14. booke after Campane.

Hereby it is manifest, that the side of an exagon inscribed in a circle being cut by an extreame and meane proportion, the greater segment thereof is the side of the decagon inscribed in the same circle. For if from the right line DC be cut of a right line equall to the line CB , we may thus reason, as the whole DB is to the whole DC , so is the part taken away DC to the part taken away CB : wherefore by the 19. of the fifth, the residue is to the residue as the whole is to the whole. Wherefore the line DC is cutte like vnto the line DB : and therefore is cut by an extreame and meane proportion.

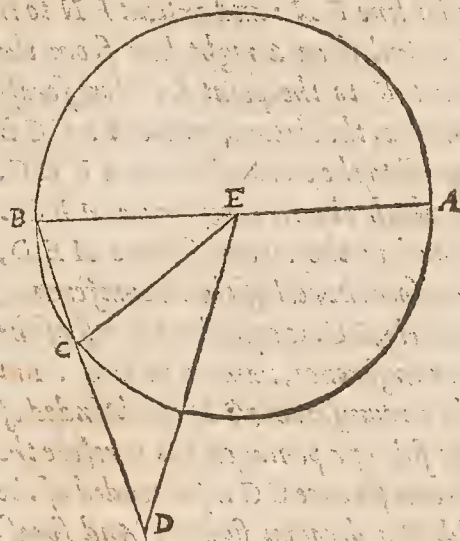
Campane

Campane putteth the conuerse of this proposition after this maner.

If a line be diuided by an extreme and meane proportion, of what circle the greater segment is the side of an equilater Hexagon, of the same shall the lesse segment be the side of an equilater Decagon. And of what circle the lesse segment is the side of an equilater Decagon, of the same is the greater segment the side of an equilater Hexagon.

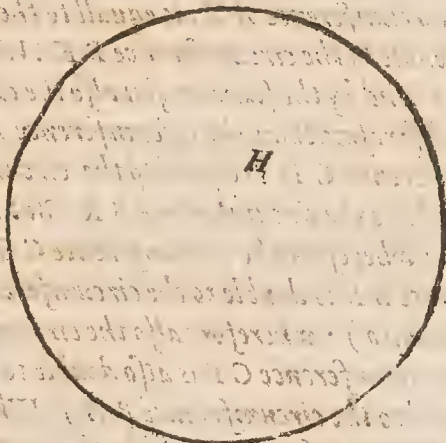
For the former figure remayning, suppose that the line BD be diuided by an extreme and meane proportion in the point C : and let the greater segment thereof be DC . The I say that of what circle the line DC is the side of an equilater Hexagon, of the same circle is the line CB the side of an equilater decagon: and of what circle, the line BC is the side of an equilater Decagon, of the same is the line DC the side of an equilater Hexagon. For if the line DC be the side of an Hexagon inscribed in the circle, then by the corollary of the 15. of the fourth, the line DC is equal to the line BE . And forasmuch as the proportion of the line BD to the line DC is as the proportion of the line DC to the line CB , by supposition: therefore (by the 7. of the fifth) the proportion of the line BD to the line BE , is as the proportion of the line BE to the line BC . Wherefore (by the 6. of the sixth) the two triangles DEB , ECB are equiangle (for the angle B is common to eche triangle). Wherefore the angle D is equall to the angle CEB : for they are subtended of sides of like proportion. And forasmuch as the angle AEC is quadruple to the angle D (by the 32. of the first twise taken, and by the 5. of the same) therefore the same angle AEC is quadruple to the angle CEB . Wherefore (by the last of the sixth) the circumference AC is quadruple to the circumference CB . Wherefore the line BC is the side of a decagon inscribed in the circle ACB .

Demonstration of the first part.



But now if the line BC be the side of a decagon inscribed in the circle ABC , the line CD shall be the side of an Hexagon inscribed in the same circle. For let DC be the side of an Hexagon inscribed in the circle H . Now by the first part of this proposition the line BC shall be the side of a decagon inscribed in the same circle. Suppose that in the two circles ABC and H be inscribed equilater decagons, all whose sides shall be equal to the line CB . And forasmuch as euery equilater figure inscribed in a circle is also equiangle, therefore bothe the decagons are equiangle. And forasmuch as all the angles of the one taken together, as it is easy to be proued by that which is added after the 32. of the first, therefore one of these decagons is equiangle to the other: and therefore the one is like to the other by the definition of like superlicies. And for that if there be two like rectiline figures inscribed in two circles, the proportion of the sides of like proportion of those figures, shall be as the proportion of the Diameters of those circles, as it is easy to proue by the corollary of the 20. of the sixth, and first of this booke: but the sides of the like decagons inscribed in the two circles ABC and H are equal: therefore their Diameters also are equal. Wherefore also their semidiameters are equal. But the semidiameters and the side of the Hexagon are equal, by the corollary of the 15. of the fourth. Wherefore the line DC is the side of an hexagon in the circle ABC , as also it is the side of an hexagon inscribed in the circle F , which is equal to the circle ABC : which was required to be proued.

Demonstration of the second part.



¶ The 10. Theoreme.

The 10. Proposition.

If in a circle be described an equilater Pentagon, the side of the Pentagon containeth in power both the side of an hexagon and the side of a decagon,
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The thirteenth Booke

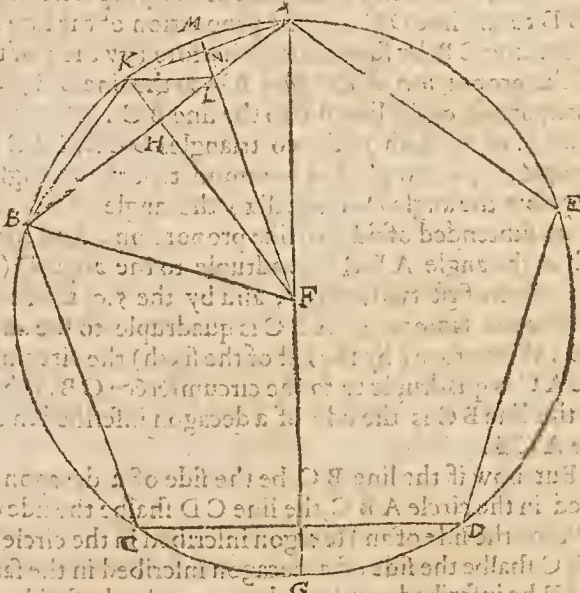
being all described in one and the selfe same circle.



Construction.

Demonstration.

Suppose that $A B C D E$ be a circle. And in the circle $A B C D E$ describe (by the 11. of the fourth) a pentagon figure $A B C D E$. Then I say, that the side of the pentagon figure $A B C D E$ containeth in power both the side of an hexagon figure; and of a decagon figure, being described in the circle $A B C D E$. Take (by the 1. of the third) the centre of the circle, and let the same be F . And drawing a right line from the point A to the point F , extend it to the point G . And drawe a right line from the point F to the point B . And from the point F drawe (by the 12. of the first) unto the line $A B$ a perpendicular line $F H$; and extend it to the point K . And drawe the right lines $A K$ and $K B$. And againe, from the point F draw (by the same) unto the line $A K$, a perpendicular line $F N$; and extend $F N$ to the point M , which line let cut the line $A B$ in the point L . And draw a right line from the point K to the point L . Now forasmuch as the circumference $A B C G$ is equall to the circumference $A E D G$, of which the circumference $A B C$ is equall to the circumference $A E D$, therefore the rest of the circumference, namely, $C G$, is equall to the rest of the circumference, namely, to $D G$. But the circumference $C D$ is subtended of the side of a pentagon: Wherefore the circumference $C G$ is subtended of the side of a decagon figure. And forasmuch as the line $B H$ is equall to the line $H A$ (by the 3. of the third) and the line $F H$ is common to them both, the angles at the point H are right angles: Wherefore (by the 4. of the first) the angle $A F K$ is equall to the angle $K F B$. Wherefore also (by the 26. of the third) the circumference $A K$, is equall to the circumference $K B$. Wherefore the circumference $A B$ is double to the circumference $B K$. Wherefore the right line $A K$ is the side of a decagon figure. And by the same reason also the circumference $A K$ is double to the circumference $K M$. And forasmuch as the circumference $A B$ is double to the circumference $B K$, but the circumference $C D$ is equall to the circumference $A B$: wherefore the circumference $C D$ is double to the circumference $B K$. But the circumference $C D$ is double to the circumference $C G$: wherefore the circumference $C G$ is equall to the circumference $B K$. But the circumference $B K$ is double to the circumference $K M$ (for that the circumference $K A$ is double therunto) : wherefore also the circumference $C G$ is double to the circumference $K M$. But the circumference $C B$ is also double to the circumference $B K$ (for the circumference $C B$ is equall to the circumference $B A$). Wherefore the whole circumference $G B$ is double to the whole circumference $B M$, by the 12. of the fifth. Wherefore also the angle $G F B$ is double to the angle $B F M$, by the last of the sixth. But the angle $G F B$ is double to the angle $F A B$ (by the 32. of the first, or 20. of the third). For the angle $F A B$ is equall to the angle $A B F$. Wherefore the angle $B F L$ is equall to the angle $F A B$, by the 15. of the fifth. And the angle $A B F$ is common to the two triangles $A B F$ and $B F L$. Wherefore (by the Corollary of the 32. of the first) the angle remayning $A F B$ is equall to the angle remayning $B L F$. Wherefore the triangle $A B F$ is equiangle to the triangle $B F L$. Wherefore (by the 4. of the sixth) proportionally as the right line $A B$ is to the right line $B F$, so is the same right line $F B$ to the



the right line BL : wherefore that which is contained under the lines AB and BL is equall to the square of the line BF , by the 17. of the sixt. Againe forasmuch as the line AL is equall to the line LK [for by the last of the sixt, the angle KFL is equall to the angle $AF L$, which equall angles are contained under the lines FK, FL , and FA, FL , & the line FK is equall to the line FA , and the line FL is common to them both. Wherefore, by the 4. of the first, the line AL is equall to the line LK] and the line LN is common to them both, & maketh right angles at the point N , and (by the 3. of the third) the base AN is equall to the base KN . Wherefore also the angle LKN is equall to the angle $L A N$. But the angle $L A N$ is equall to the angle KBL (by the 5. of the first). Wherefore the angle LKN is equall to the angle KBL . And the angle KAL is common to both the triangles AKB and AKL . Wherefore the angle remaining AKB is equall to the angle remayning ALK (by the Corollary of the 32. of the first). Wherefore the triangle KBA is equiangle to the triangle KLA . Wherefore (by the 4. of the sixt) proportionally as the right line BA is to the right line AK , so is the same right line KA to the right line AL . Wherefore that which is contained under the lines BA and AL is equall to the square of the line AK (by the 17. of the sixt). And it is proued that that which is contained under the lines AB and BL , is equall to the square of the line BF . Wherefore that which is contained under AB & BL together with that which is contained under BA and AL (which by the 2. of the second, is the square of the line BA) is equall to the squares of the lines AF and AK . But the line BA is the side of the pentagon figure, and AF the side of the hexagon figure (by the Corollary of the 15. of the fourth), and AK the side of the decagon figure. Wherefore the side of a pentagon figure, containeth in power both the side of an hexagon figure, and of a decagon figure, being described all in one and the selfe same circle: which was required to be demonstrated.

¶ A Corollary added by Flussas.

A perpendicular line from any angle drawn to the base of a pentagon, passeth by the centre.

For if we drawe a right line from the poynt A to the poynt C , and an other from the poynt A to the point D : those right lines shall be equall, by the 4. of the first: and therefore in the triangle ACD the angles at the points C and D , are by the 5. of the first, equall. But the angles made at the point where the line AG cutteth the line CD , are by supposition right angles: wherefore by the 26. of the first, the line CD is by the line AG diuided into two equall partes, and it is also diuided perpendicularly: wherefore by the corollary of the first of the third in the line AG is the centre of the circle: and therefore the line AG passeth by the centre.

¶ The 11. Theoreme.

The 11. Proposition.

If in a circle hauing a rationall line to his diameter be inscribed an equilateral pentagon: the side of the pentagon is an irrational line, and is of that kinde which is called a lesse line.



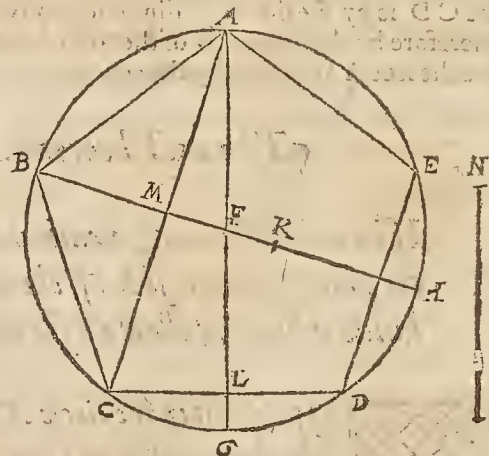
Suppose that in the circle $ABCDE$ hauing a rationall line to his diameter be inscribed a pentagon figure $ABCDE$. Then I say that the side of the pentagon figure $ABCDE$, namely, the side AB , is an irrational line of that kinde which is called a lesse line. Take (by the 1. of the third) the centre of the circle, and let the same be the point F , and draw a right line from the point A to the point F , and an other from the point F to the point B , and extend those lines to the pointes G and H . And draw a right line from the point A to the point C . And from the semidiameter FH take the fourth part (by the 9. of the sixt) and let the same be FK : But the line FH is rationall (for that it is the halfe of the diameter which is supposed to be rationall), wherefore also the line

Construction.

The thirteenth Booke

Demonstration.

line FK is rationall. And the line or semidiameter BF is rationall. Wherefore the whole line BK is rationall. And forasmuch as the circumference ACG is equall to the circumference ADG , of which the circumference ABC is equall to the circumference AED , wherefore the residue CG is equall to the residue GD . Now if we drawe a right line from the point A to the point D , it is manifest that the angles ALC and ALD are right angles. For forasmuch as the circumference CG is equall to the circumference GD , therefore (by the last of the sixth) the angle CAG is equall to the angle DAG . And the line AC is equall to the line AD , for that the circumferences which they subtend are equall, and the line AL is common to them both, therefore there are two lines AC and AD equall to two lines AD and AL , and the angle CAL is equall to the angle DAL . Wherefore (by the 4. of the first) the base CL is equall to the base LD , and the rest of the angles to the rest of the angles, and the line CD is double to the line CL . And by the same reason may it be proued, that the angles at the point M are right angles, and that the line AC is double to the line CM . Now forasmuch as the angle ALC is equall to the angle AMF , for that they are both right angles, and the angle LAC is common to both the triangles ALC and AMF : wherefore the angle remayning, namely, ACL , is equal to the angle remayning AFM , by the corollary of the 32. of the first. Wherefore the triangle ACL is equiangle to the triangle AMF . Wherefore proportionally, by the 4. of the sixth, as the line LC is to the line CA , so is the line MF to the line FA . And in the same proportion also are the doubles of the antecedents LC and MF (by the 15. of the fifth). Wherefore as the double of the line LC is to the line CA , so is the double of the line MF to the line FA . But as the double of the line MF is to the line FA , so is the line MF to the halfe of the line FA , by the 15. of the fifth, wherefore as the double of the line LC is to the line CA , so is the line MF to the halfe of the line FA , by the 11. of the fifth. And in the same proportion, by the 15. of the fifth, are the halues of the consequents, namely, of CA and of the halues of the line FA . Wherefore as the double of the line LC is to the halfe of the line CA , so is the line MF to the fourth part of the line FA . But the double of the line LC is the line DC , and the halfe of the line CA is the line CM , as hath before bene proued, and the fourth part of the line FA is the line FK (for the line FK is the fourth part of the line FA by construction). Wherefore as the line DC is to the line CM , so is the line MF to the line FK . Wherefore by composition (by the 18. of the fifth) as both the lines DC and CM are to the line CM , so is the whole line MK to the line FK . Wherefore also (by the 22. of the sixth) as the squares of the lines DC and CM are to the square of the line CM , so is the square of the line MK to the square of the line FK . And forasmuch as (by the 8. of the thirteenth) a line which is subtended vnder two sides of a pentagon figure, as is the line AC , being diuided by an extreame & meane proportion, the greater segment is equall to the side of the pentagon figure, that is, vnto the line DC : and (by the 1. of the thirteenth) the greater segment hauing added vnto it the halfe of the whole, is in power quintuple to the square made of the halfe of the whole: and the halfe of the whole line AC is the line CM . Wherefore the square that is made of the lines DC and CM , that is, of the greater segment and of the halfe of the whole, as of one line, is quintuple to the square of the line CM , that is, of the halfe of the whole. But as the square made of the lines DC and CM , as of one line, is to the square of the line CM , so is it proued, that the square of the line MK is to the square of the line FK . Wherefore the square of the line MK is quintuple to the square of the line FK . But the square of the line FK is rationall, as hath



hath before bene proued, wherefore also the square of the line $M K$ is rationall, by the 9. diffinition of the tenth), for the square of the line $M K$ hath to the square of the line $K F$ that proportion that number, hath to number, namely, that 5. hath to 1. and therefore the sayd squares are commensurable, by the 6. of the tenth). Wherefore also the line $M K$ is rationall. And forasmuch as the line $B F$ is quadruple to the line $F K$ (for the semidiameter $B F$ is equal to the semidiameter $F H$), therefore the line $B K$ is quintuple to the line $F K$. Wherefore the square of the line $B K$ is 25. times so much as the square of the line $K F$, by the corollary of the 20. of the sixth. But the square of the line $M K$, is quintuple to the square of the $F K$, as is proued. Wherefore the square of the line $B K$ is quintuple to the square of the line $K M$. Wherefore the square of the line $B K$, hath not to the square of the line $K M$, that proportion that a square number hath to a square number, by the corollary of the 25. of the eighth. Wherefore (by the 9. of the tenth) the line $B K$ is incommensurable in length to the line $K M$, and either of the lines is rationall. Wherefore the lines $B K$ and $K M$ are rationall commensurable in power onely. But if from a rationall line be taken away a rationall line being commensurable in power onely to the whole, that which remaineth is irrationall, and is (by the 73. of the tenth) called a residuall line. Wherefore the line $M B$ is a residuall line. And the line conueniently ioyned vnto it, is the line $M K$. Now I say that the line $B M$ is a fourth residuall line. Vnto the excesse of the square of the line $B K$ aboue the square of the line $K M$, let the square of the line N be equall (which excesse how to finde out, is taught in the assumpt put after the 13. proposition of the tenth). Wherefore the line $B K$ is in power more then the line $K M$ by the square of the line N . And forasmuch as the line $K F$ is commensurable in length to the line $F B$, for it is the fourth part thereof, therefore (by the 16. of the tenth) the whole line $K B$ is commensurable in length to the line $F B$. But the line $F B$ is commensurable in length to the line $B H$, namely, the semidiameter. to the diameter: wherefore the line $B K$ is commensurable in length to the line $B H$, by the 12. of the tenth. And forasmuch as the square of the line $B K$ is quintuple to the square of the line $K M$, therefore the square of the line $B K$ hath to the square of the line $K M$ that proportion that five hath to one. Wherefore by conuersion of proportion (by the corollary of the 19. of the fifth) the square of $B K$ hath to the square of the line N , that proportion that five hath to fower: & therefore it hath not that proportion that a square number hath to a square number, by the corollary of the 25. of the eighth. Wherefore, the line $B K$ is incommensurable in length to the line N (by the 9. of the tenth). Wherefore the line $B K$ is in power more then the line $K M$, by the square of a line incommensurable in length to the line $B K$. Now then forasmuch as the whole line $B K$ is in power more then the line conueniently ioyned, namely, then $K M$, by the square of a line incommensurable in length to the line $B K$, and the whole line $B K$ is commensurable in length to the rationall line geuen $B H$: therefore the line $M B$ is a fourth residuall line, by the diffinition of a fourth residuall line. But a rectangle parallelogramme contayned vnder a rationall line and a fourth residuall line, is irrationall, and the line which contayneth in power the same parallelogramme is also irrationall, and is called a lesse line (by the 94. of the tenth). But the line $A B$ contayneth in power the parallelogramme contayned vnder the lines $H B$ and $B M$ (for if we drawe a right line from the point A to the point H , the triangle $A B H$ shall be like to the triangle $A B M$, by the 8. of the sixth. For from the right angle $B A H$ is drawnen to the base $B H$ a perpendicular line. And therefore as the line $B H$ is to the line $B A$, so is the line $A B$ to the line $B M$. this followeth also of the corollary of the sayd 8. of the sixth. Wherefore the line $A B$ which is the side of the pentagon figure, is an irrationall line of that kinde which is called a lesse line. If therefore in a circle hauing a rationall line to his diameter be inscribed an equilater pentagon, the side of the pentagon is an irrationall line, and is of that kinde which is called a lesse line: which was required to be demonstrated.

¶ The 12. Theoreme.

The 12. Proposition.

If in a circle be described an equilater triangle: the square made of the side of
the
MMm.i.

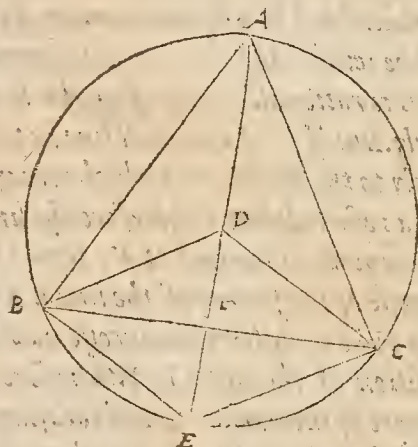
The thirtenth Booke

the triangle, is treble to the square made of the line, which is drawn from the centre of the circle to the circumference.

Construction.

Demonstration.

Suppose that ABC be a circle, and in it describe an equilateral triangle ABC . Then I say that the square made of the side of the triangle ABC is treble to the square made of the line drawn from the center of the circle ABC to the circumference. Take (by the 1. of the third) the centre of the circle, and let the same be D . And draw a right line from the point A to the point D , and extend it to the point E . And draw a right line from the point B to the point E . Now forasmuch as the triangle ABC is equilateral, therefore each of these three circumferences AB , AC , & BEC is the third part of the whole circumference of the circle ABC : wherefore the circumference BE is the sixth part of the circumference of the circle [for the circumference of the semicircle ABE is equal to the circumference of the semicircle ACE , from which taking away equal circumferences AB and AC , the circumference remaining BE shall be equal to the circumference remaining EC]: wherefore the right line BE is the side of an equilateral hexagon figure described in the circle. Wherefore it is equal to the line drawn from the centre of the circle to the circumference, that is unto the line DE (by the corollary of the 15. of the sixth). And forasmuch as the line AE is double to the line DE , therefore the square of the line AE is quadruple to the square of the line DE (by the 4. of the second): that is, to the square of the line BE . But the square of the line AE is equal to the squares of the lines AB , and BE (by the 47. of the first) for the angle ABE is (by the 31. of the third) a right angle. Wherefore the squares of the line AB & BE are quadruple to the square of the line BE . Wherefore taking away the square of the line BE , the square of the line AB shall be treble to the square of BE : but the line BE is equal to the line DE . Wherefore the square of the line AB is treble to the square of the line DE . Wherefore the square made of the side of the triangle, is treble to the square made of the line drawn from the centre of the circle to the circumference: which was required to be proved.



✱ A Corollary added by Campane.

Hereby it is manifest, that the line BC , which is the side of the equilateral triangle, divideth the semidiameter DE into two equal parts. For let the point of the division be F . And suppose a line to be drawn from the point D to the B , and another from the point D to the point C . Now it is manifest (by the 4. of the first) that the line BF is equal to the line FC , and therefore (by the 3. of the third) all the angles at the point F are right angles. Wherefore (by the 47. of the first) the square of the line BD is equal to the squares of the lines BF and FD , and by the same the square of the line BE is equal to the squares of the lines BF , and FE : but the line BD is equal to the line BE (as hath before been proved). Wherefore by the common sentence the two squares of the two lines BF and FD are equal to the two squares of the lines BF , and FE . Wherefore taking away the square of the line BF which is common to them both: the residue, namely, the square of the line DE shall be equal to the residue, namely, to the square of the line FE . Wherefore also the line FD is equal to the line FE . Wherefore hereby it is manifest that a perpendicular line drawn from the centre of a circle to the side of an equilateral triangle inscribed in it, is equal to half of the line drawn from the centre of the same circle, to the circumference thereof.

✱ A Corollary added by Fluffas.

The side of an equilateral triangle is in power sesquitercia to the perpendicular line which is drawn from one of the angles to the opposite side. For of what parts the line AB containeth in power 12. of such

such parts the line BF which is the halfe of AB containd in power 3. Wherefore the residue, namely, the perpendicular line AF containeth in power of such parts 9. (for the squares of the lines AF , and BF are by the 47. of the first equall to the square of the line AB). Now 12. to 9. is sesquitertia: wherefore the power of the line AB is to the power of the line AF in sesquitertia proportion.

Moreover the side of the triangle is the meane proportionall betwene the diameter and the perpendicular line: For (by the Corollary of the 8. of the sixth) the line AE is to the line AB as the line AB is to the line AF .

Farther the perpendicular line drawn from the angle diuideth the base into two equal parts and passeth by the center. For if there should be drawn any other right line frō the point A to the poyn^t F , the that which is drawn by the point D , two right lines should include a superficies, which is impossible. Wherefore the contrary followeth, namely, that the line, which being drawn from the angle passeth by the center, is a perpendicular line to the base (by the 3. of the third).

This Corollary is the 11. proposition of the 14. booke after Campano.

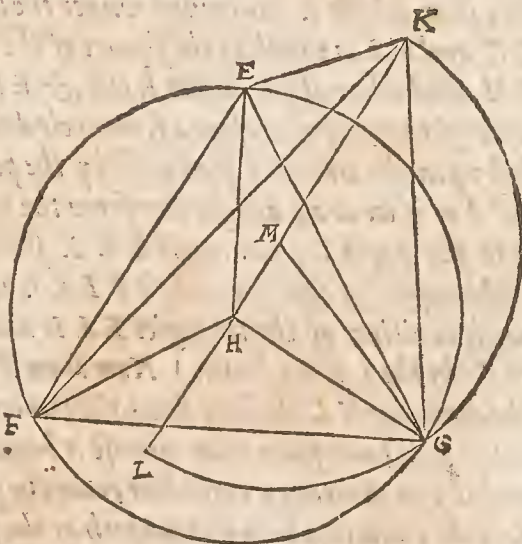
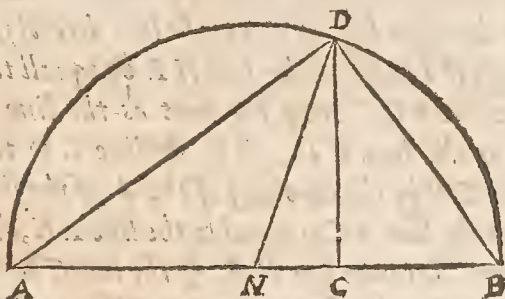
This Corollary is the 3. Corollary after the 17. proposition of the 14. booke after Campano.

The 1. Probleme.

The 13. Proposition.

To make a ^{*}Pyramis, and to comprehend it in a sphere geuen: and to proue that the diameter of the sphere is in power sesquialtera to the side of the Pyramis.

ppose that the diameter of the sphere geuen be AB , and diuide AB in the poyn^t C , so that the line AC be double to the line BC (by the 9. of the sixth). And vpon the line AB , making the center the point N , describe a semicircle ADB . And (by the 11. of the first) from the point C rayse vp vnto the line AB a perpendicular line CD . And drawe a right line from the point D to the point A . And describe a circle EFG hauing his semidiameter equall to the line CD . And describe in the circle EFG , an equilater triangle EFG (by the 2. of the fourth). And (by the 1. of the third) take the cētre of the circle, and let the same be the point H . And draw these right lines EH , HF , & HG . And (by the 12. of the eleuenth) frō the point H rayse vp vnto the playne superficies of the circle EFG a perpendicular line HK , & let the line HK be equal to the right line AC . And draw these right lines KE , KF , & KG . Now forasmuch as the line HK is erected perpendicularly to the plaine superficies of the circle EFG , therefore (by the 2. definition of the eleuenth) it maketh right angles with all the right lines that touch it, and which are in the selfe same superficies of the circle EFG . But it toucheth euery one of these right lines, HE , HF and HG : Wherefore the right line HK is erected perpendicularly to euery one of these lines HE , HF , and HG . And forasmuch as the line AC is equal to the line HK , and the line CD to the line HE , and they comprehend right angles, therefore the base DA is equall to the base KE (by the 4. of the first). And by the same reason ech of the lines KF , and KG is equall to the line DA . Wherefore the three lines KE , KF , and KG are equall the one to the other. And forasmuch as the line AC is double to the line CB , (by construction): therefore the line AB is treble to the line BC : but as the line AB is to the



^{*} By the name of a Pyramis both here & in this booke following, vnderstand a Tetrahedron. First part of the construction.

First part of the demonstration.

MMm. ij.

line

The thirteenth Booke

*This Assumpt
is againe at
the end of the
demonstratio
of the probosi-
tuen proued.*

line BC , so is the square of the line AD to the square of the line DC . [Whiche may thus be proued. It is manifest (by the Corollary of the 8. of sixth) that the line CD is the meane proportionall betwene the lines AC and CB . Wherefore (by the corollary of the 20. of the same) the square of the line AC is to the square of the line CD , as the line AC is to the line CB : wherefore by composition (by the 18. of the sixth) the squares of the lines AC and CD are to the square of the line CD , as the line AB is to the line BC . But the squares of the lines AC and CD are (by the 47. of the first) equall to the square of the line AD : wherefore (by the 7. of the sixth) the square of the line AD is to the square of the line DC , as the line AB is to the line BC .] Wherefore the square of the line AD is treble to the square of the line DC . And forasmuch as the line KE is equall to the line AD (as it hath ben proued) and the line HE is put equall to the line CD : therefore the square of the line KE is triple to the square of the line HE . But vnto the square of the same line HE is (by the 12. of the thirteenth) the square of the line FE treble: wherefore the line EF is equall to the line KE . Now the lines KE , KF , and KG are equall the one to the other, as it hath before ben proued, and so also are the lines EF , FG and GE , for that they are the sides of an equilateral triangle. Wherefore euery one of these lines EF , FG , and GE , is equall to euery one of the lines KE , KF and KG . Wherefore these fower triangles EFK , KEF , KFG and KGE are equilateral. Wherefore there is made a Pyramis consisting of fower equall and equilateral triangles, whose base is the triangle EFK and toppe the poynt K .

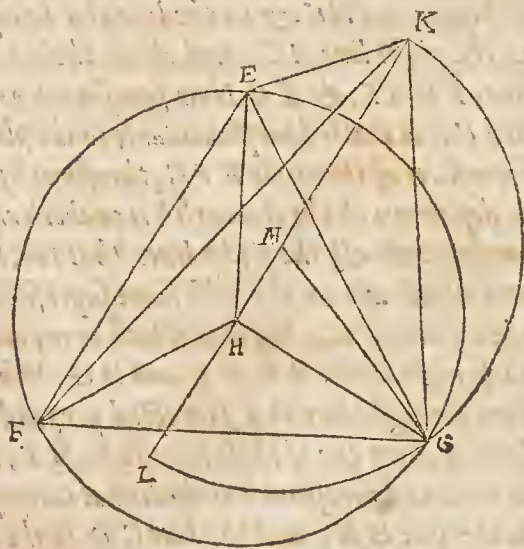
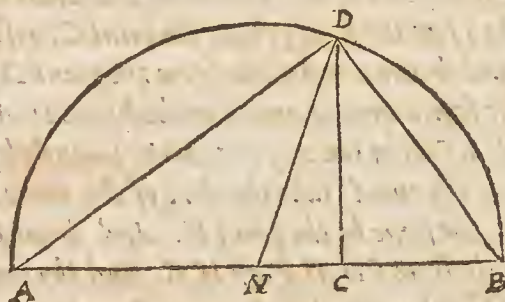
*Second part of
the Constructio.*

*Second part of
the demonstratio.*

** Looke at the
end of this de-
monstration for
an other con-
struction and
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sat.*

** Reade the two
assumpt added
by Campane
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demonstration
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tion) for the bet-
ter vnderstand-
ing of this rea-
son.*

Now it is required to comprehend the same Pyramis in the sphere geuen, and to proue that the diameter of the sphere is in power sesquialtera to the side of the pyramis. Adde vnto the right line HK a right line directly, namely HL , and let the line HK be equall to the line BC . * Now for that as the line AC is to the line CD , so is the line CD to the line CB (by the corollary of the 8. of the sixth) but the line AC is equal to the line KH , & the line CD to the line HE , & the line CB to the line HL . Therefore as the line KH is to the line HE , so is the line HE to the line HL . Wherefore that which is containd vnder the lines HK and HL is equall to the square of the line EH . And ether of the angles KHE , & EHL is a right angle, wherefore a semicircle described vpon the line KL shall passe by the poynt E . * For if we draw a right line from the point E to the poynt L , the angle LEK shalbe a right angle, for that the triagle ELK is equiangle to either of the triangles ELH and EHL (by the 8. of the sixth.) Now then if the the diameter KL abiding fixed, the semicircle be turned round about, vntil it returne vnto the selfe same place from whence it began to be moued, it shall also passe by the pointes F and G . For drawing a right line from the point F to the point L , and an other from the poynt L to the point G , which also maketh at the points F and G right angles, the pyramis shall be containd in the sphere geuen. For the line KL being the diameter of the sphere is equall to the line AB , which is the diameter of the sphere geuen, for the line KH is put equall to the line AC , and the line HL to the line CB .



[In the semicircle $A D B$ of the former figure drawe the line $D N$. And diuide the line $K L$ into two equall parts in the point M . And draw a line from M to G . And forasmuch as by construction the line $K H$ is equall to the line $A C$, and the line $H L$ to the line $C B$: therefore the whole line $A B$ is equal to the whole line $K L$. Wherefore also the halfe of the line $K L$, namely, the line $L M$, is equal to the semidiameter $B N$: wherefore taking away from those equall lines, equall parts $B C$ and $L H$, the residues $N C$, and $M H$ shalbe equall. Wherefore in the two triangles $M H G$ and $N C D$, the two sides about the equall right angles $D C N$ and $G H M$, namely, the sides $D C, C N$; and $G H, H M$, are equall: wherefore the bases $M G$ and $N D$ are equall (by the 4. of the first.) And by the same reason may it be proued that right lines drawen from the poynt M to the points E and F are equal to the line $N D$. But the right line $N D$ is equall to the line $A N$, which is drawen from the centre to the circumference: wherefore the line $M G$ is equall to the line $M K$, & also to the lines $M E, M F$ and $M L$. Wherefore making the cētre the poynt M , and the space $M K$ or $M G$ describe a semicircle $K G L$: and the diameter $K L$ abiding fixed let the sayd semicircle $K G L$ be moued rōunde about vntill it returne to the same place from whence it began to be moued: and there shalbe described a sphere about the centre M (by the 12. diffinition of the eleuenth) touching euery one of the angles of the Pyramis which are at the points K, E, F, G : for those angles are equally distāt from the centre of the sphere, namely, by the semidiameter of the sayd sphere, as hath before bene proued. Wherefore in the sphere geuen whose diameter is the line $K L$, or the line $A B$, is inscribed a Tetrahedron $E F G K$.]

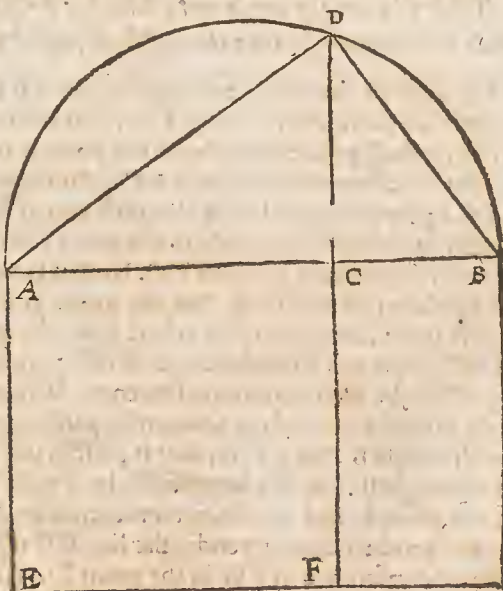
An other construction and demonstration of the second part after Flus-sa.

Now I say, that the diameter of the sphere is in power sesquialtera to the side of the Pyramis. For forasmuch as the line $A C$ is double to the line $C B$ (by cōstructiō) therefore the line $A B$ is treble to the line $B C$. Wherefore by conuerſion) by the corollary of the 19. of the ſiuenth) the line $A B$ is sesquialtera to the line $A C$. But as the line $B A$ is to the line $A C$, so is the square of the line $B A$, to the square of the line $A D$. For if we draw a right line frō the point B , to the point D , as the line $B D$ is to the line $A D$, so is the same $A D$ to the line $A C$, by reaso of the likenes of the triangles $D A B$, & $D A C$ (by the 8. of the sixth): & by reason also that as the first is to the third so is the square of the first to the square of the second (by the corollary of the 20. of the sixth). Wherefore the square of the line $B A$, is sesquialter to the square of the line $A D$. But the line $B A$ is equal to the diameter of the sphere geue, namely, to the line $K L$, as hath bene proued, & the line $A D$ is equal to the side of the pyramis inscribed in the sphere. Wherefore y diameter of the sphere is in power sesquialter to the side of the pyramis. Wherefore there is made a pyramis comprehended in a sphere geuen, and the diameter of the sphere is sesquialtera to the side of the pyramis: which was required to be done and proued.

Third part of the demonstration.

¶ An other demonstration to proue that as the line $A B$ is to the line $B C$, so is the square of the line $A D$ to the square of the line $D C$.

Let the description of the semicircle $A D B$ be as in the first description. And upon the line $A C$ describe (by the 46. of the first) a square $E C$, and make perfecte the parallelograme $F B$. Now forasmuch as the triangle $D A B$ is equiangle to the triangle $D A C$ (by the 32. of the sixth: therefore as the line $B A$ is to the line $A D$, so is the line $D A$ to the line $A C$, by the 4. of the sixth. Wherefore that which is contained vnder the lines $B A$ and $A C$, is equall to the square of the line $A D$, by the 17. of the sixth. And for that as the line $A B$ is to the line $B C$, so is the parallelogramme $E B$ to the parallelogramme $F B$, by the 1. of the sixth: and the parallelogramme $E B$ is that which is contained vnder the lines $B A$ and $A C$ (for the line $E A$ is equall to the line $A C$): and the parallelogramme $F B$ is that which is contained vnder the lines $A C$ and $B C$. Wherefore as the line $A B$ is to the line $B C$, so is that which is contained vnder the lines $B A$ and



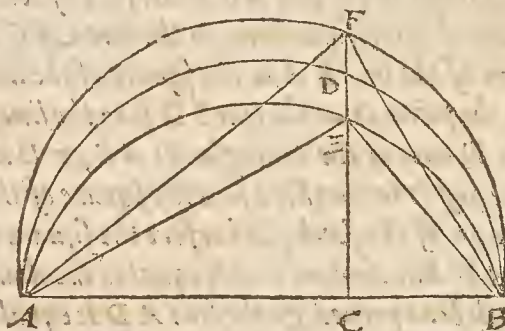
The thirteenth Booke

AC, to that which is contained under the lines *AC* and *CB*. But that which is contained under the lines *BA* and *AC*, is equall to the square of the line *AD*, by the Corollary of the 8. of the sixth, and that which is contained under the lines *AC* and *CB*, is equall to the square of the line *CD*, for the perpendicular line *DC* is the meane proportionall betwene the segmentes of the base, namely, *AC* and *CB*, by the former Corollary of the 8. of the sixth, for that the angle *ADB* is a right angle. Wherefore as the line *AB* is to the line *BC*, so is the square of the line *AD* to the square of the line *DC*, by the 11. of the fifth: which was required to be proued.

¶ Two Assumptes added by Campane.

First Assumpt.

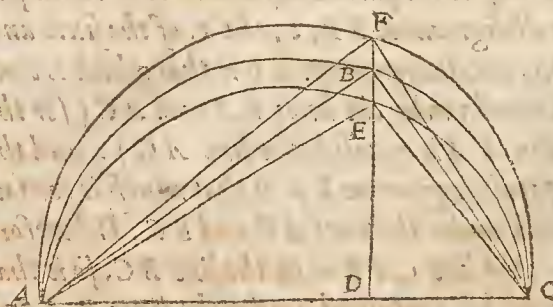
Suppose that vpon the line *AB* be erected perpendicularly the line *DC*, which line *DC* let be the meane proportionall betwene the partes of the line *AB*, namely, *AC* & *CB*: so that as the line *AC* is to the line *CD*, so let the line *CD* be to the line *CB*. And vpon the line *AB* describe a semicircle. The I say, that the circumference of that semicircle shall passe by the point *D*, which is the end of the perpendicular line. But if not, then it shall either cut the line *CD*, or it shall passe aboue it, and include it not touching it. First let it cut it in the point *E*. And draw these right lines *EB* and *EA*. Wherefore by the 31. of the third, the whole angle *AEB* is a right angle. Wherefore by the first part of the Corollary of the 8. of the sixth, the line *AC* is to the line *EC*, as the line *EC* is to the line *CB*. But by the 8. of the fifth, the proportion of the line *AC* to the line *EC*, is greater then the proportion of the same line *AC* to the line *CD* (for the line *CE* is lesse then the line *CD*). Now for that the line *CE* is to the line *CB*, as the line *AC* is to the line *CD*, and the line *CD* is to the line *CB*, as the line *AC* is to the line *CD*, therefore by the 13. of the fifth, the proportion of the line *EC* to the line *CB*, is greater then the proportion of the line *CD* to the line *CB*. Wherefore by the 10. of the fifth, the line *EC* is greater then the line *CD*, namely, the part greater then the whole: which is impossible. Wherefore the circumference shall not cut the line *CD*. Now I say, that it shall not passe aboue it, and not touch it in the point *D*. For if it be possible, let it passe aboue it, and extend the line *CD* to the circumference, and let it cut it in the point *F*. And draw the lines *FB* and *FA*, and it shall followe as before that the line *CD* is greater then the line *CF*: which is impossible. Wherefore that is manifest which was required to be proued.



¶ Second Assumpt.

If there be a right angle vnto which a base is subtended, and if vpon the same be described a semicircle: the circumference thereof shall passe by the point of the right angle.

For suppose that there be a right angle *ABC*, vnto which subtend the base *AC*, and vpon the line *AC* describe a semicircle. Then I say, that the circumference thereof shall passe by the point *B*. For if not, then it shall passe either aboue the point *B*, or vnder the point *B*. First let it passe vnder the point *B*, and let the circumference be *AEC*. And (by the 12. of the first) from the point *B* drawe vnto the line *AC* a perpendicular line *BD*, which let cut the circumference of the semicircle in the point *E*. And drawe these right lines *EA* and *EC*. Now it is manifest, by the 31. of the third, that the angle *AEC* is a right angle. But (by the 21. of the first) the angle *AEC* is greater then the angle *ABC*: which is impossible, by the 10. common sentence. Wherefore the circumference of the semicircle passeth not vnder the angle *B*. Now I say, that it passeth not aboue the angle *B*. For if it be possible, let it passe aboue the point *B*, and let the circumference be *AFC*: and produce the perpendicular line *BD* till it cut the circumference *AFC* in the point *F*. And draw these right lines *AF* and *FC*. Wherefore a-



gaine, by the 31. of the third, the angle AFC is a right angle. But by supposition, the angle ABC is a right angle, and is, by the 21. of the first, greater then the right angle AFC : which againe by the fore-
sayd common sentence, is impossible. Wherefore a semicircle described vpon the base AC , passeth nei-
ther vnder the point B , nor about it. Wherefore it passeth by it: which was required to be proued.

The conuerse of this was added after the demonstration of the 31. of the third, out of Pelitarius. And these two Assumptes of Campanè are necessary, for the better vnderstanding of the demonstration of the second part of this 13. Proposition, wherein is proued that the pyramis is contained in the Sphere geuen.

¶ Certaine Corollaryes added by *Flußas*.

✱ First Corollary.

The diameter of the Sphere is in power quadruple sesquialtera to the line which is drawn from the centre to the circumference of the circle which containeth the base of the pyramis.

For forasmuch as it hath bene proued, that the diameter KL is in power sesquialter to the side EF : and it is proued also, by the 12. of this booke, that the side EF is in power triple to the line EH (which is drawn from the centre of the circle containyng the triangle EFH). But the proportion of the extremes, namely, of the diameter to the line EH , consisteth of the proportions of the meanes, namely, of the proportion of the diameter to the line EF , and of the proportion of the line EF to the line EH , by the 5. definition of the sixt: which proportions, namely, triple, and sesquialter, added together, make quadruple sesquialter (as it is easie to proue by that which was taught in the declaration of the 5. definition of the sixt booke). Wherefore the Corollary is manifest.

¶ Second Corollary.

Onely the line which is drawn from the angle of the pyramis to the base opposite vnto it, & passing by the center of the Sphere, is perpendicular to the base, and falleth vpon the centre of the circle which containeth the base.

This Corollary is the 15. proposition of the 14. booke after Campanè.

For if any other line (then the line KMH which is drawn by the centre of the Sphere to the centre of the circle) should fall perpendicularly vpon the plaine of the base, then, from one and the selfe same point should be drawn to one and the selfe same plaine two perpendicular lines, contrary to the 13. of the eleuenth: which is impossible. Farther if from the toppe K should be drawn to the centre of the base, namely, to the point H , any other right line not passing by the centre M , two right lines shoulde include a superficies contrary to the last common sentence: which were absurde. Wherefore onely the line which is drawn by the center of the Sphere to the centre of the base, is perpendicular to the sayd base. And the line which is drawn from the angle perpendicularly to the base, shall passe by the centre of the Sphere.

✱ Third Corollary.

The perpendicular line which is drawn from the centre of the Sphere to the base of the pyramis, is equall to the sixt part of the diameter of the Sphere.

This Corollary Campanè putteth as a Corollary after the 17. proposition of the 14. booke.

For it is before proued, that the line MH (which is drawn from the centre of the Sphere to the centre of the base) is equall to the line NC : which line NC is the sixt part of the diameter AB , and therefore the line MH is the sixt part of the diameter of the Sphere. For the diameter AB is equall to the diameter of the Sphere, as hath also before bene proued.

¶ The 2. Probleme.

The 14. Proposition.

To make an octohedron, and to cōprehend it in the sphere geuen, namely, that wherein the pyramis was comprehended: and to proue that the diameter of the sphere is in power double to the side of the octahedron.

Take

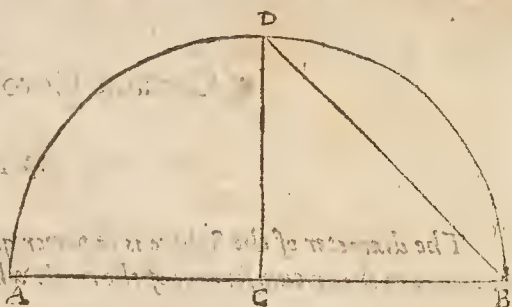
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Construction.



Ake the diameter of the former sphere geuen; which let be the line AB : and divide it (by the 10. of the first into two equall partes in the point C . And describe vpon the line AB a semicircle ADB . And (by the 11. of the first) from the point C rayse vp vnto the line AB a perpendicular line CD . And draw a right line from the point D to the point B . And describe a square $EFGH$ hauing euery one of his sides equall to the line BD .

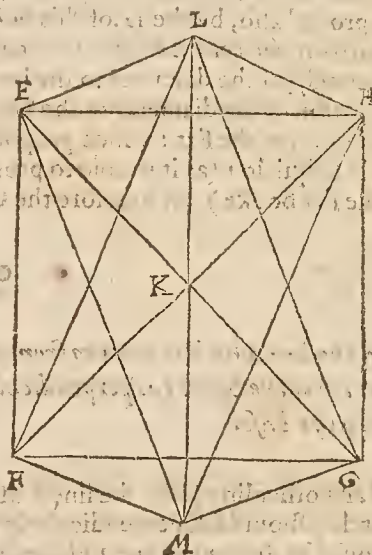
and draw the diagonal lines FH & EG , cutting the one the other in the point K . And (by the 12. of the eleuenth) from the point K (namely, the point where the lines FH and EG cut the one the other) rayse vp to the playne superficies, wherein the square $EFGH$ is, a perpendicular line KL , and extend the line KL on the other side of the playne superficies to the point M . And let eche of the lines KL and



First part of the demonstration.

* For the 4. angles at the point K are equall to fouer right angles by the Corollary of the 15. of the first: and those 4. angles are equall the one to the other by the 8. of the first: and therefore eche is a right angle.

KM be put equall to one of these lines KE , KF , KH or KG . And draw these right lines LE , LF , LG , LH , ME , MF , MG , and MH . Now forasmuch as the line KE is (by the corollary of the 34. of the first) equall to the line KH , and the angle EKH is a right angle, therefore the square of HE is double to the square of EK , by the 47. of the first: Agayne forasmuch as the line LK is equall to the line KE by position, and the angle LKE is by the second diffinition of the eleuenth, a right angle: therefore the square of the line EL is double to the square of the line EK : and it is proued that the square of the line HE is double to the square of the line EK . Wherefore the square of the line LE is equall to the square of the line EH . Wherefore also the line LE is equall to the line EH . And by the same reason the line LH is also equall to the line HE . Wherefore the triangle LHE is equilateral. In like sort may we proue that euery one of the rest of the triangles whose bases are the sides of the square $EFGH$, and toppes the pointes L and M , are equilateral. The sayd eight triangles also are equall the one to the other, for euery side of eche is equall to the side of the square $EFGH$. Wherefore there is made an octohedron cotained vnder eight triangles whose sides are equall. Now it is required to comprehend it in the sphere geuen, and to proue that the diameter of the sphere is in power double to the side of the octohedron.



Second part of the demonstration.

Forasmuch as these three lines LK , KM , and KE are equall the one to the other, therefore a semicircle described vpon the line LM shall passe also by the point E . And by the same reason, if the semicircle be turned round about, vntill it retorne vnto the selfe same place from whence first it began to be moued, it shal passe by the pointes F , G , H , and the octohedron shall be comprehended in a sphere. I say also that it is comprehended in the sphere geuen. For forasmuch as the line LK is equal to the line KM , by position, and the line KE is common to them both, and they contayne right angles by the 3. diffinition of the eleuenth, therefore (by the 4. of the first) the base LE is equall to the base EM . And forasmuch as the angle LEM is a right angle, by the 31. of the third, for it is in a semicircle, as hath bene proued, therefore the square of the line LM is double to the square of the line LE by the 47. of the first. Againe forasmuch as the line AC is equall to the line BC , therefore the line AB is double to the line BC , by the diffinition of a circle. But as the line AB is to the line BC , so is the square of the line

line AB to the square of the line BD , by the corollaries of the 8. and 20. of the sixth. Wherefore the square of the line AB is double to the square of the line BD . And it is proued that the square of the line LM is double to the square of the line LE . Wherefore the square of the line BD is equall to the square of the line LE . For the line EH which is equall to the line LF , is put to be equall to the line DB . Wherefore the square of the line AB is equall to the square of the line LM . Wherefore the line AB is equall to the line LM . And the line AB is the diameter of the sphere geuen, wherefore the line LM is equall to the diameter of the sphere geuen. Wherefore the octoedron is containned in the sphere geuen: * and it is also proued that the diameter of the sphere is in power double to the side of the octoedron. Wherefore there is made an octoedron, and it is comprehended in the sphere geuen, wherein was comprehended the Pyramis: and it is proued that the diameter of the sphere is in power double to the side of the octoedron: which was required to be doone, and to be proued.

* For the square of the line AB , which is proued equall to the square of the line LM , is double to the square of the line BD , which is also equall to the square of the line LE .

Certayne Corollaries added by Fluffas.

✱ First Corollary.

The side of a Pyramis is in power sesquitertia to the side of an octoedron inscribed in the same Sphere.

For forasmuch as the diameter is in power double to the side of the octoedron, therefore of what partes the diameter containeth in power 6. of the same, the side of the octoedron containeth in power 3: but of what partes the diameter containeth 6. of the same, the side of the pyramis containeth 4. by the 13. of this booke. Wherefore of what partes the side of the pyramis containeth 4. of the same the side of the octoedron containeth 3.

Second Corollary.

An octoedron is deuided into two equall and like Pyramids.

The common bases of these Pyramids are set vpon euery square containned of the sides of the octoedron, vpon which square are set the 8. triangles of the octoedron: which pyramids are by the 8. definition of the eleuenth equall and like. And the foresayd square common to those Pyramids, is the halfe of the square of the diameter of the sphere, for it is the square of the side of the octoedron.

This Corollary is the 16. proposition of the 14. booke after Campanus.

Third Corollary.

The three diameters of the octoedron, do cutte the one the other perpendicularly into two equall parts, in the center of the sphere which containeth the sayd octoedron.

As it is manifest by the three diameters EG , FH , and LM which cutte the one the other in the center K equall and perpendicularly.

¶ The 3. Probleme.

The 15. Proposition.

To make a solide called a cube, and to comprehend it in the sphere geuen, namely, that Sphere wherein the former two solides were comprehended: and to proue that the diameter of the sphere, is in power treble to the side of the cube.



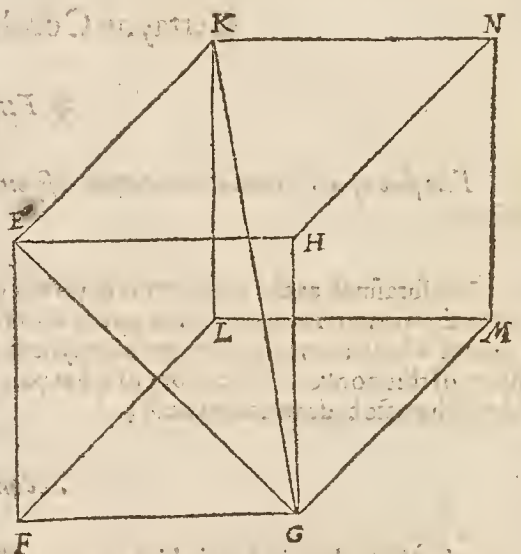
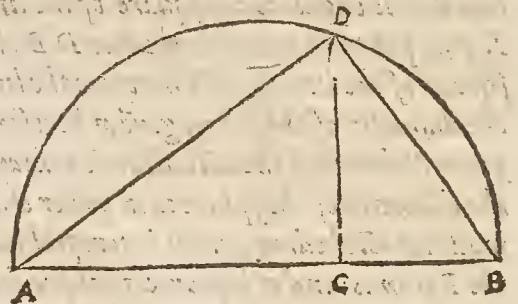
Take the diameter of the sphere geuen, namely, AB , and diuide it in the point C : So that let the line AC be double to the line BC by the 9. of the sixth. And vpon the

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line

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line AB describe a semicircle ADB . And (by the 11. of the first) from the poynt C rayse vp vnto the line AB a perpendicular line CD . And draw a right line DB . And describe a square $EFGH$, hauing euery one of his sides equall to the line DB . And from the pointes E, F, G, H , rayse vp (-by the 12. of the eleuenth) vnto the playne superficies of the square $EFGH$ perpendicular lines EK, FL, GM , and HN : and let euery one of the lines EK, FL, GM , and HN be put equall to one of the lines EF, FG, GH , or HE , which are the sides of the square, and draw these right lines KL, LM, MN , and NK . Wherefore there is made a cube namely FN which is containd vnder six equall squares. Now it is required to comprehend the same cube in the sphere geuen, and to proue that y^e diameter of the sphere is in power ble to the side of the cube. Draw these right lines KG and EG . And forasmuch as the angle KEG is a right angle, for that the line KE is erected perpendicularly to the playne superficies EG , and therefore also to the right line EG , by the 2. diffinitioⁿ of the eleuenth, wherefore a semicircle described vpon the line KG shall passe by the poynt E . Agayne forasmuch as the line FG is erected perpendicu-



larly to either of these lines FL and FE , by the diffinition of a square, & by the 2. diffinition of the eleuenth, therefore the line FG is erected perpendicularly to the playne superficies FK , by the 4. of the eleuenth. Wherefore if we draw a right line from the point F to the point K , the line GF shall be erected perpendicularly to the line KF , by the 2. diffinition of the eleuenth. And by the same reason agayne a semicircle described vpon the line GK shall passe also by the point F . And likewise shall it passe by the rest of the pointes of the angles of that cube. If now the diameter KG abiding fixed the semicircle be turned round about vntill it returne into the selfe same place from whence it began first to be moued, the cube shalbe comprehended in a sphere. I say also that it is comprehended in the sphere geuen.

For forasmuch as the line GF is equall to the line FE and the angle F is a right angle, therefore the square of the line EG is by the 47. of the first double to the square of the line EF . But the line EF is equall to the line EK . Wherefore the square of the line EG is double to the square of the line EK . Wherefore the squares of EG and EK that is the square of the line GK , by the 47. of the first, are treble to the square of the line EK . And forasmuch as the line AB is treble to the line BC , but as the line AB is to the line BC , so is the square of the line AB to the square of the line BD , by the corollaries of the 8. and 20. of the sixth. Wherefore the square of the line AB is treble to the square of the line BD . And it is proued that the square of the line GK is treble to the square of the line KE , and the line KE is put equall to the line BD . Wherefore the line KG is also equall to the line AB . And the line AB is the diameter of the sphere geuen. Wherefore the line KG is equall to the diameter of the sphere geuen. Wherefore the cube is comprehended in the sphere geuen: and it is also proued that the diameter of the Sphere is in power treble to the side of the cube: which was required to be doone, and to be proued.

An other

First part of the demonstration.

Second part of the Constructio.
Second part of the demonstration.

* By the 2. Assumpt of the 13. of this booke.

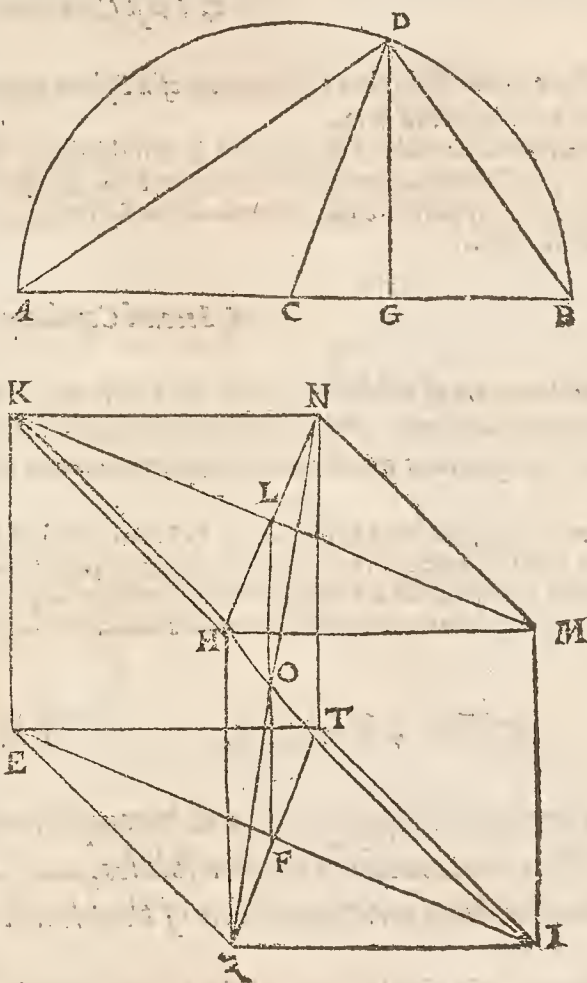
Third part of the demonstration.

✱ An other demonstration after *Flussas*.

Suppose that the diameter of the Sphere geuen in the former Propositions, be the line *A B*. And let the center be the point *C*, vpon which describe a semicircle *A D B*. And from the diameter *A B* cut of a third part *B G*, by the 9. of the sixt. And from the point *G* raise vp vnto the line *A B* a perpendicular line *D G*, by the 11. of the first. And draw these right lines *D A*, *D C*, and *D B*. And vnto the right

line *D B* put an equall right line *Z I*: and vpon the line *Z I* describe a square *E Z I T*. And frō the pointes *E*, *Z*, *I*, *T*, erecte vnto the superficies *E Z I T* perpendicular lines *E K*, *Z H*, *I M*, *T N* (by the 12. of the eleuenth): and put every one of those perpendicular lines equall to the line *Z I*. And drawe these right lines *K H*, *H M*, *M N*, and *N K*, ech of which shall be equall and parallels to the line *Z I*, and to the rest of the lines of the square, by the 33. of the first. And moreouer they shall containe equall angles (by the 10. of the eleuenth): and therefore the angles are right angles, for that *E Z I T* is a square: wherefore the rest of the bases shall be squares. Wherefore the solide *E Z I T K H M N* being cōtained vnder 6. equall squares, is a cube, by the 21. definition of the eleuenth. Extend by the opposite sides *K E* and *M I* of the cube, a plaine *K E I M*: and againe by the other opposite sides *N T* and *H Z*, extend an other plaine *H Z T N*. Now forasmuch as ech of these plaines deuide the solide into two equall partes, namely, into two Prismes equall and like (by the 8. definition of the eleuenth): therefore those plaines shall cut the cube by the centre, by the Corollary of the 39. of the eleuenth. Wherefore the cōmon section of those plaines shall passe by the centre: Let that common section be the line *L F*. And forasmuch as the sides *H N* and *K M* of the superficieses *K E I M* and *H Z T N* do diuide the one the other into two equall partes, by

the Corollary of the 34. of the first, and so likewise do the sides *Z T* and *E I*: therefore the common section *L F* is drawn by these sections, and diuideth the plaines *K E I M* and *H Z T N* into two equall partes, by the first of the sixt: for their bases are equall, and the altitude is one and the same, namely, the altitude of the cube. Wherefore the line *L F* shall diuide into two equall partes the diameters of his plaines, namely, the right lines *K I*, *E M*, *Z N*, and *N T*, which are the diameters of the cube. Wherefore those diameters shall concur and cut one the other in one and the selfe same poynt, let the same be *O*. Wherefore the right lines *O K*, *O E*, *O I*, *O M*, *O H*, *O Z*, *O T*, and *O N*, shall be equall the one to the other, for that they are the halfes of the diameters of equall and like rectangle paralllelogrāmes. Wherefore making the centre the point *O*, and the space any of these lines *O E*, or *O K*. &c. a Sphere described, shall passe by every one of the angles of the cube, namely, which are at the pointes *E*, *Z*, *I*, *T*, *K*, *H*, *M*, *N*, by the 12. definition of the eleuenth, for that all the lines drawn from the point *O* to the angles of the cube are equall. But the right line *E I* containeth in power the two equall right lines *E Z*, and *Z I*, by the 47. of the first. Wherefore the square of the line *E I* is double [to the square] of the line *Z I*. And forasmuch as the right line *K I* subtendeth the right angle *K E I* (for that the right line *K E* is erected perpendicularly to the plaine superficies of the right lines *E Z* and *Z T* (by the 4. of the eleuenth): therefore the square of the line *K I* is equall to the squares of the lines *E I* and *E K*, but the square of the line *E I* is double to the square of the line *E K* (for it is double to the square of the line *Z I*, as hath bene proued, and the bases of the cube are equall squares). Wherefore the square of the line *K I* is triple to the square of the line *K E*, that is, to the square of the line *Z I*. But the right line *Z I* is equall to the right line *D B*, by position, vnto whose square the square of the diameter *A B* is triple, by that which was demonstrated in the 13. Proposition of this booke. Wherefore the diameters *K I* & *D B* are equall. Wherefore there is described a cube *K I*, and it is comprehended in the Sphere geuen wherin the other



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solides were contained, the diameter of which Sphere is the line AB . And the diameter KI or AB of the same Sphere, is proued to be in power triple to the side of the cube, namely, to the line DB , or ZI .

¶ Corollaries added by *Flußas*.

✱ First Corollary.

Hereby it is manifest, that the diameter of a Sphere containeth in power the sides both of a pyramis and of a cube inscribed in it.

For the power of the side of the pyramis is two thirdes of the power of the diameter (by the 13. of this booke). And the power of the side of the cube is, by this Proposition, one third of the power of the sayd diameter. Wherefore the diameter of the Sphere containeth in power the sides of the pyramis and of the cube..

¶ Second Corollary.

All the diameters of a cube cut the one the other into two equall partes in the centre of the sphere which containeth the cube. And moreover those diameters do in the selfe same point cut into two equall partes the right lines which ioine together the centres of the opposite bases.

As it is manifest to see by the right line LOF . For the angles LKO , and FIO , are equall, by the 29. of the first: and it is proued, that they are contained vnder equall sides: Wherefore (by the 4. of the first) the bases LO and FO are equall. In like sort may be proued, that the rest of the right lines which ioine together the centres of the opposite bases do cut the one the other into two equall partes in the centre O .

¶ The 4. Probleme.

The 16. Proposition.

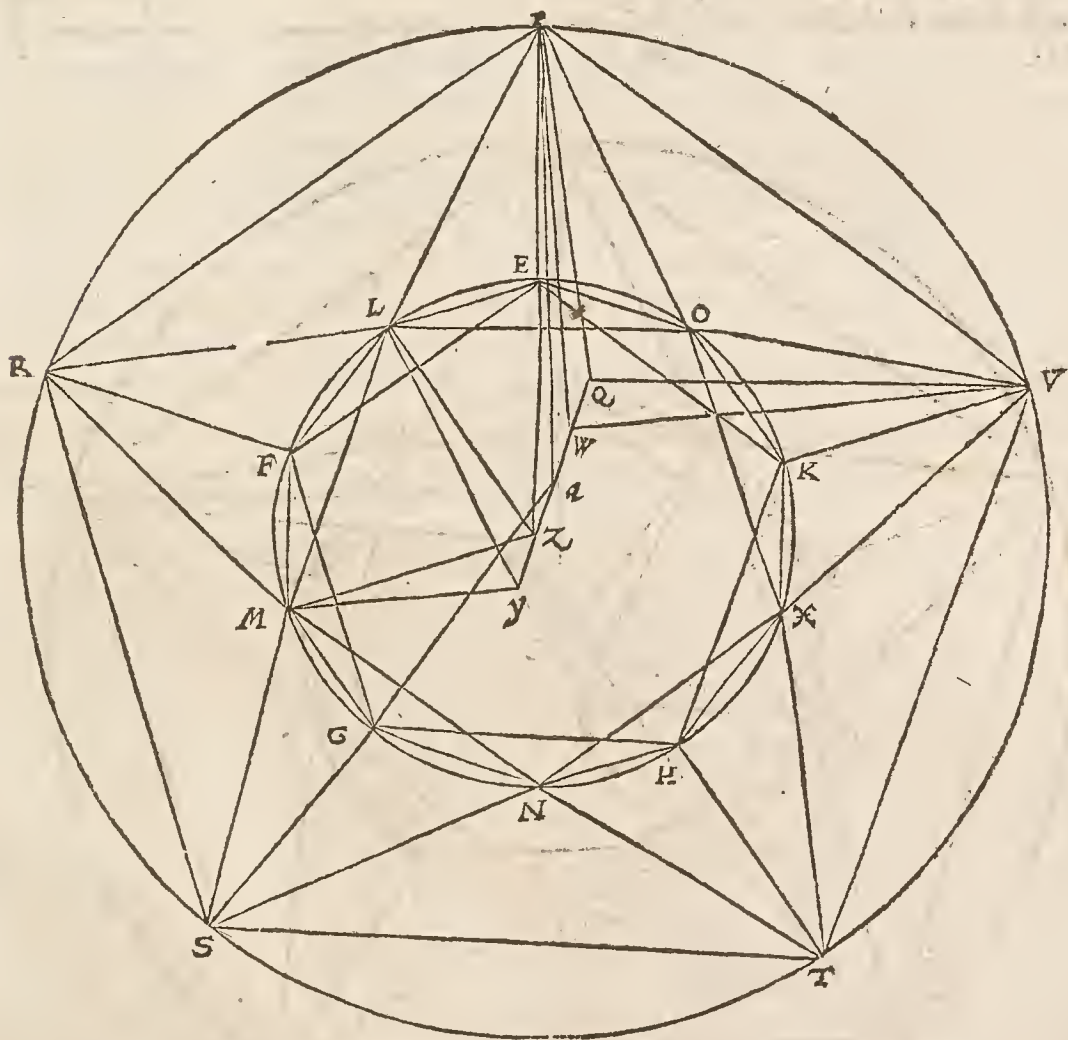
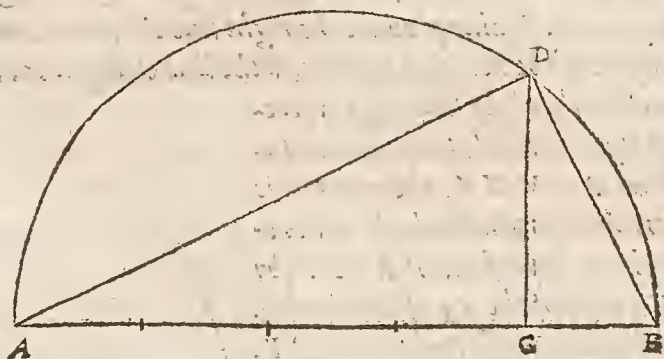
To make an Icosahedron, and to comprehend it in the Sphere geuen, wherein were contained the former solides, and to proue that the side of the Icosahedron is an irrationall line of that kinde which is called a lesse line.

First part of the construction.



Take the diameter of the Sphere, namely, the line AB : and deuide it in the point C , so that let the line AC be quadruple to the line CB , by the 9. of the sixth. And describe vpon the line AB a semicircle ADB . And (by the 11. of the first) from the point C raise vp vnto the line AB a perpendicular line CD . And draw a right line from the point D to the point B . And describe a circle $EFGHK$ whose line from the centre (which let be the point Z) to the circumference, let be equall to the line DB . And in the circle $EFGHK$ describe (by the 11. of the fourth) an equilater and equiangle Pentagon figure $EFGHK$. And deuide the circumferences EF , FG , GH , HK , and KE , into two equall partes in the pointes L, M, N, X, O . Draw also these right lines LM, MN, NX, XO , and OL : and moreover these lines $OE, EL, LF, FM, MG, GN, NH, HX, XK$, and KO , and they shall be the sides of an equilater decagon inscribed in the circle $EFGHK$, by the 29. of the third. Wherefore the figure $LMNXO$ is an equilater pentagon, by the 29. of the third, and the right line EO is the side of a decagon or ten angled figure. Raise vp (by the 12. of the eleuenth) from the pointes E, F, G, H, K , and the centre Z , vnto the plaine superficies of the circle, perpendicular lines EP, FR, GS, HT, KV , and ZW ; and let ech of them be put equall to the line drawen from the centre of the circle $EFGHK$, to the circumference, namely, to the line ZE . Wherefore right lines drawen from W to P , from W to V , from W to T , from W to S , from W to R , shall be equall and parallels to right lines drawen from Z to E , from Z to K , from Z to H , from Z to G , and from Z to F , by the 6. and 7. of the eleuenth, and 33. of the first. Wherefore the plaine superficieses $EFGHK$, and $PRSTV$, which are extended by those parallel lines, are parallel

parallel superficieses, by the 15. of the eleventh. Wherefore making the centre the point W , and the space WP , or WV , describe a circle, and it shall passe by the pointes T, S, R , and shall be equall to the circle $EFGHK$. For the semidiameters of eche are equall. And drawe these right lines PR, RS, ST, TV, VP , and they shall make a pentagon, whose sides shall be equall to the sides of the Pentagon $OLMNX$, by the 29. of the first. For ech of them doth subtend two sides of the decagon, or the fift part of equall circles. From the upper pointes P, R, S, T, V , draw these lines $PO, PL, RL, RM, SM, SN, TN, TX, VX, VO$: which shall subtend right angles contained vnder the sides of the decagon $ELFMGNHXKO$, and the perpendicular lines PE, RF, SG, TH, VK .



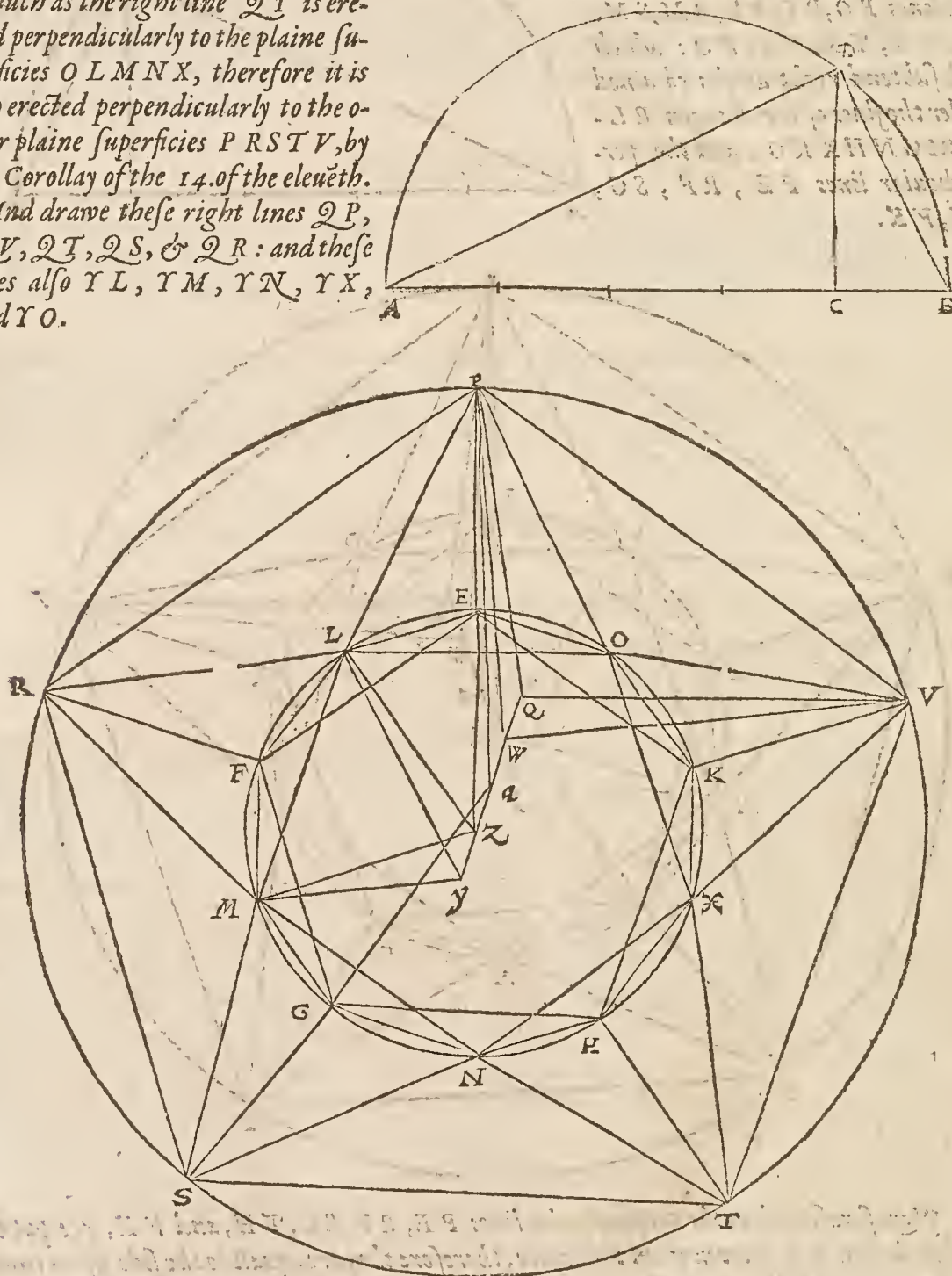
Now forasmuch as the perpendicular lines PE, RF, SG, TH , and VK , are put equall to the line ZE drawn from the centre, therefore they are equall to the side of an equilateral hexagon inscribed in the same circle (by the Corollary of the 15. of the fourth). Wherefore the right lines PO, PL, VO , and VX (which subtend the right angles contained vnder those perpendicular lines and the sides of the decagon) containe them in power, by the 47. of the first: But the side of a pentagon (namely, the side LO or PV) containeth in power the sides of an hexagon and of a decagon inscribed in one and the selfe same circle, by the 10. of

First part of the demonstration.

The thirteenth Booke

this booke. Wherefore the subtending lines $PO, PL, VO, VX, TX, TN, SN, SM, RM, RL$, containe in power the selfe same square that the sides of the pentagon $OLMNX$ containe, or that the sides of the pentagon $PRSTV$ containe: and therefore those subtending lines are equall to the sides of the foresaid pentagons. Wherefore the triangles contained of those subtending lines and of the sides of the pentagons, and which are ten in number, namely, $PLO, OVP, VOX, VXT, TXN, TNS, SNM, SMR, RML$, and RLP , are equilater. Againe produce the right line ZW on either side to the points Q & Y and unto the side of the decagon, namely, to the line OE , put the lines ZY and WZ equal. And forasmuch as the right line ZY is erected perpendicularly to the plaine superficies $OLMNX$, therefore it is also erected perpendicularly to the other plaine superficies $PRSTV$, by the Corollay of the 14. of the eleventh. And draw these right lines ZP, ZV, ZT, ZS , & ZR : and these lines also YL, YM, YN, YX , and YO .

Second part of
the construction.



Second part of
the demonstration.

Now forasmuch as the lines ZP, ZV, ZT, ZS , and ZR do eche subtend right angles containd vnder the sides of an equilater hexagon & of an equilater decagon inscribed in the circle $PQRSTV$ or in the circle $EFGHK$ (which two circles are equall) therefore the sayd lines are eche equal to the side of the pentagon inscribed in the foresayd circle by the 10. of

of this booke, and are equall the one to the other, by the 4. of the first, (for all the angles at the poynt W which they subtend are right angles). Wherefore the five triangles $\triangle P V$, $\triangle P R$, $\triangle R S$, $\triangle S T$, and $\triangle T V$, which are containd under the sayd lines $\triangle V$, $\triangle P$, $\triangle R$, $\triangle S$, $\triangle T$, and under the sides of the pentagon $V P R S T$, are equilater, and equal to the ten former triangles. And by the same reason, the five triangles opposite unto them, namely, the triangles $\triangle M L$, $\triangle M N$, $\triangle N X$, $\triangle X O$, and $\triangle O L$, are equilater and equal to the said ten triangles. For the lines $\triangle L$, $\triangle M$, $\triangle N$, $\triangle X$, and $\triangle O$ do subtend right angles cotayned under the sides of an equilater hexagon and of an equilater decagon inscribed in the circle $E F G H K$, which is equall to the circle $P R S T V$. Wherefore there is described a solide containd under 20. equilater triangles. Wherefore by the last diffinition of the eleventh there is described an Icosahedron.

Now it is required to comprehend it in the sphere geuen, and to proue that the side of the Icosahedron is an irrationall line of that kinde which is called a lesse line. Forasmuch as the line $Z W$ is the side of an hexagon, & the line $W \triangle$ is the side of a decagon, therefore the line $Z \triangle$ is diuided by an extreme and meane proportion in the point W , and his greater segmēt is $Z W$ (by the 9. of the thirtieth). Wherefore as the line $\triangle Z$ is to the line $Z W$, so is the line $Z W$ to the line $W \triangle$. But the $Z W$ is equall to the line $Z L$ by construction, and the line $W \triangle$ to the line $Z Y$ by construction also: Wherefore as the line $\triangle Z$ is to the line $Z L$, so is the line $Z L$ to the line $Z Y$, and the angles $\triangle Z L$ and $L Z Y$ are right angles (by the 2. diffinition of the eleventh). If therefore we draw a right line from the poynt L to the poynt $\triangle$, the angle $\triangle L \triangle$ shalbe a right angle, by reaso of the likenes of the triangles $\triangle L \triangle$ and $Z L \triangle$ (by the 8. of the sixth). Wherefore a semicircle described vpon the line $\triangle Y$, shal passe also by the point L (by the assumpts added by Campane after the 13. of this booke). And by the same reaso also, for that as the line $\triangle Z$ is the line $Z W$, so is the line $Z W$ to the line $W \triangle$,* but the line $Z \triangle$ is equall to the line $\triangle W$, and the line $Z W$ to the line $P W$: wherefore as the line $\triangle W$ is to the line $P W$, so is the line $P W$ to the line $W \triangle$. And therefore agayne if we draw a right line from the poynt P to the point $\triangle$, the angle $\triangle P \triangle$ shalbe a right angle. Wherefore a semicircle described vpon the line $\triangle Y$ shal passe also by the point P , by the former assumpts: & if the diameter $\triangle Y$ abiding fixed the semicircle be turned round about, until it come to the selfe same place from whence it began first to be moued, it shall passe both by the point P , and also by the rest of the pointes of the angles of the Icosahedron, and the Icosahedron shalbe comprehended in a sphere. I say also that it is containd in the sphere geuen.

Diuide (by the 10. of the first) the line $Z W$ into two equall parts in the point a . And forasmuch as the right line $Z \triangle$ is diuided by an extreme and meane proportion in the point W , and his lesse segment is $\triangle W$, therefore the segment $\triangle W$ hauing added vnto it the halfe of the greater segment, namely, the line $W a$, is (by the 3. of this booke) in power quintuple to the square made of the halfe of the greater segment: wherefore the square of the line $\triangle a$ is quintuple to the square of the line $a W$. But vnto the square of the $\triangle a$, the square of the line $\triangle Y$ is quadruple (by the corollary of the 20. of the sixth) for the line $\triangle Y$ is double to the line $\triangle a$; and by the same reason vnto the square of the $W a$ the square of the line $Z W$ is quadruple: Wherefore the square of the line $\triangle Y$ is quintuple to the square of the line $Z W$ (by the 15. of the sixth). And forasmuch as the line $A C$ is quadruple to the line $C B$, therefore the line $A B$ is quintuple to the line $C B$. But as the line $A B$ is to the line $B C$, so is the square of the line $A B$ to the square of the line $B D$ (by the 8. of the sixth, and corollary of the 20. of the same). Wherefore the square of the line $A B$ is quintuple to the square of the line $B D$. And it is is proued that the square of the line $\triangle Y$ is quintuple to the square of the line $Z W$. And the line $B D$ is equall to the line $Z W$, for either of them is by position equall to the line which is drawn from the centre of the circle $E F G H K$ to the circumference. Wherefore the line $A B$ is equall to the $\triangle Y$. But the line $A B$ is the diameter of the sphere geuen: Wherefore the line $\triangle Y$, which is proued to be the diameter of the sphere containyng the Ico-

For the line $\triangle W$, is equall to the line $\triangle Z$, & the line $Z W$, is commo to them both.

This part is againe afterward demonstrated by Flusius.

(ahedron

The thirteenth Booke

Icosahedron, is equall to the diameter of the sphere geuen. Wherefore the *Icosahedron* is containd in the sphere geuen. Now I say that the side of the *Icosahedron* is an irrational line of that kinde which is called a lesse line. For forasmuch as the diameter of the sphere is rational, and is in power quintuple to the square of the line drawen frō the centre of the circle $O L M N X$: wherefore also the line which is drawen from the centre of the circle $O L M N X$ is rationally: wherefore the diameter also being comensurable to the same line (by the 6. of the tenth) is rationally. But if in a circle having a rationally line to his diameter be described an equilateral pentagon, the side of the pentagon is (by the 11. of this booke) an irrational line, of that kinde which is called a lesse line. But the side of the pentagon $O L M N X$ is also the side of the *Icosahedron* described, as hath before ben proued. Wherefore the side of the *Icosahedron* is an irrational line of that kinde which is called a lesse line. Wherefore there is described an *Icosahedron* and it is containd in the sphere geuen, and it is proued that the side of the *Icosahedron* is an irrational line of that kind which is called a lesse line. Which was required to be done, and to be proued.

A Corollary.

Hereby it is manifest that the diameter of the sphere, is in power quintuple to the line which is drawen from the centre of the circle to the circumference, on which the *Icosahedron* is described. And that the diameter of the sphere is composed of the side of an hexagon, and of two sides of a decagon described in one and the selfe same circle.

Flussas proueth the *Icosahedron* described, to be cotayned in a sphere, by drawing right lines from the poynt a , to the poyntes P and G after this maner.

Forasmuch as the lines $Z W$, $W P$ are put equal to the line drawen from the centre to the circumference, and the line drawen from the centre to the circumference is double to the line $a W$, by construction: therefore the line $W P$ is also double to the same line $a W$. Wherefore the square of the line $W P$ is quadruple to the square of the line $a W$ (by the corollary of the 20. of the sixth). And those lines $P W$ and $W a$ containing a right angle $P W a$ (as hath before bene proued) are subtended of the right line $a P$. Wherefore (by the 47. of the first) the line $a P$ containeth in power the lines $P W$, and $W a$. Wherefore the right line $a P$ is in power quintuple to the line $W a$. Wherefore the right lines $a P$, and $a Q$ being quintuple to one and the same line $W a$, are (by the 9. of the fifth) equall. In like sort we may we proue that vnto those lines $a P$ and $a Q$, are equall the rest of the lines drawen from the poynt a to the rest of the angles R, S, T, V . For they subtend right angles containd of the line $W a$, and of the lines drawen from the centre to the circumference. And forasmuch as vnto the line $W a$ is equall the line $V a$, which is likewise erected perpendicularly vnto the other plaine superficies $O L M N X$: therefore lines drawen from the point a to the angles O, L, M, N, X , and subtending right angles at the point Z containd vnder lines drawen frō the centre to the circumference, and vnder the line $a Z$, are equall not onely the one to the other, but also to the lines, drawen frō the sayde poynt a , to the former angles at the poynts, P, R, S, T, V . For the lines drawen frō the centre to the circumference of each circle are equall, & the line $a W$ is equall to the line $a Z$. But the line $a P$ is proued equal to the line $a Q$, which is the halfe of the whole $Q Y$. Therefore the residue $a Y$ is equall to the forelayd lines $a P$, $a Q$ &c. Wherefore making the centre the poynt a , and the space one of those lines $a Q$, $a P$, &c. extende the superficies of a sphere, & it shal touch the 12. angles of the *Icosahedron*, which are at the pointes $O, L, M, N, X, P, K, S, T, V, Q, Y$: which sphere is described, if vpō the diameter $Q Y$, be drawen a semicircle, and the sayd semicircle be moued about, till it returne vnto the same place from whence it began first to be moued.

A Corollary added by *Flussas*.

The opposite sides of an *Icosahedron* are parallels. For the diameters of the sphere do fall vpon the opposite angles of the *Icosahedron*: as it was manifest by the right line $Q Y$. If therefore there be imagined to be drawen the two diameters $P N$, and $O M$ they shall concur in the point F : wherefore the right lines which ioine them together, $P V$, and $L N$, are in one and the selfe same playne superficies, by the

the 3. of the eleventh. And forasmuch as the alternate angles at the endes of the diameters are equall (by the 8. of the first): for the triangles containd vnder equall semidiameters and the side of the Ico-
sahedron are equiangle: therefore (by the 28. of the first) the lines P V and L N are paralles.

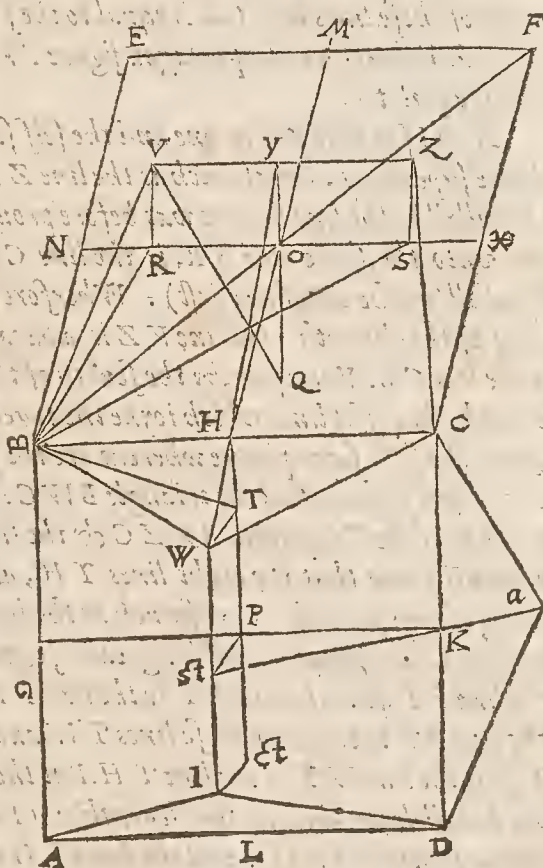
¶ The 5. Probleme.

The 17. Proposition.

To make a Dodecabedron, and to comprehend it in the sphere geuen,
wherin were comprehended the foresayd solides: and to proue that the side
of the dodecabedron is an irrationall line of that kind which is called a re-
siduall line.

TAke two playne superficieses or bases of the foresayde cube, which let be the two
squares ABCD and CBEF, cutting the one the other in the line BC per-
pendicularly according to the nature of a cube. And (by the 10. of the first) di-
vide euery one of the lines AB, BC, CD, DA, EF, EB, and FC into two equall partes in
the poyntes G, H, K, L, M, N, X. And drawe these right lines GK and HL, cutting the one
the other in the point P, and likewise draw the right lines MH and NX cutting the one the
other in the poynt O. And diuide euery one of these right lines NO, OX, HP, and LP,
by an extreme and meane proportion in the
pointes R, S, T, & et, and let their greater segments
be RO, OS, TP, and P & et. And (by the 12. of
the eleventh) frō the poynts R, O, S, rayse vp
to the outward part of the playne superficieses E-
BCF of the foresayd cube, perpendicular lines
RV, OY & SZ: and let eche of those perpēdi-
cular lines be equall to one of these lines RQ,
OS or TP, which perpendiculars shalbe paral-
lels (by the 6. of the eleventh), and likewise
from the pointes T, P, & et, rayse vp vnto the out-
ward part of the playne superficieses ABCD of
the sayd cube, these perpendicular lines TW,
P st, and et I, eche of which perpendicular lines
put equall also to the line OS, or OR or TP:
and the sayd perpendiculars shalbe parallels (by
the foresayd 6. of the eleventh). And draw
theseright linesXH, HW, BW, WC, CZ and
CB. Now I say that the pentagon figure
KBWCZ is equilater and in one and the self
same plaine superficies, and moreouer is equi-
angle. Draw these right lines TB, RB, and
SB. And forasmuch as the right line NO is
divided by an extreme and meane proportion in
the poynt R, and his greater segment is the line
RO, therefore the square of the lines NO and NR are treble to the square of the line RO
(by the 4. of this booke). But the line ON is equall to the line NB, and the line OR to the
line RV. Wherefore the squares of the lines BN and RN are treble to the square of the line
RV. But vnto the squares of the lines BN and RN is equall the square of the line BR
(by the 47. of the first). Wherefore the square of the line BR is treble to the square of the
line RV. Wherefore the squares of the lines BR and RV are quadruple to the square of
the line RV. But vnto the squares of the lines BR and RV is equall the square of the line

First part of
the construc-
tion.



The pentagon
VBWCZ,
proued eqns-
later.

100-107

A geometric diagram of a cube, labeled with letters. The vertices are labeled: E (top-left-back), M (top-left-front), F (top-right-back), V (top-left-front), Y (top-middle-front), Z (top-right-front), N (middle-left-back), R (middle-left-front), S (middle-right-back), O (center), Q (center), H (middle-left-back), C (middle-right-back), B (bottom-left-back), T (bottom-middle-back), W (bottom-left-front), P (bottom-middle-front), a (bottom-right-front), K (bottom-right-back), S (bottom-left-front), I (bottom-middle-front), L (bottom-left-back), and D (bottom-right-back). Lines connect these points to form the edges and diagonals of the cube and its internal structures.

The pentagon
V B W C R,
proued to be in
one and the selfe
same playne su-
perficies.

pentagon

pentagon figure $V B W C Z$ is in one and the selfe same playne superficies.

Now also I say that it is equiangle. For forasmuch as the right line NO is divided by an extreame and meane proportion in the point R , and his greater segment is OR , therefore as both the lines NO and OR added together is to the line ON , so (by the 5. of this booke) is the line ON to the line OR . But the line OR is equall to the line OS . Wherefore as the line SN is to the line NO , so is the line NO to the line OS . Wherefore the line SN is divided by an extreame and meane proportion in the point O , and his greater segment is the line NO . Wherefore the squares of the lines NS and SO are treble to the square of the line NO (by the 4. of this booke). But the line NO is equall to the NB , and the line SO to the line SZ : wherefore the squares of the lines NS and ZS are treble to the square of the line NB : wherefore the squares of the lines ZS , SN and NB , are quadruple to the square of the line NB . But unto the squares of the lines SN & NB (by the 47. of the first) is equal the square of the line SB : wherefore the squares of the lines BS and SZ , that is, the square of the line BZ , by the 47. of the first, (for the angle ZSB is a right angle by positio) is quadruple to the square of the line NB . Wherefore the line BZ is double to the line BN (by the Corollary of the 20. of the sixth). But the line BC is also double to the line BN . Wherefore the line BZ is equall to the line BC . Now forasmuch as these two lines BV and VZ are equall to these two lines BW and WC , and the base BZ is equall to the base BC , therefore (by the 8. of the first) the angle BVZ is equall to the angle BWC . And in like sort (by the 8. of the first) may we proue that the angle VZC is equall to the angle BWC (proving first that the lines CB and CV are equal: which are proued equal by this, that the line NS is equal to the line XR , and therefore the line CR is equal to the line BS , by the 47. of the first: wherefore also by the same γ line CV is equal to the line BZ , that is, to the line BC (for the lines BC & BZ are proued equal.) Wherefore the three angles BWC , BVZ , and VZC are equall the one to the other. But if in an equilateral pentagon figure there be thre angles equall the one to the other, the pentagon is (by the 7. of the thirteth) equiangle: wherefore the pentagon $BVZCW$ is equiangle. And it is also proued that it is equilateral: Wherefore the pentagon $BVZCW$ is both equilateral & equiangle. And it is made vpon one of the sides of the cube, namely, vpon BC . * If therefore vpon euery one of the twelue sides of the cube be vsed the like construction, there shal then be made a dodecahedron containd vnder twelue pentagons equilateral and equiangle.

Now it is required to comprehend it in the sphere geuen, and to proue that the side of the dodecahedron is an irrationall line of that kinde which is called a residuall line. Extend the line TO , and let the line extended be TQ : now then the line TQ shall light vppon the diameter of the cube, and shall diuide the one the other into two equall parts. For this is manifest to se by the 39. of the eleuenth. (For if by the two lines NX and MH be drawn two playnes perpendicularly to the bases, and cutting the cube, the common section of those playnes shalbe the line TO produced: for their common section is from the poynt O erected perpendicularly to the plaine $EBCF$, by the 19. of the eleuenth). Let them cut the one the other in the point Q : wherefore Q is the centre of the sphere which comprehendeth the cube, and TQ is the halfe diameter of the sphere by that which was demöstrated in the 15. of this booke: wherefore the right lines drawn from the centre Q to all the angles of the cube shalbe equall. And draw a right line from the point V to the point Q . Now forasmuch as the right line NS is divided by an extreame and meane proportion in the point O , and his greater segment is the line NO , as hath before ben proued, therefore the squares of the lines NS and SO are treble to the square of the line NO , by the 4. of this booke. But the line NS is equal to the line TQ (for the line NO is equal to the line OQ as hath before ben proued, & the line TO to the line OS) being both lesse segmentes: but the line OS is equall to the line TV , for the line RO is equall thereunto: wherefore the squares of the lines QI and TV are treble to the square of the line NO . But unto the squares of the lines QI & TV the square of the line VQ is equall (by the 47. of the first): wherefore the square of the line VQ is treble to the

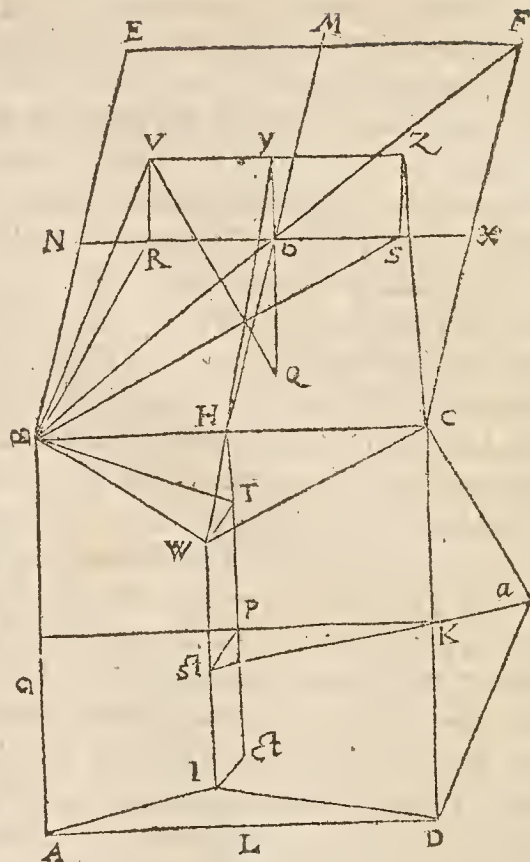
The pentagon $V B W C Z$ is proued equiangle.

* Looke for a farther construction after Flussas at the ende of the demonstration.

That the dodecahedron is containd in the sphere geuen.

The thirteenth Booke

Square of the line NO . But the semidiameter of the sphere cōprehēding the said cube is in power treble to the halfe of the side of the cube. For we haue before (in the 15. of this booke) taught how to make a cube, and to comprehend it in a sphere, and haue proued that the diameter of the sphere is in power treble to the side of the cube. Now in what proportiō y whole is to the whole, in the same is the halfe to the halfe (by the 15. of the fifth). But the line NO is the halfe of the side of the cube. Wherefore the line VZ is equall to the semidiameter of the sphere cōprehēding the cube. But the point Z is the centre of the sphere cōprehēding the cube. Wherefore the point V , which is one of the angles of the dodecahedron, toucheth the superficies of the sphere geuen. In like sort also may we proue, that euery one of the rest of the angles of the dodecahedron toucheth the superficies of the sphere. Wherefore the dodecahedron is comprehended in the sphere geuen.



That the side of the dodecahedron is a residuall line.

Now I say, that the side of the dodecahedron is an irratiōall line of that kinde which is called a residuall line. For forasmuch as the line NO is diuided by an extreme and meane proportion in the point R , and his greater segment is the line OR , and the line OX is also diuided by an extreme and meane proportion in the point S , and his greater segment is the line OS . Wherefore the whole line NX is diuided by an extreme and meane proportion, and his greater segment is the line RS . (For for that as the line ON is to the line OR , so is the line OR to the line NR , and in the same proportion also are their doubles (for the partes of equemultiplices haue one and the selfe same proportion with the whole, by the 15. of the fifth). Wherefore as the line NX is to the line RS , so is the line RS to both the lines NR and SX added together. But the line NX is greater then the line RS , by both the lines NR and SX added together. Wherefore the line NX is diuided by an extreme and meane proportion, and his greater segment is the line RS . But the line RS is equall to the line VZ , as hath before bene proued. Wherefore the line NX is diuided by an extreme and meane proportion, and his greater segment is the line VZ . And forasmuch as the diameter of the Sphere is ratiōall, and is in power treble to the side of the cube, by the 15. of this booke, therefore the line NX , being the side of the cube, is ratiōall. But if a ratiōall line be diuided by an extreme and meane proportion, either of the segmentes is (by the 6. of this booke) an irratiōall line of that kinde which is called a residuall line. Wherefore the line VZ being the side of the dodecahedron, is an irratiōall line of that kinde which is called a residuall line. Wherefore there is made a dodecahedron, and it is cōprehēded in the Sphere geuen, wherein the other solides were contained, and it is proued that the side of the dodecahedron is a residuall line: which was required to be done, and also to be proued.

¶ Corollary.

Hereby it is manifest, that the side of a cube being diuided by an extreme and

and meane proportion, the greater segment thereof is the side of the dodecahedron. As it was manifest by the line VZ which was proued to be the greater segment of the right line NX ; namely, of the side of the cube.

A further construction of the dodecahedron after *Flussas*.

Forasmuch as it hath bene proued that the pentagon $BVZCW$ is equilater and equiangle and toucheth one of the sides of the cube. Let vs show also by what meanes vpon eche of the 12. sides of the cube may in like sort be applyed pentagons ioyning one to the other, and composing the 12. bases of the dodecahedron. Draw in the former figure these right lines $AI, ID, IL, \text{and } K$. Now forasmuch as the line PL was in the point ϵ diuided like vnto the lines PH, ON . or OX , and vpon the pointes T, P, ϵ , were erected perpendicular lines equall vnto the line OY , and the rest: namely, vnto the greater segmēt: and the lines TW and ϵI were proued parallels, therefore the lines WI and $T\epsilon$ are parallels, by the 7. of the eleuenth, and 33. of the first. Wherefore also, by the 9. of the eleuenth, the lines WI and DC are parallels. Wherefore by the 7. of the eleuenth $CWID$ is a playne superficies. And the triangle AID is a playne superficies, by the 2. of the eleuenth. Now it is manifest that the right lines $ID, \text{and } IA$ are equall to the right line WC . For the right lines $AL \text{ and } L\epsilon$ (which are equall to the right lines $BH, \text{and } HT$) do make the subtēded lines $A\text{ and } BT$ equall by the 4. of the first. And agayne forasmuch as the lines BT and TW contayne a right angle BTW , as also doo the right lines $A\epsilon$ and ϵI contayne the right angle $A\epsilon I$ (for the right lines WT , and $I\epsilon$ are erected perpendicularly vnto one and the selfe same playne $ABCD$ by supposition). And the squares of the lines BT and TW are equall to the squares of the lines $A\epsilon$, and ϵI (for it is proued that the line BT is equall to the line $A\epsilon$, and the line TW to the line ϵI). And vnto the squares of the lines BT and TW is equall the square of the line BW , by the 47. of the first: likewise by the same vnto the squares of the lines $A\epsilon$ and ϵI is equall the square of the line AI . Wherefore the square of the line BW is equall to the square of the line AI , wherefore also the line BT is equall to the line AI . And by the same reason are the lines ID and WC equall to the same lines. Now forasmuch as the lines AI and ID , and the lines AL and LD are equall, and the base IL is common to them both, the angles ALI and DLI shalbe equall, by the 8. of the first: and therefore they are right angles, by the 10. diffinition of the first. And by the same reason are the angles WHB , and WHC right angles. And forasmuch as the two lines HT and TW are equall to the two lines $L\epsilon$ and ϵI , and they contayne equall angles, that is, right angles by supposition, therefore the angles WHT , and $IL\epsilon$, are equall by the 4. of the first. Wherefore the playne superficies AID is in like sort inclined to the playne superficies $ABCD$, as the playne superficies BWC is inclined to the same playne $ABCD$, by the 4. diffinition of the eleuenth. In like sort may we proue that the playne $WCDI$ is in like sort inclined to the playne $ABCD$, as the playne $BVZC$ is to the playne $EBCF$. For that in the triangles YOH and $\text{and } PK$ which consist of equall sides (eche to his correspondent side), the angles YHO , and $\text{and } KP$, which are the angles of the inclination, are equall. And now if the right line $\text{and } K$ be extended to the point a , and the pentagon $CWID$ a be made perfect, we may, by the same reason, proue that that playne is equiangle and equilater, that we proued the pentagon $BVZCW$ to be equaliter and equiangle. And likewise if the other playnes $BWIA$ and AID be made perfect, they may be proued to be equall and like pentagons and in like sort situate, and they are set vpon these common right lines BW, WC, WI, AI , and ID . And obseruing this methode, there shall vpon euery one of the 12. sides of the cube be set euery one of the 12. pentagons which compose the dodecahedron.

Draw in the former figure these lines, $A, AL, \text{and } D$.

¶ Certayne Corollaryes added by *Flussas*.

✧ First Corollary.

The side of a cube, is equall to the right line which subtendeth the angle of the pentagon of a dodecahedron contayned in one and the selfe same sphere with the cube.

For the angles BWC and AID , are subtended of the lines BC and AD . Which are sides of the Cube.

¶ Second Corollary.

In a dodecahedron there are sixe sides euery two of which are parallels and opposite, whose sections into two equall partes, are coupled by three right lines, which in the center of the sphere which contayneth the dodecahedron, denide into two equall partes and perpendicularly both them selues and also the sides.

The thirteenth Booke

For vpon the fixe bases of the cube are set sixe sides of the dodecahedron, as it hath bene proued (by the lines ZV, WI &c.) which are cutte into two equall partes by right lines, which ioine together the centers of the bases of the cube, as the line YO produced, and the other like. Which lines coupling together the centers of the bases are three in number, cutting the one the other perpendicularly (for they are parallels to the sides of the cube) and they cutte the one the other into two equall partes in the center of the sphere which containeth the cube (by that which was demonstrated in the 15. of this booke). And vnto these equall lines, ioyning together the centers of the bases of the cube, are without the bases added equall partes OY, P π , and the other like, which by supposition are equall to halfe of the side of the dodecahedron. Wherefore the whole lines, which ioine together the sectoins of the opposite sides of the dodecahedron, are equall, and they cut those sides into two equall partes and perpendicularly.

Third Corollary.

A right line ioyning together the poynts of the sections of the opposite sides of the dodecahedron into two equall partes, being diuided by an extreame and meane proportion: the greater segment thereof shalbe the side of the cube, and the lesse segment the side of the dodecahedron contained in the selfe same sphere.

For it was proued that the right line YQ is diuided by an extreame and meane proportion in the poynt O, and that his greater segment OQ is halfe the side of the cube, and his lesse segment OY is halfe of the side VZ (which is the side of the dodecahedron). Wherefore it followeth (by the 15. of the fifth) that their doubles are in the same proportion. Wherefore the double of the line YQ which ioyneth the poynt opposite vnto the line Y, is the whole: and the greater segment is the double of the line OQ which is the side of the cube: & the lesse segment is the double of the line YO, which is equall to the side of the dodecahedron, namely, to the side VZ.

The 6. Probleme.

The 18. Proposition.

To finde out the sides of the foresayd fine bodies, and to compare them together.

TAke the diameter of the Sphere geuen, and let the same be AB, and diuide it in the point C, so that let the line AC be equall to CB, by the 10. of the first: and in the point D, so that let AD be double to DB, by the 9. of the sixth. And vpon the line AB describe a semicircle AEB. And from the pointes C and D, raise up (by the 11. of the first) vnto the line AB perpendicular lines CE and DF. And draw these right lines AF, FB, and BE. Now forasmuch as the line AD is double to the line DB, therefore the line AB is treble to the line DB. Wherefore the line BA is sesquialter to the line AD (for it is as 3. to 2.). But as the line BA is to the line AD, so is the square of the line BA to the square of the line AF (by the 6. of the sixth, or by the Corollary of the same, and by the Corollary of the 20. of the same): for the triangle AFB is equiangle to the triangle AFD. Wherefore the square of the line BA is sesquialter to the square of the line AF. But the diameter of a sphere is in power sesquialter to the side of the pyramis, by the 13. of this booke, and the line AB is the diameter of the sphere. Wherefore the line AF is equall to the side of the pyramis.

The side of a
pyramis.

Again forasmuch as the line AB is treble to the line BD: but as the line AB is to the line BD, so is the square of the line AB to the square of the line FB, by the Corollaries of the 8. and 20. of the sixth. Wherefore the square of the line AB is treble to the square of the line FB. But the diameter of a sphere is in power treble to the side of the cube (by the 15. of this booke) and the diameter of the sphere is the line AB. Wherefore the line BF is the side of the cube.

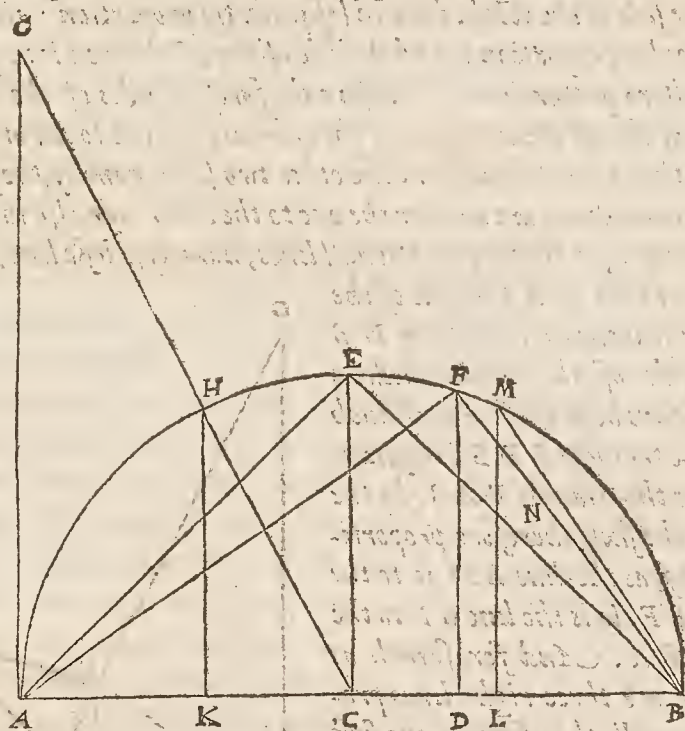
The side of a
cube.

And forasmuch as the line AC is equall to the line CB, therefore the line AB is double to the line CB. But as the line AB is to the line CB, so is the square of the line AB to the square of the line BE (by the former Corollaries). Therefore the square of the line AB is double to the

The side of an octohedron.

the square of the line BE . But the diameter of the sphere is in power double to the side of the octohedron, and AB is the diameter of the sphere given: wherefore the line BE is the side of the octohedron. Raise up (by the 11. of the first) from the point A unto the right line AB a perpendicular line AG . And put the line AG equall to the line AB . And drawe a right line from the point G to the point C . And let the line GC cut the circumference in the point H . And (by the 12. of the first) from the point H drawe unto the line AB a perpendicular line HK . Now forasmuch as the line GA is double to the line AC (for GA is equall to AB). But as GA is to AC , so is HK to KC (by the 28. of the first, and Corollary of the 2. of the sixth): wherefore the line HK is double to the line KC . Wherefore the square of the line HK is quadruple to the square of the line KC , by the Corollary of the 20. of the sixth. Wherefore the squares of the lines HK and KC , which are all one with the square of the line HC , by the 47. of the first, is quintuple to the square of the line KC . But the line HC is equall to the line CB , by the definition of a circle. Wherefore the square of the line BC is quintuple to the square of the line

CK . And forasmuch as the line AB is double to the line BC , of which the line AD is double to the line DB . Wherefore the residue, namely, BD , is double to the residue, namely, to DC (by the 19. of the fifth). Wherefore the line BC is treble to the line CD . Wherefore the square of the line BC is nonecuple to the square of the line CD , by the Corollary of the 20. of the sixth. But the square of BC is onely quintuple to the square of CK . Wherefore the square of CK is greater then the square of CD , by the 10. of the fifth. Wherefore also the line CK is greater then the line CD . Vn-



to the line CK put (by the 2. of the first) an equall line CL . And from the point L raise up unto the line AB a perpendicular line LM . And drawe a right line from the point M to the point B . Now forasmuch as the square of the line CB is quintuple to the square of the line CK , and the line AB is double to the line BC , and the line KL is double to the line CK : therefore the square of the line AB is quintuple to the square of the line KL , by the 15. of the fifth. But the diameter of a sphere is in power quintuple to the line which is drawn from the centre of the circle to the circumference on which the Icosahedron is described, by the Corollary of the 16. of the sixth. And the line AB is the diameter of the sphere: wherefore the line KL is the semidiameter of the circle on which the Icosahedron is described. Wherefore the line KL is the side of an hexagon figure described in the same circle, by the Corollary of the 15. of the fourth. And forasmuch as the diameter of the sphere is made of the side an hexagon figure, and of two sides of a decagon being each of them described in one and the selfe same circle (by the Corollary of the 16. of this booke): and the line AB is the diameter of the sphere, and the line KL is the side of the hexagon, and the line AK is equall to the line LB : wherefore either of the lines AK and LB is the side of a decagon described in the circle on which the Icosahedron is described (that is, in the circle whose semidiameter is the line KL). And forasmuch as the line LB is the side of a decagon, and the line ML of an hexagon (for ML is equall to KL , for that it is equall to KH , by the 14. of the third, for they are equally distant

The thirteenth Booke

The side of an Icosahedron.

The sides of a dodecahedron.

Comparison of the five sides of the foresayd bodies.

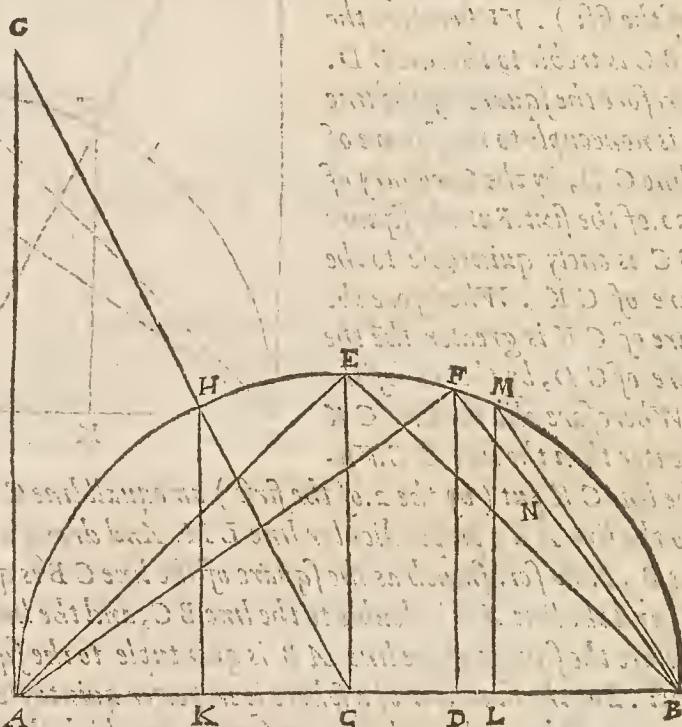
The side of the Icosahedron proued greater then the side of the dodecahedron.

distant from the centre, and each of the lines $H K$ and $K L$ is double to $K C$). Wherefore the line $M B$ is the side of a pentagon, by the 10. of this booke. But the side of the pentagon is also the side of the Icosahedron, by that which was demonstrated in the 10. of this booke. Wherefore the line $M B$ is the side of an Icosahedron.

And forasmuch as the line $F B$ is the side of a cube, let it be divided by an extreme and meane proportion in the point N , and let the greater segment thereof be $N B$. Wherefore the line $N B$ is the side of a Dodecahedron, by the Corollary of the 17. of this booke.

And forasmuch as it hath bene proued, by the 13. of this booke, that the diameter of the sphere is in power sesquialter to $A F$ the side of the pyramis, and is in power double to $B E$ the side of the octohedron, by the 14. of the same, and is in power treble to $F B$ the side of the cube, by the 15. of the same. Wherefore it followeth, that of what partes the diameter of the sphere containeth sixe, of such partes the side of the pyramis containeth fower: and the side of the octohedron three: and the side of the cube two. Wherefore the side of the pyramis is in power to the side of the octohedron in sesquitercia proportion: and is in power to the side of the cube in double proportion. And the side of the octohedron is in power to the side of the cube in sesquialtera proportion. Wherefore the foresaid sides of the three figures, that is, of the pyramis, of the octohedron, and of the cube, are the one to the other in rationall proportions. Wherefore they are rationall. But the other two sides, namely, the sides of the Icosahedron and of the Dodecahedron, are neither the one to the other, nor also to the foresaid sides in rationall proportions: for they are irrational lines, namely, a lesse line, and a residuall line.

But that $M B$ the side of the Icosahedron is greater then $N B$ the side of the Dodecahedron may thus be proued. Forasmuch as the triangle $F D B$ is equiangular to the triangle $F A B$, by the 8. of the sixth, therefore proportionally, as the line $B D$ is to the line $B F$, so is the line $B F$ to the line $B A$. And forasmuch as there are three right lines proportionall, therefore as the first is to the third, so is the square made of the first to the square made of the third, by the Corollary of the 20. of the sixth. Wherefore as the line $D B$ is to the line $B A$, so is the square of the line $D B$ to the square of the line $B A$. Wherefore (by conuersion, by the Corollary of the 4. of the fifth) as the line $A B$ is to the line $B D$, so is the square of the line $F B$ to the square of the line $B D$. But the line $A B$ is treble to the line $B D$, as hath before bene proued. Wherefore the square of the line $F B$ is treble to the square of the line $B D$. But the square of the line $A D$ is quadruple to the square of the line $D B$, by the Corollary of the 20. of the sixth, for the line $A D$ is double to the line $D B$. Wherefore the square of the line $A D$ is greater then the square of the line $F B$, by the 10. of the fifth. Wherefore also the line $A D$ is greater then the line $F B$. Wherefore the line $A L$ is much greater then the line $F B$. And the line $A L$ being divided by an extreme & meane proportion, his greater segment is the line $K L$, by the 9. of this booke: (for the line $L K$ is the side of an hexagon, and the line $K A$ is the side of a decagon inscribed in one and the same circle, as hath before bene proued): and the line $F B$ being divided



by an extreme and meane proportion, his greater segment is NB . Wherefore the line KL is greater then the line NB . (* For two lines diuided by an extreme and meane proportion, are euery way like proportionall). But the line KL is equall to the line LM . Wherefore the line LM is greater then the line NB . But the line MB is greater then the line LM . Wherefore the line MB being the side of the Icosahedron, is much greater then the line NB being the side of the Dodecahedron: which was required to be done, and to be proued.

* This Assumpt
is afterward
proued in the
14. booke and 4.
proposition.

An other way to proue that the line MB is greater then the line NB . Forasmuch as the line AD is double to the line DB , therefore the line AB is treble to the line DB . But as AB is to BD , so is the square of the line AB to the square of the line BF , by the 8. of the sixt (for the triangle FAB is equiangle to the triangle FDB). Wherefore the square of the line AB is treble to the square of the line BF . And it is before proued, that the square of the line AB is quintuple to the square of the line KL . Wherefore five squares made of the line KL , are equall to three squares made of the line BF . But three squares made of the line BF , are greater then sixe squares made of the line NB , as is straight way proued. Wherefore five squares made of the line KL , are greater then sixe squares made of the line NB . Wherefore also one square made of the line KL , is greater then one square made of the line NB . Wherefore the line KL is greater then the line NB . But the line KL is equall to the line LM . Wherefore the line LM is greater then the line NB . Wherefore the line MB is much greater then the line NB : which was required to be proued.

An other demõ-
monstration to
proue that the
side of the Ico-
sahedrõ is grea-
ter then the side
of the dodecabe-
dron.

But now let vs proue that three squares made of the line FB , are greater then sixe squares made of the line NB . Forasmuch as the line BN is greater then the line NF , for it is the greater segment of the line BF diuided by an extreme and meane proportion, therefore that which is contained vnder the lines BF , and BN , is greater then that which is contained vnder the lines BF and FN , by the 1. of the sixt. Wherefore that which is contained vnder the lines BF and BN , together with that which is contained vnder the lines BF and FN , is greater then that which is contained vnder the lines BF and FN twise. But that which is contained vnder the lines BF and FN , together with that which is contained vnder the lines BF and BN , is the square of the line BF , by the 2. of the second, and that which is contained vnder the lines BF and FN once, is equall to the square of NB . For the line FB is diuided by an extreme and meane proportion in the point N : (and (by the 17. of the sixt) that which is contained vnder the extremes, is equall to the square made of the middle line). Wherefore the square of the line FB , is greater then the double of the square of the line BN . Wherefore one of the squares made of the line BF , is greater then two squares made of the line BN . Wherefore also three squares made of the line FB , are greater then sixe squares made of the line BN : which was required to be proued.

That 3. squares
of the line FB
are greater the
6. squares of
the line NB .

A Corollary.

Now also I say that besides the five foresayd solides there can not be described any other solide cõprehended vnder figures equilater & equiangle the one to the other. For of two triangles, or of any two other playne superficieses can not be made a solide angle (for, that is cõtrary to the definition of a solide angle). Vnder three triangles is containned the solide angle of a pyramis: vnder fower, the solide angle of an octohedrõ: vnder five, the solide angle of an Icosahedrõ: of sixe, equilater & equiangle triangles set to one point can not be made a solide angle. For forasmuch as the angle of an equilater triangle is two third partes of a right angle, the sixe angles of the solide shalbe equall to fower right angles, which is impossible. For euery solide angle is (by the 21. of the eleueth) containned vnder playne angles lesse the fower right angles. And by the same reason can not be made a solide angle contained vnder more the sixe playne superficial angles of equilater triangles. Vnder three squares is contained the angle of a cube. Vnder fower squares it is impossible that a solide angle should be containned: for then

That there can
be no other so-
lids besides these
five, containned
vnder equilater
and equiangle
bases.

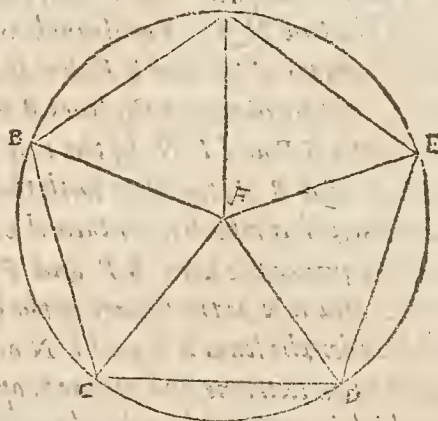
The thirteenth Booke

agayne it should be containd vnder fower right angles. Wherefore much lesse can any solide angle be containd vnder more squares then fower. Vnder three equilater and equiangle pentagons is containd the solide angle of a dodecahedron. But vnder fower it is impossible. For forasmuch as the angle of a pentagon is a right angle and the fift part more of a right angle, the fower angles shalbe greater then fower right angles: which is impossible. And therefore much lesse can a solide angle be composed of more pentagons then fower. Neither can a solide angle be containd vnder any other equilater and equiangle figures of many angles, for that that also should be absurd. For the more the sides increase, the greater are the angles which they containe, and therefore the farther of are the superficial angles containd of those sides from composing of a solide angle. Wherefore besides the foresayd five figures there can not be made any solide figure containd vnder equall sides and equall angles: which was required to be proued.

An Assumpt.

That the angle of an equilater and equiangle Pentagon is one right angle and a fifteth part ouer; which thing was also before proued in the corollary of the 32. of the first.

But now that the angle of an equilater and equiangle pentagon is a right angle and a fift part more of a right angle, may thus be proued. Suppose that $ABCDE$ be an equilater and equiangle pentagon. And (by the 14. of the fourth) describe about it a circle $ABCDE$. And take (by the 1. of the third) the center thereof, and let the same be F . And draw these right lines FA, FB, FC, FD, FE . Wherefore those lines do diuide the angles of the pentagon into two equall partes in the poyntes A, B, C, D, E , by the 4. of the first. And forasmuch as the five angles that are at the poynt F are equall to fower right angles, by the corollary of the 15. of the first, and they are equall the one to the other by the 8. of the first: therefore one of those angles, as for example sake, the angle AFB is a fift part lesse then a right angle. Wherefore the angles remayning, namely, FAB , & ABF , are one right angle and a fift part ouer. But the angle FAB is equall to the angle FCB . Wherefore the whole angle ABC being one of the angles of the pentagon is a right angle and a fift part more then a right angle: which was required to be proued.



¶ A Corollary added by Flussas.

Now, let vs teach, how those fine solides, haue eche like inclinations of theyr bases.

The sides of the angle of the inclination of the superficieses of the Tetrahedron are proued rationally.

The sides of the angle of the inclination of the superficieses of the cube proued rationally.

First let vs take a Pyramis, and diuide one of the sides thereof into two equall parts: and from the two angles opposite vnto that side, draw perpédiculars, which shall fall vpon the section, by the corollary of the 12. of the thirteenth, and at the sayd poynt of diuision (as may easily be proued). Wherefore they shal containe the angle of the inclination of the plaines, by the 4. diffinition of the eleuenth, which angle is subtended of the opposite side of the pyramis. Now forasmuch as the rest of the angles of the inclination of the plaines of the Pyramis, are containd vnder two perpédicular lines of the triangles, and are subtended of the side of the Pyramis, it foloweth, by the 8. of the first, that those angles are equall. Wherefore (by the 5. diffinition of the eleuenth) the superficieses are in like sort inclined the one to the other.

One of the sides of a Cube being diuided into two equall parts, if from the sayd section be drawn in two of the bases thereof, two perpendicular lines, they shalbe parallels and equall to the sides of the square which cotayne a right angle. And forasmuch as all the angles of the bases of the Cube are right angles: therefore those perpendiculars falling vpon the section of the side common to the two bases, shal containe a right angle (by the 10. of the eleuenth): which selfe angle is the angle of inclination (by the 4. diffinition of the eleuenth) and is subtended of the diameter of the base of the Cube. And by the

the same reason may we proue that the rest of the angles of the inclination of the bases of the cube are right angles. Wherefore the inclinations of the superficieses of the cube the one to the other, are equal (by the 5. diffinition of the eleuenth).

In an Octohedron take the diameter which coupleth the two opposite angles. And from those opposite angles draw to one and the selfe same side of the Octohedron, in two bases thereof, two perpendicular lines, which shall diuide that side into two equall parts and perpendicularly (by the Corollary of the 12. of the thirteenth). Wherefore those perpendiculars shall containe the angle of the inclination of the bases (by the 4. diffinition of the eleuenth): and the same angle is subtended of the diameter of the Octohedron. Wherefore the rest of the angles after the same maner described in the rest of the bases, being comprehended and subtended of equall sides, shall (by the 8. of the first) be equall the one to the other. And therefore the inclinations of the playnes in the Octohedron, shall (by the 5. diffinition of the eleuenth) be equall.

That the plaines of an octohedron are in like sort inclined.

In an Icosahedron let there be drawn from the angles of two of the bases, to one side common to both the sayd bases perpendiculars, which shall containe the angle of the inclination of the bases (by the 4. diffinition of the eleuenth): which angle is subtended of the right line which subtendeth the angle of the pentagon which containeth five sides of the Icosahedron, by the 16. of this booke: for it coupleth the two opposite angles of the triangles which are ioyned together. Wherefore the rest of the angles of the inclination of the bases being after the same maner found out, they shall be contained vnder equall sides, and subtended of equall bases, and therefore (by the 8. of the first) those angles shall be equall. Wherefore also all the inclinations of the bases of the Icosahedron the one to the other shall be equall, by the 5. diffinition of the eleuenth.

That the plaines of an Icosahedron are in like sort inclined.

In a Dodecahedron, from the two opposite angles of two next pentagons draw to theyr common side perpendicular lines, passing by the centres of the sayd pentagons, which shall, where they fall, diuide the side into two equall parts by the 3. of the third. (For the bases of a Dodecahedron are contained in a circle) And the angle contained vnder those perpendicular lines is the inclination of those bases (by the 4. diffinition of the eleuenth). And the foresayd opposite angles are coupled by a right line equal to the right line which coupleth the opposite sections into two equall parts of the sides of the dodecahedron (by the 33. of the first). For they couple together the halfe sides of the dodecahedron, which halves are parallels and equall, by the 3. corollary of the 17. of this booke: which coupling lines also are equall, by the same corollary. Wherefore the angles being contained of equal perpendicular lines, and subtended of equall coupling lines, shall (by the 8. of the first) be equal. And they are the angles of the inclinations. Wherefore the bases of the dodecahedron are in like sort inclined the one to the other (by the 5. diffinition of the eleuenth).

That the plaines of a Dodecahedron are in like sort inclined.

Flussas after this teacheth how to know the rationality or irrationality of the sides of the triangles, which containe the angles of the inclinations of the superficieses of the foresayd bodies.

In a Pyramis the angle of the inclinatio is contained vnder two perpendicular lines of the triangles, and is subtended of the side of the Pyramis. Now the side of the pyramis is in power sesquitercia to the perpendicular line, by the corollary of the 12. of this booke: and therefore the triangle contained of those perpendicular lines and the side of pyramis, hath his sides rational & commensurable in power the one to the other.

The sides of the angle of the inclination of the superficieses of the Tetrahedron are proued rational.

Forasmuch as the two sides of a Cube (or right lines equall to them) subtended vnder the diameter of one of the bases, doo make the angle of the inclination: and the diameter of the cube is in power sesquialter to the diameter of the base, which diameter of the base is in power double to the side (by the 47. of the first): therefore those lines are rational and commensurable in power.

The sides of the angle of the inclination of the superficieses of the cube are proued rational.

In an Octohedron, whose two perpendiculars of the bases containe the angle of the inclination of the Octohedron, which angle also is subtended of the diameter of the Octohedron, the diameter is in power double to the side of the Octohedron, & the side is in power sequitertia to the perpendicular line, by the 12. of this booke: wherefore the diameter thereof is in power duple superbiptiens tertias to the perpendicular line. Wherefore also the diameter and the perpendicular line are rational and commensurable (by the 6. of the tenth.)

The sides of the angle &c. of the octohedron are proued rational.

As touching an Icosahedron, it was proued in the 16. of this booke, that the side thereof is a lesse line, when the diameter of the sphere is rational. And forasmuch as the angle of the inclination of the bases thereof, is contained of the perpendicular lines of the triangles, and subtended of the right line which subtendeth the angle of the Pentagon which containeth five sides of the Icosahedron: and vnto the perpendicular lines the side is commensurable (namely, is in power sesquitercia vnto them, by the Corollary of the 12. of this booke): therefore the perpendicular lines which containe the angles are irrational lines, namely, lesse lines (by the 105. of the tenth booke.) And forasmuch as the diameter containeth in power both the side of the Icosahedron, and the line which subtendeth the foresayd angle, if from the power of the diameter which is rational, be taken away the power of the side of the Icosahedron which is irrational, it is manifest that the residue which is the power of the subtending line shall be irrational. For if it should be rational, the number which measureth the whole power of the diameter, and the part taken away of the subtending line, should also, by the 4. common sentence of

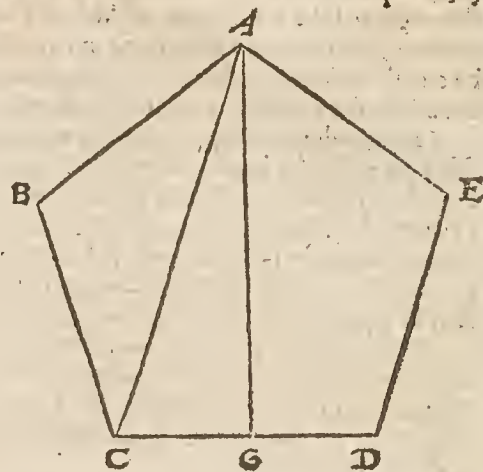
The thirteenth Booke

The sides of the angle $\text{E}\hat{\text{C}}\text{C}$. of the Icosahedron proued irrati-
onall.

the seuenth measure the residue, namely, the power of the side: which is irrationall for: that it is a lesse line, which were absurd. Wherefore it is manifest that the right lines which compose the angle of the inclination of the bases of the Icosahedron are Irrationall lines. For the subtending line hath to the line containinge, a greater proportion, then the whole hath to the greater segment.

The angle of the inclination of the bases of a dodecahedron, is containd vnder two perpendiculars of the bases of the dodecahedron, and is subtended of that right line, whose greater segment is the side of a Cube inscribed in the dodecahedron, which right line is equall to the line which coupleth the sections, into two equal parts, of the opposite sides of the dodecahedron. And this coupling line we say is an irrationall line, for that the diameter of the sphere containeth in power both the coupling line, and the side of the dodecahedron: but the side of the dodecahedron is an irrationall line, namely, a residuall line (by the 17. of this booke). Wherefore the residue namely, the coupling line is an irrationall line, as it is easy to proue by the 4. comon sentence of the seueth. And that the perpendicular lines which containe the angle of the inclination are irrationall, is thus proued.

Suppose that there be a Pentagon A B C D E , and draw in it the perpendicular line, A G , and let the line subtending the angle of the pentagon be A C . Now forasmuch as the right line A C is the side of the cube, and C D the side the Dodecahedron inscribed in one and the selfe same sphere, by the 2. Corollary of the 17. of this booke: but the line A C is commensurable to the diameter of the sphere, by the 15. of this booke, and is therefore rationally, by the 6. diffinitio of the tenth: & the right line C D which is the side of the dodecahedron is irrationall (by the 17. of this booke). Wherefore the line C G which is the halfe of the line C D is irrationall by the 103. of the 10. booke. And the right line A C containeth in power the two right lines A G and G C (by the 47. of the first). If therefore from the power of the right line A C being rationally, be take away the power of the line C G being irrationall, the power remainyng, namely, the power of the line A G , shall of necessitie be irrationall. For if the power of the line A G taken away should be rationally, and the whole power of the line A C is rationally, the residue, namely, the power of the line C G should be also rationally, and should be measured by the selfe same numbers, by the 4. common sentence of the seuenth. But it is proued that the line C G is irrationall, for it is the halfe of the whole residuall line C D (by the 17. of the thirteenth): which is impossible. Wherefore the perpendicular line A G is irrationall. Wherefore the angle of the inclination of the dodecahedron, which is containd vnder two perpendicular lines of the pentagon, and is subtended of a right line, which coupleth the sections into two equall parts of the opposite sides of the dodecahedron (by the 2. corollary of the 18. of this booke) which line we haue proued to be irrationall (for that it is equall to the two lines A C , and C D by the 4. corollary of the 17. of this booke) is containd vnder irrationall right lines.



The sides of the angle $\text{E}\hat{\text{C}}\text{C}$. of the dodecahedron proued irrati-
onall.

How to know whether the angle of the inclination be a right angle, an acute angle, or an oblique angle.

By the proportion of the subtending line (of the foresayd angles of inclination) to the lines which containe the angle, is found out the obliquitie of the angle. For if the subtending line be in power double to the line which containeth the angle, then is the angle a right angle (by the 48. of the first.) But if it be in power lesse then the double it is an acute angle (by the 13. of the second). But if it be in power more then the double, or haue a greater proportion then the whole hath to the greater segment, the angle shall be an obtuse angle (by the 12. of the second and 4. of the thirteenth). By which may be proued that the square of the whole is greater then the double of the square of the greater segment.

This is to be noted that that which *Flussus* hath here taught touching the inclinations of the bases of the five regular bodies, Hypsicles teacheth after the 5. proposition of the 15. booke. Where he confesseth, that he receiued it of one Isidorus, and seeking to make the mater more cleare, he endeouored himselfe to declare, that the angles of the inclination of the solides are geuen, and that they

are either acute or obtuse, according to the nature of the solide: although *Euclide* in all his 15. bookes hath not yet shewed, what a thing geuen is. Wherefore

Flussus framing his demonstration vpon an other ground procedeth after an other maner, which semeth more playne, and more aptly here- to be placed then there. Albeit the reader in that place shall not be frustrate of his also.

(. . .)

The ende of the thirteenth Booke of Euclides Elementes.

The fourteenth booke of Euclides Elementes.



IN this booke, which is commonly accompted the 14. booke of *Euclide* is more at large intreated of our principall purpose: namely, of the comparison and proportion of the five regular bodies (customably called the 5. figures or formes of *Pythagoras*) the one to the other, and also of their sides together, eche to other: which thinges are of most secret vse, and inestimable pleasure, and commoditie to such as diligently search for them, and attayne vnto them. Which thinges also vndoubtedly for the woorthines and hardnes thereof (for thinges of most price are most hardest) were first searched, and found out of Philosophers, not of the inferior or meane sort, but of the depest and most grounded

*The argument
of the fourteenth
booke.*

Philosophers, and best exercised in Geometry. And albeit this booke with the booke following, namely, the 15. booke, hath bene hetherto of all men for the most part, and is also at this day numbred and accompted amongst *Euclides* bookes, and supposed to be two of his, namely, the 14. and 15. in order: as all exemplars (not onely new and lately set abroad, but also old monumentes written by hand) doo manifestly witnes: yet it is thought by the best learned in these dayes, that these two bookes are none of *Euclides*, but of some other author, no lesse worthy, nor of lesse estimation and authoritie, notwithstanding, then *Euclide*. *Apollonius* a man of deepe knowledge a great Philosopher and in Geometrie maruelous (whose wöderful bookes writtē of the sections of cones, which exercise & occupy the wittes of the wisest and best learned, are yet remayning) is thought, and that not without iust cause, to be the author of them, or as some thinke *Hypsicles* him selfe. For what can be more playnely, then that which he him selfe witnesseth in the preface of this booke. *Basilides of Tire* (sayth *Hypsicles*) and my father together, scanning, and perysing a writing or booke of *Apollonius*, which was of the comparison of a dodecahedron to an Icosahedron inscribed in one and the selfe same sphere, and what proportion these figures had the one to the other, found that *Apollonius* had sayled in this matter. But afterward (sayth he) I found an other copy or booke of *Apollonius*, wherein the demonstration of that matter was full and perfect, and shewed it vnto them, whereat they much reioysed. By which wordes it semeth to be manifest that *Apollonius* was the first author of this booke, which was afterward set forth by *Hypsicles*. For so his owne wordes after in the same preface seme to import.

The Preface of *Hypsicles* before the fourteenth booke.

H Rend *Protarchus*, whē that *Basilides of Tire* came into *Alexandria*, hauing familiar frendship with my father by reason of his knowledge in the mathematicall sciences, he remayned with him a long time, yea euen all the time of the pestilence. And sometime reasoning betwene themselves of that which *Apollonius* had written touching the comparison of a dodecahedron and of an Icosahedron inscribed in one and the selfe same sphere, what proportion such bodies haue the one to the other, they iudged that *Apollonius* had somewhat erred therein. Wherefore they (as my father declared vnto me) diligently weighing it, wrote it perfectly. Howbeit afterward I happened to finde an other booke written of *Apollonius*, which contayned in it the

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right

The fourteenth Booke

right demonstration of that which they sought for : which when they saw, they much reioyced. As for that which Apollonius wrote, may be sene of all men, for that it is in enery mans hand. And that which was of vs more diligently afterward written agayne, I thought good to send and dedicate vnto you, as to one whome I thought worthy commendation, both for that deepe knowledge which I know you haue in all kindes of learning, and chiefly in Geometrie, so that you are able redily to iudge of those thinges which are spoken, and also for the greate loue and good will which you beare towards my father and me. Wherefore vouchsafegently to accept this, which I send vnto you. But now is it time to end our preface, and to begin the matter.

¶ The 1. Theoreme.

The 1. Proposition.

First proposition
after
Flussas.

A perpendicular line drawn from the centre of a circle to the side of a Pentagon described in the same circle : is the halfe of these two lines, namely, of the side of an hexagon figure, and of the side of a decagon figure being both described in the selfe same circle.

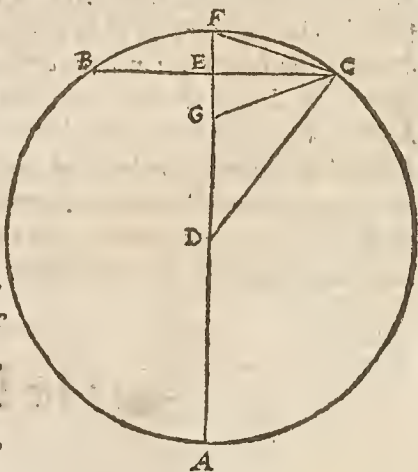
Construction.



Suppose that the circle be ABC . And let the side of an equilateral Pentagon described in the circle ABC , be BC . And (by the 1. of the third) take the centre of the circle, and let the same be D . And (by the 12. of the first) from the point D draw vnto the line BC a perpendicular line DE . And extend the right line DE directly to the point F . Then I say, that the line DE (which is drawn from the centre to BC the side of the pentagon) is the halfe of the sides of an hexagon and of a decagon taken together and described in the same circle. Draw these right lines DC and CF . And

Demonstration.

vnto the line EF put an equall line GE . And draw a right line from the point G to the point C . Now forasmuch as the circumference of the whole circle is quintuple to the circumference BCF (which is subtended of the side of the pentagon) and the circumference ACF is the halfe of the circumference of the whole circle, and the circumference CF (which is subtended of the side of the decagon) is the halfe of the circumference BCF : therefore the circumference ACF is quintuple to the circumference CF (by the 15. of the fifth). Wherefore the circumference AC is quadruple to the circumference FC . But as the circumference AC is to the circumference FC , so is the angle ADC to the angle FDG , by the last of the sixth. Wherefore the angle ADC is quadruple to the angle FDG . But the angle ADC is double to the angle EFC , by the 20. of the third: Wherefore the angle EFC is double to the angle GDC . But the angle EFC is equall to the angle EGC , by the 4. of the first. Wherefore the angle EGC is double to the angle EDC . Wherefore the line DG is equall to the line GC (by the 32. and 6. of the first). But the line GC is equall to the line CF , by the 4. of the first. Wherefore the line DG is equall to the line CF . And the line GE is equall to the line EF (by construction). Wherefore the line DE is equall to the lines EF and FC added together. Vnto the lines EF and FC adde the line DE . Wherefore the lines DF and FC added together, are double to the line DE . But the line DF is equall to the side of the hexagon: and FC to the side of the decagon.



Wherefore

Wherefore the line D E is the halfe of the side of the hexagon, and of the side of the decagon being both added together and described in one and the selfe same circle.

It is manifest * by the Propositions of the thirteenth booke, that a perpendicular line drawn from the centre of a circle to the side of an equilateral triangle described in the same circle, is halfe of the semidiameter of the circle. Wherefore by this Proposition, a perpendicular drawn from the centre of a circle to the side of a Pentagon, is equall to the perpendicular drawn from the centre to the side of the triangle, and to halfe of the side of the decagon described in the same circle.

* This is manifest by the 12. proposition of the thirteenth booke as Campane well gathereth in a Corollary of the same.

¶ The 2. Theoreme.

The 2. Proposition.

One and the selfe same circle comprehendeth both the Pentagon of a Dodecahedron, and the triangle of an Icosahedron, described in one and the selfe same Sphere.

The 4. proposition after Flusas.

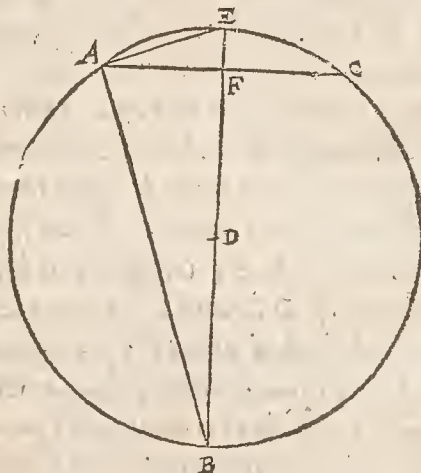


His Theoreme is described of Aristeus in that booke whose title is, The comparison of the five figures, and is described of Apollonius in his second edition of the comparison of a Dodecahedron to an Icosahedron, which is, * that as the superficies of a Dodecahedron is to the superficies of an Icosahedron, so is the Dodecahedron to an Icosahedron, for that a perpendicular line drawn from the centre of a sphere to the pentagon of a dodecahedron and to the triangle of an Icosahedron is one and the selfe same. Now must we also proue that one and the selfe same circle comprehendeth both the pentagon of a Dodecahedron, and also the triangle of an Icosahedron described in one and the selfe same sphere, first this being proued :

* This is afterward proued in the 4. proposition.

If in a circle be described an equilateral pentagon, the squares which are made of the side of the pentagon, and of that right line which is subtended vnder two sides of the pentagon, are quintuple to the square of the semidiameter of the circle. Suppose that A B C be a circle.

And let the side of a pentagon in the circle A B C, be A C. And take (by the 1. of the third) the centre of the circle and let the same be D. And (by the 12. of the first) from the point D draw vnto the line A C a perpendicular line D F. And extend the line D F on either side to the pointes B and E. And draw a right line from the point A to the point B. Now I say, that the squares of the lines B A and A C are quintuple to the square of the line D E. Drawe a right line from the point A to the point E. Wherefore the line A E is the side of a decagon figure.



This Assumpt is the 3. proposition after Flusas. Construction of the Assumpt.

And forasmuch as the line B E is double to the line D E : therefore the square of the line B E is quadruple to the square of D E (by the 20. of the sixth). But vnto the square of the line B E, are equall the squares of the lines B A and A E (by the 47. of the first, for the angle B A E is a right angle, by the 31. of the third). Wherefore the squares of the lines B A and A E, are quadruple to the square of the line D E. Wherefore the squares of the lines A B, A E, and D E, are quintuple to the square of the line D E. But the squares of the lines D E and A E, are equall to the square of the line A C (by the 10. of the thirteenth). Wherefore the squares of the lines B A and A C, are quintuple to the square of the line D E.

Demonstration of the Assumpt.

This being thus proued, now is to be demonstrated that one and the selfe same circle comprehendeth both the pentagon of a dodecahedron, & the triangle of an Icosahedron described in one & the selfe same circle. Take the diameter of the sphere, & let the same be A B. And in the same sphere describe a dodecahedron, & also an Icosahedron. And let one of the pentagons of the dodecahedron be C D E F G, & let one of the triangles of the Icosahedron be K L H. Now I say that the semidiameters of the circles which are described about them are equall, that

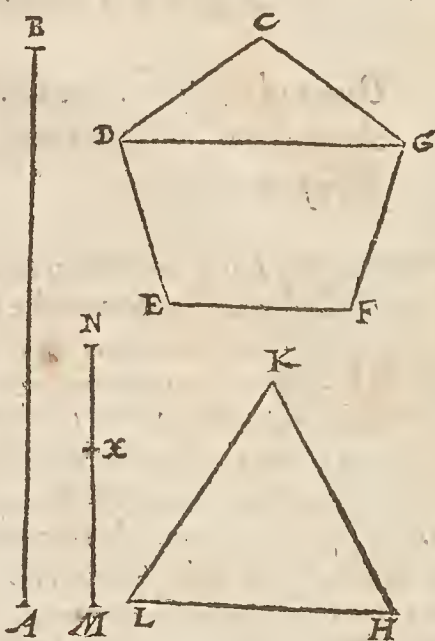
Construction of the proposition.

The fourteenth Booke

Demonstration of the proposition.

* Here is required an Assumpt which is after-ward proved in the 4. proposition of this booke: namely, that lines divided by an extreme & meane proportion, are every way proportionall: which is granted them, followeth the reason aptly. For the line AN is by supposition divided by an extreme and meane proportion, & his greater segment is the line MX , and the line DG divided by an extreme and meane proportion, his greater segment is the line CG , by the 8. of the thirteenth.

that is, that one and the selfe same circle containeth both the pentagon $CDEFG$, and the triangle KLH . Draw a right line from the point D to the point G . Wherefore the line DG is the side of a cube (by the corollary of the 17. of the thirteenth). Take a certayne right line MN . And let the square of the line AB be quintuple to the square of the line MN , by the assumpt put after the 6. proposition of the tenth. But the diameter of a sphere is in power quintuple to the square of the semidiameter of the circle, on which is described the Icosahedron (by the corollary of the 16. of the thirteenth). Wherefore the line MN is the semidiameter of the circle on which is described the Icosahedron. Divide (by the 30. of the sixth) the line MN by an extreme and meane proportion in the point X . And let the greater segment thereof be MX . Wherefore the line MX is the side of a decagon described in the same circle (by the corollary of the 9. of the thirteenth). And forasmuch as the square of the line AB is quintuple to the square of the line MN : But the square of the line BA is treble to the square of the line DG (by the corollary of the 15. of the thirteenth). Wherefore three squares of the line DG are equal to five squares of the line MN . * But as three squares of the line DG are to five squares of the line MN , so are three squares of the line CG to five squares of the line MX . Wherefore three squares of the line CG are equal to five squares of the line MX . But five squares of the line CG are equal to five squares of the line MN & to five squares of the line MX . For (by the 10. of the thirteenth) one square of the line CG is equal to one square of the line MN & to one square of the line MX . Wherefore five squares of the line CG are equal to three squares of the line DG and to three squares of the line CG (as it is not hard to proue, marking what hath before bene proued). But three squares of the line DG , together with three squares of the line CG , are equal to fiftene squares of the semidiameter of the circle described about the pentagon $CDEFG$ (for it was before proued in the assumpt put in this proposition) that the squares of DG and GC taken once, are quintuple to the square of the semidiameter of the circle described about the pentagon $CDEFG$. And five squares of the line KL are equal to fiftene squares of the semidiameter of the circle described about the triangle KLH . (For by the 12. of the thirteenth, one square of the line LK is triple to one square of the line drawne from the centre to the circumference (of the circle which containeth the pentagon $CDEFG$) are equal to fiftene squares of the line drawne from the centre to the circumference of the circle which containeth the triangle KLH): wherefore one of the squares which is drawne from the centre to the circumference of the one circle, is equal to one of the squares which is drawne from the centre to the circumference of the other circle. Wherefore the diameter is equal to the diameter, wherefore one and the selfe same circle comprehendeth both the pentagon of a dodecahedron and the triangle of an Icosahedron described in one and the selfe same circle: which was required to be proued.



¶ The 3. Theoreme.

The 3 Proposition.

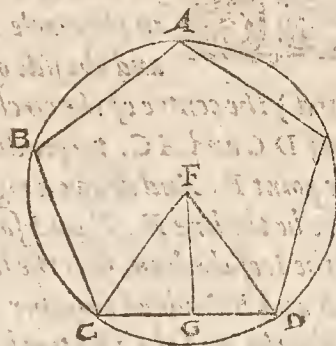
The 5. proposition after
Eustat.

If there be an equilater and equiangle pentagon, and about it be described a circle, and from the centre to one of the sides be drawne a perpendicular line, that which is containd vnder one of the sides and the perpendicular line

line thirty times, is equall to the superficies of the dodecahedron.



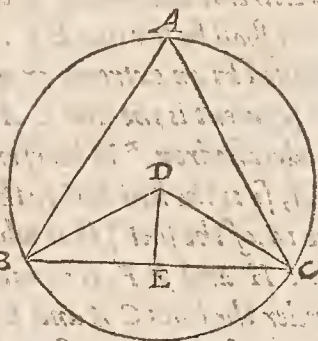
Suppose that ABCD be an equilater and equiangle pentagon. And about the same pentagon, describe (by the 14. of the fourth) a circle. And let the centre thereof be the poynt F. And from the poynt F draw (by the 12. of the first) unto the line CD a perpendicular line FG. Now I say that that which is containned under the lines CD and FG thirty times, is equall to 12. pentagons of the same quantitie that the pentagon ABCD is. Draw these right lines CF and FD. Now forasmuch as that which is containned under the lines CD and FG is double to the triangle GDF (by the 41. of the first) therefore that which is containned under the lines CD and FG five times is equall to ten of those triangles. But ten of those triangles are two pentagos, and sixe times ten of those triangles are all the pentagons. Wherefore that which is containned under the lines CD and FG thirty times is equall to 12. pentagons. But 12. pentagons are the superficies of dodecahedron. Wherefore that which is containned under the lines CD and FG thirty times is equall to the superficies of the dodecahedron.



Construction.

Demonstration.

In like sort also may we proue that if there be an equilater triangle, as for example, the triangle ABC, and about it be described a circle, and the centre of the circle be the point D, and the perpendicular line be the line DE: that which is containned under the lines BC and DE thirty times, is equall to the superficies of the Icosahedron. For agayne forasmuch as that which is containned under the lines DE and BC is double to the triangle DBC (by the 41. of the first) therefore two triangles are equall to that which is containned under the lines DE and BC, and three of those triangles containne the whole triangle. Wherefore sixe such triangles as DBC is, are equall to that which is containned under the lines DE and BC thrise. But sixe such triangles as DBC is, are equall to two such triangles as ABC is. Wherefore that which is contained under the lines DE and BC thrise, is equall to two such triangles as ABC is. But two of those triangles take ten times containeth the whole Icosahedron. Wherefore that which is containned under the lines DE and BC thirty times, is equall to twenty such triangles as the triangle ABC is, that is, to the whole superficies of the Icosahedron.



The 5. proposition after Flusas.

Demonstration.

** Wherefore as the superficies of the dodecahedron is to the superficies of the Icosahedron, so is that which is containned under the lines CD and FG to that which is containned under the lines BC and DE.*

* This is the reason of the Corollary following.

¶ Corollary.

By this it is manifest, that as the superficies of the Dodecahedron is to the superficies of the Icosahedron, so is that which is contained under the side of the Pentagon, and the perpendicular line which is drawn from the centre of the circle described about the Pentagon to the same side, to that which is contained under the side of the Icosahedron and the perpendicular line which is drawn from the centre of the circle described about the triangle to the same side: so that the Icosahedron and Dodecahedron be both described in one and the selfe same Sphere.

A Corollary which also Flusas putteth as a Corollary after the 5. proposition in his order.

¶ ¶ ¶

¶ The

The fourteenth Booke

¶ The 4. Theoreme.

The 4. Proposition.

The 6. proposition
after Eustath.

This being done, now is to be proued, that as the superficies of the Dodecahedron is to the superficies of the Icosahedron, so is the side of the cube to the side of the Icosahedron.

Construction.

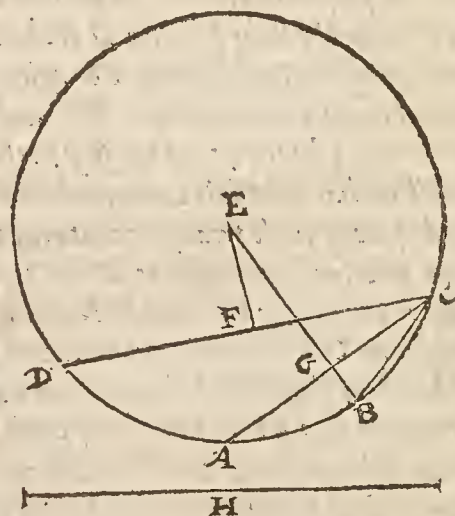


Take (by the 2. Theoreme of this booke) a circle containing both the pentagon of a Dodecahedron, and the triangle of an Icosahedron, being both described in one and the selfe same sphere, and let the same circle be DBC . And in the circle DBC describe the side of an equilater triangle, namely, CD ; and the side of an equilater pentagon, namely, AC . And take (by the 1. of the third) the centre of the circle, and let the same be E . And from the point E draw vnto the lines DC and AC , perpendicular lines EF and EG . And extend the line EG directly to the point B . And draw a right line from the point B to the point C . And let the side of the cube be the line H . Now I say, that as the superficies of the Dodecahedron is to the superficies of the Icosahedron, so is the line H to the line CD . Forasmuch as the line made of the lines EB and BC added together (namely, of the side of the hexagon, and of the side of a decagon) is (by the 9. of the thirteenth) diuided by an extreme and meane proportion, and his greater segment is the line BE : and the line EG is also (by the 1. of the fouretenth) the halfe of the same line, and the line EF is the halfe of the line BE (by the Corollary of the 12. of the thirteenth). Wherefore the line EG being diuided by an extreme and meane proportion,* his greater segment shall be the line EF . And the line H also being diuided by an extreme & meane proportion, his greater segment is the line CA , as it was proued† in the Dodecahedron.* Wherefore as the line H is to the line CA , so is the line EG to the line EF . Wherefore (by the 16. of the sixth) that which is contained vnder the lines H and EF , is equall to that which is contained vnder the lines CA and EG . And for that as the line H is to the line CD , so is that which is contained vnder the lines H and EF , to that which is contained vnder the lines CD and EF (by the 1. of the sixth). But vnto that which is contained vnder the lines H and EF , is equall that which is contained vnder the lines CA and EG . Wherefore (by the 11. of the fifth) as the line H is to the line CD , so is that which is contained vnder the lines CA and EG , to that which is contained vnder the lines CD and EF , that is (by the Corollary next going before) as the superficies of the Dodecahedron is to the superficies of the Icosahedron, so is the line H to the line CD .

* This is not hard to proue by the 15. 16. and 19. of the sixth.

† In the Corollary of the 17. of the thirtieth.

* Here againe is required the Assumpt which is afterward proued in this 4. proposition.



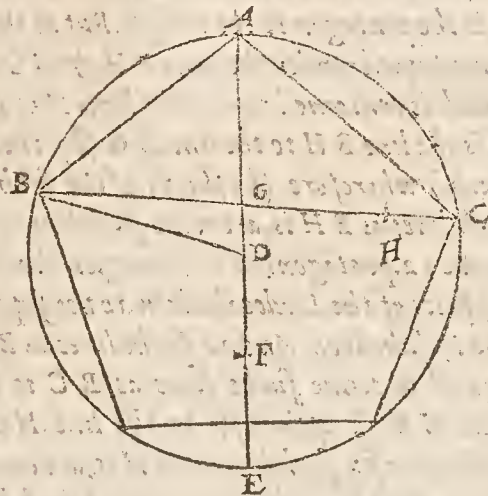
An other demonstration to proue that as the superficies of the Dodecahedron is to the superficies of the Icosahedron, so is the side of the cube to the side of the Icosahedron. †

† But first the Assumpt following, the construction whereof here beginneth, is to be proued.



Et there be a circle ABC . And in it describe two sides of an equilater pentagon (by the 11. of the fifth) namely, AB and AC : and draw a right line from the point B to the point C . And (by the 1. of the third) take the centre of the circle, and let the same be D . And draw a right line from the point A to the point D , and extend it directly to the point E , and let it cut the line BC in the point G . And let the line DF be halfe to the line DA , and let the line

the line GC be treble to the line HC , by the 9. of the sixth. Now I say, that that which is contained under the lines AF and BH , is equall to the pentagon inscribed in the circle ABC . Draw a right line from the point B to the point D . Now forasmuch as the line AD is double to the line DE , therefore the line AF is sesquialter to the line AD . Again, forasmuch as the line GC is treble to the line CH , therefore the line GH is double to the line CH . Wherefore the line GC is sesquialter to the line HG . Wherefore as the line FA is to the line AD , so is the line GC to the line GH . Wherefore (by the 16. of the sixth) that which is contained under the lines AF & HG , is equall to that which is contained under the lines DA and GC . But the line GC is equall to the line BG (by the 3. of the third). Wherefore that which is contained under the lines AD and BG , is equall to that which is contained under the lines AF and GH . But that which is contained under the lines AD and BG , is equall to two such triangles as the triangle ABD is (by the 41. of the first). Wherefore that which is contained under the lines AF and GH , is equall to two such triangles as the triangle ABD is. Wherefore that which is contained under the lines AF and GH five times, is equall to ten triangles. But ten triangles are two pentagons. Wherefore that which is contained under the lines AF and GH five times, is equall to two pentagons. And forasmuch as the line GH is double to the line HC , therefore that which is contained under the lines AF and GH , is double to that which is contained under the lines AF and HC (by the 1. of the sixth). Wherefore that which is contained under the lines AF and HC twice, is equall to that which is contained under the lines AF and GH once. Take each of those parallelogrammes five times. Wherefore that which is contained under the lines AF and HC ten times, is equall to that which is contained under the lines AF & GH five times, that is, to two pentagons. Wherefore that which is contained under the lines AF and HC five times, is equall to one pentagon. But that which is contained under the lines AF and HC five times, is equall (by the 1. of the sixth) to that which is contained under the lines AF and HB , for the line HB is quintuple to the line HC (as it is easie to see by the construction) and they are both under one & the selfe same altitude, namely, under AF . Wherefore that which is contained under the lines AF and BH , is equall to one pentagon.



The Assumpt.
which also Flus-
sas putteth as an
Assumpt after
the 6. propositiō.
Demonstration
of the Assumpt.

This being proued, now let there be drawne a Circle comprehending both the Pentagon of a Dodecahedron, and the triangle of an Icosahedron, being both described in one and the selfe same Sphere.



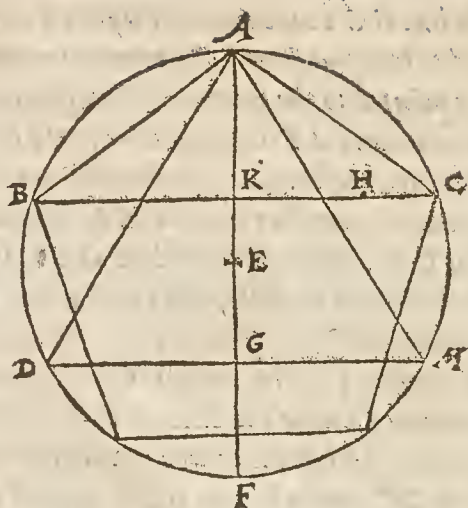
Et the circle be ABC . And in it describe as before, two sides of an equilateral pentagon, namely BA and AC : and draw a right line from the point B to the point C : and take the centre of the circle and let the same be E . And from the point A to the point E draw a right line AE : and extend the line AE to the point F . And let it cut the line BC in the point K . And let the line AE be double to the line EG , & let the line CK be treble to the line CH , by the 9. of the sixth. And from the point G raise up (by the 11. of the first) unto the line AF a perpendicular line GM : and extend the line GM directly to the point D . Wherefore the line MD is the side of an equilateral triangle, by the corollary of the 12. of the thirteenth: draw these right lines AD and AM . Wherefore ADM is an equilateral triangle. And for as much as that which is contained under the lines AG and BH is equal to the pentagon (by the former assumpt) and that which

Construction
pertaining to
the second de-
monstration of
the 4. propositiō.

Second demon-
stration of the
4. propositiō.

The fourteenth Booke

is contained under the lines AG and GD is equal to the triangle ADM : therefore as that which is contained under the lines AG and HB is to that which is contained under the lines AG and GD , so is the pentagon to the triangle. But as that which is contained under the lines BH & AG is to that which is contained under the lines AG and GD , so is the line BH to the line DG (by the 1. of the sixth) wherefore (by the 15. of the fifth) as 12. such lines as BH is, are to 20. such lines as DG is, so are 12. pentagons to 20. triangles, that is the superficies of the Dodecahedron, to the superficies of the Icosahedron. And 12. such lines as BH is, are equal to tenne such lines as BC is (for the line HB is quintuple to the line HC): and the line BC is sextuple to the line CH . Wherefore six such lines as BH is, are equal to five such lines as BC are: and in the same proportion are their doubles: and 20. such lines as the line DG is, are equal to 10. such lines as the line DM is: for the line DM is double to the line DG . Wherefore as 10. such lines as BC is, are to 10. such lines as DM is, that is, as the line BC is to the line DM , so is the superficies of the Dodecahedron to the superficies of the Icosahedron. But the line BC is the side of the cube, and the line DM the side of the Icosahedron: wherefore (by the 11. of the fifth) as the superficies of the Dodecahedron is to the superficies of the Icosahedron, so is the line BC to the line DM , that is, the side of the cube to the side of the Icosahedron.



The 7. proposition after
Euclid.

Nowe will we proue that a right line being deuided by an extreme and meane proportiō, what proportiō the line cōtaining in power the squares of the whole line and of the greater segment, hath to the line containing in power the squares of the whole line and of the lesse segment, the same proportion hath the side of the cube to the side of the Icosahedron, being both described in one and the selfe same sphere.

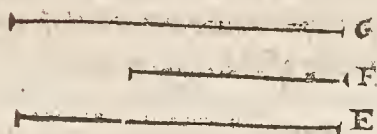
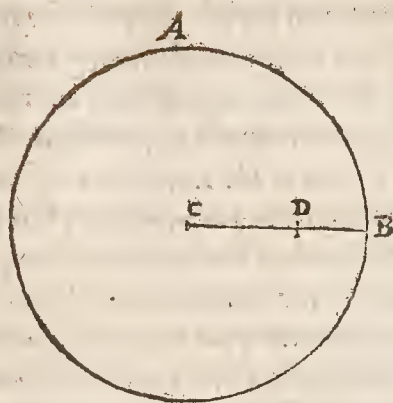
Construction.



Suppose that AB be a circle containing both the pentagon of a Dodecahedron & the triangle of an Icosahedron described bothe in one and the selfe same sphere. Take the centre of the circle, and let the same be C . And from the point C extend to the circumference a right line at all auentures, and let the same be BC . And (by the 30. of the sixth) deuide the line BC by an extreme and meane proportion in the point D , and let the greater segment thereof be CD . Wherefore the line CD is the side of a Decagō described in the same circle (by the corollary of the 9. of the thirtieth). Take the side of an Icosahedron, and let the same be the line E , and the side of a Dodecahedron, and let the same be the line F , and the side of a cube & let the same be the line G . Wherefore the line E is the side of an equilateral triangle, and F of an equaliter pentagon described in one and the selfe same circle. And the line G being deuided by an extreme and meane proportion, his greater segment is the line F , by the corollary of the 17. of the thirtieth. Now forasmuch as the line E is the side of an equilateral triangle, but (by the 12. of the thirtieth) the side of an equilateral triangle is in power treble to the line BC , (which is drawne from the center to the circumference) therefore the square of the line E is treble to the square of the line BC : but the squares of the line BC and BD are (by the 4. of the thirtieth) treble to the square of the line CD . Wherefore as the square of the line E is to the square of the line CB , so are the squares of the lines CB and BD to the square of the line CD . Wherefore alternately (by the 16. of

Demonstration.

the fifth) as the square of the line E is to the squares of the lines C B and B D, so is the square of the line C B to the square of the line C D. * But as the square of the line B C is to the square of the line C D, so is the square of the line G (the side of the cube) to the square of the line F, the side of Dodecahedron. For the line F is the greater segmēt of the line G (as was before proued.) Wherefore (by the 11. of the fifth) as the square of the line E is to the squares C B and B D, so is the square of the line G, to the square of the line F. Wherefore alternately (by the 16. of the fifth) & also by conuersion (by the corollary of the 4. of the fifth) as the square of the line G, is to the square of the line E, so is the square of the line F, to the squares of the lines C B & B D. But vnto the square of the line F are equal the squares of the lines B C & C D, for the side of a pentagon cōtaineth in power both the side of a sixe angled figure, and the side of a ten angled figure (by the 10. of the thirteenth.) Wherefore as the square of the line G, is to the square of the line E, so are the squares of the lines B C and C D to the squares of the lines C B and B D. But as the squares of the lines C B and C D are to the squares of the lines C B & B D, † so (any right line what so euer it be, being diuided by an extreme and meane proportion) is the line containing in power the squares of the whole line, and of the greater segmēt, to the line containing in power the squares of the whole line, and of the lesse segment: wherefore (by the 11. of the fifth) as the square of the line G (the side of the cube) is to the square of the line E, so (any right line being deuided by an extreme and meane proportion) is the line containing in power the squares made of the whole line, and of the greater segmēt, to the line containing in power the squares made of the whole line, and of the lesse segment: but the line G is the side of the Cube, and the line E of the Icosahedron (by supposition.) If therfore a right line be deuided by an extreeme and meane proportion, as the line cōtaining in power the squares of the whole line, and of the greater segment, is to the line containing in power the squares of the whole line and of the lesse segment: so is the side of the cube to the side of the Icosahedron, being both described in one and the selfe same sphere.



* Here againe is required the Assumpt afterward proued in this 4. proposition.

† As may by the Assumpt afterward in this proposition be plainly proued.

Now will we proue that as the side of the Cube is to the side of the Icosahedron, so is the solide of the Dodecahedron to the solide of the Icosahedron.

The 8. proposition after Flussas.



Orasmuche as equal circles comprehend both the pentagon of a Dodecahedron, and the triangle of an Icosahedron, being both described in one and the selfe same sphere, by the 2. of this booke: but in a sphere equal circles are equally distant from the centre (for the perpendicular lines drawn from the centre of the sphere to the plaine superficieses of the circles are equal, and do fall vpon the centres of the circles. † Wherefore perpendicular lines drawne from the centre of the sphere, to the centre of the circle, comprehending bothe the triangle of an Icosahedron, and the pentagon of a Dodecahedron are equal: wherefore the pyramides, whose bases are the pentagons of the Dodecahedron, are of equal altitude with the pyramides whose bases are the triangles of the Icosahedron. But pyramids of equal altitude, are in that proportion the one to the other, that their bases are (by the 5. of the twelfth) wherefore as the pentagon is to the triangle, so is the pyramis whose base is the pentagon of the Dodecahedron and toppe the centre of the sphere, to the pyramis whose base is the triangle and top the

† By the Corollary adaed by Flussas after his Assumpt put after the 17. proposition of the 12. booke.

The fourteenth Booke

centre also of the sphere. Wherefore (by the 15. of the fifth) as 12. pentagons are to 20. triangles; so are 12. pyramids hauing pentagons to their bases to 20. pyramids hauing triangles to their bases. But 12. pentagons are the superficies of the Dodecahedron, and 20. triangles are the superficies of the Icosahedron. Wherefore as the superficies of the Dodecahedron is to the superficies of the Icosahedron, so are 12. pyramids hauing pentagons to their bases to 20. pyramids, hauing triangles to their bases. But 12. pyramids hauing pentagons to their bases, are the solide of the Dodecahedron, and 20. pyramids hauing triangles to their bases are the solide of the Icosahedron. Wherefore (by the 11. of the fifth) as the superficies of the Dodecahedron is to the superficies of the Icosahedron, so is the solide of the Dodecahedron to the solide of the Icosahedron. But as the superficies of the Dodecahedron, is to the superficies of the Icosahedron, so haue we proued that the side of the cube is to the side of the Icosahedron. Wherefore, by the 11. of the fifth, as the side of the cube is to the side of the Icosahedron, so is the solide of the Dodecahedron to the solide of the Icosahedron.

Corollary of the
8. after Flusius.

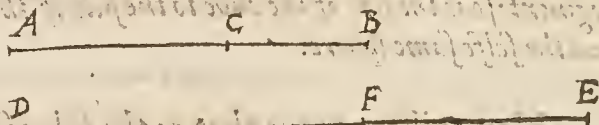
This Assumpt is
the 3. propositio
after Flusius,
and is it which
diuers times
hath bene taken
as granted in
this booke, and
once also in the
last proposition
of the 13. booke:
as we haue be-
fore noted.
Demonstra-
tion.

If two right lines be diuided by an extreame and meane proportion, they shall euery way be in like proportion: which thing is thus demonstrated.



Et the line AB be (by the 30. of the sixth) diuided by an extreame and meane proportion in the poynt C , and let the greater segment thereof be the line CA . And likewise also let the line DE be diuided by an extreame and meane proportion in the poynt F , and let the greater segment thereof be the line DF . Then I say that as the whole line AB is to the greater segment thereof AC , so is the whole line DE to the greater segment thereof DF . For forasmuch as that which is containd vnder the lines AB and BC is equall to the square of the line AC (by the diffinition of a line diuided be an extreame and meane proportion): and that which is containd vnder the lines DE and EF is also equall to the square of the line DF (by the same diffinition): therefore as that which is containd vnder the lines AB and BC is to the square of the line AC , so is that which is containd vnder the lines DE and EF to the square of the line DF . For in eche is the proportion of equalitie. Wherefore as that which is containd vnder the lines AB and BC fower times, is to the square of the line AC , so is that which is containd vnder the lines DE and EF fower times to the square of the line DF (by the 15. of the fifth).

Wherefore by composition (by the 18. of the fifth) as that which is containd vnder the lines AB and BC fower times, together with the square of the line AC , is to the square of the line AC , so is that which is containd vnder the lines DE and EF fower times, together with the square of the line DF , to the square of the line DF . Wherefore as the square which is made of the lines AB and BC added together and made one line (which square by the 8. of the second is equall to that which is containd vnder the lines AB and BC fower times together with square of the line AC) is to the square of the line AC , so is the square made of the lines DE & EF added together and made one line (which square is also, by the same, equal to that which is containd vnder the lines DE and EF fower times together with the square of the line DF) to the square of the line DF . Wherefore also as the lines AB & BC added together are to the line AC , so are the lines DE & EF added together to the line DF (by the 22. of the sixth). Wherefore by composition (by the 18. of the fifth) as both the lines AB & BC added the one to the other, together with the line AC , that is, as two such lines as AB is, are to the line AC , so are both the lines DE and EF added the one to the other together with the line DF , that is two such lines



lines as DE is to the line DF . And in the same proportion are the halues of the antecedents by the 15. of the fifth. Wherefore as the line AB is to the line AC , so is the line DE to the line DF . (And therefore by the 19. of the fifth, as the line AB is to the line BC , so is the line DF to the line FE . Wherefore also by division by the 17. of the fifth, as the line AC is to the line CB , so is the line DF to the line DE).

Now that we haue proued,* that, any right line whatsoeuer being diuided by an extreame and meane proportion, what proportion the line containyng in power the squares made of the whole line and of the greater segment added together, hath to the line containyng in power the squares made of the whole line and of the lesse segment added together, the same proportion hath the side of the cube to the side of the Icosahedron: Now also that we haue proued, † that as the side of the cube is to the side of the Icosahedron, so is the superficies of the Dodecahedron to the superficies of the Icosahedron, being both described in one and the selfe same sphere: and moreouer seing that we haue proued, ‡ that as the superficies of the Dodecahedron is to the superficies of the Icosahedron, so is the Dodecahedron to the Icosahedron, for that both the pentagon of the Dodecahedron, and the triangle of the Icosahedron are comprehended in one and the selfe same circle: All these things I say being proued, it is manifest, that if in

* In the 4. section of this proposition.

† In the 1. and 3. section of the same proposition.

‡ In the 5. section of the same proposition.

one and the selfe same sphere be described a Dodecahedron, and an Icosahedron, they shall be in proportion the one to the other, as, a right line whatsoeuer being diuided by an extreame and meane proportion, the line containyng in power the squares of the whole line and of the greater segment added together, is to the line containyng in power the squares of the whole line and of the lesse segment added together. For for that as the Dodecahedron is to the Icosahedron, so is the superficies of the Dodecahedron to the superficies of the Icosahedron, that is, the side of the cube to the side of the Icosahedron: but as the side of the cube is to the side of the Icosahedron, so, any right line what so euer being diuided by an extreame and meane proportion, is the line containyng in power the squares of the whole line and of the greater segment added together, to the line containyng in power the squares of the whole line and of the lesse segment added together.

A Corollary.

Wherefore as a Dodecahedron is to an Icosahedron described in one and the selfe same sphere, so, any right line what so euer being diuided by an extreame and meane proportion, is the line containyng in power the squares of the whole line & of the greater segment added together, to the line containyng in power the squares of the whole line and of the lesse segment added together.

(? ?)

The ende of the fourteenth Booke

of Euclides Elementes
after Hypsicles.



The fourteenth booke of Euclides Elementes after Fluffas.



Or that the fourteenth Booke, as it is set forth by Fluffas, containeth in it moe Propositions then are found in Hypsicles, & also some of those Propositions which Hypsicles hath, are by him somewhat otherwise demonstrated, I thought my labour well bestowed for the readers sake to turne it also all whole, notwithstanding my trauaile before taken in turning the same booke after Hypsicles. Where note ye, that here in this 14. booke after Fluffas, and in the other bookes following, namely, the 15. and 16. I haue in alleading of the Propositions of the same 14. booke, followed the order and number of the Propositions, as Fluffas hath placed them.

The first Proposition.

A perpendicular line drawn from the centre of a circle, to the side of a Pentagon inscribed in the same circle: is the halfe of these two lines taken together, namely, of the side of the hexagon, and of the side of the decagon inscribed in the same circle.

The first proposition after Campane.

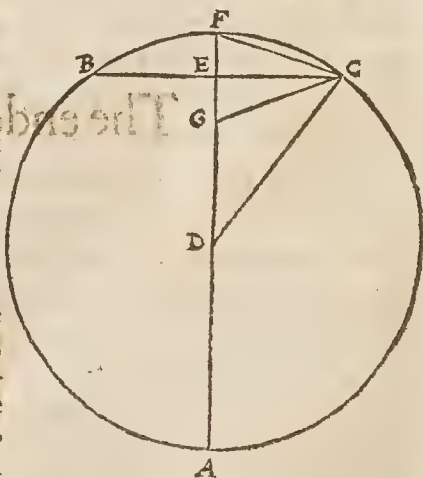
Construction.



Demonstration.

Take a circle ABC , and inscribe in it the side of a pentagon, which let be BC , and take the centre of the circle, which let be the point D : and fro it draw vnto the side BC a perpendicular line DE which produce to the point F . And vnto the line EF put the line EG equall. And draw these right lines CG , CD , and CF . Then I say, that the right line DE (which is drawn from the centre to BC the side of the pentagon) is the halfe of the sides of the decagon and hexagon, taken together. Forasmuch as the line DE is a perpendicular vnto the line BC : therefore the sections BE and EC shall be equall (by the 3. of the third): and the line EF is common vnto them both, and the angles FEC and FEB , are right angles, by supposition. Wherefore the bases BF and FC are equall (by the 4. of the first). But the line BC is the side of a pentagon, by constructi-

on. Wherefore FC which subtendeth the halfe of the side of the pentagon, is the side of the decagon inscribed in the circle ABC . But vnto the line FC is, by the 4. of the first, equall the line CG , for they subtend right angles CEG , and CEF , which are contained vnder equall sides. Wherefore also the angles CGE , and CFE , of the triangle CFG , are equall, by the 5. of the first. And forasmuch as the arke FC is subtended of the side of a decagon, the arke CA shall be quadruple to the arke CF : Wherefore also the angle CDA shall be quadruple to the angle CDF (by the last of the sixth). And forasmuch as the same angle CDA , which is set at the centre, is double to the angle CFA , which is set at the circumference, by the 20. of the third: therefore the angle CFA , or CFD , is double to the angle CDF , namely, the halfe of quadruple. But vnto the angle CFD or CFG , is proued equall the angle CGF : Wherefore the outward angle CGF , is double to the angle CDF . Wherefore the angles CDG and DCG , shall be equall. For vnto those two angles the angle CGF is equall, by the 32. of the first. Wherefore the sides GC and GD , are equall, by the 6. of the first. Wherefore also the



the line GD is equall to the line FC , which is the ſide of the decagon. But vnto the right line FE is equall the line EG , by conſtruction. Wherefore the whole line DE is equall to the two lines CF and FE . Wherefore thoſe lines taken together (namely, the lines DF and FC) ſhall be double to the line DE . Wherefore the line DE (which is drawn from the centre perpendicularly to the ſide of the pentagon) ſhal be the halfe of both theſe lines taken together, namely, of DF the ſide of the hexagon, and CF the ſide of the decagon. For the line DF which is drawn from the centre, is equall to the ſide of the hexagon, by the Corollary of the 15. of the fourth. Wherefore a perpendicular line drawn from the center of a circle, to the ſide of a pentagon inſcribed in the ſame circle: is the halfe of theſe two lines taken together, namely, of the ſide of the hexagon, and of the ſide of the decagon inſcribed in the ſame circle: which was required to be proued.

✱ A Corollary.

If a right line drawn perpendicularly from the centre of a circle to the ſide of a pentagon, be diuided by an extreme and meane proportion: the greater ſegment ſhall be the line which is drawn from the ſame centre to the ſide of an equilater triangle inſcribed in the ſame circle. For, that line (drawn to the ſide of the triangle) is (by the Corollary of the 12. of the thirteenth) the halfe of the line drawn from the centre to the circumference, that is, of the ſide of the hexagon: Wherefore the reſidue ſhall be the halfe of the ſide of the decagon. For the whole line is the halfe of the two ſides, namely, of the ſide of the hexagon, and of the ſide of the decagon. But of the ſide of a decagon and of an hexagon taken together, the greater ſegment is the ſide of the hexagon (by the 9. of the thirteenth). Wherefore the greater ſegment of their halves ſhall be the halfe of the hexagon, by the 15. of the fifth: which halfe is the perpendicular line drawn from the centre to the ſide of the triangle, by the Corollary of the 12. of the thirteenth.

¶ The ſecond Propoſition.

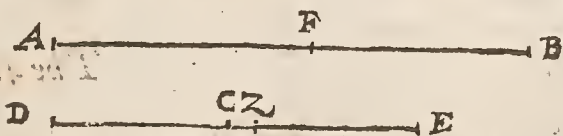
If two right lines be diuided by an extreme and meane proportion: they ſhall be diuided into the ſelfe ſame proportions.

The 2. propoſition after Campane.



Suppoſe that theſe two right lines AB and DE be eche cut by an extreme and meane proportion in the pointes F and Z . Then I ſay, that theſe two lines are diuided into the ſelfe ſame proportions, that is, that the line AB is in the point F diuided in like ſort as the line DE is in the point Z . For if they be not in like ſort cut, let one of them, namely, DE , be cut like vnto the line AB in the point C . So that let the line DE be to DC the greater part, as the greater part DC is to CE the leſſe part, by the 3. definition of the ſixt. But (by ſuppoſition) the line DE is to the line DZ , as the line DZ is to the line ZE . Wherefore the right line DE is diuided by an extreme and meane proportion in two pointes C and Z . But the proportion of DE to DC the leſſe line, is greater then the proportion of the ſame DE to DZ the greater line, by the 2. part of the 8. of the fifth. But as DE is to DC , ſo is DC to CE . Wherefore the proportion of DC to CE , is greater then the proportion of DZ to ZE . And forasmuch as DZ is greater then DC , the proportion of DZ to CE ſhall be greater then the proportion of DC to CE , by the 8. of the fifth. Wherefore the proportion of DZ to CE , is much greater then the proportion of DZ to ZE . Wherefore one and the ſelfe ſame magnitude, namely, DZ , hath to CE the greater line, a greater proportion then it hath to ZE the leſſe line, contrary to the ſecond part of the 8. of the fifth: which is impoſſible. Wherefore the right lines AB & DE , are not cut vnlike. Wherefore they are cut like, and into the ſelfe ſame proportions. And the ſame demonſtration alſo will ſerue, if the point C fall in any other place. For alwaies ſome one of them ſhall be the greater. If therefore two right lines be cut by an extreme and meane proportion: they ſhall be cut into the ſelfe ſame proportions: which was required to be proued.

Demonſtration leading to an impoſſibilitie.



¶ The third Propoſition.

If in a circle be deſcribed an equilater Pentagon: the ſquares made of the ſide of the Pentagon and of the line which ſubtendeth two ſides of the Pentagon,

The 4. propoſition after Campane.

RRr.j.

The fourteenth Booke

Pentagon, these two squares (I say) taken together, are quintuple to the square of the line drawn from the centre of the circle to the circumference.

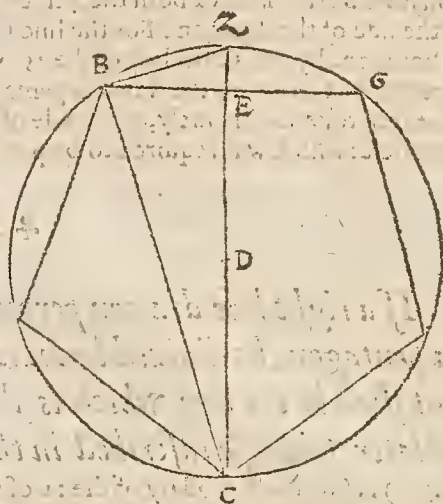
Construction.



Suppose that in the circle B C G the side of a Pentagon be B G : and let the line B C subtend two sides thereof. And let the line B G be divided into two equall partes by a right line drawn from the centre D : namely, by the diameter C D E produced to the point Z. And drawe the right line B Z . Then I say, that the right lines B C and B G, are in power quintuple to the right line D Z, which is drawn from the centre to the circumference.

Demonstration.

For forasmuch as (by the 47. of the first) the squares of the lines C B and B Z, are equall to the square of the diameter C Z : therefore they are quadruple to the square of the line D Z, by the 20. of the sixth (for the line C Z is double to the line D Z). Wherefore the right lines C B, B Z, and Z D, are in power quintuple to the line Z D. But the right line B G containeth in power the two lines B Z and Z D, by the 10. of the thirteenth. For D Z is the side of an hexagon, & B Z the side of a decagon. Wherefore the lines B C and B G (whose powers are equall to the powers of the lines C B, B Z, Z D) are in power quintuple to the line D Z. If therefore in a circle be described an equilateral Pentagon : the squares made of the side of the Pentagon and of the line which subtendeth two sides of the Pentagon, these two squares (I say) taken together, are quintuple to the square of the line drawn from the centre of the circle to the circumference.



¶ A Corollary.

This Corollary Campanè also putteth after the 4. proposition in his order.

If a Cube and a Dodecahedron be contained in one and the selfe same Sphere : the side of the Cube, and the side of the Dodecahedron, are in power quintuple to the line which is drawn from the centre of the circle which containeth the Pentagon of the Dodecahedron. For it was proued in the 17. of the thirteenth, that the side of the Cube subtendeth two sides of the Pentagon of the Dodecahedron, where the sayd solides are contained in one and the selfe same Sphere. Wherefore the side of the Cube subtending two sides of the Pentagon, and the side of the same Pentagon, are contained in one and the selfe same circle. Wherefore, by this Proposition, they are in power quintuple to the line which is drawn from the centre of the same circle, which containeth the Pentagon of the Dodecahedron.

The 4. Proposition.

The 5. proposition after Campanè.

One and the selfe same circle containeth both the Pentagon of a Dodecahedron, and the triangle of an Icosahedron described in one and the selfe same sphere.

Construction.

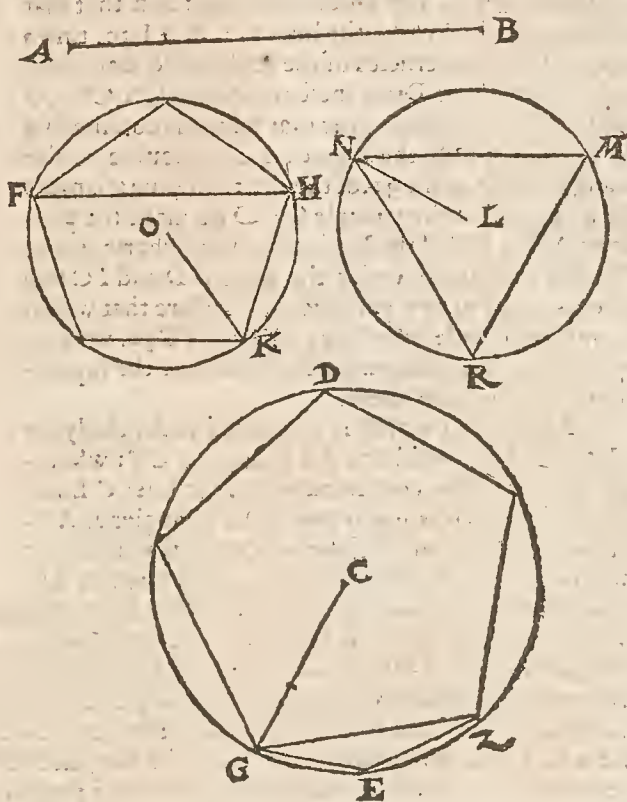


Let the diameter of the sphere geuen be A B, and let the bases of the Icosahedron and Dodecahedron described in it, be the triangle M N R, and the pentagon F K H, and about them let there be described circles. by the 5. and 14. of the fourth. And let the lines drawne from the centres of those circles to the circumferences be L N and O K. Then I say that the lines L N and O K are equal, and therefore one and the selfe same circle containeth both those figures. Let the right line A B, be in power quintuple to some one right line, as to the line C G (by the Corollary of the 6. of the tenth.) And making the centre the poynt C, & the space C G, describe a circle D Z G. And let the side of a pentagon inscribed in that circle (by the 11. of the fourth) be the line Z G. And let E G (subtending halfe of the arke Z G) be the side of a Decagon inscribed in that circle. And by the 30. of the sixth, diuide the line C G by an extreme & meane proportion in the poynt I. Now forasmuche as in the 16. of the thirteenth, it was proued, that this line C G (vnto whome the diameter A B of the sphere is in power quintuple) is the line which is drawne from the centre of the circle, which containeth five angles of the Icosahedron, and the side of the pentagon described

Demonstration.

in

in that circle D Z G, namely the line Z G is side of the Icosahedron described in the Sphere, whose diameter is the line A B: therefore the right line Z G, is equal to the line M N, which was put to be the side of the Icosahedron, or of his triagular base. Moreouer, by the 17. of the thirteenth, it was manifest that the right line F H (which subtendeth the angle of the pentagon of the Dodecahedron inscribed in the foresayde sphere) is the side of the Cube, inscribed in the self same sphere. (For vpon the angles of the cube, were made the angles of the Dodecahedron.) Wherefore the diameter A B is in power triple to F H, the side of the Cube (by the 15. of the thirteenth). But the same line A B is (by supposition) in power quintuple to the line C G. Wherefore fivesquares of the line C G, are equal to thre squares of the line F H: (for eche is equal to one and the self same square of the line A B). And forasmuche as E G the side of the Decagon, cutteth the right line C G by an extreme and meane proportion (by the collary of the 9. of the thirteenth): Likewise the line H K, cutteth the line F H, the side of the Cube by an extreeme and meane proportion (by the Collary of the 17. of the thirteenth): therefore the lines C G and F H, are deuided into the self same proportions, by the second of this booke: and the right lines C I and E G, which are the greater segmentes of one and the selfe same line C G, are equal: And forasmuche as fivesquares of the line C G are equal to thre squares of the lines F H: therefore fivesquares of the line G E, are equal to thre squares of the line H K (for the lines G E and H K are the greater segmets of the lines C G and F H). Wherefore fivesquares of the lines C G & G E are equal to thre squares of the lines F H & H K, by the 12. of the fift. But vnto the squares of the lines C G and G E, is equal the square of the line Z G, by the 10. of the thirteenth: and vnto the line Z G the line M N was equal: wherefore fivesquares of the line M N, are equal to three squares of the lines F H, H K. But the squares of the lines F H and H K, are quintuple to the square of the line O K (which is drawne from the centre) by the third of this booke. Wherefore thre squares of the lines F H and H K make 15. squares of the line O K. And forasmuch as the square of the line M N is triple to the square of the line L N (which is drawne from the centre) by the 12. of the thirteenth, therefore fivesquares of the line M N are equal to 15. squares of the line L N. But fivesquares of the line M N are equal vnto thre squares of the lines F H and H K. Wherefore one square of the line L N is equall to one square of the line O K (being eche the fuetenth part of equal magnitudes) by the 15. of the fifth. Wherefore the lines L N and O K, which are drawne from the centers, are equal. Wherefore also the circles N R M, and F K H which are described of those lines, are equal. And those circles contayne (by supposition) the bases of the Dodecahedron and of the Icosahedron described in one and the selfe same sphere. Wherefore one and the selfe same circle. &c. as in the proposition: which was required to be proued.



The 5. Proposition.

If in a circle be inscribed the pentagon of a Dodecahedron, and the triangle of an Icosahedron, and from the centre to one of theyr sides, be drawne a perpendicular line: That which is contained 30. times vnder the side, & the perpendicular line falling vpon it, is equal to the superficies of that solide, vpon whose side the perpendicular line falleth.

This is the 6th and 7. propositions after Campane.



Vppose that in the circle A G E, be described the pentagon of a Dodecahedron, which let be A B G D E, and the triangle of an Icosahedron described in the same sphere, which let be A F H. And let the centre be the poynt C. From which draw perpendicularly the line C I to the side of the Pentagon, and the line C L to the side of the triangle. Then I say that the rectangle figure contained vnder the lines C I and G D 30. times, is equal to

Construction.

RRr.ij.

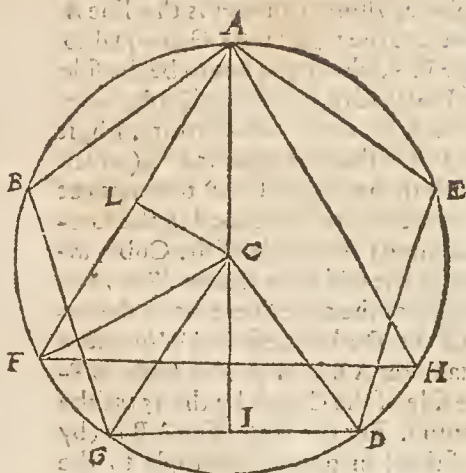
the

The fourteenth Booke

Demonstration.

the superficies of the Dodecahedron : and that that which is contained vnder the lines CL & AF 30. times is equal to the superficies of the Icosahedron described in the same sphere. Draw these right lines CA, CF, CG and CD . Now forasmuch as that which is contained vnder the base GD & the altitude IC , is double to the triangle GCD , by the 41. of the first: And five triangles like and equal to the triangle GCD do make the pentagon $ABGDE$ of the Dodecahedron: wherefore that which is contained vnder the lines GD and IC five times is equal to two pentagons. Wherefore that which is contained vnder the lines GD and IC 30. times is equal to the 12. pentagons, which containe the superficies of the Dodecahedron.

Again that which is contained vnder the lines CL and AF , is double to the triangle ACF : wherefore that which is contained vnder the lines CL and AF three times is equal to two such triangles as AFH is, which is one of the bases of the Icosahedron (for the triangle ACF , is the third part of the triangle AFH , as it is easie to proue, by the 8. & 4. of the first.) Wherefore that which is contained vnder the lines CL and AF , 30 times times, is equall to 20. such triangles as AFH is, which containe the superficies of the Icosahedron. And forasmuch as one and the selfe same sphere containeth the Dodecahedron of this pentagon, and the Icosahedron of this triangle (by the 4. of this booke :) and the line CL falleth perpendicularly vpon the side of the Icosahedron, and the line CI vpon the side of the Dodecahedron: that which is 30. times contained vnder the side, and the perpendicular line falling vpon it, is equal to the superficies of that solide, vpon whose side the perpendicular falleth. If therefore in a circle &c. as in the proposition: which was required to be demonstrated.



A Corollary.

This Corollary Campane also addeth after the 7. proposition in his order.

The superficieses of a Dodecahedron and of an Icosahedron described in one and the selfe same sphere, are the one to the other, as that which is contained vnder the side of the one and the perpendicular line drawne vnto it from the centre of his base, to that which is contained vnder the side of the other, and the perpendicular line drawne to it from the centre of his base. For as thirtie times is to thirtie times, so is once to once by the 15. of the fifth.

The 6. Proposition.

The 5. proposition after Campane.

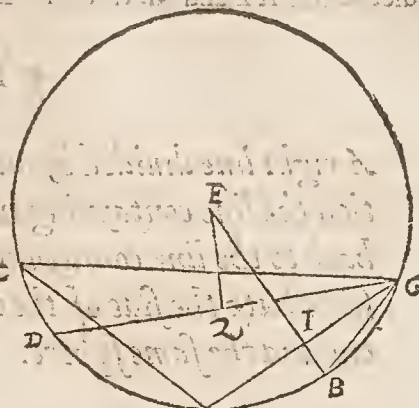
The superficies of a Dodecahedron, is to the superficies of an Icosahedron described in one and the selfe same sphere, in that proportion, that the side of the Cube is to the side of the Icosahedron contained in the self same sphere.

Construction.

Suppose that there be a circle ABG , & in it (by the 4. of this booke) let there be inscribed the sides of a Dodecahedron and of an Icosahedron contained in one and the selfe same sphere. And let the side of the Dodecahedron be AG , and the side of the Icosahedron be DG . And let the centre be the poynt E : from which draw vnto those sides, perpendicular lines EI and EZ . And produce the line EI to the poynt B , and draw the line BG . And let the side of the cube contained in the self same sphere be GC . Then I say that the superficies of the Dodecahedron is to the superficies of the Icosahedron, as the line CG , is to the line GD . For forasmuch as the line EI being diuided by an extreme and meane proportion, the greater segment thereof shall be the line EZ , by the corollary of the first of this booke: and the line CG being diuided by an extreme and meane proportion, his greater segment is the line AG , by the corollary of the 17. of the thirteenth: Wherefore the right lines EI and CG are cut proportionally by the second of this booke. Wherefore as the line CG , is to the line AG , so is the line EI to the line EZ . Wherefore that which is contained vnder the extreames CG and EZ , is equall to that which is contained vnder the meanes AG and EI . (by the 16. of the sixth.) But as that which is contained vnder the lines CG and EZ is to that which is contained vnder the lines DG and EZ , so (by the first of the sixth) is the line CG to the line

Demonstration.

line DG, for both those parallelogrames haue one and the selfe same altitude, namely the line EZ. Wherefore as that which is contained vnder the lines EI and AG (which is proued equal to that which is contained vnder the lines CG and EZ) is to that which is contained vnder the lines DG and EZ, so is the line CG to the line DG. But as that which is contained vnder the lines EI and AG is to that which is contained vnder the lines DG and EZ, so (by the corollary of the former proposition) is the superficies of the Dodecahedron, to the superficies of the Icosahedron. Wherefore as the superficies of the Dodecahedron is to the superficies of the Icosahedron, so is CG the side of the cube, to GD the side of the Icosahedron. The superficies therefore of a Dodecahedron is to the superficies. &c. as in the proposition, which was required to be proued.



An assumpt.

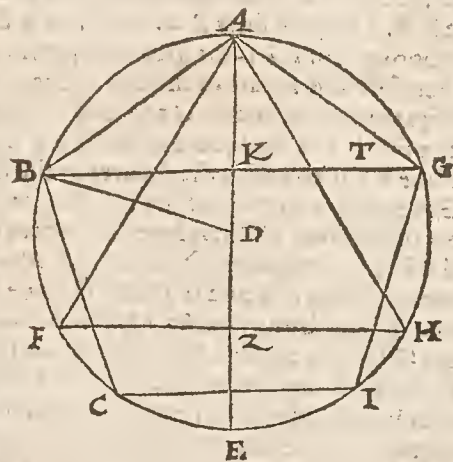
The Pentagon of a Dodecahedron, is equall to that which is contained vnder the perpendicular line which falleth vpon the base of the triangle of the Icosahedron, and five sixth partes of the side of the cube, the sayd three solids being described in one and the selfe same sphere.

This Assumpt
Campane also
hath after the
8. proposition,
in his order.

Suppose that in the circle ABEG, the pentagon of a Dodecahedron be ABCIG, and let two sides thereof AB and AG be subtended of the right line BG. And let the triangle of the Icosahedron inscribed in the selfe same sphere, by the 4. of this booke, be AFH. And let the centre of the circle be the poynt D, and let the diameter be ADE, cutting FH, the side of the triangle in the poynt Z, and cutting the line BG in the poynt K. And draw the right line BD. And from the right line KG cut off a third part TG, by the 9. of the sixth. Now then the line BG subtending two sides of the Dodecahedron, shalbe the side of the cube inscribed in the same sphere, by the 17. of the thirteenth: and the triangle of the Icosahedron of the same sphere shalbe AFH by the 4. of this booke. And the line AZ which passeth by the centre D shall fall perpendicularly vpon the side of the triangle. For forasmuch as the angles GAE & BAE are equal (by the 27. of the third, for they are set vpon equall circumferences): therefore the bases BK and KG are (by the 4. of the first) equal. Wherefore the line BT containeth 5. sixth partes of the line BG. Then I say that that which is contained vnder the lines AZ and BT, is equall to the pentagon ABCIG. For forasmuch as the line AZ is sesquialter to the line AD (for the line DE is diuided into two equall partes in the poynt Z, by the corollary of the 12. of the thirteenth). Likewise by construction, the line KG is sesquialter to the line KT: therefore as the line AZ is to the line AD, so is the line KG to the line KT. Wherefore that which is contained vnder the extreames AZ and KT, is equall to that which is contained vnder the meanes AD and KG, by the 16. of the sixth. But vnto the line KG is the line BK proued equall. Wherefore that which is contained vnder the lines AZ and KT is equall to that which is contained vnder the lines AD and BK. But that which is contained vnder the lines AD and BK is (by the 41. of the first) double to the triangle ABD. Wherefore that which is contained vnder the lines AZ and KT is double to the same triangle ABD. And forasmuch as the pentagon ABCIG containeth five triangles equal to the triangle ABD, and that which is contained vnder the lines AZ and KT containeth two such triangles: therefore the pentagon ABCIG is duple sesquialter to the rectangle parallelogramme contained vnder the lines AZ and KT. And forasmuch as by construction the line BT is duple sesquialter to the right line KT: but by 1. of the sixth, that which is contained vnder the lines AZ and BT is to that which is contained vnder the lines AZ and KT, as the base BT is to the base KT: therefore that which is contained vnder the lines AZ and BT is duple sesquialter to that which is contained vnder the line AZ & KT. But vnto that which is contained vnder the lines AZ and KT the pentagon ABCIG is proued duple sesquialter. Wherefore the pentagon ABCIG of the Dodecahedron is equall to that which is contained vnder the per-

Construction.

Demonstration.



The fourteenth Booke

pendicular line A Z, and vnder the line B T which is five fixe partes of the line B G.

¶ The 7. Proposition.

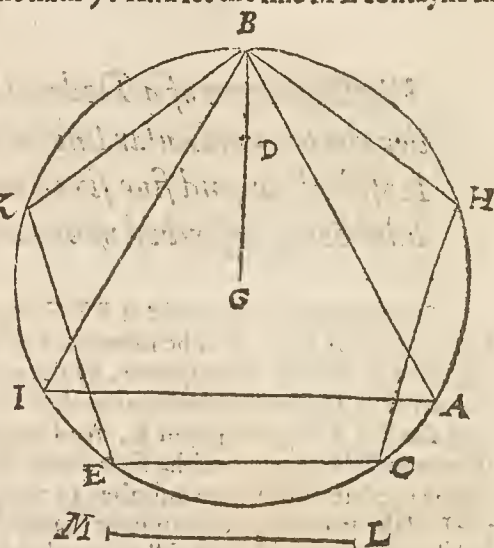
A right line diuided by an extreame and meane proportion: what proportion the line containyng in power the whole line and the greater segment, hath to the line containyng in power the whole and the lesse segment: the same hath the side of the cube to the side of the Icosahedron containyng in one and the same sphere.

The 9. proposition after Campanus.

Construction.

Take a circle A B E: and in it (by the 11. of the fourth) inscribe an equilater pentagon B Z E C H: and (by the second of the same) an equilater triangle A B I. And let the centre thereof be the poynt G. And draw a line from G to B. And diuide the line G B by an extreame and meane proportion in the poynt D (by the 30. of the sixth). And let the line M L containe in

power both the whole line G B and his lesse segment B D (by the corollary of the 13. of the tenth). And draw the right line B E subtending the angle of the pentagon, which shall be the side of the cube (by the corollary of the 17. of the thirteenth): and the line B I shall be the side of the Icosahedron, and the line B Z the side of the Dodecahedron by the 4. of this booke. Then I say that B E the side of the cube is to B I the side of the Icosahedron, as the line containyng in power the lines B G & G D is to the line containyng in power the lines G B and B D. Forasmuch as (by the 12. of the thirteenth) the line B I is in power triple to the line B G, and (by the 4. of the same) the squares of the line G B & B D are triple to the square of the line G D. Wherefore (by the 15. of the fifth) the square of the line B I is to the squares of the lines G B & B D (namely, triple to triple), as the square of the line B G is to the square of the line G D (namely, as one is to one). But as the square of the line B G is to the square of the line G D, so is the square of the line B E to the square of the line B Z. (For the lines B G, G D, and B E, B Z are in one and the same proportion, by the second of this booke. For B Z is the greater segment of the line B E, by the corollary of the 17. of the thirteenth). Wherefore the square of the line B E is to the square of the line B Z, as the square of the line B I is to the squares of the lines B G and B D. Wherefore alternately the square of the line B E is to the square of the line B I, as the square of the line B Z is to the squares of the lines G B and B D. But the square of the line B Z is equall to the squares of the lines B G and G D (by the 10. of the thirteenth). For the line B G is equall to the side of the hexagon, and the line G D to the side of the decagon, by the corollary of the 9. of the same. Wherefore the squares of the lines B G and G D, are to the squares of the lines G B and B D, as the square of the line B E is to the square of the line B I. But the line Z B containeth in power the lines B G and G D, and the line M L containeth in power the lines G B and B D by construction. Wherefore as the line Z B (which containeth in power the whole line B G and the greater segment G D) is to the line M L (which containeth in power the whole line G B and the lesse segment B D) so is B E the side of the cube to B I the side of the Icosahedron, by the 22. of the sixth. Wherefore a right line diuided by an extreame and meane proportion: what proportion the line containyng in power the whole line and the greater segment, hath to the line containyng in power the whole line and the lesse segment: the same hath the side of the cube to the side of the Icosahedron containyng in one and the same sphere: which was required to be proued.



Demonstration.

¶ The 8. Proposition.

The solide of a Dodecahedron is to the solide of an Icosahedron: as the side of a Cube is to the side of an Icosahedron, all those solides being described in one and the selfe same Sphere.

This Campanus putteth as a Corollary in the 9. proposition after his order.

Forasmuch



Forasmuch as in the 4. of this booke, it hath bene proued, that one and the self same circle containeth both the triangle of an Icosahedron, and the pentagon of a Dodecahedron described in one and the selfe same Sphere : Wherefore the circles, which containe those bases, being equall, the perpendiculars also which are drawn from the centre of the Sphere to those circles, shall be equall (by the Corollary of the Assumpt of the 16. of the twelfth) . And therefore the Pyramids set vpon the bases of those solides haue one and the selfe same altitude : For the altitudes of those Pyramids concur in the centre.

Wherefore they are in proportion as their bases are, by the 5. and 6. of the twelfth . And therefore the pyramids which compose the Dodecahedron, are to the pyramids which compose the Icosahedron, as the bases are, which bases are the superficieses of those solides . Wherefore their solides are the one to the other, as their superficieses are . But the superficies of the Dodecahedron is to the superficies of the Icosahedron, as the side of the cube is to the side of the Icosahedron, by the 6. of this booke. Wherefore by the 11. of the fifth, as the solide of the Dodecahedron is to the solide of the Icosahedron, so is the side of the cube to the side of the Icosahedron, all the said solides being inscribed in one and the selfe same Sphere . Wherefore the solide of a Dodecahedron is to the solide of an Icosahedron : as the side of a cube is to the side of an Icosahedron, all those solides being described in one and the selfe same Sphere : which was required to be proued.

✱ A Corollary.

The solide of a Dodecahedron is to the solide of an Icosahedron, as the superficieses of the one are to the superficieses of the other, being described in one and the selfe same Sphere : Namely, as the side of the cube is to the side of the Icosahedron, as was before manifest : for they are resolued into pyramids of one and the selfe same altitude.

This Corollary is the 9. proposition after Campane.

¶ The 9. Proposition.

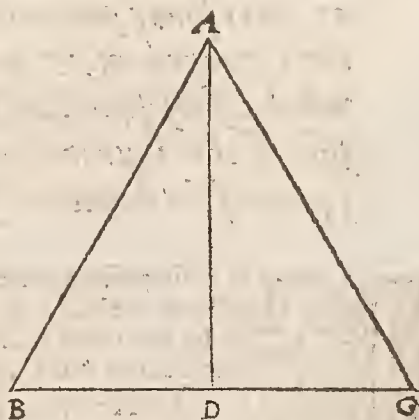
If the side of an equilater triangle be rationall: the superficies shall be irrationall, of that kinde which is called Mediall.

The 12. proposition after Campane.



Vppose that ABG be an equilater triangle: and from the point A draw vnto the side BG a perpendicular line AD : and let the line AB be rationall . Then I say, that the superficies ABG is mediall . Forasmuch as the line AB is in power sesquitertia to the line AD (by the Corollary of the 12. of the thirteenth) : of what partes the line AB containeth in power 12, of the same partes the line AD containeth in power 9 : wherefore the residue BD containeth in power of the same partes 3. (Eor the line AB containeth in power

the lines AD and BD , by the 47. of the first) . Wherefore the lines AD and DB are rationall and commensurable to the rationall line set AB , by the 6. of the tenth . But forasmuch as the power of the line AD is to the power of the line DB in that proportion that 9. a square number is to 3. a number not square : therfore they are not in the proportion of square numbers, by the Corollary of the 25. of the eighth. And therfore they are not commensurable in length, by the 9. of the tenth . Wherefore that which is contained vnder the lines AD and DB , which are rationall lines commensurable in power onely, is mediall, by the 22. of the tenth . But that which is contained vnder the lines AD and DB , is double to the triangle ABD , by the 41. of the first . Wherefore that which is contained vnder the lines AD and DB , is equall to the whole triangle ABG (which is double to the triangle ABD , by the 1. of the sixt) . Wherefore the triangle ABG is mediall . If therfore the side of an equilater triangle be rationall : the superficies shall be irrationall, of that kinde which is called Mediall : which was required to be proued.



Construction.

Demonstration.

✱ A Corollary.

If an Octohedron and a Tetrahedron be inscribed in a Sphere whose diameter is rationall: their superficieses shall be mediall . For those superficieses consist of equilater triangles, whose sides are commensurable to the diameter which is rationall, by the 13. and 14. of

The 13. proposition after Campane.

The fourteenth Booke

14. of the thirteenth, and therefore they are rationall. But they are commensurable in power onely to the perpendicular line, and therefore they containe a medially triangle, as it was before manifest.

¶ The 10. Proposition.

The 14. proposition after Campane.

If a Tetrahedron and an Octohedron be inscribed in one and the self same Sphere: the base of the Tetrahedron shall be sesquitertia to the base of the Octohedron, and the superficieses of the Octohedron shall be sesquialtera to the superficieses of the Tetrahedron.

Demonstration of the first part.



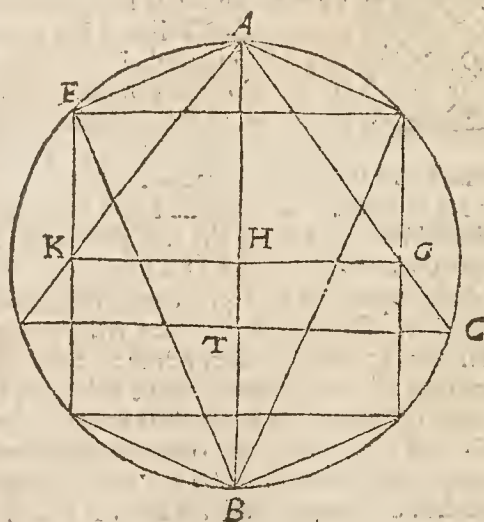
Orasmuch as the diameter of the Sphere is in power sesquialtera to the side of the Tetrahedron (by the 13. of the thirteenth) and the same diameter is in power duple to the side of the Octohedron (by the 14. of the same booke): therefore of what partes the diameter containeth in power sixe, of the same the side of the Tetrahedron containeth in power 4, and of the same the side of the Octohedron containeth in power 3. Wherefore the power of the side of the Tetrahedron, is to the power of the side of the Octohedron in the same proportion that 4. is to 3: which is sesquitertia. And like triangles (which are the bases of the solides) described of those sides, shall have the one to the other the same proportion that the squares made of those sides haue. For both the triangles are the one to the other, and also the squares are the one to the other, in double proportion of that in which the sides are, by the 10. of the sixth. Wherefore of what partes one base of the Tetrahedron was 4: of the same are fower bases of the Tetrahedron 16: likewise of what partes of the same one base of the Octohedron was 3: of the same are 8. bases of the Octohedron 24. Wherefore the bases of the Octohedron are to the bases of the Tetrahedron, in that proportion that 24. is to 16: which is sesquialtera. If therefore a Tetrahedron and an Octohedron be inscribed in one and the selfe same Sphere: the base of the Tetrahedron shall be sesquitertia to the base of the Octohedron, and the superficieses of the Octohedron shall be sesquialtera to the superficieses of the Tetrahedron: which was required to be proued.

Demonstration of the second part.

¶ The 11. Proposition.

The 17. proposition after Campane.

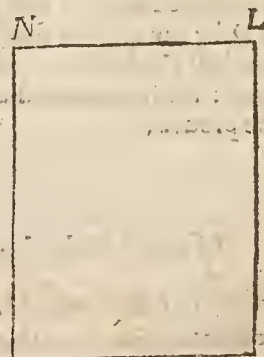
A Tetrahedron is to an Octohedron inscribed in one and the selfe same Sphere, in proportion, as the rectangle parallelograme contained vnder the line, which containeth in power 27. sixty fower partes of the side of the Tetrahedron, & vnder the line which is subsesquioctaua to the same side of the Tetrahedron, is to the square of the diameter of the sphere.



First part of the construction.



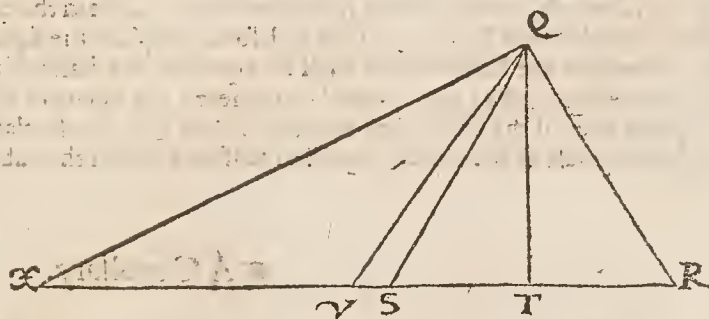
Et vs suppose a Sphere, whose diameter let be the line AB, and let the centre be the point H. And in it let there be inscribed a Tetrahedron ADC, and an Octohedron AEKBG. And let the line NL containe in power $\frac{27}{64}$ of AC the side of the Tetrahedron. And let the line ML be in length subsesquioctaua to the same side. The I say, that the Tetrahedron ACD is to the Octohedron AEB, as the rectangle parallelogramme contained vnder the lines NL and LM, is to the square of the line AB. Forasmuch as the line drawn from the angle A by the centre H perpendicularly vpon the base of the Tetrahedron, falleth vpon the center T of the circle which containeth that base, and maketh the right line HT the sixth part of the diameter AB (by the Corollary of the 13. of the thirteenth): therefore the line HA (which is drawn from the centre to the circumference) is triple to the line HT: and therefore the whole line AT is



First part of the Demonstration.

to the line A H, as 4. is to 3. Let the Tetrahedron A D C be cut by a plaine G H K passing by the centre H, and being parallel vnto the base D T C, by the Corollary of the 15. of the eleuenth. Now then the triangle A D C of the Tetrahedron, shall be cut by the right line K G, which is parallel to the line D C, by the 16. of the eleuenth. Wherefore as the line A T is to the line A H, so is the line A C to the line A G (by the 2. of the sixth). Wherefore the line A C is to the line A G sesquitercia, that is, as 4. to 3. And forasmuch as the triangles A D C, A K G, and the rest which are cut by the plaine K H G, are like the one to the other, by the 5. of the sixth: the pyramids A D C and A K G, shall be like the one to the other, by the 7. definition of the eleuenth. Wherefore they are in triple proportion of that in which the sides A C and A G are, by the 8. of the twelfth. But the proportion of the sides A C to A G is, as the proportion of 4. to 3. Now then, if, by the 2. of the eighth, ye finde out 4. of the left numbers in continuall proportion, and in that proportion that 4. is to 3: which shall be 64. 48. 36. and 27: it is manifest, by the 15. definition of the fifth, that the extremes 64. to 27. are in triple proportion of that in which the proportion geuen 4. to 3. is: Or the quantitie of the proportion of 4. to 3. (which is 1. and $\frac{1}{3}$) being twise multiplied into it selfe, there shall be produced the proportion of 64. to 27. Wherefore the Pyramid or Tetrahedron A D C is to the pyramid A K G, as 64. is to 27: which is triple to the proportion of 4. to 3. And forasmuch as the line A C is vnto the line A G in length sesquitercia: of what partes the line A C containeth in power 64: of the same partes doth the line A G containe in power 36. For (by the 2. of the sixth) the proportion of the powers or squares, is duple to the proportion of the sides which are as 64. is to 48.

Now then vpon the line R S which let be equall to the line A G, let there be an equilater triangle Q R S described (by the first of the first). And from the angle Q, draw to the base R S a perpendicular line Q T. And extend the line R S to the poynt X. And as 27. is to 64. (so by the corollary of the 6. of the tenth) let the line R S be to the line R X. And diuide the line R X into two equall partes in the poynt V, and draw the line Q V. And forasmuch as the line R S is equall to the line A G, of what partes the line A C containeth in power 64: of the same part the line R S containeth in power 36. for it is proued that the line A G containeth in power 36. of those partes: And of what partes the line R S containeth in power 36 of the same partes the line Q T containeth in power 27. by the corollary of the 12. of the thirteenth. Wherefore of what partes the line A C containeth in power 64. of the same partes the line Q T containeth in power 27. Wherefore the right line Q T shall be equall to the right line L N by supposition. Agayne forasmuch as the line R S is put equall to the line A G: and of what partes the line R S containeth in length 27. of the same partes is the line R X put to containe in length 64. and of what partes the line R X containeth in length 64. of the same the line A C (which is in length sesquitercia to the line A G or R S) containeth 36. Wherefore the line R V (which is the halfe of the line R X) containeth in length of the same partes 32. of which the line A C containe in length 36. Wherefore the line R V is to the line A C sublesquioctaua: and therefore the line R V is equall to the line L M which is also sublesquioctaua to the same line A C. And forasmuch as the line N L is equall to the line Q T, and the line L M to the line R V (as before hath bene proued) the rectangle parallelogramme containe vnder the lines Q T and R V, shall be equall to the rectangle parallelogramme, containe vnder the line N L which is in power $\frac{27}{64}$ to the side A C, and vnder the line L M, which is in length sublesquioctaua to the same side A C. But that which is containe vnder the lines Q T and R V is double to the triangle Q V R by the 41. of the first: and to the same triangle Q V R is the triangle Q X R duple by the first of the sixth. Wherefore the whole triangle Q X R is equall to that which is containe vnder the lines Q T and R V, and therefore is equall to the parallelogramme M N. And forasmuch as the line R X by supposition containe in length 64. of those partes of which the line R S containeth 27: and the triangles Q R X, and Q R S are, by the first of the sixth, in the proportion of their bases; that is, as 64. is to 27: but as 64. is to 27. so is the pyramid or tetrahedron A D C to the pyramid A K G: wherefore as the parallelogramme N M or the triangle Q R X, is to the triangle Q R S, so is the pyramid A D C to the pyramid A K G. And forasmuch as the semidiameter A H is the altitude of the pyramid A K G, and also of the two equall and like pyramids of the octohedron which haue their common base in the square of the octohedron (by the corollary of the 14. of the thirteenth): therefore as the base of the pyramid A K G (which is the triangle Q R S) is to two squares of the octohedron, that is, to the square of the diameter A B, which is equall to those squares (by the 47. of the first), so is the pyramid A K G to the octohedron A E B, by the 6. of the twelfth. And forasmuch as the parallelogramme M N is to the base Q R S, as the pyramid A D C is to the pyramid A K G, and the base Q R S is to the square of the line B E, as the pyramid A K G is to the octohedron A E B: therefore by proportion of equality taking away the meanes (by the 22. of the fifth) as the parallelogramme N M is to the square of the line B E, so is the pyramid A D C to the octo-



Second part
of the con-
struction.

Second part
of the De-
monstration.

The fourteenth Booke

hedron A E B inscribed in one and the selfe same sphere. But the parallelogramme N M is containd vnder the line N L which by supposition is in power $\frac{27}{64}$ to A C the side of the tetrahedron A D C, and vnder the line L M which is also by supposition in length subsefquioctaua to the same line A C. Wherefore a tetrahedron & an octohedron inscribed in one and the selfe same sphere, are in proportiō, as the rectāgle parallelogramme containd vnder the line, which containeth in power 27. sixty fower parts of the side of the Tetrahedron, and vnder the line which is subsefquioctaua to the same side of the Tetrahedron, is to the square of the diameter of the sphere: which was required to be proued.

¶ The 12. Proposition.

The 18. proposition after Campane.

If a cube be containd in a sphere: the square of the diameter doubled, is equall to all the superficieces of the cube taken together. And a perpendicular line drawne from the centre of the sphere to any base of the cube, is equall to halfe the side of the cube.

Demonstration of the first part.



Demonstration of the second part.

Or forasmuch as (by the 15. of the thirteenth) the diameter of the sphere is in power triple to the side of the cube: therefore the square of the diameter doubled is sextuple to the base of the same cube. But the sextuple of the power of one of the sides containeth the whole superficies of the cube. For the cube is composed of six square superficieses (by the 21. diffinition of the eleuenth) whose sides therefore are equall: wherefore the square of the diameter doubled is equall to the whole superficieses of the cube. And forasmuch as the diameter of the cube, and the line which falleth perpendicularly vpon the opposite bases of the cube, do cut the one the other into two equall partes in the centre of the sphere which containeth the cube (by the 2. corollary of the 15. of the thirteenth) and the whole right line which completh the centres of the opposite bases, is equall to the side of the cube by the 33. of the first, for it completh the equall and parallel semidiameters of the bases: therefore the halfe thereof shall be equall to the halfe of the side of the cube by the 15. of the fifth. If therefore a cube be containd in a sphere: the square of the diameter doubled is equall to all the superficieces of the cube taken together. And a perpendicular line drawne from the centre of the sphere to any base of the cube, is equall to halfe the side of the cube: which was required to be proued.

¶ A Corollary.

The Corollary of the 8. proposition after Campane.

If two thirds of the power of the diameter of the sphere be multiplyed into the perpendicular line equall to halfe the side of the cube, there shall be produced a solide equall to the solide of the cube. For it is before manifest that two third partes of the power of the diameter of the sphere are equall to two bases of the cube. If therefore vnto eche of those two thirds be applyed halfe the altitude of the cube, they shall make eche of those solides equall to halfe of the cube, by the 31. of the eleuenth: for they haue equall bases. Wherefore two of those solides are equall to the whole cube.

You shall vnderstand (gentle reader) that *Campane* in his 14. booke of *Euclides Elementes* hath 18. propositions with diuers corollaries following of them. Some of which propositions and corollaries I haue before in the twelfth and thirteenth bookes added out of *Flussas* as corollaries (which thing also I haue noted on the side of those corollaries, namely, with what proposition or corollary of *Campanes* 14. booke they doo agree). The rest of his 18. propositions and corollaries are contained in the twelue former propositions and corollaries of this 14. booke after *Flussas*: where ye may see on the side of eche proposition and corollary with what proposition and corollary of *Campanes* they agree. But the eight propositions following together with their corollaries, *Flussas* hath added of him selfe, as he him selfe affirmeth.

The

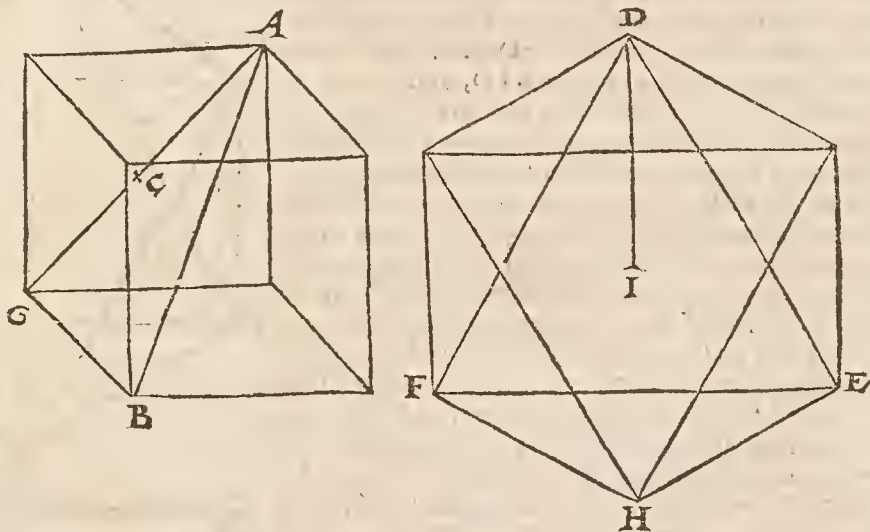
The 13. Proposition.

One and the self same circle containeth both the square of a cube, and the triangle of an Octohedron described in one and the selfe same sphere.



Vppose that there be a cube A B G, and an Octohedron D E F described in one and the selfe same sphere, whose diameter let be A B, or D H: And let the lines drawne from the centres (that is the semidiameters of the circles which containe the bases of those solides) be C A and I D. Then I say that the lines C A and I D are equal. Forasmuche as A B the diameter of the sphere which containeth the cube, is in power triple to B G the side of the cube (by the 15. of the thirteenth) vnto which side, A G the diameter of the base of the cube, is in power double (by the 47. of the first): which line A G is also the diameter of the circle, which containeth the base (by the 9. of the fourth): therefore A B the diameter of the sphere is in power sesquialter to the line A G: namely, of what partes the line A B, containeth in power 12. of the same the line A G,

Demonstration.



shall containe in power 8. And therefore the right line A C which is drawn from the centre of the circle to the circumference, containeth in power of the same partes 2. Wherefore the diameter of the sphere is in power sextuple to the lyne which is drawne from the centre to the circumference of the circle which containeth the square of the cube. But the Diameter of the selfe same Sphere which containeth the Octohedron, is one

and the selfe same with the diameter of the cube, namely, D H, is equall to A B: and the same diameter is also the diameter of the square which is made of the sides of the Octohedron: wherefore the saide diameter is in power double to the side of the same Octohedron, by the 14. of the thirteenth. But the side D F is in power triple to the line drawne from the centre to the circumference of the circle which containeth the triangle of the octohedron (namely to the line I D) by the 12. of the thirteenth. Wherefore the selfe same diameter A B or D H, which was in power sextuple to the line drawne from the centre to the circumference of the circle which containeth the square of the cube, is also sextuple to the line I D drawne from the centre to the circumference of the circle, which containeth the triangle of the Octohedron: Wherefore the lines drawne from the centres of the circles to the circumferences which containe the bases of the cube and of the octohedron are equal. And therefore the circles are equal, by the first diffinition of the third. Wherefore one and the selfe same circle containeth &c. as in the proposition: which was required to be proued.

A Corollary.

Hereby it is manifest, that perpendiculars coupling together in a sphere, the centres of the circles which containe the opposite bases of the cube and of the Octohedron, are equal. For the circles are equal, by the second corollary of the assumpt of the 16. of the twelfth: and the lines which passing by the centre of the sphere, couple together the centres of the bases, are also equal, by the first corollary of the same. Wherefore the perpendicular which coupleth together the opposite bases of the Octohedron, is equal to the side of the cube. For either of them is the altitude erected.

The 14. Proposition.

An Octohedron is to the triple of a Tetrahedron contained in one and the selfe

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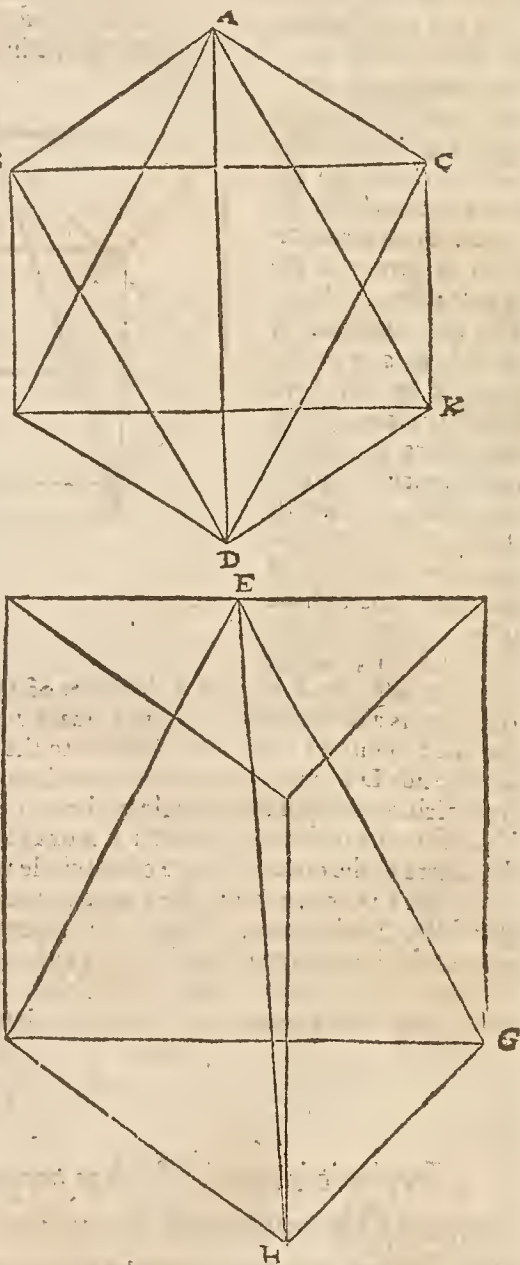
The fourteenth Booke

selfe same Sphere, in that proportion that their sides are.

Construction.

Demonstration.

Suppose that there be an octohedron $ABCD$, and a Tetrahedron $EFGH$: vpon whose base $EFGH$ erect a Prisme, which is done by erecting from the angles of the base perpendicular lines equal to the altitude of the Tetrahedron: which prisme shalbe triple to the Tetrahedron $EFGH$, by the first corollary of the 7. of the twelfth. Then I say that the octohedron $ABCD$ is to the prisme which is triple to the Tetrahedron, $EFGH$, as the side BC is to the side FG . For forasmuch as the sides of the opposite bases of the octohedron, are right lines touching the one the other, and are parallels to other right lines touching the one the other, for the sides of the squares which are composed of the sides of the octohedron, are opposite: Wherefore the opposite plaine triangles, namely, ABC & KID , shalbe parallels, and so the rest by the 15. of the eleuenth. Let the diameter of the Octohedron, be the line AD . Now then the whole Octohedron is cut into foure equal and like pyramids set vpon the bases of the octohedron, and hauing the same altitude with it, & being about the Diameter AD : namely the pyramis set vpon the base BID , and hauing his toppe the poynt A , and also the pyramis set vpon the base BCD , hauing his top the same poynt A . Likewise the pyramis set vpon the base IKD , & hauing his toppe the same poynt A : and moreouer the pyramis set vpon the base CKD , and hauing his toppe the former poynt A : which pyramids shalbe equal by the 8. definition of the eleuenth (for they eche consist of two bases of the octohedron, and of two triangles contained vnder the diameter AD and two sides of the octohedron). Wherefore the prisme which is set vpon the base of the Octohedron, and hauing the same altitude with it, namelye, the altitude of the parallel bases, as it is manifest by the former, is equal to thre of those pyramids of the Octohedron, by the first corollary of the seuenth of the twelfth. Wherefore that prisme shall haue to the other prisme vnder the same altitude, composed of the 4 pyramids of the whole octohedron, the proportion of the triangular bases, by the 3. corollary of the same. And forasmuch as 4. pyramids are vnto 3. pyramids in sesquitercia proportion, therefore the triangular base of the prisme which containeth 4. pyramids, is in sesquitercia proportion to the base of the prisme which containeth thre pyramids of the same octohedron, and are set vpon the base of the Octohedron and vnder the altitude thereof: that is, in sesquitercia proportion to the base of the Octohedron. But the base of the same octohedron is in sesquitercia proportion to the base of the pyramis, by the tenth of this booke: Wherefore the triangular bases, namely, of the prisme which containeth four pyramids of the octohedron, and is vnder the altitude thereof, are equal to the triangular bases of the prisme, which containeth thre pyramids vnder the altitude of the pyramis $EFGH$. But the prisme of the octohedron is equal to the octohedron: and the prisme of the pyramis $EFGH$ is proued triple to the same pyramis $EFGH$. Now then the prismes set vpon equal bases, are the one to the other as their altitudes are (by the corollary of the 25. of the eleuenth) namely, as are the parallelipedons their doubles, by the corollary of the 31. of the eleuenth. But the altitude of the Octohedron is equal to the side of the cube contained in the same sphere, by the corollary of the 13. of this booke. And the side of the cube is in power to the altitude of the Tetrahedon in that proportion that 12. is to 16, by the 18. of the thirteenth: And the side of the octohedron is to the side of the pyramis in that proportion that 18. is to 24. (by the same 18. of the thirteenth) which proportion is one & the selfe same with the proportiō of 12. to 16. Wherefore that prisme which is equal to the Octohedron, is to the prisme which is triple to the Tetrahedon, in that proportiō that the altitudes, or that the sides are. Wherefore an octohedron is to the triple of a Tetrahedon contained in one and the selfe same Sphere, in that proportion that their sides are: which was required to be demonstrated.



A Corollary.

The ſides of a Tetrahedron & of an Octohedrō are proportionall with their altitudes. For the ſides & altitudes were in power ſeſquitercia. Moreouer the diameter of the ſphere is to the ſide of the Tetrahedron, as the ſide of the Octohedron is to the ſide of the cube, namely, the powers of eche is in ſeſquialter proportion, by the 18. of the thirteenth.

The 15. Proposition.

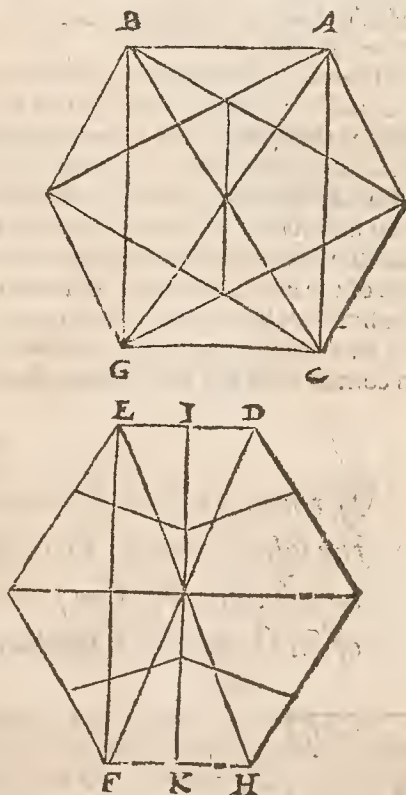
If a rational line containing in power two lines, make the whole and the greater ſegment; and again containing in power two lines, make the whole and the leſſe ſegment: the greater ſegment ſhalbe the ſide of the Icoſahedron, and the leſſe ſegment ſhalbe the ſide of the Dodecahedron contained in one and the ſelfe ſame ſphere.



Vppoſe that A G be the diameter of the ſphere which containeth the Icoſahedron A B G C. And let B G ſubtend the ſides of the pentagon deſcribed of the ſides of the Icoſahedron (by the 16. of the thirtēth.) Moreouer vpon the ſame diameter A G, or D F equal vnto it, let ther be deſcribed a dodecahedron D E F H, by the 17. of the thirteenth, whoſe oppoſites ſides E D and F H let be cut into two equal partes in the poynts I and K, and draw a line from I to K. And let the line E F couple two of the oppoſite angles of the baſes which are ioyned together. Thē I ſay that A B the ſide of the Icoſahedron is the greater ſegment which the diameter A G containeth in power together with the whole line, and line E D is the leſſe ſegment, which the ſame diameter A G or D F containeth in power together with the whole. For forasmuche as the oppoſite ſides A B and G C of the Icoſahedron being coupled by the diameters A G and B C, are equal & parallels, by the 2. corollary of the 16. of the thirtēth: the right lines B G & A C which couple thē together are equal & parallels by the 33. of the firſt. Moreouer the angles B A C & A B G being ſubtended of equal diameters, ſhall by the 8. of the firſt be equal, & by the 29. of the firſt, they ſhal be right angles. Wherefore, the right line A G cōtaineth in power the two lines A B and B G, by the 47. of the firſt. And forasmuch as the line B G ſubtendeth the angle of the pentagon compoſed of the ſides of the Icoſahedron, the greater ſegment of the right line B G, ſhalbe the right line A B, by the 8. of the thirteenth: which line A B, together with the whole line B G, the line A G containeth in power. And forasmuch as the line I K coupling the oppoſite and parallel ſides E D and F H of the Dodecahedron, maketh at thoſe poyntes right angles, by the 3. corollary of the 17. of the thirteenth: the right line E F which coupleth together equal and parallel lines E I & F K, ſhalbe equal to the ſame line I K, by the 33. of the firſt. Wherefore the angle D E F ſhalbe a right angle by the 29. of the firſt. Wherefore the diameter D F cōtaineth in power the two lines E D and E F. But the leſſe ſegment of the line I K is E D the ſide of the Dodecahedron, by the 4. corollary of the 17. of the thirteenth. Wherefore the ſame line E D is alſo the leſſe ſegment of the line E F (which is equal vnto the line I K): wherefore the diameter D F containing in power the two lines E D and E F (by the 47. of the firſt) containeth in power E D the ſide of the dodecahedron, the leſſe ſegment, together with the whole. If therefore a rational line A G or D F containing in power two lines A B and B G, doo make the whole line and the greater ſegment, and againe containing in power two lines E F and E D, do make the whole line and the leſſe ſegment: the greater ſegment A B, ſhall be the ſide of the Icoſahedron, and the leſſe ſegment E D ſhall be the ſide of the Dodecahedron contained in one and the ſelfe ſame ſphere.

Construction.

Demonstration.



The 16. Proposition.

If the power of the ſide of an Octohedron be expreſſed by two right lines ioyned

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*ioyned together by an extreme and meane proportion: the side of the Ico-
sahedron contained in the same sphere, shalbe duple to the lesse segment.*

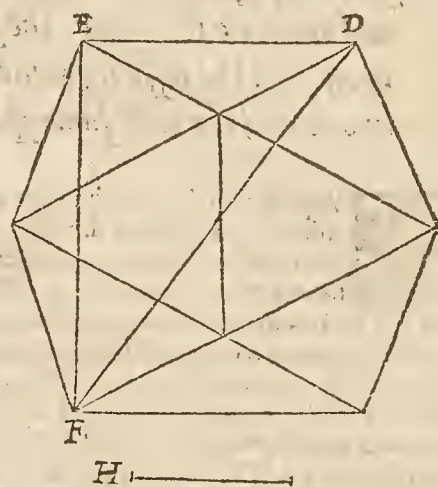
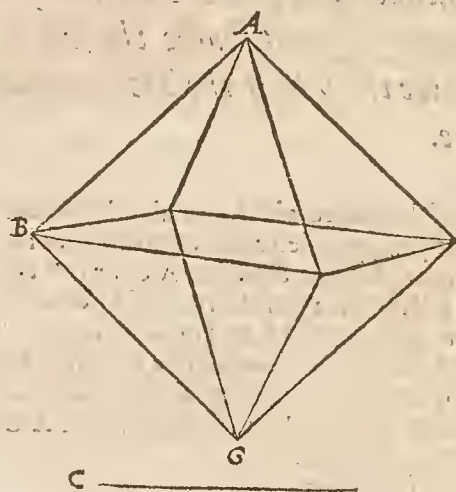
Construction.



Demonstration.

Et A B the side of the Octohedron A B G containe in power the two lines C and H, which let haue that proportion that the whole hath to the greater segment (by the corollarye of the first proposition added by *Flussas* after the last proposition of the sixth booke). And let the Icosahedron contained in the same sphere be D E F, whose side let be D E, and let the right line subtending the angle of the pentagon made of the sides of the Icosahedron be the line E F. Then I say that the side E D is in power double to the line H the lesse of those segmentes. Forasmuch as by that which was demonstrated in the 15. of this booke, it was manifest that E D the side of the Icosahedron is the greater segment of the line E F, and that the diameter D F containeth in power the two lines E D and E F, namely, the whole and the greater segment: but by supposition the side A B

containeth in power the two lines C & H ioyned together in the self same proportio. Wherefore the line E F is to the line E D, as the line C, is to the line H, by the 2. of this booke. And alternaty by the 16. of the fifth, the line E F is to the line C, as the line E D, is to the line H. And forasmuche as the



line D F containeth in power the two lines E D and E F, and the line A B containeth in power the two lines C and H: therefore the squares of the lines E F and E D are to the square of the line D F, as the squares of the lines C and H to the square of the line A B. And alternately, the squares of the lines E F and E D, are to the squares of the lines C and H, as the square of the line D F is to the square of the line A B. But D F the diameter is (by the 14. of the thirteenth) in power double to A B the side of the octohedron inscribed (by supposition) in the same sphere. Wherefore the squares of the lines E F and E D, are double to the squares of the lines C and H. And therefore one square of the line E D is double to one square of the line H by the 12. of the fifth. Wherefore E D the side of the Icosahedron is in power duple to the line H, which is the lesse segment. If therefore the power of the side of an octohedron be expressed by two right lines ioyned together by an extreme and meane proportion: the side of the Icosahedron contained in the same sphere, shalbe duple to the lesse segment.

The 17. Proposition.

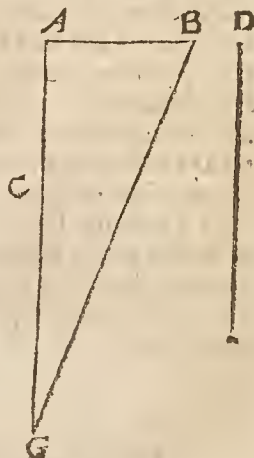
If the side of a dodecahedron, and the right line, of whome the said side is the lesse segment, be so set that they make a right angle: the right line which containeth in power halfe the line subtending the angle, is the side of an Octohedron contained in the selfe same sphere.

Construction.



Demonstration.

Vppose that A B be the side of a Dodecahedron, and let the right line of which that side is the lesse segment be A G, namely which coupleth the opposite sides of the Dodecahedron, by the 4. corollary of the 17. of the thirteenth: and let those lines be so set that they make a right angle at the point A. And draw the right line B G. And let the line D containe in power halfe the line B G (by the first proposition added by *Flussas* after the last of the sixth). Then I say that the line D is the side of an Octohedron containy in the same sphere. Forasmuche as the line A G maketh the greater segment G C the side of the cube contained in the same sphere (by the same 4. corollary of the 17. of the thirteenth): and the squares of the whole line A G. and of the lesse segment A B are triple to the square of the greater segment G C, by the 4. of the thirteenth: Moreouer the diameter of the sphere, is in power triple to the same line G C the side of the cube (by the 15. of the thirteenth):



Wherefore

Wherefore the line B G is equal to the diameter. For it containeth in power the two lines A B and A G (by the 47. of the first,) and therefore it containeth in power the triple of the line G C. But the side of the Octohedron contained in the same Sphere, is in power triple to halfe the diameter of the Sphere by the 14. of the thirteenth. And by supposition the line D containeth in power the halfe of the line B G. Wherefore the line D (containing in power the halfe of the same diameter) is the side of an Octohedron. If therefore the side of a Dodecahedron and the right line of whome the said side is the lesse segment, be so set that they make a right angle: the right line which containeth in power halfe the line subtending the angle, is the side of an Octohedron contained in the selfe same Sphere: Which was required to be proued.

✱ A Corollary.

Vnto what right line the side of the Octohedron is in power sesquialter: vnto the same line the side of the Dodecahedron inscribed in the same Sphere, is the greater segment. For the side of the Dodecahedron is the greater segment of the segment C G, vnto which D the side of the Octohedron is in power sesquialter, that is, is halfe of the power of the line B G, which was triple vnto the line C G.

¶ The 18. Proposition.

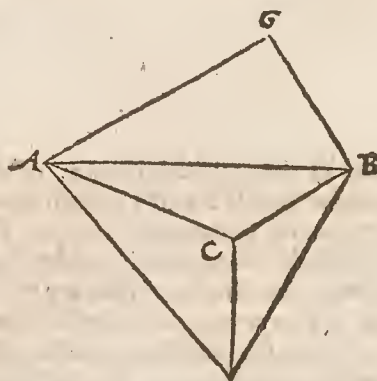
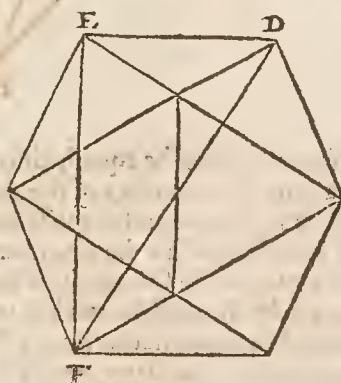
If the side of a Tetrahedron containe in power two right lines ioyned together by an extreme and meane proportion: the side of an Icosahedron described in the selfe same Sphere, is in power sesquialter to the lesse right line.



Suppose that A B C be a Tetrahedron, and let his side be A B, whose power let be diuided into the lines A G and G B, ioyned together by an extreme and meane proportion: namely, let it be diuided into A G the whole line, and G B the greater segment (by the Corollary of the first Proposition added by Flußas after the last of the sixth). And let E D be the side of the Icosahedron E D F contained in the selfe same Sphere. And let the line which subtendeth the angle of the Pentagon described of the sides of the Icosahedron be E F.

Construction.

Then I say, that E D the side of the Icosahedron is in power sesquialter to the lesse line G B. Forasmuch as (by that which was demonstrated in the 15. of this booke) the side E D is the greater segment of the line E F which subtendeth the angle of the Pentagon. But as the whole line E F is to the greater segment E D, so is the same greater segment to the lesse (by the 30. of the sixth): and by supposition, A G, was the whole line, and G B the greater segment:



Demonstration.

Wherefore as E F is to E D, so is A G to G B, by the second of the fourth. And alternately, the line E F is to the line A G, as the line E D is to the line G B. And forasmuch as (by supposition) the line A B containeth in power the two lines A G and G B: therefore (by the 48. of the first) the angle A G B is a right angle. But the angle D E F is a right angle, by that which was demonstrated in the 15. of this booke. Wherefore the triangles A G B and F E D, are equiangle, by the 5. of the sixth. Wherefore their sides are proportionall: namely, as the line E D is to the line G B, so is the line F D to the line A B, by the 4. of the sixth. But by that which hath before bene demonstrated, F D is the diameter of the Sphere which containeth the Icosahedron: which diameter is in power sesquialter to A B the side of the Tetrahedron inscribed in the same Sphere, by the 13. of the thirteenth. Wherefore the line E D the side of the Icosahedron, is in power sesquialter to G B the greater segment or lesse line. If therefore the side of a Tetrahedron containe in power two right lines ioyned together by an extreme and meane proportion: the side of an Icosahedron described in the selfe same Sphere, is in power sesquialter to the lesse right line.

¶ The

The fourteenth Booke

¶ The 19. Proposition.

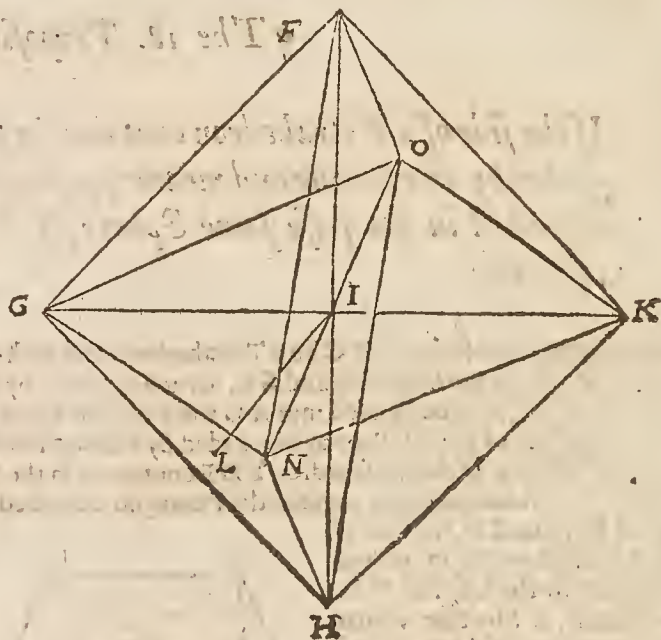
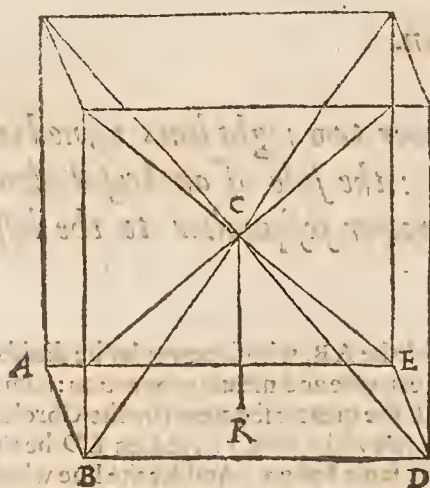
The superficies of a Cube is to the superficies of an Octohedron inscribed in one and the selfe same Sphere, in that proportion that the solides are.

Construction.



Vppose that A B C D E be a Cube, whose fower diameters let be the lines A C, B C, D C, and E C produced on each side. Let also the Octohedron inscribed in the selfe same Sphere be F G H K: whose three diameters let be F H, G K, and O N. Then I say, that the cube A B D is to the Octohedron F G H, as the superficies of the cube is to the superficies of the Octohedron. Drawe from the centre of the cube to the base A B E D, a perpendicular line C R. And from the centre of the Octohedron draw to the base G N H, a perpendicular line I L. And forasmuch as, the three diameters of the cube do passe by the centre C, therefore, by the 2. Corollary of the 15. of the thirteenth, there shall be made of the cube sixe pyramids, as thys pyramis A B D E C, equall to the whole cube. For there are in the cube sixe bases, vpon which fall equall perpendiculars from the centre, by the Corollary of the Assumpt of the 16. of the twelfth, for the bases are contained in equall circles of the Sphere. But in the Octohedron the three diameters do make vpon the 8. bases, 8. pyramids, hauing their toppes in the centre, by the 3. Corollary of the 14. of the thirteenth. Now the bases of the cube and of the Octohedron are contained in equall circles of the

Demonstration.



Sphere, by the 13. of this booke. Wherefore they shall be equally distant from the centre, and the perpendicular lines C R and I L, shall be equall, by the Corollary of the Assumpt of the 16. of the twelfth. Wherefore the pyramids of the cube shall be vnder one and the selfe same altitude with the pyramids of the Octohedron, namely, vnder the perpendicular line drawn from the centre to the bases. Wherefore sixe pyramids of the cube, are to 8. pyramids of the Octohedron being vnder one and the same altitude, in that proportion that their bases are, by the 6. of the twelfth: that is, one pyramis set vpon sixe bases of the cube, and hauing to his altitude the perpendicular line, which pyramis is equall to the sixe pyramids, by the same 6. of the twelfth, is to one pyramis set vpon the 8. bases of the Octohedron, being equall to the Octohedron, and also vnder one and the selfe same altitude, in that proportion that sixe bases of the cube, which containe the whole superficies of the cube, are to 8. bases of the Octohedron, which containe the whole superficies of the Octohedron. For the solides of those pyramids are in proportion the one to the other, as their bases are, by the selfe same 6. of the twelfth. Wherefore the superficies of the cube is to the superficies of the Octohedron inscribed in one and the selfe same Sphere, in that proportion, that the solides are: which was required to be proued.

¶ The 20. Proposition.

If a Cube and an Octohedron be contained in one & the selfe same Sphere: they shall be in proportion the one to the other, as the side of the Cube is to the

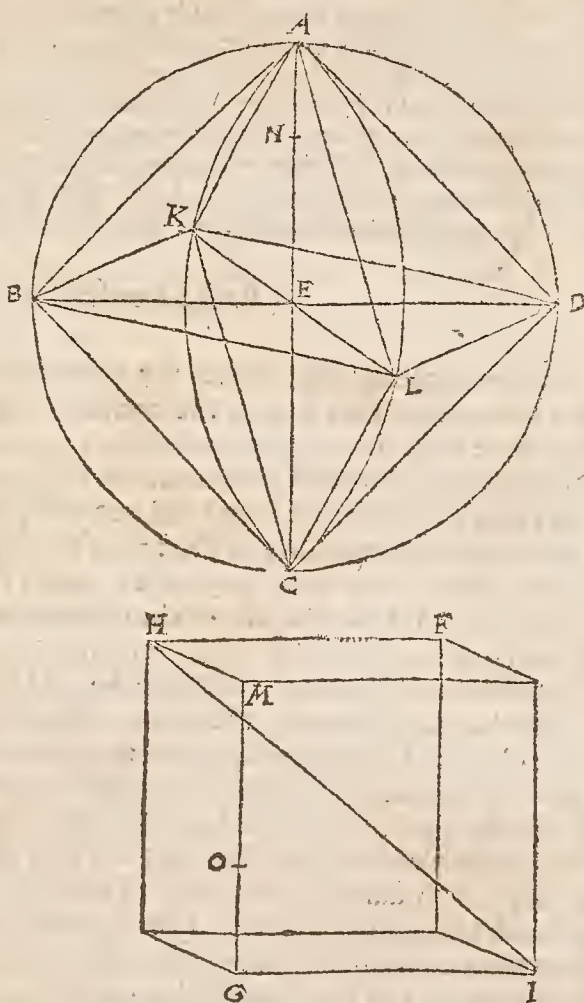
the semidiameter of the Sphere.



Vppose that the Octohedron $AECDB$ be inscribed in the Sphere $ABCD$: and let the cube inscribed in the same Sphere be $FGHIM$: whose diameter let be HI , which is equall to the diameter AC , by the 15. of the thirteenth: let the halfe of the diameter be AE . Then I say, that the cube $FGHIM$ is to the Octohedron $AECDB$, as the side MG is to the semidiameter AE . Forasmuch as the diameter AC is in power double to BK the side of the Octohedron (by the 14. of the thirteenth) and is in power triple to MG the side of the cube (by the 15. of the same): therefore the square $BKDL$ shall be sesquialter to FM the square of the cube. From the line AE cut of a third part AN , and frō the line MG cut of likewise a third part GO , by the 9. of the sixth. Now then the line EN shall be two third partes of the line AE , and so also shall the line MO be of the line MG . Wherefore the parallelipedon set vpon the base $BKDL$, and hauing his altitude the line EA , is triple to the parallelipedon set vpon the same base, and hauing his altitude the line AN , by the Corollary of the 31. of the eleuenth: but it is also triple to the pyramis $ABKDL$ which is set vpon the same base, and is vnder the same altitude (by the second Corollary of the 7. of the twelfth). Wherefore the pyramis $ABKDL$ is equall to the parallelipedon, which is set vpon the base $BKDL$, and hath to his altitude the line AN . But vnto that parallelipedō, is double the parallelipedon which is set vpon the same base $BKDL$, and hath to his altitude a line double to the line EN , by the Corollary of the 31. of the first: and vnto the pyramis is double the Octohedron $ABKDL C$, by the 2. Corollary of the 14. of the thirteenth. Wherefore the Octohedron $ABKDL C$ is equall to the parallelipedon set vpon the base $BKDL$, & hauing his altitude the line EN (by the 15. of the fifth). But the parallelipedon set vpon the base $BKDL$, which is sesquialter to the base FM , and hauing to his altitude the line MO , which is two third partes of the side of the cube MG , is equall to the cube FG : by the 2. part of the 34. of the eleuenth. (For it was before proued that the base $BKDL$ is sesquialter to the base FM). Now then these two parallelipedons, namely, the parallelipedon which is set vpon the base $BKDL$ (which is sesquialter to the base of the cube) and hath to his altitude the line MO (which is two third partes of MG the side of the cube) which parallelipedon is proued equall to the cube, and the parallelipedon set vpon the same base $BKDL$, and hauing his altitude the line EN (which parallelipedon is proued equall to the Octohedron): these two parallelipedons (I say) are the one to the other, as the altitude MO , is to the altitude EN , so is the cube $FGHIM$, to the Octohedron $ABKDL C$, by the 7. of the fifth. But as the line MO is to the line EN , so is the whole line MG to the whole line EA , by the 18. of the fifth. Wherefore as MG the side of the cube, is to EA the semidiameter, so is the line $FGHIM$ to the Octohedron $ABKDL C$ inscribed in one & the selfe same Sphere. If therefore a cube and an Octohedron be contained in one and the selfe same Sphere: they shall be in proportion the one to the other, as the side of the cube is to the semidiameter of the Sphere: which was required to be demonstrated.

Construction.

Demonstration.



A Corollary.

Distinctly to notefie the powers of the sides of the five solides by the power of the diameter of the sphere.

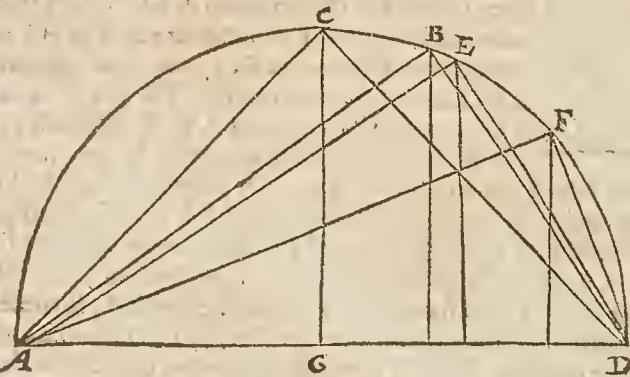
The sides of the tetrahedron and of the cube doo cut the power of the diameter of the sphere into two squares which are in proportion double the one to the other. The octohedron cutteth the

Tt.j.

power

The fourteenth Booke

power of the diameter into two equall squares. The Icosahedron into two squares, whose proportion is duple to the proportion of a line diuided by an extreame and meane proportion, whose lesse segmēt is the side of the Icosahedron. And the dodecahedron into two squares, whose proportion is quadruple to the proportion of a line diuided by an extreame and meane proportion; whose lesse segment is the side of the dodecahedron. For A D the diameter of the sphere, containeth in power A B the side of the tetrahedron, and B D the side of the cube, which B D is in power halfe of the side A B. The diameter also of the sphere containeth in power A C and C D two equall sides of the octohedron. But the diameter containeth in power the whole line A E and the greater segment thereof E D, which is the side of the Icosahedron, by the 15. of this booke.



Wherefore their powers being in duple proportiō of that in which the sides are, by the first corollary of the 20. of the sixth, haue their proportion duple to the proportion of an extreame & meane proportiō. Farther the diameter cōtaineth in power the whole line A F, and his lesse segment F D, which is the side of the dodecahedron, by the same 15. of this booke. Wherefore the whole hauing to the lesse, a double proportion of that which the extreame hath to the meane, namely, of the whole to the greater segment, by the 10. diffinition of the fifth, it followeth that the proportion of the power is double to the doubled proportion of the sides, by the same first corollary of the 20. of the sixth: that is, is quadruple to the proportion of the extreame and of the meane, by the diffinition of the sixth.

An aduertisment added by *Flussas*.

By this meanes therefore, the diameter of a sphere being geuen, there shall be geuen the side of euery one of the bodies inscribed. And forasmuch as three of those bodies haue their sides commensurable in power onely, and not in length, vnto the diameter geuen (for their powers are in the proportion of a square number to a number not square: wherefore they haue not the proportion of a square number to a square number, by the corollary of the 25. of the eighth: wherefore also their sides are incommensurable in length by the 9. of the tenth): therefore it is sufficient to compare the powers and not the lengths of those sides the one to the other: which powers are contained in the power of the diameter: namely, from the power of the diameter, let there be taken away the power of the cube, and there shall remayne the power of the Tetrahedron: and taking away the power of the Tetrahedron, there remaineth the power of the cube: and taking away from the power of the diameter halfe the power thereof, there shall be left the power of the side of the octohedron. But forasmuch as the sides of the dodecahedron and of the Icosahedron are proued to be irracionall (for the side of the Icosahedron is a lesse line, by the 16. of the thirteenth: and the side of the dodecahedron is a residuall line, by the 17. of the same): therefore those sides are vnto the diameter which is a racionall line set, incommensurable both in length and in power. Wherefore their comparison can not be diffined or described by any proportion expressed by numbers, by the 8. of the tenth: neither can they be compared the one to the other: for irracionall lines of diuers kindes are incōmēsurable the one to the other: for if they should be commensurable, they should be of one and the selfe same kinde, by the 103. and 105. of the tenth, which is impossible. Wherefore we seeking to compare them to the power of the diameter, thought they could not be more aptly expressed, then by such proportions, which cutte that racionall power of the diameter according to their sides: namely, diuiding the power of the diameter by lines which haue that proportiō, that the greater segment hath to the lesse, to put the lesse segment to be the side of the Icosahedron: & deuiding the sayd power of the diameter by lines hauing the proportion of the whole to the lesse segment, to expresse the side of the dodecahedron by the lesse segment: which thing may well be done betwene magnitudes incommensurable.

The ende of the fourteenth Booke
of Euclides Elementes
after *Flussas*.

The fifteenth booke of Euclides Elementes.



His fiftenth and last booke of Euclide, or rather the second booke of *Appollonius* or *Hypsicles*, teacheth the inscription and circumscription of the five regular bodies, one within and about an other: a thing vndoubtedly pleasant and delectable in minde to contemplate, and also profitable and necessary in act to practise. For without practise in act, it is very hard to se and conceiue the constructions and demonstrations of the propositions of this booke, vnles a man haue a very depe, sharpe, & fine imagination. Wherefore I would wish the diligent student in this booke, (to make the study thereof more pleasant vnto him) to haue presently before his eyes, the bodies formed & framed of pasted paper (as I taught after the diffinitions of the eleuenth booke.) And then to drawe

*The argument of the
15. booke.*

and describe the lines and diuisions, and superficieses, according to the constructions of the propositions. In which descriptions if he be wary and diligent, he shall finde all things in these solide matters, as clere and as manifest vnto the eye, as were things before taught only in plaine or superficial figures. And although I haue before in the twelfth booke admonished the reader hereof, yet bicause in this booke chiefly that thing is required, I thought it should not be irkesome vnto him, againe to be put in minde thereof.

Farther this is to be noted, that in the Greke exemplars are found in this 15. booke only 5. propositions, which 5. are also only touched and set forth by *Hypsicles*: vnto which *Campane* addeth 8. and so maketh vp the number of 13. *Campane* vndoubtedly although he were very well lerned, and that generally in all kinds of learning, yet assuredly being brought vp in a time of rudenes, when all good letters were darkned, & barberousnes had ouerthrowne and ouerwhelmed the whole world, he was vtterly rude and ignorant in the Greke tongue, so that certenly he neuer redde *Euclide* in the Greke; nor (of like) translated out of the Greke: but had it translated out of the Arabike tonge. The Arabians were men of great study, and industry, and commonly great Philosophers, notable Phisitions, and in mathematicall Artes most expert, so that all kinds of good learning flourished and raigned amongst them in a manner only. These men turned whatsoeuer good author was in the Greke tonge (of what Art and knowledge soeuer it were) into the Arabike tonge. And fro thence were many of the turned into the Latine, and by that meanes many Greeke authors came to the handes of the Latines, and not from the first fountaine the Greke tonge, wherein they were first written. As appeareth by many words of the Arabike tonge yet remaining in such booke: as are *Zenith*, *nadir*, *helmuayn*, *helmuariphe*, and infinite such other. Which Arabians also in translating such Greke workes, were accustomed to adde, as they thought good, & for the fuller vnderstanding of the author, many things: as is to be sene in diuers authors, as, namely, in *Theodosius de Sphera*, where you see in the olde translation (which was vndoubtedly out of the Arabike) many propositions, almost euery third or fourth leafe. Some such cotype of *Euclide*, most likely, did *Campanus* follow, wherein he founde those propositions, which he hath more & aboue those which are found in the Greke set out by *Hypsicles*: and that not only in this 15. booke, but also in the 14. booke, wherein also ye finde many propositions more the are founde in the Greeke, set out also by *Hypsicles*. Likewise in the booke before, ye shall finde many propositions added, and manye inuerted, and set out of order farre otherwise, then they are placed in the Greeke exemplars. *Flussas* also a diligent restorer of *Euclide*, a man also which hath well deserued of the whole Art of Geometrie, hath added moreouer in this booke (as also in the former

The fiftenth Booke

14. boke he added 8. propositiōs) 9. propositiōs of his owne, touching the inscription, and circumscription of these bodies, very singular vndoubtedly, and wittye. All which, for that nothing should want to the desirous louer of knowledge, I haue faithfully with no small paines turned. And whereas *Flussas* in the beginning of the eleuenth booke, namely, in the end of the diffinitions there set, putteth two diffinitions, of the inscription and circumscription of solides or corporall figures, within or about the one the other, which certainly are not to be reiectēd: yet for that vntill this present 15. boke, there is no mention made of the inscription or circumscription of these bodies, I thought it not so conuenient there to place them, but to referre the to the beginning of this 15. booke: where they are in maner of necessitie required to the elucidation of the Propositions and demonstrations of the same. The diffinitions are these.

Diffinition. 1.

A solide figure, is then said to be inscribed in a solide figure, when the angles of the figure inscribed touche together at one time, either the angles of the figure circumscribed, or the superficieses, or the sides.

Diffinition. 2.

A solide figure is then said to be circumscribed about a solide figure: when together at one time either the angles, or the superficieses, or the sides of the figure circumscribed, touch the angles of the figure inscribed.

IN the fourth booke in the diffinitions of the inscription or circumscription of playne rectiline figures one with in or about an other, was required that all the angles of the figure inscribed, should at one time touch all the sides of the figure circumscribed: but in the five regular solides (to whome chesely these two diffinitions pertaine) for that the number of their angles, superficieses, & sides are not equal, one compared to an other: it is not of necessitie, that all the angles of the solide inscribed should together at one time touch either all the angles, or all the superficieses, or all the sides of the solide circumscribed: but it is sufficient, that those angles of the inscribed solide which touch, doe at one time together eche touch some one angle of the figure circumscribed, or some one base, or some one side: so that if the angles of the inscribed figure do at one time touche the angles of the figure circumscribed, none of them may at the same time touche either the bases or the sides of the same circumscribed figure: and so if they touch the bases, they may touche neither angles nor sides: and likewise if they touche the sides, they may touch neither angles nor bases. And although sometimes all the angles of the figure inscribed can not touch either the angles, or the bases, or the sides of the figure circumscribed, by reason the number of the angles, bases or sides of the said figure circumscribed, wanteth of the number of the angles of the figure inscribed yet shall those angles of the inscribed figure which touch, so touch, that the void places left betwene the inscribed and circumscribed figures shal on euery side be equal and like. As ye may afterwarde in this fiftenth booke most plainly perceiue.

¶ The 1. Proposition.

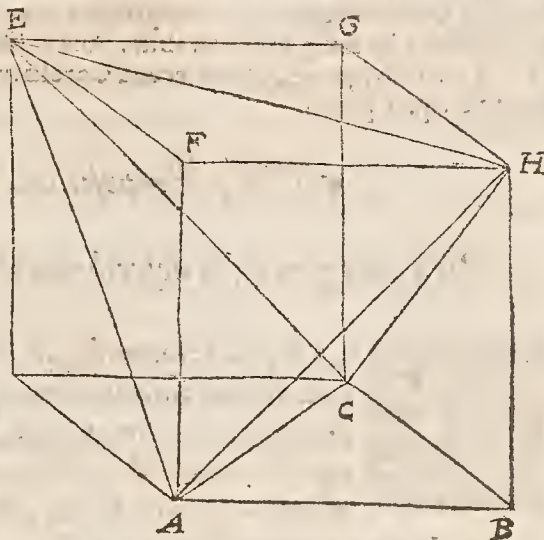
The 1. Probleme.

In a Cube geuen to describe + a trilater equilater Pyramis.



Vppose that the cube geuen be $AB-CDEFGH$. In the same cube it is required to in-scribe a Tetrahedron. Drawe these right lines AC , CE , AE , AH ,

EH , HC . Now it is manifest, that the triangles AEC , AHE , AHC , and CHE , are equilater, for their sides are the diameters of equall squares. Wherefore $AECH$ is a trilater equilater pyramis, or Tetrahedron, & it is in-scribed in the cube geue (by the first definition of this booke): which was required to be done.



† In this proposition as also in all the other following, by the name of a pyramis vnderstand a tetrahedron: as I haue before admonished.

Construction.

Demonstration.

¶ The 2. Proposition.

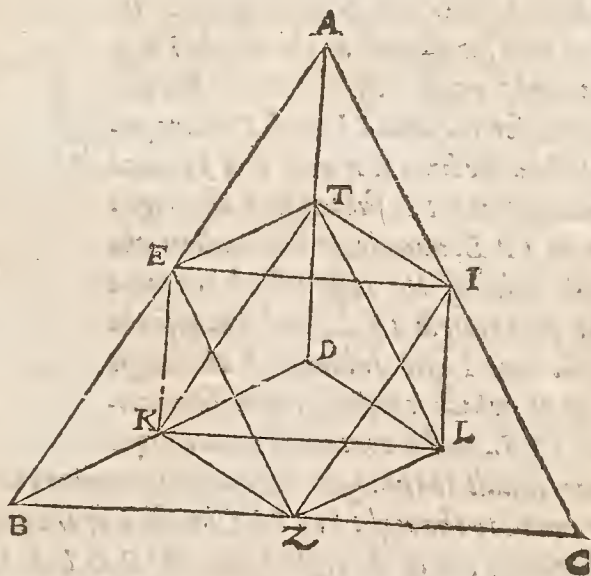
The 2. Probleme.

In a trilater equilater Pyramis geuen to describe an Octohedron.



Vppose that the trilater equilater pyramis geue be $ABCD$, whose sides let be diuided into two equall partes in the pointes E, Z, I, K, L, T . And drawe these 12. right lines $EZ, ZI, IE, KL, LT, TK, EK, KZ, ZL, LI, IT$, and TE . Which 12. right lines are, by the 4. of the first, equall. For they subtend equall plaine angles of the bases of the pyramis, and those equall angles are contained vnder equall sides, namely, vnder the halfes of the sides of the pyramis. Wherefore the triangles TKL , TLI , TIE , TEK , ZKL , ZLI , ZIE , ZEK , are equilater: and they limitate, and containe the solide $TKLEZI$.

Wherefore the solide $TKLEZI$ is an Octohedron: by the 23. definition of the eleuenth. And the angles of the same Octohedron do touch the sides of the pyramis $ABCD$ in the pointes E, Z, I, T, K, L . Wherefore the Octohedron is in-scribed in the pyramis (by the 1. definition of this booke). Wherefore in the trilater equilater pyramis geuen, is in-scribed an Octohedron: which was required to be done.



Construction.

Demonstration.

* A Corollary added by Flusas.

Hereby it is manifest, that a pyramis is cut into two equall partes, by euery

TTt.ij.

one

The fiftenth Booke

one of the three equall squares, which diuide $\S$ Octohedron into two equall partes and perpendicularly. For the three diameters of those squares do in the centre cut the one the other into two equall partes and perpendicularly, by the third Corollary of the 14. of the thirteenth, which squares, as for example, the square $E K L I$, do diuide in sunder the pyramids and the prismes, namely, the pyramis $K L T D$ and the prisme $K L T E I A$ from the pyramis $E K Z B$, and the prisme $E K Z I L G$, which pyramids are equall the one to the other, and so also are the prismes equall the one to the other: by the 3. of the twelfth. And in like sort do the rest of the squares, namely, $K Z I T$ and $Z L T E$: which squares, by the second Corollary of the 14. of the thirteenth, do diuide the Octohedron into two equall partes.

¶ The 3. Proposition.

The 3. Probleme.

In a cube geuen, to describe an Octohedron.

Construction.

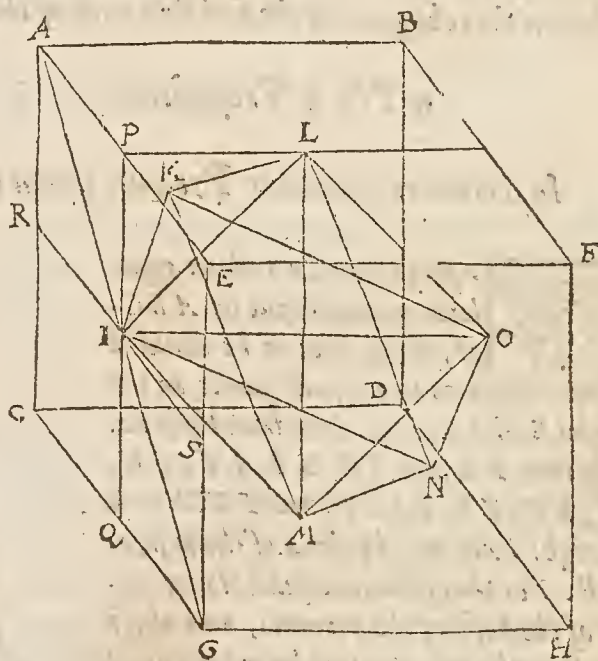


Take a Cube, namely, $A B C D E F G H$. And diuide euery one of the sides thereof into two equall partes. And drawe right lines coupling together the sections, as for example, these right lines, $P Q$ and $R S$, which shall be equall vnto the side of the cube (by the 33. of the first) and shall diuide the one the other into two equall parts in the middest of the diameter $A G$ in the

point I (by the Corollary of the 34. of the first). Wherefore the point I is the centre of the base of the cube. And by the same reason may be found out the centres of the rest of the bases, which let be the pointes K, L, O, N, M . And drawe these right lines $L I, I M, M O, O L, K I, K L, K M, K O, N I, N L, N M, \& N O$.

Demonstration.

And now forasmuch as the angle $I P L$ is a right angle (by the 10. of the eleuenth, for the lines $I P$ and $P L$ are parallels to the lines $R A$ and $A B$). And the right line $I L$ subtendeth the right angle $I P L$, namely, it subtendeth the halfe sides of the cube which containe the right angle $I P L$, and likewise the right line $I M$ subtendeth the angle $I Q M$ which is equall to the same angle $I P L$, and is contained vnder right lines equall to the right lines which containe the angle $I P L$. Wherefore the right line $I M$ is equall to the right line $I L$ (by the 4. of the first). And by the same reason may we proue, that euery one of the right lines $M O, O L, K I, K L, K M, K O, N I, N L, N M$, and $N O$, which subtend angles equall to the selfe same angle $I P L$, and are contained vnder sides equall to the sides which containe the angle $I P L$, are equall to the right line $I L$. Wherefore the triangles $K L I, K L O, K M I, K M O$, and $N L I, N L O, N M I, N M O$, are equilater and equall: and they containe the solide $I K L O N M$. Wherefore $I K L O N M$ is an Octohedron, by the 23. definition of the eleuenth. And forasmuch as the angles thereof do altogether in the pointes I, K, L, O, N, M , touch the bases of the cube which containeth it, it followeth that the Octohedron is inscribed in the cube (by the first definition of this booke). Wherefore in the cube geuen, is described an Octohedron: which was required to be done.



¶ A Corollary

¶ A Corollary added by Fluffas.

Hereby it is manifest, that right lines ioyning together the centres of the opposite bases of the cube, do cut the one the other into two equall parts, and perpendicularly, in the centre of the cube, or in the centre of the Sphere which containeth the cube.

For forasmuch as the right lines LM and IO which knit together the centres of the opposite bases of the cube, do also knit together the opposite angles of the Octohedron inscribed in the cube, it followeth (by the 3. Corollary of the 14. of the thirteenth) that those lines LM and IO , do cut the one the other into two equall partes in a point. But the diameters of the cube do also cut the one the other into two equall partes, by the 39. of the eleuenth. Wherefore that point shall be the centre of the sphere which containeth the cube. For making that point the centre, and the space some one of the semidiameters, describe a sphere, and it shall passe by the angles of the cube: and likewise making the same point the centre, and the space halfe of the line LM , describe a sphere, and it shall also passe by the angles of the Octohedron.

¶ The 4. Proposition.

The 4. Probleme.

In an Octohedron geuen, to describe a Cube.

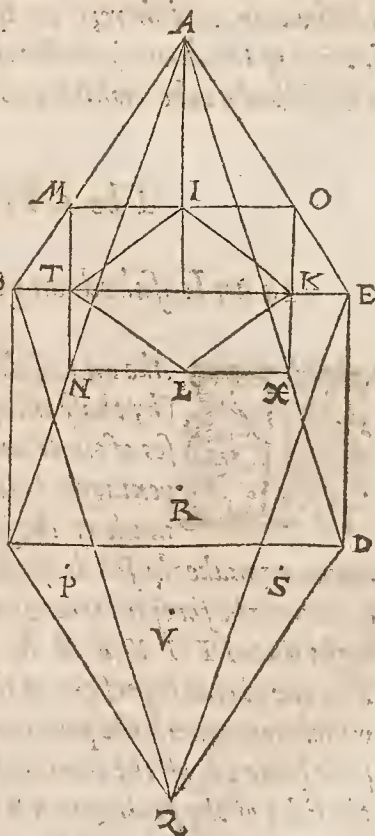


Suppose that the Octohedron geuen be $ABGDEZ$. And let the two pyramids thereof be $ABGDE$, and $ZBGDE$. And take the centres of the triangles of the pyramis $ABGDE$, that is, take the centres of the circles which containe those triangles: and let those centres be the pointes T, I, K, L . And by those

Construction.

centres let there be drawn parallel lines to the sides of the square $BGDE$: which parallel right lines let be MTN , NLX , XKO , & OIM . And forasmuch as those parallel right lines do (by the 2. of the sixth) cut the equall right lines AB, AG, AD , and AE , proportionally, therefore they concur in the pointes M, N, X, O . Wherefore the right lines MN, NX, XO , and OM , which subtend equall angles set at the point A , & contained under equall right lines, are equall (by the 4. of the first). And moreover, seing that they are parallels vnto the lines BG, GD, DE, EB , which make a square, therefore $MNXO$ is also a square, by the 10. of the eleuenth. Wherefore also, by the 15. of the same, the square $MNXO$ is parallel to the square $BGDE$. For all the right lines touch the one the other in the pointes of their sections: From the centres T, I, K, L , draw these right lines TI, IK, KL, LT . And drawe the right line AIC . And forasmuch as I is the centre of the equilater triangle ABE , therefore the right line AI being extended, cutteth the right line BE into two equall partes (by the Corollary of the 12. of the thirteenth). And forasmuch as MO is a parallel to BE , therefore the triangle AIO is like to the whole triangle ACE (by the Corollary of the 2. of the sixth). And the right line MO is diuided into two equall partes in the point I (by the 4. of the sixth). And by the same reason may we proue, that the right lines MN, NX, XO , are diuided into two equall

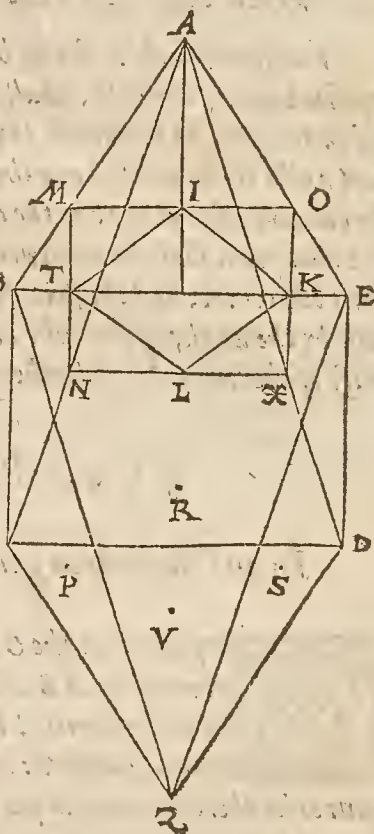
partes



Demonstration.

The fifteenth Booke

partes in the pointes T, L, K . Wherefore also againe, the bases TI, IK, KL, LT , which subtend the angles set at the pointes M, O, X, N , which angles are right angles, and are contained vnder equall sides, those bases, I say, are equall. And forasmuch as TIM is an Isosceles triangle, therefore the angles set at the base, namely, the angles MTI and MIT , are equal (by the 5. of the first). But the angle M is a right angle: wherefore eche of the angles MIT and MTI , is the halfe of a right angle. And by the same reason the angles OKI & OKL , are equall. Wherefore the angle remayning, namely, TIK , is a right angle (by the 13. of the first). For the right lines TI and IK are set vpon the line MO . And by the same reason may the rest of the angles, namely, IKL, KLT, LTI , be proued right angles, and they are in one and the self same plaine superficies, namely, $MNOX$ (by the 7. of the eleuenth). Wherefore the right lines which ioine together the centres of the plaine superficiall triangles which make the solide angle B , do make the square $ITKL$. And by the same reason may be proued, that the plaine superficiall triangles of the rest of the five solide angles of the Octohedron set at the pointes B, G, Z, D, E , do in the centres of their bases receaue squares, So that there are in number sixe squares, for euery Octohedron hath sixe solide angles: and those squares are equall: for their sides do containe equall angles of inclinations contained vnder equall sides, namely, vnder those sides which are drawn from the centre to the side of the equall triangles (by the 2. Corollary of the 18. of the thirteenth). Wherefore $ITKL RPS$ is a cube (by the 21. definition of the eleuenth) and hath his angles in the centres of the bases of the Octohedron, and therefore is inscribed in it (by the first definition of this booke). Wherefore in an Octohedron geuen is described a cube: which was required to be done.



The 5. Proposition.

The 5. Probleme.

In an Icosahedron geuen, to describe a Dodecahedron.

Construction.

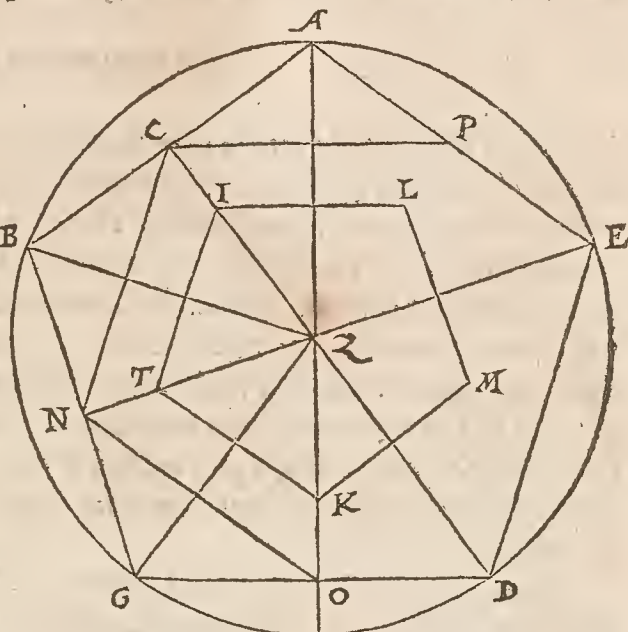


Iake an Icosahedron, one of whose solide angles let be Z . Now forasmuch as (by those things which haue bene proued in the 16. of the thirteenth) the bases of the triangles which containe the angle of the Icosahedron doo make a pentagon inscribed in a circle, let that pentagon be $ABGDE$, which is made of the five bases of the triangles, whose playne superficiall angles remayning make the solide angle geuen, namely, Z . And take the centres of the circles which containe the foresaid triangles, which centers let be the poyntes I, T, K, M, L : and draw these right lines IT, TK, KM, ML, LI . Now then a perpendicular line drawne from the poynt Z to the playne superficies of the pentagon $ABGDE$, shall fall vpon the centre of the circle which containeth the pentagon $ABGDE$ (by those things which haue bene proued in the selfe same 16. of the thirteenth). Moreouer perpendicular lines drawne from the centre to the sides of the pentagon $ABGDE$ shall in the poyntes C, N, O where they fall cut the right lines AB, BG, GD into two equall partes (by the 3. of the third). Draw these right lines CN and NO . And forasmuch as the angles CBN and NGO are equall, and are contained vnder equall sides, therefore the base CN is equall to the base NO (by the 4. of the first). More-

Demonstration.

uer

ouer perpendicular lines drawne from the poynt Z to the bases of the pentagon $ABGDE$, shall likewise cutte the bases into two equall partes (by the 3. of the third). For the perpendiculars passe by the centre (by the corollary of the 12. of the thirteth): Wherefore those perpendicular lines shall fal vpo the points C, N, O . And now forasmuch as the right lines ZI, IC are equall to the right lines ZT, TN , & also to the right lines ZK, KO (by reason of the likenes of the equall triangles): therefore the line IT is a parallell to the line CN , and so also is the line TK to the line NO (by the 2. of the sixth). Wherefore the angles ITK , and CNO are equal (by the 11. of the eleuenth). Agayne forasmuch as the triangles CBN , and NGO are Isoscels triangles, therefore the angles BCN and BNC are equall (by the 5. of the first). And by the same reason the angles GNO , and GON are equall. And morcouer the angles BCN and BNC are equall to the angles GNO , and GON , for that the triangles CBN and NGO are equall and like. But the three angles BNC, CNO, ONG , are equall to two right angles (by the 13. of the first): for that vpon the right line BG are set the right lines CN & ON . And the three angles of the triangle CBN , namely, the angles BNC, BCN or GNO (for the angle GNO is equall to the angle BCN as it hath bene proued) and BNC are also equall to two right angles (by the 32. of the first). Wherefore taking away the angles BNC & GNO , the angle remayning, namely, CNO is equall to the angle remayning, namely, to CBN . Wherefore also the angle ITK (which is proued to be equall to the angle CNO) is equall to the angle CBN . Wherefore ITK is the angle of a pentagon. And by the same reason may be proued that the rest of the angles, namely the angles TIL, ILM, LMK, MKT , are equall to the rest of the angles, namely to BAE, AED, EDG, DGB . Wherefore $ITKML$ is an equilater and equiangle pentagon (by the 4. of the first) For the equall bases of the pentagon $ITKML$ doo subtend equall angles set at the point Z , and comprehended vnder equall sides. Moreouer it is manifest that the pentagon $ITKML$ is in one and the selfe same playne superficies. For forasmuch as the angles ONC and NCP are in one and the selfe same playne superficies, namely in the superficies $ABGDE$: But vnto the same playne superficies the playne superficieses of the angles KTI and TIL are parallels (by the 15. of the eleuenth). And the triangles KTI and TIL concur: wherefore they are in one and the selfe same playne superficies (by the corollary of the 16. of the eleuenth). And by the same reason so may we proue that the triangles ILM, LMK, MKT are in the selfe same playne superficies wherein are the triangles KTI and TIL . Wherefore the pentagon $ITKML$ is in one and the selfe same playne superficies. Wherefore the solide angle of the Icosahedron, namely the solide angle at the poynt Z subtendeth an equilater and equiangle pentagon plaine superficies, which pentagon hath his plaine superficiall angles in the centres of the triangles which make the solide angle Z . And in like sort may we proue that the other eleuen solide angles of the Icosahedron, eche of which eleuen solide angles are equall and like to the solide angle Z (by the 16. of the thirtenth) are subtended vnto pentagons equall, and like, and in like sort set to the pentagon $ITKML$. And forasmuch as in those pentagons the right lines, which ioine together the centers of the bases, are common sides, it followeth that those 12. pentagons include a solide which solide is therefore a dodecahedron (by the 24. diffinition of the eleuenth): and is, by the first diffinition of this booke, described in the Icosahedron, five



The fourteenth Booke

sides whereof are set vpon the pentagon *ABGDE*. Wherefore in an Icosahedron geuen is inscribed a dodecahedron: which was required to be done.

An annotation of Hypsicles.

This is to be noted, that if a man should demaund how many sides an Icosahedron hath, we may thus answer: It is manifest that an Icosahedron is containd vnder 20. triangles, and that euery triangle consisteth of three right lines. Now then multiply the 20. triangles into the sides of one of the triangles, and so shall there be produced 60. the halfe of which is 30. And so many sides hath an Icosahedron. And in like sort in a dodecahedron, forasmuch as 12. pentagons make a dodecahedron, and euery pentagon containeth 5. right lines, multiply 5 into 12. and there shall be produced 60. the halfe of which is 30. And so many are the sides of a dodecahedron. And the reason why we take the halfe, is, for that euery side whether it be of a triangle or of a pentagon, or of a square as in a cube, is taken twise. And by the same reason may you finde out how many sides are in a cube, and in a pyrmis, and in an octohedron.

But now agayne if ye will finde out the number of the angles of euery one of the solide figures, when ye haue done the same multiplication that ye did before, diuide the same sides, by the number of the plaine superficieses which comprehend one of the angles of the solides. As for example, forasmuch as 5. triangles containe the solide angle of an Icosahedron, diuide 60. by 5. and there will come forth 12. and so many solide angles hath an Icosahedron. In a dodecahedron, forasmuch as three pentagons comprehend an angle, diuide 60. by 3. and there will come forth 20. and so many are the angles of a dodecahedron. And by the same reason may you finde out how many angles are in eche of the rest of the solide figures.

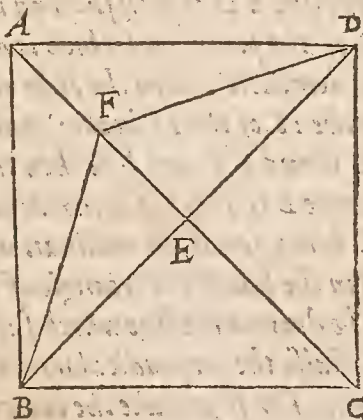
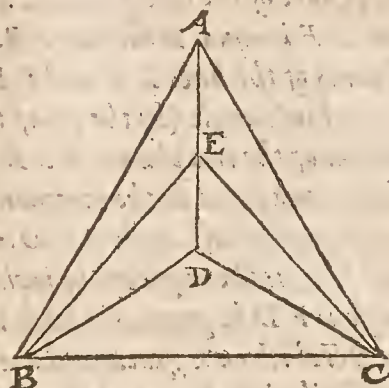
If it be required to be knowne, how one of the plaines of any of the fve solides being geuen, there may be found out the inclination of the sayd plaines the one to the other, which containe eche of the solides. This (as sayth Isidorus our greate master) is found out after this maner. It is manifest that in a cube, the plaines which containe it, doo cut the one the other by a right angle. But in a Tetrahedron, one of the triangles being geuen, let the endes of one of the sides of the sayd triangle be the centers, and let the space be the perpendicular line drawne from the toppe of the triangle to the base, and describe circumferences of a circle, which shall cutte the one the other: and from the intersection to the centers draw right lines, which shall containe the inclination of the plaines cotayning the Tetrahedron. In an Octohedron, take one of the sides of the triangle thereof, and vpon it describe a square, and draw the diagonall line, and making the centres, the endes of the diagonall line, and the space likewise the perpendicular line drawne from the toppe of the triangle to the base, describe circumferences: and agayne from the common section to the centres draw right lines, and they shall containe the inclination sought for. In an Icosahedron, vpon the side of one of the triangles thereof, describe a pentagon, and draw the line which subtendeth one of the angles of the sayd pentagon, and making the centres the endes of that line, and the space the perpendicular line of the triangle, describe circumferences: and draw from the common intersection of the circumferences, vnto the centres right lines: and they shall containe likewise the inclination of the plaines of the icosahedron. In a dodecahedron, take one of the pentagons, and draw likewise the line which subtendeth one of the angles of the pentagon, and making the centres the endes of that line, and the space, the perpendicular line drawne from the section into two equall partes of that line to the side of the pentagon, which is parallel vnto it, describe circumferences: and from the point of the intersection of the circumferences draw vnto the centres right lines: and they shall also containe the inclination of the plaines of the dodecahedron. Thus did this most singular learned man reason, thinking the demonstration in euery one of them to be plaine and cleare. But to make the demonstration of them manifest

That which here followeth concerning the inclination of the plaines of the fve solides, was before taught (though not altogether after the same maner) out of Flussas in the latter ende of the 13. booke.

fest, I think it good to declare and make open his wordes, and first in a Tetrahedron.

Suppose that there be a Pyramis or Tetrahedron $ABCD$ contained under 4. equilateral triangles: & let the toppe thereof be the point D . And (by the 10. of the first) diuide the side AD into two equall parts in the point E : & draw the lines BE and EC . And forasmuch as ADB and ADC are equilateral triangles, and the line AD is diuided into two equall partes, therefore the lines BE and EC fall perpendicularly vpon the line AD , by the 8. of the first. Now I say that the angle $BE C$ is an acute angle. For forasmuch as the line AC is double to the line AE (for by construction the line AD , which is equal to the line AC , is diuided into two equall partes in the point E): therefore the square of the line AC is quadruple to the square of the line AE (by the corollary of the 20. of the sixth). But the square of the line AC is equall to the squares of the lines AE and EC (by the 47. of the first): and the square of the line AC is to the square of the line CE (sesquitercia) as 4. to 3: (for the square of the line AC is proued quadruple to the square of the line AE): wherefore the square of the line BC (which is equall to the square of the line AC) is lesse then the squares of the two perpendiculars BE & EC (for it is vnto them in subsesquialter proportio, namely, as 4. to 6. or 2. to 3.) Wherefore (by the 13. of the second) the angle $BE C$ is an acute angle. Now forasmuch as the line AD is the common intersection of the two plaines ABD , and ADC , and in either of those plaines to one point of the common section are drawne perpendicular lines BE and EC which containe an acute angle $BE C$, therefore (by the 5. diffinition of the eleuenth) the angle $BE C$ is the inclination of the plaines, and it is geuen. For the line BC , which is the side of the triangle, being geuen, and any one of the lines BE or EC , which is the perpendicular of the equilateral triangle, being also geuen: make the centres the poyntes B and C , that is, the endes of one of the sides, and the space the perpendicular of the triangle, and describe circumferences, and they shall cutte the one the other in the poynt E . And from the poynt E draw to the centres B and C right lines, and they shall containe the inclination of the plaines: and this is it which Isidorus before sayd. And now that making the centres the poynts B and C , and the space the perpendicular of the triangle, the circles described shall cutte the one the other, it is manifest, for either of the lines BE and EC is greater then halfe of the line BC . Now if the centers were the poynts B and C , and the space the halfe of the line BC , the circles described shall touch the one the other. But if the space be lesse then the halfe, they shal neither touch nor cut the one the other: but if it be greater, they shall undoubtedly cut.

Again suppose that vpon the square $ABCD$ be set a pyramis, hauing his altitude the poynt E , and let the triangles which containe it, be equilateral: wherefore the pyramis $ABCE$ shalbe the halfe of the Octohedron (by the 2 corollary of the 14. of the thirteenth.) Deuide by the 10. of the first one side of one of the triangles, namely, the line AE , into two equal partes in the poynt F : and draw the lines BF and DF : wherefore the lines BF and DF are equal and fall perpendicularly vpon the line AE (by the 4. and 8. of the first.) Then I say that the angle $BF D$, is an obtuse angle. For draw the line BD . And forasmuch as AC is a square, and the diameter is BD : therefore the square of the line BD is double to the square of the line DA (by the 47. of the first.) But the square of the line DA is to the square of the line

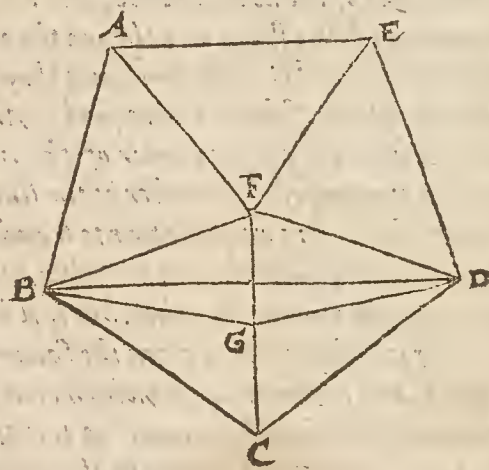


The reason of this you shall most plainly see in that which is added out of Proclus after the 22 proposition of the first booke.

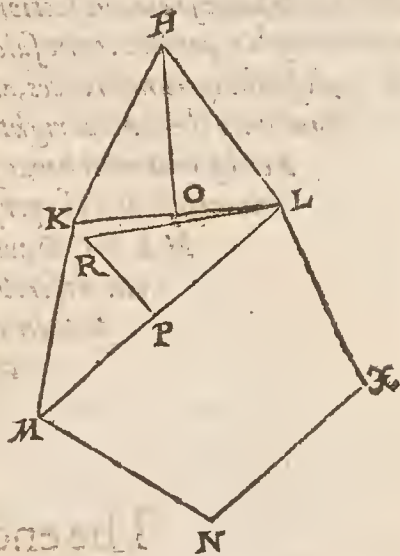
The fiftenth Booke

D F, as 4. is to 3.) as was in the former proued.) Wherefore the square of the line *D B* is to the square of the line *F D*, as 8. is to 3. (namely, as 2. to 1. and 4. to 3. added together), but the line *D F* is equal to the line *F B*. Wherefore the square of the line *D B*, is greater then the squares of the lines *D F* and *F B* (for it is to them, as 8. is to 6.) Wherefore the angle *B F D*, is an obtuse angle (by the 12. of the second.) And forasmuche as the line *A E* is the common section of the two plaines *A B E* and *A D E* cutting the one the other, and in either of those plaines to a poynt in the common section are drawne perpendicular lines, *B F* and *D F*, containing an obtuse angle *B F D*: wherefore the angle *B F D* (contained of the right lines *B F* and *D F*) is the angle of the inclination of the plaines *A B E* and *A D E*. If therefore the angle *B F D* be geuen, the saide inclination also is geuen. For forasmuch as the triangle of the Octohedron is geuen, and one of the sides of the Octohedron is the line *A D*, and vpon it is described the square *A C*, and *B D* the diameter of that square being geuen, and the lines *B F* and *F D* are the perpendiculars of that triangle: wherefore also the angle *B F D* is geuen. Now then if vpon the side of the triangle be described a square: as the square *A C*, and the diameter *B D* be drawne, if also making the centres the poyntes *B* and *D*, and the space, the saide perpendicular of the triangle, we describe circles, they shall cut the one the other in the poynt *F*: And the right lines which are drawne from the centres to the poynt *F* shal containe that inclination, which is comprehended vnder the angle *B F D*, which is the angle of the inclination of those plaines. And it is manifest that either of the lines *B F* and *F D*, is greater then the halfe of the line. For for that, by the demonstration, it was proued that the square of the line *B D* is to the square of the line *F D*, as 8. is to 3: therefore the square of halfe the line *B D* is to the square of the line *F D*, as 2. is to thre (for the square of halfe the line *B D* is the fourth part of the square of the whole line *B D*, by the 4. of the second). Wherefore either of the lines *B F* and *F D*, is greater then the line *B D*: wherefore the circles which are described by those lines *B F* and *F D*, and hauing their centres the poynts *B* and *D* shall cut the one the other. And thus much touching the octohedron.

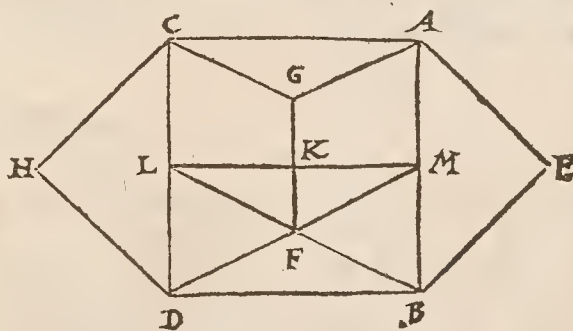
As touching the Icosahedron, suppose an equilater pētagon *A B C D E*, & vpon it let there be set a pyramis hauing his toppē the poynt *F*: and let the triangles which cōtaine it, be equilater. Now thē the pyramis *A B C D E F*, shal be a part of the Icosahedrō. Let *F C* one side of one of the triangles be deuided into two equal partes in the poynt *G*. And draw the lines *B G* & *G D*, which shal be equal & fal perpēdicularly vpon the line *F C* (as it is easie to se by the demōstratiō of the former). Thē I say that y angle *B G D* is an obtuse angle: which thing is manifest. For drawing the line *B D*, it shall subtend the obtuse angle *B C D* of the pentagon (which is obtuse, by that which was demonstrated in the ende of the first corollary of the 18. of the 13. booke:) But the angle *B G D* is greater then the angle *B C D*, for the lines *B G* and *G D*, are lesse then the lines *B C* and *C D*: wherefore likewise as in the former the angle *B G D* is the inclination of the triangles *B F C*, and *C F D*. Wherefore the angle *B G D* being geuen, the inclination also of the plaines of the Icosahedron shall be geuen. For if vpon the side of the triangle of the Icosahedron be described an equilater pentagon, and then be drawne the line which subtendeth two sides of the pentagon, as in this figure the line *B D*, if also the perpendiculars *B G* and *G D* of the triangles be drawne, the angle *B G D* shal be geuen. For if ye make the centres the endes of the line which subtendeth two sides of the pentagon, as the poynts *B* and *D*, and the space the perpendicular of the triangle, and so describe circles, they shall cut the one the other in the poynt *G*, and from the poynt of the intersection



tersection G, drawe vnto the centres B and D right lines, and they shal containe the angle of the inclination B G D. And it is manifest, by the description of the figure, that either of the lines B G and G D is greater then the line B D. Which thing may also thus be proued. Suppose an equilater H K L, and vpon K L (one of the sides thereof) describe an equilater pentagon K M N X L, and draw the line M L. And diuide (by the 10. of the first) the side K L into two equal partes in the poynt O, & draw the line H O, which shall be the perpendicular of the triangle H K L (by the 8. of the first.) Then I say that the line H O is greater then half of the line M L, which subtendeth the inclination of the plaines. For from the poynt K draw (by the 12. of the first) vnto the line M L a perpendicular line K P: and forasmuche as the angle K L P is greater then the third part of a right angle, that is, then the angle K H O (For the angle K L M is two fifteth partes of a right angle, by the 4. of the first, and by the assumpt put after the first corollary of the 18. of the thirteenth booke, and the angle K H O is one third part of one right angle, for the whole angle K H L, wherof the angle K H O is the half, by the 4. of the first, is one third part of two right angles, by the 32. of the first.) vpon the line M L, and at the poynt L put vnto the angle K H O an equal angle P L R (by the 23. of the first.) Wherefore the triangles P L R & O H K, shalbe equiangle, by the corollary of the 32. of the first. Wherefore also the line P L shalbe the perpendicular of the equilater triangle described vpon the line R L. Wherefore (by the corollarye added by Flussas after the 12. proposition of the thirteenth booke) the line R L is in power sesquitercia, that is, as 4. is to 3. to the perpendicular L P. But the line K L is greater then the line L R (by the 19. of the first. Wherefore $\frac{KL^2}{LR^2}$ hath to the square of the line L P a greater proportion then 4. to 3: but it hath to the square of the line H O that proportion that 4. hath to 3. Wherefore the line K L hath to the line L P a greater proportion then it hath to the line H O. Wherefore the line H O is greater then the line L P, by the 10. of the fifth.



As concerning a Dodecahedron. Take one of the squares of the cube wheron the Dodecahedron is described (by the 17. of the thirteenth): and let the same be A B C D: and let the two plaines of the Dodecahedron set vpon it be A E B F G, and G F D H C. Then I say that here also is geuen the inclination of the two pentagons. Diuide (by the 10. of the first) the side F G into two equal partes in the poynt K. And from the poynt K draw vnto the line F G in either of the plaines A E B F G and G F D H C perpendicular lines K L and K M (by the 11. of the first.) And draw the line M L. First I say that the angle M K L is an obtuse angle. For, by the discourse of the demonstration of the 17. proposition of the 13. booke, where is taught the description of the Dodecahedron, it is manifest, that the line drawne perpendicularly from the poynt K to the square A B C D, is equal to halfe the side of the pentagon. Wherefore it is lesse then halfe of the line M L. Wherefore the angle M K L is an obtuse angle. Moreover by the former discourse of the 17. proposition of the 13. booke, it was manifest that the square of the line K L is equal to the square of halfe the side of the cube, and to the square of halfe the side of the pentagon. And forasmuche as



the lines $K L$ and $K M$ are equal, and are eche greater then halfe of the line $M L$: wherfore the angle $M K L$ being geuen, there shall also be geuen the inclination of the two plaines of the Dodecahedron. For forasmuch as the side of the square $A B C D$ subtendeth two sides of the pentagon geuen, the pentagon also is geuen, and therefore also is geuen the line $M L$. But there is also geuen either of the lines $M K$ and $K L$: for they are drawne perpendicularly fro the section into two equal partes of the line $A B$, which subtendeth two sides of the pentagon vnto the side of the pentagon, which is a parallel vnto the line $A B$: namely, to the side $F G$. Wherfore there is geuen the angle $L K M$, which is the angle of the inclination sought for. And now touching Isidorus wordes, he sayeth, that the pentagon being geuen, we must draw the line which subtendeth two sides of the pentagon, which line is equal to the side of the cube: and making the centres the endes of that line, and the space the perpendicular line, which is drawne from the section of the same line into two equal parts to the side of the pentagon which is parellel to the said line, as in the former description the line $K L$, or the line $K M$, describe circumferences, and from the poynt of the intersection of the circumferences draw vnto the centres right lines which shall containe the angle of the inclination.

For by that which was sayd before, namely, touching the Icosahedron, it is manifest that the perpendicular $K L$, is greater then halfe of the line $M L$ or $C D$, which is equal vnto it. And therefore the circles described by those perpendiculars, and hauing to their centres the end of the line $C D$, shall cut the one the other, as was before proued.

The ende of the fuetenth Booke

of Euclides Elementes

after Hypsicles.

The end of the fiftenth Booke of Euclides Elementes, after Hypsicles, is the description of the Dodecahedron, which is a solid figure composed of twelve pentagonal faces. The text describes the construction of this solid figure, showing how it can be derived from a cube and a dodecahedron. The construction involves drawing a cube, and then a dodecahedron, and showing how they are related. The text also describes the properties of the Dodecahedron, such as its volume and surface area.



The Dodecahedron is a solid figure composed of twelve pentagonal faces. It is one of the five regular polyhedra, along with the tetrahedron, cube, octahedron, and icosahedron. The Dodecahedron has 20 vertices and 30 edges. It is a dual polyhedron to the icosahedron, meaning that each vertex of the Dodecahedron corresponds to a face of the icosahedron, and vice versa. The Dodecahedron is also a space-filling polyhedron, meaning that it can be used to fill space without any gaps or overlaps. The text describes the construction of the Dodecahedron, showing how it can be derived from a cube and a dodecahedron. The construction involves drawing a cube, and then a dodecahedron, and showing how they are related. The text also describes the properties of the Dodecahedron, such as its volume and surface area.

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¶ The 6. Proposition.

The 6. Probleme.

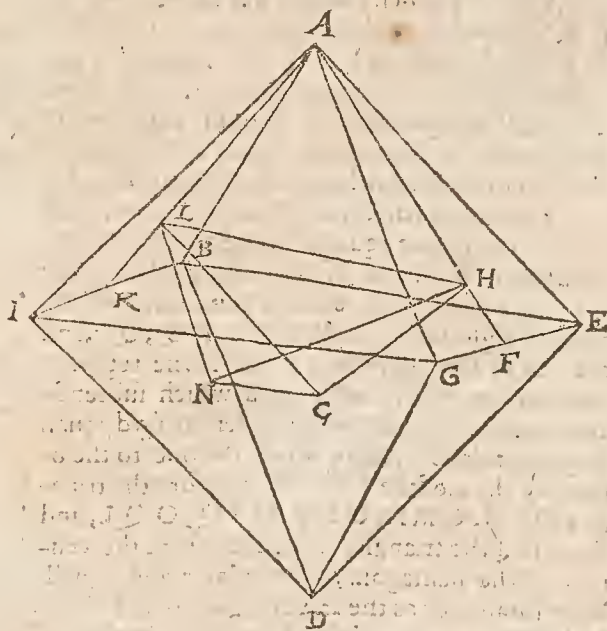
In an Octohedron geuen, to inscribe a trilater equilater Pyramis.



Vppose that the Octohedron wherein the Tetrahedron is required to be inscribed, be $ABGDEI$. Take fower bases of the Octohedron, that is, three which close in the lowest triangle EGD , namely, AGE , BED , IGD : and let the fourth be ABI , which is opposite to the lowest triangle before put, namely, to EGD . And take the centres of those fower bases, which let be the pointes H, C, N, L . And vpon the triangle HCL erecte a pyramis $HCLN$. Now forasmuch as these two bases of the Octohedron, namely, AGE and ABI are set vpon the right lines EG and BI which are opposite the one to the other, in the square $GEBI$ of the Octohedron, from the point A drawe by the centres of the bases, namely, by the centres H, L , perpendicular lines AHF , ALK , cutting the lines EG and BI into two equall partes in the pointes F, K (by the

Construction.

Corollary of the 12. of the thirteenth). Wherefore a right line drawn frō the point F to the point K , shall be a parallel and equall to the sides of the Octohedron, namely, to EB and GI (by the 33. of the first). And the right line HL which cutteth the equall sides AF, AK , proportionally (for AH and AL are drawn from the centres of equall circles to the circumferences) is a parallel to the right line FK (by the 2. of the sixth) and also to the sides of the Octohedron, namely, to EB and IG (by the 9. of the eleuenth). Wherefore as the line AF is to the line AH , so is the line FK to the line HL (by the 4. of the sixth): For the triangles AFK and AHL are like (by the Corollary of the 2. of the sixth). But the line AF is in sesquialter proportion to the line AH : (for the side EG maketh HF the halfe of the right line AH , by the Corollary of the 12. of the thirteenth). Wherefore FK or GI the side of the Octohedron, is sesquialter to the right line HL . And by the same reason may we proue that the sides of the Octohedron are sesquialter to the rest of the right lines which make the pyramis $HNCN$, namely, to the right lines HN, NC, CL, LN , and CH : wherefore those right lines are equall, and therefore the triangles which are described of them, namely, the triangles HCL, HNL, NCL , and CHL , which make the pyramis $HNCN$, are equall and equilater. And forasmuch as the angles of the same pyramis, namely, the angles H, C, N, L , do end in the centres of the bases of the Octohedron, therefore it is inscribed in the same Octohedron, by the first definition of this booke. Wherefore in an Octohedron geuen, is inscribed a trilater equilater pyramis: which was required to be done.



Demonstration.

✱ A Corollary.

The bases of a Pyramis inscribed in an Octohedron, are parallels to the bases of the Octohedron. Forasmuch as the sides of the bases of the Pyramis touching the one the other, are parallels to the sides of the Octohedron which also touch the one the other, as for example, HL was proued to be a parallel to GI , and LC to DI , therefore, by the 15. of the eleuenth, the plaine superficies which is drawn by the lines HL and LC , is a parallel to the plaine superficies drawn by the lines GI and DI . And so likewise of the rest.

✱ Second Corollary.

A right line ioyning together the centres of the opposite bases of the Octohedron, is sesquialter to the perpendicular line drawn from the angle of the inscribed

The fiftenth Booke

scribed pyramis to the base thereof. For forasmuch as the pyramis and the cube which containeth it do in the selfe same pointes end their angles (by the 1. of this booke): therefore they shall both be inclosed in one and the selfe same Octohedron (by the 4. of this booke). But the diameter of the cube ioyneth together the centres of the opposite bases of the Octohedron, and therefore is the diameter of the Sphere which containeth the cube and the pyramis inscribed in the cube (by the 13. and 14. of the thirteenth): which diameter is sesquialter to the perpendicular which is drawn from the angle of the pyramis to the base thereof: for the line which is drawn from the centre of the sphere to the base of the pyramis, is the sixth part of the diameter (by the 3. Corollary of the 13. of the thirteenth). Wherefore of what partes the diameter containeth fixe, of the same partes the perpendicular containeth fower.

¶ The 7. Proposition.

The 7. Probleme.

In a dodecahedron geuen, to inscribe an Icosahedron.

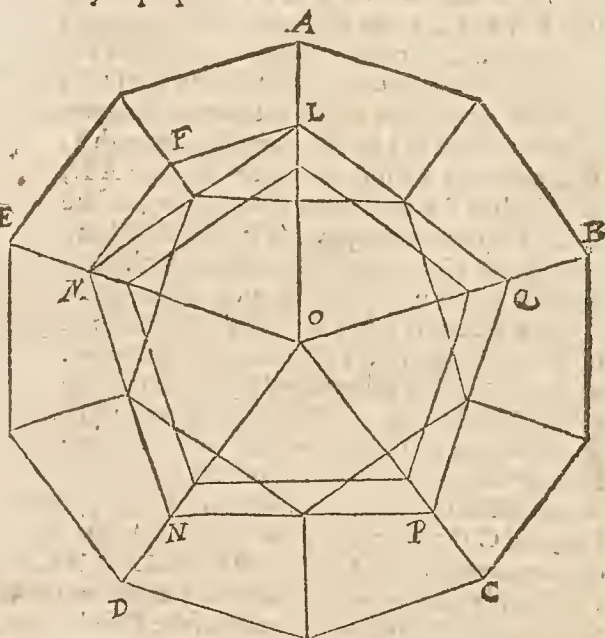
Construction.



Suppose that the dodecahedron geuen, be A B C D E. And let the centres of the circles which cōtaine fixe bases of the same dodecahedron be the points L, M, N, P, Q, O. And draw these right lines O L, O M, O N, O P, O Q, and moreover these right lines L M, M N, N P, P Q, Q L. And now forasmuch as equall and equilater pentagons are contained in equall circles, therefore perpendicular lines drawne from their centres to the

Demonstration.

sides shall be equall (by the 14. of the third), and shall diuide the sides of the dodecahedron into two equall partes (by the 3. of the same). Wherefore the foresayde perpendicular lines shall concur in the point of the section, wherein the sides are diuided into two equall partes, as L F and M F doo. And they also containe equall angles, namely, the inclination of the bases of the dodecahedron, (by the 2. corollary of the 18. of the thirteenth). Wherefore the right lines L M, M N, N P, P Q, Q L, and the rest of the right lines which ioyn together two centres of the bases, and which subtende the equall angles contained vnder the sayd equall perpendicular lines, are equall the one to the other (by the 4. of the first). Wherefore the triangles O L M, O M N, O N P, O P Q, O Q L, and the rest of the triangles which are set at the centres of the pentagons, are equilater and equall. Now forasmuch as the 12. pentagons of a dodecahedron containe 60. plaine superficiall angles, of which 60. euery thre make one solide angle of the dodecahedron, it followeth that a dodecahedron hath 20. solide angles: but eche of those solide angles is subtended of ech of the triangles of the Icosahedron, namely, of ech of those triangles which ioyn together the centres of the pentagons which make the solide angle, as we haue before proued. Wherefore the 20. equall and equilater triangles which subtende the 20. solide angles of the dodecahedron, and haue their sides which are drawne from the centres of the pentagons common, doo make an Icosahedron (by the 25. diffinition of the eleuenth): and it is inscribed in the dodecahedron geuen (by the first diffinition of this booke) for that the angles thereof doo all at one time touch the bases of the dodecahedron. Wherefore in a dodecahedron geuen, is inscribed an Icosahedron: which was required to be done.



¶ The 8. Proposition.

The 8. Probleme.

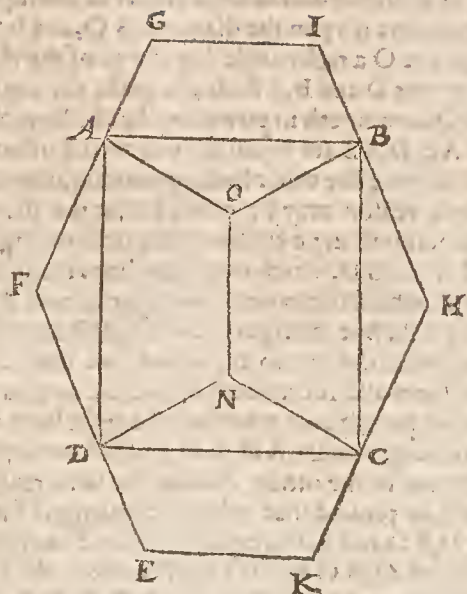
In a dodecahedron geuen, to include a cube.

Construction.



Describe (by the 17. of the thirteenth) a dodecahedron. And by the same, take the 12. sides of the cube, eche of which subtend one angle of eche of the 12. bases of the dodecahedron: for the side of the cube subtendeth the angle of the pentagon of the dodecahedron, by the 2. corollary of the 17. of the thirteenth. If therefore in the dodecahedron described (by the selfe same 17. proposition) we draw the 12. right lines subtended vnder the foresayd 12. angles, and ending in 8. angles of the dodecahedron, and concurring

ring together in such sort that they be in like sort situate, as it was plainly proued in that proposition, then shall it be manifest, that the right lines drawne in this dodecahedron from the foresayd 8. angles thereof doo make the foresayd cube, which therefore is included in the dodecahedron, for that the sides of the cube are drawne in the sides of the dodecahedron, and the angles of the same cube are set in the angles of the said dodecahedron. As for example take 4. pentagons of a dodecahedron, namely $AGIBO$, $BHCNO$, $CKEDN$ and $DFAON$. And draw these right lines AB , BC , CD , DA . Which four right lines make a square: for that eche of those right lines doo subtend equall angles of equall pentagons, & the angles which those 4. right lines containe are right angles, as we proued in the construction of the dodecahedron, in the 17. proposition before alledged. Wherefore the sixe bases being squares, do make a cube (by the 21. diffinition of the eleuenth) and for that the 8. angles of the sayd cube are set in 8. angles of the dodecahedron, therefore is the sayd cube inscribed in the dodecahedron (by the first diffinition of this booke). Wherefore in a dodecahedron is inscribed a cube: which was required to be doone.



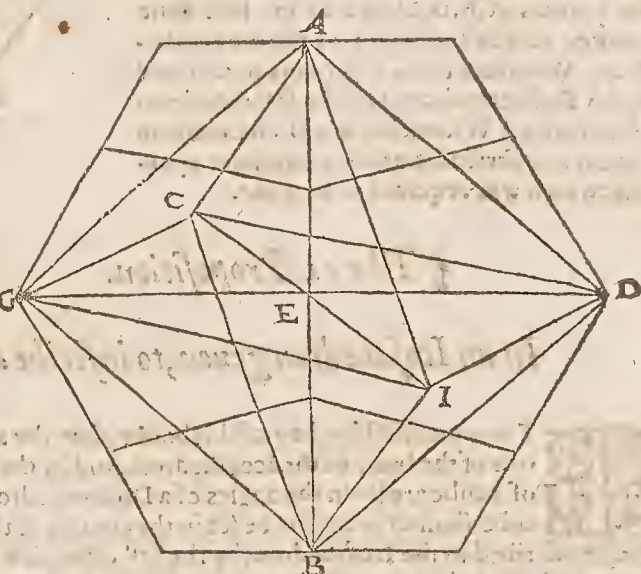
Demonstration.

¶ The 9. Proposition. The 9. Probleme.

In a Dodecahedron geuen to include an Octohedron.



Vppose that the dodecahedron geuen be $ABGD$. Now (by the 3. corollary of the 17. of the thirteenth) take the 6. sides which are opposite the one to the other, those 6. sides, I saye whose sections wherein they are deuided into two equal partes, are coupled by three right lines which in the centre of the sphere, wherein the Dodecahedron is contained, doe cut the one the other perpendicularly. And let the poyntes wherein the foresayde sides are cut into two equal partes be A, B, G, D, C, I . And let the foresaid three right lines ioyning together the saide sections be AB , GD and CI . And let the centre of the sphere be E . Now forasmuch as (by the foresaid corollary) those three right lines are equal, it followeth (by the 4. of the first) that the right lines subtending the right angles which they make at the centre of the sphere, whiche right angles are contained vnder the halues of the said three right lines, are equal the one to the other: that is, the right lines $AG, GB, BD, DA, CA, CG, CB, CD$, and IA, IG, IB, ID are equal the one to the other. Wherefore also the 8. triangles $CAG, CGB, CBD, CDA, IAG, IGB, IBD, IDA$ are equal and equilateral. And therefore $AGBD CI$ is an Octohedron by the 23. definition of the eleuenth. And the sayd Octahedron is included in the dodecahedron (by the first definition of this booke:) for that all the angles thereof doe at one time touch the sides of the dodecahedron. Wherefore in the dodecahedron geuen, is included an Octohedron: which was required to be done.



Construction.

Demonstration.

¶ The 10. Proposition. The 10. Probleme:

In a Dodecahedron geuen, to inscribe an equilater trilater Pyramis.



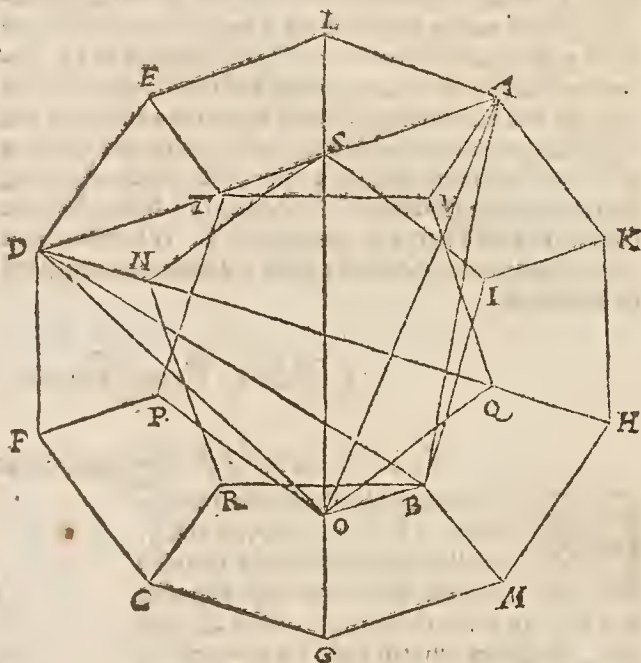
Vppose that the Dodecahedron geuen, be $ABCD$, of which Dodecahedron take thre bases meeting at the poynt S , namely these thre bases $ALSIK$, $DNSLE$ and $SIBRN$: and of those thre bases take the three angles at the poynts A, B, D : and draw these right lines AB , BD and DA : and let the diameter of the sphere containing the dodecahedron be SO , and then

Construction.

The fiftenth Booke

Demonstration.

then draw the right lines AO, BO and DO : Now forasmuch as (by the 17. of the thirteenth) the angles of the dodecahedron are set in the superficies of the sphere described about the Dodecahedron: therefore if upon the diameter SO , and by the poynt A , be described a semicircle, it shall make the angle SAO a right angle (by the 31. of the third.) And likewise if the same semicircle be drawne by the poyntes D and B , it shall also make the angles SBO , and SDO right angles. Wherefore the diameter SO containeth in power bothe the lines SA, AO , or the lines SB, BO , or els SD, DO , but the lines SA, SD, SB are equal the one to the other, for they eche subtend one of the angles of equal pentagons. Wherefore the other lines remaining, namely, AO, BO, DO are equal the one to the other. And by the same reason may be proued that the diameter HD which subtendeth the two right lines HA, AD , containeth in power both the said two right lines, and also containeth in power bothe the right lines HB and BD , which two right lines it also subtendeth. And moreover by the same reason the diameter AC , which subtendeth the right lines CB and BA , containeth in power both the said right lines CB and BA . But the right lines HA, HB and CB are equal the one to the other, for that eche of them also subtendeth one of the angles of equal pentagons: wherefore the right lines remaining, namely, AD, BD , and BA are equal the one to the other. And by the same reason may be proued that eche of those right lines AD, BD and BA is equal to eche of the right lines AO, BO and DO . Wherefore the sixe right lines AB, BD, DA, AO, BO , & DO are equal the one to the other. And therefore the triangles which are made of the, namely, the triangles ABD, AOB, AOD and BOA are equal and equilateral: which triangles therefore do make a pyramis $ABDO$, whose base is ABD and toppes the poynt O . Eche of the angles of which pyramis, namely, the angles at the pointes A, B, D, O , doe in the selfe same pointes touche the angles of the Dodecahedron. Wherefore the said pyramis is inscribed in the Dodecahedron, (by the first diffinition of this booke,) Wherefore in a Dodecahedron geuen, is inscribed a trilater equilateral pyramis: which was required to be done.



¶ *The 11. Proposition.*

The 11. Probleme.

In an Icosahedron geuen, to inscribe a cube.

IT was manifest by the 7. of this booke, that the angles of a Dodecahedron are set in the centres of the bases of the Icosahedron. And by the 8. of this booke, it was proued, that the angles of a cube are set in the angles of a Dodecahedron. Wherefore the selfe same angles of the cube, shall of necessitie be set in the centres of the bases of Icosahedron. Wherefore the cube shall be inscribed in the Icosahedron (by the first diffinition of this booke.) Wherefore in an Icosahedron geuen, is included a cube: which was required to be done.

¶ *The 12. Proposition.*

The 12. Probleme.

In an Icosahedron geuen, to inscribe a trilater equilateral pyramis.

BY the former proposition it was manifest, that the angles of a cube are set in the centres of the bases of the Icosahedron. And (by the first of this booke) it was playne that the foure angles of a pyramis are set in foure angles of a cube. Wherefore it is euident, by the first diffinition of this booke, that a pyramis described of right lines ioyning together these foure centres of the bases of the Icosahedron, shall be inscribed in the same Icosahedron. Wherefore in an Icosahedron geuen, is inscribed an equilateral trilater pyramis: which was required to be done.

The

¶ The 13. Probleme

The 13. Proposition.

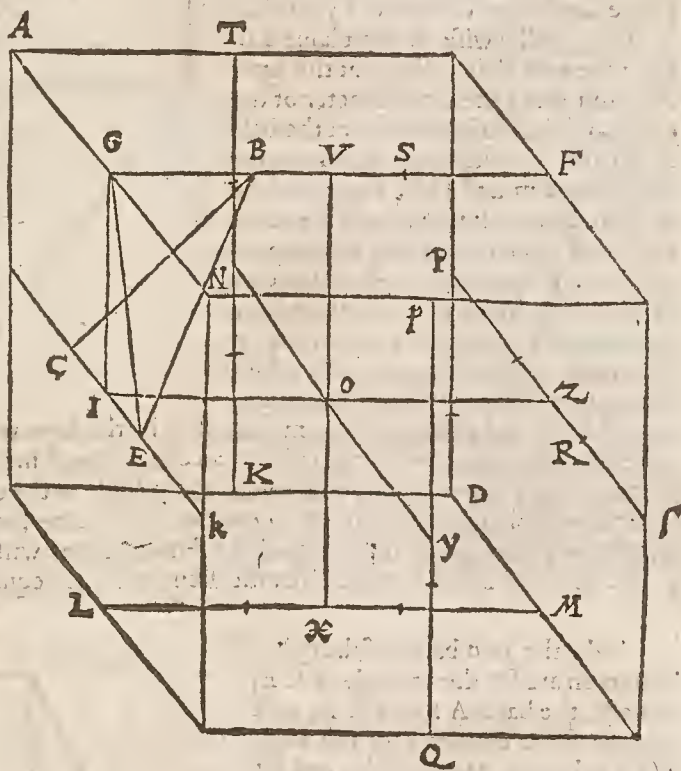
In a Cube geuen, to inscribe a Dodecahedron.



Ake a Cube A D F L. And diuide euery one of the sides therof into two equall partes in the pointes T, H, K, P : G, L, M, F : and p k Q s. And drawe these right lines T K, G F, p Q, H k, P f, and L M : which lines againe diuide into two equall partes in the pointes N, V, Y, I, Z, X. And drawe these right lines N Y, V X, and I Z : Now the three lines N Y, V X, and I Z, together with the diameter of the cube, shall cut the one the other into two equall partes in the centre of the cube, by the 39. of the eleuenth : let

First part of the construction.

that centre be the point O. And not to stand long about the demonstration, vnderstand all these right lines to be equall and parallels to the sides of the cube and to cut the one the other right angled wise, by the 29. of the first. Let their halfes, namely, F V, G V, H I, and k I, and the rest such like, be diuided by an extreme and meane proportion, by the 30. of the sixth : whose greater segmets let be the lines F S, G B, H C, and k E, &c. and drawe these right lines G I, G E, B C, and B E. Now forasmuch as the line G I is equall to the whole line G V, which is the halfe of the side of the cube : and the line I E is equall to the line B V, that is, to the lesse segmet : therefore, the squares of the lines G I and I E, are triple to the square of the line G B, by the 4. of the thirteenth : But vnto the squares of the lines G I and I E, the square of the line G E is equall, by the 47. of the first : for the angle G I E is a right angle : Wherefore the square of the line G E is, triple to the square of the line G B. And forasmuch as the line F G is erected perpendicularly to the plaine A G k L, by the 4. of the eleuenth : for it is erected perpendicularly to the two lines A G and G I : therefore the angle B. G E is a right angle : for the line G E is drawn in the plaine A G k L. Wherefore the line B E, containing in power the two lines B G and G E, by the 47. of the first, is in power quadruple to the line G B (for the line G E was proued to be in power triple to the same line G B) : Wherefore the line B E is in length double to the line B G, by the 20. of the sixth. But (by construction) the line C E is double to the line I E : Wherefore the halfes G B and I E, are in proportion the one to the other, as their doubles B E and C E : by the 15. of the fifth. Wherefore the line C E is the greater segment of the line B E diuided by an extreme and meane proportion. And forasmuch as the selfe same thing may be proued touching the line B C : therefore the lines B E and B C, are equall, making an Isosceles triangle. Now let vs proue that three angles of the Pentagon of the Dodecahedron are set at the pointes B, C, E : and the other two angles are set betwene the lines B C and B E.



First part of the demonstration.

Forasmuch as the circle which containeth the triangle B C E circumscribeth the Pentagon whose side is the line C E, by the 11. of the fourth : Extend the plaine of the triangle B C E, by the parallel lines d B and H E, cutting the line A D, namely, the diameter of A D the base of the cube in the point I : and let it cut the line A h the diameter of the cube in the point m. And by the point I drawe in the base A D, a parallell line vnto the line A d : which let be I l. And forasmuch as from the triangle A H N is, by the parallel line I l, taken away the triangle A l I, like vnto the whole triangle A H N, by the Corollary of the 2. of the sixth : the lines A l, and I l, shall be equall. But as the line H A is to the line A d, so (by the 2. of the sixth) is the line H l to the line I l, or to the line l A, which is equall to the line I l. And the greater segment of the line H A (which is halfe the side of the cube) is, as before hath bene proued, the line A d, that is, the line G B, which is equall to the line A d (by the 33. of the first).

Second part of the construction.

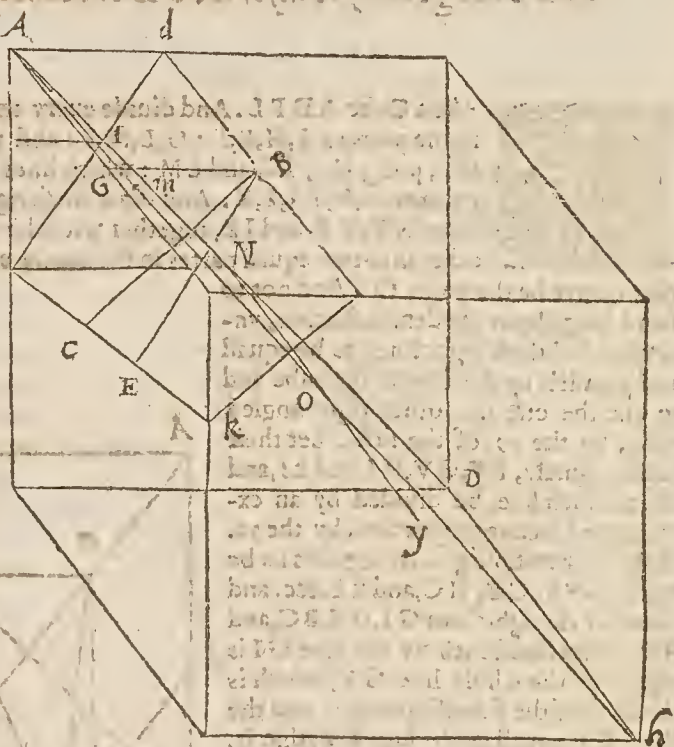
Second part of the Demonstration.

The fourteenth Booke

Wherefore the greater segment of the line $H I$ is the line $I A$. And as the whole line $H I$ is to the greater segment, so shall the same greater segment $H I$ be to the lesse segment $I A$, by the 5. of the thirteenth. Wherefore the line $H A$ is divided by an extreme and meane proportion in the point I . But in the triangle $A H N$, the line $N A$, which is drawn from the centre of the base $A D$, is in the point I cut like into the line $A H$, by the parallel line $I I$ (by the same second of the sixth): for the lines $H N$ and $I I$ are parallels; by construction.

Wherefore the line $N A$ is in the point I divided by an extreme and meane proportion by the superficies $d B E H$. And forasmuch as the line $Y O N$ which coupleth the centres of the opposite bases, is a parallel to the line $H E$: A plaine superficies extended by the line $Y O N$, parallel wise to the plaine $d B E H$: the two plaines shall cut the lines $A O$ and $A N$ (the semidiameter of the cube, and the semidiameter of the base $A D$) into the selfe same proportions in the pointes m and I , by the 17. of the eleventh. But the line $A N$ is in the point I divided by an extreme & meane proportion: Wherefore the semidiameter of the cube is in the point m divided by an extreme and meane proportion by the plaine of the triangle $B C E$. And forasmuch as the rest of the triangles described in the cube after the like manner, may by the same reasons be proued to be in a plaine which cutteth the semidiameter of the cube by an extreme and meane proportion: it is manifest that three plaines of the Dodecahedron shall vnder euery angle of the cube concur in one & the selfe same point of the semidiameter being cut by an extreme and meane proportion.

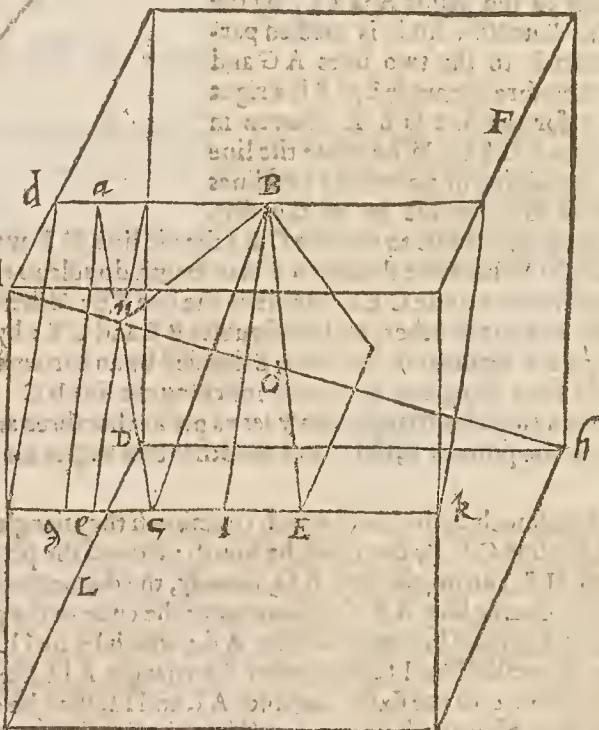
Now resteth to proue that the right lines which couple that point of the semidiameter with the angles of the triangle $B E C$, are equall: whereby may be proued that the Pentagons are equilater, and equiangle.



Third part of the construction.

Take the two bases of the cube.

Whereon are set the triangle $B C E$, namely, the bases $A F$ and $A k$, take also the same diameter of the cube that was before, namely, $A h$: and let the side set at the poynnt n , of the section of the diameter by an extreame & meane proportion, be the line $C n$ or $B n$: and let the centre of the cube be as before the point O . And extend the line $C n$ to the line $B d$, and let it concur with it in the point a . And forasmuch as the plaine which passeth by the line $H C E$ and the centre O (cutting the cube into two equall partes) is parallel to $A F$ the base of the cube by construction: imagine that by the poynnt n , be extended a playne superficies parallel to the former parallel plaines, which shall cutte the semidiameter $O A$ & the line $C a$, proportionally in the point n , by the 17. of the eleventh: For those lines doo touch the extreame parallel plaines extended by the lines $H E$ and $E O$, and by the lines $A d$ and $d B$. But it is proued that the line $O A$ is divided by an extreame and meane proportion in the



poynnt

point n : wherefore the line Ca , is also diuided by an extreame and meane proportion in the point n . Agayne forasmuch as BCE is an Ifofcels triangle, and it is proued that the line BI cutteth the base CE into two equall partes in the point I , the angles BIC and BIE shall be right angles. Imagine by the line BI and the centre O a plaine to passe (cutting the cube into two equall partes) parallel to the base AD . And vnto those plaines let there be imagined an other parallel plaine passing by the point n : which let be ne : which shall cutte the semidiameter AO and the halfe side of the cube, namely, the line IH , like, in the pointes n and e by the 17. of the eleuenth. Wherefore the line IH is in the point e diuided by an extreame & meane proportio. Wherefore the line He is equall to the line CI or IE : namely, ech are lesse segmets. And forasmuch as the line Ie is to the line IC (which is equall to the line EH) as the whole is to the greater segment, take away from the whole line Ie the greater segmet IC : there shall remayne the lesse segment Ce by the 5. of the thirteenth. Wherefore the line Ie is diuided by an extreame & meane proportion in the point C . Agayne vnto the same playnes imagine an other playne to passe by the point a , parallel wise, and let the same be ag . Now then (by the same 17. of the eleuenth) the lines Ca and Cg are in like sort cut in the pointes n and e . But the line Ca was in the point n cutte by an extreame and meane proportion, wherefore the line Cg shall be cutte in the point e , by an extreame & meane proportion. But the line IC is to the line Ce , as the greater segment is to the lesse: wherefore the line Ce , is to the line eg , as the greater segment to the lesse: and therefore their proportion is as the whole line IC is to the greater segment Ce , and as the greater segment Ce is to the lesse segment eg : wherefore the whole line Ceg which maketh the greater segment and the lesse, is equall to the whole line IC or IE . And forasmuch as two parallel plaine superficieses (namely, that which is extended by IOB and that which is extended by the line ag) are cutte by the playne of the triangle BCE , which passeth by the lines ag and IB , their common sections ag and IB shall be parallels (by the 16. of the eleuenth). But the angle BIE or BIC is a right angle, wherefore the angle agC is also a right angle (by the 29. of the first) and those right angles are containd vnder equall sides, namely, the line gC is equall to the line CI , and the line ag to the line BI , by the 33. of the first: wherefore the bases Ca and CB are equall, by the 4. of the first. But of the line CB the line CE was proued to be the greater segment: wherefore the same line CE is also the greater segment of the line Ca : but cn was also the greater segment of the same line Ca . Wherefore vnto the line CE , the line cn which is the side of the dodecahedron, and is set at the diameter, is equall. And by the same reason the rest of the sides, which are set at the diameter may be proued equall to lines equall to the line CE . Wherefore the pentagon inscribed in the circle where in is contained the triangle BCE is, by the 11. of the fourth equiangle, and equilater. And forasmuch as two pentagons, set vpon euery one of the bases of the cube doo make a dodecahedron, and sixe bases of the cube doo receaue twelue angles of the dodecahedron: and the 8. semidiameters doo in the pointes where they are cutte by an extreame and meane proportion receaue the rest: therefore the 12. pentagon bases containing 20. solide angles doo inscribe the dodecahedron in the cube: by the 1. diffinition of this booke. Wherefore in a cube geuen is inscribed a dodecahedron: which was required to be done.

First Corollary.

The diameter of the sphere, which containeth the dodecahedron, containeth in power these two sides, namely, the side of the Dodecahedron, and the side of the cube wherein the Dodecahedron is inscribed. For in the first figure a line drawne from the centre O , to the point B the angle of the Dodecahedron, namely the line OB , containeth in power these two lines OV the halfe side of the cube, and VB the halfe side of the dodecahedron, by the 47. of the first. Wherefore by the 15. of the fifth, the double of the line OB , which is the diameter of the sphere containing the Dodecahedron, containeth in power the double of the other lines OV and VB , which are the sides of the cube, and of the dodecahedron.

¶ Second Corollary.

The side of a cube diuided by an extreme and meane proportion, maketh the lesse segment the side of the dodecahedron inscribed in it: and the greater segment the side of the cube inscribed in the same Dodecahedron. For it was before proued, that the side of the dodecahedron is the greater segment of BE the side of the triangle BEC : but the side BE (which is equall to the lines GB and SF) is the greater segmet of GF the side of the cube: which line BE (subtending the angle of the pentagon) was (by the 8. of this booke) the side of the cube inscribed in the dodecahedron.

The fiftenth Booke

Third Corollary.

The side of a cube, is equal to the sides of a Dodecahedron inscribed in it, and circumscribed about it. For it was manifest by this proposition, that the side of a cube maketh the lesse segment, the side of a Dodecahedron inscribed in it, namely, as in the first figure the line B S the side of the Dodecahedron inscribed, is the lesse segment of the line G F the side of the cube. And it was proued in the 17. of the thirteenth, that the same side of the cube subtendeth the angle of the pentagon of the Dodecahedron circumscribed: and therefore it maketh the greater segment the side of the Dodecahedron or of the pentagon, by the first corollary of the same. Wherefore it is equal to bothe those segments.

The 14. Probleme.

The 14. Proposition.

In a cube geuen, to inscribe an Icosahedron.

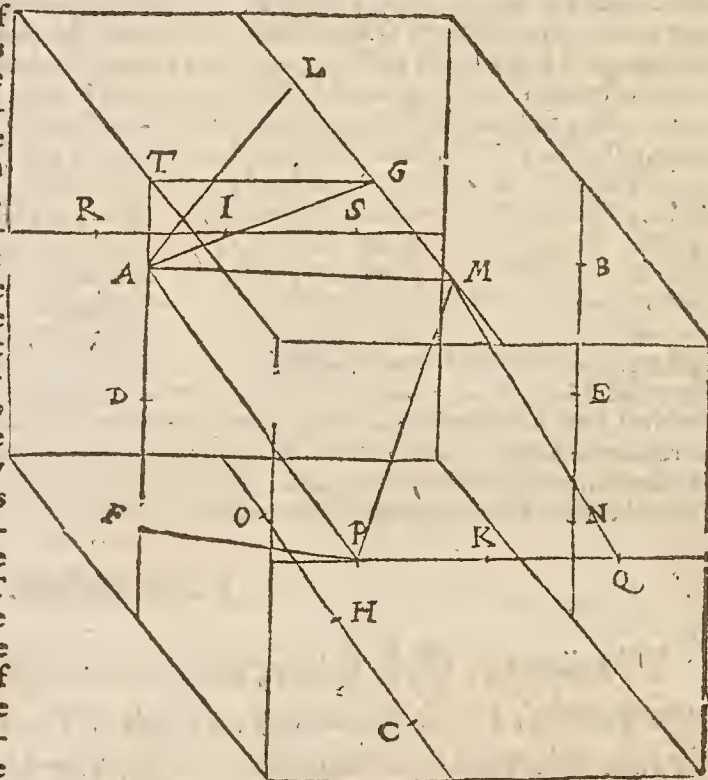
Construction.

Suppose that the cube geuen be A B C, the Centres of whose bases let be the points D, E, G, H, I, K: by whiche

poyntes draw in the bases vnto the other sides parallels not touching the one the other. And deuide the lines drawn from the centres, as the line D T. &c. by an extreme and meane proportion in the poyntes A, F: L, M: N, B: P, Q: R, S: C, O: by the 30. of the sixth: and let the greater segmentes be about the centres. And draw these right lines, A L, A G, A M, and T G. And forasmuch as the lines cut are parallels to the sides of the cube: they shall make right angles the one with the other by the 29. of the first: and forasmuch as they are equal: their sections shall be equal, for that the sections are like by the 2. of the fourteenth. Wherefore the line T G is equal to the line D T, for they are eche, halfe sides of the cube. Wherefore the square of the whole line T G, and of the lesse segment T A, is triple to the square of the line A D the greater segment (by the 4. of the thirteenth). But the line A G containeth in power the lines

Demonstration.

A T & T G, for the angle A T G is a right angle. Wherefore the square of the line A G is triple to the square of the line A D. And forasmuch as the line M G L is erected perpendicularly to the plain passing by the lines A T, & which is parallel to the bases of the cube (by the corollary of the 14. of the eleventh) therefore the angle A G L is a right angle. But the line L G is equal to the line A D, for they are the greater segments of equal lines: Wherefore the line A G (which is in power triple to the line A D) is in power triple to the line L G. Wherefore adding vnto the same square of the line A G, the square of the line L G, the square of the line A L, which (by the 47. of the first) containeth in power the two lines A G and L G, shall be quadruple to the line A D or L G. Wherefore the line A L is double to the line A D (by the 20. of the sixth:) and therefore is equal to the line A F, or to the line L M. And by the same reason may we proue that euery one of the other lines which couple the next sections of the lines cut, as the lines A M, P F, P M, M Q, and the rest are equal. Wherefore the triangles A L M, A P F, A M P, P M Q and the rest such like, are equal, equiangle, and equilateral, by the 4. and eighth of the first. And forasmuch as vpon euery one of the lines cut of the cube are set two triangles, as the triangles A L M, and B L M, there shall be made 12. triangles. And forasmuch as vnder euery one of the 8. angles of the cube, are subtended the other 8. triangles, as the triangle A M P. &c. of 12. and 8. triangles, shall be produced 20. triangles equal and equilateral containing the solide of an Icosahedron, by the 25. definition of the eleventh, which shall be inscribed in the cube geuen A B C by the first definition of this booke. The inuention of the demonstration of this dependeth of the ground of the former. Wherefore in a cube geuen, we haue described an Icosahedron: which was required to be done.



✱ First Corollary.

The diameter of a sphere which containeth an Icosahedron, containeth two sides, namely the side of the Icosahedron, and the side of the cube which containeth the Icosahedron. For if we drawe the line A B, it shall make the angles at the poynt A right angles: for that it is a parallel to the sides of the cube: wherefore the line which coupleth the opposite angles of the Icosahedron, as the poynts F and B, containeth in power the line A B (the side of the cube) and the line A F (the side of the Icosahedron) by the 47. of the first. Which line F B is equal to the diameter of the sphere, which containeth the Icosahedron, by the demonstration of the 18. of the thirteenth.

✱ Second Corollary.

The six opposite sides of the Icosahedron devided into two equal partes: their sections are coupled by three equal right lines, cutting the one the other into two equal partes, and perpendicularly in the centre of the sphere which containeth the Icosahedron. For those three lines are the three lines which couple the centres of the bases of the cube, which do in suche sort in the centre of the cube, cut the one the other, by the corollary of the third of this booke, and therefore are equal to the sides of the cube. But right lines drawne from the centre of the cube to the angles of the Icosahedron, every one of them shall subtend the halfe side of the cube, and the halfe side of the Icosahedron (which halfe sides containe a right angle) wherefore those lines are equal. Wherby it is manifest that the foresaid centre is the centre of the sphere which containeth the Icosahedron.

✱ Third Corollary.

The side of a cube devided by an extreme and meane proportion, maketh the greater segment the side of an Icosahedron described in it. For the halfe side of the cube maketh the halfe of the side of the Icosahedron the greater segment: wherefore also the whole side of the cube, maketh the whole side of the Icosahedron the greater segment by the 15. of the fiftie, for the sections are like by the 2. of the fourteenth.

¶ Fourth Corollary.

The sides and bases of the Icosahedron, which are opposite the one to the other, are parallels. Forasmuch as every one of the opposite sides of the Icosahedron, may be in the parallel lines of the cube, namely, in those parallels which are opposite in the cube: and the triangles which are made of parallel lines, are parallels, by the 15. of the eleventh: therefore the opposite triangles of the Icosahedron, as also the sides, are parallels the one to the other.

¶ The 15. Probleme.

The 15. Proposition.

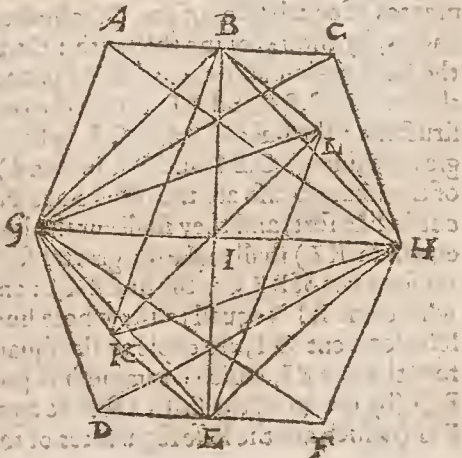
In an Icosahedron geuen, to inscribe an Octohedron.



Suppose that the Icosahedron geuen be A C D F: and by the former second Corollary, let there be take the three right lines which cut the one the other into two equall partes perpendicularly, and which couple the sections into two equall partes of the sides of the Icosahedron: which let be B E, G H, and K L, cutting the one the other in the point I. And drawe these right lines B G, G E, E H, and H B. And forasmuch as the angles at the point I are (by construction) right angles, and are contained vnder equall lines: the bases G B and H E shall make a square, by the 4. of the first. Likewise vnto those bases shall be equall the lines drawn from the pointes K and L, to every one of the pointes B, G, E, H: And therefore the triangles which make the pyramis B G E H K, shall be equall and equilater: And by the same reason shall the rest of the triangles which make the other pyramis B G E H L vpon the same base B G E H, be equall and equilater. Wherefore B G E H K L shall be an Octohedron (by the 23. definition of the eleventh): And shall be inscribed in the Icosahedron, by the first definition of this booke. Wherefore in an Icosahedron geuen, is inscribed an Octohedron: which was required to be done.

Construction.

Demonstration.



¶ The

The fiftenth Booke

¶ The 16. Probleme.

The 16. Proposition.

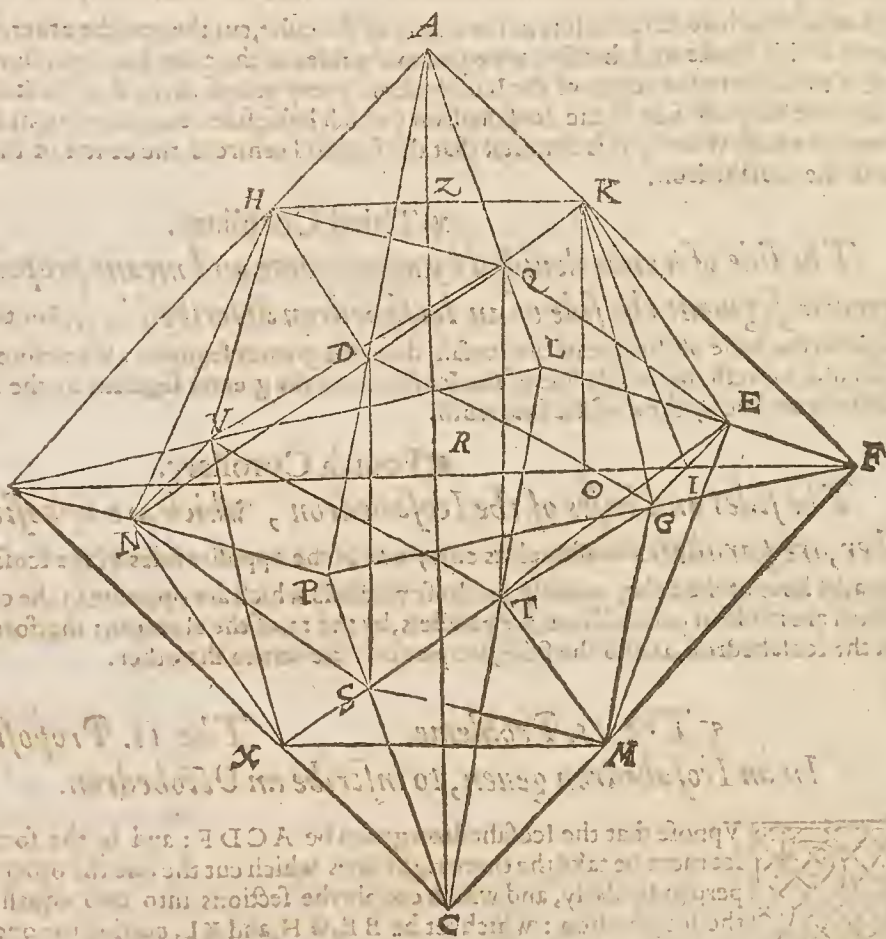
In an Octohedron geuen, to inscribe an Icosahedron.

Construction.



Demonstration.

Et there be taken an Octohedron, whose 6. angles, let be A, B, C, F, P, L. And draw the lines A C, B F, P L, cutting the one the other perpendicularly in the point R (by the 2. Corollary of the 14. of the thirteenth). And let euery one of the 12. sides of the Octohedron be diuided by an extreme and meane proportion, in the pointes H, X, M, K, D, S, N, G, V, E, Q, T. And let the greater segmentes be the lines B H, B X, F M, F K, A D, A Q, C S, C T, P N, P G, L V, L E: And draw these lines H K, X M, G E, N V, D S, Q T. Now forasmuch as in the triangle A B F, the sides are cut proportionally, namely, as the line B H is to the line H A, so is the line F K to the line K A (by the 2. of the fourteenth): therefore the line H K shall be a parallel to the line B F (by the 2. of the sixth). And forasmuch as the line A C cutteth the line H K in the point Z, and the line Z K is a parallel vnto the line R F, the line R A shall be cut by an extreme and meane proportion in the point Z: by the 2. of the sixth: namely, shall be cut like vnto the line F A: and the greater segmēt thereof shall be the line Z R. Vnto the line Z K put the line R O equall, by the 3. of the first: and drawe the line K O: now then, the line K O shall be equall to the line Z R, by the 33. of the first. Draw the lines K G, K E, and K L. And forasmuch as the triangles A R F, and A Z K, are equiangle (by the 6. of the sixth), the sides A Z and Z K, shall be equall the one to the other, by the 4. of the sixth, for the sides A R and R F, are equall. Wherefore the line Z K shall be the lesse segment of the line R A. But if the greater segment R Z be diuided by an extreme & meane proportion, the greater segment thereof shall be the line Z K, which was the lesse segment of the whole line R A, by the 5. of the thirteenth. And forasmuch as the two lines F E and F G, are equall to the two lines A H and A K, namely, ech are lesse segmentes of equall sides of the Octohedron, and the angles H A K and E F G are equall, namely, are right angles, by the 14. of the thirteenth: the bases H K and G F shall be equall, by the 4. of the first: And by the same reason vnto them may be proued equall the lines X M, N V, D S, and Q T. And forasmuch as the lines A C, B F, and P L, do cut the one the other into two equall partes, and perpendicularly, by construction: the lines H K and G E (which subtend angles of triangles like vnto the triangles whose angles the lines A C, B F, and P L subtend) are cut into two equall partes in the pointes Z and I, by the 4. of the sixth, so also are the other lines N V, X M, D S, Q T (which are equall vnto the lines H K & G E) cut in like sort, and they shall cut the lines A C, B F, and P L like. Wherefore the line K O (which is equall to R Z) shall make the greater segment the line R O, which is equall to the line Z K (for the greater segment of R Z was the line Z K): and therefore the line O I shall be the lesse segment, when as the whole line R I is equall to the whole line R Z. Wherefore the squares of the whole line K O, and of the lesse segment O I, are triple to the square of the greater segment R O, by the 4. of the thirteenth. Wherefore the line K I, which containeth in power the two lines K O and O I, is in power triple to the line R O (by the 47. of the first): for the angle K O I is a right angle. And forasmuch as the lines F E and F G (which are the lesse segmentes of the sides of the Octohedron) are equall: and the line F K is common



mon to them both : and the angles KFG and KFE (of the triangles of the Octohedron) are equall : the bases KG and KE shall (by the 4. of the first) be equall : and therefore the angles KIE and KIG which they subtend, are equall (by the 8. of the first) : Wherefore they are right angles, by the 13. of the first. Wherefore the right line KE (which containeth in power the two lines KI and IE , by the 47. of the first) is in power quadruple to the line RO (or IE) : for the line KI is proued to be in power triple to the same line RO : But the line GE is double to the line IE : Wherefore the line GE is also in power quadruple to the line IE (by the 20. of the sixth) . Wherefore the two lines KE and GE are equall. And by the same reason, may the rest of the lines, namely, HK , HN , NV , VX , XS , and the other lines which couple the sections of the sides of the Octohedron be proued equall to the same lines KE and GE . Wherefore the triangles described of them, namely, GFK , GKD , GDS , GSM , GME , shall be equall and equilateral, by the 8. of the first, making a solide angle at the point G : which is therefore the angle of an Icosahedron, by the 16. of the thirteenth, and is set in the section G of the side PF . And by the same reason may be proued, that the rest of the eleuen solide angles of the Icosahedron, are set in the sections of euery one of the sides of the Octohedron, namely, in the pointes E , N , V , H , K , M , X , D , S , Q , T . Wherefore there are 12. angles of the Icosahedron. Moreouer, forasmuch as euery one of the bases of the Octohedron, do eche containe triangles of the Icosahedron, as in the pyramis ABC , FP (which is the halfe of the Octohedron) the triangle FCP receaueth in the sections of his sides the triangle GMS : and the triangle CPB containeth the triangle NXS : and the triangle BAP containeth the triangle HND : and moreouer the triangle APF containeth the triangle KDG , and the same may be proued in the opposite pyramis $ABCF$: Wherefore there shall be eight triangles. And forasmuch as besides these triangles, to euery one of the solide angles of the Octohedron are subtended two triangles, as the triangles KEG and MEG , to the angle F : and the triangles HNH and XNV , to the angle B : also the triangles NDS and GDS , to the angle P : likewise the triangles DHK and QHK , to the angle A : Moreouer the triangles EQT and VQT , to the angle L : and finally the triangles SXM and TXM , to the angle C : these 12. triangles being added to the 8. former triangles, shall produce 20. triangles equall and equilateral coupled together : which shall make an Icosahedron, by the 25. definition of the eleuenth : and it shall be inscribed in the Octohedron geuen $ABCFPL$, by the first definition of this booke : for the 12. angles thereof, are set in 12. like sections of the sides of the Octohedron. Wherefore in an Octohedron geuen, is inscribed an Icosahedron.

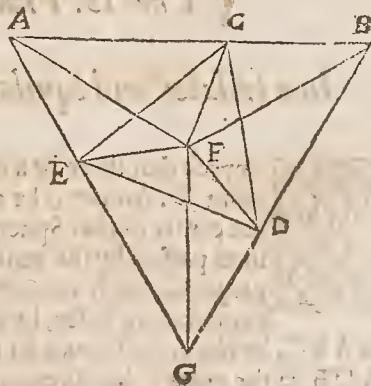
¶ First Corollary.

The side of an equilateral triangle being diuided by an extreme and meane proportion : a right line subtending within the triangle, the angle which is contained vnder the greater segment and the lesse : is in power duple to the lesse segment of the same side. For the line KE , which subtendeth the angle KFE of the triangle $AEFL$, which angle KFE is contained vnder the two segmentes KF & FE , was proued equall to the line HK , which containeth in power the two lesse segmentes HA and AK , by the 47. of the first, for the angle HAK is a right angle. Wherefore the line KE or HK , is in power duple to the line AK .

* Second Corollary.

The bases of the Icosahedron are concentricall (that is, haue one and the selfe same centre) with the bases of the Octohedron which containeth it.

For suppose that ABG be the base of an Octohedron containing ECD the base of an Icosahedron : and let the centre of the base ABG be the point F . And drawe these right lines FA , FB , FC , and FE . Now then the two lines FA and AE shall be equall to the two lines FB and BC : for they are lines drawn from the centre, and are also lesse segmentes : and they containe the halfe of equall angles. Wherefore (by the 4. of the first) the bases FC and FE are equall : and by the same reason vnto them shall be equall the other line FD . Wherefore making the centre the point F : and the space FE describe a circle and it shall be circumscribed about the triangle CED : and so shall the point F the centre of the base of the Octohedron be the centre of CE the base of the Icosahedron.



My.j.

¶ The

The fifteenth Booke

¶ The 17. Probleme.

The 17. Proposition.

In an Octohedron geuen, to inscribe a Dodecahedron.

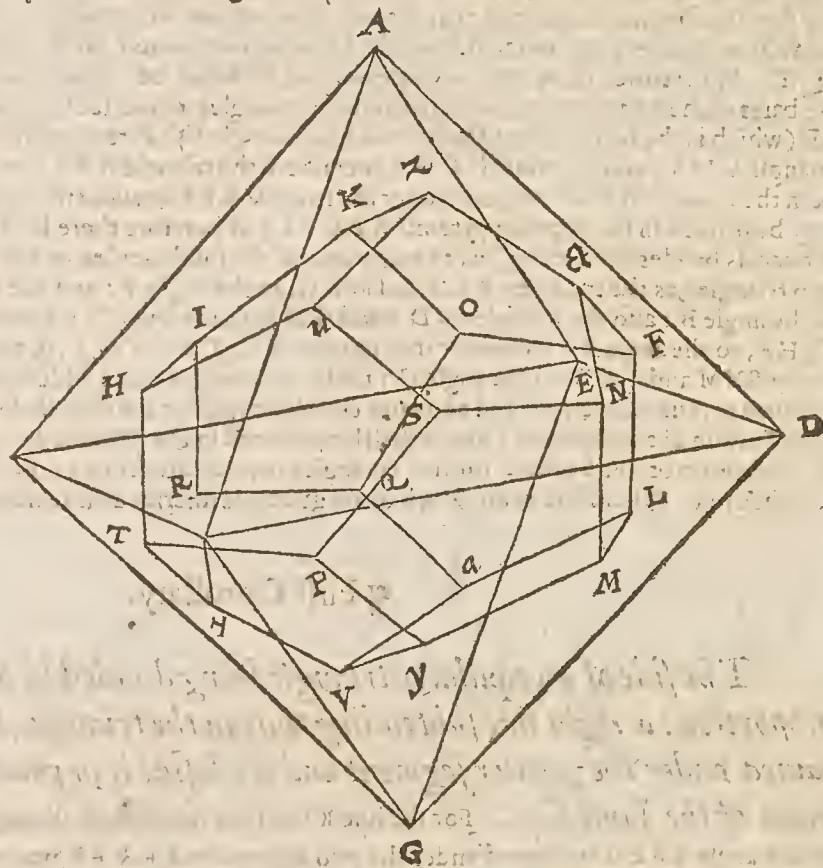
Construction.



Vppose that the Octohedron geuen be $ABGDEC$: whose 12. sides let be cut by an extreme and meane proportion, as in the former Proposition. It was manifest that of the right lines which couple these sections, are made 20. triangles, of which 8. are concentricall with the bases of the Octohedron, by the second Corollary of the former Proposition. If therefore in euery one of the centres of the 20. triangles be inscribed (by the 5. of this booke) euery one of the 12. angles of the Dodecahedron, we shall finde, that 8. an-

Demonstration.

gles of the Dodecahedron are set in the 8. centres of the bases of the Octohedron: namely, these angles I, u, α, O, M, a, P , and X : and of the other 12. solide angles there are two in the centres of the two triangles which haue one side common vnder euery one of the solide angles of the Octohedron: namely, vnder the solide angle A , the two solide angles K, Z : vnder the solide angle B , the two solide angles H, T : vnder the solide angle G , the two solide angles Y, V : vnder the solide angle D , the two solide angles F, L : vnder the solide angle E , the two solide angles S, N : vnder the solide angle C , the two solide angles Q, R : and forasmuch as in the Octohedron are sixe solide angles, vnder them shall be subtended 12. solide angles of the Dodecahedron: and so are made 20. solide angles composed of 12. equall and equilater superficiall pentagons (as it was manifest, by the 5. of this booke) which therefore containe a Dodecahedron (by the 14. definition of the eleuenth). And it is inscribed in the Octohedron (by the 1. definition of this booke): for that euery one of the bases of the Octohedron do receaue angles therof. Wherefore in an Octohedron geuen, is inscribed a Dodecahedron.



¶ The 18. Probleme.

The 18. Proposition.

In a trilater and equilater Pyramis, to inscribe a Cube.

Construction.



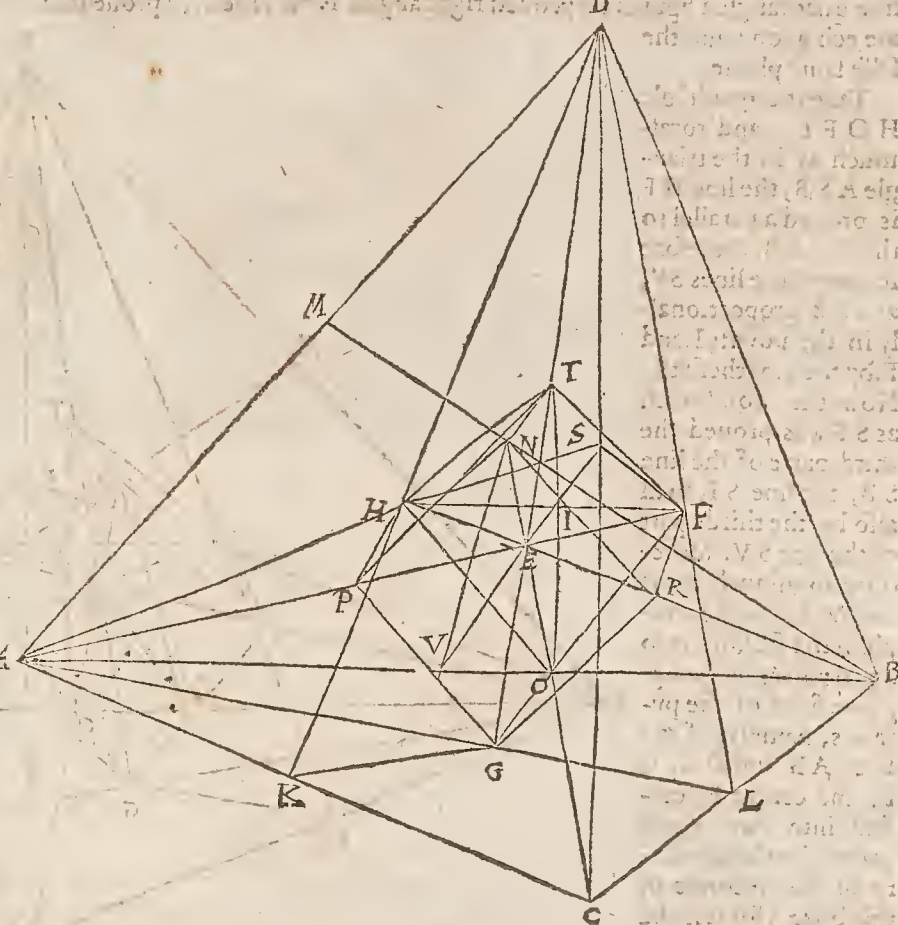
Vppose that there be a trilater equilater Pyramis, whose base let be ABC , and toppe the point D . And let it be comprehended in a Sphere, by the 13. of the thirteenth. And let the centre of that Sphere be the point E . And from the solide angles A, B, C, D , draw right lines passing by the centre E , vnto the opposite bases of the pyramis, and they shall fall perpendicularly vpon the bases, and shall also fall vpon the centres of the circles which containe the bases, by the Corollary of the 13. of the thirteenth. Let the centre of the triangle ABC , be the point G , and let the centre of the triangle ADC be the point H , and of the triangle ADB let the point N be the centre, and finally, let the point F be the centre of the other triangle DBC . And let the right lines falling vpon those centres be DEG, BEH, CEN , & AEF . And by those centres G, H, N, F , let there be drawn from the angles to the opposite sides these right lines, $AGL, DHK,$

DH, KBNM, and DFL, which shall fall perpendicularly vpon the sides BC, CA, AD, and CB, by the Corollary of the 12. of the thirteenth, and therefore they shall cut them into two equall partes in the pointes K, L, M, by the 3. of the third. Agayne let the lines which wer drawn from the solide angles to the opposite bases be divided into two equal partes, namely, the line DG in the point T, the line CN in the point O, the line AF in the point P, and the line BH in the point R: and drawe the lines HT, FT, HO, and FO. Now forasmuch as the lines GK, and GL, which are drawn from the centre of one and the self same triangle ABC to the sides, are equall, and the lines DK and DL are equall, for they are the perpendiculars of equal & like triangles: and the line DG is common to the. Wherefore, by the 8. of the first, the angles KDG & LDG, are equall. And forasmuch as the lines HD & DF are drawn from the centre of equal circles which containe the equal triangles ADC & DBC, therefore they are equal, & the line DT is comon to the both, and they containe equal angles, as before hath bene proued. Wherefore the bases HT and FT are equal by the 4. of the first. And by the same reason if we drawe the lines CF and CH, may we proue that the other lines HO and FO, are equal to the same lines HT and FT, and also the one to the other. Wherefore also after the same maner may be proued that the rest of the lines, which couple the centres of the triangles and the sections of the perpendiculars into two equal partes, as the lines NP, GR, GP, RN: NT, PH, GO, and RF, are equal. And forasmuch as from euery one of the centres of the bases are drawne thre right lines to the sections into two equal parts of the perpendiculars, and there are foure centres, it followeth, that these equal right lines so drawne, are twelue in number, of which euery three and three make a solide angle in the foure centres of the bases, and in the foure sections into two equal partes of the perpendiculars: wherefore that solide hath 8. angles, contained vnder 12. equal sides, which make fixe quadrangled figures, namely, HOF T, PGRN, PHOG, GOF R, FRNT, and TNPH. Now let vs proue that those quadrangled figures are rectangle.

Forasmuch as vpon DC the common base of the triangles ADC and BDC falleth the perpendiculars AS and BS, which are drawne by the centres H and F: either of these lines HS and SF shalbe the third part of either of these lines AS and SB: for the line AH is duple to the line HS, and deuideth the base DC into two equal partes by the corollary of the 12. of the thirteenth. Wherefore in the triangle ABS, the sides AS and BS are cut proportionally in the poynts H and F: and therefore the line HF is a parallel to the side AB, by the 2. of the sixth. Wherefore the triangles ASB and HSF are equiangle, by the 6. of the sixth. Wherefore the base HF shalbe the third part of the base AB, by the 4. of the sixth. We may also proue that the line TO is the third part of the line DC, for the lines EC and ED, which are drawne from the centre of the sphere which containeth the pyramis are equal: and the line EN, (which is drawne from the centre to the base) is the third part of the line EC, so also is the line GE the third part of the line ED (by the corollary of the 13. of the thirteenth) for it is the sixth parte of the diameter of the sphere which containeth the pyramis: And the line ON, is the half of the whole line NC wherefore the residue EO is the third part of the line EC, and so also is the line ET the third part of the line ED. Wherefore the line TO in the triangle DEC is a parallel to the line DC, and is a third parte of the same, by the former 2. and 4. of the sixth, as the line HF was proued the third part of the line AB. But AB and DC being sides of the pyramis are equal. Wherefore the lines HF and TO, being the third partes of equal lines, are equal, by the 15. of the sixth. Wherefore by the 8. of the first the angles HTF, and TFO are equal: and by the same reason, the angles opposite vnto them, namely, the angles FOH

YYy.ij.

and



Demonstration.

Produce in the figure the line TF to the point B.

The fiftenth Booke

and $\angle H T$ are equal the one to the other, and also are equal to the said angles $H T F$ and $T F O$: but these foure angles are equal to 4. right angles by the corollary of the 32. of the first: wherefore the angles of the quadrangle $H O F T$ are right angles. And by the same reason may the angles of the other five quadrangled figures be proued right angles. Now resteth to proue that the foresayde quadrangles are ech in one and the selfe same plaine.

Take the quadrangle
H O F T : and foras-
much as in the trian-
gle A S B, the line H F
is proued a parallel to
the line A B, therefore
it cutteth the lines S V,
and S B proportionally
in the poynts I. and
F. by the 2. of the sixth:
Now then forasmuch
as S F was proued the
third parte of the line
S B, the line S I, shall
also be the thirde part
of the line S V. More-
ouer forasmuch as the
line V S, whiche cou-
pleth the sections into
equal partes of the op-
posite sides of the py-
ramis, namely, of the
sides A B and D C, is
by the centre E di-
uided into two equal
partes, by the corollary
of the seconde of
this booke (for it is the
diameter of the octo-

hedron inscribed in the pyramis): therefore the line SI is two third partes of the halfe line SE . And by the same reason, forasmuch as in the triangle DEC the line TO is proued to be a parallel to the side DC , it shall in the selfe same triangle cut the lines CE and SE , proportionally in the poynts O and I by the same 2. of the sixth: but the line EO is proued to be a third parte of the line EC . Wherefore the lynce EI is also a third part of the line ES . Wherefore the residue IS shalbe two third partes of the whole line ES . Wherefore the point I cutteth either of the lines TO and HF . Wherefore the two lines HIF and TIO cutting the one the other, are in one and the selfe same plaine, by the 2. of the eleuenth. And therefore the poyntes H, T, F, O are in one & the selfe same plaine. Wherefore the rectangle figure $HOFT$ being quadrilater and equilater, and in one and the selfe same playne, is a square, by the definition of a square. And by the same reason may the rest of the bases of the solide be proued to be squares equall and plaine or superficial: Now then the solide is comprehended of 6. equal squares (which are contained of 12. equal sides) which squares make 8. solide angles, of which foure are in the centres of the bases of the pyramis, and the other 4. are in the middle sections of the foure perpendiculars. Wherefore the solide $HOFT PGRN$, is a cube by the 21. definition of the eleuenth, and is inscribed in the pyramis, by the first definition of this booke. Wherefore in a trilater equilater pyramis given, is inscribed a cube.

¶ A Corrollary.

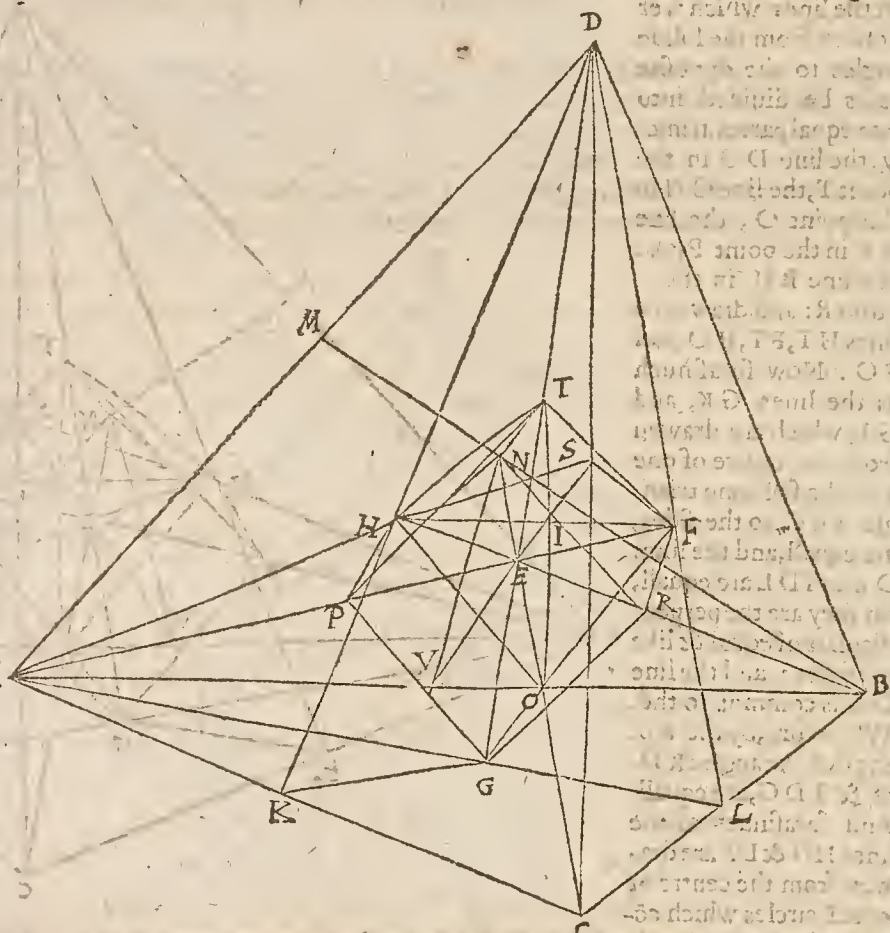
The line which cutteth into two equall partes the opposite sides of the Pyramis, is triple to the side of the cube inscribed in the pyramis, and passeth by the centre of the cube. For the line SEV , whose third part the line SI is, cutteth the opposite sides CD and AB into two equall partes: but the line EI (which is drawne from the centre of the cube to the base) is proued to be a third part of the line ES : wherefore the side of the cube which is double to the line EI shall be a third part of the whole line VS , which is (as hath bene proued) double to the line ES .

The 19. Probleme

The 19. Proposition.

In a trilater equilater Pyramis geuen, to inscribe an Icosahedron.

Suppose





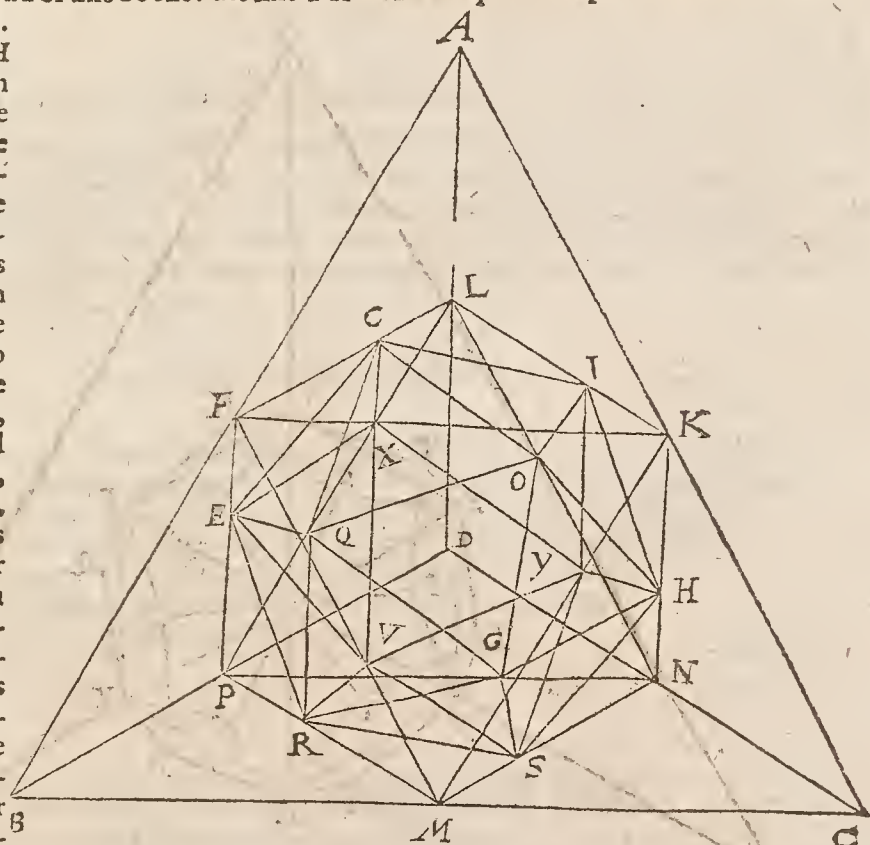
Vppose that the pyramis geuen, be $ABGD$: euery one of whose sides let be diuided into two equall partes in the poyntes E, M, K, L, P, N . And in euery one of the bases of that pyramis, describe the triangles EFP, PMN, NKL , and FMK : which triangles shall be equilater by the 4. of the first; for the sides subtend equall angles of the pyramis, containd vnder the halues of the sides of the same pyramis: wherefore the sides of the said triangles are equall. Let those sides be diuided by an extreame and meane proportion (by the 30. of

Construction.

the sixth) in the poyntes $C, E, Q, R, S, T, H, I, O, V, Y, X$. Now then those sides are cutte into the selfe same proportions, by the 2. of the fourth: and therefore they make the like sectiōs, equall, by the 2. part of the ninth of the sixth. Now I say, that the foresayd poyntes doo receaue the angles of the Icosahedron inscribed in the pyramis $ABGD$. In the foresayd triangles let there agayne be made other triangles by coupling the sectiōs, and let those triangles be TRS, IOH, CEQ , and VXY ; which shall be equilater: for euery one of their sides doo subtend equall angles of equilater triangles, and those sayd equall angles are containd vnder equall sides (namely, vnder the greater segment and the lesse): and therefore the sides which subtend those angles are equall by the 4. of the first. Now let vs proue that at eche of the foresayd poynts, as for example at T , is set the solide angle of an Icosahedron. Forasmuch as the triangles TRS and TQO are equilater and equall, the 4. right lines TR, TS, TQ and TO shall be equall. And forasmuch as $FPNK$ is a square cutting the pyramis $ABGD$ into two equall partes, by the corollay of the second of this booke: the line TH shall be in power duple to the line TN or NH by the 47. of the first.

Demonstration.

For the lines TN or NH are equall, for that by construction they are eche lesse segmentes: and the line RT or TS is in power duple to the same line TN or NH (by the corollary of the 16. of this booke) for it subtendeth the angle of the triangle containd vnder the two segmentes. Wherefore the lines TH, TS, TR, TQ , and TO are equall: and so also are the lines HS, SR, RQ, QO , and OH , which subtend the angles at the poynt T , equall. For the line QR containeth in power the two lines PQ and PR the lesse segmentes, which two lines the line TH also containd in power. And the rest of the lines doo subtend angles (of equilater triangles) containd vnder the greater segment and the lesse. Wherefore the fixe triangles TRS, TSH, TSH, TOQ, TQR are equilater and equall making the solide angle of an Icosahedron at the poynt T , by the 16. of the thirteenth, in the side PN of the triangle PNM . And by the same reason in the other sides of the 4. triangles PNM, NKL, FMK , & LFP (which are inscribed in the bases of the pyramis) which sides are 12. in nūber shal be set 12. angles of the Icosahedro cōtaind vnder 20. equal & equilater triangles of which fower are set in the 4. bases of the pyramis, namely, these fower triangles, TRS, IOH, CEQ, VXY : 4. triangles are vnder 4. angles of the pyramis: that is, the fower triangles CIX, YSH, ERV, TQO : and vnder euery one of the fixe sides of the pyramis are set two triangles, namely, vnder the side, of the triangles THS and THO : vnder the side DB the triangles RQE and RQT : vnder the side DA the triangles COQ , and COI : vnder the side AB , the triangles EXC and EXV : vnder the side BG the triangles SVR and SVY : and vnder the side AG the triangles IYH and IYX . Wherefore the solide being containd vnder 20. equilater and equall triangles shall be an Icosahedron by the 23. diffinition of the eleuenth: and shall be inscribed in the pyramis $ABGD$ by the first diffinition of this booke, for all his angles doo at one time touch the bases of the pyramis. Wherefore in a trilater equilater pyramis geuen, we haue inscribed an Icosahedron.



¶ The 20. Proposition. The 20. Probleme.
In a trilater equilater Pyramis geuen, to inscribe a dodecabedron.

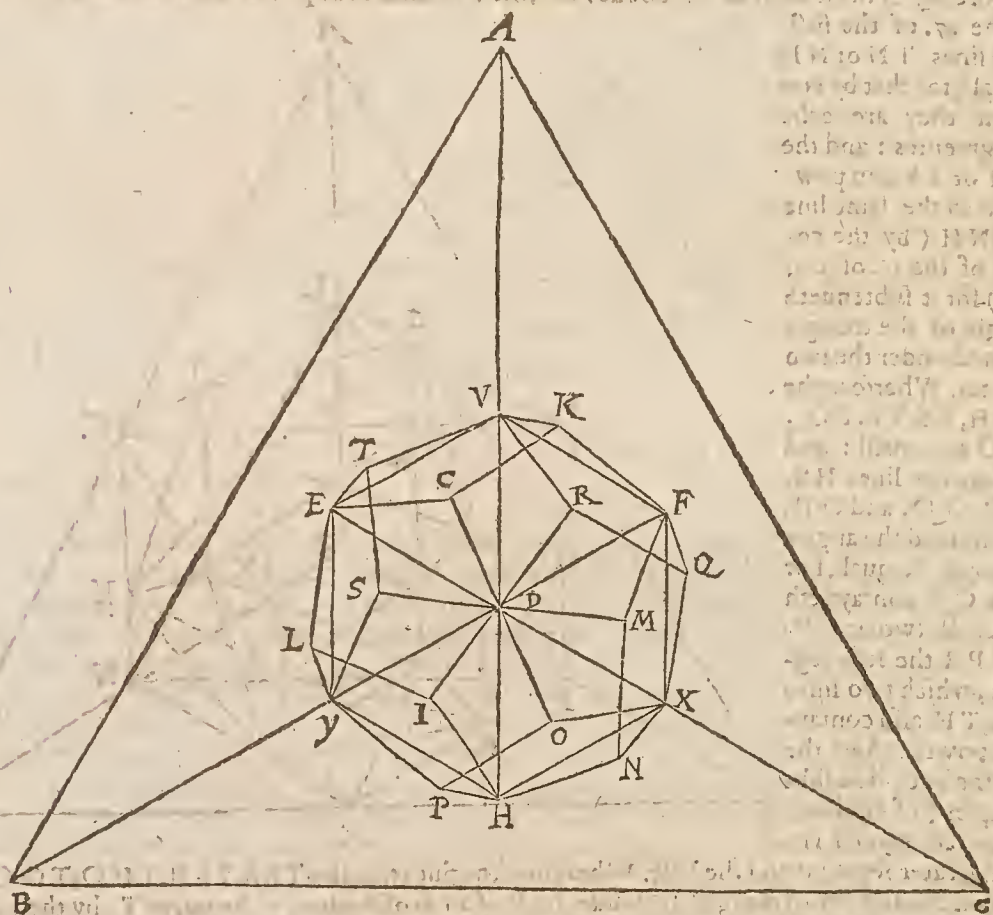
YYy.iii.

Suppose

The fiftenth Booke



Vppose that the pyramis geuen be $ABGD$, eche of whose sides let be cutte into two equall partes: and draw the lines which couple the sections, which being divided by an extreame and meane proportion, and right lines being drawne by the sections, shall receaue 20. triangles making an Icosahedron, as in the former proposition it was manifest. Now then if we take the centres of those triangles, we shall there finde the 20. angles of the dodecahedron inscribed in it by the 5. of this booke. And forasmuch as 4. bases of the foresayd Icosahedron are cōcentricall with the bases of the pyramis, as it was proued in the 2. corollary of the 6. of this booke: there shal be placed 4. angles of the dodecahedron, namely, the 4. angles E, F, H, D , in the 4. centres of the bases: and of the other 16. angles, vnder euery one of the 6. sides of the pyramis are subtended two: namely, vnder the side AD , the angles C, K : vnder the side BD the angles L, I : vnder the side GD the angles M, N : vnder the side AB the angles T, S : vnder the side BG the angles P, O : and vnder the side AG the angles R, Q : so thererest 4. angles, whose true place we will now appoynt. Forasmuch as a cube containd in one and the selfe same sphere with a dodecahedron, is inscribed in the same dodecahedron, as it was manifest by the 17. of the thirteenth, and 8. of this booke: it followeth that a cube and a dodecahedron circumscribed about it, are containd in one and the selfe same bodies, for that their angles concur in one and the selfe same poyntes. And it was proued in the 18. of this booke, that 4. angles of the cube inscribed in the pyramis are set in the middle sections of the



perpendiculars which are drawne from the solide angles of the pyramis to the opposite bases: wherefore the other 4. angles of the dodecahedron are also, as the angles of the cube, set in those middle sections of the perpendiculars. Namely, the angle V is set in the midst of the perpendicular AH : the angle Y in the midst of the perpendicular BF : the angle X in the midst of the perpendicular GE : and lastly the angle D in the midst of the perpendicular D which is drawne from the toppe of the pyramis to the opposite base. Wherefore those 4. angles of the dodecahedron may be sayd to be directly vnder the solide angles of the pyramis, or they may be sayd to be set at the perpendiculars. Wherefore the dodecahedron after this maner set, is inscribed in the pyramis geuen (by the first diffinition of this booke) for that vpo euery one of the bases of the pyramis are set an angle of the dodecahedron inscribed. Wherefore in a trilater equilater pyramis is inscribed a dodecahedron.

The 21. Probleme.

The 21. Proposition.

In euery one of the regular solides to inscribe a Sphere.



IN the 13. of the thirteenth and the other 4. propositions following, it was declared that the 5. regular solides, are so containd in a sphere, that right lines drawne from the centre of the sphere or of the solide inscribed, to euery one of the angles of the solide inscribed, are equall. Which right lines therefore make pyramids, whose toppes are the centre of the sphere, or of the solide, and the bases are euery one of the bases of those solides. And forasmuch as those bases are in euery solide equall and like the one to the other, and described in equall circles: those circles shall cutte the sphere: for the angles which touch the circumference of the circle, touch also the superficies of the sphere. Wherefore perpendiculars drawne from the centre of the sphere to the bases, or to the playne superficieses of the equall circles, are equall, by the corollary of the assumpt of the 16. of the twelfth. Wherefore making the centre the centre of the sphere which containeth the solide, and the space some one of the equall perpendiculars, describe a sphere, and it shall touch euery one of the bases of that solide: neither shall the superficies of the sphere passe beyond those bases: when as those perpendiculars are the left lines which are drawne from the centre to the bases, by the 3. corollary of the same assumpt. Wherefore we haue in euery one of the regular bodies inscribed a sphere: which regular bodies are in number onely fve, by the corollary of the 18. of the thirteenth.

This proposition on Campane hath, & is the last also in order of the 15. booke with him.

A Corollary.

The regular figures inscribed in spheres, and also the spheres circumscribed about them, or containning them, haue one and the selfe same centre. Namely, their pyramids, the angles of whose bases touch the superficies of the sphere, doo from those angles cause equall right lines to be drawne to one and the selfe same poynt, making the toppes of the pyramids in the same poynt: and therefore they make the centres of the spheres in the selfe same toppes: when as the right lines drawne from those angles to the crooked superficies, wherein are set the angles of the bases of the pyramids, are equall.

An aduertisement of *Flussas*

Of these solides, onely the Octohedron receaueth the other solides inscribed one within another. For the Octohedron containeth the Icosahedron inscribed in it: and the same Icosahedron containeth the Dodecahedron inscribed in the same Icosahedron: and the same dodecahedron containeth the cube inscribed in the same Octohedron, and finally the same cube circumscribeth the Pyramis inscribed in the sayd Octohedron. But this happeneth not in the other solides.

(s.)

The ende of the fuetenth Booke

of Euclides Elementes after

Campane and Flussas.



¶ The sixtenth booke of the Elementes of Geometrie

added by *Flussas*.

The argu-
ment of the
16. booke.



IN the former fuetenth booke hath bene taught how to inscribe the fine regular solides one with in an other. Now semeth to rest, to cōpare those solids so inscribed, one to an other, and to set forth their passions and proprieties: which thing, *Flussas* considering, in this sixtēth booke added by him, hath excellently well and most conningly performed. For which vndoubtedly he hath of all them which haue a loue to the Mathematical, deserued much prayse and commendacion: both for the great trauailes and paynes (which it is most likely) he hath taken in inuenting such straunge and wonderfull propositions with their demonstrations, in this booke containd, as also for participating and communicating abroad the same to others. Which booke also, that the reader should want nothing conducing to the perfection of Euclides Elementes: I haue with some trauaile translated, & for the worthines thereof haue added it, as a sixtenth booke to the 15. bookes of Euclide. Vouchsafe therefore gentle reader diligently to read and peyse it, for in it shall you finde not onely matter strange and delectable, but also occasion of inuention of greater things pertayning to the natures of the fine regular solides.

¶ The 1. Proposition.

A Dodecahedron, and a cube inscribed in it, and a Pyramis inscribed in the same cube, are contained in one and the selfe same sphere.



Or the angles of the pyramis are set in the angles of the cube wherein it is inscribed (by the first of the fuetenth: and all the angles of the cube are set in the angles of the dodecahedron circumscribed about it (by the 8. of the fuetenth): And all the angles of the Dodecahedron, are set in the superficies of the sphere, by the 17. of the thirteenth. Wherefore those three solides inscribed one within an other, are contained in one and the selfe same sphere, by the first diffinition of the fuetenth. A dodecahedron therfore and a cube inscribed in it, and a pyramis inscribed in the same cube, are contained in one and the selfe same sphere.

¶ A Corollary.

These three solides likewise are set in one and the selfe same Icosahedron, or Octohedron, or Pyramis. For they are inscribed in one and the same Icosahedron, by the 5. 11. & 12. of the fuetenth: and they are inscribed in one and the selfe same Octohedron, by the 4. 6. and 16. of the same: lastly they are inscribed in one and the selfe same pyramis, by the first, 18. and 19. of the same. For the angles of all these solides are set in the centres of the bases of the circumscribed Icosahedron, or octohedron, or pyramis.

¶ The 2. Proposition.

The proportion of a Dodecahedron circumscribed about a cube, to a Dodecahedron

hedron inſcribed in the ſame cube, is triple to an extreme & meane proportion.



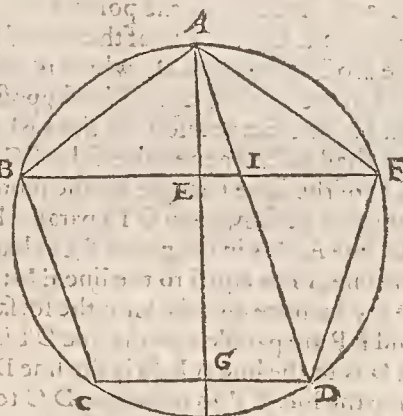
Forasmuch as in the 2. corollary of the 13. of the ſiuententh, it was proued, that the ſide of a Dodecahedron inſcribed in a cube, is the leſſe ſegment of the ſide of that cube deuided by an extreme and meane proportion: and the ſide of the dodecahedron circumscribed about the ſame cube, is the greater ſegment of the ſide of the ſame cube (which thing alſo was taught in the 13. of the ſiuententh) the ſide of the Dodecahedron circumscribed, ſhalbe to the ſide of the Dodecahedron inſcribed, as the greater ſegment of a right line deuided by an extreme and meane proportion, is to the leſſe ſegment of the ſame, which proportion is called an extreme and meane proportion, by the definition, and by the 30. of ſixth. But the proportion of like ſolide prolihedrons, is triple to the proportion of the ſides of like proportion, by the corollary of the 17. of the twelueth. Wherefore the proportion of the Dodecahedron circumscribed about the cube, is to the dodecahedron inſcribed in the ſame cube, in triple proportion of the ſides ioyned together by an extreme and meane proportion. The proportion therefore of a Dodecahedron circumscribed about a cube to a dodecahedron inſcribed in the ſame cube, is triple to an extreme and meane proportion.

The 3. Propoſition.

In euery equiangle, and equilater Pentagon, a perpendicular drawne from one of the angles to the baſe, is deuided by an extreme and meane proportion by a right line ſubtending the ſame angle.



Suppoſe that A B C D E be an equiangle and equilater pentagon: and from one of the angles namely, from A, let there be drawne to the baſe C D a perpendicular A G: and let the line B F ſubtend the angle B A F, which line B F let the line A D cut in the poynt I. Then I ſay that the line B F, cutteth the line A G by an extreme and meane proportion. For forasmuch as the angles G A F and G A B are equal by the 27. of the third, and the angles A B F and A F B, are equal by the 5. of the firſt: therefore the angles remaining at the poynt E, of the triangles A E B and A E F are equal: for that they are the residues of two right angles by the corollary of the 32. of the firſt. But the angle E G C, is by conſtruction a right angle: wherefore the lines B F & C D are parallels by the 28. of the firſt. Wherefore as the line D I is to the line I A, ſo is the line G E to the line E A, by the 2. of the ſixth. But the line D A, is in the point I deuided by an extreme and meane proportion, by the 8. of the thirteenth. Wherefore the line G A is in the poynt E, deuided by an extreme and meane proportion (by the 2. of the fourteenth). Wherefore in euery equiangle and equilater pentagon, a perpendicular drawne from one of the angles to the baſe, is deuided by an extreme and meane proportion by a right line ſubtending the ſame angle.



Conſtruction.

Demonſtration.

A Corollary.

The line which ſubtendeth the angle of a pentagon, is a parallel to the ſide oppoſite vnto the angle. As it was manifeſt in the lines B F and C D.

The 4. Propoſition.

If from the angles of the baſe of a * Pyramis, be drawne to the oppoſite ſides, right lines cutting the ſayd ſides by an extreme and meane proportion: they ſhall containe the baſe of the Icoſahedron inſcribed in the Pyramis, which baſe ſhalbe inſcribed in an equilater triangle, whoſe angles cut the ſides of the baſe of the Pyramis by an extreme and meane proportion.

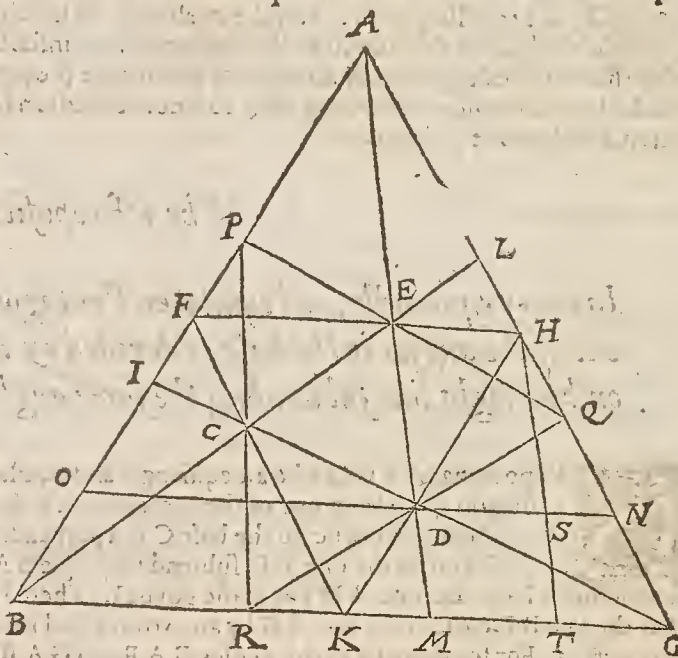
* By a Pyramis vnderſtand a Tetrahedron throughout all this booke.

The sixteenth Booke of the

Construction.

Demonstration.

Suppose that ABG be the base of a pyramis, in which let be inscribed an equilateral triangle FKH , which is done by deviding the sides into two equal partes. And in this triangle let there be inscribed the base of the Icosahedron inscribed in the pyramis: which is described by deviding the sides FK, KH, HF , by an extreme & meane proportio in the poynts C, D, E , by the 19. of the first. Again let the sides of the pyramis, namely, AB, BG , and GA be deviding by an extreme and meane proportion in the poynts I, M, L , by the 30. of the sixth. And drawe these right lines AM, BL, GI . Then I say that those lines describe the triangle CDE of the Icosahedron. For forasmuch as the lines BG and FH are parallels, by the 2. of the sixth: by the point D let the line ODN be drawne parallel to either of the lines BG & FH . Wherefore the triangle HDN shalbe like to the triangle HKG , by the corollary of the 2. of the sixth. Wherefore either of these lines DN and NH shall be equal to the line DH , the greater segment of the line KH or FH . And forasmuch as the line FO is a parallel to the line HK , and the line OD to the line FH : the line OD shall be equal to the whole line FH in the parallelogramme $FODH$, by the 34. of the first. Wherefore as the whole line FH is to the greater segment FE , so shall the lines equal to them be, namely, the line OD to the line DN , by the 7. of the fifth. Wherefore the line ON is deviding by an extreme and meane proportion in the poynt D , by the 2. of the fourteenth. But the triangles AOD, AFE , and ABM , are like the one to the other, and so also are the triangles ADN, AEH , and AMG , by the corollary of the second of the sixth: Wherefore as FE is to EH , so is OD to DN , and BM to MG . Wherefore the line AM cutting the lines FH and ON , lyke vnto the line BG in the pointes E, D, M , describeth ED the side of the triangle of the Icosahedron ECD , which is described in the sections E, C, D , by suppositio. And by the same reason the lines BL and GI shall describe the other sides EC and CD of the same triangle. By the point E , let there be drawne to G a parallel line PEQ . Now forasmuch as the lines BM and FE are parallels, the line AM is in the poynt E , cut like to the line AB in the poynt F , by the 2. of the sixth. Wherefore the line AE is equal to the line EM : and vnto the line EM also are equal, either of the lines GD and DI : which are cut like vnto the forsaide lines. Again forasmuche as in the triangle ADI the lines DI and EP are parallels, as the line DI is to the line EP , so is the line AD to the line AE : but as the line AD is to the line AE , so is the line DG to the line EQ by the 2. of the sixth: wherefore as the line DI is to the line EP , so is the line DG to the line EQ : and alternately as the line DI is to the line DG , so is the line EP to the line EQ : but the lines DI and IG are equal: wherefore also the lines EP and EQ are equal. And forasmuch as the line AH is equal to the line FH , whose greater segmet is the line HN : therefore the whole line AN , is deviding by an extreme and meane proportion in the poynt H , by the 3. of the thirteenth. But as the line AN is to the line AH , so is the line AD to the line AE , by the 2. of sixth (for the lines FH and ON are parallels:) and againe as the line AD is to the line AE , so (by the same) is the line AG to the line AQ , and the line AI to the line AP : for the lines PQ , and GI are parallels: Wherefore the lines AG and AI are deviding by an extreme and meane proportion in the pointes Q & P : & the line AQ shalbe the greater segmet of the line AG or AB . And forasmuch as the whole line AG is to the greater segment AQ , as the greater segment AI is to the residue AP : the line AP shalbe the lesse segment of the whole line AB or AG . Wherefore the line PEQ (which by the poynt E passeth parallelwise to the line GI) cutteth the lines AG and BA by an extreme and meane proportion in the poynts Q and P . And by the same reason the line PR (which by the poynt C , passeth parallelwise to the line AM) shall fall vpon the sections P and R : so also shal the line RQ (which by the poynt D passeth parallelwise to the line BL) fall vpon the sections RQ . Wherefore either of the lines PE and EQ shalbe equal to the line CD , in the parallelogrammes PD , and QC , by the 34. of the first. And forasmuch as the lines PE and EQ are equal, the lines PC, CR, RD and DQ shalbe likewise equal. Wherefore the triangle PRQ is equilateral, and cutteth the sides of the base of the pyramis in the poyntes P, Q, R , by an extreme and meane proportion. And in it is inscribed the base ECD of the Icosahedron contained in the foresayd pyramis. If therefore from the angles of the base of a pyramis, be drawne to the opposite sides, right lines cutting the sayde sides by an extreme and meane proportion: they shall containe the base of the Icosahedron inscribed in the pyramis, which base shall be inscribed in an equilateral triangle, whose angles cut the sides of the base of the pyramis by an extreme & meane proportion.



¶ A Corollary.

The ſide of an Icoſahedron inſcribed in an Octohedron, is the greater ſegment of the line, which being drawn from the angle of the baſe of the Octohedron cutteth the oppoſite ſide by an extreame and meane proportion.

For, by the 16. of the ſiuenth, FKH is the baſe of the Octohedron, which containeth the baſe of the Icoſahedron CDE : vnto which triangle FKH, the triangle HKG is equall, as hath bene proued. By the point H draw vnto the line ME a parallel line HT, cutting the line DN in the point S. Wherefore ES, DT, and ET, are parallelogrammes : and therefore the lines EH and MT are equall : and the lines EM and HT are like cut in the pointes D and S, by the 34. of the firſt. Wherefore the greater ſegment of the line HT is the line HS, which is equall to ED the ſide of the Icoſahedron. But (by the 2. of the ſixth) the line TK is cut like to the line HK by the parallel DM. And therefore (by the 2. of the fourteenth) it is diuided by an extreme and meane proportion. But the line TM is equall to the line EH. Wherefore alſo the line TK is equall to the line EF or DH. Wherefore the reſidues EH and TG are equall. For the whole lines FH and KG are equall. Wherefore KG the ſide of the triangle HKG is in the point T diuided by an extreme and meane proportion in the point T, by the right line HT, and the greater ſegment thereof is the line ED the ſide of the Icoſahedron inſcribed in the Octohedron, whoſe baſe is the triangle HKG (or the triangle FKH which is equall to the triangle HKG) by the 16. of the ſiuenth.

¶ The 5. Propoſition.

The ſide of a Pyramis diuided by an extreme and meane proportion, maketh the leſſe ſegment in power double to the ſide of the Icoſahedron inſcribed in it.

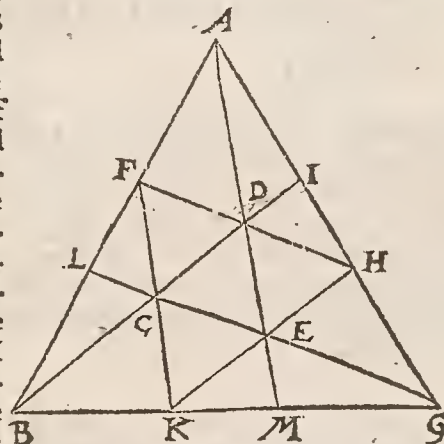


Suppoſe that ABG be the baſe of a pyramis : and let the baſe of the Icoſahedron inſcribed in it, be CDE, deſcribed of three right lines, which being drawn from the angles of the baſe ABG cut the oppoſite ſides by an extreme and meane proportion, by the former Propoſition : namely, of theſe three lines AM, BI, and GL. Then I ſay, that AI the leſſe ſegment of the ſide AG, is in power duple to CE the ſide of the Icoſahedron. For,

foraſmuch as by the former Propoſition, it was proued that the triangle CDE is inſcribed in an equilateral triangle, whoſe angles cut the ſides of ABG the baſe of the pyramis by an extreme and meane proportion, let that triangle be FHK, cutting the line AB in the point F. Wherefore the leſſe ſegment FA is equall to the ſegment AI, by the 2. of the fourteenth : (for the lines AB and AG are cut like). Moreouer the ſide FH of the triangle FHK is in the point D cut into two equall partes, as in the former Propoſition it was proued, and FCED alſo by the ſame is a parallelogramme : Wherefore the lines CE and FD are equall, by the 33. of the firſt. And for aſmuch as the line FH ſubtendeth the angle BAG of an equilateral triangle, which angle is contained vnder the greater ſegment AH and the leſſe ſegment AF : therefore the line FH is in power double to the line AF or to the line AI the leſſe ſegment, by the Corollary of the 16. of the ſiuenth. But the ſame line FH is in power quadruple to the line CE, by the 4. of the ſecond : (for the line FH is double to the line CE). Wherefore the line AI being the halfe of the ſquare of the line FH is in power duple to the line CE, to which the line FH was in power quadruple. Wherefore the ſide AG of the pyramis being diuided by an extreme and meane proportion, maketh the leſſe ſegment AI in power duple to the ſide CE of the Icoſahedron inſcribed in it.

Conſtruction.

Demonſtration.



¶ A Corollary.

The ſide of an Icoſahedron inſcribed in a pyramis, is a reſiduall line.

For the diameter of the Sphere which containeth the ſiue regular bodies, being rationally, is in power ſeſquialtera to the ſide of the pyramis, by the 13. of the thirtieth : and therefore the ſide of the pyramis is rationally, by the definition : which ſide being diuided by an extreme and meane proportion, maketh

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The sixteenth Booke of the

the lesse segment a residuall line, by the 6. of the thirteenth. Wherefore the side of the Icosahedron being commensurable to the same lesse segment (for the square of the side of the Icosahedron is the halfe of the square of the said lesse segment) is a residuall line, by that which was added after the 103. of the tenth booke.

¶ The 6. Proposition.

The side of a Cube containeth in power halfe the side of an equilater triangular Pyramis inscribed in the said Cube.

OR forasmuch as the side of the pyramis inscribed in the cube subtédeth two sides of the cube which containe a right angle, by the 1. of the fuetenth: it is manifest, by the 47. of the first, that the side of the pyramis subtéding the said sides, is in power duple to the side of the cube: Wherefore also the square of the side of the cube is the halfe of the square of the side of the pyramis. The side therefore of a cube containeth in power halfe the side of an equilater triangular pyramis inscribed in the said cube.

¶ The 7. Proposition.

The side of a Pyramis is duple to the side of an Octohedron inscribed in it.

ORasmuch as by the 2. of the fuetenth it was proued, that the side of the Octohedron inscribed in a pyramis coupleth the middle sections of the sides of the pyramis. Wherefore the sides of the pyramis and of the Octohedron are parallels, by the Corollary of the 39. of the first: and therefore, by the Corollary of the 2. of the sixth, they subtend like triangles. Wherefore (by the 4. of the sixth) the side of the pyramis is double to the side of the Octohedron, namely, in the proportion of the sides. The side therefore of a pyramis is duple to the side of an Octohedron inscribed in it.

¶ The 8. Proposition.

The side of a Cube is in power duple to the side of an Octohedron inscribed in it.

IT was proued in the 3. of the fuetenth, that the diameter of the Octohedron inscribed in the cube, coupleth the centres of the opposite bases of the cube. Wherefore the said diameter is equall to the side of the cube. But the same is also the diameter of the square made of the sides of the Octohedron, namely, is the diameter of the Sphere which containeth it, by the 14. of the thirteenth. Wherefore that diameter being equall to the side of the cube, is in power double to the side of that square, or to the side of the Octohedron inscribed in it, by the 47. of the first. The side therefore of a Cube, is in power duple to the side of an Octohedron inscribed in it: which was required to be proued.

¶ The 9. Proposition.

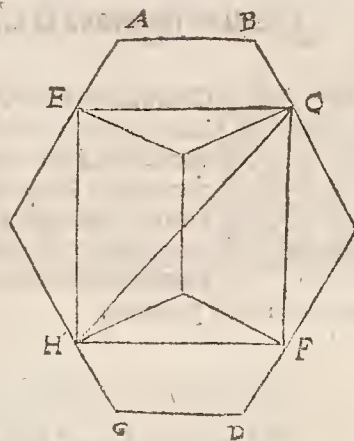
The side of a Dodecahedron, is the greater segment of the line which containeth in power halfe the side of the Pyramis inscribed in the sayd Dodecahedron.

Construction.



SVppose that of the Dodecahedron $ABGD$ the side be AB : and let the base of the cube inscribed in the Dodecahedron be $ECHF$, by the 8. of the fuetenth. And let the side of the pyramis inscribed in the cube be CH , by the 1. of the fuetenth. Wherefore the same pyramis is inscribed in the Dodecahedron, by the 10. of the fuetenth. Then I say, that AB the side of the Dodecahedron is the greater segment of the line which containeth in power halfe the line CH , which is the side of the pyramis inscribed in the Dodecahedron.

Dodecahedron . For forasmuch as EC the side of the cube being diuided by an extreme and meane proportion maketh the greater segment the line AB , the side of the Dodecahedron, by the first Corollary of the 17. of the thirteenth : (For they are contained in one and the selfe same Sphere (by the first of this booke) : and the line EC the side of the cube containeth in power the halfe of the side CH , by the 6. of this booke . Wherefore AB the side of the Dodecahedron, is the greater segment of the line EC , which containeth in power the halfe of the line CH , which is the side of the Dodecahedron inscribed in the pyramis . The side therefore of a Dodecahedron, is the greater segment of the line which containeth in power halfe the side of the Pyramis inscribed in the said Dodecahedron.



Demonstration.

¶ *The 10. Proposition.*

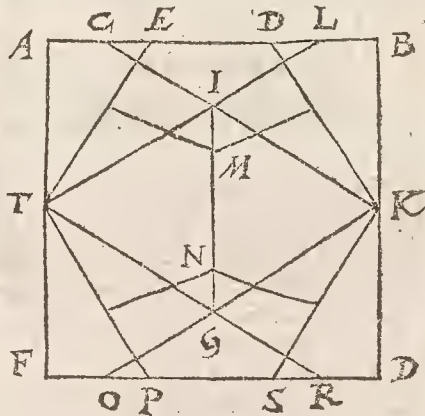
The side of an Icosahedron, is the meane proportionall betwene the side of the Cube circumscribed about the Icosahedron, and the side of the Dodecahedron inscribed in the same Cube.



Vppose that there be a cube $ABFD$, in which let there be inscribed an Icosahedron $CLIGOR$, by the 14. of the fuetenth. Let also the Dodecahedron inscribed in the same be $EDMNPS$, by the 13. of the same. Now forasmuch as CL the side of the Icosahedron is the greater segmēt of AB the side of the cube circumscribed about it, by the 3. Corollary of the 14. of the fuetenth : and the side ED of the Dodecahedron inscribed in the same cube is the lesse segmēt of the same side AB of the cube, by the 2. Corollary of the 13. of the fuetenth : it followeth that AB the side of the cube being diuided by an extreme and meane proportion, maketh the greater segment CL the side of the Icosahedron inscribed in it, and the lesse segment ED the side of the Dodecahedron likewise inscribed in it . Wherefore as the whole line AB the side of the cube, is to the greater segment CL the side of the Icosahedron, so is the greater segment CL the side of the Icosahedron, to the lesse segment ED the side of the Dodecahedron, by the third definition of the sixth . Wherefore the side of an Icosahedron, is the meane proportionall betwene the side of the cube circumscribed about the Icosahedron, and the side of the Dodecahedron inscribed in the same cube.

Construction.

Demonstration.



¶ *The 11. Proposition.*

The side of a Pyramis, is in power † Octodecuple to the side of the cube inscribed in it .

† That is, as 18. to 1.



Or, by that which was demonstrated in the 18. of the fuetenth, the side of the pyramis is triple to the diameter of the base of the cube inscribed in it : and therefore it is in power nonecuple to the same diameter (by the 20. of the sixth) . But the diameter is in power double to the side of the cube, by the 47. of the first . And the double of nonecuple maketh Octodecuple . Wherefore the side of the pyramis is in power Octodecuple to the side of the cube inscribed in it .

Demonstration.

¶ *The 12. Proposition.*

The side of a Pyramis, is in power Octodecuple to that right line, whose
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The sixteenth Booke of the

greater segment is the side of the Dodecahedron inscribed in the Pyramis.



Orasmuch as the Dodecahedron and the cube inscribed in it, are set in one and the selfe same pyramis, by the Corollary of the first of this booke: and the side of the pyramis circumscribed about the cube is in power octodecuple to the side of the cube inscribed, by the former Proposition: but the greater segment of the selfe same side of the cube, is the side of the Dodecahedron which containeth the cube, by the Corollary of the 17. of the thirteenth. Wherefore the side of the pyramis is in power octodecuple to that right line, namely, to the side of the cube, whose greater segment is the side of the Dodecahedron inscribed in the pyramis.

¶ The 13. Proposition.

The side of an Icosahedron inscribed in an Octohedron, is in power duple to the lesse segment of the side of the same Octohedron.



Orasmuch as in the 17. of the thirteenth, it was proued, that the side of an Icosahedron inscribed in a pyramis, coupleth together the two sections (which are produced by an extreme and meane proportion) of the side of the Octohedron which make a right angle: and that right angle is contained vnder the lesse segmentes of the sides of the Octohedron, and is subtended of the side of the Icosahedron inscribed: it followeth therefore, that the side of the Icosahedron which subtendeth the right angle, being in power equall to the two lines which containe the said angle, by the 47. of the first, is in power duple to euery one of the lesse segmētes of the side of the Octohedron which containe a right angle. Wherefore the side of an Icosahedron inscribed in an Octohedron, is in power duple to the lesse segment of the side of the same Octohedron.

¶ The 14. Proposition.

The sides of the Octohedron, and of the Cube inscribed in it, are in power the one to the other + in quadruple sesquialter proportion.

† That is, as 9.
to 2.

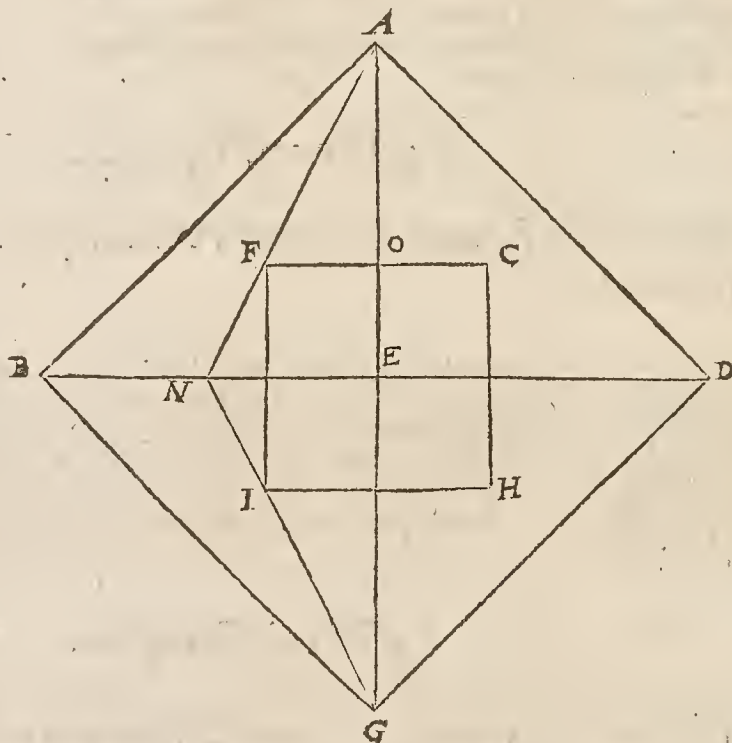


Vppose that A B G D E be an Octohedron, and let the cube inscribed in it be F C H I. Then I say, that A B the side of the Octohedron, is in power quadruple sesquialter to F I the side of the cube. Let there be drawen to B E the base of the triangle A B E a perpendicular A N: and againe let there be drawen to the same base in the triangle G B E the perpendicular G N:

which A N & G N shall passe by the centres F and I: and the line A F is duple to the line F N, by the Corollary of the 12. of the thirteenth.

Wherefore the line A O is duple to the line O E, by the 2. of the sixth. For the lines F O and N E are parallels. And therefore the diameter A G is triple to the line F I. Wherefore the power of A G is * noncuple to the power of F I. But the line A G is in power duple to the side A B, by the 14. of the thirteenth. Wherefore the square of the line A B, being ing the halfe of the square of the line A G, which is noncuple to the square of the line F I, is quadruple sesquialter

* That is, as
18. to 2. or 9.
to 1.



alter to the square of the line FI. The sides therefore of the Octohedron and of the cube inscribed in it, are in power the one to the other, in quadruple sesquialter proportion.

¶ *The 15. Proposition.*

The side of the Octohedron, is in power quadruple sesquialter to that right line, whose greater segment is the side of the Dodecahedron inscribed in the same Octohedron.

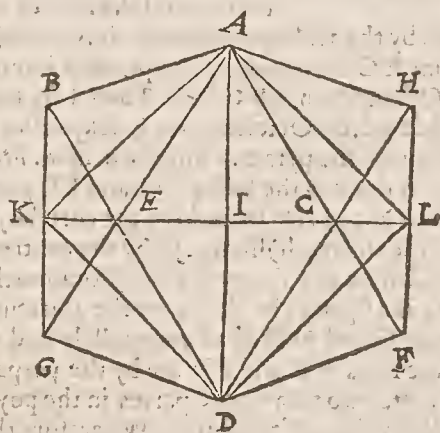
Forasmuch as in the 14. of this booke, it was proued, that the side of the Octohedron is in power quadruple sesquialter to the side of the cube inscribed in it: but the side of the cube being cut by an extreme and meane proportion, maketh the greater segment the side of the Dodecahedron circumscribed about it, by the 3. Corollary of the 13. of the fuetenth: therefore the side of the Octohedron is in power quadruple sesquialter to that right line (namely, to the side of the cube) whose greater segment is the side of the Dodecahedron inscribed in the cube. But the Dodecahedron and the cube inscribed one within an other, are inscribed in one and the selfe same Octohedron, by the Corollary of the first of this booke. The side therefore of the Octohedron, is in power quadruple sesquialter to that right line, whose greater segment is the side of the Dodecahedron inscribed in the same Octohedron.

¶ *The 16. Proposition.*

The side of an Icosahedron, is the greater segment of that right line, which is in power duple to the side of the Octohedron inscribed in the same Icosahedron.

Suppose that there be an Icosahedron $ABGD FHEC$: whose side let be BG or EC : and let the Octohedron inscribed in it be $AKDL$: and let the side thereof be AL . Then I say, that the side EC is the greater segment of that right line which is in power duple to the side AL . For forasmuch as figures inscribed and circumscribed haue one & the selfe same centre, by the Corollary of the 21. of the fuetenth, let the same be the point I . Now right lines drawn by that centre to the middle sections of the opposite sides, namely, the lines

AID and KIL , do in the point I cut the one the other into two equall partes, and perpendicularly, by the Corollary of the 14. of the fuetenth: and forasmuch as they couple the middle sections of the opposite lines BG and HF , therefore they cut them perpendicularly: wherefore also the lines BG and HF , are parallels, by the 4. Corollary of the 14. of the fuetenth. Now then draw a line from B to H : and the sayd line BH shall be equall and parallel to the line KL , by the 33. of the first. But the line BH subtendeth two sides of the pentagon which is composed of the sides of the Icosahedron, namely, the sides BA and AH : Wherefore the line BH being cut by an extreme and meane proportion maketh the greater segment the side of the pentagon; by the 8. of the thirteenth: which side is also the side of the Icosahedron, namely, EC . And vnto the line BH the line KL , is equall: and the line KL is in power duple to AL the side of the Octohedron, by the 47. of the first: for in the square $AKDL$ the angle KAL is a right angle. Wherefore EC the side of the Icosahedron, is the greater segment of the line BH or KL , which is in power duple to AL the side of the Octohedron inscribed in the Icosahedron. Wherefore the side of an Icosahedron, is the greater segment of that right line, which is in power duple to the side of the Octohedron inscribed in the same Icosahedron.



Draw in the figure a line from B to H.

¶ *The 17. Proposition.*

The side of a Cube is to the side of a Dodecahedron inscribed in it, in duple proportion of an extreame and meane proportion.

For

The sixteenth Booke of the

OR it was manifest by the 2. corollary of the 13. of the fiftenth, that the side of a cube diuided by an extreame and meane proportion, maketh the lesse segment, the side of the dodecahedron inscribed in it: but the whole is to the lesse segment in duple proportion of that in which it is to the greater, by the 10. diffinitio of the fifth. For the whole, the greater segment, and the lesse, are lines in continuall proportion, by the 3. diffinition of the sixth. Wherefore the whole, namely the side of the cube, is to the side of the dodecahedron inscribed in it, namely, to his lesse segment, in duple proportion of an extreame and meane proportion, namely, * of that which the whole hath to the greater segment, by the 2. of the fourteenth.

* What the duple of an extreame and meane proportion is.

¶ The 18. Proposition.

The side of a Dodecahedron is, to the side of a Cube inscribed in it, in conuerse proportion of an extreame and meane proportion.

IT was proued in the 3. corollary of the 13. of the fiftenth, that the side of a Dodecahedron circumscribed about a Cube, is the greater segment of the side of the same Cube. Wherefore the whole side of the Cube inscribed is to the greater segment, namely, to the side of the dodecahedron circumscribed, in an extreame and meane proportion: wherefore by conuersion, the greater segment, that is, the side of the dodecahedron, is to the whole, namely, to the side of the Cube inscribed, in the conuerse proportion of an extreame and meane proportion, by the 13. diffinition of the fiftenth.

¶ The 19. Proposition.

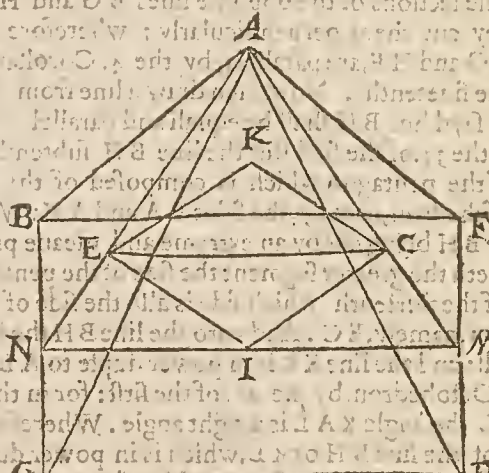
The side of an Octohedron, is sesquialter to the side of a Pyramis inscribed in it.

Construction.



OR (by the corollary of the 14. of the thirteenth) the Octohedron is cutte into two quadrilater pyramids, one of which let be A B G D F: and let the centres of the circles which containe the 4. bases of the Octohedron be K, E, I, C. And draw these right lines K E, E I, I C, C K, and E C. Wherefore K E I C is a square, and one of the bases of the cube inscribed in the Octohedron, by the 4. of the fiftenth. And forasmuch as the angles of a cube and of the pyramis inscribed in it, are set in the centres of the bases of the Octohedron circumscribed about the cube, by the 6. of the fiftenth: and the side of the pyramis coupleth the opposite angles of the base of the

cube, by the 1. of the fiftenth, it is manifest that the line E C is the side of the pyramis inscribed in the Octohedron A B G D F. Then I say that G D the side of the Octohedron, is sesquialter to E C the side of the pyramis inscribed in it. From the poynt A draw to the bases B G and F D perpendiculars A N and A M: which (by the corollary of the 12. of the thirteenth) shall passe by the centres E and C. And draw the line N M. Now forasmuch, as B G D F is a square, by the 14. of the thirteenth, the lines N G and M D shall be parallels and equall. For the lines B G and F D are by the perpendiculars cutte into two equall partes in the poyntes N and M (by the 3. of the third). Wherefore the lines N M and G D shall be parallels and equall, by the 33. of the first. And forasmuch as the lines A N and A M, which are the perpendiculars of equall and like triangles are cut a like in the poyntes E and C, the lines E C and N M, shall be parallels, by the 2. of the sixth: and therefore by the corollary of the same, the triangles A E C, and A N M shall be like. Wherefore as the line A N is to the line A E, so is the line N M to the line E C by the 4. of the sixth. But the line A N is sesquialter to the line A E, for the line A E is duple to the line E N, by the corollary of the 12. of the thirteenth: wherefore the line N M, or the line G D which is equall vnto it, is sesquialter to the line E C. Wherefore G D the side of the Octohedron, is sesquialter to E C the side of the pyramis inscribed in it.



Demonstration.

¶ The

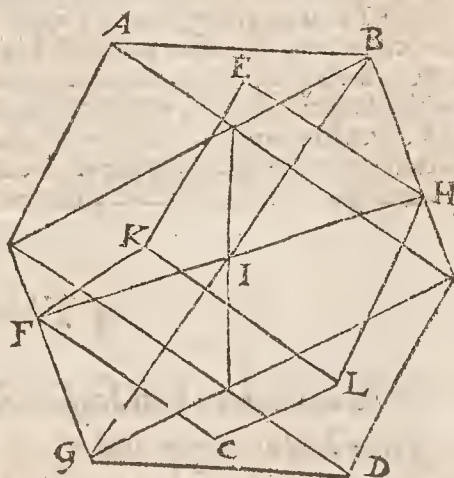
¶ *The 20. Propoſition.*

If from the power of the diameter of an Icoſahedron, be taken away the power tripled of the ſide of the cube inſcribed in the Icoſahedron: the power remayning ſhall be ſeſquitertia to the power of the ſide of the Icoſahedron.



Et there be taken an Icoſahedron $ABGD$: and let two baſes of the cube inſcribed in it, ioyned together be EHL and $LKFC$: and let the diameter of the cube be FH and the ſide be EH , and let the diameter of the Icoſahedron be BG , and the ſide be AB . Then I ſay, that if from the power of the diameter BG , be taken away the power tripled of EH the ſide of the cube: the power remayning, ſhall be ſeſquitertia to the power of AB the ſide of the Icoſahedron. For forasmuch as the centres of inſcribed and circumscribed figures, are in one & the ſelfe ſame poynt, by the corollary of the 21. of the ſiuetéth: the diameters BG and FH ſhall in one and the ſelfe ſame poynt cutte the one the other into two equall partes: for we haue before by the ſame corollary taught, that the toppes of equall and like pyramids doo in that poynt concurre, let that poynt be the centre I . Now the angles of the cube, which are at the poyntes F and H are ſet at the centres of the baſes of the Icoſahedron, by the 11. of the ſiuetenth. Wherefore the line FH ſhall be perpendicular to both the baſes of the Icoſahedron, by the corollary of the aſſupt of the 16. of the twelfth. Wherefore the line IB containeth in power the two lines IH and HB , by the 47. of the firſt. But the line HB , is drawne from the centre of the circle which containeth the baſe of the Icoſahedron, namely, the angle B is placed in the circumference, and the poynt H is the centre. Wherefore the whole line BG containeth in power the whole lines FH and the diameter of the circle (namely, the double of the line BH) by the 15. of the ſiueteth. But the diameter which is double to the line HB is in power ſeſquitertia to the ſide of the equilateral triangle inſcribed in the ſame circle, by the corollary of the 12. of the thirtieth. For it is in proportion to the ſide, as the ſide is to the perpendicular, by the corollary of the 8. of the ſixth. And FH the diameter of the cube, is in power triple to EH the ſide of the ſame cube, by the 15. of the thirtieth. If therefore from the power of the diameter BG , be taken away the power tripled of EH the ſide of the cube inſcribed; that is, the power of the line FH : the reſidue (namely, the power of the diameter of the circle which is duple to the line HB) ſhall be ſeſquitertia to the ſide of the triangle inſcribed in that circle: which ſelfe ſide is AB the ſide of the Icoſahedron. If therefore from the power of the diameter of an Icoſahedron, be také away the power tripled of the ſide of the cube inſcribed in the Icoſahedron, the power remayning ſhall be ſeſquitertia to the power of the ſide of the Icoſahedron.

Demonſtration.



A Corollary.

The diameter of the Icoſahedron, containeth in power two lines, namely, the diameter of the cube inſcribed, which coupleth the centres of the oppoſite baſes, and the diameter of the circle which containeth the baſe of the Icoſahedron. For it was manifeſt, that BG the diameter containeth in power the line FH which coupleth the centres, and the double of the line BH , that is, the diameter of the circle containing the baſe wherein is the centre H .

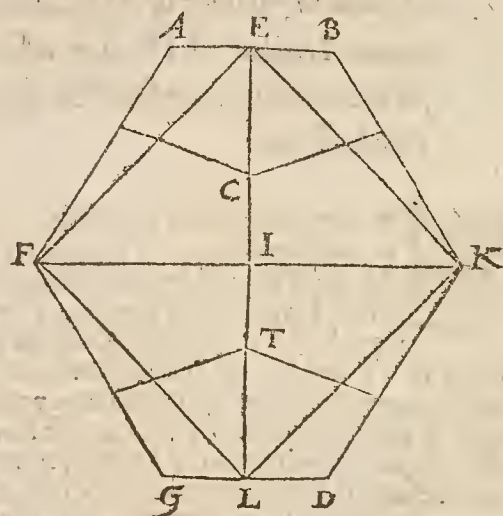
¶ *The 21. Propoſition.*

The ſide of a Dodecahedron is the leſſe ſegment of that right line, which is in power duple to the ſide of the Octohedron inſcribed in the ſame Dodecahedron.

The sixteenth Booke of the



Et ther be taken a Dodecahedron $ABGDC T$, one of whose sides let be AB . And let the Octohedron inscribed in the Dodecahedron be $EFLKI$: one of whose sides let be EF . Then I say that AB the side of the Dodecahedron, is the lesse segment of a certayne right line (cut by an extreame and meane proportion) which is in power duple to EF the side of the Octohedron inscribed in the Dodecahedron. Draw the diameters EL and FK of the Octohedron. Now they couple the middle sections of the opposite sides of the dodecahedron AB and GD , (by the 9. of the fuetenth, & 3. corollary of the 17. of the thirtéth) & every one of those diameters being diuided by an extreame and meane proportion, doo make the lesse segment, the side of the dodecahedron, by the 4. corollary of the same. Wherefore the side AB is the lesse segment of the line FK . But the line FK containeth in power the two equall lines EF & EK , by the 47. of the first: for the angle FEK is a right angle of the square $FEKL$ of the Octohedron. Wherefore the line FK is in power duple to the line EF . Wherefore the line AB (the side of the dodecahedron) is the lesse segment of the line FK , which is in power duple to EF the side of the Octohedron. The side therefore of a Dodecahedron is the lesse segment of that right line, which is in power duple to the side of the Octohedron inscribed in the same Dodecahedron.



¶ The 22. Proposition.

The diameter of an Icosahedron is in power sesquitertia to the side of the same Icosahedron, and also is in power sesquialter to the side of the Pyramis inscribed in the Icosahedron.



Or forasmuch as it hath bene proued (by the 20. of this booke) that if frō the power of the diameter of the Icosahedron be taken away the triple of the power of the side of the cube inscribed in it, there shalbe left a square sesquitertia to the square of the side of the Icosahedron: But the power of the side of the cube tripled, is the diameter of the same cube, by the 15. of the thirtéth: And the cube, & the pyramis inscribed in it are contained in one & the self same sphere, by the first of this booke, and in one & the self same Icosahedron by the corollary of the same. Wherefore one and the selfe same diameter of the cube, or of the sphere which containeth the cube and the pyramis, is in power sesquialter to the side of the pyramis by the 13. of the thirteenth. Wherefore it followeth, that if from the diameter of the Icosahedron, be taken away the triple power of the side of the cube, or the sesquialter power of the side of the pyramis, which are the powers of one and the selfe same diameter, there shall be left the sesquitertia power of the side of the Icosahedron. The diameter therefore of an Icosahedron is in power sesquitertia to the side of the same Icosahedron, and also is in power sesquialter to the side of the Pyramis inscribed in the Icosahedron.

The 23. Proposition.

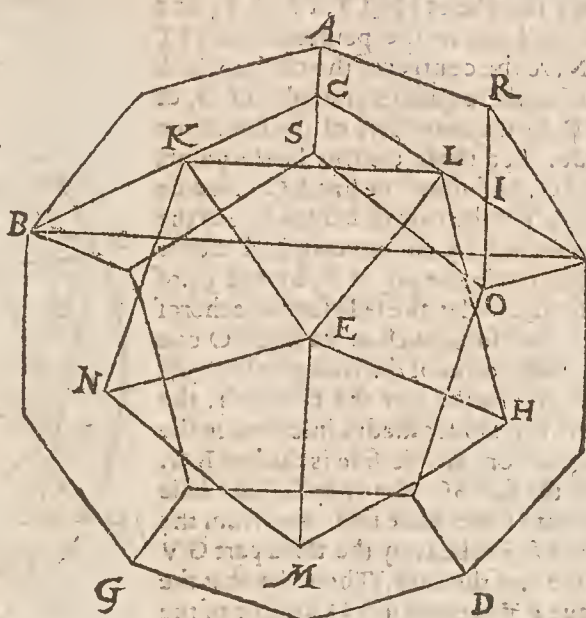
The side of a Dodecahedron is to the side of an Icosahedron inscribed in it, as the lesse segment of the perpendicular of the Pentagō, is to that line which is drawne from the centre to the side of the same pentagon.

Construction.



Et there be taken a Dodecahedron $ABGDFS O$. Whose side let be AS or SO : and let the Icosahedron inscribed in it be $KLHMNE$, whose side let be KL . From the two angles of the pentagons BAS and EAS of the Dodecahedron, namely, from the angles B and F , let there be drawne to the common base AS perpendicular lines BC & FC : which shal passe by the centres K & L of the sayd pentagons, by the corollary of the 10. of the thirtéth. Draw the

the lines B F and R O. Now forasmuche as the line R O subtendeth the angle O F R of the pentagon of the dodecahedron, it shall cut the line F C by an extreme and meane proportion, by the 3. of this booke, let it cut it in the poynt I. And forasmuche as the line K L is the side of the Icosahedron inscribed in the Dodecahedron, it coupleth the centres of the bases of the dodecahedron: for the angles of the Icosahedron are set in the centres of the bases of the dodecahedron, by the 7. of the fuetenth. Now I say that S O, the side of the dodecahedron is to K L the side of the Icosahedron, as the lesse segment I F of the perpendicular line C E, is to the line L C which is drawne from the centre L to A S the side of the pentagon. For forasmuche as in the triangle B C F the two sides C B and C F are in the centres L and K cut like proportionally, the lines B F and K L shalbe parallels, by the 2. of the sixth. Wherefore the triangles B C F, and K C L shall be equiangle, by the corollary of the same. Wherefore as the line C L is to the line K L, so is the line C F to the line B F, by the 4. of the sixth. But C F maketh the lesse segment the line I F, by the 3. of this booke, and the line B F maketh the lesse segment the line S O, namely, the side of the Dodecahedron, by the 2. corollary of the 13. of the fuetenth. For the line B F which coupleth the angles B and F of the bases of the dodecahedron, is equall to the side of the cube, which containeth the dodecahedron, (by the 13. of the fuetenth). Wherefore as the whole line C F, is to the whole line B F, so is the lesse segment I F to the lesse segment S O (by the 2. of the 14). But as the line C F is to the line B F, so is the line C L proued to be to the line K L. Wherefore as the line I F is to the line S O, so is the line C L to the line K L. Wherefore alternately by the 16. of the fuetenth, as the line I F the lesse segment of the perpendicular of the pentagon F A S, is to the line L C which is drawne from the centre of the pentagon, to the base, so is the line S O the side of the Dodecahedron to the line K L the side of the Icosahedron inscribed in it. The side therfore of a Dodecahedron is to the side of an Icosahedron inscribed in it, as the lesse segment of the perpendicular of the pentagon, is to that line which is drawne from the centre to the side of the same pentagon.



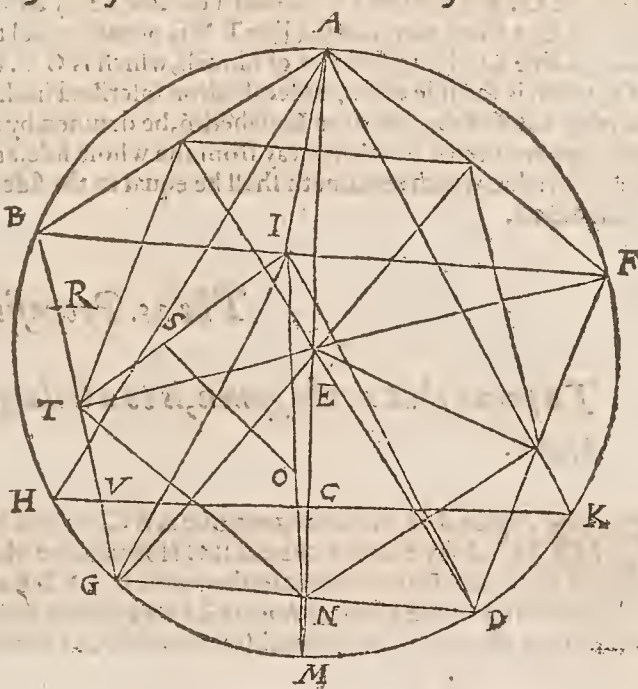
Demonstration.

¶ The 24. Proposition.

If halfe of the side of an Icosahedron be deuided by an extreme & meane proportion: and if the lesse segment thereof be taken away from the whole side, and againe from the residue be taken away the third part: that which remaineth shall be equal to the side of the Dodecahedron inscribed in the same Icosahedron.



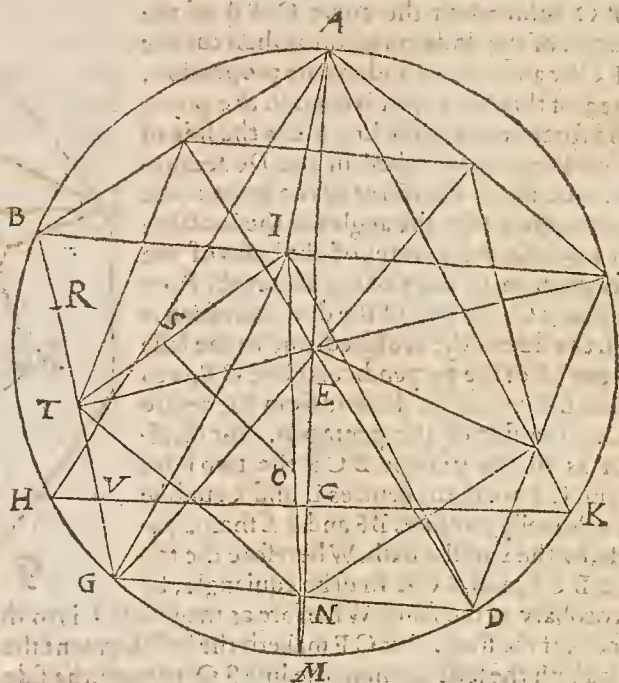
Suppose that A B G D F be a pentagon, containing five sides of the Icosahedron by the 16. of the thirteenth, and let it be inscribed in a circle, whose centre let be the point E. And vpon the sides of the pentagon, let there be reared vp triangles, making a solide angle of the Icosahedron at the poynt I, by the 16. of the thirteenth. And in the circle A B D, inscribe an equilater triangle A H K. From the centre E drawe to H K the side of the triangle, and G D the side of the pentagon, a perpendicular line, which let be E C N M. And draw these right lines E G, E D, I G and I D. And deuide the line B G into two equal parts in the poynt T. And draw



Construction.

The sixteenth Booke of the

drawe these lines IN, IT, TN, ET . And forasmuche as in the perpendiculars IT & IN are the centres of the circles which containe the equilater triangles IBG , & IGD , by the corollarye of the first of the thirde. Let those centres be the points S and O . And draw the line SO . Deuide the line TB the half of BG the side of the Icosahedron by an extreme and meane proportion in the poynt R , by the 30. of the sixth, and let the lesse segment thereof be RB . And forasmuch as the line SO coupleth the centres of the triangles IBG , & IGD , it is by the 5. of the fuetenth, the side of the Dodecahedron inscribed in the Icosahedron, whose side is the line BG . From the side BG take away BR the lesse segment of the halfe side. And from the residue GR take away the third part GV (by the 9. of the sixth.) Then I say that the residue RV is equal to SO the side of the Dodecahedron inscribed. For forasmuch as the perpendicular EN is in the poynt C deuided by an extreme and meane



Demonstration.

proportion, by the corollary of the first of the fourteenth, and the greater segment thereof is the line EC , and vnto the line EC the line CM is equal, by the corollary of the 12. of the thirteenth: wherefore the line EC is to the line CM , as the line CM is to the same line EN , by the 7. of the fuetth. But as the line EC is to the line CM , so is the whole line EN to the greater segment EC , by the 3. diffinition of the sixth. Wherefore (by the 11. of the fuetth), as the whole line EN is to the greater segment EC , so is the line CM to the line EN . Wherefore the line CM , is deuided by an extreme and meane proportion in the poynt N , namely, is deuided like vnto the line EN , by the 2. of the fourteenth. Wherefore the line EM exceedeth the line EN by the lesse segment of his halfe, namely, by MN . And forasmuch as EGD is the triangle of an equilater and equiangle pentagon $ABGDF$, and ETN is likewise the triangle of the like pentagon inscribed in the pentagon $ABGDF$: Therefore by the 20. of the sixth, the triangle ETN is like to the triangle EGD . Wherefore as the line EG is to the line EN , so by the 4. of the sixth, is the line GD to the line NT . Wherefore the line GD (or BG which is equal vnto it) exceedeth the line NT by the lesse segment of the halfe of BG . For the line EG did in like sort exceede the line EN . But that lesse segment is the line BR . Wherefore the residue RG is equal to the line TN . And forasmuch as IBG is an equilater triangle: the perpendicular ST shalbe the halfe of the line SI which is drawne from the centre, by the corollary of the 12. of the thirteenth: wherefore the line IT exceedeth the line IS by his third part. And forasmuch as the line SO which coupleth the sections, is a parallel to the line TN , by the 2. of the sixth. For the equal perpendiculars IT , and IN are cut like in the poynts S & O : therefore the triangles ITN & ISO , are like by the corollary of the second of the sixth. Wherefore as the line IT is to the line IS , so by the 4. of the sixth is the line TN to the line SO . But the line IT exceedeth the line IS by a third part: wherefore the line TN , exceedeth the line SO by a third part: but the line TN is proued equal to the line RG . Wherefore the line RG exceedeth the line SO by a third part of himself, which is GV . Wherefore the residue RV , is equal to the line SO , which is the side of the dodecahedron inscribed in the Icosahedron, whose side is the line BG . If therefore halfe of the side of an Icosahedron, be deuided by an extreme & meane proportion: and if the lesse segment thereof be taken away from the whole side, and againe from the residue be take away the third part: that which remaineth shall be equal to the side of the dodecahedron inscribed in the same Icosahedron.

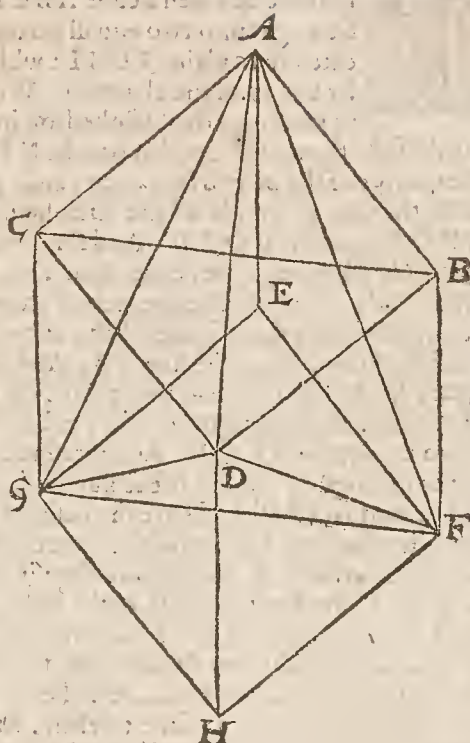
The 25. Proposition.

To proue that a cube geuen, is to a trilater equilater pyramis inscribed in it, triple.



Suppose that the cube geuen, be $ABCH$: and let the pyramis inscribed in it be $AGDF$. Then I say that the cube $ABCH$ is triple to the pyramis $AGDF$. For forasmuch as the base AFD is common to the pyramis $AFDB$ and $AFDG$, the pyramis $AFDB$ shalbe set without the pyramis $AFDG$. Likewise the rest of the bases of the inscribed pyramis are common to the rest of the pyramids sette without: which are these: the pyramis AGD C yppon the

the base AGD : the pyramis $AGFE$ vpon the base AGF : and the pyramis $GDFH$ vpon the base GDF . Which pyramids taken without, are foure in number, equal and like the one to the other, by the 8. diffinition of the eleuenth. For euery one of them is contained vnder thre halfe squares of the cube, and one of the bases of the pyramis inscribed. Wherefore euery one of the is cotained vnder the halfe base of the cube, & the altitude of the cube. As the pyramis $AEGF$, hath to his base halfe of the square $E H$, namely, the triangle $E G F$, & hath to his altitude, the altitude of the cube, namely, the line $A E$. Wherefore the sayd pyramis is the sixth part of the cube. For if the cube be deuided into two prismes, by the plaine $C B F G$, the prisme $ACBGEF$, shalbe triple to the pyramis $AEGF$, hauing one & the selfe same base with it EGF , and one and the selfe same altitude $E A$, by the first corollary of the 7. of the twelueh. Wherefore the sayd outward pyramis $AEGF$ is the sixth part of the whole cube. Wherefore also the same pyramis together wyth the other thre outwarde pyramids $A F D B$, $A G D C$, and $G D F H$, shal containe two third partes of the cube. Wherefore the residue, namely, the pyramis inscribed $A G D F$, shal contain one third part of the cube. And therefore conuerfely the cube shall be triple to it: wherefore we haue proued that a cube geue triple to a trilater & equilater pyramis inscribed in it.

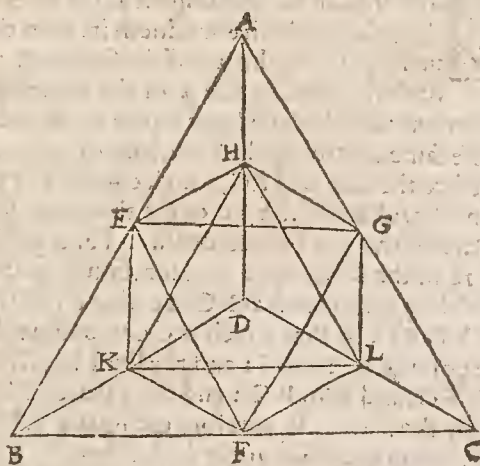


¶ The 26. Proposition.

To proue that a trilater equilater Pyramis is duple to an Octohedron inscribed in it.



Et there be taken a trilater Pyramis $A B C D$: whose fixe sides let be cut into two equall partes, in the pointes E, K, F, L, G , and H : inscribing thereby an Octohedron in the pyramis, by the 2. of the fuetenth. Wherefore the pyramids $A E G H$, $B E F K$, $C F G L$, & $D K H L$, fall without the Octohedron inscribed, by the same second of the fuetenth. But the outward Pyramids (namely, $A E G H$, and the three other) are like vnto the whole pyramis, by the 7. definition of the eleuenth. For the bases of the whole pyramis are by parallel lines drawn in them cut into like triangles, by the Corollary of the 2. of the sixth, of which the foresayd pyramids are made. Wherefore the whole pyramis is to euery one of them in treble proportion of that in which the sides of like proportion are, by the 8. of the twelfth. But by construction, the proportion of the side $A B$ to the side $A E$ is duple. Wherefore the whole pyramis $A B C D$ is octuple to the pyramis $A E G H$, and so is it to euery one of the pyramids which are equall to $A E G H$. For duple proportion multiplied into it selfe twise maketh octuple. Wherefore it followeth that the 4. pyramids $A E G H$, $B E F K$, $C F G L$, and $D K H L$, taken together, make the halfe of the whole pyramis $A B C D$. Wherefore the residue, namely, the Octohedron $E G L K H F$, is the other halfe of the pyramis. Wherefore the pyramis is duple to the Octohedron. Wherefore we haue proued that a trilater equilater pyramis is duple to an Octohedron inscribed in it.



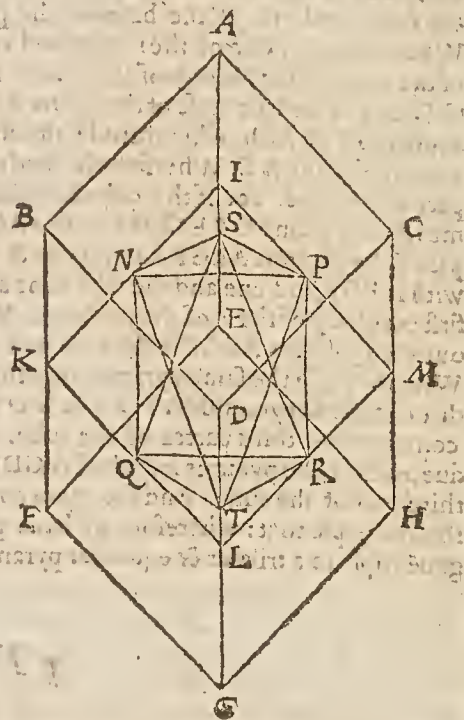
¶ The 27. Proposition.

To proue that a Cube is sextuple to an Octohedron inscribed in it.

The sixteenth Booke of the

Et there be taken a cube $ABCD, EFGH$: whose 4 standing lines $AE, BF, CH, & DG$, let be cut into two equall partes in the pointes I, K, M, L : and by those pointes let there be extended a plaine $KLM I$: which shall be a square, and parallel to the squares BC & FH , by the 15. of the eleuenth. Wherefore in it shall be the base which is common to the two pyramids of the Octohedron inscribed in the cube, by that which was demonstrated in the third of the fuetenth. Let that base be $NP R Q$, coupling the centres of the bases of the cube: and vpon that base let be set the two pyramids of the Octohedron, which let be $NP QRS$, and $NP QRT$. And forasmuch as those two pyramids taken together, haue their altitude equall with the altitude of the whole cube, ech of them a part hath to his altiude halfe the altitude of the cube, namely, halfe of the side of the cube, as the line KB . And forasmuch as the square $KLM I$ is double to the square $NRQP$, by the 47. of the first: the other squares of the cube shall also be double to the square $NRQP$. And forasmuch as the cube, as it was manifest by the last of the fuetenth, is resoluéd into sixe pyramids, whose bases are the bases of the cube, & the altitudes the lines drawn frō the centre to the bases, which are equall to halfe the side of the cube: it followeth that euery one of the sixe pyramids of the cube, hauing his base double to the base of eche of the pyramids of the Octohedron, and the selfe same altitude that the said pyramids of the Octohedron haue, is double to either of the pyramids of the octohedron, by the 6. of the twelfth. And forasmuch as euery one of the pyramids of the cube is equall to the two pyramids of the Octohedron, the sixe pyramids of the cube shall be sextuple to the whole Octohedron. Wherefore it is manifest, that a cube is sextuple to an Octohedron inscribed in it.

Demonstration.



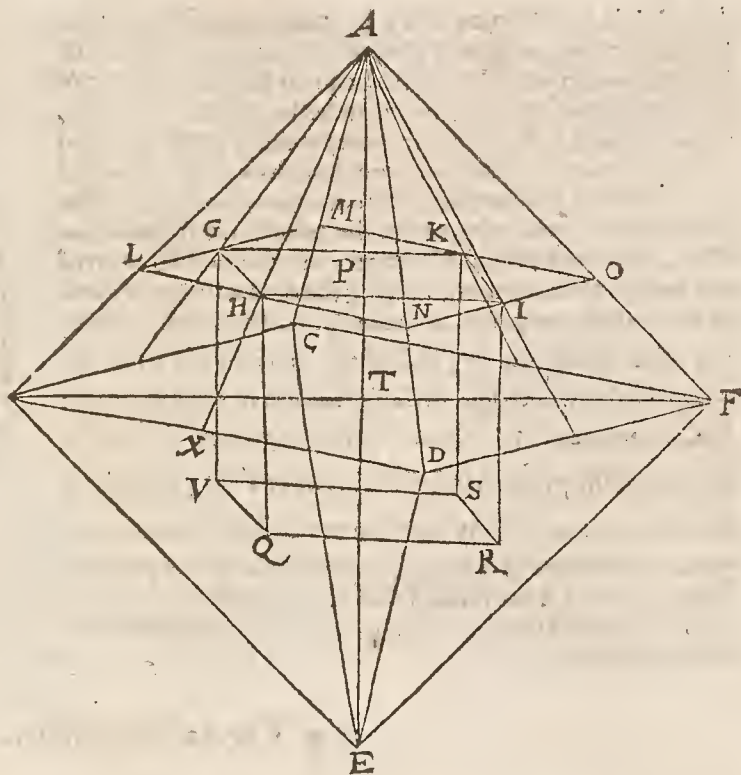
¶ The 28. Proposition.

To proue that an Octohedron is quadruple sesquialter to a Cube inscribed in it.



Suppose that the Octohedron geuen be $AB C D E F$: and let the cube inscribed in it be $G H I K, V Q R S$. Then I say, that the Octohedron is quadruple sesquialter to the cube inscribed in it. Forasmuch as the lines drawn from the centre of the Octohedron, or of the Sphere which containeth it, vnto the centres of the bases of the Octohedron, are proued equall, by the 21. of the fuetenth: and the angles of the cube are set in the centres of those bases, by the 4. of the fuetenth: it followeth, that the selfe same right lines are drawn from one and the selfe same centre of the cube and of the Octohedron: for they haue eche one and the selfe same centre, by the Corollary of the 21. of the fuetenth. Let that centre be the point T . Wherefore the base $B D F C$, which cutteth the Octohedron into two equall and quadrilater pyramids, by the Corollary of the 14. of the thirteenth, shall also cut the cube into two equall partes, by the Corollary of the 39. of the eleuenth. For it passeth by the centre T , by that which was demonstrated in the 14. of the thirteenth. And forasmuch as the base of the cube is in the 4. centres G, H, I, K , of the bases of the pyramis $AB D F C$, a plaine $L N O M$, extended by those pointes, shall be parallel to the plaine $B D F C$, by that which was demonstrated in the 4. of the fuetenth, and shall cut the pyramis in the pointes L, N, O, M : and the lines LN, BD , and NO, DF , shall be parallels, so also shall the lines OM, FC ; and LM, BC : and the square $G H I K$ of the cube shall be inscribed in the square $L N O M$, by the same. Wherefore the square $L N O M$ is duple to the square $G H I K$, by the 47. of the first. From the solide angle A , let there be drawn to the plaine superficies $B D F C$, a perpendicular, which let fall vpon it in the point T , and let the same perpendicular be AT , cutting the plaine $L N O M$ in the point P . And it shall also be a perpendicular to the plaine $L N O M$, by the Corollary of the 14. of the eleuenth. Againe from the angle $B A D$ of the triangle $A D B$, let there be drawn by the centre H of the triangle, to the base a line $A H X$. Wherefore the line $A X$ is sesquialter to the line $A H$, by the Corollary of the 12. of the thirteenth. Wherefore the line $A H$ is duple to the line $H X$. But the other lines AB, AD, AF, AC , and the perpendicular APT , are cutlike vnto the line $A H X$, by the 17. of the eleuenth: Wherefore the line AP is double to the line PT . Wherefore the line AP is the altitude of the cube, for the line PT is the halfe thereof. And

And forasmuch as vpon the base GHIK of the cube, and vnder the altitude AP of the ſame cube, is ſet the pyramis AGHIK: the ſaid pyramis is the third part of the cube, by the Corollary of the 7. of the twelfth. But vnto the pyramis AGHIK the pyramis ALNOM is duple, by the 6. of the twelfth, for the baſe of the one is double to the baſe of the other. Wherefore the pyramis ALNOM is two third partes of the cube. And forasmuch as the pyramids ALNOM, and ABDFC, are like, by the 7. definition of the eleuenth: therefore they are in triple proportion of that in which the ſides of like proportion AH to AX, or AL to AB, are, by the Corollary of the 8. of the twelfth. But the ſide AB is proued to be ſeſquialter to the ſide AL. Wherefore the pyramis ABDFC is to the pyramis ALNOM, as 27. is to 8. (that is, in ſeſquialter proportion tripled: for the quantitie or denomination of ſeſquialter proportion, namely,



$1\frac{1}{2}$ multiplied into it ſelfe once maketh $2\frac{1}{4}$, which againe multiplied by $1\frac{1}{2}$ maketh $3\frac{3}{4}$, that is, 27. to 8.) But of what partes the pyramis ALNOM containeth 8, of the ſame the cube containeth 12: namely, is ſeſquialter to the pyramis. Wherefore of what partes the cube containeth 12, of the ſame the whole Octohedron (which is double to the pyramis ABDFC) containeth 54. Which 54. hath to 12. quadruple ſeſquialter proportion. Wherefore the whole Octohedron is to the cube inſcribed in it, in quadruple ſeſquialter proportion. Wherefore we haue proued that an Octohedron geuen is quadruple ſeſquialter to a cube inſcribed in it.

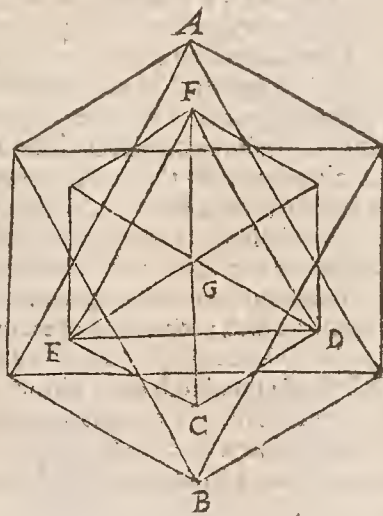
¶ A Corollary.

An Octohedron is to a cube inſcribed in it, in that proportion that the ſquares of their ſides are. For by the 14. of this booke, the ſide of the Octohedron is in power quadruple ſeſquialter to the ſide of the cube inſcribed in it.

¶ The 29. Proposition.

To proue that an octohedrö geuē, is ^{*}tredecuple ſeſquialter to a trilater equilateral pyramis inſcribed in it.

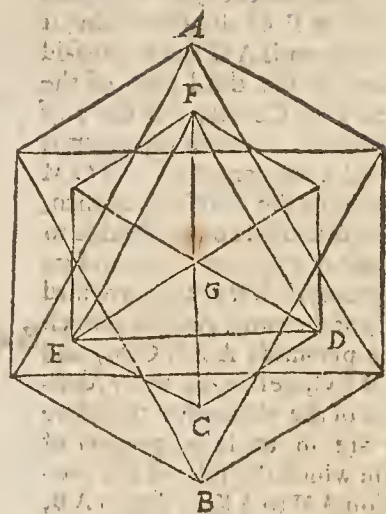
Let the octohedron geuen, be AB: in which let there be inſcribed a cube FCED, by the 4. of the ſiuetenth, and in the cube let there be inſcribed a pyramis FEGD, by the 1. of the ſiuetenth. And forasmuche as the angles of the pyramis are (by the ſame fiſt of the ſiuetenth) ſet in the angles of the cube: and the angles of the cube are ſet in the centres of the baſes of the Octohedron, namely, in the poyntes F, E, C, D, G by the 4. of the ſiuetenth. Wherefore the angles of the pyramis, are ſet in the centres F, C, E, D of the octohedron. Wherefore the pyramis FEDG is inſcribed in the octohedron (by the 6. of the ſiuetenth.) And forasmuche as the octohedron



^{*} That is as
 $13\frac{1}{2}$ is to 1.

The sixteenth Booke of the

dron AB is to the cube FCED, inscribed in it quadruple sesquialter (by the former propositiō): and the cube CDEF is to the pyramis FEDG inscribed in it triple, by the 25. of booke: wherefore three magnitudes being geuen, namely, the octohedron, the cube and the pyramis, the proportion of the extremes (namely, of the octohedron to the pyramis) is made of the proportions of the meanes, (namely, of the octohedron to the cube, and of the cube to the pyramis,) as it is easie to see by the declaration vpon the 10. diffinition of the fifteth. Now then multiplying the quantities or denominations of the proportions (namely, of the octohedron to the cube which is $4\frac{1}{2}$, and of the cube to the pyramis, which is 3) as was taught in the diffinition of the sixth, there shalbe produced $13\frac{1}{2}$, namely, the proportion of the octohedron to the pyramis inscribed in it. For $4\frac{1}{2}$, multiplied by 3, produce $13\frac{1}{2}$. Wherefore the Octohedron is to the pyramis inscribed in it in tredecuple sesquialter proportion. Wherefore we haue proued that an Octohedron is to a trilater equilater pyramis inscribed in it, in tredecuple sesquialter proportion.



The 30. Proposition.

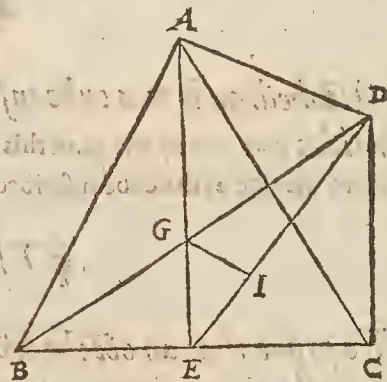
To proue that a trilater equilater Pyramis, is noneuple to a cube inscribed in it.



Suppose that the pyramis geuen, be ABCD, whose two bases let be ABC, and DBC, and let their centres be the poynts G and I. And from the angle A, draw vnto the base BC, a perpendicular AE: likewise from the angle D draw vnto the same base BC, a perpendicular DE: and they shal concur in the section E by the 3. of the third: and in them shalbe the centres G and I, by the corollary of the first of the third. And forasmuch as the line AD is the side of the pyramis, the same AD shall be the diameter of the base of the

Demonstration.

cube which containeth the pyramis, by the 1. of the fifteth. Draw the line GI. And forasmuch as the line GI coupleth the centres of the bases of the pyramis: the saide line GI shalbe the diameter of the base of the cube inscribed in the pyramis by the 18. of the fifteth. And forasmuch as the line AG is double to the line GE, by the corollary of the twelue of the thirteenth: the whole line AE shal be triple to the line GE: and so is also the line DE to the line IE. Wherefore the lines AD and GI are parallels, by the 2. of the sixth. And therefore the triangles AED, and GEI are like, by the corollary of the same. And forasmuch as the triangles AED, and GEI are like, the line AD shalbe triple to the line GI, by the 4. of the sixth. But the line AD is the diameter of the base of the cube circumscribed about the pyramis ABCD; and the line GI is the diameter of the base of the cube inscribed in the pyramis ABCD:



but the diameters of the bases are equemultiples to the sides (namely, are in power duple). Wherefore the side of the cube circumscribed about the pyramis ABCD, is triple to the side of the cube, inscribed in the same pyramis, by the 15. of the fifteth: but like cubes are in triple proportion the one to the other of that in which their sides are, by the 33. of the eleuenth: and the sides are in triple proportion the one to the other: Wherefore triple taken three times bringeth forth twenty seuenuple, which is 27. to 1: for the 4. termes 27. 9. 3. 1, being set in triple proportion: the proportion of the first to the fourth, namely, of 27. to 1. shalbe triple to the proportion of the first to the second, namely, of 27. to 9, by the 10. diffinition of the fifteth: which proportion of 27. to 1. is the proportiō of the sides tripled, which proportiō also is found in like solides. Wherefore of what partes the cube circumscribed containeth 27. of the same, the cube inscribed containeth one: but of what partes the cube circumscribed, containeth 27. of the same, the pyramis inscribed in it, containeth 9. by the 25. of this booke: wherefore of what partes the pyramis ABCD containeth 9. of the same, the cube inscribed in the pyramis, containeth one. Wherefore we haue proued that a trilater and equilater pyramis, is noneuple to a cube inscribed in it.

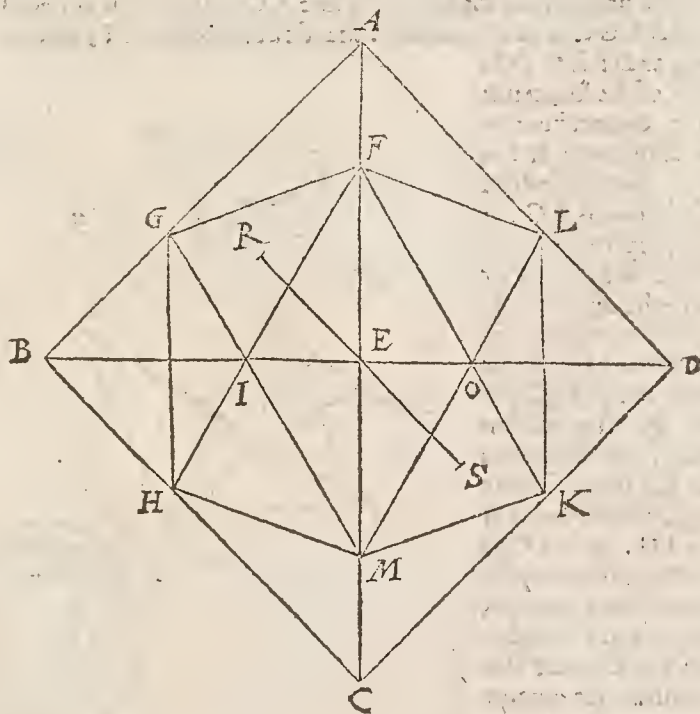
The

¶ The 31. Proposition.

An Octohedron hath to an Icoſahedron inſcribed in it, that proportion, which two baſes of the Octohedron haue to five baſes of the Icoſahedron.

Suppoſe that the octohedron geuen be A B C D, and let the Icoſahedron inſcribed in it, be F G H M K L I O. Then I ſay that the octohedron is to the Icoſahedron, as two baſes of the octohedron, are to five baſes of the Icoſahedron. For forasmuche as the ſolide of the octohedron conſiſteth of eight pyramids, ſet vpon the baſes of the octohedron, and hauing to their altitude a perpendicular line drawne from the centre to the baſe: let that perpendicular be E R, or E S, being drawne from the centre E (which centre is common to either of the ſolides, by the corollary of the 21. of the ſiuententh) to the centres of the baſes, namely, to the poyntes R and S. Wherefore for that thre pyramids are equal and like, they ſhalbe equal to a priſme ſet vpon the ſelfe ſame baſe, and vnder the ſelfe ſame altitude, by the corollary of the ſeuenth of the twelueh. But vnto this priſme is double that priſme which is ſet vpon the ſelfe ſame baſe, and hath his altitude duple, namely, the whole line R S by the corollary of the 25. of the eleuenth: for it is equal to the two equal and like priſmes whereof it is compoſed. Wherefore the priſme ſet vpon the baſe of the octohedron, and hauing to his altitude the line R S is equal to ſix pyramids, ſet vpon ſix baſes of the Octohedron, and hauing to their altitude the line E R. So there remaine two pyramids (for in the octohedron are 8. baſes) which ſhall be equal to the priſme which is ſet vpon the third part of the baſe of the octohedron, and vnder the altitude R S. For priſmes vnder one and the ſelfe ſame altitude, are in proportion the one to the other, as are their baſes, by the corollary of the 7. of the twelueh.

Demonſtration.



Wherefore the two priſmes which are ſet vpon the baſe of the octohedron, and vpon a third part thereof, and vnder the altitude R S, are equal to the 8. pyramids of the Octohedron, or to the whole ſolide of the octohedron. And forasmuch as the Icoſahedron inſcribed in the octohedron, hath his baſes ſet in the baſes of the Octohedron, by the 17. of the ſiuententh: it followeth that the pyramids ſet vpon the baſes of the Icoſahedron, & hauing to their toppes one and the ſelfe ſame centre E, are contained vnder the ſelfe ſame altitude, that the pyramids of the octohedron are contained vnder, namely, vnder the line E R, or E S. And therefore a priſme, ſet vpon the baſe of the Icoſahedron, and hauing his altitude double to the altitude of the pyramid, namely, the whole line R S, is equal to ſixe pyramids ſet vpon the baſe of the Icoſahedron, and vnder the altitude E R or E S, as we haue proved in the octohedron. Wherefore the 20. pyramids, ſet vpon the 20. baſes of the Icoſahedron, are equal to thre priſmes ſet vpon the baſe of the Icoſahedron, and vnder the altitude R S, and moreover to another priſme ſet vpon a thirde part of the baſe of the Icoſahedron and vnder the ſame altitude R S, which priſme is a thirde part of the former priſme, by the corollary of the 7. of the twelueh: for their proportion is as the proportion of the baſes. Wherefore two priſmes ſet vpon the baſe of the octohedron, and a third part thereof, and vnder the altitude R S, is to 4. priſmes ſet vpon three baſes of the Icoſahedron, and a third part thereof, and vnder the ſame altitude R S, in the ſame proportion that the baſes are, that is, as 4. third partes of the baſe of the Octodron (which are equal to one baſe, and $\frac{1}{3}$) to ten third partes of the baſe of the Icoſahedron (which are equal to thre baſes & $\frac{1}{3}$) or as two third partes of the baſe of the Octohedron, are to five thirde partes of the baſe of the Icoſahedron. But two thirde partes of the baſe of the Octohedron, are to five thirde partes of the baſe of the Icoſahedron, as two baſes are to five baſes (by the 15. of the fifth, for they are partes of equemultiplices:) And two priſmes of the Octohedron are to 4. priſmes of the Icoſahedron, as the ſolide of the Octohedron is to the ſolide of the Icoſahedron, when as eche are equal to eche of the ſolides: Wherefore (by the 11. of the ſiuenth) the ſolide of the Octohedron, is to the ſolide of the Icoſahedron inſcribed in it, as two baſes of the Octohedron, are to five baſes of the Icoſahedron. An Octohedron,

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hedron therefore is to an Icosahedron inscribed in it, in that proportion, that two bases of the Octahedron, are to sixe bases of the Icosahedron.

¶ The 32. Proposition.

The proportiō of the solide of an Icosahedron to the solide of a Dodecahedron inscribed in it, consisteth of the proportion of the side of the Icosahedron to the side of the Cube containd in the same sphere, and of the proportion tripled of the diameter to the line which completh the centers of the opposite bases of the Icosahedron.

Construction.

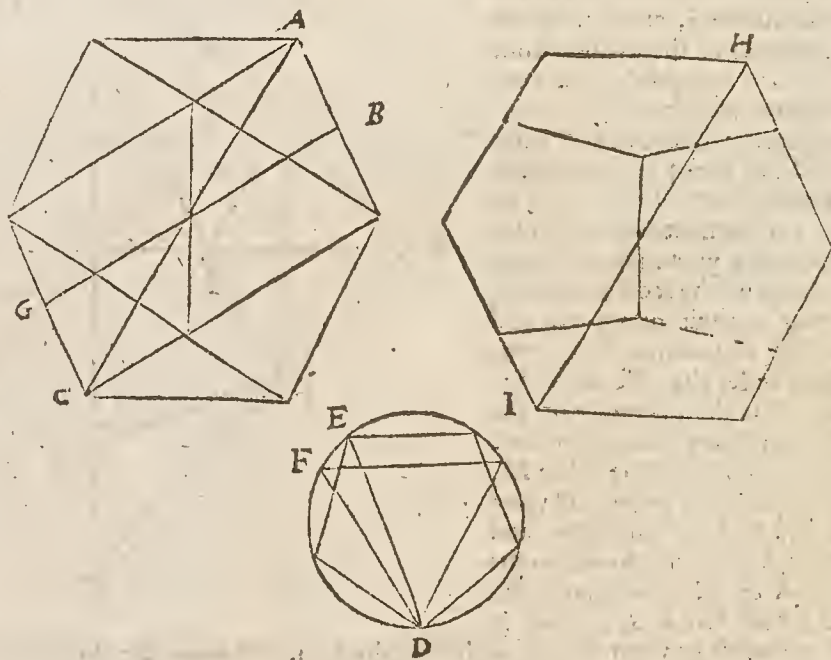


Vppose that there be a Dodecahedron, whose diameter let be HI , and let the Icosahedron contained in the same sphere be $ABGC$, whose dimetient let be AC . And let the right line which coupleth the centres of the opposite bases be BG . And let the dodecahedron inscribed in the Icosahedron be that which is set vpon the diameter BG , by the 5. of the fuetenth. And let the side of the cube be DE , and let the side of the Icosahedron be DF , both the sayd solides being described in one and the selfe same sphere. The

Demonstration.

I say that the proportion of the solide of the Icosahedron $ABGC$ to the solide of the dodecahedron set vpon the diameter BG , inscribed in it, consisteth of the proportion of the line DF to the line DE , and of the proportion tripled of the line AC to the line BG . For forasmuch as the solide of the Icosahedron $ABGC$ is to the solide of the dodecahedron HI , being containd in one and the selfe same

sphere, as DF is to DE , by the 8. of the fourteenth. But the dodecahedron whose diameter is HI , is to the dodecahedron whose diamer is BG , in treble proportiō of that in which the diameter HI is to the diameter BG , by the corollary of the 17. of the twelfth: & the lines HI and AC are equall by supposition (namely, the diameters of one and the selfe same sphere). Wherefore as HI is to BG , so is AC to BG . Wherefore the proportion of the extremes, namely, of the Icosahedron $ABGC$ to the Dodecahedron set vpon the diameter BG which coupleth the centres, consisteth (by the 5. diffinition of the sixt) of the proportions of the meanes, namely, of the proportiō



of the Icosahedron $ABGC$ to the dodecahedron HI (which is one and the same with the proportion of DF to DE) and of the proportion of the same HI to the other dodecahedron set vpon the diameter BG , inscribed in the same Icosahedron $ABGC$, by the same 5. of the fuetenth: which proportion is triple to the proportiō of the line HI (or the line AC) to BG which coupleth the centres of the opposite bases of the Icosahedron. The proportion therefore of the solide of the Icosahedron to the solide of a Dodecahedron inscribed in it, consisteth of the proportion of the side of the Icosahedron to the side of the Cube containd in the same sphere, and of the proportion tripled of the diameter to the line which coupleth the centres of the opposite bases of the Icosahedron.

¶ The 33. Proposition.

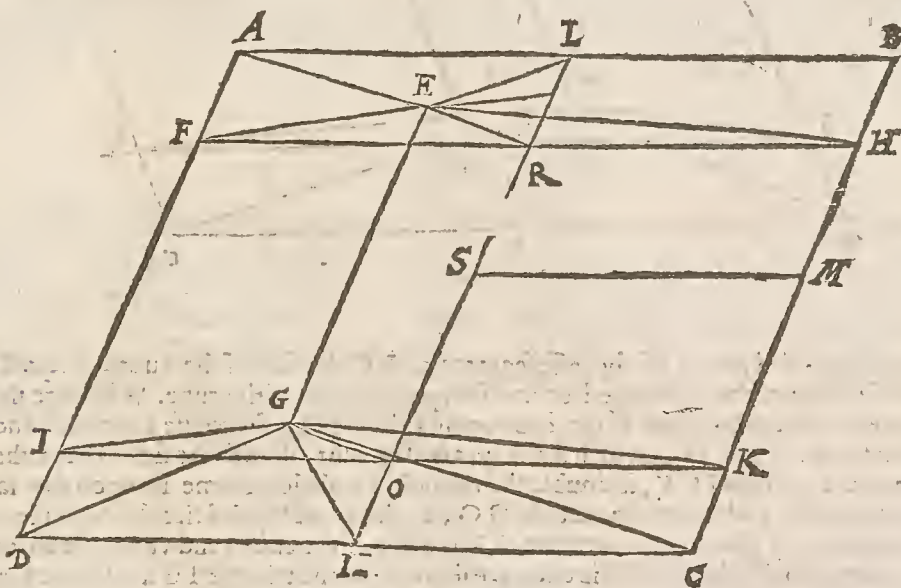
The solide of a Dodecahedron exceedeth the solide of a Cube inscribed in it

it, by a parallelipipedon, whose base wanteth of the base of the Cube by a third part of the lesse segment, and whose altitude wanteth of the altitude of the Cube, by the lesse segment of the lesse segment, of halfe the side of the Cube.

Forasmuch as by the 17. of the thirteenth, and 8. of the fiveth, it was manifest, that the base of a cube inscribed in a dodecahedron, doth with his sides subtend the angles of 4. pentagons cōcurring at one and the selfe same side of the dodecahedron: let that base of the cube be A B C D: and let the side wherat 4. bases of the dodecahedron circumscribed concur, be E G: which shall containe a solide A E B D G C set vpon the base A B C D. Diuide the sides A B and D C into two equall partes in the poyntes L and N. And draw the line L N, which is a parallel to the side E G, as it was manifest by the 17. of the thirteenth. The perpendiculars also E R and G O which couple those parallels, are eche equall, to halfe of the side E G, and eche is the greater segment of halfe the side of the cube, and therefore the whole line E G is the greater segment of the whole line L N the side of the cube (by the foresayd 17. of the thirteenth). By the poyntes R and O, draw vnto the sides A B and C D parallel lines F H and I K. And draw these right lines E F, E H, G I and G K. Now forasmuch as the two lines F H & E R touching the one the other are parallels to the two lines I K and G O touching also the one the other, & not being in the selfe same playne with the two first lines: therefore the playne superficieses E F H and G I K passing by those lines are parallels, by the 15. of the eleventh: which playnes do cutte the solide A E B D G C. Wherefore there are made fower quadrangled pyramids set vpon the rectangle parallelogrames L H, L F, N K, and N I, and hauing their toppes the poyntes E and G. And forasmuch as the triangles G O K and E R H are equall and like, by the 4. of the first, namely, they con-

Construction

Demonstration.



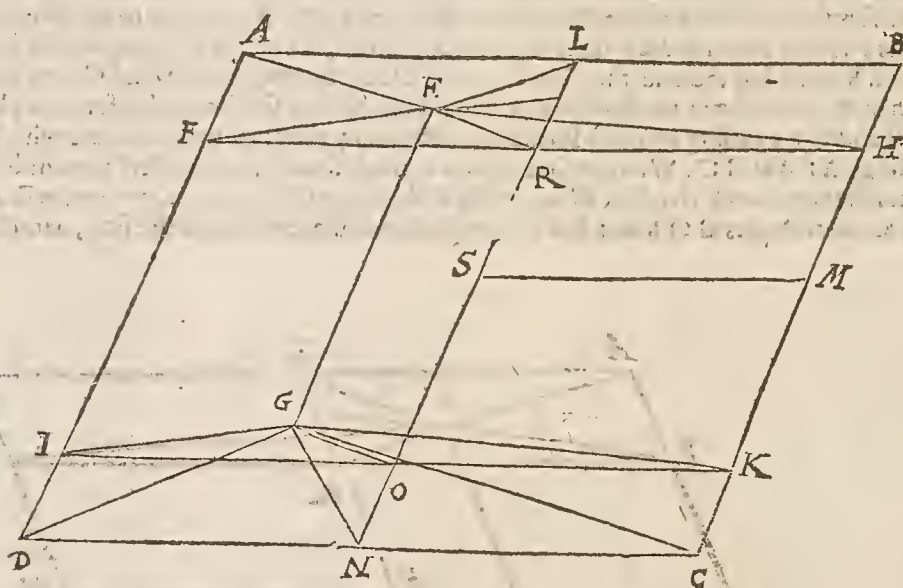
Extend in the figure a line from the point E to the point B.

tainne equall angles comprehended vnder equall sides, and they are parallels by construction, being set in the playnes G I K and E F H: the figures G K H E, O K H R, and G O R E shall be parallelogrammes, by the definition of a parallelogramme, and therefore the solide G O K E R H is a prisme, by the 11. definition of the eleventh. And by the same reason may the solide G O I E R F be proued to be a prisme. And forasmuch as vpon equall bases N O K C, and R L B H, and vnder equall altitudes O G and R E are set pyramids; those pyramids shall be equall to that pyramid which is set vpon the base C K I D (which is double to either of the bases N O K C, and R L B H) and vnder the same altitude O G, by the 6. of the twelfth. And forasmuch as the side G E is the greater segment of the line C B, the line K H, which by the 33. of the first, is equall to the line G E, shall be the greater segment of the same line C B, by the 2. of the fourteenth. Wherefore the residues C K and H B shall make the lesse segment of the whole line C B. But as the greater segment K H is to the two lines C K and H B the lesse segment, so is the rectangle parallelogramme O H to the two rectangle parallelogrammes O C and H L, by the 1. of the sixth. Wherefore the lesse segment of the parallelogramme N B, shall be the two parallelogrammes O C and H L. Put the line K M double to the line K C and draw the line M S parallel to the line C N. Wherefore the parallelogramme O K M S is equall to the parallelogrammes O C and H L, by the 1. of the sixth. Wherefore the pyramid set vpon the base O K M S containeth two third partes of the prisme set vpon the selfe same base, by the 4. corollary of the 7. of the twelfth. Wherefore the prisme which is set vpon two third partes

The sixteenth Booke of the

partes of the base $OKMS$ is equall to the two pyramids $NOKCG$ and $RLBHE$. For the sections of a prisme are one to the other, as the sections of the base are, by the first corollary of the 25. of the eleuenth. But the sections of the base are as the sections of the line CB or KM , by the 1. of the sixt. Wherefore the two pyramids $NOKCG$ and $RLBHE$, adde vnto the prisme $GOKERH$ two third partes of the prisme set vpon the base $OKMS$. And forasmuch as the line KM is the lesse segmēt of the whole line BC (for it is equall to to the two lines CK and HB), and the prisme set vpon the base $OKHR$ is cutte like vnto the line KM , namely, in eche are taken two thirdes, as hath before bene proued: the prisme equall to the two pyramids, shall adde vnto the prisme $GOKERH$, which is set vpon the greater segment KH , two thirds of the lesse segment. Wherefore in the line BC there shall remayne one third part of the lesse segment: and therefore in the rectangle parallelogramme NB which is halfe the base of the cube, there shall remayne the same third part of the lesse segment. And by the same reason may we proue that in the other pyramids $ONDIG$, and $RLAFE$, and in the prisme $GOKERH$ is left the selfe same excesse of the base $LAND$, namely, the third part of the lesse segment. Wherefore the whole prisme containned betwene the triangles IGK and FEH , and vnder the length of the greater

Extend in the figure a line from the point E to the point B .



segment and two third partes of the lesse segment of BC the side of the cube, is equall to the solide composed of 4. bases of the dodecahedron and set vpon the base of the cube. Wherefore the base of that prisme wanteth of the whole base of the cube onely a third part of the lesse segment: and the altitude of that prisme was the line GO , which is the greater segment of halfe the side of the cube. And forasmuch as vnto the triangle IGK , is double the rectangle parallelogramme set vpon the same base IK , (the side of the cube) and vnder the altitude GO , by the 41. of the first: it followeth that three rectangle parallelogrammes set vpon the same base IK , the side of the cube, and vnder the altitude OG the greater segment of halfe the side of the cube, are sextuple to the triangle IGK . Wherefore those three rectangle parallelogrammes doo make one rectangle parallelogramme set vpon the base IK and vnder the altitude of the line GO tripled. But by the 7. diffinition of the eleuenth, there are sixe prismes equal and like vnto the foresayd prisme, being set vpon euery one of the sixe bases of the cube: which prismes are in proportion the one to the other as their bases are by the 3. corollary of the 7. of twelfth. Wherefore the solide composed of these sixe prismes, shall want of the base $ABCD$ the third part of the lesse segment, and taking his altitude of the foresayd rectangle parallelogramme, the sayd altitude shall be equall to three greater segmentes (one of which is GO) of halfe the side of the cube.

Now resteth to proue that these three segmentes want of the side of the cube by the lesse segment of the lesse segment of halfe the side of the cube. Suppose that AB the side of the cube be diuided into the greater segment AC , and into the lesse segment CB (by the 30. of the sixt). And diuide into two equall partes the line AC in the poynt G , and the line CB in the poynt E . And vnto the line CG put the line CL equall. Now forasmuch as the lines AG and GC are the greater segmentes of halfe the line AB , for eche of them is the halfe of the greater segment of the whole line AB : the lines EB and EC shall be the lesse segmentes of halfe the line AB . Wherefore the whole line CL is the greater segment, and the line CE is

E is the leſſe ſegment. But as the line C L is to the line C E, ſo is the line C E to the reſidue E L. Wherefore the line E L is the greater ſegment of the line C E, or of the line E B which is equal vnto it. Wherefore the reſidue L B is the leſſe ſegment of the ſame E B (which is the leſſe ſegment of halfe the ſide of the cube). But the lines A G, G C, and C L are three greater ſegmentes of the halfe of the whole line A B: which three greater ſegmentes make the altitude of the foreſayd ſolide: wherefore the altitude of the ſayd ſolide wanteth of A B the ſide of the cube by the line L B, which is the leſſe ſegment of the line B E. Which line B E againe is the leſſe ſegment of halfe the ſide A B of the cube. Wherefore the foreſayd ſolide conſiſting of the ſixe ſolides, whereby the dodecahedron exceedeth the cube inſcribed in it, is ſet vpon a baſe which wanteth of the baſe of the cube by a third part of the leſſe ſegment, and is vnder an altitude wanting of the ſide of the cube by the leſſe ſegment of the leſſe ſegment of halfe the ſide of the cube. The ſolide therefore of a dodecahedron exceedeth the ſolide of a cube inſcribed in it, by a parallelipedon, whoſe baſe wanteth of the baſe of the cube by a third part of the leſſe ſegment, and whoſe altitude wanteth of the altitude of the cube, by the leſſe ſegment of the leſſe ſegment of halfe the ſide of the cube.

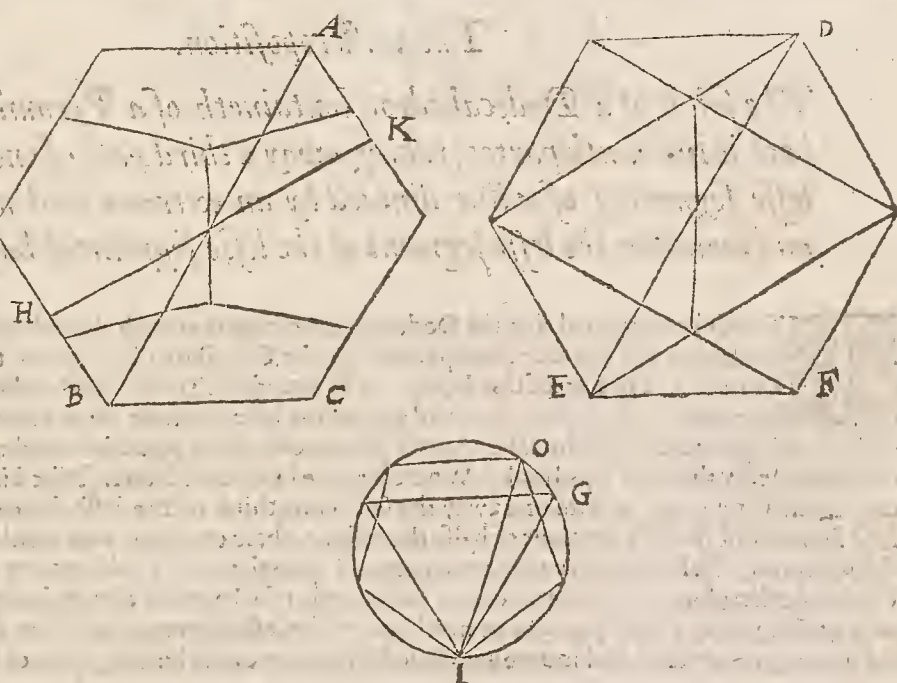
¶ A Corollary.

A Dodecahedron is double to a Cube inſcribed in it, taking away the third part of the leſſe ſegment of the cube, and moreouer the leſſe ſegment of the leſſe ſegment of halfe of that exceſſe. For if there be geuen a cube, from which is cut of a ſolide ſet vpon a third part of the leſſe ſegment of the baſe, and vnder one and the ſame altitude with the cube: that ſolide taken away hath to the whole ſolide the proportion of the ſection of the baſe to the baſe, by the 32. of the eleuenth. Wherefore from the cube is taken away a third part of the leſſe ſegment. Farther, forasmuch as the reſidue wanteth of the altitude of the cube, by the leſſe ſegment of the leſſe ſegment of halfe the altitude or ſide, and that reſidue is a parallelipedon, if it be cut by a plaine ſuperficiēs parallel to the oppoſite plaine ſuperficiēs, cutting the altitude of the cube by a point, it ſhall take away from that parallelipedon a ſolide, hauing to the whole the proportion of the ſection to the altitude, by the 3. Corollary of the 25. of the eleuenth. Wherefore the exceſſe wanteth of the ſame cube by the third part of the leſſe ſegment, and moreouer by the leſſe ſegment of the leſſe ſegment of halfe of that exceſſe.

¶ The 34. Propoſition.

The proportion of the ſolide of a Dodecahedron to the ſolide of an Icoſahedron inſcribed in it, conſiſteth of the proportion tripled of the diameter to that line which completh the oppoſite baſes of the Dodecahedron, and of the proportion of the ſide of the Cube to the ſide of the Icoſahedron inſcribed in one and the ſelfe ſame Sphere.

Suppoſe that A H B C K be a Dodecahedron, whose diameter let be A B: and let the line which completh the centres of the oppoſite baſes be K H: and let the Icoſahedron inſcribed in the Dodecahedron A B C, be D E F: whose diameter let be D E. Now forasmuch as one and the ſelfe ſame circle containeth the pentagon of a Dodecahedron, & the triangle of an Icoſahedron deſcribed in one and the ſelfe ſame Sphere, by the 14. of the fourthenth: Let that cir-



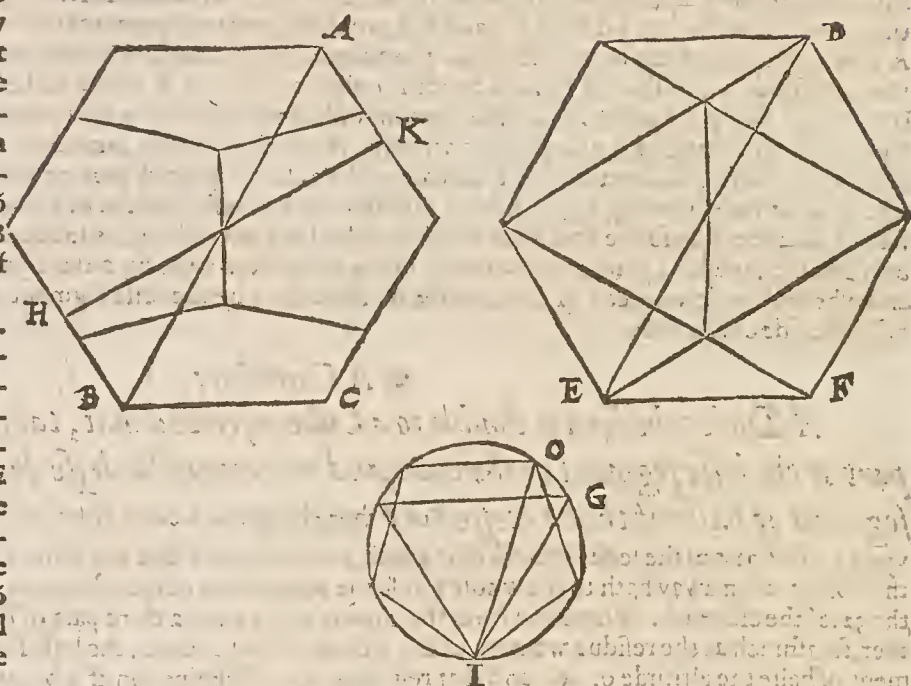
The sixteenth Booke of the

cle be I G O. Wherefore I O is the side of the cube, and I G the side of the Icosahedron, by the same. The I say, that the proportion of the Dodecahedron A H B C K to the Icosahedron D E F inscribed in it, consisteth of the proportion tripled of the line A B to the line K H; and of the proportion of the line I O to the line I G.

Demonstration.

For forasmuch as the Icosahedron D E F is inscribed in the Dodecahedron A B C, by supposition, the diameter D E shalbe equal to the line K H, by the 7. of the five tenth. Wherefore the Dodecahedron set vpon the diameter K H shall be inscribed in the same Sphere, wherein the Icosahedron D E F is inscribed:

but the Dodecahedron A H B C K is to the Dodecahedron vpon the diameter K H in triple proportion of that in which the diameter A B is to the diameter K H, by the Corollary of the 17. of the twelfth: and the same Dodecahedron which is set vpon the diameter K H, hath to the Icosahedron D E F (which is set vpon the same diameter, or vpon a diameter equall vnto it, namely, D E) that proportion which I O the side of the cube hath to I G the side of the Icosahedron, inscribed in one & the selfe same Sphere, by the 8 of the fourteenth. Wherefore the proportion of the Dodecahedron A H B C K to the Icosahedron D E F inscribed in it, consisteth of the proportion tripled of the diameter A B to the line K H, which coupleth the centres of the opposite bases of the Dodecahedron (which proportion is that which the Dodecahedron A H B C K hath to the Dodecahedron set vpon the diameter K H) and of the proportion of I O the side of the cube to I G the side of the Icosahedron (which is the proportion of the Dodecahedron set vpon the diameter K H to the Icosahedron D E F described in one and the selfe same Sphere) by the 5. definition of the sixth. The proportion therefore of the solide of a Dodecahedron to the solide of an Icosahedron inscribed in it, consisteth of the proportion tripled of the diameter to that line which coupleth the opposite bases of the Dodecahedron, and of the proportion of the side of the cube to the side of the Icosahedron inscribed in one and the selfe same Sphere.



The 35. Proposition.

The solide of a Dodecahedron containeth of a Pyramis circumscribed about it two ninth partes, taking away a third part of one ninth part of the lesse segment (of a line divided by an extreme and meane proportion) and moreover the lesse segment of the lesse segment of halfe the residue.

IT hath bene proued that the Dodecahedron, together with the cube inscribed in it is contained in one and the selfe same pyramis, by the Corollary of the first of this booke. And by the Corollary of the 33. of this booke, it is manifest, that the Dodecahedron is double to the same cube, taking away the third part of the lesse segment, and moreover the lesse segment of the lesse segment of halfe the residue, or of this excessse. But a pyramis is to the same cube inscribed in it noneuple, by the 30. of this booke. Wherefore the Dodecahedron inscribed in the pyramis, and containing the same cube twice, taking away the selfe same third of the lesse segment, and moreover the lesse segment of the lesse segment of halfe the residue, shall containe two ninth partes of the solide of the pyramis (of which ninth partes eche is equall vnto the cube) taking away this selfe same excessse. The solide therefore of a Dodecahedron containeth of a Pyramis circumscribed about it two ninth partes, taking away a third part of one ninth part of the lesse segment (of a line divided by an extreme and meane proportion) and moreover the lesse segment of the lesse segment of halfe the residue.

¶ The

¶ The 36. Proposition.

An Octohedron exceedeth an Icoſahedron inſcribed in it, by a parallelipedon ſet vpon the ſquare of the ſide of the Icoſahedron, and hauing to his altitude the line which is the greater ſegment of halfe the ſemidiameter of the Octohedron.

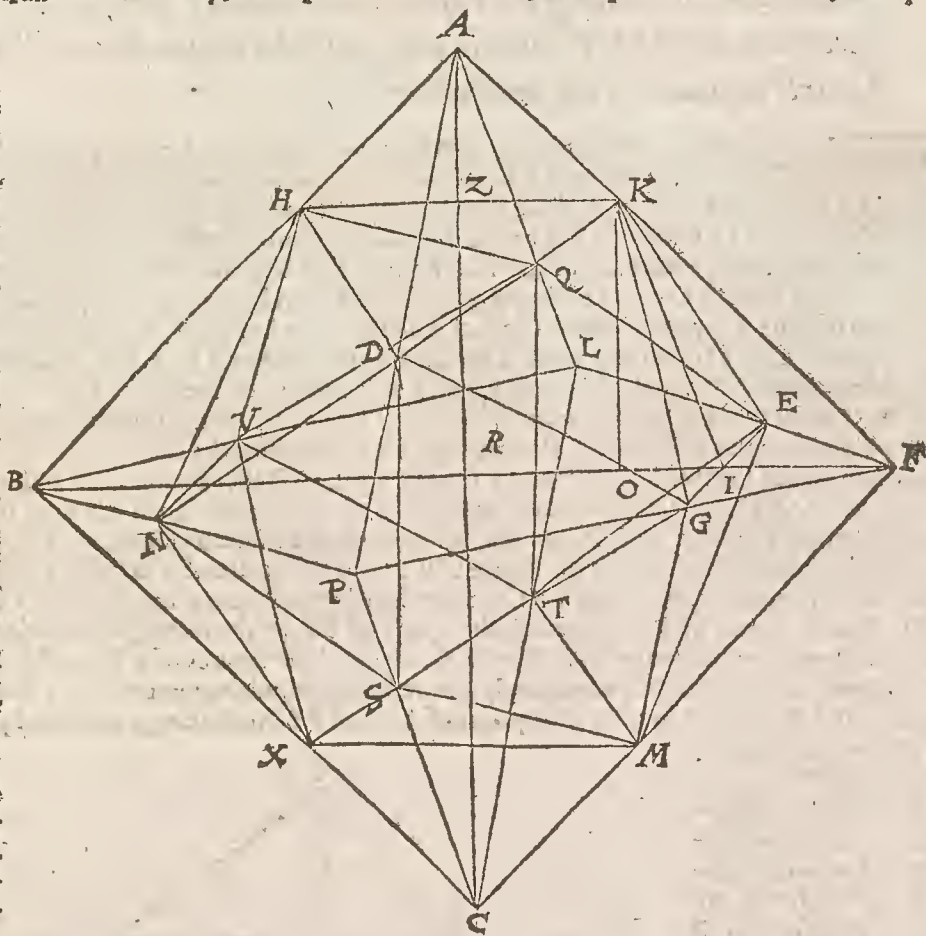


Vppoſe that there be an Octohedron $ABCFPL$: in which let there be inſcribed an Icoſahedron $HKEGMXNVDSQT$, by the 16. of the ſiuetenth. And draw the diameters $AZRC$, $BROIF$, and the perpendicular KO parallel to the line AZR . Then I ſay, that the Octohedron $ABCFPL$ is greater then the Icoſahedron inſcribed in it, by a parallelipedon ſet vpon the ſquare of the ſide HK or GE , and hauing to his altitude the line KO or RZ : which is the greater ſegment of the ſemidiameter AR . Forasmuch

Construction.

Demonstration.

as in the ſame 16. it hath bene proued, that the triangles KDG and KEQ are deſcribed in the baſes APF and ALF of the Octohedron: therefore about the ſolide angle there remaine vpon the baſe $FE G$ three triangles KEG , KFE , and KFG , which containe a pyramis $KEFG$. Vnto which pyramis ſhall be equall and like the oppoſite pyramis $MEFG$ ſet vpon the ſame baſe $FE G$, by the 8. definition of the eleuenth. And by the ſame reaſon ſhall there at euery ſolide angle of the Octohedron remayne two pyramids equall and like: namely, two vpon the baſe AHK , two vpon the baſe BNV , two vpon the baſe DPS , and moreover two vpon the baſe QUT . Now theſe there ſhall be made twelue pyramids, ſet vpon a baſe contained of the ſide of the Icoſahedron, and vnder two leſſe ſegmentes of the ſide of the Octohedron containing a right angle, as for example the baſe GEF , And forasmuch as the ſide GE ſubtending a right angle, is, by the 47. of the firſt, in power duple to either of the lines EF and FG , and ſo the ſide KH is in power duple to either of the ſides AH and AK : and either of the lines AH , AK , or EF , FG , is in power duple to eyther of the lines AZ or ZK which cōtaine



a right angle, made in the triangle or baſe AHK by the perpendicular AZ . Wherefore it followeth that the ſide GE or HK , is in power quadruple to the triangles GEF or AHK . But the pyramis $KEFG$, hauing his baſe $FE G$ in the plane $FLBP$ of the Octohedron, ſhall haue to his altitude the perpendicular KO (by the 4. definition of the ſixth) which is the greater ſegment of the ſemidiameter of the Octohedron, by the 16. of the ſiuetenth. Wherefore three pyramids ſet vnder the ſame altitude and vpon equall baſes, ſhall be equall to one priſme ſet vpon the ſame baſe, and vnder the ſame altitude, by the 1. Corollary of the 7. of the twelfth. Wherefore 4. priſmes ſet vpon the baſe GEF quadrupled (which is equall to the ſquare of the ſide GE) and vnder the altitude KO (or RZ the greater ſegment which is equall to KO) ſhall containe a ſolide equall to the twelue pyramids, which twelue pyramids make the exceſſe of the Octohedron about the Icoſahedron inſcribed in it. An Octohedron therefore exceedeth an Icoſahedron inſcribed in it, by a parallelipedon ſet vpon the ſquare of the ſide of the Icoſahedron, and hauing to his altitude the line which is the greater ſegment of halfe the ſemidiameter of the Octohedron.

¶ A Corollary.

The sixteenth Booke of the

A Corollary.

A Pyramis exceedeth the double of an Icosahedron inscribed in it, by a solide set vpon the square of the side of the Icosahedron inscribed in it, and hauing to his altitude that whole line of which the side of the Icosahedron is the greater segmēt. For it is manifest by the 19. of the fuetēth, that an octohedrō & an Icosahedrō inscribed in it are inscribed in one & the self same pyramis. It hath moreouer bene proued in the 26. of this boke, that a pyramis is double to an octohedrō inscribed in it. Wherefore the two excesses of the two octohedrons (vnto which the pyramis is equal) aboue the two Icosahedrons (inscribed in the said two octohedrons) being brought into an solide, the said solide shalbe set vpon the selfe same square of the side of the Icosahedron, and shall haue to his altitude the perpendicular K O doubled: whose double coupling the opposite sides H K and X M maketh the greater segmeat the same side of the Icosahedron, by the first and second corollary of the 14. of the fuetēth.

The 37. Proposition.

If in a triangle hauing to his base a rational line set, the sides be commensurable in power to the base, and from the toppe be drawn to the base a perpendicular line cutting the base: The sections of the base shall be commensurable in length to the whole base, and the perpendicular shall be commensurable in power to the said whole base.

Construction.



Suppose that there be a triangle A B G, whose base B G let be a rational line set of purpose.

And let the sides A B and A G be vnto the same B G commensurable at the least in power.

And from the toppe A, draw vnto the base B G, a perpendicular, cutting the base in the point

P. Then I say that the sections of the base, are commensurable in lengthe to the whole line

B G, and that the perpendicular A P, is vnto the same base B G cōmensurable at the least in power. Pro-

duce on either side the line B G to the poyntes C and E. And vnto the line A G put the line G E equal,

and vnto the line A B put the line B C equal. And vpon the lines C B, B G and G E describe squares B-

K, B D, and G L. And from the greater of the squares of the lines A B or A G, which let be G L cut of a

parallelogramme E M equal to the lesse square B K (by the 45. of the first:) And (by the same) vnto the

residue G M, let there be applied vpon the line G D an equall rectangle parallelogramme QD. Now for-

asmuch as the angles A P B and A P G are right angles, therefore (by the 47. of the first) the line A G con-

taineth in power the two lines A P and P G, and the line A B the two lines A P and P B. Wherefore how

much the line A G containeth in power more then the line A B, so much also doth the line P G contain

in power more then the line B P: namely, taking away the common square of A P, there is left the ex-

cesse of the square of P G aboue the square of B P. But the square of A G (which is G L) exceedeth the

square of A B (namely, the square B K) by the rectangle parallelogramme G M or O D, by construction.

Wherefore the square of P G exceedeth the square of B P, by the rectangle parallelogramme O D. And

forasmuch as vnto the squares of A B and A G, are equal the squares of A P and P B, and of A P and P-

G: and their excesse is taken away, namely, the rectangle parallelogramme O D: there shall be left the

squares of A P and P O equal to the squares of A P and P B. And taking away the square of A P which

is cōmon, the residues

namely, the squares of

B P and P O shall be e-

qual: and therefore

their sides (namely, the

lines B P and P O) are

equal. And forasmuch

as the squares G L and

B K are (by suppositiō)

rational, and therefore

cōmensurable their ex-

cesse O D, shall be com-

mensurable vnto the by

the 15. of the tēth. And

therefore it is rationall

by the 9. diffinition of

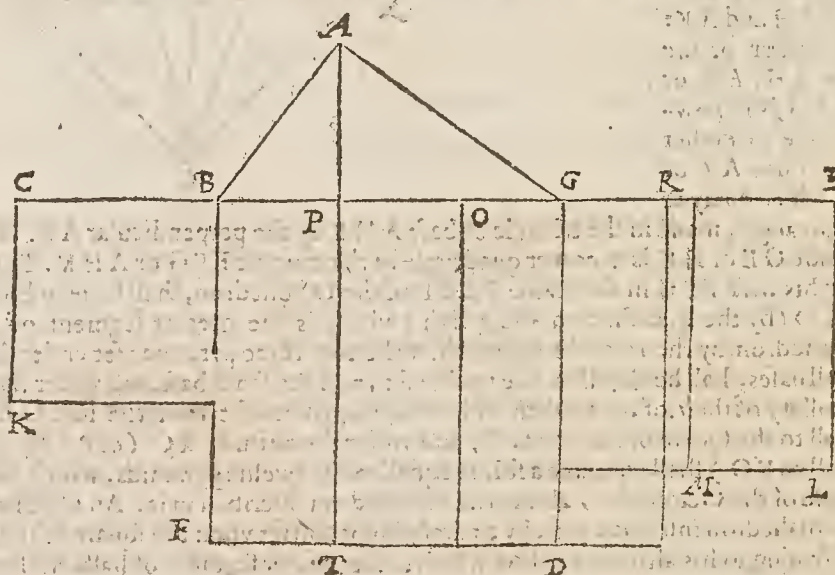
the tēth. Wherefore the

rational parallelogram

O D, being applied vp-

on the rational line G-

D (or B G) maketh the



breadth

breadth OG rational and cōmensurable in length to the whole line BG , by the 20. of the tenth. But if the whole line BG be cōmensurable to one of the partes OG , the lines BO , OG , and BG shall be cōmensurable, by the same 15. of the tenth. Wherefore also the line OG shall be cōmensurable to the half of the line BO , namely, to the line PO , or PB , by the 12. of the tenth. And forasmuch as the two lines PO and OG are cōmensurable, the whole line PG shall be cōmensurable to the line PO , or to the line PB , by the same 15. of the tenth. Wherefore either of the lines PG and PB shall be cōmensurable vnto the whole line PB , by the same. Wherefore the lines BG , PB and PG haue the one to the other that proportion which numbers haue, by the 5. of thirteenth. Wherefore the sections PB and PG of the base BG are cōmensurable in length to the same base, by the 6. of the tenth.

And now that the perpendicular AP is cōmensurable in power to the base BG , is thus proued. Forasmuch as the square of AB is by supposition, cōmensurable to the square of BG : and vnto the rational square of AB is cōmensurable the rational square of BP (by the 12. of the eleuenth) Wherefore the residue, namely, the square of PA is cōmensurable to the same square of BP , by the 2. part of the 15. of the eleuenth. Wherefore by the 12. of the tenth, the square of PA is cōmensurable to the whole square of BG . Wherefore the perpendicular AP is cōmensurable in power to the base BG , by the 3. definition of the tenth: which was required to be proued.

In demonstrating of this, we made no mention at all of the length of the sides AB and AG , but only of the length of the base BG : for that the line BG is the rational line first set: and the other lines AB and AG are supposed to be cōmensurable in power only to the line BG . Wherefore if that be plainly demonstrated, when the sides are cōmensurable in power only to the base, much more easily wil it follow, if the same sides be supposed to be cōmensurable both in length and in power to the base: that is, if their lengthes be expressed by the rootes of square numbers.

*Second part
of the De-
monstration.*

¶ A Corollary. 1.

By the former things demonstrated, it is manifest that if from the powers of the base, and of one of the sides, be taken away the power of the other side, and if the halfe of the power remaining, be applied vpon the whole base, it shall make the breadth that section of the base which is coupled to the first side. For from the powers of the base BG , and of one of the sides AG , that is, from the squares BD and GL , the power of the other side AB , namely the square BK (or the parallelogramme EM) is taken away. And of the residue, (namely, of the square BD , and of the parallelogramme OD , or DR , which by supposition is equal vnto OD) the halfe (namely, of the whole FR , which is PD , for the lines GR and PB are equal to the lines GO and PO) is applied to the whole line BG or GD : and maketh the breadth the line PG the section of the base BG , which section is coupled to the first side AG . And by the same reason in the other side, if from the squares BD and BK be taken away the square GL , there shall remaine the rectangle parallelogramme FO : For the parallelogramme EM is equal to the square BK , and the parallelogramme GM to the parallelogramme OD . Wherefore FP the halfe of the residue FO , maketh the breadth BP , which is coupled to the first side taken AB .

* A Corollary. 2.

If a perpendicular drawne from an angle of a triangle do cut the base: the sections are to the other sides in power proportionall by an Arithmetical proportion. For it was proued that the excesse of the powers of the lines AG and AB is one and the same with the excesse of the powers of the lines PG and PB . If therefore the powers do equally exceede the one the other, they shall by an Arithmetical proportion, be proportionall.

The ende of the sixtenth Booke
of the Elementes of Geometrie added

by Flusſas.

Flußas, of mixt and

 *A brieft treatise, added by Flußas, of mixt and
composed regular solides.*



Regular solides are sayd to be composed and mixt, when ech of them is transformed into other solides, keeping still the forme, number, and inclination of the bases, which they before had one to the other: some of which yet are transformed into mixt solides, and other some into simple. Into mixt, as a Dodecahedron and an Icosahedron: which are transformed or altered, if ye diuide their sides into two equall partes, and take away the solide angles subtended of plaine superficiall figures made by the lines coupling those

Icosidodecahedron.

Exoctohedron.

middle sections: for the solide remainyng after the taking away of those solide angles, is called an Icosidodecahedron. If ye diuide the sides of a cube and of an Octohedron into two equall partes, and couple the sections, the solide angles subtended of the plaine superfiieces made by the coupling lines, being taken away, there shall be left a solide, which is called an Exoctohedron. So that both of a Dodecahedron and also of an Icosahedron, the solide which is made, shall be called an Icosidodecahedron: and likewise the solide made of a Cube & also of an Octohedron, shall be called an Exoctohedron. But the other solide, namely, a Pyramis (or Tetrahedron) is transformed into a simple solide: for if ye diuide into two equall partes euery one of the sides of the pyramis, triangles described of the lines which couple the sections, and subtending, and taking away solide angles of the pyramis, are equall and like vnto the equilater triangles left in euery one of the bases: of all which triangles is produced an Octohedron, namely, a simple and not a composed solide. For the Octohedron hath fower bases, like in number, forme, and mutuall inclination with the bases of the pyramis: and hath the other fower bases with like situation opposite and parallel to the former. Wherefore the application of the pyramis taken twise, maketh a simple Octohedron, as the other solides make a mixt compound solide.

¶ First Definition.

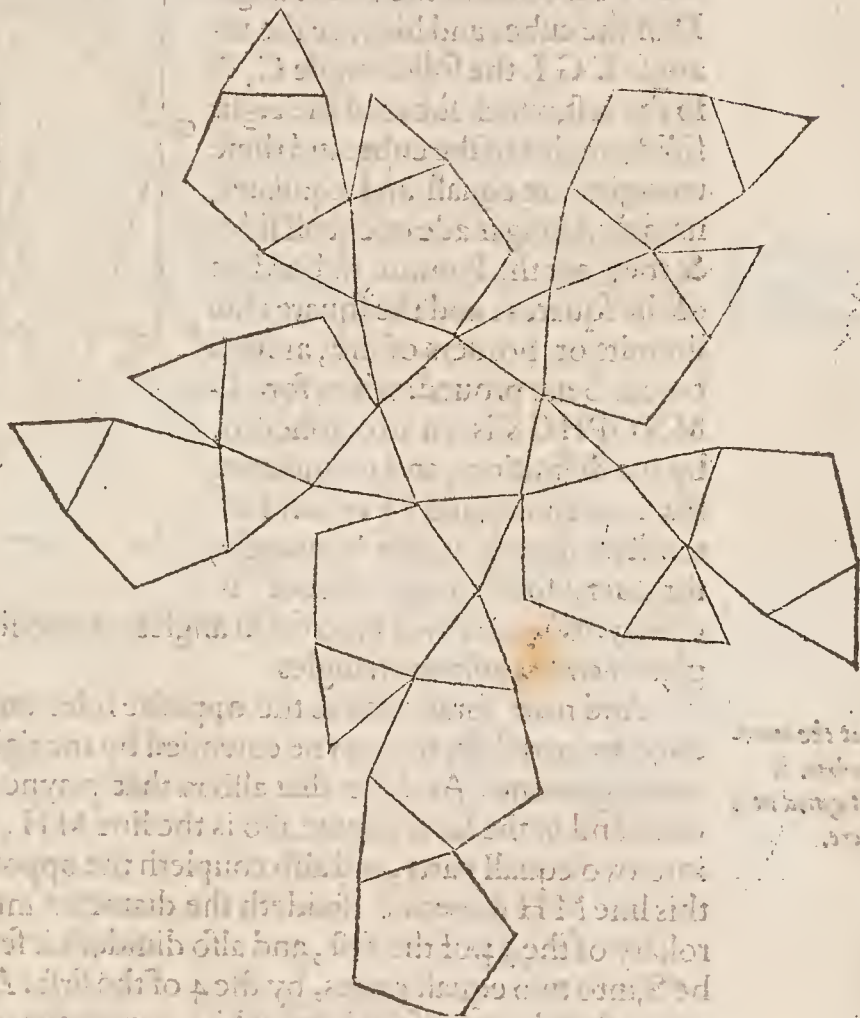
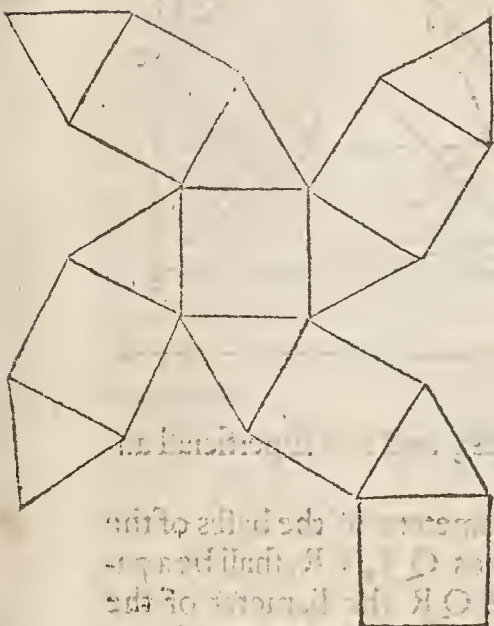
An Exoctohedron is a solide figure contained of sixe equall squares, and eight equilater and equall triangles.

¶ Second Definition.

An Icosidodecahedron is a solide figure, contained vnder twelue equilater equall and equiangle Pentagons, and twentie equall and equilater triangles.

For the better vnderstanding of the two former definitions, and also of the two Propositions following, I haue here set two figures, whose formes, if ye first describe

describe vpon pasted paper or such like matter, and then cut them and folde them accordingly, they will represent vnto you the perfect formes of an Exoctohedron and of an Icosidodecahedron.



The first Probleme.

To describe an equilater and equiangle exoctohedron, and to contayne it in a sphere geuen: and to proue that the diameter of the sphere is double to the side of the sayd exoctohedron.



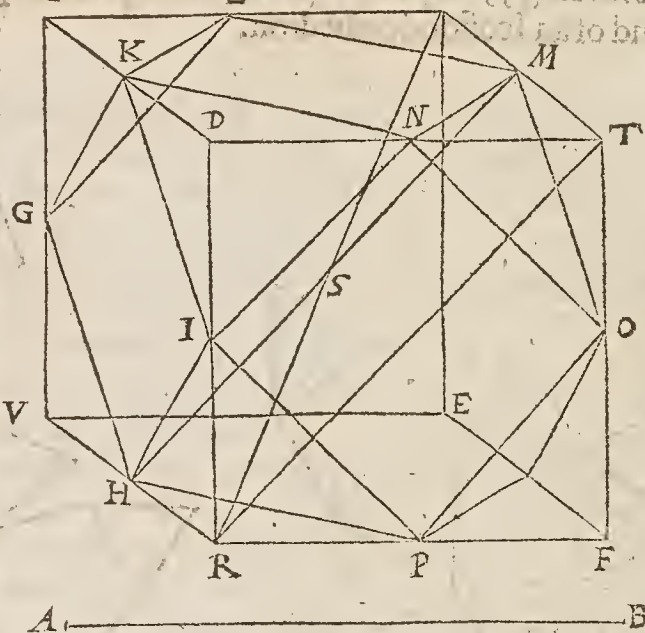
Vppose that there be a sphere geuen, whose diameter let be AB . And about the diameter AB let there be described a square by the sixth of the fourth: and vpon the square let there be described a cube by the 15. of the thirteenth: which let be $CDEFQTVR$: and let the diameter thereof be QR , and the centre S . And diuide the sides of the cube into two equall partes, in the poyntes $G, H, I, K, L, M, N, O, P$. &c. And couple the middle sections by the right lines IN, NO, OP, PI and such like, which subtend the angles of the squares or bases of the cube: and they are equall by the 4. first, and contayne right angles, as the angle NIP . For the angle NDI which is at the base of the Isosceles triangle NDI , is the halfe of a right angle, and so likewise is the opposite angle RIP . Wherefore the residue NIP is a right angle, and so the rest. Wherefore $NIP-O$ is a square. And by the same reason shall the rest $NMLK, K GHI$ &c. inscribed

Construction
of the exocto-
hedron.

Flussas, of mixt and

in the bases of the cube, be squares: and they shall be fixe in number, according to the nūber of the bases of the cube.

Agayne forasmuch as the triangle KIN subtendeth the solide angle D of the cube, and likewise the triangle $KG L$ the solide angle C , & so the rest, which subtend the eight solide angles of the cube: and these triangles are equall and equilater, namely, being made of equall sides & they are the limmits or borders of the squares, and the squares the limmits or borders of the, as hath before bene proued: wherefore $L-MNOPH G K$ is an exoctohedrō, by the diffinition, and is equilater, for it is containd of equall subtendent lines: it is also equiangle, for euery solide angle thereof, is containd vnder two superficial angles of two squares, and two superficial angles of two equilater triangles.



That the exoctohedron is containd in a sphere.

And now forasmuch as the opposite sides and diameters of the bases of the cube are parallels, the playne extended by the right lines $Q T$, $V R$, shall be a parallelogramme. And for that also in that playne lyeth $Q R$ the diameter of the cube, and in the same playne also is the line $M H$, which diuideth the sayd playne into two equall partes, and also coupleth the opposite angles of the exoctohedron this line $M H$ therefore diuideth the diameter into two equall partes, by the collary of the 34. of the first, and also diuideth it selfe in the same poynt, which let be S , into two equall partes, by the 4 of the first. And by the same reason may we proue that the rest of the lines, which couple the opposite angles of the exoctohedron, doo in S the centre of the cube diuide the one the other into two equall partes. For euery one of the angles of the exoctohedron are set in euery one of the bases of the cube. Wherefore making the centre the poynt S , and the space $S H$ or $S M$, describe a sphere, and it shall touch euery one of the angles equedistant from the poynt S .

That the exoctohedron is containd in the sphere geuen.

And forasmuch as $A B$ the diameter of the sphere geuen, is put equall to the diameter of the base of the cube, namely, to the line $R T$, and the same line $R T$ is equall to the line $M H$, by the 33. of the first: which line $M H$ coupling the opposite angles of the exoctohedron, is drawne by the centre: wherefore it is the diameter of the sphere geuen which containeth the exoctohedron.

That the diameter of the sphere is double to the side of the exoctohedron.

Finally forasmuch as in the triangle $R F T$, the line $P O$ doth cutte the sides into two equall partes, it shall cutte them proportionally with the bases, namely, as $F R$ is to $F P$, so shall $R T$ be to $P O$, by the 2. of the sixth. But $F R$ is double to $F P$, by supposition: wherefore $R T$, or the diameter $H M$, is also double to the line $P O$ the side of the exoctohedron. Wherefore we haue described an equilater & equiangle exoctohedron, and comprehended it in a sphere geuen, and haue proued that the diameter of the sphere is double to the side of the exoctohedron.

The

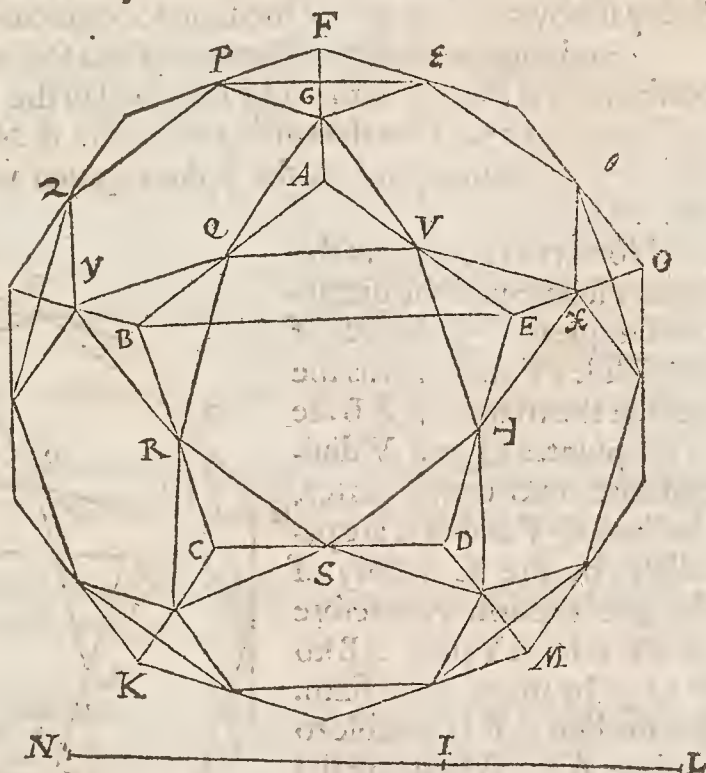
¶ The 2. Probleme.

To describe an equilater & equiangle Icosidodecahedron, & to comprehend it in a sphere geuen: and to proue that the diameter being diuided by an extreame and meane proportion, maketh the greater segment double to the side of the Icosidodecahedron.



Vppose that the diameter of the sphere geuen be NL , and (by the 30. of the sixth) diuide the line NL by an extreame and meane proportion in the poynt I : and the greater segment thereof let be NI . And vpo the line NI describe a cube by the 15. of the thirteenth: and about this cube let there be circumscribed a dodecahedron, by the 17. of the thirteenth: & let the same be $ABCDEFHKMO$.

And diuide euery one of the sides into two equall parts in the poynts $Q, R, S, T, V, X, Y, Z, P, \epsilon, \theta, G$. &c: and couple the sections with right lines, which shall subtend the angles of the pentagons, as the lines $PG, GV, VQ, QY, YR, RQ, VT, TX, XV$, and so the rest. Now forasmuch as these lines subtend equall angles of the pentagons, and those equall angles are containd of equall sides (namely of the halues of the sides of the pentagons: therefore those subtending lines are equall, by the 4. of the first. Wherefore the triangles GQV, YQR, VXT , and the rest which take away solide angles of the dodecahedron, are equilater. Agayne forasmuch as in euery pentagon are described fise equall right lines, coupling the middle sections of the sides, as are the lines QV, VT, TS, SR, RQ : they describe a pentagon in the playne of the pentagon of the dodecahedron: and the sayd pentagon is containd in a circle, namely, whose centre is the centre of a pentagon of the dodecahedron. For the lines drawne from that centre to the angles of this pentagon are equall, for that they are perpendiculars vpon the bases cutte, by the 12. of the fourth. Wherefore the pentagon $QRSTV$ is equiangle, by the 11. of the same. And by the same reason may the rest of the pentagons described in the bases of the dodecahedron be proued equall and like. Wherefore those pentagons are 12. in number: And forasmuch as the equall and like triangles, doo subtend and take away 20. solide angles of the dodecahedron, therefore the sayd triangles shall be 20. in nuber. Wherefore we haue described an Icosidodecahedron by the diffinitio, which Icosidodecahedron is equilater, for that all the sides of the triangles are equal & comon with the pentagons: and it is also equiangle. For euery one of the solide angles is made of two superficial angles of an equilater pentagon, and of two superficial angles of an equilater triangle.



Construction
of the Icosi-
dodecahedron.

Flussas, of mixt and

That the Icosidodecahedron is containd in the sphere geuen.

Now let vs proue that it is contained in the Sphere geuen, whose diameter is NL . Forasmuch as perpendiculars drawn from the centres of the Dodecahedron, to the middle sections of his sides, are the halves of the lines, which couple the opposite middle sections of the sides of the Dodecahedron, by the 3. Corollary of the 17. of the thirteenth: which lines also, by the same Corollary, do in the centre diuide the one the other into two equall partes: therefore right lines drawn from that point to the angles of the Icosidodecahedron (which are set in those middle sections) are equall: which lines are 30. in number according to the number of the sides of the Dodecahedron: for euery one of the angles of the Icosidodecahedron are set in the middle sections of euery one of the sides, of the Dodecahedron. Wherefore making the centre the centre of the Dodecahedron, and the space any one of the lines drawn from the centre to the middle sections, describe a Sphere, and it shall passe by all the angles of the Icosidodecahedron, and shall containe it.

And forasmuch as the diameter of this solide, is that right line, whose greater segment is the side of the cube inscribed in the Dodecahedron, by the 14. Corollary of the 17. of the thirteenth, which side is NI , by supposition. Wherefore that solide is containd in the Sphere geuen whose diameter is put to be the line NL .

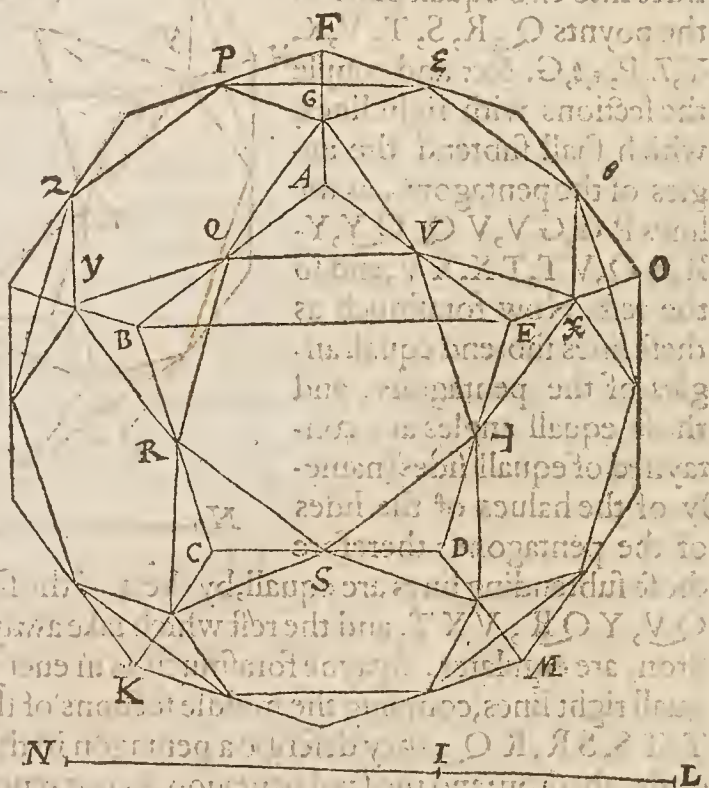
That the diameter being deuided by an extreme and meane proportion. &c.

Now let vs proue that the greater segment of the diameter is duple to QV the side of this solide. Forasmuch as the sides of the triangle AEB are in the pointes Q and V diuided into two equall partes, the lines QV and BE are parallels, by the Corollary of the 39. of the first. Wherefore as AE is to AV , so is EB to VQ , by the 2. of the sixth. But the line AE is double to the line AV . Wherefore the line BE is double to the line QV : by the 4. of the sixth. Now the line BE is equal to NI , or to the side of the cube, by the 2. Corollary of the 17. of the thirteenth, which line NI is the greater segment of the diameter NL . Wherefore, the greater segment of the diameter geuen, is double to the side of the Icosidodecahedron inscribed in the Sphere geuen. Wherefore we haue described an equilater and equiangle Icosidodecahedron, and contained it in a Sphere geuen, and haue proued that the diameter thereof being diuided by an extreme and meane proportion, maketh hys greater segment double to the side of the Icosidodecahedron.

¶ An aduertisment of Flussas.

To the vnderstanding of the nature of this Icosidodecahedron, ye must well conceaue the passions and proprieties of both those solides, of whose bases it consisteth, namely, of the Icosahedron and of the Dodecahedron. And although in it

the



the bases are placed oppositely, yet haue they one to the other one & the same inclination. By reason wherof there lie hidden in it the actions and passions of the other regular solides. And I would haue thought it not impertinent to the purpose to haue set forth the inscriptions and circumscriptions of this solide, if want of time had not hindred. But to the end the reader may the better attaine to the vnderstanding therof, I haue here following briefly set forth, how it may in or about euery one of the fiue regular solides be inscribed or circumscribed: by the helpe whereof he may, with small trauaile or rather none at all, so that he haue well peysed and considered the demonstrations pertayning to the foresayd fiue regular solides, demonstrate both the inscription of the sayd solides in it, and the inscription of it in the sayd solides.

¶ Of the inscriptions and circumscriptions of an Icosidodecahedron.

An Icosidodecahedron may containe the other fiue regular bodyes. For it will receaue the angles of a Dodecahedron, in the centres of the triangles which subtend the solide angles of the Dodecahedron: which solide angles are 20. in number, and are placed in the same order in which the solide angles of the Dodecahedron taken away or subtended by them, are. And by that reason it shall receaue a Cube and a Pyramis containd in the Dodecahedron: when as the angles of the one are set in the angles of the other.

An Icosidodecahedron receaueth an Octohedron, in the angles cutting the fixe opposite sections of the Dodecahedron, euen as if it were a simple Dodecahedron.

And it containeth an Icosahedron, placing the 12. angles of the Icosahedron in the selfe same centres of the 12. Pentagons.

It may also by the same reason be inscribed in euery one of the fiue regular bodies: namely, in a Pyramis, if ye place 4. triangular bases concentricall with 4. bases of the Pyramis, after the same maner, that ye inscribed an Icosahedron in a Pyramis. So likewise may it be inscribed in an Octohedron, if ye make 8. bases thereof concentricall with the 8. bases of the Octohedron. It shall also be inscribed in a Cube, if ye place the angles which receaue the Octohedron inscribed in it, in the centres of the bases of the Cube. Moreouer, ye shall inscribe it in an Icosahedron, when the triangles compased in. of the Pentagon bases, are concentricall with the triangles, which make a solide angle of the Icosahedron. Finally, it shall be inscribed in a Dodecahedron, if ye place euery one of the angles thereof in the middle sections of the sides of the Dodecahedron, according to the order of the construction thereof.

The opposite plaine superficieses also of this solide are parallels. For the opposite solide angles are subtended of parallel plaine superficieses, as well in the angles of the Dodecahedron subtended by triangles, as in the angles of the Icosahedron subtended of Pentagons, which thing may easily be demonstrated. Moreouer in this solide are infinite properties & passions, springing of the solides wherof it is composed.

Wherefore it is manifest that a Dodecahedron & an Icosahedron, mixed, are transformed

Flussas, of mixt and

transformed into one & the selfe same solide of an Icosidodecahedron. A cube also and an octohedrō are mixed and altered into an other solide, namely, into one and the same Exoctohedron. But a pyramis is transformed into a simple and perfect solide, namely into an Octohedron.

If we will frame these two solides ioyned together into one solide, this onely must we obserue.

In the pentagon of a dodecahedron inscribe a like pentagon, so that let the angles of the pentagon inscribed be set in the middle sections of the sides of the pentagon circumscribed, and then vpon the said pentagon inscribed, let there be set a solide angle of an Icosahedron, and so obserue the selfe same order in euery one of the bases of the Dodecahedron: and the solide angles of the Icosahedron set vpon these pentagons shall produce a solide consisting of the whole Dodecahedron, and of the whole Icosahedron. In like sort, if in euery base of the Icosahedrō, the sides being diuided into two equall partes be inscribed an equilater triangle, and vpon euery one of those equilater triangles be set a solide angle of a Dodecahedron: there shall be produced the selfe same solide consisting of the whole Icosahedron, & of the whole Dodecahedron.

And after the same order, if in the bases of a cube, be inscribed squares subtending the solide angles of an Octohedron, or in the bases of an Octohedron, be inscribed equilater triangles subtending the solide angles of a cube, there shall be produced a solide consisting of either of the whole solides, namely, of the whole cube and of the whole Octohedron.

But equilater triangles inscribed in the bases of a pyramis, hauing their angles set in the middle sections of the sides of the pyramis, and the solide angles of a pyramis set vpon the sayd equilater triangles, there shall be produced a solide, consisting of two equal and like pyramids.

And now if in these solides thus composed, ye take away the solide angles, there shalbe restored againe the first composed solides: namely, the solide angles taken away from a Dodecahedron and an Icosahedron composed into one, there shalbe left an Icosidodecahedron: the solide angles takē away from a cube, and an octohedrō cōposed into one solide, there shalbe left an exocthedrō. Moreover the solide angles taken away from two pyramids composed into one solide, there shalbe left an Octohedron.

Flussas after this setteth forth certaine passions and properties of the five simple regular bodies: which although he demonstrateth not, yet are they not hard to be demonstrated, if we wel please and conceiue that, which in the former bookes hath bene taught touching those solides.

Of the nature of a trilater and equilater Pyramis.

1. A trilater equilater Pyramis, is deuided into two equal partes, by three equal squares, which in the centre of the pyramis cutte the one the other into two equal partes, and perpendicularly, and whose angles are set in the middle sections of the sides of the pyramis.
2. From a pyramis are taken away 4. pyramids like vnto the whole, which vtterly take away the sides of the pyramis, and that which is left is

is an octohedrō inscribed in the pyramis in which all the solides inscribed in the pyramis are contained. A perpendicular drawne from the angle of the pyramis to the base, is double to the diameter of the cube inscribed in it. And a right line coupling the middle sections of the opposite sides of the pyramis, is triple to the side of the selfe same cube. The side also of the pyramis is triple to the diameter of the base of the cube. Wherefore the same side of the pyramis is in power duple to the right line which coupleth the middle sections of the opposite sides. And it is in power sesquialter to the perpendicular which is drawne from the angle to the base. Wherefore the perpendicular is in power sesquitertia to the line which coupleth the middle sections of the opposite sides. A pyramis, and an Octohedron inscribed in it, also an Icosahedron inscribed in the same Octohedron, doo containe one and the selfe same sphere.

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Of the nature of an Octohedron.

Foure perpendiculars of an Octohedron, drawne in 4. bases therof from two opposite angles of the said Octohedron, and coupled together by those 4. bases, describe a Rhombus, or diamond figure: one of whose diameters is in power duple to the other diameter. For it hath the same proportiō that the diameter of the Octohedron, hath to the side of the Octohedron. An Octohedron & an Icosahedron inscribed in it, do containe one and the selfe same sphere. The diameter of the solide of the Octohedron, is in power sesquialter to the diameter of the circle which containeth the base: and is in power triple to the right line which coupleth the centres of the opposite bases: and is in power *duple superbipartiens tercius to the perpendicular or side of the foresaid Rhombus: and moreover is in length triple to the line which coupleth the centres of the next bases. The angle of the inclination of the bases of the Octohedron, doth with the angle of the inclination of the bases of the pyramis, make angles equal to two right angles.

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*That is as 3.
103.
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Of the nature of a Cube.

The diameter of a cube, is in power sesquialter to the diameter of his base: and is in power triple to his side: and vnto the line which coupleth the centres of the next bases, it is in power sextuple. Moreover the side of the cube is to the side of the Icosahedron inscribed in it, as the whole is to the greater segment: vnto the side of the Dodecahedron, it is as the whole is to the lesse segment: vnto the side of the Octohedron, it is in power duple: and vnto the side of the pyramis, it is in power subduple. Moreover the cube is triple to the pyramis: but to the cube the Dodecahedron is in a maner duple. Wherefore the same Dodecahedron is in a maner sextuple to the sayd pyramis.

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Of the nature of an Icosahedron.

Fiue triangles of an Icosahedron, do make a solide angle, the bases of which triangles make a pentagon. If therefore from the opposite bases of the Icosahedron

1.

Flußas, of mixt and

be taken the other pentagon by them described, these pentagons shall in such sort cut the diameter of the Icosahedron which coupleth the forsaide opposite angles, that that part which is contained betweene the plaines of those two pentagons, shalbe the greater segment: and the residue which is drawne from the plaine to the angle shall be the lesse segment. If the opposite angles of two bases ioyned together, be coupled by a right line, the greater segment of that right line is the side of the Icosahedron. A line drawne from the centre of the Icosahedron to the angles, is in power quintuple, to halfe that line which is take betweene the pentagons, or of the halfe of that line which is drawne from the centre of the circle which containeth the foresaide pentagon: which two lines are therefore equall. The side of the Icosahedron containeth in power either of them, and also the lesse segment, namely, the line which falleth from the solide angle to the pentagon. The diameter of the Icosahedron containeth in power the whole line, which coupleth the opposite angles of the bases ioyned together, and the greater segment thereof, namely, the side of the Icosahedron. The diameter also is in power quintuple to the line which was taken betweene the pentagons, or to the line which is drawne from the centre to the circumference of the circle which containeth the pentagon composed of the sides of the Icosahedron. The dimetient containeth in power the right line which coupleth the centres of the opposite bases of the Icosahedron, and the diameter of the circle which containeth the base. Moreouer the sayd dimetient containeth in power the diameter of the circle, which containeth the pentagon, and also the line which is drawne from the centre of the same circle to the circumference: That is, it is quintuple to the line drawne from the centre to the circumference. The line which coupleth the centres of the opposite bases, containeth in power the line which coupleth the centres of the next bases, and also the rest of that line of which the side of the cube inscribed in the Icosahedron is the greater segment. The line which coupleth the middle sections of the opposite sides, is triple to the side of the dodecahedron inscribed in it. Wherefore if the side of the Icosahedron, and the greater segment thereof be made one line, the third part of the whole, is the side of the dodecahedron inscribed in the Icosahedron.

Of the nature of a Dodecahedron.

The diameter of a dodecahedron containeth in power the side of the dodecahedron, and also that right line, vnto which the side of the dodecahedron is the lesse segment, and the side of the cube inscribed in it is the greater segment: which line is that which subtendeth the angle of the inclination of the bases, containd vnder two perpendiculars of the bases of the dodecahedron. If there be taken two bases of the dodecahedron distant the one from the other by the length of one of the sides, a right line coupling their centres, being diuided by an extreame and meane proportion, maketh the greater segment the right line which coupleth the centres of the next bases. If by the centres of fiue bases set vpon one base, be drawne a playne superficies, and by the centres of the bases which are set vpon the opposite base be drawne also a playne superficies, and then be drawne a right line coupling the centres of the opposite bases, that right line is so cut, that eche of his partes set without the playne superficies, is the greater segment of that part which is containd betweene the playnes. The side of the dodecahedron is the greater segment

segment of the line which subtendeth the angle of the pentagon. A perpendicular line drawne from the centre of the dodecahedron to one of the bases, is in power quintuple to half the line which is betwene the playnes : And therefore the whole line which coupleth the centres of the opposite bases, is in power quintuple to the whole line which is betwene the sayd playnes. The line which subtendeth the angle of the base of the dodecahedron, together with the side of the base, are in power quintuple to the line which is drawne from the centre of the circle, which containeth the base, to the circumference. A section of a sphere containing three bases of the dodecahedron taketh a third part of the diameter of the sayd sphere.

The side of the dodecahedron, and the line which subtendeth the angle of the pentagon, are equall to the right line which coupleth the middle sections of the opposite sides of the dodecahedron.

(32)

¶ The ende of the Elementes of Geometrie,
of the most auncient Philosopher
Euclide of Megara.

EEE.ij.



Faultes escaped.

M Aruaile not (gentle reader), that faultes here following, haue escaped in the correction of this booke. For, for that the matter in it containied is fraunge to our Printers here in England, not hauing bene accustomed to Print many, or rather any bookes containying such matter, which causeth them to be vnfurnished of a corrector skilfull in that art: I was forced, to my great trauaile and paine, to correcte the whole booke my selfe. And in deede sometimes for want of *Argus* eyes, and iuste consideration, notwithstanding my diligence in correcting, faultes escaped through me: sometimes also for lacke of diligence in the Printer to amend my corrections, faultes remayned vncorrected by his meanes. So that betwene vs both these faultes haue escaped vncorrected: which faultes yet, to say the trouth, for the most part are such, as a very young student without noting them vnto him, mought easily of him selfe finde and correcte. And this I dare boldly affirme, that not many bookes, if any, concerning this art in other tounes, Greke, Latine, or Italian, are with so fewe faultes of importance Printed, as this booke is. The triall wherof I referre to them which haue read any bookes of this arte in other tounes, & shall happen hereafter to read this. And as touching these faultes to be corrected, I would wishe you (good reader) before you beginne to read any of the 16. bookes in this volume contained, first to amend the faultes in that booke which you will read, according as they are here signified vnto you. Where you shall finde in what booke, leafe, side, and line, both the fault escaped is, & also how it is to be corrected. And if you happen in reading to finde any more faultes not here mentioned, as peraduenture you may, for that diuers faultes were vtterly so easie and light to correcte, that I would not note them, & besides that, no one man though he be neuer so diligent and circumspecte can espie all thinges, I trust you will therefore impute no blame either vnto me or to the Printer, but gently amend and correct them, accepting our good minde, which was to haue had the booke passed to your handes vtterly without fault, as touching the Printing.

Fol.	Page.	Line.	Faultes.	Corrected.
Errata Lib. 1.				
1	2	41	point B, as Campane	point C, as Campane
3	1	22	all lines drawne	all right lines
3	1	28	lines drawnen to the superficies	right lines drawen to the circumference
9	1	42	lines A B and A C,	lines A B and B C
15	1	35	are equall	are proued equall
20	2	28	(by the first	(by the fourth
21	1	39	the centre C.	the centre E
24	2	6	If fower right	If two right
25	2	3	first petition	fourth petition
49	2	7	14. 16. 32. 64. &c.	4. 8. 16. 32. 64. &c.
53	1	39	the triangle N G	the triangle K G
54	2	25	(by the 44.	(by the 42.
57	2	23	and C B G is the	and C G B is the
In stede of Flusates through out the whole booke read Flusas.				

Errata Lib. 2.				
60	2	29	Gnomon F G E H	Gnomon A H K D
		30	Gnomon E H F G	Gnomon E C K D
69	1	18	the whole line	the whole figure
76	2	9	the diameter C D	the diameter A H F

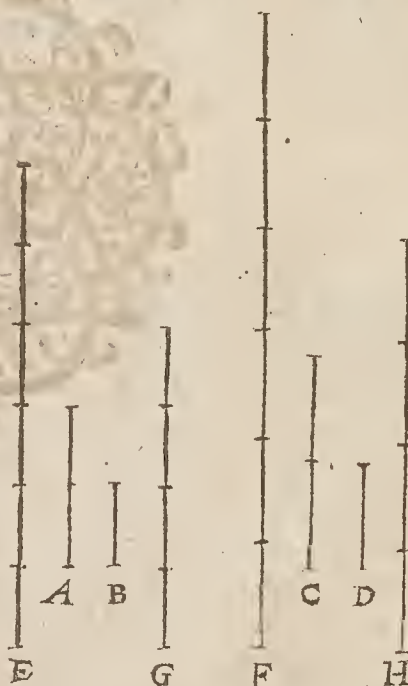
Errata Lib. 3.				
82	2	36	angle	equall to the angle
92	1	last	the line A C is	the line A F is

Errata Lib. 4.				
110	2	10	C D toucheth the	E D toucheth the
		12	side of the other	angle of the other
115	1	21	and H B	and H E
117	2	44	the angle A C D	the angle A C B
118	1	2	into ten equall	into two equall

Fol.	Page.	Line.	Faultes.	Corrected.
121	1	32	C D, and E A,	C D, D E, & E A,
124	1	29	the first	the third

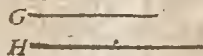
Errata Lib. 5.

126	1	43	it maketh 22.	it maketh 24.
			more then 17. by 5.	more then 17. by 7.
129	1		In stede of the figure of the 6. definition	draw in the magnet a figure like vnto this.



134	2	4	As A B is to A, so is	As A B is to B, so is
			C D to C	C D to D
141	2	last	But if K exceede M,	But if H exceede M,

Fol.	Page.	Line.	Faultes.	Corrected.
143	2		In the figure adde to the line N a line equall to the line C. Also diuide the line GH into three equall partes. And in the line D put out the pricke towards one of the endes thereof.	
151	1	29	unto as is to D	unto B as C is to D
	2	13	before this particule: [then by euersion] is left out this sentēce [and if also the proportion of the second to the third in the first order, be greater then the proportion of the second to the third in the latter order]	
153	1		In the figure of the 7. Proposition, in stede of the lines H, C, G, which are put vnder the lines A, B, C, drawe two lines according to the measure of these two lines.	



Errata Lib. 6.

155	1	35	grāme A B G D	grāme A C G D
181	1	3	the segment	the sector
		4	the segment	the sector

Errata Lib. 7. 8. and 9.

207	2	2	17. of the sixth	17. of the seventh
216	1	last	measure the least	measure the last
222	1	first	the lesse shall	the last shall
224	2	23	an odde number.	an euen number

Errata Lib. 10.

241	1	28	to a number	to a square number
242	1	3	be commensurable	be incommensurable
245	1	39	of lesse	of the lesse
253	2	36	a fencircle	a semicircle
258	1	25	lines A B and D B	lines A D and D B
258	2	16	lines A B and B G	lines A B and B C
259	1	40	H E and A F	H E and H F
261	1	21	D K is ratiōnall	D K is irrationall
			D E is irrationall	D E is ratiōnall
284	2	12	E is a ratiōnall	I is a ratiōnall
290	1	34	C D nor the line	E D nor the line
300	1	37	lines C D and D E	lines C F and D F
302	1	36	commensurable	incommensurable
306	1	24	πορισματιώδης,	πορισματιώδης,
		25	that is, Acquisitively	that is, Acquisitively

Errata Lib. 11.

314	2	33	the side B F	the side D F
320	1	12	Under foure plaine	Under sixe plaine
		17	Under foure plaine	Under sixe plaine
330	2	33	L H B M, and so	L G B M, and so
338			In the figure in the line L R A E H M N in stede of the letter R put the letter K	
347	2	46	W, I, d, and Q	W, I, h, and Q

Fol.	Page.	Line.	Faultes.	Corrected.
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Errata Lib. 12.

363	2	10	B A, D A, D C,	B A, B C, D C,
368	2		In the figure in stede of the letter R put the letter Q	
380	1			
388			In the second figure extend the line D G to the point B	
	2	19	Earthmeasures	Earthmeasurers

Errata Lib. 13.

400	1	53	in the circle F	in the circle H
406	2	17	power ble to	power treble to
407	1	48	lines K I, E M, Z N, and N T.	lines K I, E M, Z N, and N H
		last	K I and D B	K I and A B
409	2	34	line V a, which	line Z a, which
410	1	33	Y H, H W, B W, W C,	Y H, H W, B W, W C,
		34	C Z, and C B.	C Z, Z V, & V B.
412	1	16	lines A f	lines A f
		24	line B T is equall	line B W is equall
413	2	22	containeth sixe, of such parts the side of the pyramis containeth fower	containeth in power sixe, of such partes the side of the pyramis containeth in power fower

Errata Lib. 14. Hyps.

417	1	45	selfe same circle	selfe same Sphere
	2	43	same circle	same Sphere

Errata Lib. 14. Fluffas.

427	2	42	in sesquitertia	in sub sesquitertia
430	1	20	to the line E N	to the line A N
		51	the line F G H I M	the cube F G H I M

Errata Lib. 15.

440	1	10	line E H)	line c H)
	2	42	lines A T, and	lines. A T, & T G,
444	1		In the figure in stede of the letter G put the letter T	
		54	Under the side, of	Under the side D G

Errata Lib. 16.

447	1		wherefore the line A I	wherefore the square of the line A I.
448	2	14	in a pyramis	in an Octohedron
458	1	7	whole line P B,	whole line G B



EEE. iij.



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